Problem 12-1

A truck traveling along a straight road at speed v_1, increases its speed to v_2 in time t. If its acceleration is constant, determine the distance traveled.

Given:

$$v_1 = 20\ \frac{\text{km}}{\text{hr}} \qquad v_2 = 120\ \frac{\text{km}}{\text{hr}} \qquad t = 15\ \text{s}$$

Solution:

$$a = \frac{v_2 - v_1}{t} \qquad a = 1.852\ \frac{\text{m}}{\text{s}^2}$$

$$d = v_1 t + \frac{1}{2} a t^2 \qquad d = 291.67\ \text{m}$$

Problem 12-2

A car starts from rest and reaches a speed v after traveling a distance d along a straight road. Determine its constant acceleration and the time of travel.

Given: $v = 80\ \frac{\text{ft}}{\text{s}}$ $\quad d = 500\ \text{ft}$

Solution:

$$v^2 = 2ad \qquad a = \frac{v^2}{2d} \qquad a = 6.4\ \frac{\text{ft}}{\text{s}^2}$$

$$v = at \qquad t = \frac{v}{a} \qquad t = 12.5\ \text{s}$$

Problem 12-3

A baseball is thrown downward from a tower of height h with an initial speed v_0. Determine the speed at which it hits the ground and the time of travel.

Given:

$$h = 50\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad v_0 = 18\ \frac{\text{ft}}{\text{s}}$$

Solution:

$$v = \sqrt{v_0^2 + 2gh} \qquad v = 59.5\ \frac{\text{ft}}{\text{s}}$$

$$t = \frac{v - v_0}{g} \qquad t = 1.29\ \text{s}$$

***Problem 12–4**

Starting from rest, a particle moving in a straight line has an acceleration of $a = (bt + c)$. What is the particle's velocity at t_1 and what is its position at t_2?

Given: $b = 2\ \dfrac{\text{m}}{\text{s}^3}$ $\quad c = -6\ \dfrac{\text{m}}{\text{s}^2}$ $\quad t_1 = 6\ \text{s}$ $\quad t_2 = 11\ \text{s}$

Solution:

$$a(t) = bt + c \qquad v(t) = \int_0^t a(t)\ dt \qquad d(t) = \int_0^t v(t)\ dt$$

$$v(t_1) = 0\ \frac{\text{m}}{\text{s}} \qquad d(t_2) = 80.7\ \text{m}$$

Problem 12-5

Traveling with an initial speed v_0 a car accelerates at rate a along a straight road. How long will it take to reach a speed v_f? Also, through what distance does the car travel during this time?

Given: $v_0 = 70\ \dfrac{\text{km}}{\text{hr}}$ $\quad a = 6000\ \dfrac{\text{km}}{\text{hr}^2}$ $\quad v_f = 120\ \dfrac{\text{km}}{\text{hr}}$

Solution:

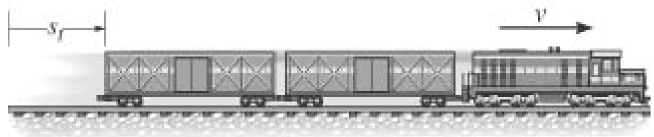
$$v_f = v_0 + at \qquad t = \frac{v_f - v_0}{a} \qquad t = 30\ \text{s}$$

$$v_f^2 = v_0^2 + 2as \qquad s = \frac{v_f^2 - v_0^2}{2a} \qquad s = 792\ \text{m}$$

Problem 12-6

A freight train travels at $v = v_0\left(1 - e^{-bt}\right)$ where t is the elapsed time. Determine the distance traveled in time t_1, and the acceleration at this time.

Given:



$v_0 = 60\ \frac{ft}{s}$

$b = \frac{1}{s}$

$t_1 = 3\ s$

Solution:

$v(t) = v_0\left(1 - e^{-b\,t}\right) \qquad a(t) = \frac{d}{dt}v(t) \qquad d(t) = \int_0^t v(t)\ dt$

$d(t_1) = 123.0\ ft \qquad a(t_1) = 2.99\ \frac{ft}{s^2}$

Problem 12-7

The position of a particle along a straight line is given by $s_p = at^3 + bt^2 + ct$. Determine its maximum acceleration and maximum velocity during the time interval $t_0 \le t \le t_f$.

Given: $a = 1\ \frac{ft}{s^3} \qquad b = -9\ \frac{ft}{s^2} \qquad c = 15\ \frac{ft}{s} \qquad t_0 = 0\ s \qquad t_f = 10\ s$

Solution:

$$s_p = a\,t^3 + b\,t^2 + c\,t$$

$$v_p = \frac{d}{dt}s_p = 3a\,t^2 + 2b\,t + c$$

$$a_p = \frac{d}{dt}v_p = \frac{d^2}{dt^2}s_p = 6a\,t + 2b$$

Since the acceleration is linear in time then the maximum will occur at the start or at the end. We check both possibilities.

$a_{max} = \max\left(6a\,t_0 + b, 6a\,t_f + 2b\right) \qquad a_{max} = 42\ \frac{ft}{s^2}$

The maximum velocity can occur at the beginning, at the end, or where the acceleration is zero. We will check all three locations.

$t_{cr} = \frac{-b}{3a} \qquad t_{cr} = 3\ s$

$$v_{max} = \max\left(3a\,t_0^{\,2} + 2b\,t_0 + c\,, 3a\,t_f^{\,2} + 2b\,t_f + c\,, 3a\,t_{cr}^{\,2} + 2b\,t_{cr} + c\right) \qquad v_{max} = 135\,\frac{\text{ft}}{\text{s}}$$

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*__Problem 12-8__

From approximately what floor of a building must a car be dropped from an at-rest position so that it reaches a speed v_f when it hits the ground? Each floor is a distance h higher than the one below it. (Note: You may want to remember this when traveling at speed v_f)

Given: $v_f = 55\ \text{mph}$ $\quad h = 12\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$a_c = \text{g} \qquad v_f^{\,2} = 0 + 2a_c\,s \qquad H = \frac{v_f^{\,2}}{2a_c} \qquad H = 101.124\,\text{ft}$$

Number of floors $\quad N$

Height of one floor $\quad h = 12\,\text{ft}$

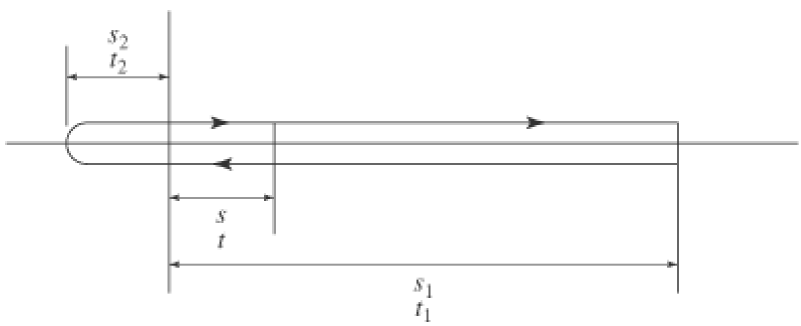
$$N = \frac{H}{h} \qquad N = 8.427 \qquad N = \text{ceil}(N)$$

The car must be dropped from floor number $N = 9$

Problem 12–9

A particle moves along a straight line such that its position is defined by $s_p = at^3 + bt^2 + c$. Determine the average velocity, the average speed, and the acceleration of the particle at time t_1.

Given: $a = 1\ \frac{\text{m}}{\text{s}^3}$ $\quad b = -3\ \frac{\text{m}}{\text{s}^2}$ $\quad c = 2\ \text{m}$ $\quad t_0 = 0\ \text{s}$ $\quad t_1 = 4\ \text{s}$



Solution:

$$s_p(t) = a\,t^3 + b\,t^2 + c \qquad v_p(t) = \frac{d}{dt}s_p(t) \qquad a_p(t) = \frac{d}{dt}v_p(t)$$

Find the critical velocity where $v_p = 0$.

$t_2 = 1.5\ \text{s}$ Given $v_p(t_2) = 0$ $t_2 = \text{Find}(t_2)$ $t_2 = 2\ \text{s}$

$$v_{ave} = \frac{s_p(t_1) - s_p(t_0)}{t_1} \qquad v_{ave} = 4\ \frac{\text{m}}{\text{s}}$$

$$v_{avespeed} = \frac{|s_p(t_2) - s_p(t_0)| + |s_p(t_1) - s_p(t_2)|}{t_1} \qquad v_{avespeed} = 6\ \frac{\text{m}}{\text{s}}$$

$$a_1 = a_p(t_1) \qquad a_1 = 18\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–10

A particle is moving along a straight line such that its acceleration is defined as $a = -kv$. If $v = v_0$ when $d = 0$ and $t = 0$, determine the particle's velocity as a function of position and the distance the particle moves before it stops.

Given: $k = \frac{2}{\text{s}}$ $v_0 = 20\ \frac{\text{m}}{\text{s}}$

Solution: $a_p(v) = -kv$ $v\frac{\text{d}}{\text{d}s}v = -kv$ $\int_{v_0}^{v} 1\ \text{d}v = -ks_p$

Velocity as a function of position $v = v_0 - ks_p$

Distance it travels before it stops $0 = v_0 - ks_p$

$$s_p = \frac{v_0}{k} \qquad s_p = 10\ \text{m}$$

Problem 12-11

The acceleration of a particle as it moves along a straight line is given by $a = bt + c$. If $s = s_0$ and $v = v_0$ when $t = 0$, determine the particle's velocity and position when $t = t_1$. Also, determine the total distance the particle travels during this time period.

Given: $b = 2\ \frac{\text{m}}{\text{s}^3}$ $c = -1\ \frac{\text{m}}{\text{s}^2}$ $s_0 = 1\ \text{m}$ $v_0 = 2\ \frac{\text{m}}{\text{s}}$ $t_1 = 6\ \text{s}$

Solution:

$$\int_{v_0}^{v} 1\,dv = \int_0^t (b\,t + c)\,dt \qquad v = v_0 + \frac{b\,t^2}{2} + c\,t$$

$$\int_{s_0}^{s} 1\,ds = \int_0^t \left(v_0 + \frac{b\,t^2}{2} + c\,t\right) dt \qquad s = s_0 + v_0\,t + \frac{b}{6}t^3 + \frac{c}{2}t^2$$

When $t = t_1$ $\quad v_1 = v_0 + \frac{b\,t_1^2}{2} + c\,t_1 \qquad v_1 = 32\,\frac{\text{m}}{\text{s}}$

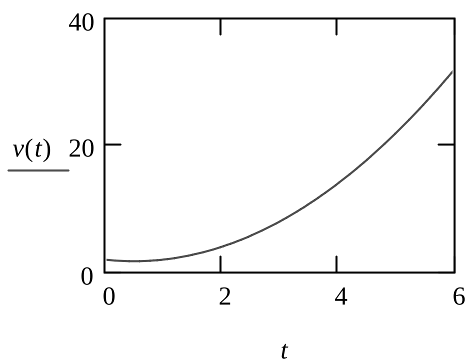
$$s_1 = s_0 + v_0\,t_1 + \frac{b}{6}t_1^3 + \frac{c}{2}t_1^2 \qquad s_1 = 67\ \text{m}$$

The total distance traveled depends on whether the particle turned around or not. To tell we will plot the velocity and see if it is zero at any point in the interval

$t = 0, 0.01t_1 .. t_1 \qquad v(t) = v_0 + \frac{b\,t^2}{2} + c\,t$

If v never goes to zero then

$d = s_1 - s_0 \qquad d = 66\ \text{m}$



***Problem 12–12**

A particle, initially at the origin, moves along a straight line through a fluid medium such that its velocity is defined as $v = b(1 - e^{-ct})$. Determine the displacement of the particle during the time $0 < t < t_1$.

Given: $\quad b = 1.8\,\frac{\text{m}}{\text{s}} \qquad c = \frac{0.3}{\text{s}} \qquad t_1 = 3\ \text{s}$

Solution:

$v(t) = b\left(1 - e^{-c\,t}\right)$ $\quad s_p(t) = \int_0^t v(t)\,dt$ $\quad s_p(t_1) = 1.839\ \text{m}$

Problem 12–13

The velocity of a particle traveling in a straight line is given $v = bt + ct^2$. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = t_1$. How far has the particle traveled during the time t_1, and what is its average speed?

Given: $b = 6\ \frac{\text{m}}{\text{s}^2}$ $\quad c = -3\ \frac{\text{m}}{\text{s}^3}$ $\quad t_0 = 0\ \text{s}$ $\quad t_1 = 3\ \text{s}$

Solution: $v(t) = b\,t + c\,t^2$ $\quad a(t) = \frac{d}{dt}v(t)$ $\quad s_p(t) = \int_0^t v(t)\,dt$

Deceleration $\quad a_1 = a(t_1)$ $\quad a_1 = -12\ \frac{\text{m}}{\text{s}^2}$

Find the turning time t_2

$t_2 = 1.5\ \text{s}$ $\quad$ Given $\quad v(t_2) = 0$ $\quad t_2 = \text{Find}(t_2)$ $\quad t_2 = 2\ \text{s}$

Total distance traveled $\quad d = \left|s_p(t_1) - s_p(t_2)\right| + \left|s_p(t_2) - s_p(t_0)\right|$ $\quad d = 8\ \text{m}$

Average speed $\quad v_{avespeed} = \frac{d}{t_1 - t_0}$ $\quad v_{avespeed} = 2.667\ \frac{\text{m}}{\text{s}}$

Problem 12–14

A particle moves along a straight line such that its position is defined by $s = bt^2 + ct + d$. Determine the average velocity, the average speed, and the acceleration of the particle when $t = t_1$.

Given: $b = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad c = -6\ \frac{\text{m}}{\text{s}}$ $\quad d = 5\ \text{m}$ $\quad t_0 = 0\ \text{s}$ $\quad t_1 = 6\ \text{s}$

Solution:

$s_p(t) = b\,t^2 + c\,t + d$ $\quad v(t) = \frac{d}{dt}s_p(t)$ $\quad a(t) = \frac{d}{dt}v(t)$

Find the critical time $\quad t_2 = 2s$ $\quad$ Given $\quad v(t_2) = 0$ $\quad t_2 = \text{Find}(t_2)$ $\quad t_2 = 3\ \text{s}$

$v_{avevel} = \frac{s_p(t_1) - s_p(t_0)}{t_1}$ $\quad v_{avevel} = 0\ \frac{\text{m}}{\text{s}}$

$$v_{avespeed} = \frac{\left|s_p(t_1) - s_p(t_2)\right| + \left|s_p(t_2) - s_p(t_0)\right|}{t_1} \qquad v_{avespeed} = 3\ \frac{\text{m}}{\text{s}}$$

$$a_1 = a(t_1) \qquad a_1 = 2\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–15

A particle is moving along a straight line such that when it is at the origin it has a velocity v_0. If it begins to decelerate at the rate $a = bv^{1/2}$ determine the particle's position and velocity when $t = t_1$.

Given:

$$v_0 = 4\ \frac{\text{m}}{\text{s}} \qquad b = -1.5\ \sqrt{\frac{\text{m}}{\text{s}^3}} \qquad t_1 = 2\ \text{s} \qquad a(v) = b\sqrt{v}$$

Solution:

$$a(v) = b\sqrt{v} = \frac{d}{dt}v \qquad \int_{v_0}^{v} \frac{1}{\sqrt{v}}\, dv = 2\left(\sqrt{v} - \sqrt{v_0}\right) = b\,t$$

$$v(t) = \left(\sqrt{v_0} + \frac{1}{2}b\,t\right)^2 \qquad v(t_1) = 0.25\ \frac{\text{m}}{\text{s}}$$

$$s_p(t) = \int_0^t v(t)\, dt \qquad s_p(t_1) = 3.5\ \text{m}$$

***Problem 12-16**

A particle travels to the right along a straight line with a velocity $v_p = a / (b + s_p)$. Determine its deceleration when $s_p = s_{p1}$.

Given: $a = 5\ \frac{\text{m}^2}{\text{s}} \qquad b = 4\ \text{m} \qquad s_{p1} = 2\ \text{m}$

Solution: $$v_p = \frac{a}{b + s_p} \qquad a_p = v_p \frac{dv_p}{ds_p} = \frac{a}{b + s_p}\,\frac{-a}{\left(b + s_p\right)^2} = \frac{-a^2}{\left(b + s_p\right)^3}$$

$$a_{p1} = \frac{-a^2}{\left(b + s_{p1}\right)^3} \qquad a_{p1} = -0.116\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–17

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (at - b)$ and $a_B = (ct_2 - d)$, where t is in seconds. Determine the distance between them at t and the total distance each has traveled in time t.

Given:

$a = 6\ \frac{\text{ft}}{\text{s}^3}$ $b = 3\ \frac{\text{ft}}{\text{s}^2}$ $c = 12\ \frac{\text{ft}}{\text{s}^3}$ $d = 8\ \frac{\text{ft}}{\text{s}^2}$ $t = 4\ \text{s}$

Solution:

$$\frac{dv_A}{dt} = at - b \qquad v_A = \left(\frac{at^2}{2} - bt\right)$$

$$s_A = \left(\frac{at^3}{6} - \frac{bt^2}{2}\right)$$

$$\frac{dv_B}{dt} = ct^2 - d \qquad v_B = \left(\frac{ct^3}{3\ \text{s}} - dt\right) \qquad s_B = \left(\frac{ct^4}{12\ \text{s}} - \frac{dt^2}{2}\right)$$

Distance between A and B

$$d_{AB} = \left|\frac{at^3}{6} - \frac{bt^2}{2} - \frac{ct^4}{12\ \text{s}} + \frac{dt^2}{2}\right| \qquad d_{AB} = 46.33\ \text{m}$$

Total distance A and B has travelled.

$$D = \frac{at^3}{6} - \frac{bt^2}{2} + \frac{ct^4}{12\ \text{s}} - \frac{dt^2}{2} \qquad D = 70.714\ \text{m}$$

Problem 12–18

A car is to be hoisted by elevator to the fourth floor of a parking garage, which is at a height h above the ground. If the elevator can accelerate at a_1, decelerate at a_2, and reach a maximum speed v, determine the shortest time to make the lift, starting from rest and ending at rest.

Given: $h = 48\ \text{ft}$ $a_1 = 0.6\ \frac{\text{ft}}{\text{s}^2}$ $a_2 = 0.3\ \frac{\text{ft}}{\text{s}^2}$ $v = 8\ \frac{\text{ft}}{\text{s}}$

Solution: Assume that the elevator never reaches its maximum speed.

Guesses $t_1 = 1\ \text{s}$ $t_2 = 2\ \text{s}$ $v_{max} = 1\ \frac{\text{ft}}{\text{s}}$ $h_1 = 1\ \text{ft}$

Given $v_{max} = a_1 t_1$

$$h_1 = \frac{1}{2}a_1 t_1^2$$

$$0 = v_{max} - a_2(t_2 - t_1)$$

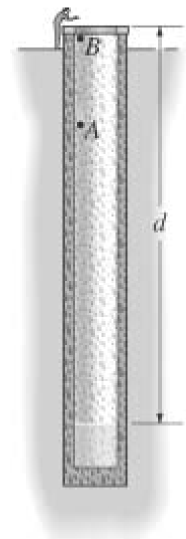
$$h = h_1 + v_{max}(t_2 - t_1) - \frac{1}{2}a_2(t_2 - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t_2 \\ v_{max} \\ h_1 \end{pmatrix} = \text{Find}(t_1, t_2, v_{max}, h_1) \qquad t_2 = 21.909\ \text{s}$$

Since $v_{max} = 4.382\frac{\text{ft}}{\text{s}} < v = 8\frac{\text{ft}}{\text{s}}$ then our original assumption is correct.

Problem 12-19

A stone A is dropped from rest down a well, and at time t_1 another stone B is dropped from rest. Determine the distance between the stones at a later time t_2.



Given: $d = 80\ \text{ft}$ $\quad t_1 = 1\ \text{s}$ $\quad t_2 = 2\ \text{s}$ $\quad g = 32.2\frac{\text{ft}}{\text{s}^2}$

Solution:

$a_A = g \qquad v_A = g t \qquad s_A = \frac{g}{2}t^2$

$a_B = g \qquad v_B = g(t - t_1) \qquad s_B = \frac{g}{2}(t - t_1)^2$

At time t_2

$s_{A2} = \frac{g}{2}t_2^2 \qquad s_{A2} = 64.4\,\text{ft}$

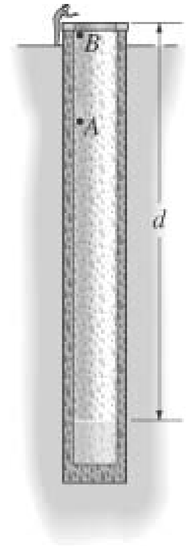
$s_{B2} = \frac{g}{2}(t_2 - t_1)^2 \qquad s_{B2} = 16.1\,\text{ft}$

$d = s_{A2} - s_{B2} \qquad d = 48.3\,\text{ft}$

***Problem 12-20**

A stone A is dropped from rest down a well, and at time t_1 another stone B is dropped from rest. Determine the time interval between the instant A strikes the water and the instant B strikes the water. Also, at what speed do they strike the water?

Given: $d = 80\ \text{ft}$ $t_1 = 1\ \text{s}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$a_A = g$ $v_A = g\,t$ $s_A = \frac{g}{2}t^2$

$a_B = g$ $v_B = g(t - t_1)$ $s_B = \frac{g}{2}(t - t_1)^2$

Time to hit for each particle

$t_A = \sqrt{\frac{2d}{g}}$ $t_A = 2.229\ \text{s}$

$t_B = \sqrt{\frac{2d}{g}} + t_1$ $t_B = 3.229\ \text{s}$

$\Delta t = t_B - t_A$ $\Delta t = 1\ \text{s}$

Speed

$v_A = g\,t_A$ $v_B = v_A$ $v_A = 71.777\ \frac{\text{ft}}{\text{s}}$ $v_B = 71.777\ \frac{\text{ft}}{\text{s}}$

Problem 12–21

A particle has an initial speed v_0. If it experiences a deceleration $a = bt$, determine the distance traveled before it stops.

Given: $v_0 = 27\ \frac{\text{m}}{\text{s}}$ $b = -6\ \frac{\text{m}}{\text{s}^3}$

Solution:

$a(t) = b\,t$ $v(t) = b\frac{t^2}{2} + v_0$ $s_p(t) = b\frac{t^3}{6} + v_0\,t$

$t = \sqrt{\frac{2v_0}{-b}}$ $t = 3\ \text{s}$ $s_p(t) = 54\ \text{m}$

Problem 12-22

The acceleration of a rocket traveling upward is given by $a_p = b + c\,s_p$. Determine the rocket's velocity when $s_p = s_{p1}$ and the time needed to reach this altitude. Initially, $v_p = 0$ and $s_p = 0$ when $t = 0$.

Given: $b = 6 \frac{\text{m}}{\text{s}^2}$ $c = 0.02 \frac{1}{\text{s}^2}$ $s_{p1} = 2000 \text{ m}$

Solution:

$$a_p = b + c s_p = v_p \frac{dv_p}{ds_p}$$

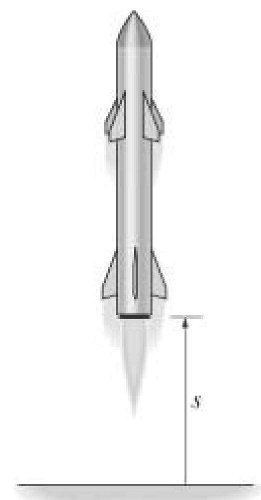
$$\int_0^{v_p} v_p \, dv_p = \int_0^{s_p} (b + c s_p) \, ds_p$$

$$\frac{v_p^2}{2} = b s_p + \frac{c}{2} s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b s_p + c s_p^2} \qquad v_{p1} = \sqrt{2b s_{p1} + c s_{p1}^2} \qquad v_{p1} = 322.49 \frac{\text{m}}{\text{s}}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b s_p + c s_p^2}} \, ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b s_p + c s_p^2}} \, ds_p \qquad t_1 = 19.274 \text{ s}$$

Problem 12-23



The acceleration of a rocket traveling upward is given by $a_p = b + c\, s_p$. Determine the time needed for the rocket to reach an altitute s_{p1}. Initially, $v_p = 0$ and $s_p = 0$ when $t = 0$.

Given: $b = 6 \frac{\text{m}}{\text{s}^2}$ $c = 0.02 \frac{1}{\text{s}^2}$ $s_{p1} = 100 \text{ m}$

Solution:

$$a_p = b + c s_p = v_p \frac{dv_p}{ds_p}$$

$$\int_0^{v_p} v_p \, dv_p = \int_0^{s_p} (b + c s_p) \, ds_p$$

$$\frac{v_p^2}{2} = b s_p + \frac{c}{2} s_p^2$$

$$v_p = \frac{ds_p}{dt} = \sqrt{2b s_p + c s_p^2} \qquad v_{p1} = \sqrt{2b s_{p1} + c s_{p1}^2} \qquad v_{p1} = 37.417 \frac{\text{m}}{\text{s}}$$

$$t = \int_0^{s_p} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\,ds_p \qquad t_1 = \int_0^{s_{p1}} \frac{1}{\sqrt{2b\,s_p + c\,s_p^2}}\,ds_p \qquad t_1 = 5.624\ \text{s}$$

***Problem 12–24**

A particle is moving with velocity v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -k\,v^3$, where k is a constant, determine its velocity and position as functions of time.

Solution:

$$a = \frac{dv}{dt} = -kv^3 \qquad \int_{v_0}^{v} v^{-3}\,dv = \int_0^t -k\,dt \qquad \frac{-1}{2}\left(v^{-2} - v_0^{-2}\right) = -kt$$

$$v(t) = \frac{1}{\sqrt{2kt + \frac{1}{v_0^2}}}$$

$$ds = v\,dt \qquad \int_0^s 1\,ds = \int_0^t \frac{1}{\sqrt{2kt + \left(\frac{1}{v_0^2}\right)}}\,dt$$

$$s(t) = \frac{1}{k}\left[\sqrt{2kt + \left(\frac{1}{v_0^2}\right)} - \frac{1}{v_0}\right]$$

Problem 12–25

A particle has an initial speed v_0. If it experiences a deceleration $a = bt$, determine its velocity when it travels a distance s_1. How much time does this take?

Given: $v_0 = 27\ \frac{\text{m}}{\text{s}}$ $\quad b = -6\ \frac{\text{m}}{\text{s}^3}$ $\quad s_1 = 10\ \text{m}$

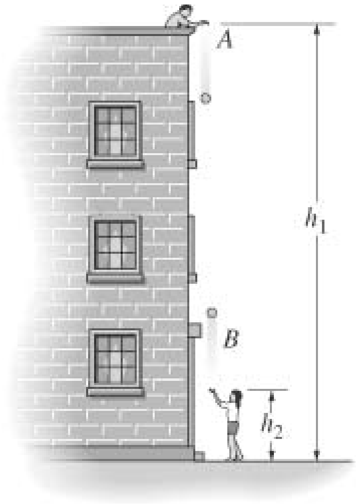
Solution:

$$a(t) = b\,t \qquad v(t) = b\frac{t^2}{2} + v_0 \qquad s_p(t) = b\frac{t^3}{6} + v_0\,t$$

Guess $t_1 = 1\ \text{s}$ Given $s_p(t_1) = s_1$ $\quad t_1 = \text{Find}(t_1)$ $\quad t_1 = 0.372\ \text{s}$

$$v(t_1) = 26.6\ \frac{\text{m}}{\text{s}}$$

Problem 12-26



Ball A is released from rest at height h_1 at the same time that a second ball B is thrown upward from a distance h_2 above the ground. If the balls pass one another at a height h_3 determine the speed at which ball B was thrown upward.

Given:

$h_1 = 40\ \text{ft}$

$h_2 = 5\ \text{ft}$

$h_3 = 20\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

For ball A:

$a_A = -g$

$v_A = -g\,t$

$s_A = \left(\dfrac{-g}{2}\right)t^2 + h_1$

For ball B:

$a_B = -g$

$v_B = -g\,t + v_{B0}$

$s_B = \left(\dfrac{-g}{2}\right)t^2 + v_{B0}\,t + h_2$

Guesses $t = 1\ \text{s}$ $v_{B0} = 2\ \dfrac{\text{ft}}{\text{s}}$

Given $h_3 = \left(\dfrac{-g}{2}\right)t^2 + h_1$ $h_3 = \left(\dfrac{-g}{2}\right)t^2 + v_{B0}\,t + h_2$

$\begin{pmatrix} t \\ v_{B0} \end{pmatrix} = \text{Find}(t, v_{B0})$ $t = 1.115\ \text{s}$ $v_{B0} = 31.403\ \dfrac{\text{ft}}{\text{s}}$

Problem 12–27

A car starts from rest and moves along a straight line with an acceleration $a = k\,s^{-1/3}$. Determine the car's velocity and position at $t = t_1$.

Given: $k = 3\left(\dfrac{\text{m}^4}{\text{s}^6}\right)^{\frac{1}{3}}$ $t_1 = 6\ \text{s}$

Solution:

$$a = v\frac{d}{ds_p}v = k s_p^{\frac{-1}{3}} \qquad \int_0^v v\,dv = \frac{v^2}{2} = \int_0^{s_p} k s_p^{\frac{-1}{3}}\,ds = \frac{3}{2}k s_p^{\frac{2}{3}}$$

$$v = \sqrt{3k}\,s_p^{\frac{1}{3}} = \frac{d}{dt}s_p \qquad \sqrt{3k}\,t = \int_0^{s_p} s_p^{\frac{-1}{3}}\,ds_p = \frac{3}{2}s_p^{\frac{2}{3}}$$

$$s_p(t) = \left(\frac{2\sqrt{3k}t}{3}\right)^{\frac{3}{2}} \qquad s_p(t_1) = 41.6\ \text{m}$$

$$v(t) = \frac{d}{dt}s_p(t) \qquad v(t_1) = 10.39\ \frac{\text{m}}{\text{s}}$$

***Problem 12-28**

The acceleration of a particle along a straight line is defined by $a_p = b\,t + c$. At $t = 0$, $s_p = s_{p0}$ and $v_p = v_{p0}$. When $t = t_1$ determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

Given: $b = 2\ \frac{\text{m}}{\text{s}^3}$ $\quad c = -9\ \frac{\text{m}}{\text{s}^2}$ $\quad s_{p0} = 1\ \text{m}$ $\quad v_{p0} = 10\ \frac{\text{m}}{\text{s}}$ $\quad t_1 = 9\ \text{s}$

Solution:

$$a_p = b\,t + c$$

$$v_p = \left(\frac{b}{2}\right)t^2 + c\,t + v_{p0}$$

$$s_p = \left(\frac{b}{6}\right)t^3 + \left(\frac{c}{2}\right)t^2 + v_{p0}\,t + s_{p0}$$

a) The position $\quad s_{p1} = \left(\frac{b}{6}\right)t_1^3 + \left(\frac{c}{2}\right)t_1^2 + v_{p0}\,t_1 + s_{p0} \qquad s_{p1} = -30.5\ \text{m}$

b) The total distance traveled - find the turning times $\quad v_p = \left(\frac{b}{2}\right)t^2 + c\,t + v_{p0} = 0$

$$t_2 = \frac{-c - \sqrt{c^2 - 2b\,v_{p0}}}{b} \qquad t_2 = 1.298\ \text{s}$$

$$t_3 = \frac{-c + \sqrt{c^2 - 2b\,v_{p0}}}{b} \qquad t_3 = 7.702\ \text{s}$$

$$s_{p2} = \left(\frac{b}{6}\right)t_2^{\,3} + \left(\frac{c}{2}\right)t_2^{\,2} + v_{p0}\,t_2 + s_{p0} \qquad s_{p2} = 7.127\ \text{m}$$

$$s_{p3} = \left(\frac{b}{6}\right)t_3^{\,3} + \frac{c}{2}t_3^{\,2} + v_{p0}\,t_3 + s_{p0} \qquad s_{p3} = -36.627\ \text{m}$$

$$d = \left|s_{p2} - s_{p0}\right| + \left|s_{p2} - s_{p3}\right| + \left|s_{p1} - s_{p3}\right| \qquad d = 56.009\ \text{m}$$

c) The velocity $v_{p1} = \left(\frac{b}{2}\right)t_1^{\,2} + c\,t_1 + v_{p0}$ $\qquad v_{p1} = 10\ \frac{\text{m}}{\text{s}}$

Problem 12–29

A particle is moving along a straight line such that its acceleration is defined as $a = k\,s^2$. If $v = v_0$ when $s = s_{p0}$ and $t = 0$, determine the particle's velocity as a function of position.

Given: $k = 4\ \frac{1}{\text{m}\,\text{s}^2}$ $\qquad v_0 = -100\ \frac{\text{m}}{\text{s}}$ $\qquad s_{p0} = 10\ \text{m}$

Solution:

$$a = v\frac{\text{d}}{\text{d}s_p}v = k\,s_p^{\,2} \qquad \int_{v_0}^{v} v\ \text{d}v = \int_{s_{p0}}^{s_p} k\,s_p^{\,2}\ \text{d}s_p$$

$$\frac{1}{2}\left(v^2 - v_0^{\,2}\right) = \frac{1}{3}k\left(s_p^{\,3} - s_{p0}^{\,3}\right) \qquad v = \sqrt{v_0^{\,2} + \frac{2}{3}k\left(s_p^{\,3} - s_{p0}^{\,3}\right)}$$

Problem 12–30

A car can have an acceleration and a deceleration a. If it starts from rest, and can have a maximum speed v, determine the shortest time it can travel a distance d at which point it stops.

Given: $a = 5\ \frac{\text{m}}{\text{s}^2}$ $\qquad v = 60\ \frac{\text{m}}{\text{s}}$ $\qquad d = 1200\ \text{m}$

Solution: Assume that it can reach maximum speed

Guesses $t_1 = 1\ \text{s}$ $\quad t_2 = 2\ \text{s}$ $\quad t_3 = 3\ \text{s}$ $\quad d_1 = 1\ \text{m}$ $\quad d_2 = 2\ \text{m}$

Given $a\,t_1 = v$ $\qquad \frac{1}{2}a\,t_1^{\,2} = d_1$ $\qquad d_2 = d_1 + v\left(t_2 - t_1\right)$

$$d = d_2 + v(t_3 - t_2) - \frac{1}{2}a(t_3 - t_2)^2 \qquad 0 = v - a(t_3 - t_2)$$

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ d_1 \\ d_2 \end{pmatrix} = \text{Find}(t_1, t_2, t_3, d_1, d_2) \qquad \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 32 \end{pmatrix} \text{s} \qquad \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 360 \\ 840 \end{pmatrix} \text{m}$$

$$t_3 = 32 \text{ s}$$

Problem 12-31

Determine the time required for a car to travel a distance d along a road if the car starts from rest, reaches a maximum speed at some intermediate point, and then stops at the end of the road. The car can accelerate at a_1 and decelerate at a_2.

Given: $d = 1 \text{ km}$ $\quad a_1 = 1.5 \frac{\text{m}}{\text{s}^2}$ $\quad a_2 = 2 \frac{\text{m}}{\text{s}^2}$

Let t_1 be the time at which it stops accelerating and t the total time.

Solution: Guesses $t_1 = 1 \text{ s}$ $\quad d_1 = 1 \text{ m}$ $\quad t = 3 \text{ s}$ $\quad v_1 = 1 \frac{\text{m}}{\text{s}}$

Given $\quad d_1 = \frac{a_1}{2}t_1^2 \qquad v_1 = a_1 t_1 \qquad v_1 = a_2(t - t_1)$

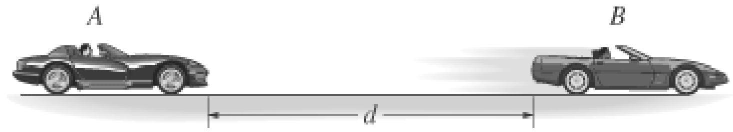
$$d = d_1 + v_1(t - t_1) - \frac{1}{2}a_2(t - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t \\ v_1 \\ d_1 \end{pmatrix} = \text{Find}(t_1, t, v_1, d_1) \qquad t_1 = 27.603 \text{ s} \quad v_1 = 41.404 \frac{\text{m}}{\text{s}} \quad d_1 = 571.429 \text{ m}$$

$$t = 48.305 \text{ s}$$

***Problem 12-32**

When two cars A and B are next to one another, they are traveling in the same direction with speeds v_{A0} and v_{B0} respectively. If B maintains its constant speed, while A begins to decelerate at the rate a_A, determine the distance d between the cars at the instant A stops.



Solution:

Motion of car A:

$$-a_A = \text{constant} \qquad 0 = v_{A0} - a_A t \qquad s_A = v_{A0} t - \frac{1}{2} a_A t^2$$

$$t = \frac{v_{A0}}{a_A} \qquad s_A = \frac{v_{A0}^2}{2a_A}$$

Motion of car B:

$$a_B = 0 \qquad v_B = v_{B0} \qquad s_B = v_{B0} t \qquad s_B = \frac{v_{B0} v_{A0}}{a_A}$$

The distance between cars A and B is

$$d = |s_B - s_A| = \left| \frac{v_{B0} v_{A0}}{a_A} - \frac{v_{A0}^2}{2a_A} \right| = \left| \frac{2v_{B0} v_{A0} - v_{A0}^2}{2a_A} \right|$$

$$d = \left| \frac{2v_{B0} v_{A0} - v_{A0}^2}{2a_A} \right|$$

Problem 12-33

If the effects of atmospheric resistance are accounted for, a freely falling body has an acceleration defined by the equation $a = g(1 - c v^2)$, where the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity at time t_1 and (b) the body's terminal or maximum attainable velocity as $t \rightarrow \infty$.

Given: $t_1 = 5 \text{ s} \qquad g = 9.81 \frac{\text{m}}{\text{s}^2} \qquad c = 10^{-4} \frac{\text{s}^2}{\text{m}^2}$

Solution:

(a) $$a = \frac{dv}{dt} = g(1 - c v^2)$$

Guess $v_1 = 1 \frac{\text{m}}{\text{s}}$

Given $$\int_0^{v_1} \frac{1}{1 - c v^2} dv = \int_0^{t_1} g \, dt \qquad v_1 = \text{Find}(v_1) \qquad v_1 = 45.461 \frac{\text{m}}{\text{s}}$$

(b) Terminal velocity means $a = 0$

$$0 = g\left(1 - c\,v_{term}^{2}\right) \qquad v_{term} = \sqrt{\frac{1}{c}} \qquad v_{term} = 100\ \frac{\text{m}}{\text{s}}$$

Problem 12-34

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g[R^2/(R+y)^2]$, where g is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g = 9.81$ m/s^2 and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

Solution: $g = 9.81\ \frac{\text{m}}{\text{s}^2} \qquad R = 6356\ \text{km}$

$$v\,dv = a\,dy = \frac{-gR^2}{(R+y)^2}\,dy$$

$$\int_v^0 v\,dv = -gR^2\int_0^\infty \frac{1}{(R+y)^2}\,dy \qquad \frac{-v^2}{2} = -gR$$

$$v = \sqrt{2gR} \qquad v = 11.2\ \frac{\text{km}}{\text{s}}$$

Problem 12-35

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12-34), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude y_0. Use the numerical data in Prob. 12-34.

Solution: $g = 9.81\ \frac{\text{m}}{\text{s}^2} \qquad R = 6356\ \text{km} \qquad y_0 = 500\ \text{km}$

$$v\,dv = a\,dy = \frac{-gR^2}{(R+y)^2}\,dy$$

$$\int_0^v v\,dv = -gR^2\int_{y_0}^{y}\frac{1}{(R+y)^2}\,dy$$

$$\frac{v^2}{2} = gR^2\left(\frac{1}{R+y} - \frac{1}{R+y_0}\right) = \frac{gR^2(y_0-y)}{(R+y)(R+y_0)} \qquad v = \sqrt{\frac{2gR^2(y_0-y)}{(R+y)(R+y_0)}}$$

When it hits, $y = 0$ $\qquad v_{earth} = \sqrt{\frac{2gRy_0}{R+y_0}} \qquad v_{earth} = 3.016\,\frac{\text{km}}{\text{s}}$

***Problem 12-36**

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v_f^2)(v_f^2 - v^2)$, determine the time needed for the velocity to become $v < v_f$. Initially the particle falls from rest.

Solution:

$$\frac{dv}{dt} = a = \frac{g}{v_f^2}\left(v_f^2 - v^2\right) \qquad \int_0^v \frac{1}{v_f^2 - v^2}\,dv = \frac{g}{v_f^2}\int_0^t 1\,dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f+v}{v_f-v}\right) = \left(\frac{g}{v_f^2}\right)t \qquad t = \frac{v_f}{2g}\ln\left(\frac{v_f+v}{v_f-v}\right)$$

Problem 12-37

An airplane starts from rest, travels a distance d down a runway, and after uniform acceleration, takes off with a speed v_r. It then climbs in a straight line with a uniform acceleration a_a until it reaches a constant speed v_a. Draw the *s-t*, *v-t*, and *a-t* graphs that describe the motion.

Given: $\quad d = 5000\text{ ft} \qquad v_r = 162\,\frac{\text{mi}}{\text{hr}}$

$\quad a_a = 3\,\frac{\text{ft}}{\text{s}^2} \qquad v_a = 220\,\frac{\text{mi}}{\text{hr}}$

Solution: First find the acceleration and time on the runway and the time in the air

$$a_r = \frac{v_r^2}{2d} \qquad a_r = 5.645\,\frac{\text{ft}}{\text{s}^2} \qquad t_r = \frac{v_r}{a_r} \qquad t_r = 42.088\text{ s}$$

$$t_a = \frac{v_a - v_r}{a_a} \qquad t_a = 28.356 \text{ s}$$

The equations of motion

$$t_1 = 0, 0.01t_r .. t_r$$

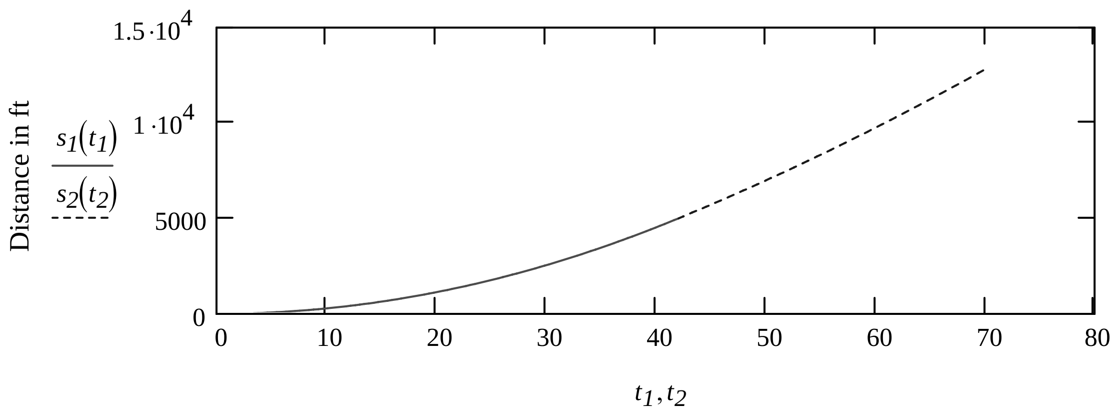
$$a_1(t_1) = a_r \frac{\text{s}^2}{\text{ft}} \qquad v_1(t_1) = a_r t_1 \frac{\text{s}}{\text{ft}} \qquad s_1(t_1) = \frac{1}{2} a_r t_1^2 \frac{1}{\text{ft}}$$

$$t_2 = t_r, 1.01t_r .. t_r + t_a$$

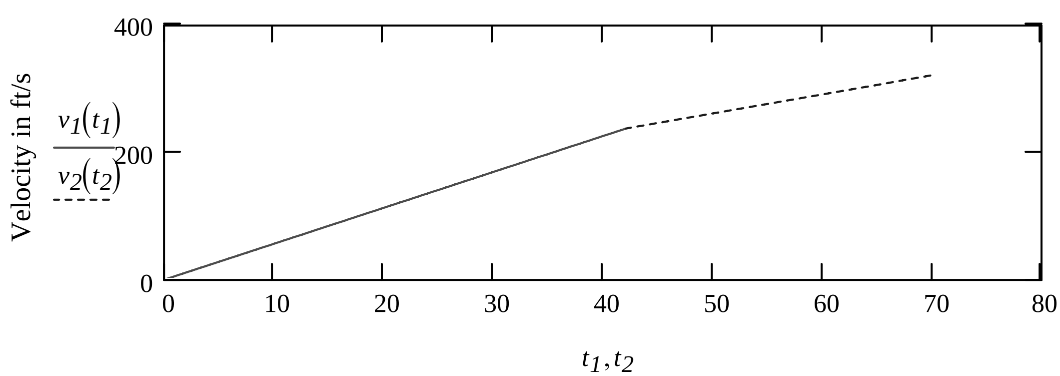
$$a_2(t_2) = a_a \frac{\text{s}^2}{\text{ft}} \qquad v_2(t_2) = \left[a_r t_r + a_a(t_2 - t_r)\right] \frac{\text{s}}{\text{ft}}$$

$$s_2(t_2) = \left[\frac{1}{2} a_r t_r^2 + a_r t_r(t_2 - t_r) + \frac{1}{2} a_a (t_2 - t_r)^2\right] \frac{1}{\text{ft}}$$

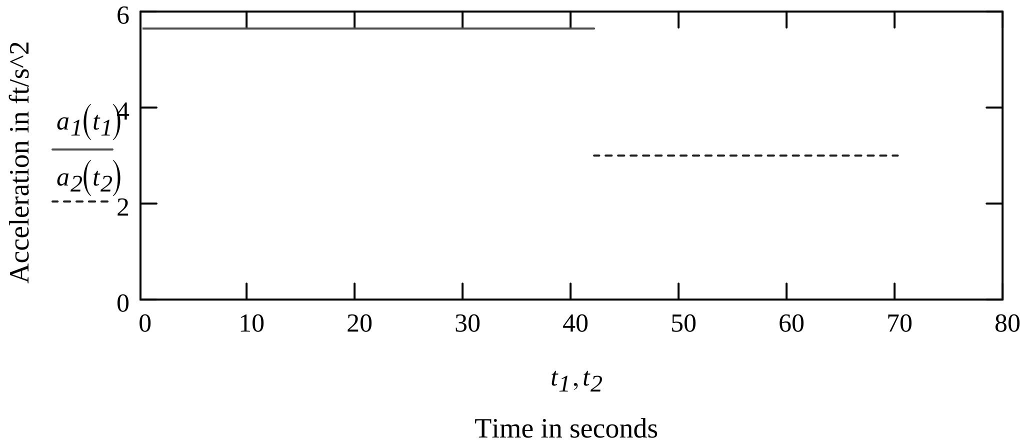
The plots



Time in seconds

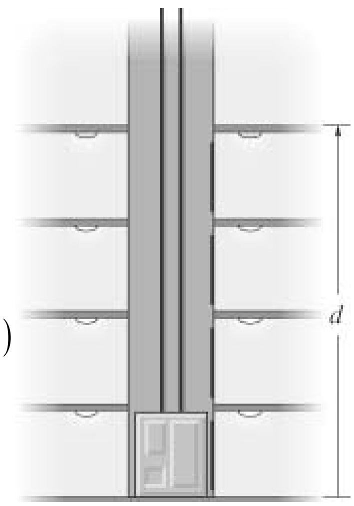


Time in seconds



Problem 12-38

The elevator starts from rest at the first floor of the building. It can accelerate at rate a_1 and then decelerate at rate a_2. Determine the shortest time it takes to reach a floor a distance d above the ground. The elevator starts from rest and then stops. Draw the *a-t*, *v-t*, and *s-t* graphs for the motion.



Given: $a_1 = 5\ \frac{\text{ft}}{\text{s}^2}$ $a_2 = 2\ \frac{\text{ft}}{\text{s}^2}$ $d = 40\ \text{ft}$

Solution: Guesses $t_1 = 1\ \text{s}$ $t = 2\ \text{s}$

$d_1 = 20\ \text{ft}$ $v_{max} = 1\ \frac{\text{ft}}{\text{s}}$

Given $v_{max} = a_1 t_1$ $d_1 = \frac{1}{2} a_1 t_1^2$ $v_{max} = a_2(t - t_1)$

$$d = d_1 + v_{max}(t - t_1) - \frac{1}{2} a_2 (t - t_1)^2$$

$$\begin{pmatrix} t_1 \\ t \\ d_1 \\ v_{max} \end{pmatrix} = \text{Find}(t_1, t, d_1, v_{max})$$

$d_1 = 11.429\ \text{ft}$ $t_1 = 2.138\ \text{s}$ $v_{max} = 10.69\ \frac{\text{ft}}{\text{s}}$

$t = 7.483\ \text{s}$

The equations of motion

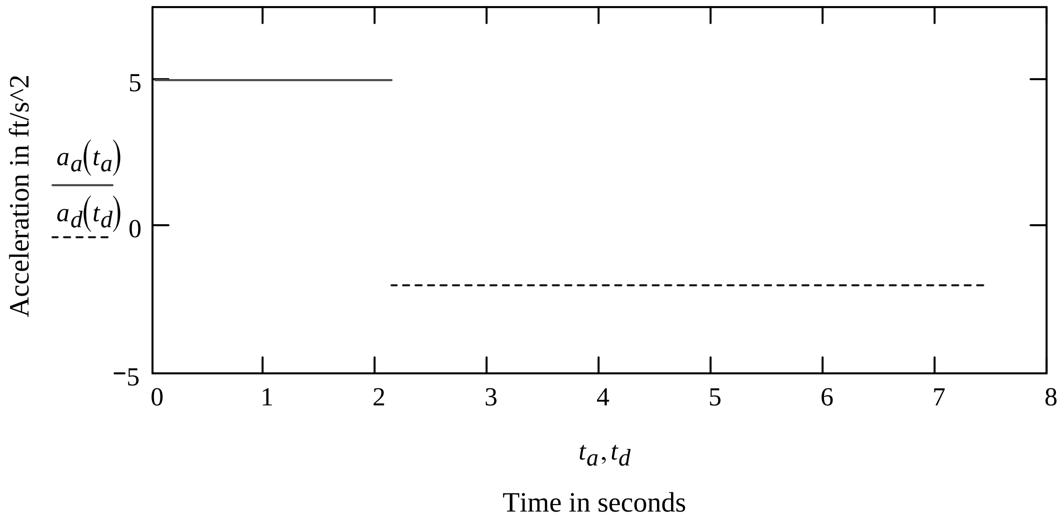
$t_a = 0, 0.01 t_1 .. t_1$ $t_d = t_1, 1.01 t_1 .. t$

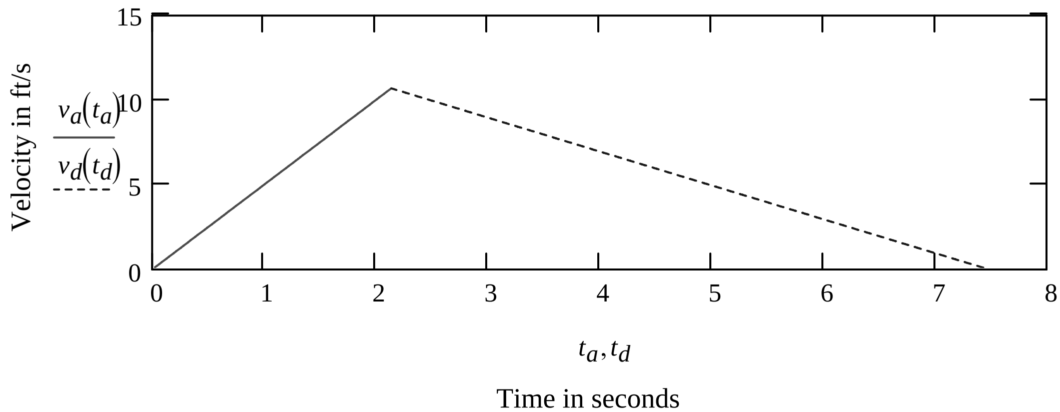
$a_a(t_a) = a_1 \frac{\text{s}^2}{\text{ft}}$ $a_d(t_d) = -a_2 \frac{\text{s}^2}{\text{ft}}$

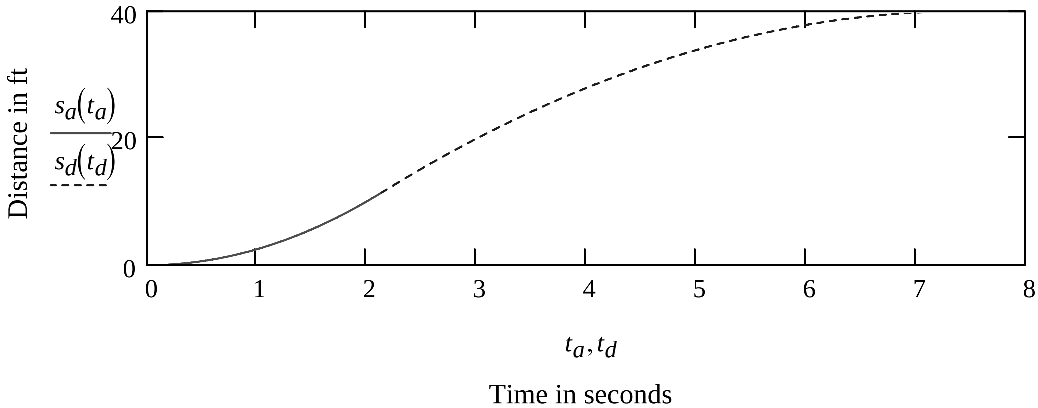
$$v_a(t_a) = a_1 t_a \frac{s}{ft} \qquad v_d(t_d) = \left[v_{max} - a_2(t_d - t_1)\right]\frac{s}{ft}$$

$$s_a(t_a) = \frac{1}{2} a_1 t_a^2 \frac{1}{ft} \qquad s_d(t_d) = \left[d_1 + v_{max}(t_d - t_1) - \frac{1}{2} a_2 (t_d - t_1)^2\right]\frac{1}{ft}$$

The plots





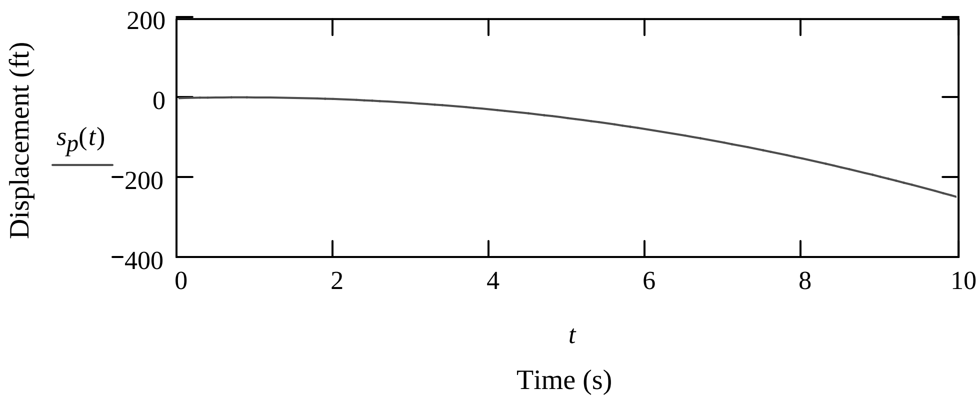


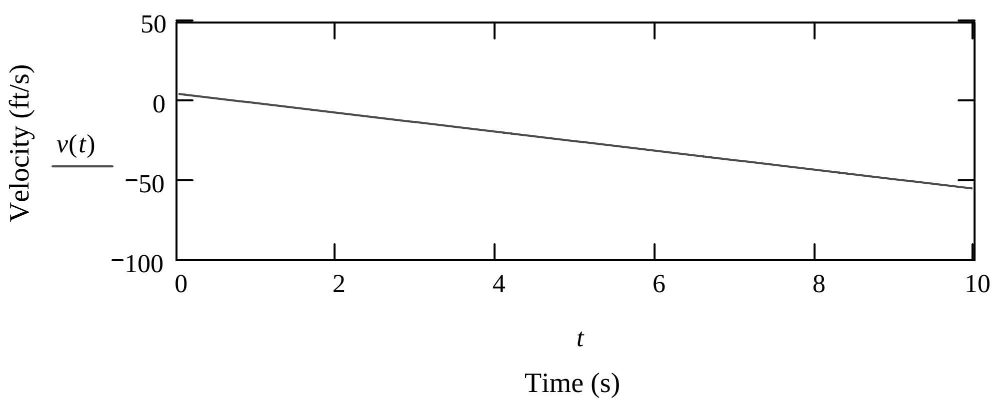
Problem 12–39

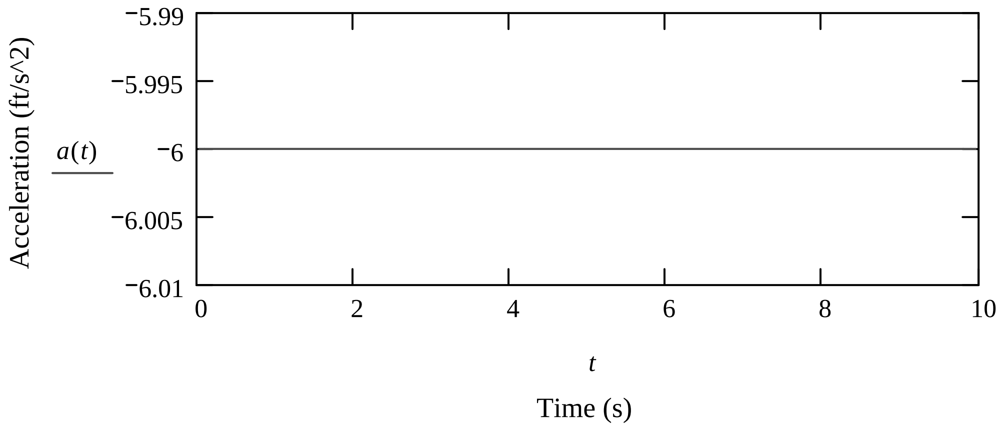
If the position of a particle is defined as $s = bt + ct^2$, construct the s–t, v–t, and a–t graphs for $0 \le t \le T$.

Given: $b = 5\ \text{ft}$ $c = -3\ \text{ft}$ $T = 10\ \text{s}$ $t = 0, 0.01T .. T$

Solution: $s_p(t) = \left(bt + ct^2\right)\frac{1}{\text{ft}}$ $v(t) = (b + 2ct)\frac{\text{s}}{\text{ft}}$ $a(t) = (2c)\frac{\text{s}^2}{\text{ft}}$







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*__Problem 12-40__

If the position of a particle is defined by $s_p = b \sin(ct) + d$, construct the *s-t*, *v-t*, and *a-t* graphs for $0 \le t \le T$.

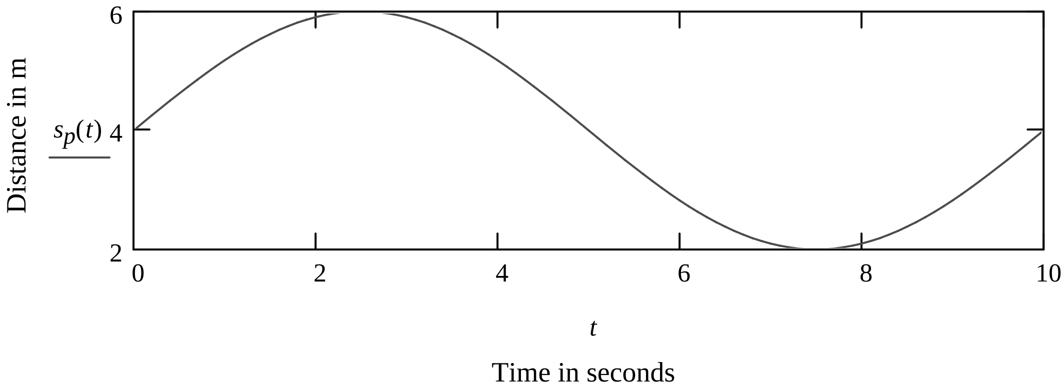
Given: $b = 2\ \text{m}$ $\quad c = \frac{\pi}{5}\ \frac{1}{\text{s}}$ $\quad d = 4\ \text{m}$ $\quad T = 10\ \text{s}$ $\quad t = 0, 0.01T .. T$

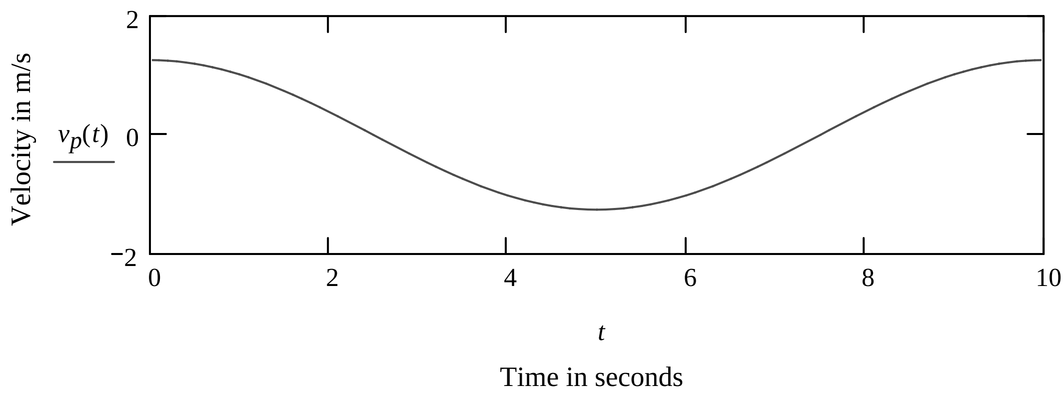
Solution:

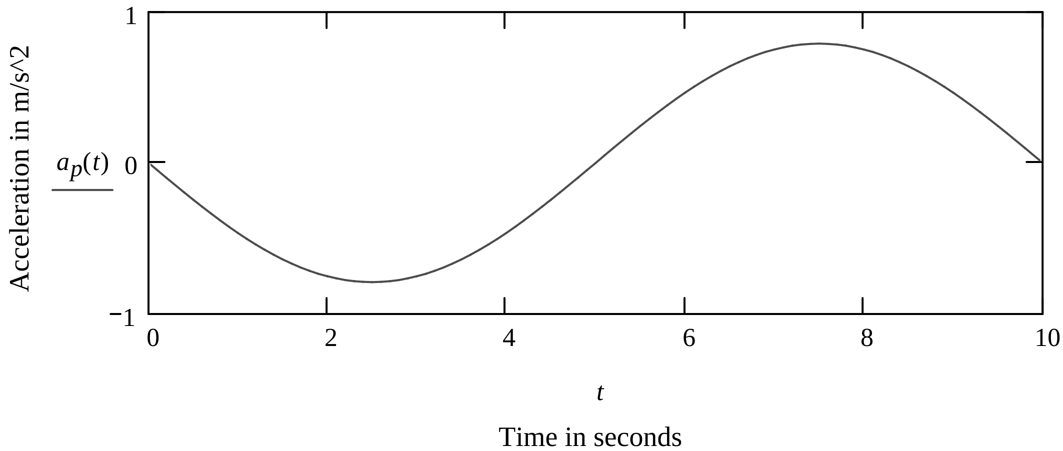
$$s_p(t) = (b \sin(c\,t) + d)\frac{1}{\text{m}}$$

$$v_p(t) = b\,c \cos(c\,t)\,\frac{\text{s}}{\text{m}}$$

$$a_p(t) = -b\,c^2 \sin(c\,t)\,\frac{\text{s}}{\text{m}^2}$$







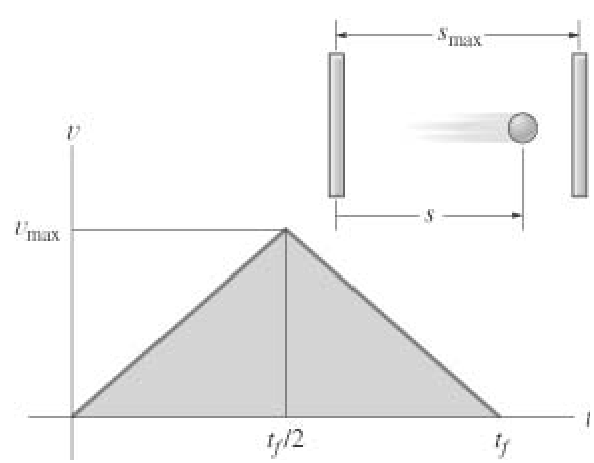
Problem 12-41

The *v*-*t* graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude *a*. If the plates are spaced s_{max} apart, determine the maximum velocity v_{max} and the time t_f for the particle to travel from one plate to the other. Also draw the *s*-*t* graph. When $t = t_f/2$ the particle is at $s = s_{max}/2$.

Given:

$$a = 4\ \frac{\text{m}}{\text{s}^2}$$

$$s_{max} = 200\ \text{mm}$$



Solution:

$$s_{max} = 2\left[\frac{1}{2}a\left(\frac{t_f}{2}\right)^2\right]$$

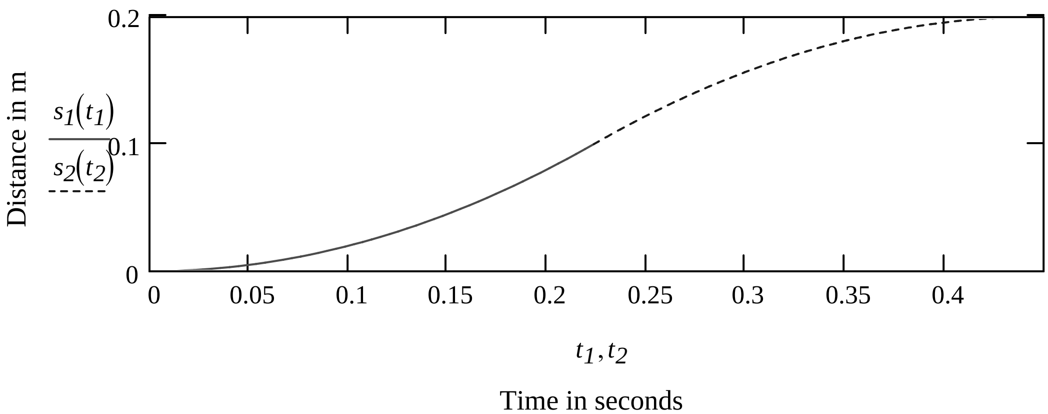
$t_f = \sqrt{\frac{4 s_{max}}{a}}$ $t_f = 0.447$ s

$v_{max} = a \frac{t_f}{2}$ $v_{max} = 0.894 \frac{m}{s}$

The plots

$t_1 = 0, 0.01 t_f .. \frac{t_f}{2}$ $s_1(t_1) = \frac{1}{2} a t_1^2 \frac{1}{m}$

$t_2 = \frac{t_f}{2}, 1.01 \frac{t_f}{2} .. t_f$ $s_2(t_2) = \left[\frac{1}{2} a \left(\frac{t_f}{2}\right)^2 + a \frac{t_f}{2}\left(t_2 - \frac{t_f}{2}\right) - \frac{1}{2} a \left(t_2 - \frac{t_f}{2}\right)^2\right] \frac{1}{m}$



Problem 12-42

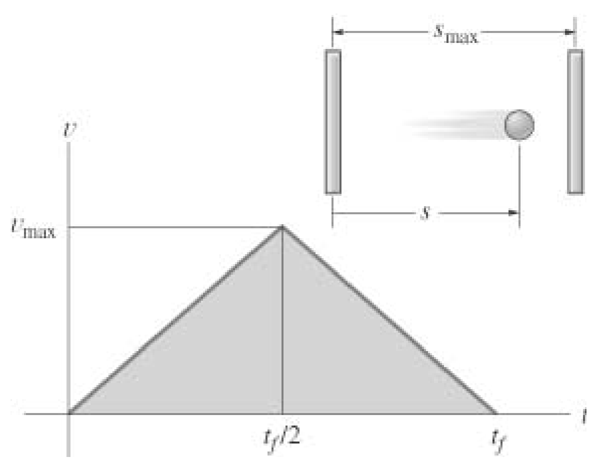
The *v-t* graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where t_f and v_{max} are given. Draw the *s-t* and *a-t* graphs for the particle. When $t = t_f/2$ the particle is at $s = s_c$.

Given:

$t_f = 0.2$ s

$v_{max} = 10 \frac{m}{s}$

$s_c = 0.5$ m



Solution:

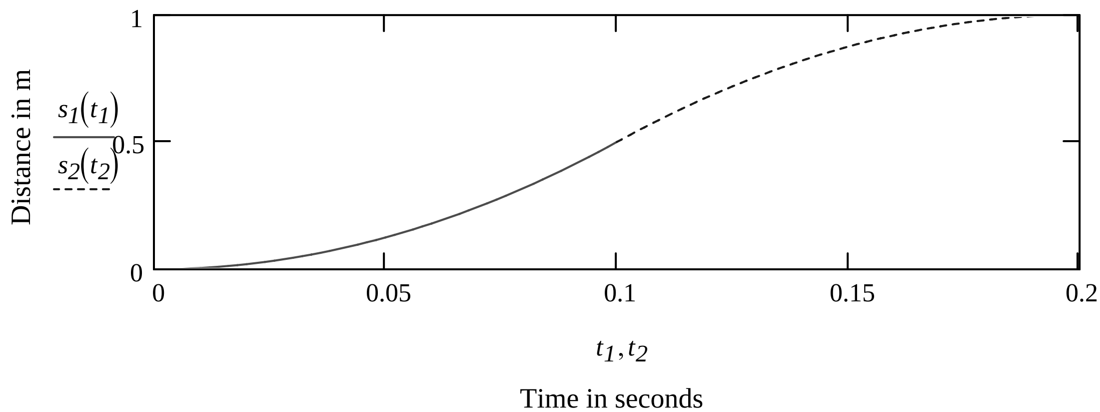
$a = \frac{2 v_{max}}{t_f}$ $a = 100 \frac{m}{s^2}$

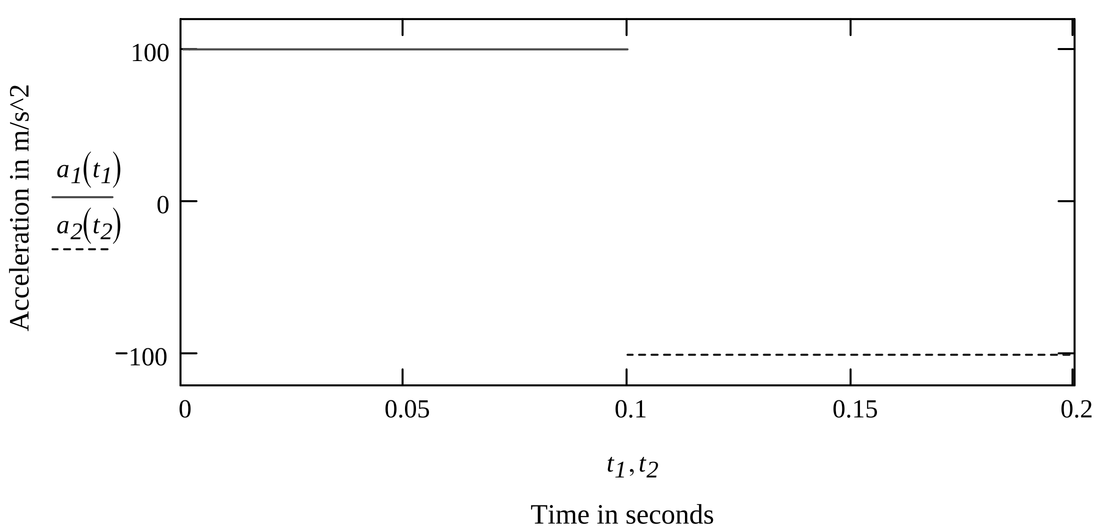
The plots

$$t_1 = 0, 0.01t_f .. \frac{t_f}{2} \qquad s_1(t_1) = \frac{1}{2}a t_1^2 \frac{1}{\text{m}} \qquad a_1(t_1) = a\frac{\text{s}^2}{\text{m}}$$

$$t_2 = \frac{t_f}{2}, 1.01\frac{t_f}{2} .. t_f \qquad s_2(t_2) = \left[\frac{1}{2}a\left(\frac{t_f}{2}\right)^2 + a\frac{t_f}{2}\left(t_2 - \frac{t_f}{2}\right) - \frac{1}{2}a\left(t_2 - \frac{t_f}{2}\right)^2\right]\frac{1}{\text{m}}$$

$$a_2(t_2) = -a\frac{\text{s}^2}{\text{m}}$$





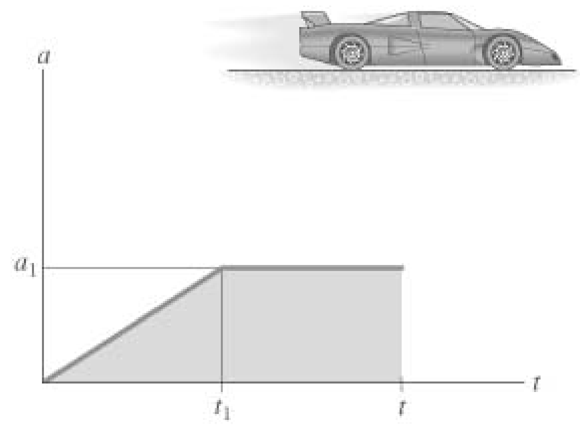
Problem 12–43

A car starting from rest moves along a straight track with an acceleration as shown. Determine the time t for the car to reach speed v.

Given: $v = 50\ \frac{m}{s}$

$a_1 = 8\ \frac{m}{s^2}$

$t_1 = 10\ s$



Solution:

Assume that $t > t_1$

Guess $t = 12\ s$

Given $v = \frac{a_1}{t_1}\frac{t_1^2}{2} + a_1(t - t_1)$ $t = \text{Find}(t)$ $t = 11.25\ s$

***Problem 12-44**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v-t graph. Determine the motorcycle's acceleration and position when $t = t_4$ and $t = t_5$.

Given:

$v_0 = 5\ \frac{m}{s}$

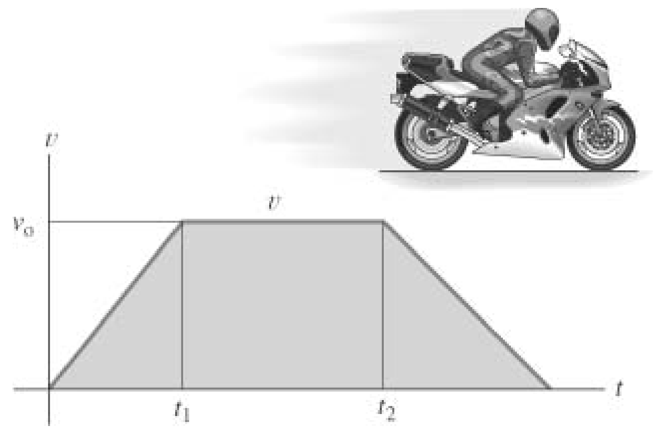
$t_1 = 4\ s$

$t_2 = 10\ s$

$t_3 = 15\ s$

$t_4 = 8\ s$

$t_5 = 12\ s$



Solution: At $t = t_4$ Because $t_1 < t_4 < t_2$ then $a_4 = \frac{dv}{dt} = 0$

$s_4 = \frac{1}{2}v_0 t_1 + (t_4 - t_1)v_0$ $s_4 = 30\ m$

At $t = t_5$ Because $t_2 < t_5 < t_3$ then

$a_5 = \frac{-v_0}{t_3 - t_2}$ $a_5 = -1\ \frac{m}{s^2}$

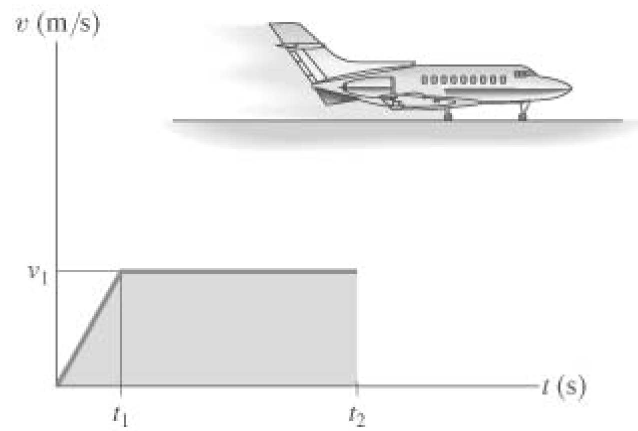
$$s_5 = \frac{1}{2}t_1 v_0 + v_0(t_2 - t_1) + \frac{1}{2}v_0(t_3 - t_2) - \frac{1}{2}\frac{t_3 - t_5}{t_3 - t_2}v_0(t_3 - t_5)$$

$$s_5 = 48\ \text{m}$$

Problem 12–45

From experimental data, the motion of a jet plane while traveling along a runway is defined by the v–t graph shown. Construct the s-t and a-t graphs for the motion.

Given:



$v_1 = 80\ \frac{\text{m}}{\text{s}}$

$t_1 = 10\ \text{s}$

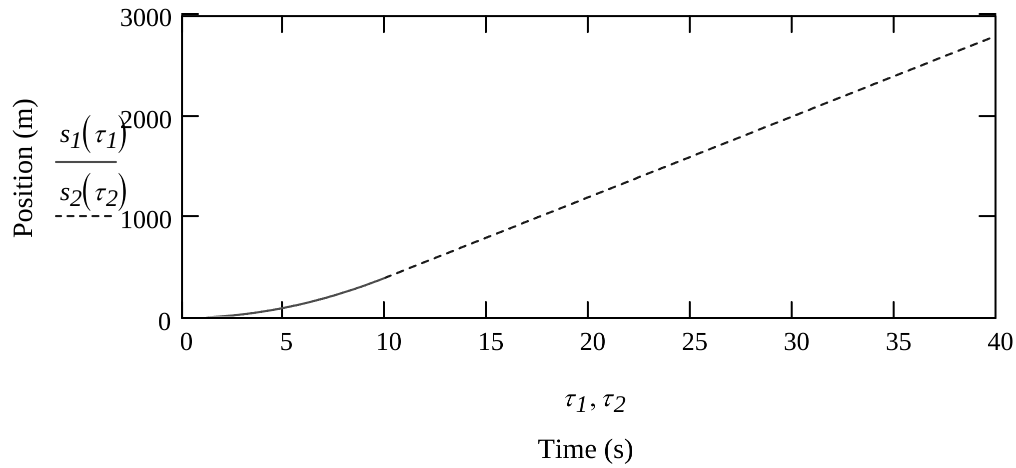
$t_2 = 40\ \text{s}$

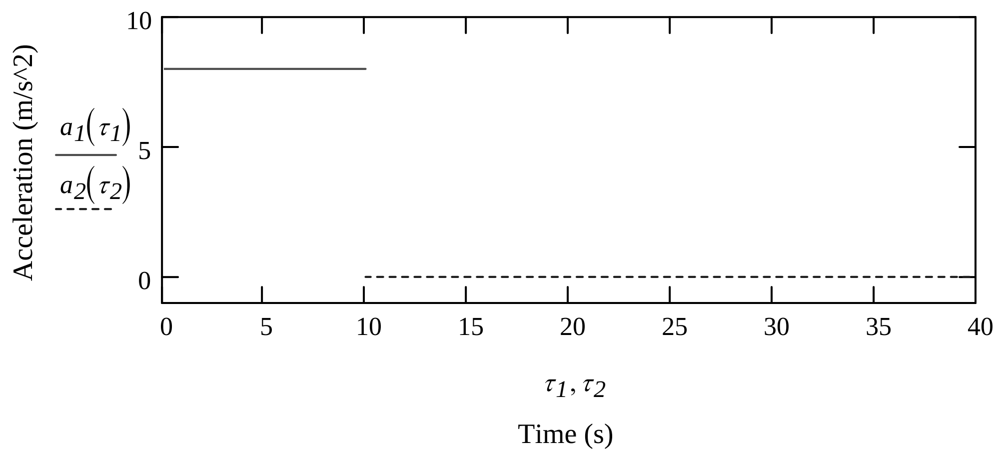
Solution:

$k_1 = \frac{v_1}{t_1}$ $\qquad k_2 = 0\ \frac{\text{m}}{\text{s}^2}$

$\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad s_1(\tau_1) = \left(\frac{1}{2}k_1 \tau_1^{\,2}\right)\ \text{m}$ $\qquad a_1(\tau_1) = k_1 \frac{\text{s}^2}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\qquad s_2(\tau_2) = \left(v_1 \tau_2 - \frac{1}{2}k_1 t_1^{\,2}\right)\ \text{m}$ $\qquad a_2(\tau_2) = k_2 \frac{\text{s}^2}{\text{m}}$





Problem 12–46

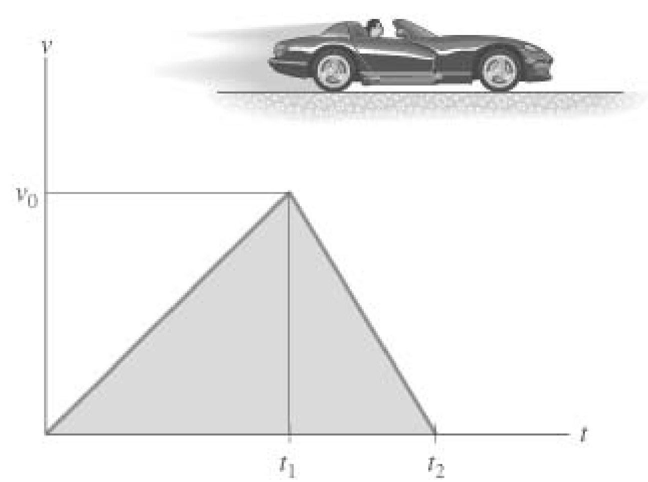
A car travels along a straight road with the speed shown by the v–t graph. Determine the total distance the car travels until it stops at t_2. Also plot the s–t and a–t graphs.

Given:

$t_1 = 30\ \text{s}$

$t_2 = 48\ \text{s}$

$v_0 = 6\ \frac{\text{m}}{\text{s}}$



Solution:

$k_1 = \frac{v_0}{t_1}$

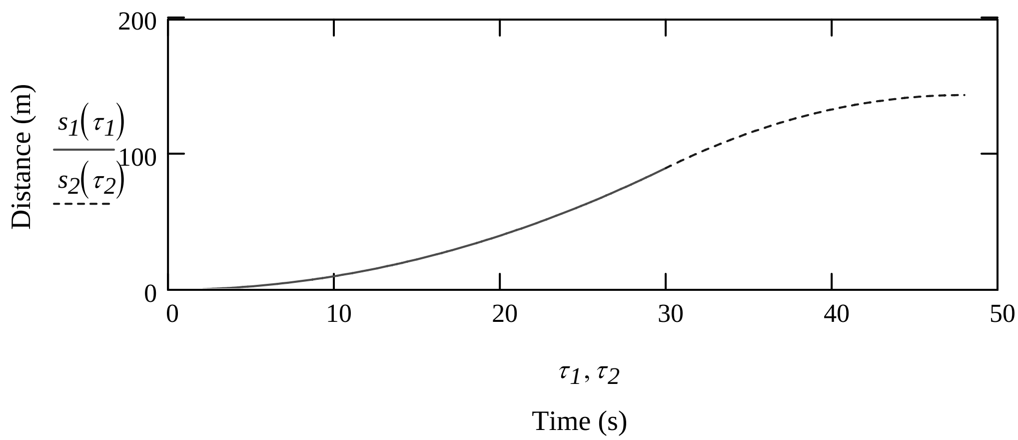
$$k_2 = \frac{v_0}{t_2 - t_1}$$

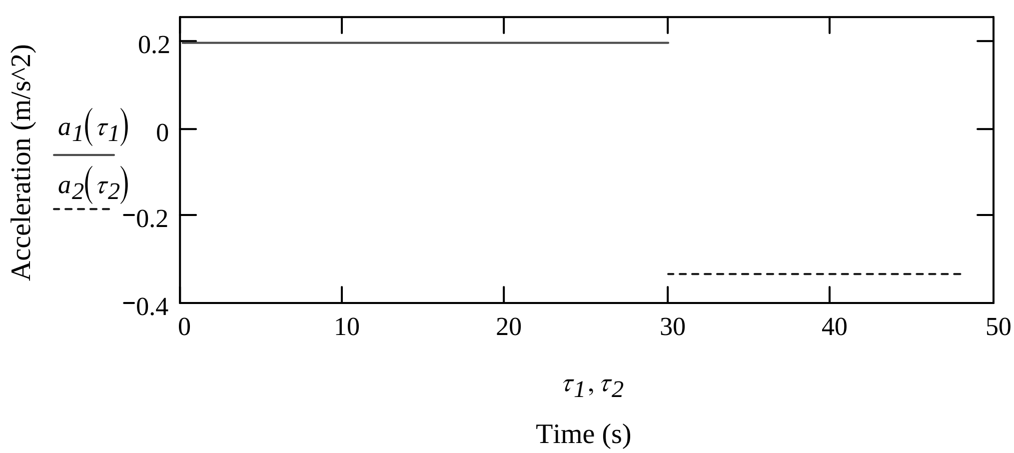
$\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad s_1(t) = \left(\frac{1}{2}k_1 t^2\right)$

$a_1(t) = k_1$ $\qquad a_2(t) = -k_2$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\qquad s_2(t) = \left[s_1(t_1) + (v_0 + k_2 t_1)(t - t_1) - \frac{k_2}{2}\left(t^2 - t_1^2\right)\right]$

$d = s_2(t_2)$ $\qquad d = 144\ \text{m}$





Problem 12–47

The *v–t* graph for the motion of a train as it moves from station *A* to station *B* is shown. Draw the *a–t* graph and determine the average speed and the distance between the stations.

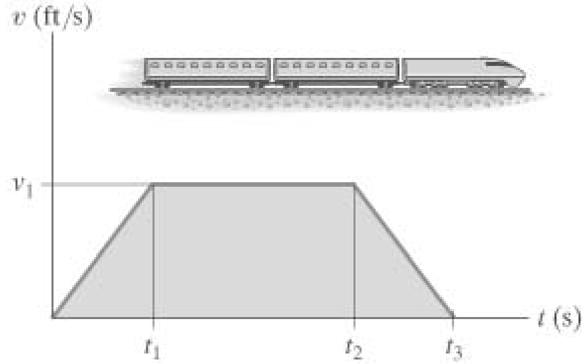
Given:

$t_1 = 30\ \text{s}$

$t_2 = 90\ \text{s}$

$t_3 = 120\ \text{s}$

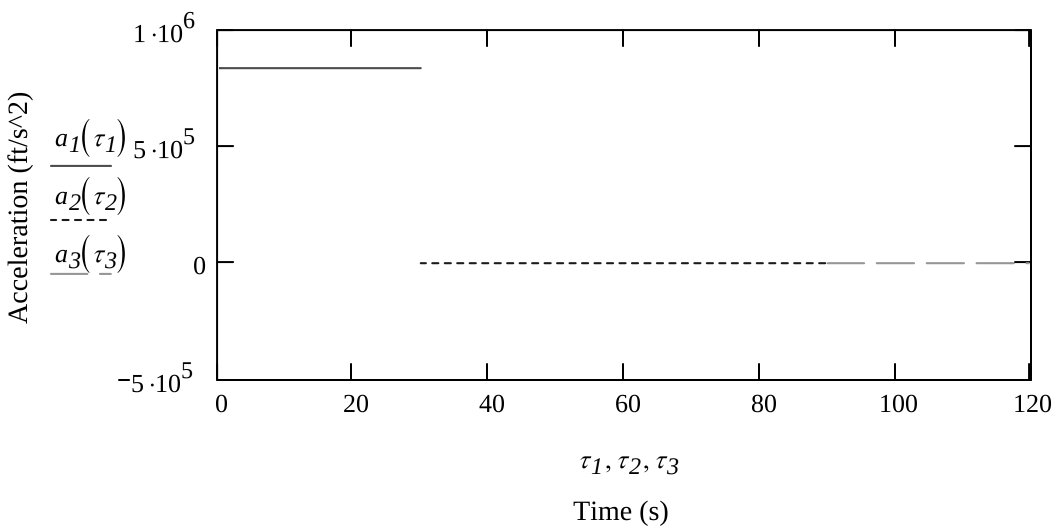
$v_1 = 40\ \frac{\text{ft}}{\text{s}}$



Solution:

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t_2 \qquad \tau_3 = t_2, 1.01t_2 .. t_3$

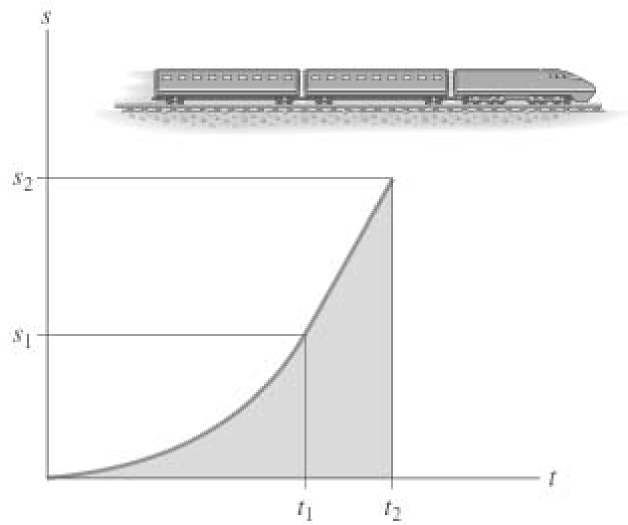
$a_1(t) = \frac{v_1}{t_1}\frac{\text{s}^2}{\text{ft}} \qquad a_2(t) = 0 \qquad a_3(t) = \frac{-v_1}{t_3 - t_2}\frac{\text{s}^2}{\text{ft}}$



$d = \frac{1}{2}v_1 t_1 + v_1(t_2 - t_1) + \frac{1}{2}v_1(t_3 - t_2) \qquad d = 3600\,\text{ft}$

$speed = \frac{d}{t_3} \qquad speed = 30\,\frac{\text{ft}}{\text{s}}$

***Problem 12–48**



The *s–t* graph for a train has been experimentally determined. From the data, construct the *v–t* and *a–t* graphs for the motion; $0 \le t \le t_2$. For $0 \le t \le t_1$, the curve is a parabola, and then it becomes straight for $t \ge t_1$.

Given:

$t_1 = 30 \text{ s}$

$t_2 = 40 \text{ s}$

$s_1 = 360 \text{ m}$

$s_2 = 600 \text{ m}$

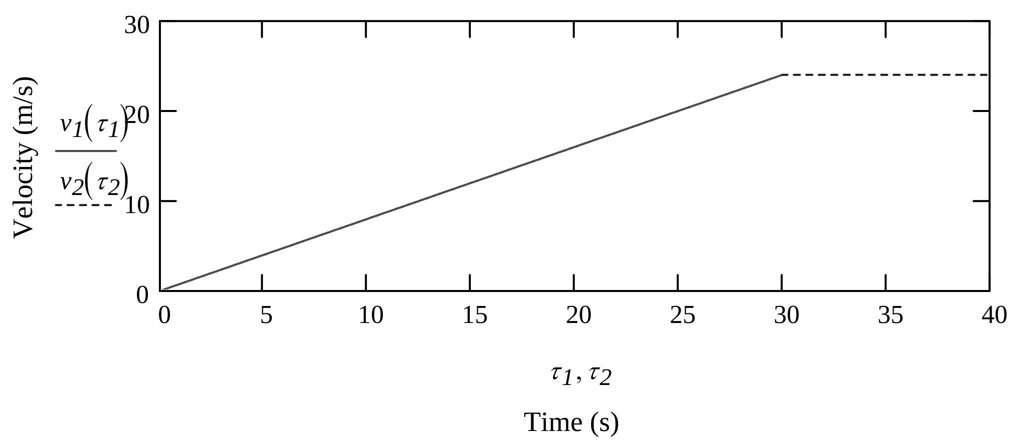
Solution:

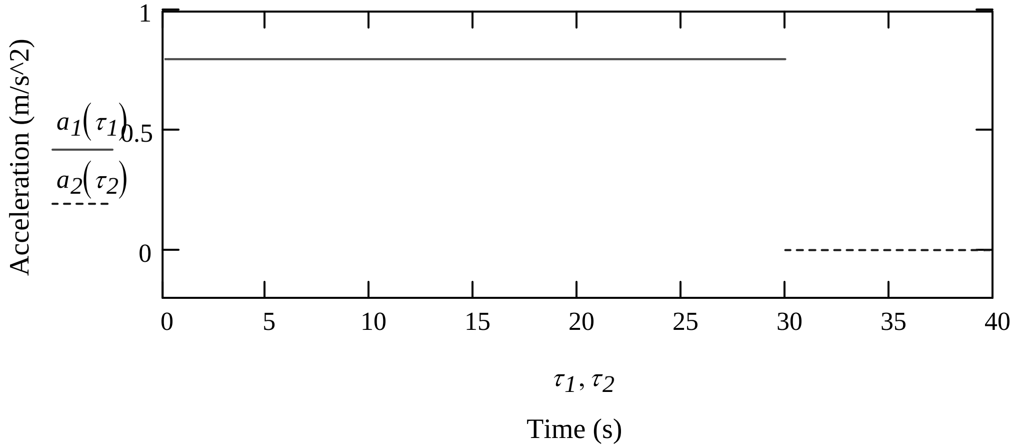
$$k_1 = \frac{s_1}{t_1^{\,2}} \qquad k_2 = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t_2$$

$$s_{p1}(t) = k_1 t^2 \qquad v_1(t) = 2k_1 t \qquad a_1(t) = 2k_1$$

$$s_{p2}(t) = s_{p1}(t_1) + k_2(t - t_1) \qquad v_2(t) = k_2 \qquad a_2(t) = 0$$





Problem 12-49

The *v*-*t* graph for the motion of a car as if moves along a straight road is shown. Draw the *a*-*t* graph and determine the maximum acceleration during the time interval $0 < t < t_2$. The car starts from rest at $s = 0$.

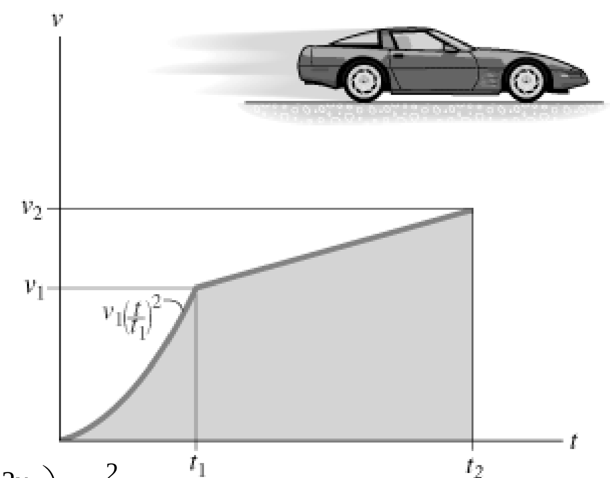
Given:

$t_1 = 10 \text{ s}$

$t_2 = 30 \text{ s}$

$v_1 = 40 \frac{\text{ft}}{\text{s}}$

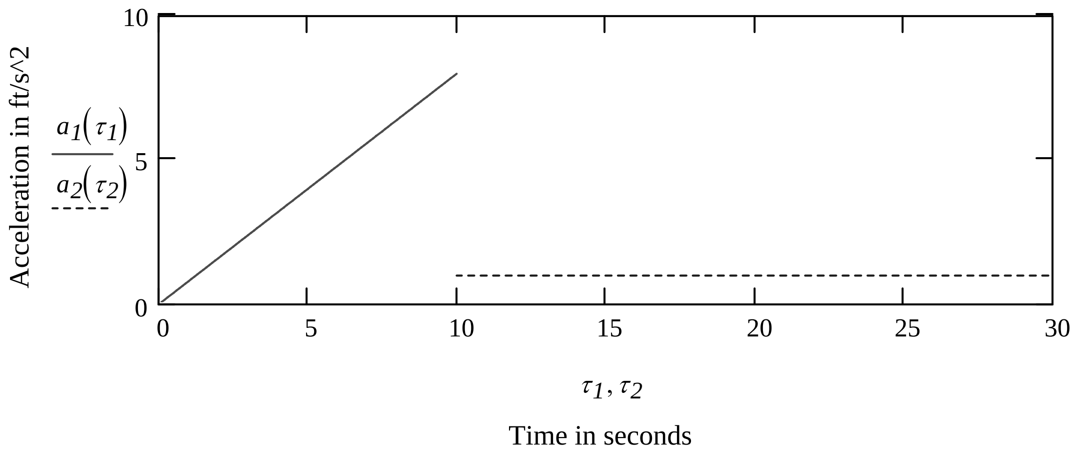
$v_2 = 60 \frac{\text{ft}}{\text{s}}$



Solution:

$\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad a_1(\tau_1) = \left(\frac{2v_1}{t_1^2}\right)\tau_1 \frac{\text{s}^2}{\text{ft}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\qquad a_2(\tau_2) = \frac{v_2 - v_1}{t_2 - t_1} \frac{\text{s}^2}{\text{ft}}$



$$a_{max} = 2\left(\frac{v_1}{t_1^{\,2}}\right)t_1 \qquad a_{max} = 8\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-50

The v-t graph for the motion of a car as it moves along a straight road is shown. Draw the s-t graph and determine the average speed and the distance traveled for the time interval $0 < t < t_2$. The car starts from rest at $s = 0$.

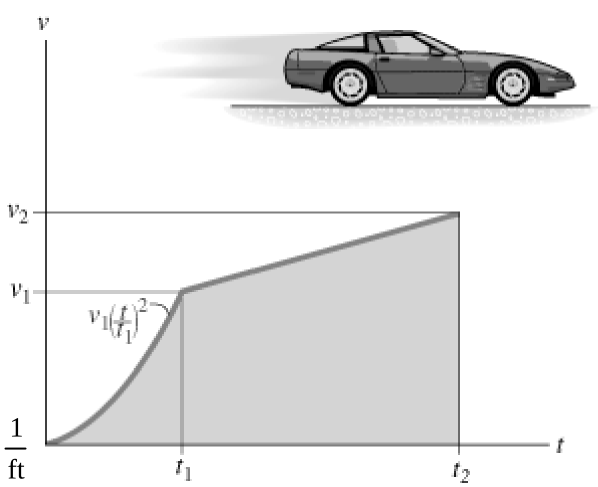
Given:

$$t_1 = 10\ \text{s}$$

$$t_2 = 30\ \text{s}$$

$$v_1 = 40\,\frac{\text{ft}}{\text{s}}$$

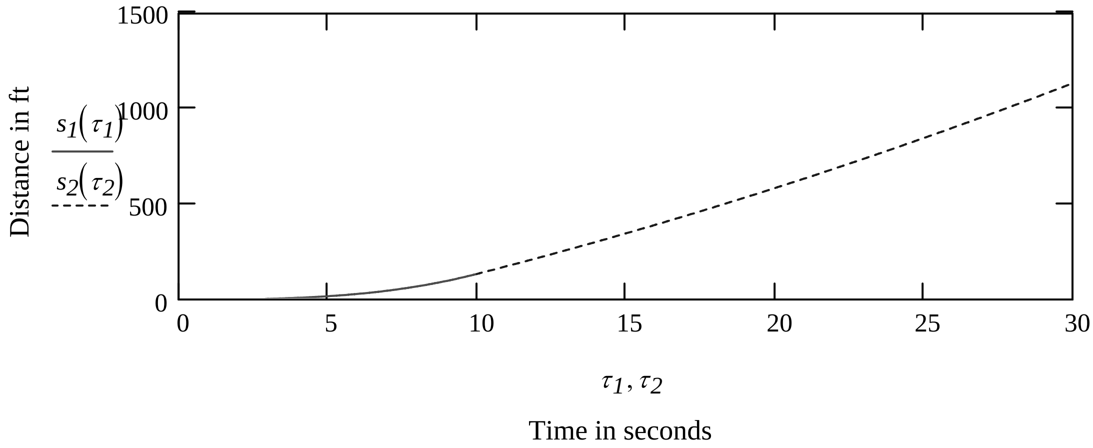
$$v_2 = 60\,\frac{\text{ft}}{\text{s}}$$



Solution: The graph

$$\tau_1 = 0, 0.01t_1 .. t_1 \qquad s_1(\tau_1) = \frac{v_1}{t_1^{\,2}}\,\frac{\tau_1^{\,3}}{3}\,\frac{1}{\text{ft}}$$

$$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad s_2(\tau_2) = \left[\frac{v_1\,t_1}{3} + v_1(\tau_2 - t_1) + \frac{v_2 - v_1}{t_2 - t_1}\,\frac{(\tau_2 - t_1)^2}{2}\right]\frac{1}{\text{ft}}$$



Distance traveled $\quad d = \frac{v_1 t_1}{3} + v_1(t_2 - t_1) + \frac{v_2 - v_1}{t_2 - t_1}\frac{(t_2 - t_1)^2}{2} \quad d = 1.133 \times 10^3\ \text{ft}$

Average speed $\quad v_{ave} = \frac{d}{t_2} \quad v_{ave} = 37.778\ \frac{\text{ft}}{\text{s}}$

Problem 12-51

The a–s graph for a boat moving along a straight path is given. If the boat starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = s_2$, and s_3, respectively. Use Simpson's rule with n to evaluate v at $s = s_3$.

Given:

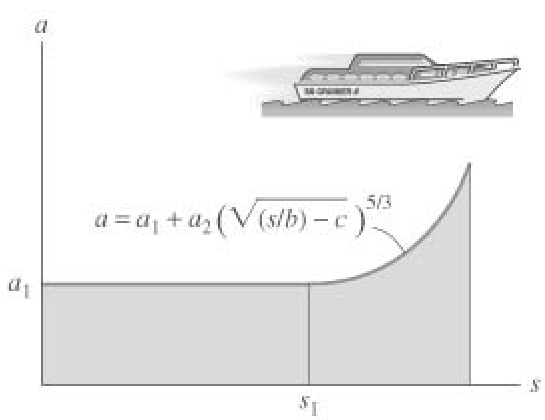
$a_1 = 5\ \frac{\text{ft}}{\text{s}^2} \quad b = 1\ \text{ft}$

$a_2 = 6\ \frac{\text{ft}}{\text{s}^2} \quad c = 10$

$s_1 = 100\ \text{ft}$

$s_2 = 75\ \text{ft}$

$s_3 = 125\ \text{ft}$



Solution:

Since $s_2 = 75\,\text{ft} < s_1 = 100\,\text{ft}$

$$a = v\frac{d}{ds}v \qquad \frac{v_2^2}{2} = \int_0^{s_2} a\, ds \qquad v_2 = \sqrt{2\int_0^{s_2} a_1\, ds} \qquad v_2 = 27.386\,\frac{ft}{s}$$

Since $s_3 = 125\,ft > s_1 = 100\,ft$

$$v_3 = \sqrt{2\int_0^{s_1} a_1\, ds + 2\int_{s_1}^{s_3} a_1 + a_2\left(\sqrt{\frac{s}{b}} - c\right)^{\frac{5}{3}} ds} \qquad v_3 = 37.444\,\frac{ft}{s}$$

***Problem 12-52**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is a height h from the ground. If the elevator maintains a constant upward speed v_0, determine how high the elevator is from the ground the instant the package hits the ground. Draw the *v-t* curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

Given: $h = 100\ ft \qquad v_0 = 4\,\frac{ft}{s} \qquad g = 32.2\,\frac{ft}{s^2}$

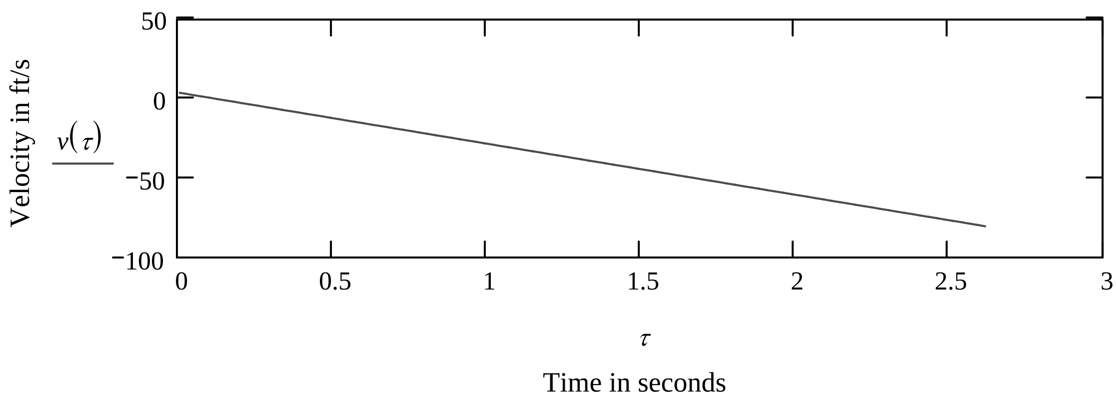
For the package $\qquad a = -g \qquad v = v_0 - g\,t \qquad s = h + v_0 t - \frac{1}{2}g\,t^2$

When it hits the ground we have

$$0 = h + v_0 t - \frac{1}{2}g\,t^2 \qquad t = \frac{v_0 + \sqrt{v_0^2 + 2g\,h}}{g} \qquad t = 2.62\ s$$

For the elevator $\qquad s_y = v_0 t + h \qquad s_y = 110.5\,ft$

The plot $\qquad \tau = 0, 0.01t\,..\,t \qquad v(\tau) = (v_0 - g\tau)\frac{s}{ft}$



Problem 12-53

Two cars start from rest side by side and travel along a straight road. Car *A* accelerates at the rate a_A for a time t_1, and then maintains a constant speed. Car *B* accelerates at the rate a_B until reaching a constant speed v_B and then maintains this speed. Construct the *a-t*, *v-t*, and *s-t* graphs for each car until $t = t_2$. What is the distance between the two cars when $t = t_2$?

Given: $a_A = 4\ \frac{\text{m}}{\text{s}^2}$ $\quad t_1 = 10\ \text{s}$ $\quad a_B = 5\ \frac{\text{m}}{\text{s}^2}$ $\quad v_B = 25\ \frac{\text{m}}{\text{s}}$ $\quad t_2 = 15\ \text{s}$

Solution:

Car *A*:

$\tau_1 = 0, 0.01t_1 .. t_1$ $\quad a_1(t) = a_A \frac{\text{s}^2}{\text{m}}$ $\quad v_1(t) = a_A t \frac{\text{s}}{\text{m}}$ $\quad s_1(t) = \frac{1}{2} a_A t^2 \frac{1}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $\quad a_2(t) = 0 \frac{\text{s}^2}{\text{m}}$ $\quad v_2(t) = v_1(t_1) \frac{\text{s}}{\text{m}}$

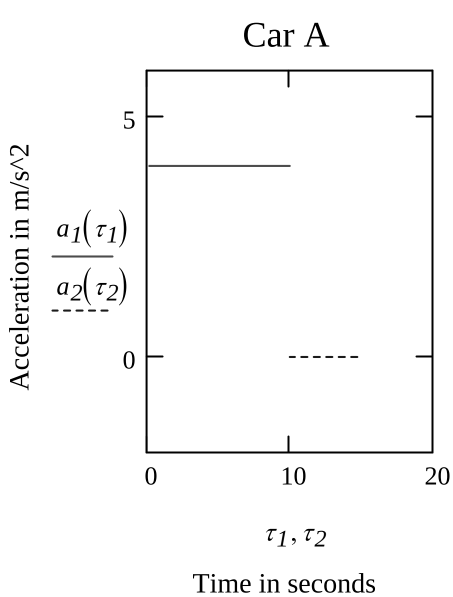
$$s_2(t) = \left[\frac{1}{2} a_A t_1^2 + a_A t_1 (t - t_1)\right] \frac{1}{\text{m}}$$

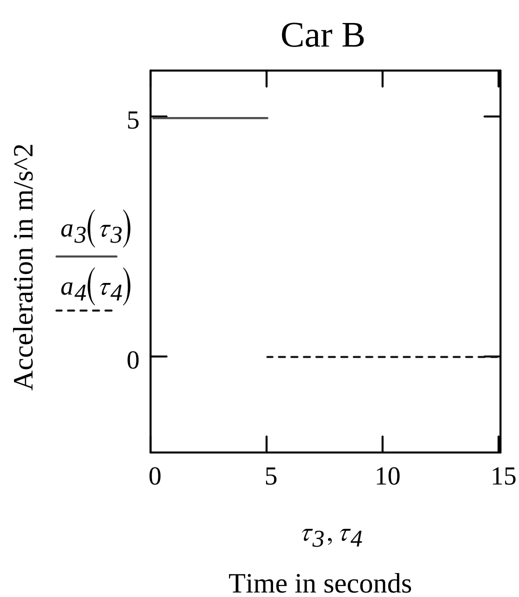
Car *B*: $\quad t_3 = \frac{v_B}{a_B}$

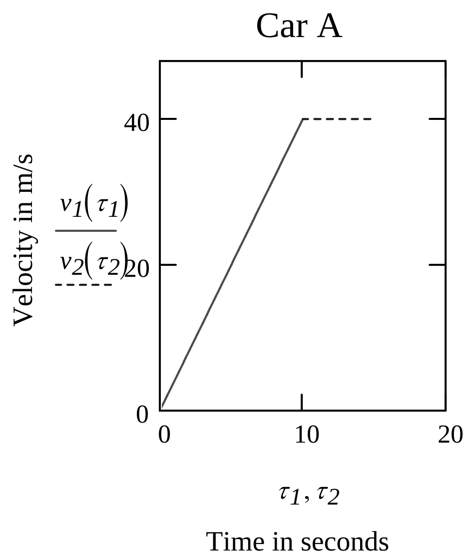
$\tau_3 = 0, 0.01t_3 .. t_3$ $\quad a_3(t) = a_B \frac{\text{s}^2}{\text{m}}$ $\quad v_3(t) = a_B t \frac{\text{s}}{\text{m}}$ $\quad s_3(t) = \frac{1}{2} a_B t^2 \frac{1}{\text{m}}$

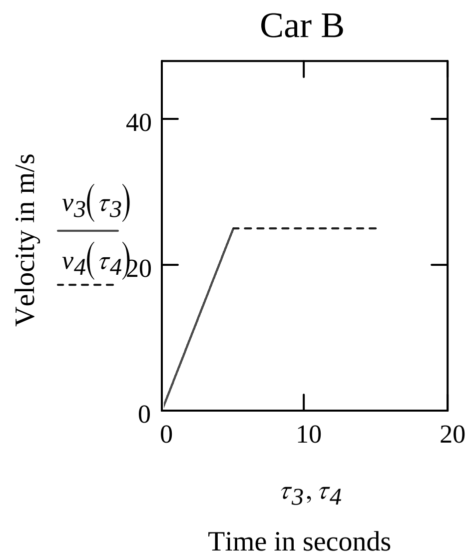
$\tau_4 = t_3, 1.01t_3 .. t_2$ $\quad a_4(t) = 0$ $\quad v_4(t) = a_B t_3 \frac{\text{s}}{\text{m}}$

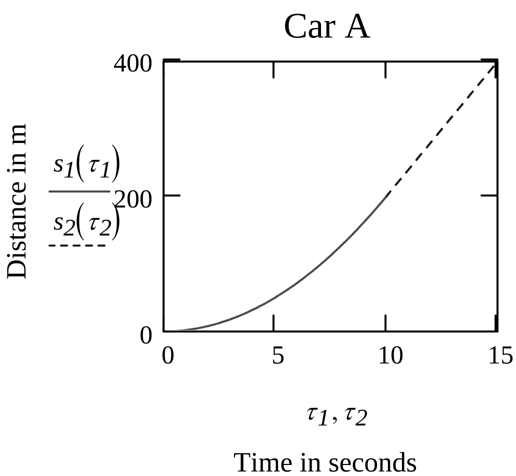
$$s_4(t) = \left[\frac{1}{2} a_B t_3^2 + a_B t_3 (t - t_3)\right] \frac{1}{\text{m}}$$

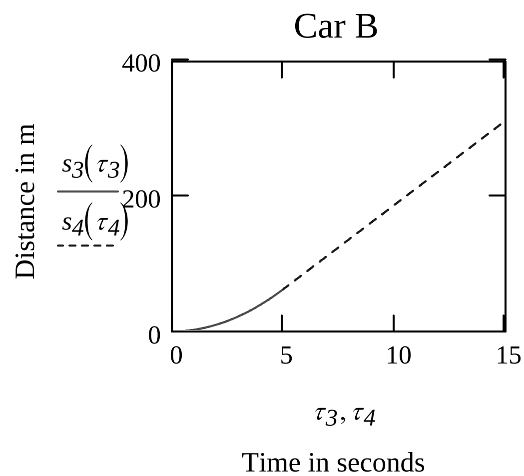












When $t = t_2$

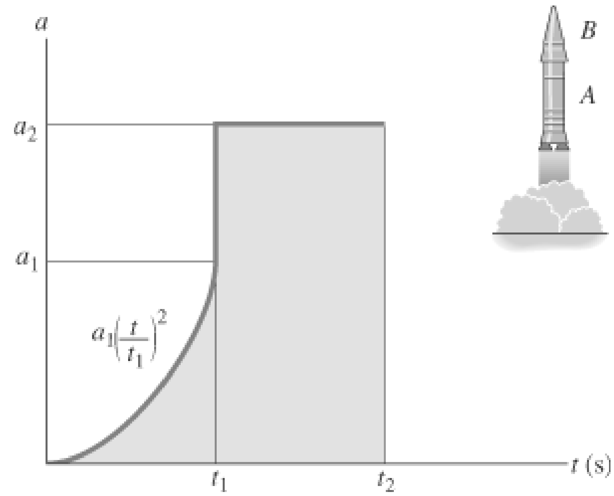
$$d = \left|\frac{1}{2}a_A t_1^{\,2} + a_A t_1(t_2 - t_1) - \left[\frac{1}{2}a_B t_3^{\,2} + a_B t_3(t_2 - t_3)\right]\right|$$

$d = 87.5\,\text{m}$

Problem 12-54

A two-stage rocket is fired vertically from rest at $s = 0$ with an acceleration as shown. After time t_1 the first stage A burns out and the second stage B ignites. Plot the v-t and s-t graphs which describe the motion of the second stage for $0 < t < t_2$.

Given:



$t_1 = 30\ \text{s}$

$t_2 = 60\text{s}$

$a_1 = 9\ \dfrac{\text{m}}{\text{s}^2}$

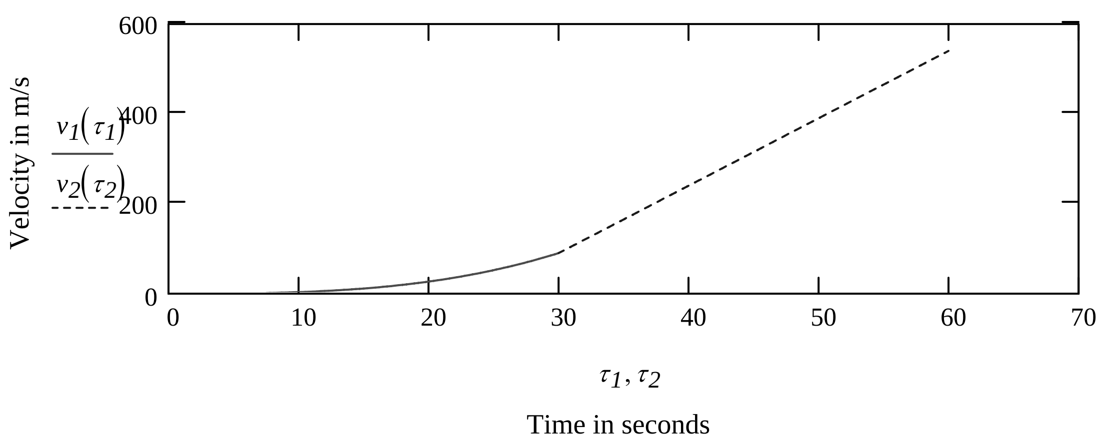
$a_2 = 15\ \dfrac{\text{m}}{\text{s}^2}$

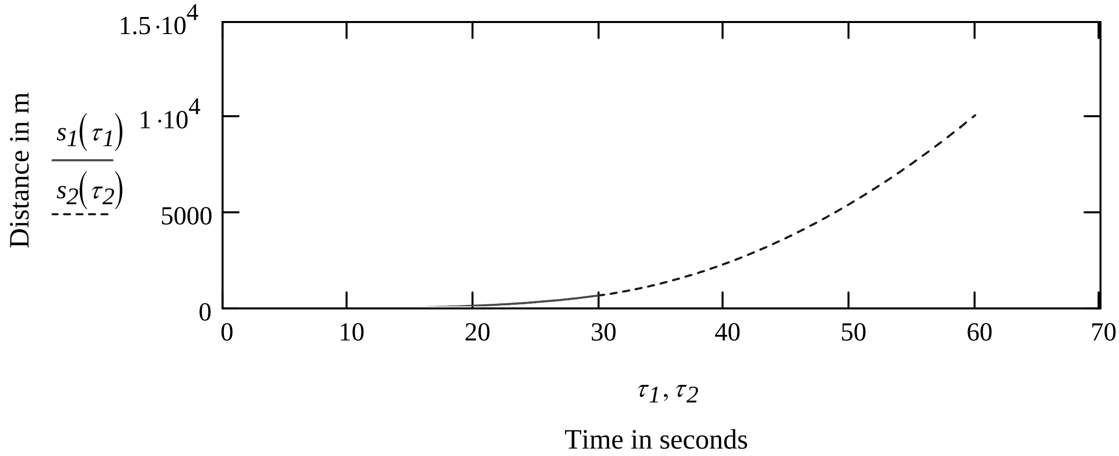
Solution:

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad v_1(\tau_1) = \dfrac{a_1}{t_1^2}\dfrac{\tau_1^3}{3}\dfrac{\text{s}}{\text{m}} \qquad s_1(\tau_1) = \dfrac{a_1}{t_1^2}\dfrac{\tau_1^4}{12}\dfrac{1}{\text{m}}$

$\tau_2 = t_1, 1.01t_1 .. t_2 \qquad v_2(\tau_2) = \left[\dfrac{a_1\, t_1}{3} + a_2(\tau_2 - t_1)\right]\dfrac{\text{s}}{\text{m}}$

$$s_2(\tau_2) = \left[\frac{a_1\, t_1^2}{12} + \frac{a_1\, t_1}{3}(\tau_2 - t_1) + a_2\frac{(\tau_2 - t_1)^2}{2}\right]\frac{\text{s}^2}{\text{m}}$$





Problem 12–55

The *a–t* graph for a motorcycle traveling along a straight road has been estimated as shown. Determine the time needed for the motorcycle to reach a maximum speed v_{max} and the distance traveled in this time. Draw the *v–t* and *s–t* graphs.The motorcycle starts from rest at *s* = 0.

Given:

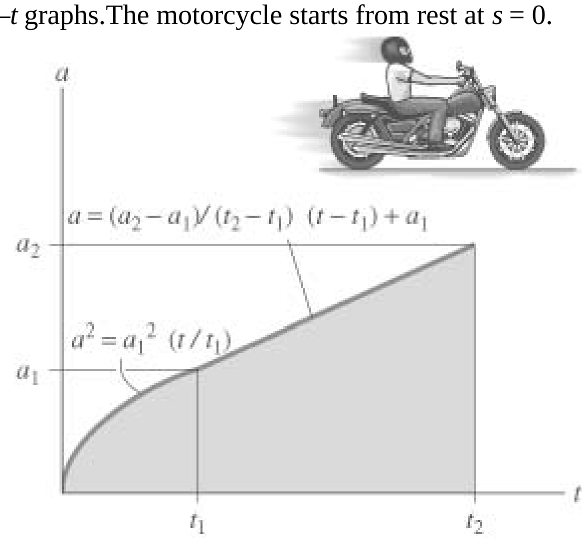
$t_1 = 10\ \text{s}$

$t_2 = 30\ \text{s}$

$a_1 = 10\ \frac{\text{ft}}{\text{s}^2}$

$a_2 = 20\ \frac{\text{ft}}{\text{s}^2}$

$v_{max} = 100\ \frac{\text{ft}}{\text{s}}$



Solution: Assume that $t_1 < t < t_2$

$\tau_1 = 0, 0.01t_1 .. t_1 \qquad \tau_2 = t_1, 1.01t_1 .. t_2$

$a_{p1}(t) = a_1\sqrt{\frac{t}{t_1}} \qquad v_{p1}(t) = \left(\frac{2a_1}{3\sqrt{t_1}}\right)\sqrt{t^3} \qquad s_{p1}(t) = \left(\frac{4a_1}{15\sqrt{t_1}}\right)\sqrt{t^5}$

$a_{p2}(t) = (a_2 - a_1)\frac{t - t_1}{t_2 - t_1} + a_1$

$$v_{p2}(t) = \frac{a_2 - a_1}{2}\frac{(t - t_1)^2}{t_2 - t_1} + a_1(t - t_1) + v_{p1}(t_1)$$

$$s_{p2}(t) = \frac{a_2 - a_1}{6}\frac{(t - t_1)^3}{t_2 - t_1} + \frac{a_1}{2}(t - t_1)^2 + v_{p1}(t_1)(t - t_1) + s_{p1}(t_1)$$

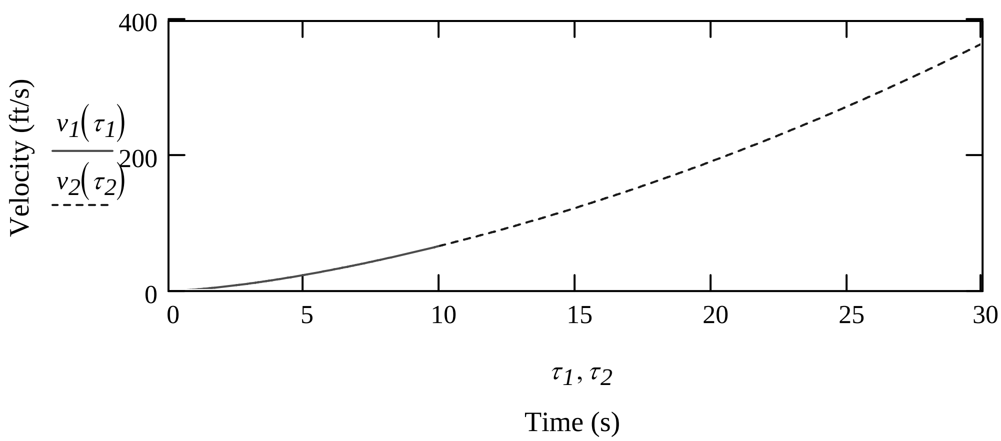
Guess $t = 1\ \text{s}$ Given $v_{p2}(t) = v_{max}$

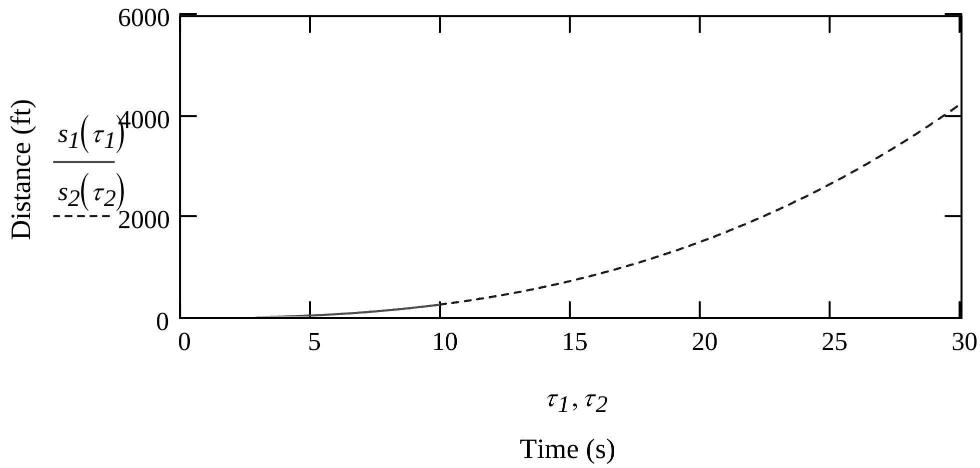
$t = \text{Find}(t)$ $t = 13.09\ \text{s}$

$d = s_{p2}(t)$ $d = 523\,\text{ft}$

$v_1(t) = v_{p1}(t)\frac{\text{s}}{\text{ft}}$ $v_2(t) = v_{p2}(t)\frac{\text{s}}{\text{ft}}$

$s_1(t) = s_{p1}(t)\frac{1}{\text{ft}}$ $s_2(t) = s_{p2}(t)\frac{1}{\text{ft}}$





*__Problem 12-56__

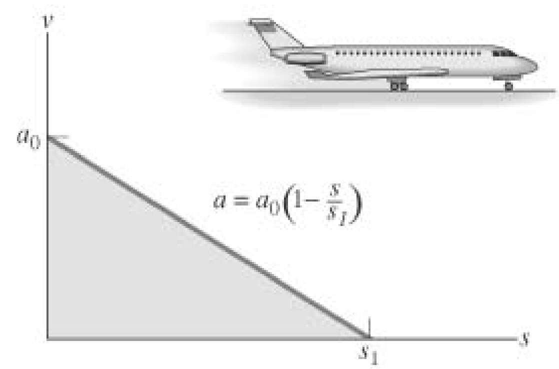
The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled a distance d. Also, how much time is required for it to travel the distance d?

Given:

$d = 200\ \text{ft}$

$a_0 = 75\ \frac{\text{ft}}{\text{s}^2}$

$s_1 = 500\ \text{ft}$



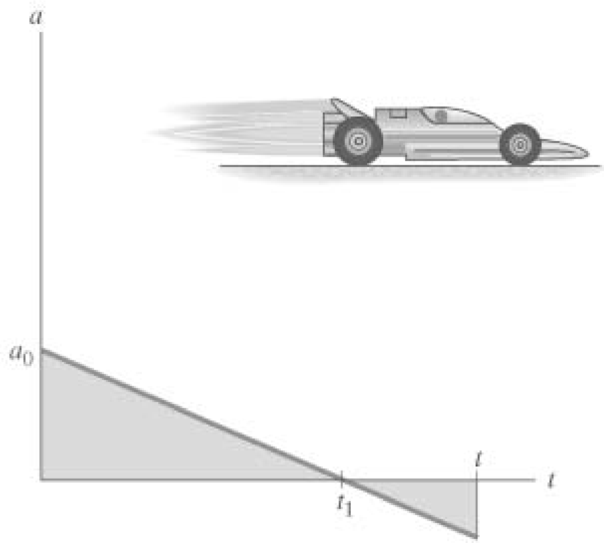
Solution:

$$a = a_0\left(1 - \frac{s}{s_1}\right) \qquad \int_0^{v_2} v\,dv = \int_0^{s} a_0\left(1 - \frac{s}{s_1}\right) ds \qquad \frac{v^2}{2} = a_0\left(s - \frac{s^2}{2s_1}\right)$$

$$v_d = \sqrt{2a_0\left(d - \frac{d^2}{2s_1}\right)} \qquad v_d = 155\ \frac{\text{ft}}{\text{s}}$$

$$v = \frac{ds}{dt} \qquad \int_0^{t} 1\,dt = \int_0^{d} \frac{1}{v}\,ds \qquad t = \int_0^{d} \frac{1}{\sqrt{2a_0\left(s - \frac{s^2}{2s_1}\right)}}\,ds \qquad t = 2.394\ \text{s}$$

Problem 12–57



The jet car is originally traveling at speed v_0 when it is subjected to the acceleration shown in the graph. Determine the car's maximum speed and the time *t* when it stops.

Given:

$$v_0 = 20\ \frac{\text{m}}{\text{s}}$$

$$a_0 = 10\ \frac{\text{m}}{\text{s}^2}$$

$$t_1 = 20\ \text{s}$$

Solution:

$$a(t) = a_0\left(1 - \frac{t}{t_1}\right) \qquad v(t) = v_0 + \int_0^t a(t)\ dt \qquad s_p(t) = \int_0^t v(t)\ dt$$

Guess $t_{stop} = 30\ \text{s}$ Given $v(t_{stop}) = 0$ $t_{stop} = \text{Find}(t_{stop})$

$$v_{max} = v(t_1) \qquad v_{max} = 120\ \frac{\text{m}}{\text{s}} \qquad t_{stop} = 41.909\ \text{s}$$

Problem 12-58

A motorcyclist at *A* is traveling at speed v_1 when he wishes to pass the truck *T* which is traveling at a constant speed v_2. To do so the motorcyclist accelerates at rate *a* until reaching a maximum speed v_3. If he then maintains this speed, determine the time needed for him to reach a point located a distance d_3 in front of the truck. Draw the *v*-*t* and *s*-*t* graphs for the motorcycle during this time.

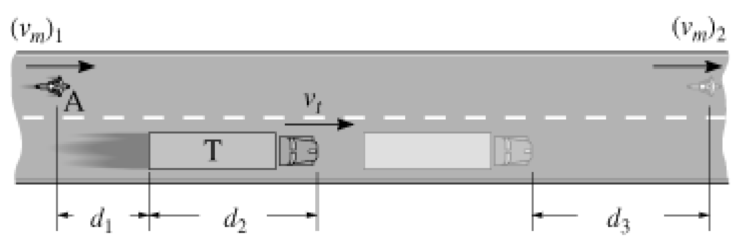
Given:

$$v_1 = 60\ \frac{\text{ft}}{\text{s}} \qquad d_1 = 40\ \text{ft}$$

$$v_2 = 60\ \frac{\text{ft}}{\text{s}} \qquad d_2 = 55\ \text{ft}$$

$$v_3 = 85\ \frac{\text{ft}}{\text{s}} \qquad d_3 = 100\ \text{ft}$$

$$a = 6\ \frac{\text{ft}}{\text{s}^2}$$



Solution: Let t_1 represent the time to full speed, t_2 the time to reache the required distance.

Guesses $t_1 = 10\ \text{s}$ $t_2 = 20\ \text{s}$

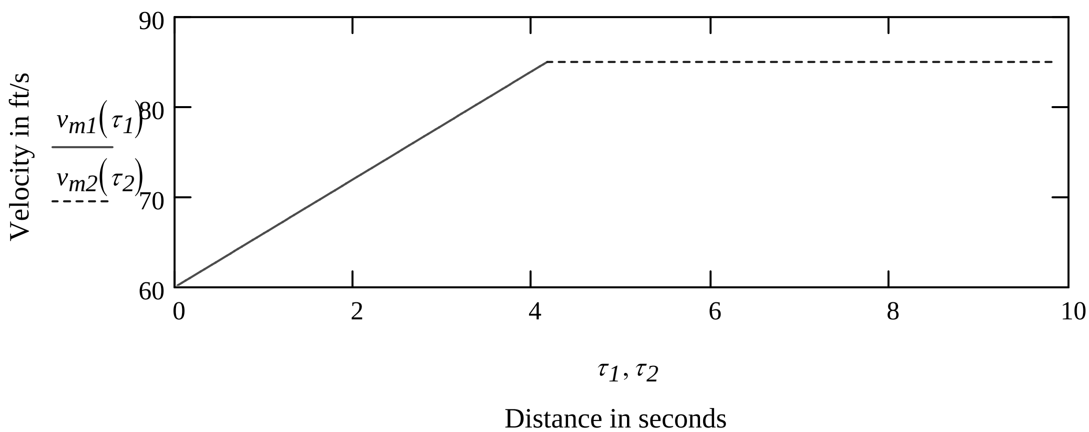
Given $v_3 = v_1 + a t_1$ $d_1 + d_2 + d_3 + v_2 t_2 = v_1 t_1 + \frac{1}{2} a t_1^2 + v_3(t_2 - t_1)$

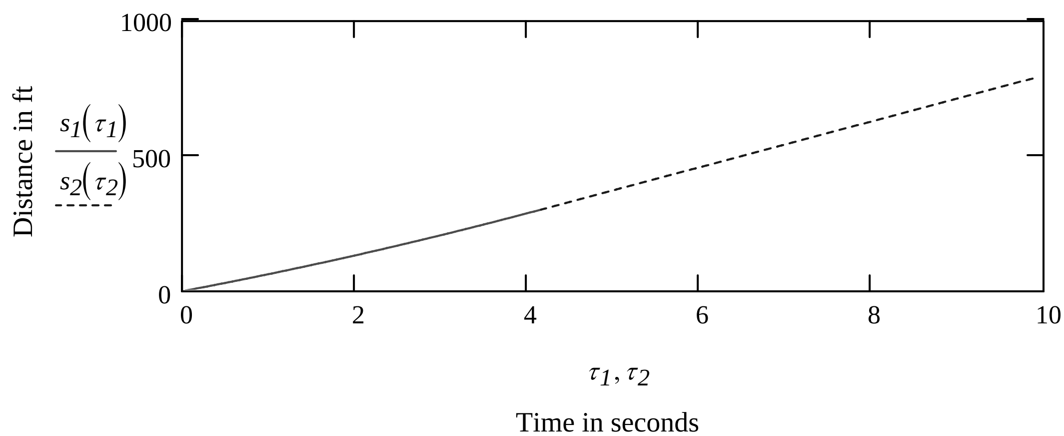
$\begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \text{Find}(t_1, t_2)$ $t_1 = 4.167\ \text{s}$ $t_2 = 9.883\ \text{s}$

Now draw the graphs

$\tau_1 = 0, 0.01t_1 .. t_1$ $s_1(\tau_1) = \left(v_1 \tau_1 + \frac{1}{2} a \tau_1^2\right)\frac{1}{\text{ft}}$ $v_{m1}(\tau_1) = (v_1 + a\tau_1)\frac{\text{s}}{\text{ft}}$

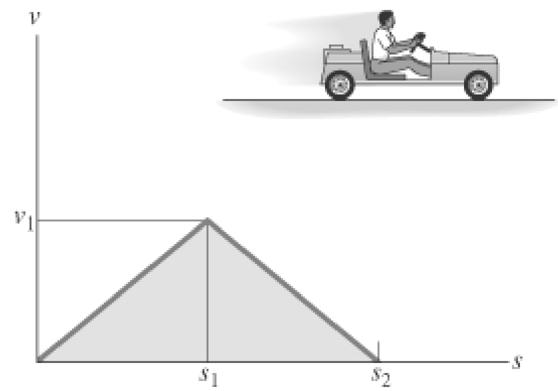
$\tau_2 = t_1, 1.01t_1 .. t_2$ $s_2(\tau_2) = \left[v_1 t_1 + \frac{1}{2} a t_1^2 + v_3(\tau_2 - t_1)\right]\frac{1}{\text{ft}}$ $v_{m2}(\tau_2) = v_3 \frac{\text{s}}{\text{ft}}$





Problem 12-59

The v-s graph for a go-cart traveling on a straight road is shown. Determine the acceleration of the go-cart at s_3 and s_4. Draw the a-s graph.



Given:

$v_1 = 8 \frac{m}{s}$ $\quad s_3 = 50 \text{ m}$

$s_1 = 100 \text{ m}$ $\quad s_4 = 150 \text{ m}$

$s_2 = 200 \text{ m}$

Solution:

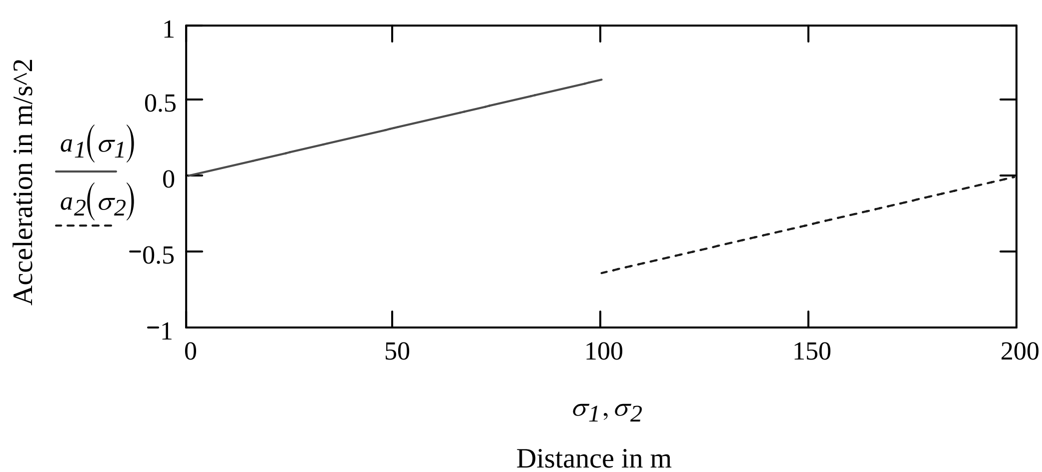
For $0 < s < s_1$ $\quad a = v\frac{dv}{ds} = v\frac{v_1}{s_1}$ $\quad a_3 = \frac{s_3}{s_1} v_1 \frac{v_1}{s_1}$ $\quad a_3 = 0.32 \frac{m}{s^2}$

For $s_1 < s < s_2$ $\quad a = v\frac{dv}{ds} = -v\frac{v_1}{s_2 - s_1}$

$$a_4 = -\frac{s_2 - s_4}{s_2 - s_1} v_1 \frac{v_1}{s_2 - s_1} \qquad a_4 = -0.32 \frac{m}{s^2}$$

$\sigma_1 = 0, 0.01 s_1 .. s_1$ $\quad a_1(\sigma_1) = \frac{\sigma_1}{s_1} \frac{v_1^2}{s_1} \frac{s^2}{m}$

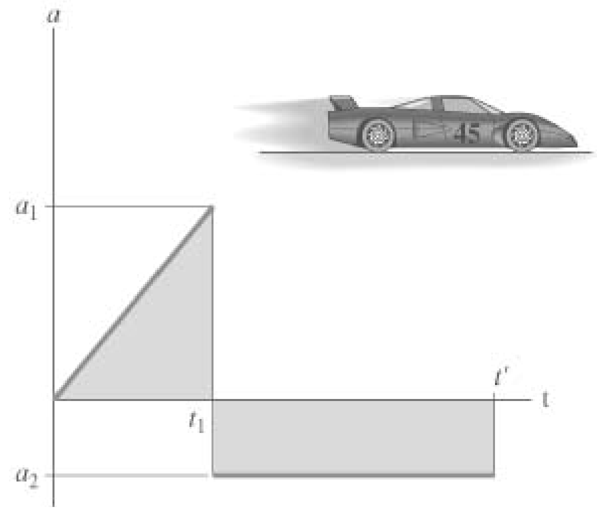
$\sigma_2 = s_1, 1.01 s_1 .. s_2$ $\quad a_2(\sigma_2) = -\frac{s_2 - \sigma_2}{s_2 - s_1} \frac{v_1^2}{s_2 - s_1} \frac{s^2}{m}$



***Problem 12–60**

The a–t graph for a car is shown. Construct the v–t and s–t graphs if the car starts from rest at t = 0. At what time t' does the car stop?

Given:



$a_1 = 5\ \frac{m}{s^2}$

$a_2 = -2\ \frac{m}{s^2}$

$t_1 = 10\ s$

Solution:

$$k = \frac{a_1}{t_1}$$

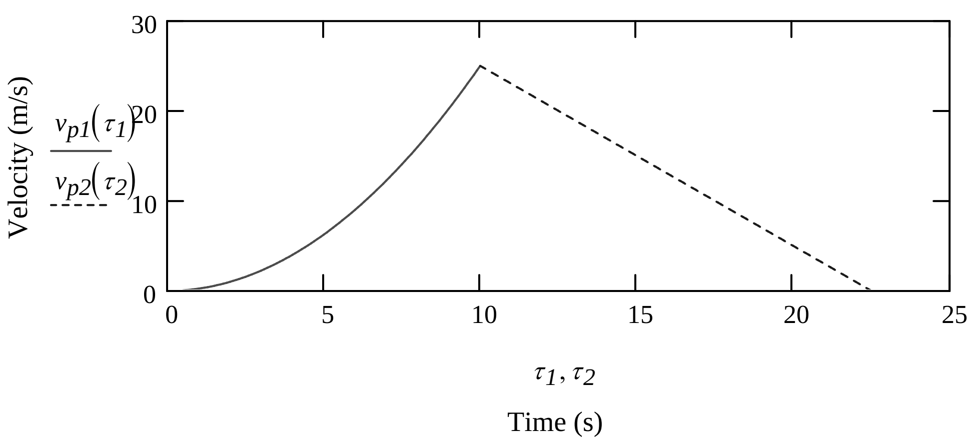
$a_{p1}(t) = kt$ $\qquad v_{p1}(t) = k\frac{t^2}{2}$ $\qquad s_{p1}(t) = k\frac{t^3}{6}$

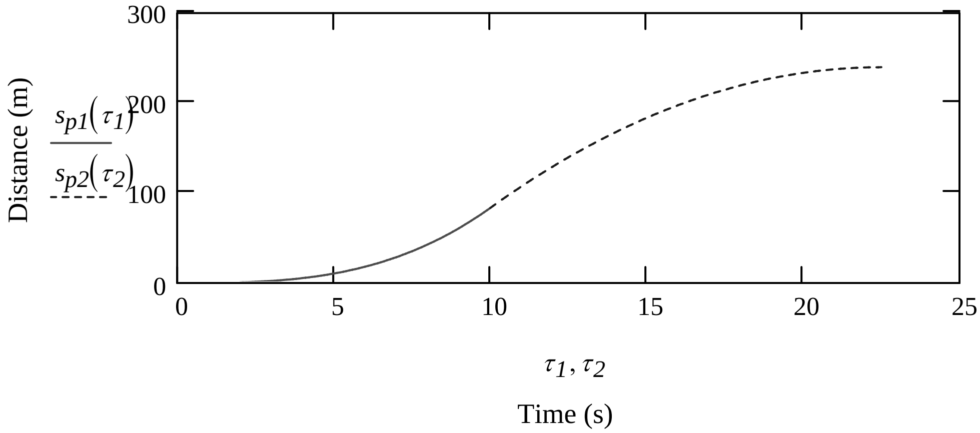
$a_{p2}(t) = a_2$ $\qquad v_{p2}(t) = v_{p1}(t_1) + a_2(t - t_1)$

$$s_{p2}(t) = s_{p1}(t_1) + v_{p1}(t_1)(t - t_1) + \frac{1}{2}a_2(t - t_1)^2$$

Guess $t' = 12\ s$ Given $v_{p2}(t') = 0$ $t' = \text{Find}(t')$ $t' = 22.5\ s$

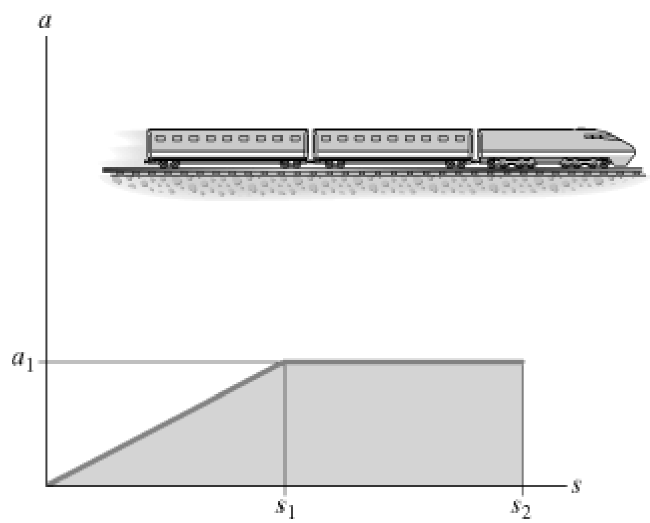
$\tau_1 = 0, 0.01t_1 .. t_1$ $\qquad \tau_2 = t_1, 1.01t_1 .. t'$





Problem 12-61

The *a*-*s* graph for a train traveling along a straight track is given for $0 \le s \le s_2$. Plot the *v*-*s* graph. $v = 0$ at $s = 0$.



Given:

$$s_1 = 200 \text{ m}$$

$$s_2 = 400 \text{ m}$$

$$a_1 = 2 \frac{\text{m}}{\text{s}^2}$$

Solution:

$$\sigma_1 = 0, 0.01s_1 .. s_1$$

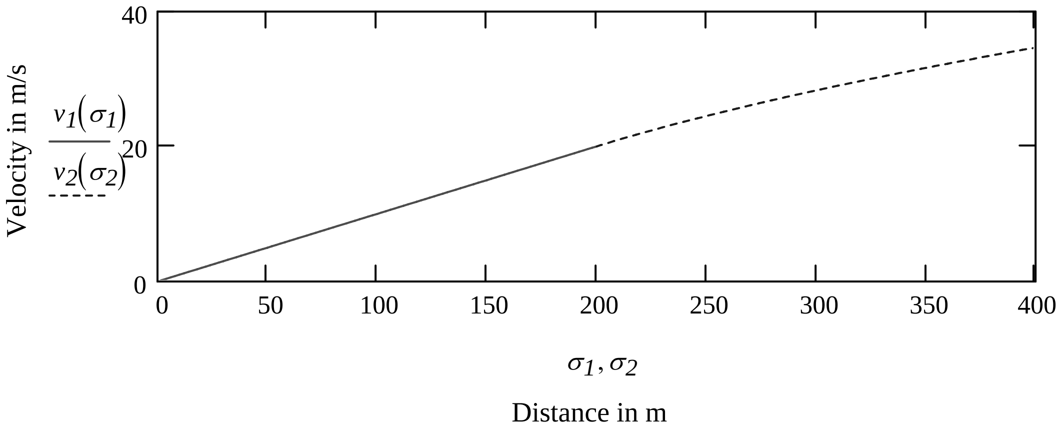
$$\sigma_2 = s_1, 1.01s_1 .. s_2$$

For $0 < s < s_1$ $\quad k = \frac{a_1}{s_1} \quad a_{c1} = ks$

$$a = ks = v\frac{dv}{ds} \qquad \int_0^v v\,dv = \int_0^s ks\,ds \qquad \frac{v^2}{2} = \frac{k}{2}s^2 \qquad v_1(\sigma_1) = \sqrt{k}\,\sigma_1 \frac{\text{s}}{\text{m}}$$

For $s_1 < s < s_2$ $\quad a_{c2} = a_1 \quad a = ks = v\frac{dv}{ds} \quad \int_{v_1}^v v\,dv = \int_{s_1}^s a_1\,ds$

$$\frac{v^2}{2} - \frac{v_1^2}{2} = a_1(s - s_1) \qquad v_2(\sigma_2) = \sqrt{2a_1(\sigma_2 - s_1) + ks_1^2}\,\frac{\text{s}}{\text{m}}$$



Problem 12-62

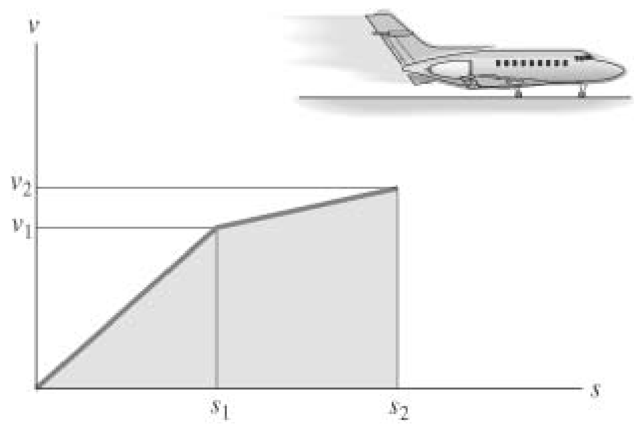
The v-s graph for an airplane traveling on a straight runway is shown. Determine the acceleration of the plane at $s = s_3$ and $s = s_4$. Draw the a-s graph.

Given:

$s_1 = 100\text{ m}$ $\quad s_4 = 150\text{ m}$

$s_2 = 200\text{ m}$ $\quad v_1 = 40\frac{\text{m}}{\text{s}}$

$s_3 = 50\text{ m}$ $\quad v_2 = 50\frac{\text{m}}{\text{s}}$



Solution:

$$a = v\frac{dv}{ds}$$

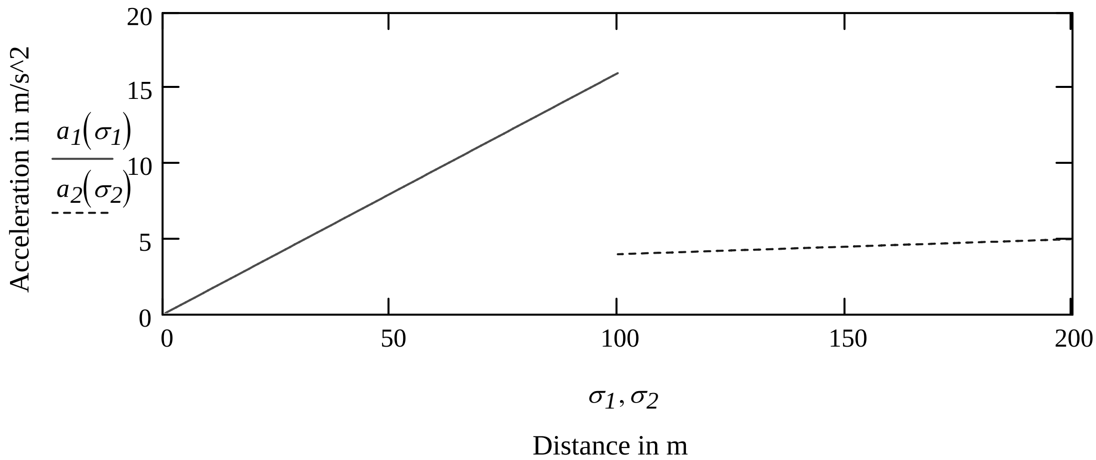
$0 < s_3 < s_1$ $\qquad a_3 = \left(\frac{s_3}{s_1}\right)v_1\left(\frac{v_1}{s_1}\right)$ $\qquad a_3 = 8\frac{\text{m}}{\text{s}^2}$

$s_1 < s_4 < s_2$ $\qquad a_4 = \left[v_1 + \frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}$ $\qquad a_4 = 4.5\frac{\text{m}}{\text{s}^2}$

The graph

$\sigma_1 = 0, 0.01s_1 .. s_1$ $\qquad a_1(\sigma_1) = \frac{\sigma_1}{s_1}\frac{v_1^2}{s_1}\frac{\text{s}^2}{\text{m}}$

$\sigma_2 = s_1, 1.01s_1 .. s_2$ $\qquad a_2(\sigma_2) = \left[v_1 + \frac{\sigma_2 - s_1}{s_2 - s_1}(v_2 - v_1)\right]\frac{v_2 - v_1}{s_2 - s_1}\frac{\text{s}^2}{\text{m}}$



Problem 12-63

Starting from rest at $s = 0$, a boat travels in a straight line with an acceleration as shown by the *a*-*s* graph. Determine the boat's speed when $s = s_4$, s_5, and s_6.

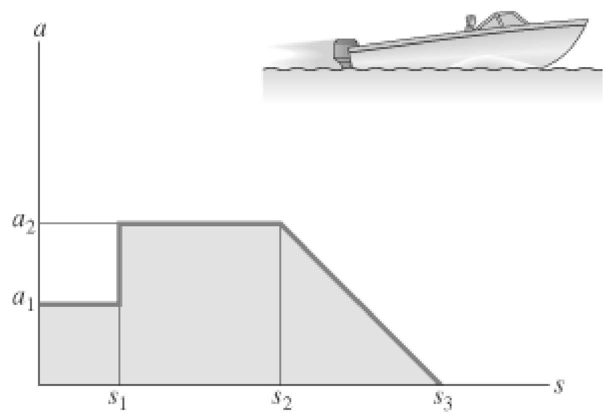
Given:

$s_1 = 50\ \text{ft}$ $\quad s_5 = 90\ \text{ft}$

$s_2 = 150\ \text{ft}$ $\quad s_6 = 200\ \text{ft}$

$s_3 = 250\ \text{ft}$ $\quad a_1 = 2\ \frac{\text{ft}}{\text{s}^2}$

$s_4 = 40\ \text{ft}$ $\quad a_2 = 4\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$0 < s_4 < s_1 \qquad a_4 = a_1 \qquad v_4 = \sqrt{2a_4 s_4} \qquad v_4 = 12.649\ \frac{\text{ft}}{\text{s}}$

$v_1 = \sqrt{2a_1 s_1}$

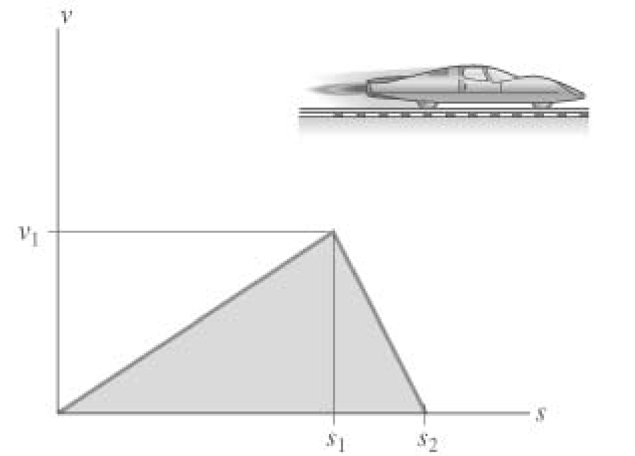
$s_1 < s_5 < s_2 \qquad a_5 = a_2 \qquad v_5 = \sqrt{2a_5(s_5 - s_1) + v_1^2} \qquad v_5 = 22.804\ \frac{\text{ft}}{\text{s}}$

$v_2 = \sqrt{2a_2(s_2 - s_1) + v_1^2}$

$s_2 < s_6 < s_3 \qquad v_6 = \sqrt{v_2^2 + 2\int_{s_2}^{s_6} \frac{s_3 - s}{s_3 - s_2} a_2\, ds} \qquad v_6 = 36.056\ \frac{\text{ft}}{\text{s}}$

***Problem 12–64**

The *v–s* graph for a test vehicle is shown. Determine its acceleration at $s = s_3$ and s_4.



Given:

$$v_1 = 50\ \frac{\text{m}}{\text{s}}$$

$$s_1 = 150\ \text{m} \quad s_3 = 100\ \text{m}$$

$$s_2 = 200\ \text{m} \quad s_4 = 175\ \text{m}$$

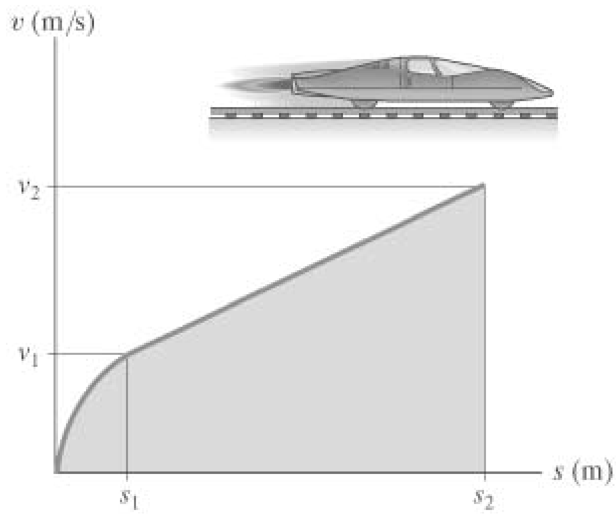
Solution:

$$a_3 = \left(\frac{s_3}{s_1}\right) v_1 \left(\frac{v_1}{s_1}\right) \qquad a_3 = 11.11\ \frac{\text{m}}{\text{s}^2}$$

$$a_4 = \left(\frac{s_2 - s_4}{s_2 - s_1}\right) v_1 \left(\frac{0 - v_1}{s_2 - s_1}\right) \qquad a_4 = -25\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–65

The *v–s* graph was determined experimentally to describe the straight-line motion of a rocket sled. Determine the acceleration of the sled at $s = s_3$ and $s = s_4$.



Given:

$$v_1 = 20\ \frac{\text{m}}{\text{s}} \qquad s_1 = 50\ \text{m}$$

$$v_2 = 60\ \frac{\text{m}}{\text{s}} \qquad s_2 = 300\ \text{m}$$

$$s_3 = 100\ \text{m} \qquad s_4 = 200\ \text{m}$$

Solution:

$$a = v\frac{dv}{ds}$$

$$a_3 = \left[\frac{s_3 - s_1}{s_2 - s_1}(v_2 - v_1) + v_1\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_3 = 4.48\ \frac{\text{m}}{\text{s}^2}$$

$$a_4 = \left[\frac{s_4 - s_1}{s_2 - s_1}(v_2 - v_1) + v_1\right]\frac{v_2 - v_1}{s_2 - s_1} \qquad a_4 = 7.04\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-66

A particle, originally at rest and located at point (a, b, c), is subjected to an acceleration $\mathbf{a_c} = \{d\,t\,\mathbf{i} + e\,t^2\,\mathbf{k}\}$. Determine the particle's position (x, y, z) at time t_1.

Given: $a = 3\ \text{ft}$ $\quad b = 2\ \text{ft}$ $\quad c = 5\ \text{ft}$ $\quad d = 6\ \dfrac{\text{ft}}{\text{s}^3}$ $\quad e = 12\ \dfrac{\text{ft}}{\text{s}^4}$ $\quad t_1 = 1\ \text{s}$

Solution:

$a_x = d\,t \qquad v_x = \left(\dfrac{d}{2}\right)t^2 \qquad s_x = \left(\dfrac{d}{6}\right)t^3 + a \qquad x = \left(\dfrac{d}{6}\right)t_1^{\,3} + a \qquad x = 4\,\text{ft}$

$a_y = 0 \qquad v_y = 0 \qquad s_y = b \qquad y = b \qquad y = 2\,\text{ft}$

$a_z = e\,t^2 \qquad v_z = \left(\dfrac{e}{3}\right)t^3 \qquad s_z = \left(\dfrac{e}{12}\right)t^4 + c \qquad z = \left(\dfrac{e}{12}\right)t_1^{\,4} + c \qquad z = 6\,\text{ft}$

Problem 12-67

The velocity of a particle is given by $\mathbf{v} = [at^2\mathbf{i} + bt^3\mathbf{j} + (ct + d)\mathbf{k}]$. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = t_1$. Also, what is the x, y, z coordinate position of the particle at this instant?

Given: $a = 16\ \dfrac{\text{m}}{\text{s}^3}$ $\quad b = 4\ \dfrac{\text{m}}{\text{s}^4}$ $\quad c = 5\ \dfrac{\text{m}}{\text{s}^2}$ $\quad d = 2\ \dfrac{\text{m}}{\text{s}}$ $\quad t_1 = 2\ \text{s}$

Solution:

Acceleration

$a_x = 2a\,t_1 \qquad a_x = 64\ \dfrac{\text{m}}{\text{s}^2}$

$a_y = 3b\,t_1^{\,2} \qquad a_y = 48\ \dfrac{\text{m}}{\text{s}^2}$

$a_z = c \qquad a_z = 5\ \dfrac{\text{m}}{\text{s}^2}$

$a_{mag} = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}} \qquad a_{mag} = 80.2\ \dfrac{\text{m}}{\text{s}^2}$

Postition

$x = \dfrac{a}{3}t_1^{\,3} \qquad x = 42.667\ \text{m}$

$y = \dfrac{b}{4}t_1^{\,4} \qquad y = 16\ \text{m}$

$z = \frac{c}{2} t_1^{\,2} + d\,t_1 \qquad z = 14\ \text{m}$

*****Problem 12-68**

A particle is traveling with a velocity of $\mathbf{v} = \left(a\sqrt{t}\,e^{b\,t}\,\mathbf{i} + c\,e^{d\,t^2}\,\mathbf{j}\right)$. Determine the magnitude of the particle's displacement from $t = 0$ to t_1. Use Simpson's rule with n steps to evaluate the integrals. What is the magnitude of the particle's acceleration when $t = t_2$?

Given: $a = 3\ \frac{\text{m}}{\text{s}^{\frac{3}{2}}} \quad b = -0.2\ \frac{1}{\text{s}} \quad c = 4\ \frac{\text{m}}{\text{s}} \quad d = -0.8\ \frac{1}{\text{s}^2} \quad t_1 = 3\ \text{s} \quad t_2 = 2\ \text{s}$

$n = 100$

Displacement

$x_1 = \int_0^{t_1} a\sqrt{t}\,e^{b\,t}\,dt \qquad x_1 = 7.34\ \text{m} \qquad y_1 = \int_0^{t_1} c\,e^{d\,t^2}\,dt \qquad y_1 = 3.96\ \text{m}$

$d_1 = \sqrt{x_1^{\,2} + y_1^{\,2}} \qquad d_1 = 8.34\ \text{m}$

Acceleration

$a_x = \frac{d}{dt}\left(a\sqrt{t}\,e^{b\,t}\right) = \frac{a}{2\sqrt{t}}e^{bt} + a\,b\sqrt{t}\,e^{b\,t} \qquad a_{x2} = \frac{a}{\sqrt{t_2}}e^{b\,t_2}\left(\frac{1}{2} + b\,t_2\right)$

$a_y = \frac{d}{dt}\left(c\,e^{d\,t^2}\right) = 2c\,d\,t\,e^{d\,t^2} \qquad a_{y2} = 2c\,d\,t_2\,e^{d\,t_2^{\,2}}$

$a_{x2} = 0.14\ \frac{\text{m}}{\text{s}^2} \qquad a_{y2} = -0.52\ \frac{\text{m}}{\text{s}^2} \qquad a_2 = \sqrt{a_{x2}^{\,2} + a_{y2}^{\,2}} \qquad a_2 = 0.541\ \frac{\text{m}}{\text{s}^2}$

Problem 12-69

The position of a particle is defined by $r = \{a\cos(bt)\,\mathbf{i} + c\sin(bt)\,\mathbf{j}\}$. Determine the magnitudes of the velocity and acceleration of the particle when $t = t_1$. Also, prove that the path of the particle is elliptical.

Given: $a = 5\ \text{m} \quad b = 2\ \frac{\text{rad}}{\text{s}} \quad c = 4\ \text{m} \quad t_1 = 1\ \text{s}$

Velocities

$v_{x1} = -a\,b\sin\left(b\,t_1\right) \qquad v_{y1} = c\,b\cos\left(b\,t_1\right) \qquad v_1 = \sqrt{v_{x1}^{\,2} + v_{y1}^{\,2}}$

$v_{x1} = -9.093\ \frac{m}{s}$ $v_{y1} = -3.329\ \frac{m}{s}$ $v_1 = 9.683\ \frac{m}{s}$

Accelerations

$a_{x1} = -a b^2 \cos(b t_1)$ $a_{y1} = -c b^2 \sin(b t_1)$ $a_1 = \sqrt{{a_{x1}}^2 + {a_{y1}}^2}$

$a_{x1} = 8.323\ \frac{m}{s^2}$ $a_{y1} = -14.549\ \frac{m}{s^2}$ $a_1 = 16.761\ \frac{m}{s^2}$

Path

$\frac{x}{a} = \cos(b t)$ $\frac{y}{c} = \sin(b t)$ Thus $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{c}\right)^2 = 1$ QED

Problem 12–70

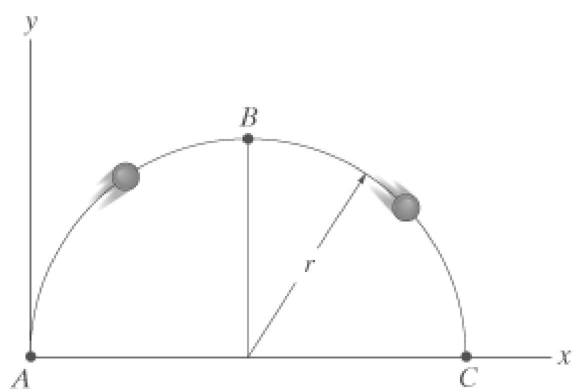
A particle travels along the curve from *A* to *B* in time t_1. If it takes time t_2 for it to go from *A* to *C*, determine its *average velocity* when it goes from *B* to *C*.

Given:

$t_1 = 1\ s$

$t_2 = 3\ s$

$r = 20\ m$



Solution:

$\mathbf{r_{AC}} = \begin{pmatrix} 2r \\ 0 \end{pmatrix}$

$\mathbf{r_{AB}} = \begin{pmatrix} r \\ r \end{pmatrix}$

$\mathbf{v_{ave}} = \frac{\mathbf{r_{AC}} - \mathbf{r_{AB}}}{t_2 - t_1}$ $\mathbf{v_{ave}} = \begin{pmatrix} 10 \\ -10 \end{pmatrix} \frac{m}{s}$

Problem 12-71

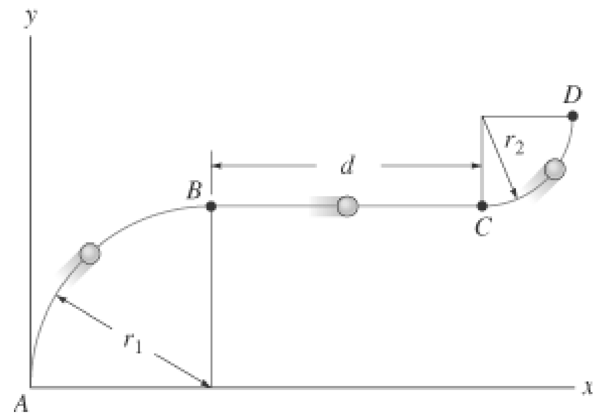
A particle travels along the curve from *A* to *B* in time t_1. It takes time t_2 for it to go from *B* to *C* and then time t_3 to go from *C* to *D*. Determine its average speed when it goes from *A* to *D*.

Given:

$t_1 = 2\ s$ $r_1 = 10\ m$

$t_2 = 4\ s$ $d = 15\ m$

$t_3 = 3\ \text{s} \quad r_2 = 5\ \text{m}$



Solution:

$$d = \left(\frac{\pi r_1}{2}\right) + d + \left(\frac{\pi r_2}{2}\right)$$

$$t = t_1 + t_2 + t_3 \qquad v_{ave} = \frac{d}{t}$$

$$v_{ave} = 4.285\ \frac{\text{m}}{\text{s}}$$

***Problem 12-72**

A car travels east a distance d_1 for time t_1, then north a distance d_2 for time t_2 and then west a distance d_3 for time t_3. Determine the total distance traveled and the magnitude of displacement of the car. Also, what is the magnitude of the average velocity and the average speed?

Given: $d_1 = 2\ \text{km} \quad d_2 = 3\ \text{km} \quad d_3 = 4\ \text{km}$

$t_1 = 5\ \text{min} \quad t_2 = 8\ \text{min} \quad t_3 = 10\ \text{min}$

Solution:

Total Distance Traveled and Displacement: The total distance traveled is

$$s = d_1 + d_2 + d_3 \qquad s = 9\,\text{km}$$

and the magnitude of the displacement is

$$\Delta r = \sqrt{\left(d_1 - d_3\right)^2 + d_2^{\,2}} \qquad \Delta r = 3.606\,\text{km}$$

Average Velocity and Speed: The total time is $\Delta t = t_1 + t_2 + t_3 \quad \Delta t = 1380\ \text{s}$

The magnitude of average velocity is

$$v_{avg} = \frac{\Delta r}{\Delta t} \qquad v_{avg} = 2.61\ \frac{\text{m}}{\text{s}}$$

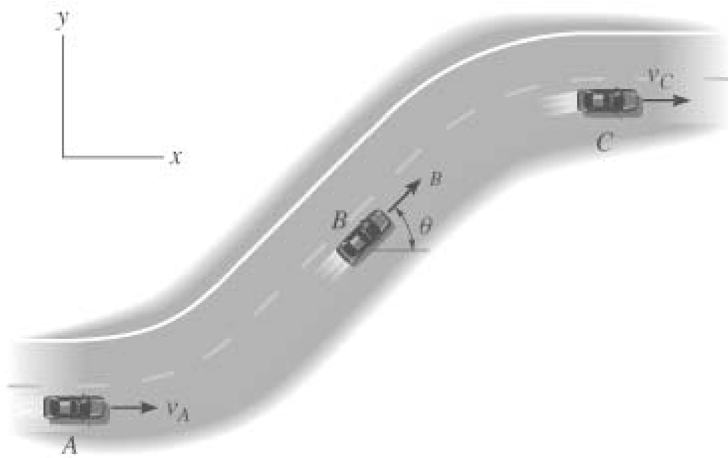
and the average speed is

$$v_{spavg} = \frac{s}{\Delta t} \qquad v_{spavg} = 6.522\ \frac{\text{m}}{\text{s}}$$

Problem 12-73

A car traveling along the straight portions of the road has the velocities indicated in the figure when it arrives at points *A*, *B*, and *C*. If it takes time t_{AB} to go from *A* to *B*, and then time t_{BC} to go from *B* to *C*, determine the average acceleration between points *A* and *B* and between points *A* and *C*.

Given:



$t_{AB} = 3\ \text{s}$

$t_{BC} = 5\ \text{s}$

$v_A = 20\ \frac{\text{m}}{\text{s}}$

$v_B = 30\ \frac{\text{m}}{\text{s}}$

$v_C = 40\ \frac{\text{m}}{\text{s}}$

$\theta = 45\ \text{deg}$

Solution:

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \qquad \mathbf{v_{Av}} = v_A\begin{pmatrix}1\\ 0\end{pmatrix} \qquad \mathbf{v_{Cv}} = v_C\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$\mathbf{a_{ABave}} = \frac{\mathbf{v_{Bv}} - \mathbf{v_{Av}}}{t_{AB}} \qquad \mathbf{a_{ABave}} = \begin{pmatrix}0.404\\ 7.071\end{pmatrix}\frac{\text{m}}{\text{s}^2}$$

$$\mathbf{a_{ACave}} = \frac{\mathbf{v_{Cv}} - \mathbf{v_{Av}}}{t_{AB} + t_{BC}} \qquad \mathbf{a_{ACave}} = \begin{pmatrix}2.5\\ 0\end{pmatrix}\frac{\text{m}}{\text{s}^2}$$

Problem 12-74

A particle moves along the curve $y = ae^{bx}$ such that its velocity has a constant magnitude of $v = v_0$. Determine the *x* and *y* components of velocity when the particle is at $y = y_1$.

Given: $a = 1\ \text{ft}$ $\quad b = \frac{2}{\text{ft}}$ $\quad v_0 = 4\ \frac{\text{ft}}{\text{s}}$ $\quad y_1 = 5\ \text{ft}$

In general we have

$$y = a e^{b x} \qquad v_y = a b e^{b x} v_x$$

$$v_x^2 + v_y^2 = v_x^2\left(1 + a^2 b^2 e^{2bx}\right) = v_0^2$$

$$v_x = \frac{v_0}{\sqrt{1 + a^2 b^2 e^{2bx}}} \qquad v_y = \frac{a b e^{bx} v_0}{\sqrt{1 + a^2 b^2 e^{2bx}}}$$

In specific case

$$x_1 = \frac{1}{b}\ln\left(\frac{y_1}{a}\right)$$

$$v_{x1} = \frac{v_0}{\sqrt{1 + a^2 b^2 e^{2bx_1}}} \qquad v_{x1} = 0.398\ \frac{\text{ft}}{\text{s}}$$

$$v_{y1} = \frac{a b e^{bx_1} v_0}{\sqrt{1 + a^2 b^2 e^{2bx_1}}} \qquad v_{y1} = 3.980\ \frac{\text{ft}}{\text{s}}$$

Problem 12-75

The path of a particle is defined by $y^2 = 4kx$, and the component of velocity along the y axis is $v_y = ct$, where both k and c are constants. Determine the x and y components of acceleration.

Solution:

$$y^2 = 4kx$$

$$2yv_y = 4kv_x$$

$$2v_y^2 + 2ya_y = 4ka_x$$

$$v_y = ct \qquad a_y = c$$

$$2(ct)^2 + 2yc = 4ka_x$$

$$a_x = \frac{c}{2k}\left(y + ct^2\right)$$

---

*__Problem 12-76__

A particle is moving along the curve $y = x - (x^2/a)$. If the velocity component in the x direction is $v_x = v_0$. and changes at the rate a_0, determine the magnitudes of the velocity and acceleration when $x = x_1$.

Given: $a = 400\ \text{ft}$ $v_0 = 2\ \frac{\text{ft}}{\text{s}}$ $a_0 = 0\ \frac{\text{ft}}{\text{s}^2}$ $x_1 = 20\ \text{ft}$

Solution:

Velocity: Taking the first derivative of the path $y = x - \left(\frac{x^2}{a}\right)$ we have,

$$v_y = v_x\left(1 - \frac{2x}{a}\right) = v_0\left(1 - \frac{2x}{a}\right)$$

$$v_{x1} = v_0 \qquad v_{y1} = v_0\left(1 - \frac{2x_1}{a}\right) \qquad v_1 = \sqrt{v_{x1}^2 + v_{y1}^2}$$

$$v_{x1} = 2\frac{\text{ft}}{\text{s}} \qquad v_{y1} = 1.8\frac{\text{ft}}{\text{s}} \qquad v_1 = 2.691\frac{\text{ft}}{\text{s}}$$

Acceleration: Taking the second derivative:

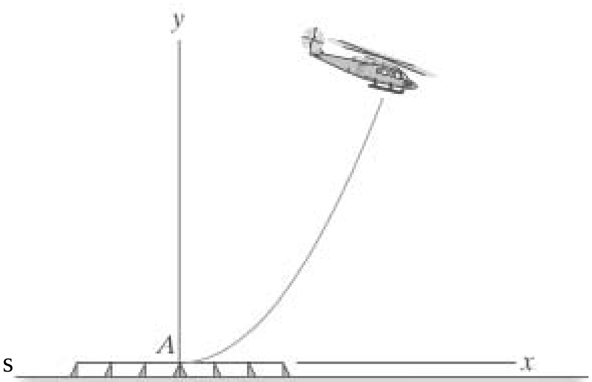
$$a_y = a_x\left(1 - \frac{2x}{a}\right) - 2\left(\frac{v_x^2}{a}\right) = a_0\left(1 - \frac{2x}{a}\right) - 2\left(\frac{v_0^2}{a}\right)$$

$$a_{x1} = a_0 \qquad a_{y1} = a_0\left(1 - \frac{2x_1}{a}\right) - 2\left(\frac{v_0^2}{a}\right) \qquad a_1 = \sqrt{a_{x1}^2 + a_{y1}^2}$$

$$a_{x1} = 0\frac{\text{ft}}{\text{s}^2} \qquad a_{y1} = -0.0200\frac{\text{ft}}{\text{s}^2} \qquad a_1 = 0.0200\frac{\text{ft}}{\text{s}^2}$$

Problem 12–77

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = bt^2$ and $y = ct^3$. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = t_1$.



Given:

$b = 2\ \frac{\text{m}}{\text{s}^2}$ $c = 0.04\ \frac{\text{m}}{\text{s}^3}$ $t_1 = 10\ \text{s}$

Solution:

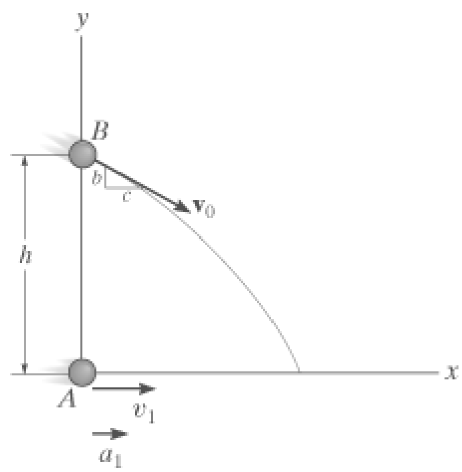
$$\mathbf{r_1} = \begin{pmatrix} b\,t_1^{\,2} \\ c\,t_1^{\,3} \end{pmatrix} \qquad \mathbf{v_1} = \begin{pmatrix} 2b\,t_1 \\ 3c\,t_1^{\,2} \end{pmatrix} \qquad \mathbf{a_1} = \begin{pmatrix} 2b \\ 6c\,t_1 \end{pmatrix}$$

$$\mathbf{r_1} = \begin{pmatrix} 200 \\ 40 \end{pmatrix}\,\text{m} \qquad \mathbf{v_1} = \begin{pmatrix} 40 \\ 12 \end{pmatrix}\frac{\text{m}}{\text{s}} \qquad \mathbf{a_1} = \begin{pmatrix} 4 \\ 2.4 \end{pmatrix}\frac{\text{m}}{\text{s}^2}$$

$$\left|\mathbf{r_1}\right| = 204\ \text{m} \qquad \left|\mathbf{v_1}\right| = 41.8\ \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{a_1}\right| = 4.66\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–78

At the instant shown particle *A* is traveling to the right at speed v_1 and has an acceleration a_1. Determine the initial speed v_0 of particle *B* so that when it is fired at the same instant from the angle shown it strikes *A*. Also, at what speed does it strike *A*?



Given:

$$v_1 = 10\ \frac{\text{ft}}{\text{s}} \qquad a_1 = 2\ \frac{\text{ft}}{\text{s}^2}$$

$$b = 3 \qquad c = 4$$

$$h = 100\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

Guesses $\quad v_0 = 1\ \frac{\text{ft}}{\text{s}} \qquad t = 1\ \text{s}$

Given $\quad v_1 t + \frac{1}{2}a_1 t^2 = \left(\frac{c}{\sqrt{b^2+c^2}}\right)v_0 t \qquad h - \frac{1}{2}g t^2 - \left(\frac{b}{\sqrt{b^2+c^2}}\right)v_0 t = 0$

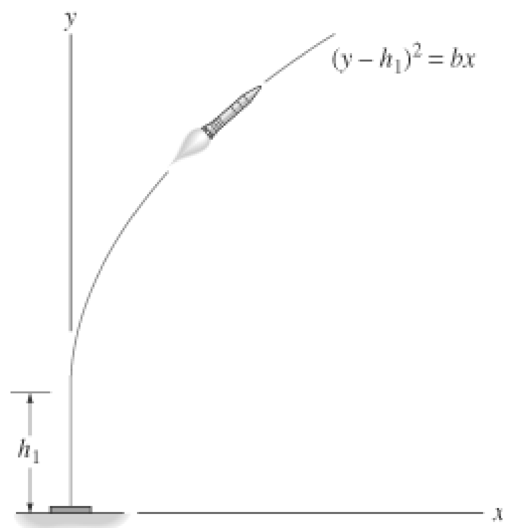
$$\begin{pmatrix} v_0 \\ t \end{pmatrix} = \text{Find}(v_0, t) \qquad t = 2.224\ \text{s} \qquad v_0 = 15.28\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_B} = \begin{pmatrix} \dfrac{c}{\sqrt{b^2+c^2}}v_0 \\ -g\,t - \dfrac{b}{\sqrt{b^2+c^2}}v_0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 12.224 \\ -80.772 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_B}\right| = 81.691\ \frac{\text{ft}}{\text{s}}$$

Problem 12-79

When a rocket reaches altitude h_1 it begins to travel along the parabolic path $(y - h_1)^2 = b\,x$. If the component of velocity in the vertical direction is constant at $v_y = v_0$, determine the magnitudes of the rocket's velocity and acceleration when it reaches altitude h_2.

Given:



$h_1 = 40\ \text{m}$

$b = 160\ \text{m}$

$v_0 = 180\ \frac{\text{m}}{\text{s}}$

$h_2 = 80\ \text{m}$

Solution:

$$b\,x = (y - h_1)^2$$

$$b\,v_x = 2(y - h_1)v_y$$

$$b\,a_x = 2v_y^{\,2}$$

$v_{x2} = \frac{2}{b}(h_2 - h_1)v_0$ $\quad v_{y2} = v_0$ $\quad v_2 = \sqrt{v_{x2}^{\,2} + v_{y2}^{\,2}}$ $\quad v_2 = 201.246\ \frac{\text{m}}{\text{s}}$

$a_{x2} = \frac{2}{b}v_0^{\,2}$ $\quad a_{y2} = 0\ \frac{\text{m}}{\text{s}^2}$ $\quad a_2 = \sqrt{a_{x2}^{\,2} + a_{y2}^{\,2}}$ $\quad a_2 = 405\ \frac{\text{m}}{\text{s}^2}$

***Problem 12–80**

Determine the minimum speed of the stunt rider, so that when he leaves the ramp at *A* he passes through the center of the hoop at *B*. Also, how far *h* should the landing ramp be from the hoop so that he lands on it safely at *C* ? Neglect the size of the motorcycle and rider.

Given:

$a = 4\ \text{ft}$

$b = 24\ \text{ft}$

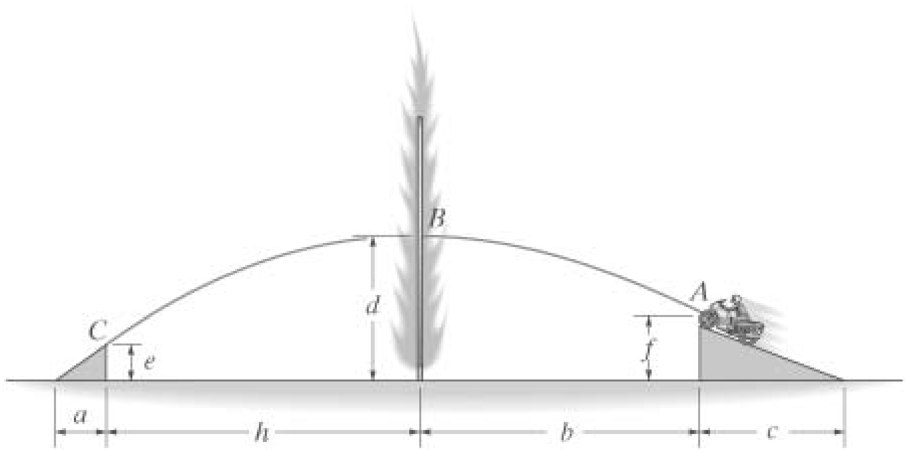
$c = 12\ \text{ft}$

$d = 12\ \text{ft}$

$e = 3\ \text{ft}$

$f = 5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution: $\theta = \operatorname{atan}\left(\dfrac{f}{c}\right)$

Guesses $v_A = 1\ \dfrac{\text{ft}}{\text{s}}$ $t_B = 1\ \text{s}$ $t_C = 1\ \text{s}$ $h = 1\ \text{ft}$

Given $b = v_A \cos(\theta) t_B$ $f + v_A \sin(\theta) t_B - \dfrac{1}{2} g t_B^2 = d$

$b + h = v_A \cos(\theta) t_C$ $f + v_A \sin(\theta) t_C - \dfrac{1}{2} g t_C^2 = e$

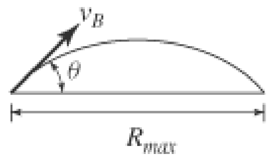
$\begin{pmatrix} t_B \\ t_C \\ v_A \\ h \end{pmatrix} = \text{Find}(t_B, t_C, v_A, h)$ $\begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.432 \\ 1.521 \end{pmatrix} \text{s}$ $v_A = 60.2\ \dfrac{\text{ft}}{\text{s}}$

$h = 60.5\ \text{ft}$

Problem 12–81

Show that if a projectile is fired at an angle θ from the horizontal with an initial velocity v_0, the *maximum* range the projectile can travel is given by $R_{max} = v_0^2/g$, where g is the acceleration of gravity. What is the angle θ for this condition?

Solution: After time t,



$$x = v_0 \cos(\theta) t \qquad t = \frac{x}{v_0 \cos(\theta)}$$

$$y = (v_0 \sin(\theta))t - \frac{1}{2} g t^2 \qquad y = x \tan(\theta) - \frac{g x^2}{2 v_0^2 \cos(\theta)^2}$$

Set $y = 0$ to determine the range, $x = R$:

$$R = \frac{2 v_0^2 \sin(\theta) \cos(\theta)}{g} = \frac{v_0^2 \sin(2\theta)}{g}$$

R_{max} occurs when $\sin(2\theta) = 1$ or, $\theta = 45\text{deg}$

This gives: $R_{max} = \dfrac{v_0^2}{g}$ Q.E.D

Problem 12-82

The balloon A is ascending at rate v_A and is being carried horizontally by the wind at v_w. If a ballast bag is dropped from the balloon when the balloon is at height h, determine the time needed for it to strike the ground. Assume that the bag was released from the balloon with the same velocity as the balloon. Also, with what speed does the bag strike the ground?

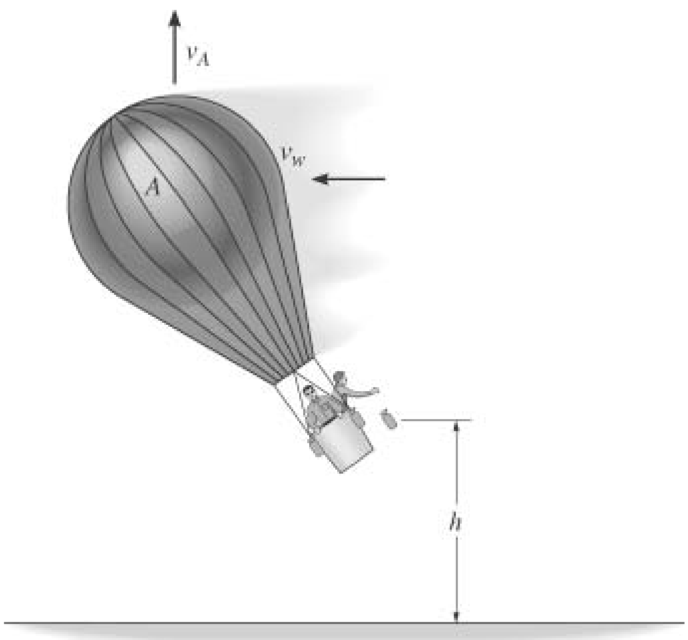
Given:

$$v_A = 12 \frac{\text{km}}{\text{hr}}$$

$$v_w = 20 \frac{\text{km}}{\text{hr}}$$

$$h = 50 \text{ m}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



Solution:

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_w \qquad v_y = -g t + v_A$$

$s_x = v_w t$ $\qquad s_y = \frac{-1}{2} g t^2 + v_A t + h$

Thus $\quad 0 = \frac{-1}{2} g t^2 + v_A t + h \qquad t = \frac{v_A + \sqrt{v_A^2 + 2gh}}{g}$ $\qquad t = 3.551 \text{ s}$

$v_x = v_w \qquad v_y = -g t + v_A \qquad v = \sqrt{v_x^2 + v_y^2}$ $\qquad v = 32.0 \frac{\text{m}}{\text{s}}$

Problem 12-83

Determine the height h on the wall to which the firefighter can project water from the hose, if the angle θ is as specified and the speed of the water at the nozzle is v_C.

Given:

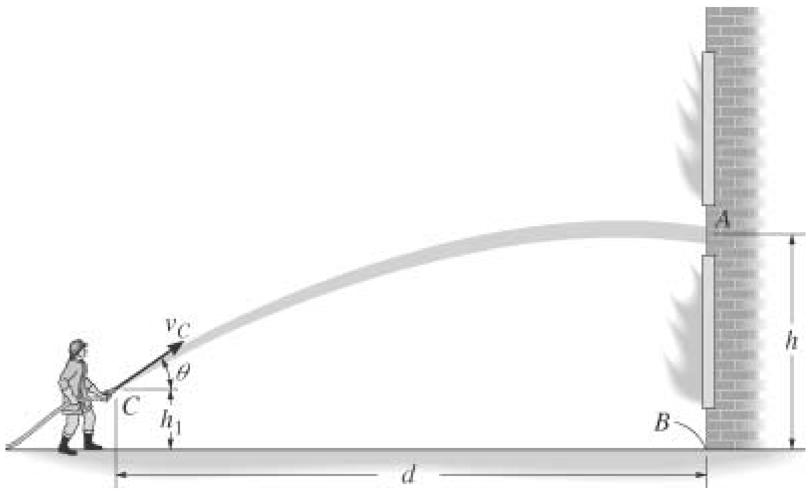
$v_C = 48 \frac{\text{ft}}{\text{s}}$

$h_1 = 3 \text{ ft}$

$d = 30 \text{ ft}$

$\theta = 40 \text{ deg}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$a_x = 0 \qquad a_y = -g$

$v_x = v_C \cos(\theta) \qquad v_y = -g t + v_C \sin(\theta)$

$s_x = v_C \cos(\theta) t \qquad s_y = \left(\frac{-g}{2}\right) t^2 + v_C \sin(\theta) t + h_1$

Guesses $\quad t = 1 \text{ s} \qquad h = 1 \text{ ft}$

Given $\quad d = v_C \cos(\theta) t \qquad h = \frac{-1}{2} g t^2 + v_C \sin(\theta) t + h_1$

$\begin{pmatrix} t \\ h \end{pmatrix} = \text{Find}(t, h) \qquad t = 0.816 \text{ s} \qquad h = 17.456 \text{ ft}$

***Problem 12-84**

Determine the smallest angle θ, measured above the horizontal, that the hose should be directed so that the water stream strikes the bottom of the wall at B. The speed of the water at the nozzle is v_C.

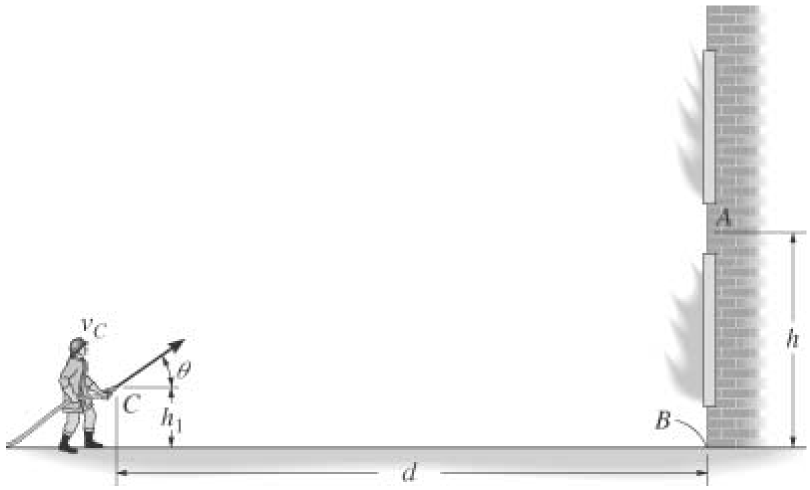
Given:

$v_C = 48\ \frac{ft}{s}$

$h_1 = 3\ ft$

$d = 30\ ft$

$g = 32.2\ \frac{ft}{s^2}$



Solution:

$a_x = 0$ $\qquad a_y = -g$

$v_x = v_C \cos(\theta)$ $\qquad v_y = -g\,t + v_C \sin(\theta)$

$s_x = v_C \cos(\theta) t$ $\qquad s_y = \frac{-g}{2} t^2 + v_C \sin(\theta) t + h_1$

When it reaches the wall $\qquad d = v_C \cos(\theta) t \qquad t = \frac{d}{v_C \cos(\theta)}$

$$0 = \frac{-g}{2}\left(\frac{d}{v_C \cos(\theta)}\right)^2 + v_C \sin(\theta) \frac{d}{v_C \cos(\theta)} + h_1 = \frac{d}{2\cos(\theta)^2}\left(\sin(2\theta) - \frac{d\,g}{v_C^2}\right) + h_1$$

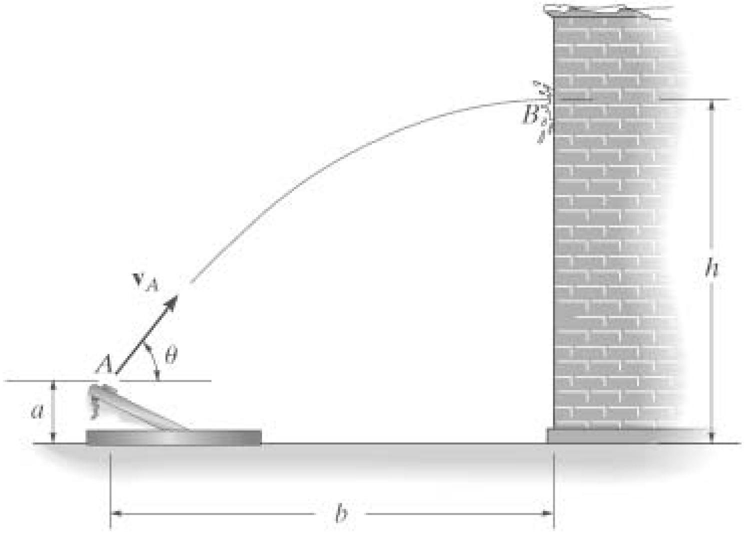
Guess $\qquad \theta = 10\ deg$

Given $\qquad 0 = \frac{d}{2\cos(\theta)^2}\left(\sin(2\theta) - \frac{d\,g}{v_C^2}\right) + h_1 \qquad \theta = \text{Find}(\theta) \qquad \theta = 6.406\ deg$

Problem 12–85

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes time t_1 to travel from A to B, determine the velocity v_A at which it was launched, the angle of release θ, and the height h.

Given:



$a = 3.5\ \text{ft}$

$b = 18\ \text{ft}$

$t_1 = 1.5\ \text{s}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v_A = 1\ \frac{\text{ft}}{\text{s}}$ $\theta = 1\ \text{deg}$ $h = 1\ \text{ft}$

Given $v_A\cos(\theta)t_1 = b$ $v_A\sin(\theta) - g\,t_1 = 0$

$$a + v_A\sin(\theta)t_1 - \frac{1}{2}g\,t_1^{\,2} = h$$

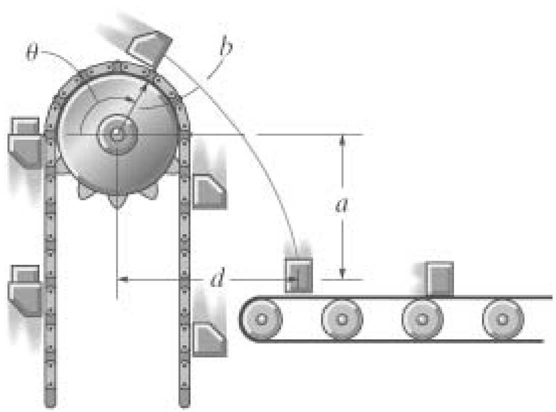
$$\begin{pmatrix} v_A \\ \theta \\ h \end{pmatrix} = \text{Find}(v_A, \theta, h)$$

$v_A = 49.8\,\frac{\text{ft}}{\text{s}}$ $\theta = 76\,\text{deg}$ $h = 39.7\,\text{ft}$

Problem 12–86

The buckets on the conveyor travel with a speed *v*. Each bucket contains a block which falls out of the bucket when $\theta = \theta_1$. Determine the distance *d* to where the block strikes the conveyor. Neglect the size of the block.

Given:



$a = 3\ \text{ft}$

$b = 1\ \text{ft}$

$\theta_1 = 120\ \text{deg}$

$v = 15\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $d = 1\ \text{ft}$ $t = 1\ \text{s}$

Given $-b\cos(\theta_1) + v\sin(\theta_1)t = d$

$$a + b\sin(\theta_1) + v\cos(\theta_1)t - \frac{1}{2}gt^2 = 0$$

$\begin{pmatrix} d \\ t \end{pmatrix} = \text{Find}(d, t)$ $t = 0.31\ \text{s}$ $d = 4.52\ \text{ft}$

Problem 12-87

Measurements of a shot recorded on a videotape during a basketball game are shown. The ball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.

Given:

$a = 7\ \text{ft}$

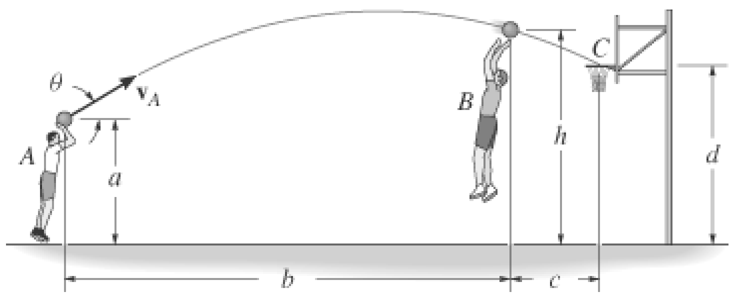
$b = 25\ \text{ft}$

$c = 5\ \text{ft}$

$d = 10\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $v_A = 10\ \frac{\text{ft}}{\text{s}}$ $t_B = 1\ \text{s}$ $t_C = 1\ \text{s}$ $h = 12\ \text{ft}$

Given $b + c = v_A\cos(\theta)t_C$ $b = v_A\cos(\theta)t_B$

$$d = \frac{-g}{2}t_C^2 + v_A\sin(\theta)t_C + a \qquad h = \frac{-g}{2}t_B^2 + v_A\sin(\theta)t_B + a$$

$\begin{pmatrix} v_A \\ t_B \\ t_C \\ h \end{pmatrix} = \text{Find}(v_A, t_B, t_C, h)$ $\begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.786 \\ 0.943 \end{pmatrix}\ \text{s}$ $v_A = 36.7\ \frac{\text{ft}}{\text{s}}$ $h = 11.489\ \text{ft}$

***Problem 12-88**

The snowmobile is traveling at speed v_0 when it leaves the embankment at A. Determine the time of flight from A to B and the range R of the trajectory.

Given:

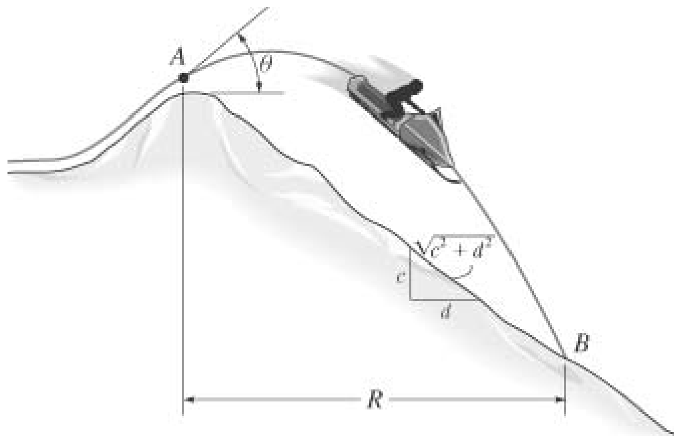
$v_0 = 10\ \frac{m}{s}$

$\theta = 40\ deg$

$c = 3$

$d = 4$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guesses $R = 1\ m$ $t = 1\ s$

Given $R = v_0 \cos(\theta) t$ $\left(\frac{-c}{d}\right) R = \left(\frac{-g}{2}\right) t^2 + v_0 \sin(\theta) t$

$\begin{pmatrix} R \\ t \end{pmatrix} = \text{Find}(R, t)$ $t = 2.482\ s$ $R = 19.012\ m$

Problem 12–89

The projectile is launched with a velocity v_0. Determine the range R, the maximum height h attained, and the time of flight. Express the results in terms of the angle θ and v_0. The acceleration due to gravity is g.

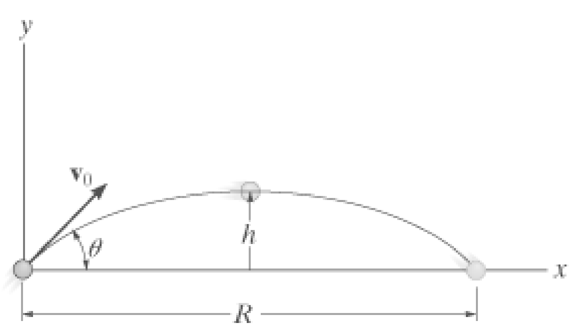
Solution:

$a_x = 0$ $a_y = -g$

$v_x = v_0 \cos(\theta)$ $v_y = -g t + v_0 \sin(\theta)$

$s_x = v_0 \cos(\theta) t$ $s_y = \frac{-1}{2} g t^2 + v_0 \sin(\theta) t$

$0 = \frac{-1}{2} g t^2 + v_0 \sin(\theta) t$ $t = \frac{2 v_0 \sin(\theta)}{g}$



$R = v_0 \cos(\theta) t$ $\qquad$ $R = \dfrac{2v_0^{\,2}}{g} \sin(\theta) \cos(\theta)$

$h = \dfrac{-1}{2} g \left(\dfrac{t}{2}\right)^2 + v_0 \sin(\theta) \dfrac{t}{2}$ $\qquad$ $h = \dfrac{v_0^{\,2} \sin(\theta)^2}{g}$

Problem 12–90

The fireman standing on the ladder directs the flow of water from his hose to the fire at B. Determine the velocity of the water at A if it is observed that the hose is held at angle θ.

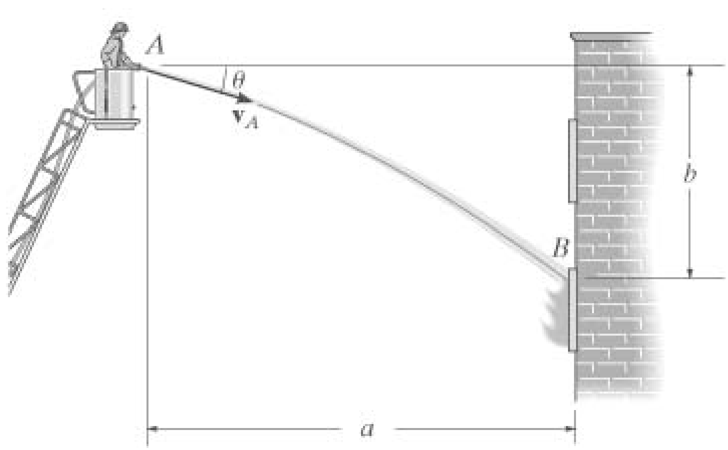
Given:

$\theta = 20 \text{ deg}$

$a = 60 \text{ ft}$

$b = 30 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad v_A = 1 \dfrac{\text{ft}}{\text{s}}$

$t = 1 \text{ s}$

Given $\quad v_A \cos(\theta) t = a \qquad \dfrac{-1}{2} g t^2 - v_A \sin(\theta) t = -b$

$\begin{pmatrix} v_A \\ t \end{pmatrix} = \text{Find}(v_A, t) \qquad t = 0.712 \text{ s} \qquad v_A = 89.7 \dfrac{\text{ft}}{\text{s}}$

Problem 12–91

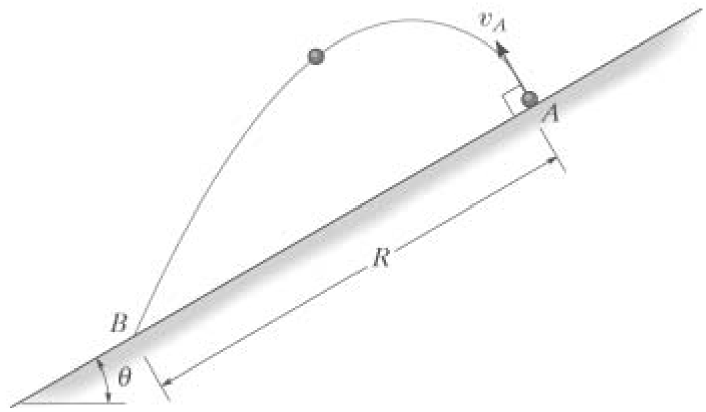
A ball bounces on the θ inclined plane such that it rebounds perpendicular to the incline with a velocity v_A. Determine the distance R to where it strikes the plane at B.

Given:

$\theta = 30 \text{ deg}$

$v_A = 40 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $t = 10 \text{ s}$

$R = 1 \text{ ft}$

Given $v_A \sin(\theta) t = R\cos(\theta)$ $\frac{-1}{2} g t^2 + v_A \cos(\theta) t = -R\sin(\theta)$

$\begin{pmatrix} t \\ R \end{pmatrix} = \text{Find}(t, R)$ $t = 2.87 \text{ s}$ $R = 66.3 \text{ ft}$

---

*__Problem 12-92__

The man stands a distance d from the wall and throws a ball at it with a speed v_0. Determine the angle θ at which he should release the ball so that it strikes the wall at the highest point possible. What is this height? The room has a ceiling height h_2.

Given:

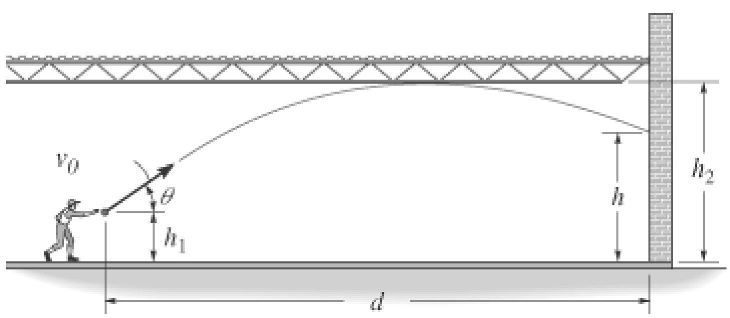
$d = 60 \text{ ft}$

$v_0 = 50 \frac{\text{ft}}{\text{s}}$

$h_1 = 5 \text{ ft}$

$h_2 = 20 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution: Guesses $t_1 = 1 \text{ s}$ $t_2 = 2 \text{ s}$ $\theta = 20 \text{ deg}$ $h = 10 \text{ ft}$

Given $d = v_0 \cos(\theta) t_2$ $h = \left(\frac{-g}{2}\right) t_2^2 + v_0 \sin(\theta) t_2 + h_1$

$$0 = -g t_1 + v_0 \sin(\theta) \qquad h_2 = \left(\frac{-g}{2}\right) t_1^{\,2} + v_0 \sin(\theta) t_1 + h_1$$

$$\begin{pmatrix} t_1 \\ t_2 \\ \theta \\ h \end{pmatrix} = \text{Find}(t_1, t_2, \theta, h) \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0.965 \\ 1.532 \end{pmatrix} \text{s} \qquad \theta = 38.434\ \text{deg} \qquad h = 14.83\ \text{ft}$$

Problem 12–93

The stones are thrown off the conveyor with a horizontal velocity v_0 as shown. Determine the distance d down the slope to where the stones hit the ground at B.

Given:

$$v_0 = 10\ \frac{\text{ft}}{\text{s}} \qquad h = 100\ \text{ft}$$

$$c = 1$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad d = 10$$

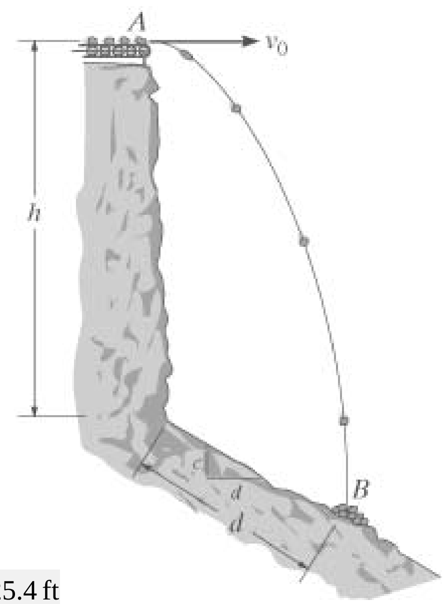
Solution:

$$\theta = \text{atan}\left(\frac{c}{d}\right)$$

Guesses $\quad t = 1\ \text{s} \quad d = 1\ \text{ft}$

Given $\quad v_0 t = d \cos(\theta)$

$$\frac{-1}{2} g t^2 = -h - d \sin(\theta)$$



$$\begin{pmatrix} t \\ d \end{pmatrix} = \text{Find}(t, d) \qquad t = 2.523\ \text{s} \qquad d = 25.4\ \text{ft}$$

Problem 12–94

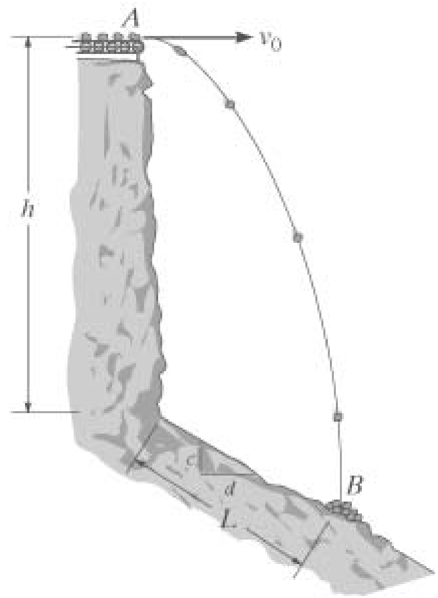
The stones are thrown off the conveyor with a horizontal velocity $v = v_0$ as shown. Determine the speed at which the stones hit the ground at B.

Given:

$$v_0 = 10\ \frac{\text{ft}}{\text{s}} \qquad h = 100\ \text{ft}$$

$g = 32.2\ \frac{ft}{s^2}$ $c = 1$

$d = 10$



Solution:

$\theta = \operatorname{atan}\left(\frac{c}{d}\right)$

Guesses $t = 1\ s$ $L = 1\ ft$

Given $v_0 t = L\cos(\theta)$

$\frac{-1}{2} g t^2 = -h - L\sin(\theta)$

$\begin{pmatrix} t \\ L \end{pmatrix} = \operatorname{Find}(t, L)$ $t = 2.523\ s$ $L = 25.4\,ft$

$\mathbf{v_B} = \begin{pmatrix} v_0 \\ -g t \end{pmatrix}$ $\mathbf{v_B} = \begin{pmatrix} 10 \\ -81.256 \end{pmatrix} \frac{ft}{s}$ $|\mathbf{v_B}| = 81.9\,\frac{ft}{s}$

Problem 12–95

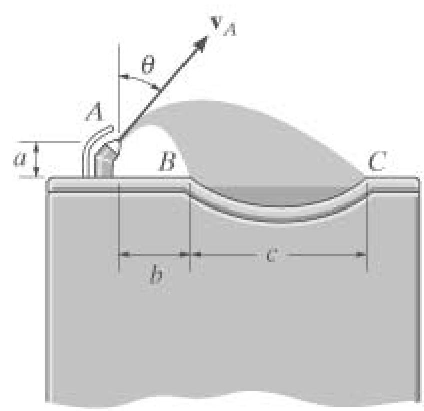
The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which water can be ejected from the nozzle so that it does not splash over the sides of the basin at *B* and *C*.

Given:

$\theta = 40\ deg$ $a = 50\ mm$

$g = 9.81\ \frac{m}{s^2}$ $b = 100\ mm$

$c = 250mm$



Solution:

Guesses $v_{min} = 1\ \frac{m}{s}$ $t_{min} = 1\ s$

$v_{max} = 1\ \frac{m}{s}$ $t_{max} = 1\ s$

Given $b = v_{min}\sin(\theta)t_{min}$ $a + v_{min}\cos(\theta)t_{min} - \frac{1}{2} g\, t_{min}^{\ 2} = 0$

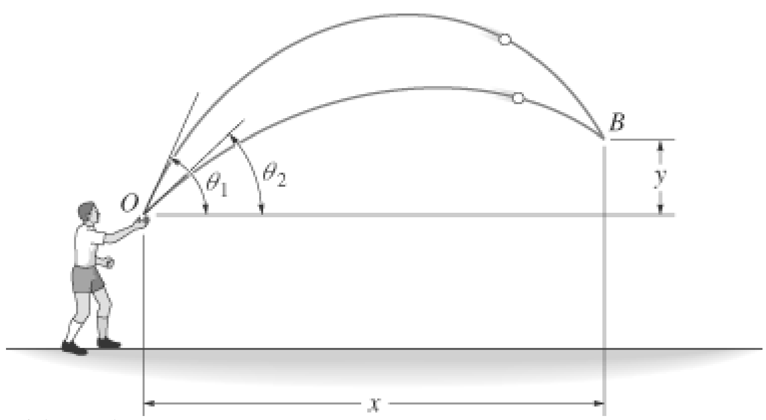
$b + c = v_{max}\sin(\theta)t_{max}$ $a + v_{max}\cos(\theta)t_{max} - \frac{1}{2} g\, t_{max}^{\ 2} = 0$

$$\begin{pmatrix} t_{min} \\ t_{max} \\ v_{min} \\ v_{max} \end{pmatrix} = \text{Find}(t_{min}, t_{max}, v_{min}, v_{max}) \qquad \begin{pmatrix} t_{min} \\ t_{max} \end{pmatrix} = \begin{pmatrix} 0.186 \\ 0.309 \end{pmatrix} \text{s}$$

$$\begin{pmatrix} v_{min} \\ v_{max} \end{pmatrix} = \begin{pmatrix} 0.838 \\ 1.764 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

****Problem 12-96**

A boy at *O* throws a ball in the air with a speed v_0 at an angle θ_1. If he then throws another ball at the same speed v_0 at an angle $\theta_2 < \theta_1$, determine the time between the throws so the balls collide in mid air at *B*.



Solution:

$$x = v_0 \cos(\theta_1)t = v_0 \cos(\theta_2)(t - \Delta t)$$

$$y = \left(\frac{-g}{2}\right)t^2 + v_0 \sin(\theta_1)t = \left(\frac{-g}{2}\right)(t - \Delta t)^2 + v_0 \sin(\theta_2)(t - \Delta t)$$

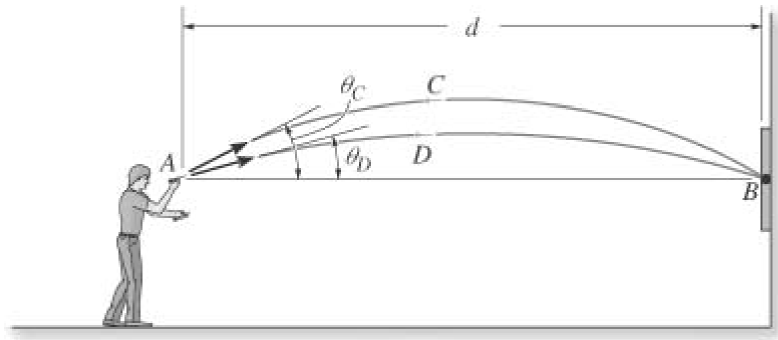
Eliminating time between these 2 equations we have

$$\Delta t = \frac{2v_0}{g}\left(\frac{\sin(\theta_1 - \theta_2)}{\cos(\theta_1) + \cos(\theta_2)}\right)$$

Problem 12-97

The man at *A* wishes to throw two darts at the target at *B* so that they arrive at the *same time.* If each dart is thrown with speed v_0, determine the angles θ_C and θ_D at which they should be thrown and the time between each throw. Note that the first dart must be thrown at $\theta_C > \theta_D$ then the second dart is thrown at θ_D.

Given:



$v_0 = 10\ \dfrac{\text{m}}{\text{s}}$

$d = 5\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Guesses $\theta_C = 70\ \text{deg}$ $\theta_D = 15\ \text{deg}$ $\Delta t = 2\ \text{s}$ $t = 1\ \text{s}$

Given $d = v_0 \cos(\theta_C) t$ $0 = \dfrac{-g}{2} t^2 + v_0 \sin(\theta_C) t$

$d = v_0 \cos(\theta_D)(t - \Delta t)$ $0 = \dfrac{-g}{2}(t - \Delta t)^2 + v_0 \sin(\theta_D)(t - \Delta t)$

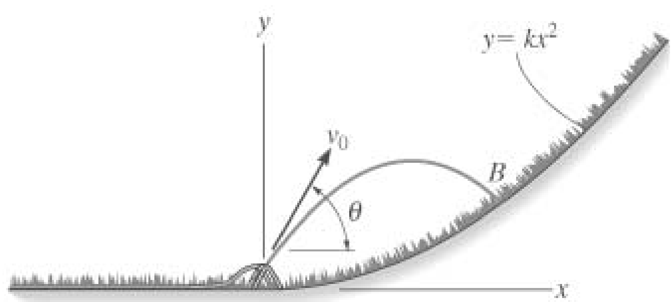
$\begin{pmatrix} \theta_C \\ \theta_D \\ t \\ \Delta t \end{pmatrix} = \text{Find}(\theta_C, \theta_D, t, \Delta t)$ $t = 1.972\ \text{s}$ $\Delta t = 1.455\ \text{s}$ $\begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix} = \begin{pmatrix} 75.313 \\ 14.687 \end{pmatrix} \text{deg}$

Problem 12–98

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity v_0 as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill.

Assume that the hill is defined by the equation $y = kx^2$ and neglect the size of the sprinkler.

Given:



$v_0 = 15\ \dfrac{\text{ft}}{\text{s}}$ $k = \dfrac{0.05}{\text{ft}}$

$\theta = 60\ \text{deg}$

Solution:

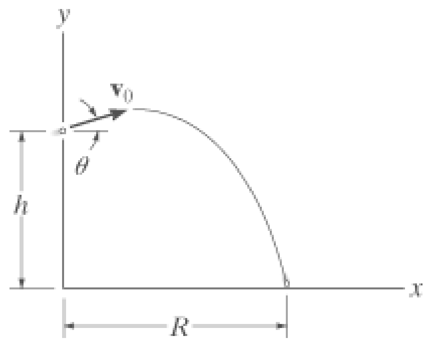
Guesses $x = 1\ \text{ft}$ $y = 1\ \text{ft}$ $t = 1\ \text{s}$

Given $x = v_0 \cos(\theta) t$ $y = v_0 \sin(\theta) t - \dfrac{1}{2} g t^2$ $y = k x^2$

$\begin{pmatrix} x \\ y \\ t \end{pmatrix} = \text{Find}(x, y, t)$ $t = 0.687\ \text{s}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.154 \\ 1.328 \end{pmatrix} \text{ft}$

Problem 12–99

The projectile is launched from a height h with a velocity $\mathbf{v}_0$. Determine the range R.



Solution:

$a_x = 0 \qquad a_y = -g$

$v_x = v_0 \cos(\theta) \qquad v_y = -g\,t + v_0 \sin(\theta)$

$s_x = v_0 \cos(\theta) t \qquad s_y = \frac{-1}{2} g\,t^2 + v_0 \sin(\theta) t + h$

When it hits

$R = v_0 \cos(\theta) t \qquad t = \frac{R}{v_0 \cos(\theta)}$

$$0 = \frac{-1}{2} g\,t^2 + v_0 \sin(\theta) t + h = \frac{-g}{2}\left(\frac{R}{v_0 \cos(\theta)}\right)^2 + v_0 \sin(\theta) \frac{R}{v_0 \cos(\theta)} + h$$

Solving for R we find

$$R = \frac{v_0^2 \cos(\theta)^2}{g}\left(\tan(\theta) + \sqrt{\tan(\theta)^2 + \frac{2g\,h}{v_0^2 \cos(\theta)^2}}\right)$$

---

*__Problem 12-100__

A car is traveling along a circular curve that has radius ρ. If its speed is v and the speed is increasing uniformly at rate a_t, determine the magnitude of its acceleration at this instant.

Given: $\rho = 50\ \text{m} \qquad v = 16\ \frac{\text{m}}{\text{s}} \qquad a_t = 8\ \frac{\text{m}}{\text{s}^2}$

Solution:

$a_n = \frac{v^2}{\rho} \qquad a_n = 5.12\ \frac{\text{m}}{\text{s}^2} \qquad a = \sqrt{a_n^2 + a_t^2} \qquad a = 9.498\ \frac{\text{m}}{\text{s}^2}$

Problem 12-101

A car moves along a circular track of radius ρ such that its speed for a short period of time $0 \le t \le t_2$, is $v = b\,t + c\,t^2$. Determine the magnitude of its acceleration when $t = t_1$. How far has it traveled at time t_1?

Given: $\rho = 250\ \text{ft}$ $t_2 = 4\ \text{s}$ $b = 3\ \frac{\text{ft}}{\text{s}^2}$ $c = 3\ \frac{\text{ft}}{\text{s}^3}$ $t_1 = 3\ \text{s}$

Solution: $v = bt + ct^2$ $a_t = b + 2ct$

At t_1 $v_1 = bt_1 + ct_1^2$ $a_{t1} = b + 2ct_1$ $a_{n1} = \frac{v_1^2}{\rho}$

$a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $a_1 = 21.63\ \frac{\text{ft}}{\text{s}^2}$

Distance traveled $d_1 = \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3$ $d_1 = 40.5\ \text{ft}$

Problem 12-102

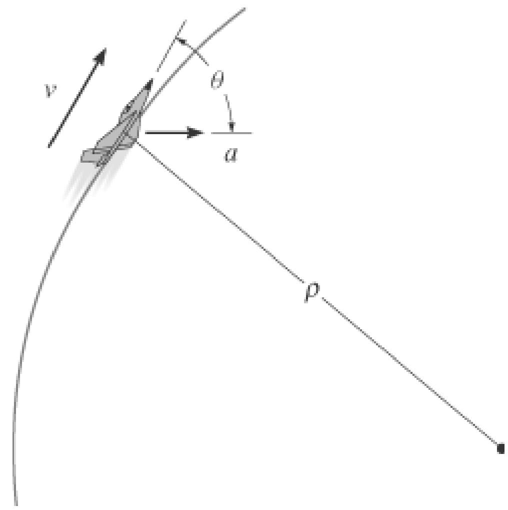
At a given instant the jet plane has speed v and acceleration a acting in the directions shown. Determine the rate of increase in the plane's speed and the radius of curvature ρ of the path.

Given:

$v = 400\ \frac{\text{ft}}{\text{s}}$

$a = 70\ \frac{\text{ft}}{\text{s}^2}$

$\theta = 60\ \text{deg}$



Solution:

Rate of increase

$a_t = (a)\cos(\theta)$ $a_t = 35\ \frac{\text{ft}}{\text{s}^2}$

Radius of curvature

$a_n = (a)\sin(\theta) = \frac{v^2}{\rho}$ $\rho = \frac{v^2}{(a)\sin(\theta)}$ $\rho = 2639\ \text{ft}$

Problem 12–103

A particle is moving along a curved path at a constant speed v. The radii of curvature of the path at points P and P' are ρ and ρ', respectively. If it takes the particle time t to go from P to P', determine the acceleration of the particle at P and P'.

Given: $v = 60\ \frac{\text{ft}}{\text{s}}$ $\rho = 20\ \text{ft}$ $\rho' = 50\ \text{ft}$ $t = 20\ \text{s}$

Solution: $a = \frac{v^2}{\rho}$ $a = 180\ \frac{\text{ft}}{\text{s}^2}$

$a' = \frac{v^2}{\rho'}$ $a' = 72\ \frac{\text{ft}}{\text{s}^2}$

Note that the time doesn't matter here because the speed is constant.

***Problem 12-104**

A boat is traveling along a circular path having radius ρ. Determine the magnitude of the boat's acceleration when the speed is v and the rate of increase in the speed is a_t.

Given: $\rho = 20\ \text{m}$ $v = 5\ \frac{\text{m}}{\text{s}}$ $a_t = 2\ \frac{\text{m}}{\text{s}^2}$

Solution:

$a_n = \frac{v^2}{\rho}$ $a_n = 1.25\ \frac{\text{m}}{\text{s}^2}$ $a = \sqrt{a_t^2 + a_n^2}$ $a = 2.358\ \frac{\text{m}}{\text{s}^2}$

Problem 12-105

Starting from rest, a bicyclist travels around a horizontal circular path of radius ρ at a speed $v = b\, t^2 + c\, t$. Determine the magnitudes of his velocity and acceleration when he has traveled a distance s_1.

Given: $\rho = 10\ \text{m}$ $b = 0.09\ \frac{\text{m}}{\text{s}^3}$ $c = 0.1\ \frac{\text{m}}{\text{s}^2}$ $s_1 = 3\ \text{m}$

Solution: Guess $t_1 = 1\ \text{s}$

Given $s_1 = \left(\frac{b}{3}\right) t_1^3 + \left(\frac{c}{2}\right) t_1^2$ $t_1 = \text{Find}(t_1)$ $t_1 = 4.147\ \text{s}$

$v_1 = b t_1^2 + c t_1$ $v_1 = 1.963\ \frac{\text{m}}{\text{s}}$

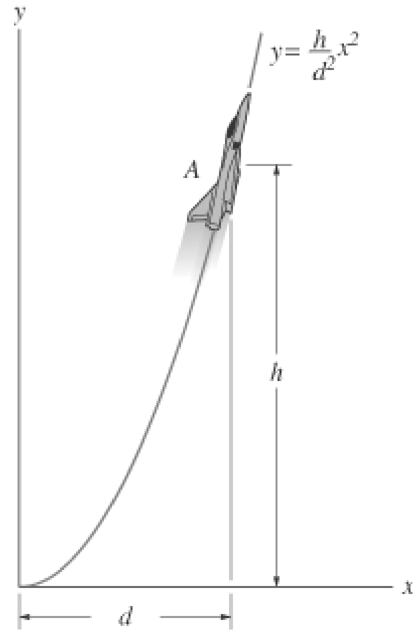
$a_{t1} = 2b t_1 + c$ $a_{t1} = 0.847\ \frac{\text{m}}{\text{s}^2}$

$a_{n1} = \frac{v_1^2}{\rho}$ $a_{n1} = 0.385\ \frac{\text{m}}{\text{s}^2}$

$a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $a_1 = 0.93\ \frac{\text{m}}{\text{s}^2}$

Problem 12-106

The jet plane travels along the vertical parabolic path. When it is at point A it has speed v which is increasing at the rate a_t. Determine the magnitude of acceleration of the plane when it is at point A.



Given:

$$v = 200\ \frac{\text{m}}{\text{s}}$$

$$a_t = 0.8\ \frac{\text{m}}{\text{s}^2}$$

$$d = 5\ \text{km}$$

$$h = 10\ \text{km}$$

Solution:

$$y(x) = h\left(\frac{x}{d}\right)^2$$

$$y'(x) = \frac{d}{dx}y(x)$$

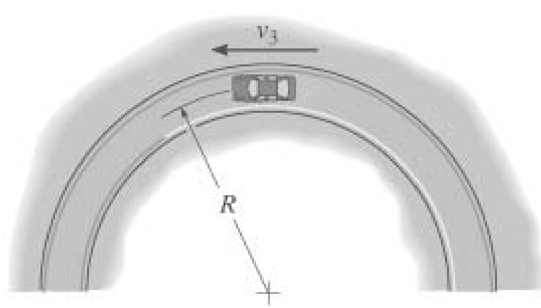
$$y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$a_n = \frac{v^2}{\rho(d)} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 0.921\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–107

The car travels along the curve having a radius of R. If its speed is uniformly increased from v_1 to v_2 in time t, determine the magnitude of its acceleration at the instant its speed is v_3.



Given:

$$v_1 = 15\ \frac{\text{m}}{\text{s}} \qquad t = 3\ \text{s}$$

$v_2 = 27\ \frac{m}{s}$ $\quad R = 300\ m$

$v_3 = 20\ \frac{m}{s}$

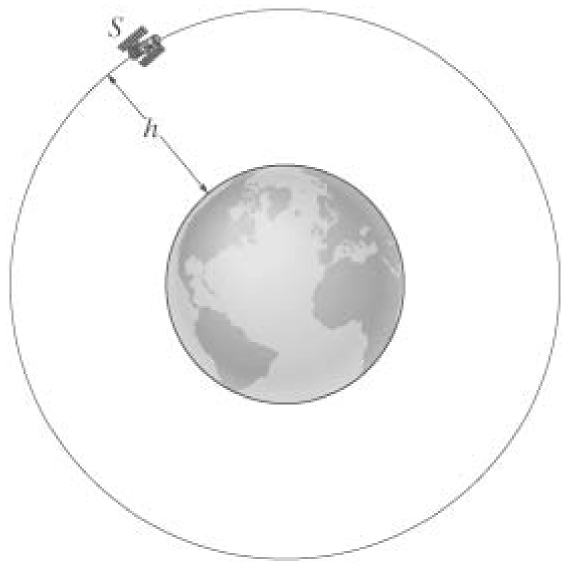
Solution:

$a_t = \frac{v_2 - v_1}{t}$ $\quad a_n = \frac{v_3^2}{R}$ $\quad a = \sqrt{a_t^2 + a_n^2}$ $\quad a = 4.22\ \frac{m}{s^2}$

***Problem 12–108**

The satellite S travels around the earth in a circular path with a constant speed v_1. If the acceleration is a, determine the altitude h. Assume the earth's diameter to be d.

Units Used: $\quad Mm = 10^3\ km$



Given:

$v_1 = 20\ \frac{Mm}{hr}$

$a = 2.5\ \frac{m}{s^2}$

$d = 12713\ km$

Solution:

Guess $\quad h = 1\ Mm$

Given $\quad a = \frac{v_1^2}{h + \frac{d}{2}}$ $\quad h = \text{Find}(h)$ $\quad h = 5.99\ Mm$

Problem 12–109

A particle P moves along the curve $y = b\,x^2 + c$ with a constant speed v. Determine the point on the curve where the maximum magnitude of acceleration occurs and compute its value.

Given: $\quad b = 1\ \frac{1}{m}$ $\quad c = -4\ m$ $\quad v = 5\ \frac{m}{s}$

Solution: Maximum acceleration occurs where the radius of curvature is the smallest which occurs at $x = 0$.

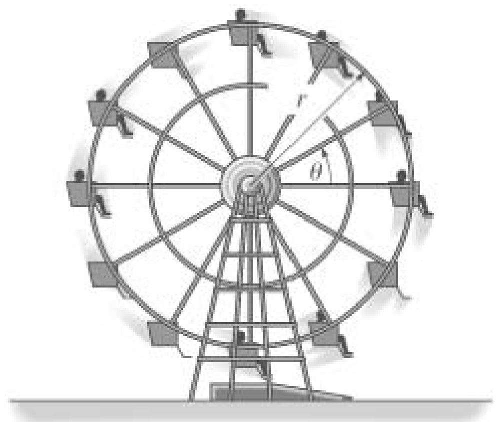
$$y(x) = bx^2 + c \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)} \qquad \rho_{min} = \rho(0\text{m}) \qquad \rho_{min} = 0.5\ \text{m}$$

$$a_{max} = \frac{v^2}{\rho_{min}} \qquad a_{max} = 50\ \frac{\text{m}}{\text{s}^2}$$

Problem 12–110

The Ferris wheel turns such that the speed of the passengers is increased by $a_t = bt$. If the wheel starts from rest when $\theta = 0°$, determine the magnitudes of the velocity and acceleration of the passengers when the wheel turns $\theta = \theta_1$.



Given:

$$b = 4\ \frac{\text{ft}}{\text{s}^3} \qquad \theta_1 = 30\ \text{deg} \qquad r = 40\ \text{ft}$$

Solution:

Guesses $t_1 = 1\ \text{s}$ $\quad v_1 = 1\ \frac{\text{ft}}{\text{s}}$ $\quad a_{t1} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $a_{t1} = b\,t_1$ $\quad v_1 = \left(\frac{b}{2}\right)t_1^2$ $\quad r\theta_1 = \left(\frac{b}{6}\right)t_1^3$

$$\begin{pmatrix} a_{t1} \\ v_1 \\ t_1 \end{pmatrix} = \text{Find}\left(a_{t1}, v_1, t_1\right) \qquad t_1 = 3.16\ \text{s} \qquad v_1 = 19.91\ \frac{\text{ft}}{\text{s}} \qquad a_{t1} = 12.62\ \frac{\text{ft}}{\text{s}^2}$$

$$a_1 = \sqrt{a_{t1}^2 + \left(\frac{v_1^2}{r}\right)^2} \qquad v_1 = 19.91\ \frac{\text{ft}}{\text{s}} \qquad a_1 = 16.05\ \frac{\text{ft}}{\text{s}^2}$$

Problem 12-111

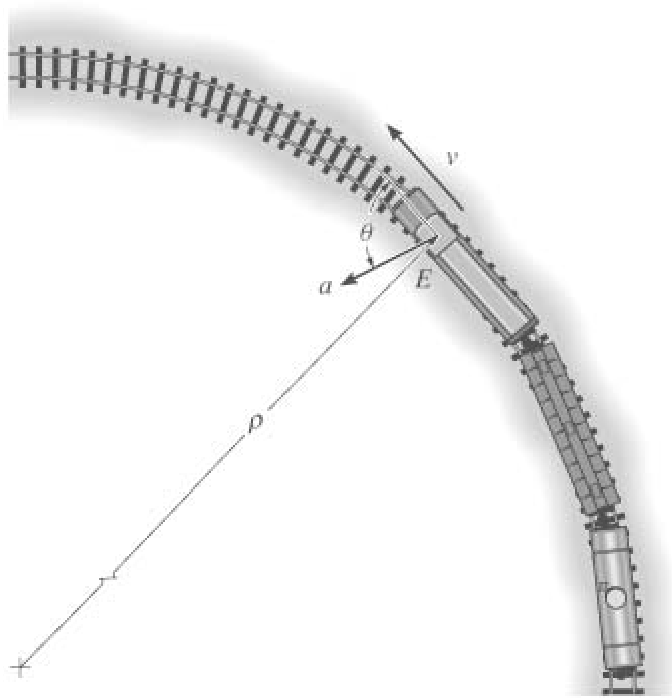
At a given instant the train engine at E has speed v and acceleration a acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature ρ of the path.

Given:

$v = 20 \frac{m}{s}$

$a = 14 \frac{m}{s^2}$

$\theta = 75 \text{ deg}$



Solution:

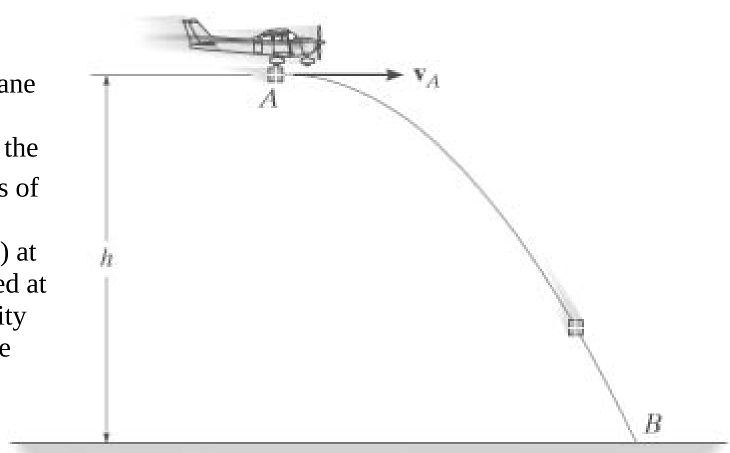
$a_t = (a)\cos(\theta)$ $\quad a_t = 3.62 \frac{m}{s^2}$

$a_n = (a)\sin(\theta)$ $\quad a_n = 13.523 \frac{m}{s^2}$

$\rho = \frac{v^2}{a_n}$ $\quad \rho = 29.579 \text{ m}$

***Problem 12–112**

A package is dropped from the plane which is flying with a constant horizontal velocity v_A. Determine the normal and tangential components of acceleration and the radius of curvature of the path of motion (a) at the moment the package is released at A, where it has a horizontal velocity v_A, and (b) *just before* it strikes the ground at B.



Given: $\quad v_A = 150 \frac{ft}{s} \quad h = 1500 \text{ ft} \quad g = 32.2 \frac{ft}{s^2}$

Solution:

At A:

$a_{An} = g \quad \rho_A = \frac{v_A^2}{a_{An}} \quad \rho_A = 699 \text{ ft}$

At *B*:

$$t = \sqrt{\frac{2h}{g}} \qquad v_x = v_A \qquad v_y = g\,t \qquad \theta = \operatorname{atan}\left(\frac{v_y}{v_x}\right)$$

$$v_B = \sqrt{v_x^{\,2} + v_y^{\,2}} \qquad a_{Bn} = g\cos(\theta) \qquad \rho_B = \frac{v_B^{\,2}}{a_{Bn}} \qquad \rho_B = 8510\,\text{ft}$$

Problem 12-113

The automobile is originally at rest at $s = 0$. If its speed is increased by $dv/dt = bt^2$, determine the magnitudes of its velocity and acceleration when $t = t_1$.

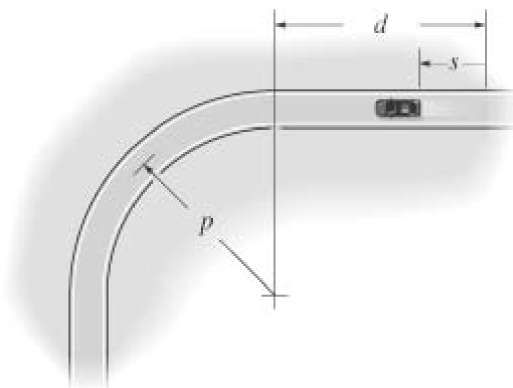
Given:

$$b = 0.05\,\frac{\text{ft}}{\text{s}^4}$$

$$t_1 = 18\text{ s}$$

$$\rho = 240\text{ ft}$$

$$d = 300\text{ ft}$$



Solution:

$$a_{t1} = b\,t_1^{\,2} \qquad a_{t1} = 16.2\,\frac{\text{ft}}{\text{s}^2}$$

$$v_1 = \left(\frac{b}{3}\right)t_1^{\,3} \qquad v_1 = 97.2\,\frac{\text{ft}}{\text{s}}$$

$$s_1 = \left(\frac{b}{12}\right)t_1^{\,4} \qquad s_1 = 437.4\,\text{ft}$$

If $s_1 = 437.4\,\text{ft} > d = 300\,\text{ft}$ then we are on the curved part of the track.

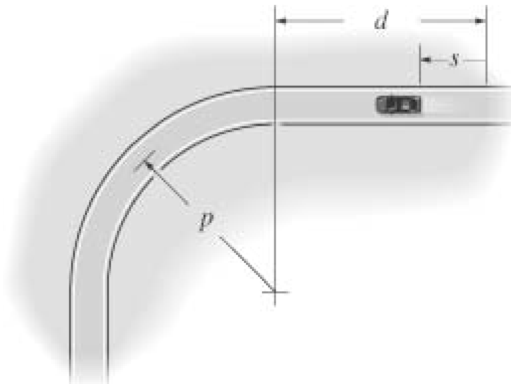
$$a_{n1} = \frac{v_1^{\,2}}{\rho} \qquad a_{n1} = 39.366\,\frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_{n1}^{\,2} + a_{t1}^{\,2}} \qquad a = 42.569\,\frac{\text{ft}}{\text{s}^2}$$

If $s_1 = 437.4\,\text{ft} < d = 300\,\text{ft}$ then we are on the straight part of the track.

$$a_{n1} = 0\,\frac{\text{ft}}{\text{s}^2} \qquad a_{n1} = 0\,\frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_{n1}^{\,2} + a_{t1}^{\,2}} \qquad a = 16.2\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-114

The automobile is originally at rest at $s = 0$. If it then starts to increase its speed at $dv/dt = bt^2$, determine the magnitudes of its velocity and acceleration at $s = s_1$.



Given:

$d = 300 \text{ ft}$

$\rho = 240 \text{ ft}$

$b = 0.05 \frac{\text{ft}}{\text{s}^4}$

$s_1 = 550 \text{ ft}$

Solution:

$a_t = bt^2 \qquad v = \left(\frac{b}{3}\right)t^3 \qquad s = \left(\frac{b}{12}\right)t^4 \qquad t_1 = \left(\frac{12s_1}{b}\right)^{\frac{1}{4}} \qquad t_1 = 19.061 \text{ s}$

$v_1 = \left(\frac{b}{3}\right)t_1^3 \qquad v_1 = 115.4 \frac{\text{ft}}{\text{s}}$

If $s_1 = 550 \text{ ft} > d = 300 \text{ ft}$ the car is on the curved path

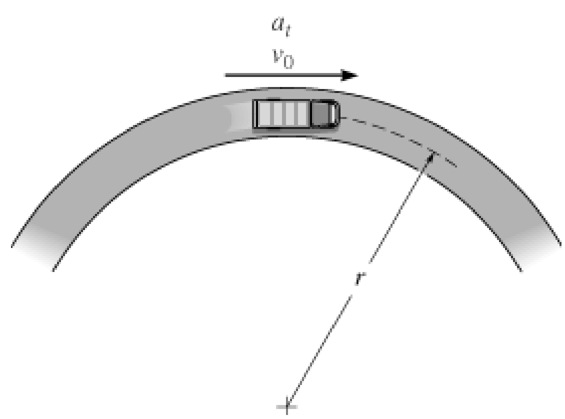
$a_t = bt_1^2 \qquad v = \left(\frac{b}{3}\right)t_1^3 \qquad a_n = \frac{v^2}{\rho} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 58.404 \frac{\text{ft}}{\text{s}^2}$

If $s_1 = 550 \text{ ft} < d = 300 \text{ ft}$ the car is on the straight path

$a_t = bt_1^2 \qquad a_n = 0 \frac{\text{ft}}{\text{s}^2} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 18.166 \frac{\text{ft}}{\text{s}^2}$

Problem 12-115

The truck travels in a circular path having a radius ρ at a speed v_0. For a short distance from $s = 0$, its speed is increased by $a_t = bs$. Determine its speed and the magnitude of its acceleration when it has moved a distance $s = s_1$.



Given:

$\rho = 50 \text{ m} \qquad s_1 = 10 \text{ m}$

$v_0 = 4 \frac{\text{m}}{\text{s}} \qquad b = 0.05 \frac{1}{\text{s}^2}$

Solution:

$$a_t = b\,s \qquad \int_{v_0}^{v_1} v\,dv = \int_0^{s_1} b\,s\,ds \qquad \frac{v_1^2}{2} - \frac{v_0^2}{2} = \frac{b}{2}s_1^2$$

$$v_1 = \sqrt{v_0^2 + b\,s_1^2} \qquad v_1 = 4.583\ \frac{\text{m}}{\text{s}}$$

$$a_{t1} = b\,s_1 \qquad a_{n1} = \frac{v_1^2}{\rho} \qquad a_1 = \sqrt{a_{t1}^2 + a_{n1}^2} \qquad a_1 = 0.653\ \frac{\text{m}}{\text{s}^2}$$

***Problem 12–116**

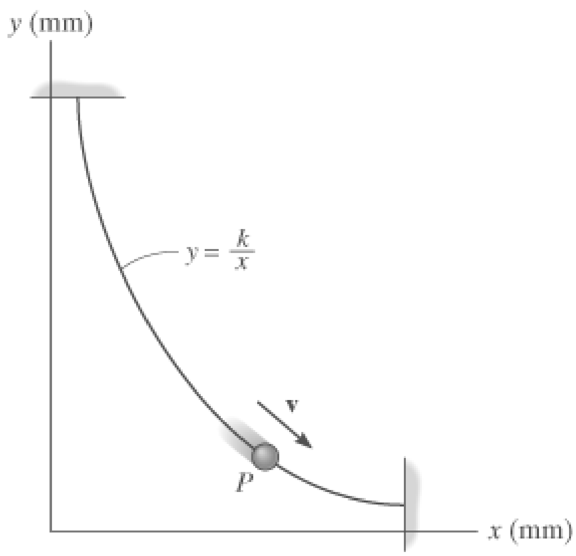
The particle travels with a constant speed v along the curve. Determine the particle's acceleration when it is located at point $x = x_1$.

Given:

$$v = 300\ \frac{\text{mm}}{\text{s}}$$

$$k = 20 \times 10^3\ \text{mm}^2$$

$$x_1 = 200\ \text{mm}$$



Solution:

$$y(x) = \frac{k}{x}$$

$$y'(x) = \frac{d}{dx}y(x)$$

$$y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \text{atan}(y'(x)) \qquad \theta_1 = \theta(x_1) \qquad \theta_1 = -26.6\,\text{deg}$$

$$\mathbf{a} = \frac{v^2}{\rho(x_1)}\begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix} \qquad \mathbf{a} = \begin{pmatrix} 144 \\ 288 \end{pmatrix}\frac{\text{mm}}{\text{s}^2} \qquad |\mathbf{a}| = 322\,\frac{\text{mm}}{\text{s}^2}$$

Problem 12–117

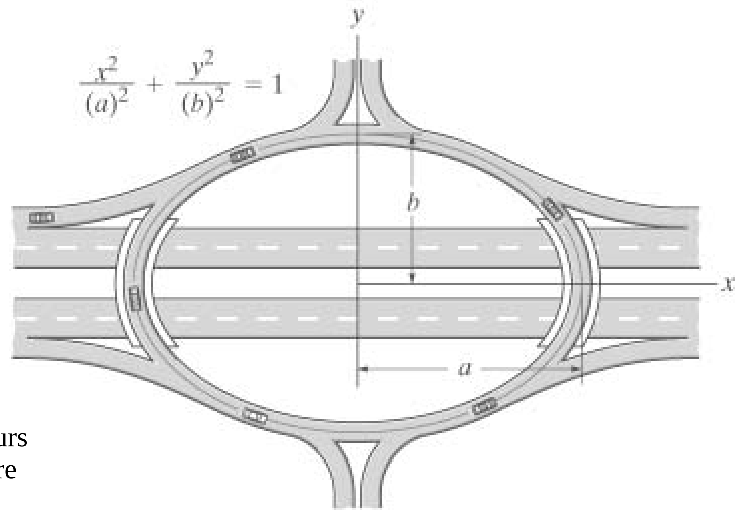
Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at *v*, determine the maximum acceleration experienced by the passengers.

Given:

$v = 60\ \frac{\text{km}}{\text{hr}}$

$a = 60\ \text{m}$

$b = 40\ \text{m}$



Solution:

Maximum acceleration occurs where the radius of curvature is the smallest. In this case that happens when $y = 0$.

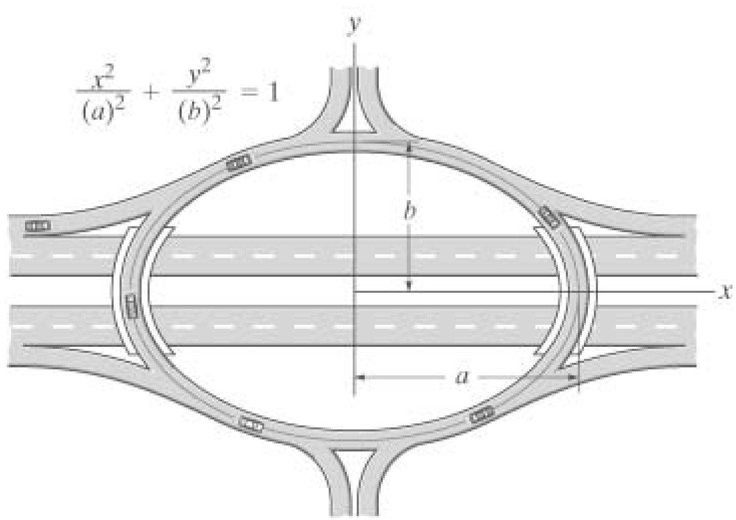
$x(y) = a\sqrt{1 - \left(\frac{y}{b}\right)^2}$ $\qquad x'(y) = \frac{d}{dy}x(y)$ $\qquad x''(y) = \frac{d}{dy}x'(y)$

$\rho(y) = \frac{-\sqrt{\left(1 + x'(y)^2\right)^3}}{x''(y)}$ $\qquad \rho_{min} = \rho(0\text{m})$ $\qquad \rho_{min} = 26.667\ \text{m}$

$a_{max} = \frac{v^2}{\rho_{min}}$ $\qquad a_{max} = 10.42\ \frac{\text{m}}{\text{s}^2}$

Problem 12–118

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at *v*, determine the minimum acceleration experienced by the passengers.



Given:

$v = 60\ \frac{\text{km}}{\text{hr}}$

$a = 60\ \text{m}$

$b = 40\ \text{m}$

Solution:

Minimum acceleration occurs where the radius of curvature is the largest. In this case that happens when $x = 0$.

$$y(x) = b\sqrt{1-\left(\frac{x}{a}\right)^2} \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho(x) = \frac{-\sqrt{\left(1+y'(x)^2\right)^3}}{y''(x)} \qquad \rho_{max} = \rho(0\text{m}) \qquad \rho_{max} = 90\text{ m}$$

$$a_{min} = \frac{v^2}{\rho_{max}} \qquad a_{min} = 3.09\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-119

The car B turns such that its speed is increased by $dv_B/dt = be^{ct}$. If the car starts from rest when $\theta = 0$, determine the magnitudes of its velocity and acceleration when the arm AB rotates to $\theta = \theta_1$. Neglect the size of the car.

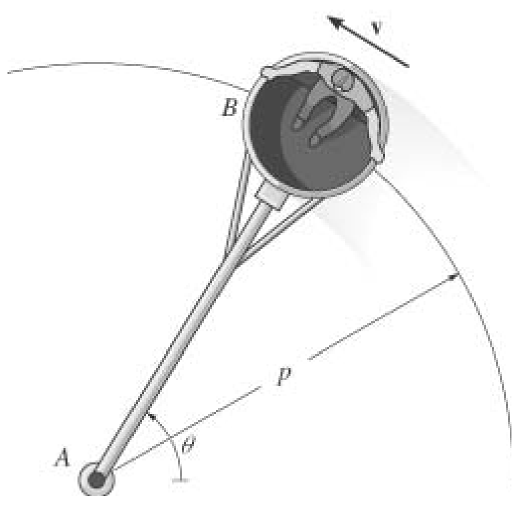
Given:

$$b = 0.5\ \frac{\text{m}}{\text{s}^2}$$

$$c = 1\ \text{s}^{-1}$$

$$\theta_1 = 30\text{ deg}$$

$$\rho = 5\text{ m}$$



Solution:

$$a_{Bt} = b\,e^{c\,t}$$

$$v_B = \frac{b}{c}\left(e^{c\,t}-1\right)$$

$$\rho\,\theta = \left(\frac{b}{c^2}\right)e^{c\,t} - \left(\frac{b}{c}\right)t - \frac{b}{c^2}$$

Guess $\quad t_1 = 1\text{ s}$

Given $\rho\,\theta_1 = \left(\frac{b}{c^2}\right)e^{c\,t_1} - \left(\frac{b}{c}\right)t_1 - \frac{b}{c^2}$ $\quad t_1 = \text{Find}(t_1) \quad t_1 = 2.123\ \text{s}$

$$v_{B1} = \frac{b}{c}\left(e^{c\,t_1} - 1\right) \qquad v_{B1} = 3.68\ \frac{\text{m}}{\text{s}}$$

$$a_{Bt1} = b\,e^{c\,t_1} \qquad a_{Bn1} = \frac{v_{B1}^2}{\rho} \qquad a_{B1} = \sqrt{a_{Bt1}^2 + a_{Bn1}^2}$$

$$a_{Bt1} = 4.180\ \frac{\text{m}}{\text{s}^2} \qquad a_{Bn1} = 2.708\ \frac{\text{m}}{\text{s}^2} \qquad a_{B1} = 4.98\ \frac{\text{m}}{\text{s}^2}$$

***Problem 12-120**

The car B turns such that its speed is increased by $dv_B/dt = b\,e^{ct}$. If the car starts from rest when $\theta = 0$, determine the magnitudes of its velocity and acceleration when $t = t_1$. Neglect the size of the car. Also, through what angle θ has it traveled?

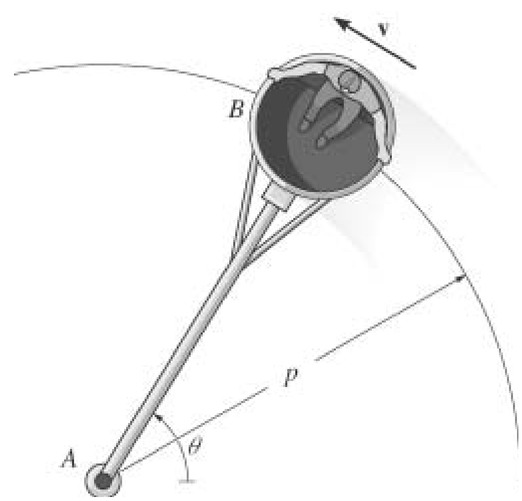
Given:

$$b = 0.5\ \frac{\text{m}}{\text{s}^2}$$

$$c = 1\ \text{s}^{-1}$$

$$t_1 = 2\ \text{s}$$

$$\rho = 5\ \text{m}$$



Solution:

$$a_{Bt} = b\,e^{c\,t}$$

$$v_B = \frac{b}{c}\left(e^{c\,t} - 1\right)$$

$$\rho\,\theta = \left(\frac{b}{c^2}\right)e^{c\,t} - \left(\frac{b}{c}\right)t - \frac{b}{c^2}$$

$$v_{B1} = \frac{b}{c}\left(e^{c\,t_1} - 1\right) \qquad v_{B1} = 3.19\ \frac{\text{m}}{\text{s}}$$

$$a_{Bt1} = b\,e^{c\,t_1} \qquad a_{Bn1} = \frac{v_{B1}^2}{\rho} \qquad a_{B1} = \sqrt{a_{Bt1}^2 + a_{Bn1}^2}$$

$a_{Bt1} = 3.695\ \frac{m}{s^2}$ $a_{Bn1} = 2.041\ \frac{m}{s^2}$ $a_{B1} = 4.22\ \frac{m}{s^2}$

$$\theta_1 = \frac{1}{\rho}\left[\left(\frac{b}{c^2}\right)e^{c\,t_1} - \left(\frac{b}{c}\right)t_1 - \frac{b}{c^2}\right]$$ $\theta_1 = 25.1\ \text{deg}$

Problem 12–121

The motorcycle is traveling at v_0 when it is at A. If the speed is then increased at dv/dt = a_t, determine its speed and acceleration at the instant $t = t_1$.

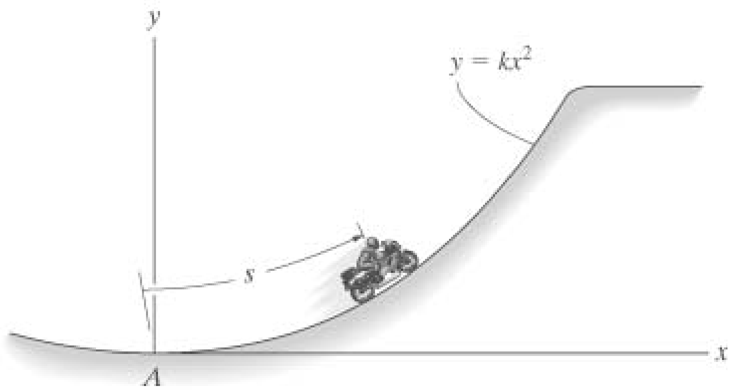
Given:

$k = 0.5\ \text{m}^{-1}$

$a_t = 0.1\ \frac{m}{s^2}$

$v_0 = 1\ \frac{m}{s}$

$t_1 = 5\ \text{s}$



Solution:

$y(x) = kx^2$ $y'(x) = 2kx$ $y''(x) = 2k$ $\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$

$v_1 = v_0 + a_t t_1$ $s_1 = v_0 t_1 + \frac{1}{2}a_t t_1^2$ $v_1 = 1.5\ \frac{m}{s}$

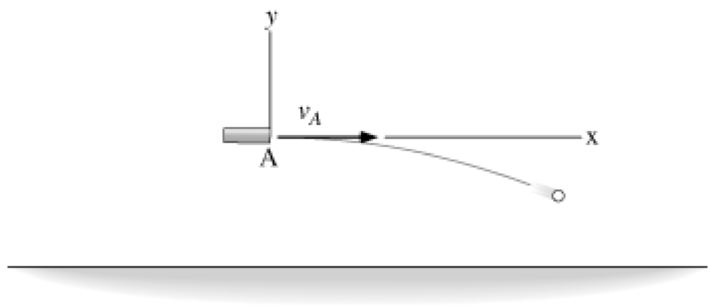
Guess $x_1 = 1\ \text{m}$ Given $s_1 = \int_0^{x_1} \sqrt{1 + y'(x)^2}\, dx$ $x_1 = \text{Find}(x_1)$

$a_{1t} = a_t$ $a_{1n} = \frac{v_1^2}{\rho(x_1)}$ $a_1 = \sqrt{a_{1t}^2 + a_{1n}^2}$ $a_1 = 0.117\ \frac{m}{s^2}$

Problem 12-122

The ball is ejected horizontally from the tube with speed v_A. Find the equation of the path $y = f(x)$, and then find the ball's velocity and the normal and tangential components of acceleration when $t = t_1$.

Given:



$v_A = 8\ \frac{m}{s}$

$t_1 = 0.25\ s$

$g = 9.81\ \frac{m}{s^2}$

Solution:

$x = v_A t \qquad t = \frac{x}{v_A} \qquad y = \frac{-g}{2} t^2 \qquad y = \frac{-g}{2 v_A^2} x^2$ parabola

when $t = t_1$

$v_x = v_A \qquad v_y = -g t_1 \qquad \theta = \operatorname{atan}\left(\frac{-v_y}{v_x}\right) \qquad \theta = 17.044\ \deg$

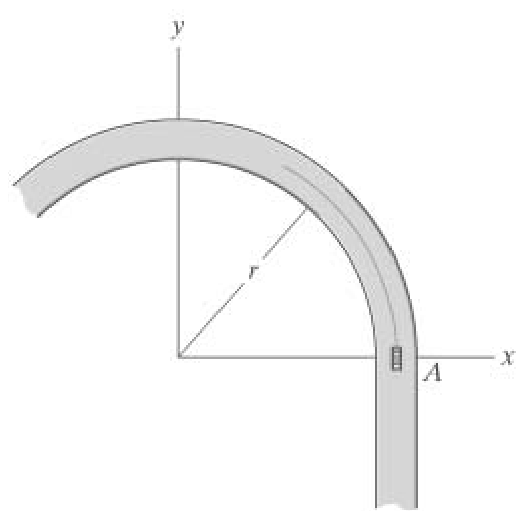
$a_n = g \cos(\theta) \qquad a_n = 9.379\ \frac{m}{s^2}$

$a_t = g \sin(\theta) \qquad a_t = 2.875\ \frac{m}{s^2}$

Problem 12–123

The car travels around the circular track having a radius r such that when it is at point A it has a velocity v_1 which is increasing at the rate dv/dt = kt. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

Given:



$k = 0.06\ \frac{m}{s^3}$

$r = 300\ m$

$v_1 = 5\ \frac{m}{s}$

Solution:

$a_t(t) = k t$

$v(t) = v_1 + \frac{k}{2} t^2$

$s_p(t) = v_1 t + \frac{k}{6} t^3$

Guess $t_1 = 1\ \text{s}$ Given $s_p(t_1) = \frac{2\pi r}{3}$ $t_1 = \text{Find}(t_1)$ $t_1 = 35.58\ \text{s}$

$v_1 = v(t_1)$ $a_{t1} = a_t(t_1)$ $a_{n1} = \frac{v_1^2}{r}$ $a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$

$v_1 = 43.0\ \frac{\text{m}}{\text{s}}$ $a_1 = 6.52\ \frac{\text{m}}{\text{s}^2}$

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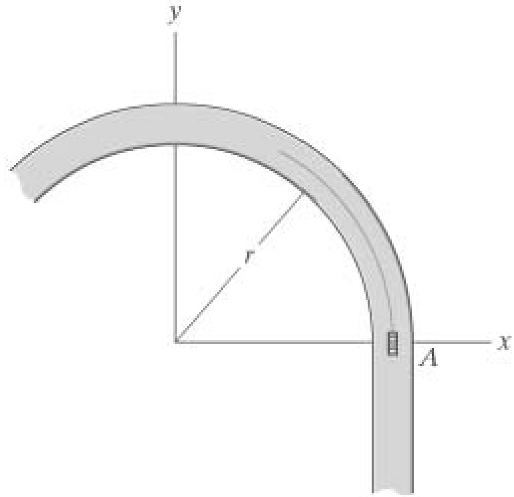
*__Problem 12–124__

The car travels around the portion of a circular track having a radius r such that when it is at point A it has a velocity v_1 which is increasing at the rate of $dv/dt = ks$. Determine the magnitudes of its velocity and acceleration when it has traveled three-fourths the way around the track.



Given:

$k = 0.002\ \text{s}^{-2}$

$r = 500\ \text{ft}$

$v_1 = 2\ \frac{\text{ft}}{\text{s}}$

Solution: $s_{p1} = \frac{3}{4} 2\pi r$ $a_t = v\frac{d}{ds_p}v = k s_p$

Guess $v_1 = 1\ \frac{\text{ft}}{\text{s}}$ Given $\int_0^{v_1} v\ dv = \int_0^{s_{p1}} k s_p\ ds_p$ $v_1 = \text{Find}(v_1)$

$a_{t1} = k s_{p1}$ $a_{n1} = \frac{v_1^2}{r}$ $a_1 = \sqrt{a_{t1}^2 + a_{n1}^2}$ $v_1 = 105.4\ \frac{\text{ft}}{\text{s}}$

$a_1 = 22.7\ \frac{\text{ft}}{\text{s}^2}$

Problem 12-125

The two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds v_A and v_B respectively. Determine at $t = t_1$, (a) the displacement along the path of each particle, (b) the position vector to each particle, and (c) the shortest distance between the particles.

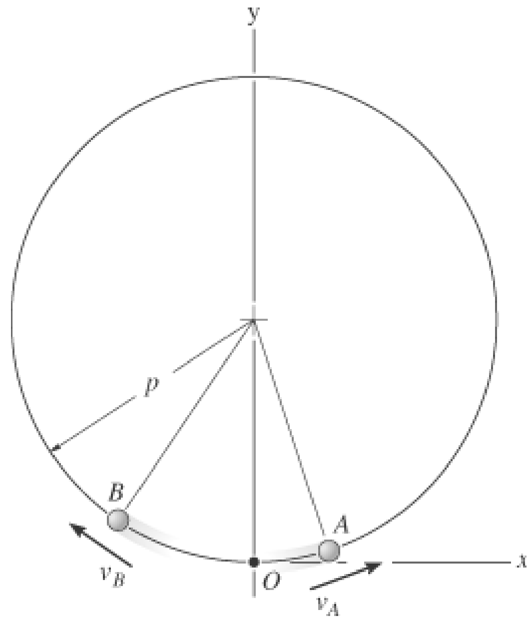
Given:

$v_A = 0.7 \frac{m}{s}$

$v_B = 1.5 \frac{m}{s}$

$t_1 = 2\ s$

$\rho = 5\ m$



Solution:

(a) The displacement along the path

$s_A = v_A t_1$ $\quad s_A = 1.4\ m$

$s_B = v_B t_1$ $\quad s_B = 3\ m$

(b) The position vector to each particle

$\theta_A = \frac{s_A}{\rho}$ $\quad \mathbf{r_A} = \begin{pmatrix} \rho \sin(\theta_A) \\ \rho - \rho \cos(\theta_A) \end{pmatrix}$ $\quad \mathbf{r_A} = \begin{pmatrix} 1.382 \\ 0.195 \end{pmatrix} m$

$\theta_B = \frac{s_B}{\rho}$ $\quad \mathbf{r_B} = \begin{pmatrix} -\rho \sin(\theta_B) \\ \rho - \rho \cos(\theta_B) \end{pmatrix}$ $\quad \mathbf{r_B} = \begin{pmatrix} -2.823 \\ 0.873 \end{pmatrix} m$

(c) The shortest distance between the particles

$d = |\mathbf{r_B} - \mathbf{r_A}|$ $\quad d = 4.26\ m$

Problem 12-126

The two particles *A* and *B* start at the origin *O* and travel in opposite directions along the circular path at constant speeds v_A and v_B respectively. Determine the time when they collide and the magnitude of the acceleration of *B* just before this happens.

Given:

$v_A = 0.7 \frac{m}{s}$

$v_B = 1.5 \frac{m}{s}$

$\rho = 5\ m$

Solution:

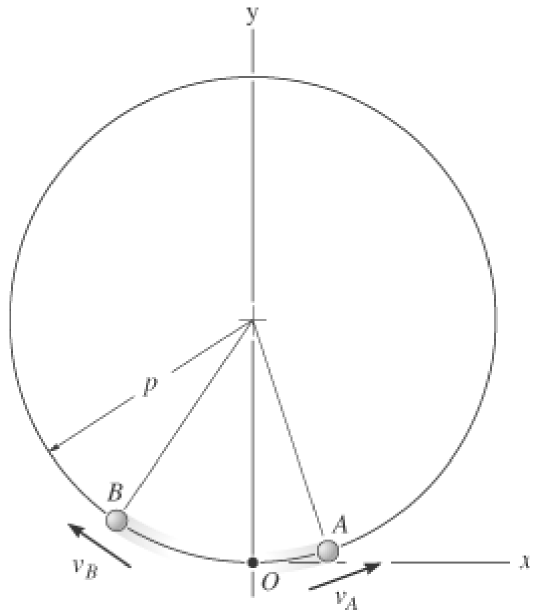
$$(v_A + v_B)t = 2\pi\rho$$

$$t = \frac{2\pi\rho}{v_A + v_B}$$

$$t = 14.28 \text{ s}$$

$$a_B = \frac{v_B^2}{\rho}$$

$$a_B = 0.45 \frac{\text{m}}{\text{s}^2}$$



Problem 12-127

The race car has an initial speed v_A at A. If it increases its speed along the circular track at the rate $a_t = bs$, determine the time needed for the car to travel distance s_1.

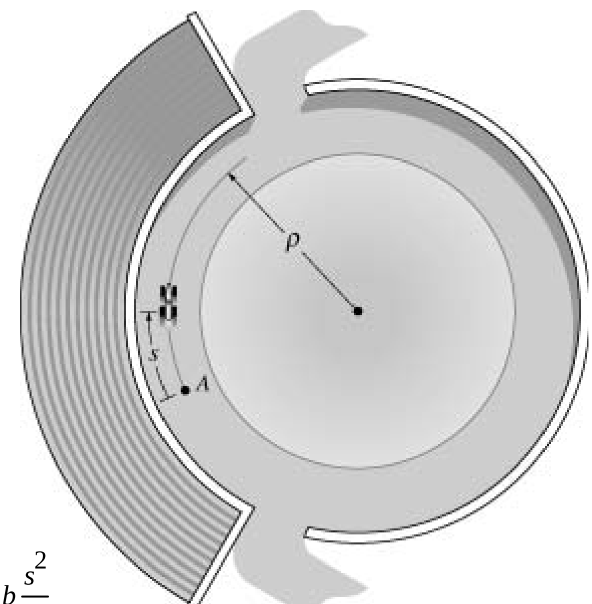
Given:

$$v_A = 15 \frac{\text{m}}{\text{s}}$$

$$b = 0.4 \text{ s}^{-2}$$

$$s_1 = 20 \text{ m}$$

$$\rho = 150 \text{ m}$$



Solution:

$$a_t = bs = v\frac{d}{ds}v$$

$$\int_{v_A}^{v} v\,dv = \int_0^s bs\,ds \qquad \frac{v^2}{2} - \frac{v_A^2}{2} = b\frac{s^2}{2}$$

$$v = \frac{d}{dt}s = \sqrt{v_A^2 + bs^2}$$

$$\int_0^s \frac{1}{\sqrt{v_A^2 + b s^2}}\, ds = \int_0^t 1\, dt \qquad t = \int_0^{s_1} \frac{1}{\sqrt{v_A^2 + b s^2}}\, ds \qquad t = 1.211\ \text{s}$$

***Problem 12-128**

A boy sits on a merry-go-round so that he is always located a distance r from the center of rotation. The merry-go-round is originally at rest, and then due to rotation the boy's speed is increased at the rate a_t. Determine the time needed for his acceleration to become a.

Given: $r = 8\ \text{ft}$ $\quad a_t = 2\ \frac{\text{ft}}{\text{s}^2}$ $\quad a = 4\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$a_n = \sqrt{a^2 - a_t^2} \qquad v = \sqrt{a_n r} \qquad t = \frac{v}{a_t} \qquad t = 2.63\ \text{s}$$

Problem 12–129

A particle moves along the curve $y = b\sin(cx)$ with a constant speed v. Determine the normal and tangential components of its velocity and acceleration at any instant.

Given: $v = 2\ \frac{\text{m}}{\text{s}}$ $\quad b = 1\ \text{m}$ $\quad c = \frac{1}{\text{m}}$

Solution:

$$y = b\sin(cx) \qquad y' = bc\cos(cx) \qquad y'' = -bc^2\sin(cx)$$

$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''} = \frac{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}}{-bc^2\sin(cx)}$$

$$a_n = \frac{v^2 bc\sin(cx)}{\left[1 + (bc\cos(cx))^2\right]^{\frac{3}{2}}} \qquad a_t = 0 \qquad v_t = 0 \qquad v_n = 0$$

Problem 12–130

The motion of a particle along a fixed path is defined by the parametric equations $r = b$, $\theta = ct$

and $z = dt^2$. Determine the unit vector that specifies the direction of the binormal axis to the osculating plane with respect to a set of fixed x, y, z coordinate axes when $t = t_1$. *Hint:* Formulate the particle's velocity v_p and acceleration a_p in terms of their **i**, **j**, **k** components. Note that $x = r\cos(\theta)$ and $y = r\sin(\theta)$. The binormal is parallel to $v_p \times a_p$. Why?

Given: $b = 8\ \text{ft}$ $c = 4\ \frac{\text{rad}}{\text{s}}$ $d = 6\ \frac{\text{ft}}{\text{s}^2}$ $t_1 = 2\ \text{s}$

Solution:

$$\mathbf{r_{p1}} = \begin{pmatrix} b\cos(c\,t_1) \\ b\sin(c\,t_1) \\ d\,t_1^{\,2} \end{pmatrix} \qquad \mathbf{v_{p1}} = \begin{pmatrix} -b\,c\sin(c\,t_1) \\ b\,c\cos(c\,t_1) \\ 2d\,t_1 \end{pmatrix} \qquad \mathbf{a_{p1}} = \begin{pmatrix} -b\,c^2\cos(c\,t_1) \\ -b\,c^2\sin(c\,t_1) \\ 2d \end{pmatrix}$$

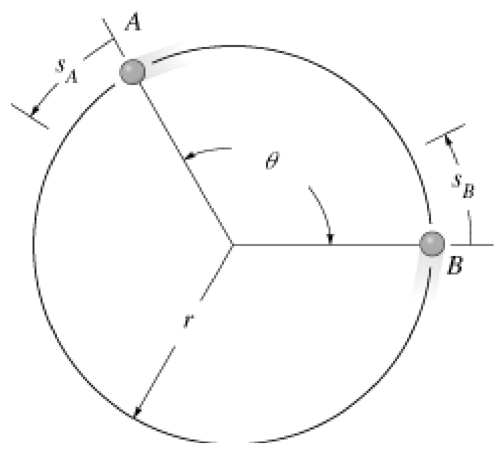
Since v_p and a_p are in the normal plane and the binormal direction is perpendicular to this plane then we can use the cross product to define the binormal direction.

$$\mathbf{u} = \frac{\mathbf{v_{p1}} \times \mathbf{a_{p1}}}{\left|\mathbf{v_{p1}} \times \mathbf{a_{p1}}\right|} \qquad \mathbf{u} = \begin{pmatrix} 0.581 \\ 0.161 \\ 0.798 \end{pmatrix}$$

Problem 12-131

Particles A and B are traveling counter-clockwise around a circular track at constant speed v_0. If at the instant shown the speed of A is increased by $dv_A/dt = bs_A$, determine the distance measured counterclockwise along the track from B to A when $t = t_1$. What is the magnitude of the acceleration of each particle at this instant?

Given:



$v_0 = 8\ \frac{\text{m}}{\text{s}}$

$b = 4\ \text{s}^{-2}$

$t_1 = 1\ \text{s}$

$r = 5\ \text{m}$

$\theta = 120\ \text{deg}$

Solution: Distance

$$a_{At} = v_A \frac{dv_A}{ds_A} = b\,s_A \qquad \int_{v_0}^{v_A} v_A\, dv_A = \int_0^{s_A} b\,s_A\, ds_A$$

$$\frac{v_A^2}{2} - \frac{v_0^2}{2} = \frac{b}{2}s_A^2 \qquad v_A = \sqrt{v_0^2 + b\,s_A^2} = \frac{ds_A}{dt}$$

Guess $s_{A1} = 1\ \text{m}$ Given $\displaystyle\int_0^{t_1} 1\,dt = \int_0^{s_{A1}} \frac{1}{\sqrt{v_0^2 + b\,s_A^2}}\,ds_A$

$s_{A1} = \text{Find}(s_{A1})$ $s_{A1} = 14.507\ \text{m}$

$s_{B1} = v_0 t_1$ $s_{B1} = 8\ \text{m}$ $s_{AB} = s_{A1} + r\theta - s_{B1}$ $s_{AB} = 16.979\ \text{m}$

$$a_A = \sqrt{(b\,s_{A1})^2 + \left(\frac{v_0^2 + b\,s_{A1}^2}{r}\right)^2} \qquad a_A = 190.24\ \frac{\text{m}}{\text{s}^2}$$

$$a_B = \frac{v_0^2}{r} \qquad a_B = 12.8\ \frac{\text{m}}{\text{s}^2}$$

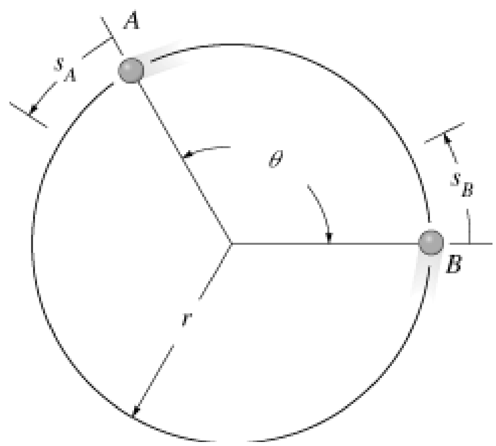
Problem 12-132

Particles A and B are traveling around a circular track at speed v_0 at the instant shown. If the speed of B is increased by $dv_B/dt = a_{Bt}$, and at the same instant A has an increase in speed $dv_A/dt = bt$, determine how long it takes for a collision to occur. What is the magnitude of the acceleration of each particle just before the collision occurs?

Given: $v_0 = 8\ \frac{\text{m}}{\text{s}}$ $r = 5\ \text{m}$

$a_{Bt} = 4\ \frac{\text{m}}{\text{s}^2}$ $\theta = 120\ \text{deg}$

$b = 0.8\ \frac{\text{m}}{\text{s}^3}$



Solution:

$$v_B = a_{Bt}\,t + v_0$$

$$s_B = \frac{a_{Bt}}{2}t^2 + v_0\,t$$

$a_{At} = b\,t$ $v_A = \frac{b}{2}t^2 + v_0$ $s_A = \frac{b}{6}t^3 + v_0\,t$

Assume that B catches A Guess $t_1 = 1\ \text{s}$

Given $\frac{a_{Bt}}{2}t_1^{\,2} + v_0 t_1 = \frac{b}{6}t_1^{\,3} + v_0 t_1 + r\theta$ $\quad t_1 = \text{Find}(t_1) \quad t_1 = 2.507\ \text{s}$

Assume that A catches B Guess $t_2 = 13\ \text{s}$

Given $\frac{a_{Bt}}{2}t_2^{\,2} + v_0 t_2 + r(2\pi - \theta) = \frac{b}{6}t_2^{\,3} + v_0 t_2$ $\quad t_2 = \text{Find}(t_2) \quad t_2 = 15.642\ \text{s}$

Take the smaller time $t = \min(t_1, t_2)$ $\quad t = 2.507\ \text{s}$

$$a_A = \sqrt{(b\,t)^2 + \left[\frac{\left(\frac{b}{2}t^2 + v_0\right)^2}{r}\right]^2} \qquad a_B = \sqrt{a_{Bt}^{\,2} + \left[\frac{(a_{Bt}\,t + v_0)^2}{r}\right]^2}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 22.2 \\ 65.14 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 12-133

The truck travels at speed v_0 along a circular road that has radius ρ. For a short distance from $s = 0$, its speed is then increased by $\text{d}v/\text{d}t = bs$. Determine its speed and the magnitude of its acceleration when it has moved a distance s_1.

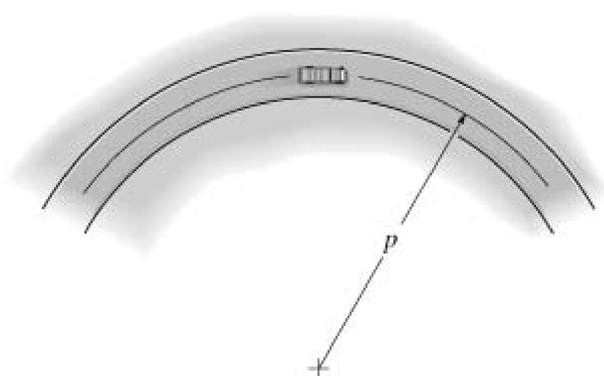
Given:

$v_0 = 4\ \frac{\text{m}}{\text{s}}$

$\rho = 50\ \text{m}$

$b = \frac{0.05}{\text{s}^2}$

$s_1 = 10\ \text{m}$



Solution:

$$a_t = v\left(\frac{\text{d}}{\text{d}s}v\right) = b\,s \qquad \int_{v_0}^{v_1} v\ \text{d}v = \int_0^{s_1} b\,s\ \text{d}s \qquad \frac{v_1^{\,2}}{2} - \frac{v_0^{\,2}}{2} = \frac{b}{2}s_1^{\,2}$$

$$v_1 = \sqrt{v_0^{\,2} + b\,s_1^{\,2}} \qquad v_1 = 4.58\ \frac{\text{m}}{\text{s}}$$

$a_t = b s_1 \qquad a_n = \frac{v_1^2}{\rho} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 0.653 \, \frac{\text{m}}{\text{s}^2}$

Problem 12-134

A go-cart moves along a circular track of radius ρ such that its speed for a short period of time, $0 < t < t_1$, is $v = b\left(1 - e^{c t^2}\right)$. Determine the magnitude of its acceleration when $t = t_2$. How far has it traveled in $t = t_2$? Use Simpson's rule with n steps to evaluate the integral.

Given: $\rho = 100 \text{ ft} \qquad t_1 = 4 \text{ s} \qquad b = 60 \, \frac{\text{ft}}{\text{s}} \qquad c = -1 \text{ s}^{-2} \qquad t_2 = 2 \text{ s} \qquad n = 50$

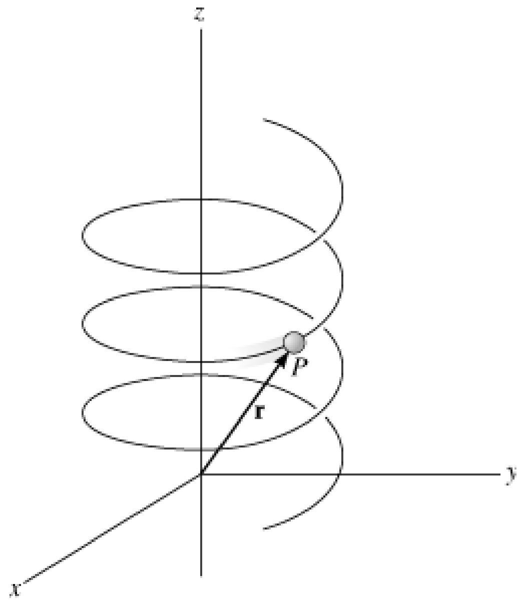
Solution: $t = t_2 \qquad v = b\left(1 - e^{c t^2}\right)$

$a_t = -2b c t e^{c t^2} \qquad a_n = \frac{v^2}{\rho} \qquad a = \sqrt{a_t^2 + a_n^2} \qquad a = 35.0 \, \frac{\text{ft}}{\text{s}^2}$

$$s_2 = \int_0^{t_2} b\left(1 - e^{c t^2}\right) \text{d}t \qquad s_2 = 67.1 \text{ ft}$$

Problem 12-135

A particle P travels along an elliptical spiral path such that its position vector $\mathbf{r}$ is defined by $\mathbf{r} = (a \cos bt \, \mathbf{i} + c \sin dt \, \mathbf{j} + et \, \mathbf{k})$. When $t = t_1$, determine the coordinate direction angles α, β, and γ, which the binormal axis to the osculating plane makes with the x, y, and z axes. *Hint:* Solve for the velocity $\mathbf{v_p}$ and acceleration $\mathbf{a_p}$ of the particle in terms of their $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components. The binormal is parallel to $\mathbf{v_p} \times \mathbf{a_p}$. Why?



Given:

$a = 2\ \text{m}$ $\quad d = 0.1\ \text{s}^{-1}$

$b = 0.1\ \text{s}^{-1}$ $\quad e = 2\ \frac{\text{m}}{\text{s}}$

$c = 1.5\ \text{m}$ $\quad t_1 = 8\ \text{s}$

Solution: $t = t_1$

$$\mathbf{r_P} = \begin{bmatrix} (a)\cos(b\,t) \\ c\sin(d\,t) \\ e\,t \end{bmatrix}$$

$$\mathbf{v_P} = \begin{pmatrix} -a\,b\sin(b\,t) \\ c\,d\cos(d\,t) \\ e \end{pmatrix} \qquad \mathbf{a_P} = \begin{pmatrix} -a\,b^2\cos(b\,t) \\ -c\,d^2\sin(d\,t) \\ 0 \end{pmatrix}$$

$$\mathbf{u_b} = \frac{\mathbf{v_P} \times \mathbf{a_P}}{|\mathbf{v_P} \times \mathbf{a_P}|} \qquad \mathbf{u_b} = \begin{pmatrix} 0.609 \\ -0.789 \\ 0.085 \end{pmatrix} \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \operatorname{acos}(\mathbf{u_b}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 52.5 \\ 142.1 \\ 85.1 \end{pmatrix} \text{deg}$$

***Problem 12-136**

The time rate of change of acceleration is referred to as the *jerk,* which is often used as a means of measuring passenger discomfort. Calculate this vector, **a'**, in terms of its cylindrical components, using Eq. 12-32.

Solution:

$$\mathbf{a} = (r'' - r\theta'^2)\mathbf{u_r} + (r\theta'' + 2r'\theta')\mathbf{u_\theta} + z''\mathbf{u_z}$$

$$\mathbf{a'} = (r''' - r'\theta'^2 - 2r\theta'\theta'')\mathbf{u_r} + (r'' - r\theta'^2)\mathbf{u'_r} \ldots$$
$$+ (r'\theta'' + r\theta''' + 2r''\theta' + 2r'\theta'')\mathbf{u_\theta} + (r\theta'' + 2r'\theta')\mathbf{u'_\theta} + z'''\mathbf{u_z} + z''\mathbf{u'_z}$$

But $\quad \mathbf{u_r} = \theta'\mathbf{u_\theta} \qquad \mathbf{u'_\theta} = -\theta'\mathbf{u_r} \qquad \mathbf{u'_z} = 0$

Substituting and combining terms yields

$$\mathbf{a'} = (r''' - 3r'\theta'^2 - 3r\theta'\theta'')\mathbf{u_r} + (r\theta''' + 3r'\theta'' + 3r''\theta' - r\theta'^3)\mathbf{u_\theta} + (z''')\mathbf{u_z}$$

Problem 12-137

If a particle's position is described by the polar coordinates $r = a(1 + \sin bt)$ and $\theta = ce^{dt}$, determine the radial and tangential components of the particle's velocity and acceleration when $t = t_1$.

Given: $a = 4\text{ m}$ $b = 1\text{ s}^{-1}$ $c = 2\text{ rad}$ $d = -1\text{ s}^{-1}$ $t_1 = 2\text{ s}$

Solution: When $t = t_1$

$r = a(1 + \sin(bt))$ $r' = ab\cos(bt)$ $r'' = -ab^2\sin(bt)$

$\theta = ce^{dt}$ $\theta' = cde^{dt}$ $\theta'' = cd^2e^{dt}$

$v_r = r'$ $v_r = -1.66\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $v_\theta = -2.07\ \frac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $a_r = -4.20\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $a_\theta = 2.97\ \frac{\text{m}}{\text{s}^2}$

Problem 12–138

The slotted fork is rotating about O at a constant rate θ'. Determine the radial and transverse components of the velocity and acceleration of the pin A at the instant $\theta = \theta_1$. The path is defined by the spiral groove $r = b + c\theta$, where θ is in radians.

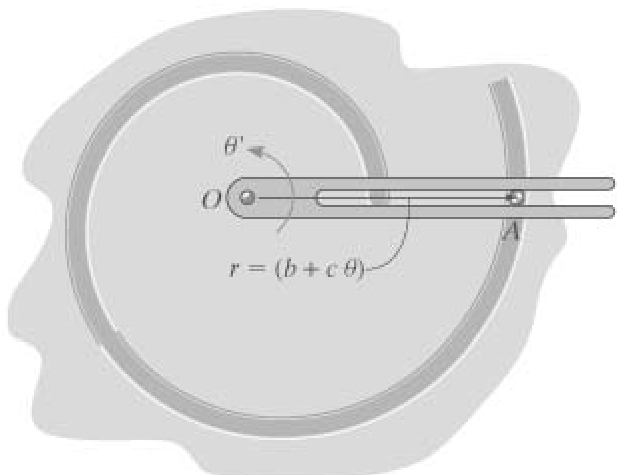
Given:

$\theta' = 3\ \frac{\text{rad}}{\text{s}}$

$b = 5\text{ in}$

$c = \frac{1}{\pi}\text{ in}$

$\theta_1 = 2\pi\text{ rad}$



Solution: $\theta = \theta_1$

$r = b + c\theta$ $\quad r' = c\theta'$ $\quad r'' = 0\,\frac{\text{in}}{\text{s}^2}$ $\quad \theta'' = 0\,\frac{\text{rad}}{\text{s}^2}$

$v_r = r'$ $\quad v_\theta = r\theta'$ $\quad a_r = r'' - r\theta'^2$ $\quad a_\theta = r\theta'' + 2r'\theta'$

$v_r = 0.955\,\frac{\text{in}}{\text{s}}$ $\quad v_\theta = 21\,\frac{\text{in}}{\text{s}}$ $\quad a_r = -63\,\frac{\text{in}}{\text{s}^2}$ $\quad a_\theta = 5.73\,\frac{\text{in}}{\text{s}^2}$

Problem 12–139

The slotted fork is rotating about *O* at the rate θ' which is increasing at θ'' when $\theta = \theta_1$. Determine the radial and transverse components of the velocity and acceleration of the pin *A* at this instant. The path is defined by the spiral groove $r = (5 + \theta/\pi)$ in., where θ is in radians.

Given:

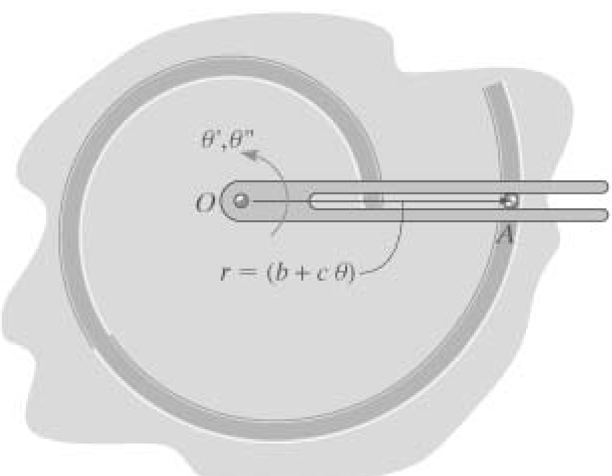
$\theta' = 3\,\frac{\text{rad}}{\text{s}}$

$\theta'' = 2\,\frac{\text{rad}}{\text{s}^2}$

$b = 5\text{ in}$

$c = \frac{1}{\pi}\text{ in}$

$\theta_1 = 2\,\pi\text{ rad}$



Solution: $\quad \theta = \theta_1$

$r = b + c\theta$ $\quad r' = c\theta'$ $\quad r'' = c\theta''$

$v_r = r'$ $\quad v_\theta = r\theta'$

$a_r = r'' - r\theta'^2$ $\quad a_\theta = r\theta'' + 2r'\theta'$

$v_r = 0.955\,\frac{\text{in}}{\text{s}}$ $\quad v_\theta = 21\,\frac{\text{in}}{\text{s}}$ $\quad a_r = -62.363\,\frac{\text{in}}{\text{s}^2}$ $\quad a_\theta = 19.73\,\frac{\text{in}}{\text{s}^2}$

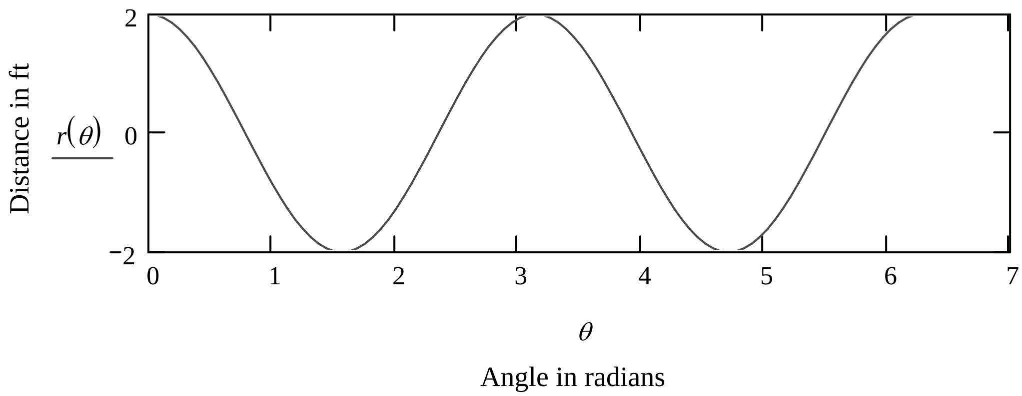
***Problem 12-140**

If a particle moves along a path such that $r = a\cos(bt)$ and $\theta = ct$, plot the path $r = f(\theta)$ and determine the particle's radial and transverse components of velocity and acceleration.

Given: $a = 2\ \text{ft}$ $b = 1\ \text{s}^{-1}$ $c = 0.5\ \frac{\text{rad}}{\text{s}}$

The plot $t = \frac{\theta}{c}$ $r = (a)\cos\left(b\frac{\theta}{c}\right)$

$\theta = 0, 0.01(2\pi)..2\pi$ $r(\theta) = (a)\cos\left(b\frac{\theta}{c}\right)\frac{1}{\text{ft}}$



$r = (a)\cos(b\,t)$ $r' = -a\,b\sin(b\,t)$ $r'' = -a\,b^2\cos(b\,t)$

$\theta = c\,t$ $\theta' = c$ $\theta'' = 0$

$v_r = r' = -a\,b\sin(b\,t)$ $a_r = r'' - r\theta'^2 = -a\left(b^2 + c^2\right)\cos(b\,t)$

$v_\theta = r\theta' = a\,c\cos(b\,t)$ $a_\theta = r\theta'' + 2r'\theta' = -2a\,b\,c\sin(b\,t)$

Problem 12-141

If a particle's position is described by the polar coordinates $r = a\sin b\theta$ and $\theta = ct$, determine the radial and tangential components of its velocity and acceleration when $t = t_1$.

Given: $a = 2\ \text{m}$ $b = 2\ \text{rad}$ $c = 4\ \frac{\text{rad}}{\text{s}}$ $t_1 = 1\ \text{s}$

Solution: $t = t_1$

$r = (a)\sin(b\,c\,t)$ $r' = a\,b\,c\cos(b\,c\,t)$ $r'' = -a\,b^2\,c^2\sin(b\,c\,t)$

$\theta = c\,t$ $\theta' = c$ $\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

$v_r = r'$ $v_r = -2.328\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $v_\theta = 7.915\ \frac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $\qquad$ $a_r = -158.3\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $\qquad$ $a_\theta = -18.624\ \frac{\text{m}}{\text{s}^2}$

Problem 12-142

A particle is moving along a circular path having a radius r. Its position as a function of time is given by $\theta = bt^2$. Determine the magnitude of the particle's acceleration when $\theta = \theta_1$. The particle starts from rest when $\theta = 0°$.

Given: $\quad r = 400\ \text{mm} \qquad b = 2\ \frac{\text{rad}}{\text{s}^2} \qquad \theta_1 = 30\ \text{deg}$

Solution: $\quad t = \sqrt{\frac{\theta_1}{b}} \qquad t = 0.512\ \text{s}$

$\theta = bt^2 \qquad \theta' = 2bt \qquad \theta'' = 2b$

$$a = \sqrt{\left(-r\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 2.317\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-143

A particle moves in the x - y plane such that its position is defined by $\mathbf{r} = at\mathbf{i} + bt^2\mathbf{j}$. Determine the radial and tangential components of the particle's velocity and acceleration when $t = t_1$.

Given: $\quad a = 2\ \frac{\text{ft}}{\text{s}} \qquad b = 4\ \frac{\text{ft}}{\text{s}^2} \qquad t_1 = 2\ \text{s}$

Solution: $\quad t = t_1$

Rectangular

$x = at \qquad v_x = a \qquad a_x = 0\ \frac{\text{ft}}{\text{s}^2}$

$y = bt^2 \qquad v_y = 2bt \qquad a_y = 2b$

Polar

$\theta = \text{atan}\left(\frac{y}{x}\right) \qquad \theta = 75.964\ \text{deg}$

$$v_r = v_x \cos(\theta) + v_y \sin(\theta) \qquad v_r = 16.007\ \frac{\text{ft}}{\text{s}}$$

$$v_\theta = -v_x \sin(\theta) + v_y \cos(\theta) \qquad v_\theta = 1.94\ \frac{\text{ft}}{\text{s}}$$

$$a_r = a_x \cos(\theta) + a_y \sin(\theta) \qquad a_r = 7.761\ \frac{\text{ft}}{\text{s}^2}$$

$$a_\theta = -a_x \sin(\theta) + a_y \cos(\theta) \qquad a_\theta = 1.94\ \frac{\text{ft}}{\text{s}^2}$$

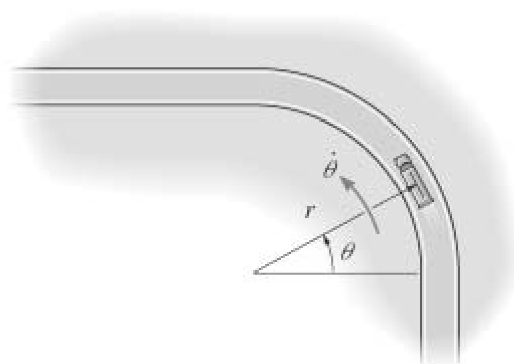
***Problem 12-144**

A truck is traveling along the horizontal circular curve of radius r with a constant speed v. Determine the angular rate of rotation θ' of the radial line r and the magnitude of the truck's acceleration.

Given:

$$r = 60\ \text{m}$$

$$v = 20\ \frac{\text{m}}{\text{s}}$$



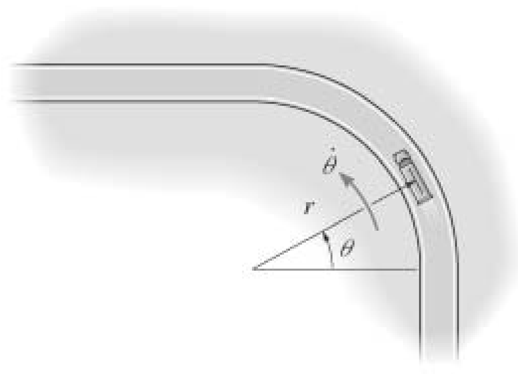
Solution:

$$\theta' = \frac{v}{r} \qquad \theta' = 0.333\ \frac{\text{rad}}{\text{s}}$$

$$a = \left|-r\,\theta'^2\right| \qquad a = 6.667\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-145

A truck is traveling along the horizontal circular curve of radius r with speed v which is increasing at the rate v'. Determine the truck's radial and transverse components of acceleration.



Given:

$$r = 60\ \text{m}$$

$$v = 20\ \frac{\text{m}}{\text{s}}$$

$$v' = 3\ \frac{\text{m}}{\text{s}^2}$$

Solution:

$$a_r = \frac{-v^2}{r} \qquad a_r = -6.667\ \frac{\text{m}}{\text{s}^2}$$

$$a_\theta = v' \qquad a_\theta = 3\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-146

A particle is moving along a circular path having radius *r* such that its position as a function of time is given by $\theta = c \sin bt$. Determine the acceleration of the particle at $\theta = \theta_1$. The particle starts from rest at $\theta = 0°$.

Given: $r = 6\ \text{in}$ $\quad c = 1\ \text{rad}$ $\quad b = 3\ \text{s}^{-1}$ $\quad \theta_1 = 30\ \text{deg}$

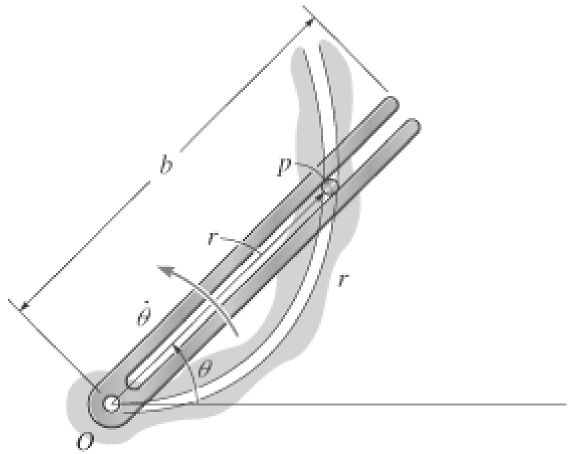
Solution: $t = \frac{1}{b}\,\text{asin}\left(\frac{\theta_1}{c}\right)$ $\qquad t = 0.184\ \text{s}$

$$\theta = c\sin(b\,t) \qquad \theta' = c\,b\cos(b\,t) \qquad \theta'' = c\,b^2\sin(b\,t)$$

$$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2} \qquad a = 48.329\ \frac{\text{in}}{\text{s}^2}$$

Problem 12-147

The slotted link is pinned at *O*, and as a result of the constant angular velocity θ' it drives the peg *P* for a short distance along the spiral guide $r = a\theta$. Determine the radial and transverse components of the velocity and acceleration of *P* at the instant $\theta = \theta_1$.



Given:

$\theta' = 3\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta_1 = \frac{\pi}{3}\ \text{rad}$

$a = 0.4\ \text{m}$ $\qquad b = 0.5\ \text{m}$

Solution: $\theta = \theta_1$

$r = a\theta$ $\qquad r' = a\theta'$ $\qquad r'' = 0\ \frac{\text{m}}{\text{s}^2}$

$v_r = r'$ $\qquad v_r = 1.2\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $\qquad v_\theta = 1.257\ \frac{\text{m}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $\qquad a_r = -3.77\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = 2r'\theta'$ $\qquad a_\theta = 7.2\ \frac{\text{m}}{\text{s}^2}$

***Problem 12-148**

The slotted link is pinned at O, and as a result of the angular velocity θ' and the angular acceleration θ'' it drives the peg P for a short distance along the spiral guide $r = a\theta$. Determine the radial and transverse components of the velocity and acceleration of P at the instant $\theta = \theta_1$.

Given:

$\theta' = 3\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta_1 = \frac{\pi}{3}\ \text{rad}$

$\theta'' = 8\ \frac{\text{rad}}{\text{s}^2}$ $\qquad a = 0.4\ \text{m}$

$b = 0.5\ \text{m}$

Solution: $\theta = \theta_1$

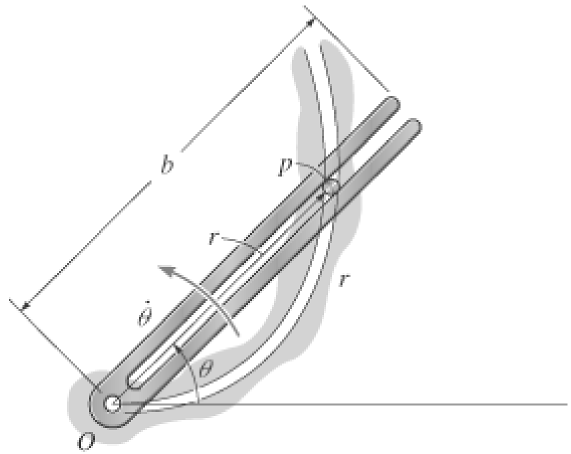
$r = a\theta \qquad r' = a\theta' \qquad r'' = a\theta''$

$v_r = r' \qquad v_r = 1.2\ \frac{m}{s}$

$v_\theta = r\theta' \qquad v_\theta = 1.257\ \frac{m}{s}$

$a_r = r'' - r\theta'^2 \qquad a_r = -0.57\ \frac{m}{s^2}$

$a_\theta = r\theta'' + 2r'\theta' \qquad a_\theta = 10.551\ \frac{m}{s^2}$



Problem 12-149

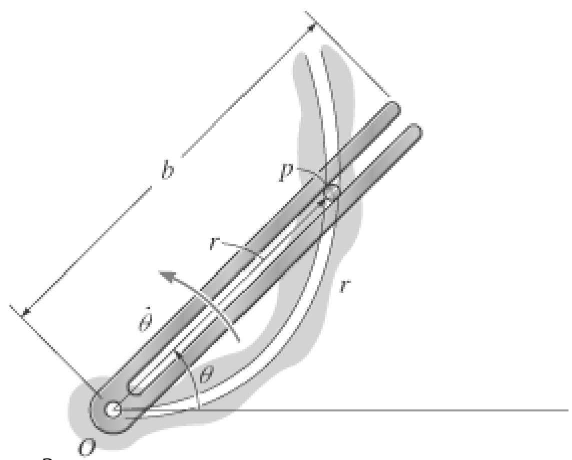
The slotted link is pinned at *O*, and as a result of the constant angular velocity θ' it drives the peg *P* for a short distance along the spiral guide $r = a\theta$ where θ is in radians. Determine the velocity and acceleration of the particle at the instant it leaves the slot in the link, i.e., when $r = b$.

Given:

$\theta' = 3\ \frac{rad}{s}$

$a = 0.4\ m$

$b = 0.5\ m$



Solution: $\theta = \frac{b}{a}$

$r = a\theta \qquad r' = a\theta' \qquad r'' = 0\ \frac{m}{s^2}$

$v_r = r' \qquad v_\theta = r\theta' \qquad a_r = r'' - r\theta'^2 \qquad a_\theta = 2r'\theta'$

$v = \sqrt{v_r^2 + v_\theta^2} \qquad a = \sqrt{a_r^2 + a_\theta^2} \qquad v = 1.921\ \frac{m}{s} \qquad a = 8.491\ \frac{m}{s^2}$

Problem 12–150

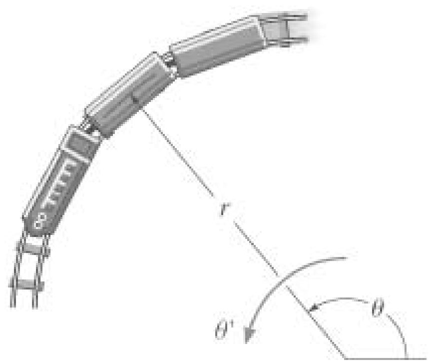
A train is traveling along the circular curve of radius *r*. At the instant shown, its angular rate of rotation is θ', which is decreasing at θ''. Determine the magnitudes of the train's velocity and acceleration at this instant.

Given:

$r = 600\ \text{ft}$

$\theta' = 0.02\ \frac{\text{rad}}{\text{s}}$

$\theta'' = -0.001\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$v = r\theta'$ $\qquad v = 12\ \frac{\text{ft}}{\text{s}}$

$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta''\right)^2}$ $\qquad a = 0.646\ \frac{\text{ft}}{\text{s}^2}$

Problem 12–151

A particle travels along a portion of the "four-leaf rose" defined by the equation $r = a\cos(b\theta)$. If the angular velocity of the radial coordinate line is $\theta' = ct^2$, determine the radial and transverse components of the particle's velocity and acceleration at the instant $\theta = \theta_1$. When $t = 0$, $\theta = 0°$.

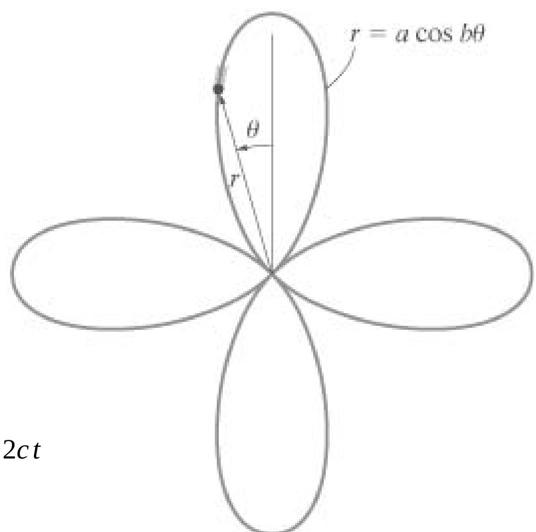
Given:

$a = 5\ \text{m}$

$b = 2$

$c = 3\ \frac{\text{rad}}{\text{s}^3}$

$\theta_1 = 30\ \text{deg}$



Solution:

$\theta(t) = \frac{c}{3}t^3 \qquad \theta'(t) = ct^2 \qquad \theta''(t) = 2ct$

$r(t) = (a)\cos\left(b\,\theta(t)\right) \qquad r'(t) = \frac{d}{dt}r(t) \qquad r''(t) = \frac{d}{dt}r'(t)$

When $\theta = \theta_1$ $\qquad t_1 = \left(\frac{3\theta_1}{c}\right)^{\frac{1}{3}}$

$v_r = r'(t_1)$ $\qquad v_r = -16.88\ \frac{\text{m}}{\text{s}}$

$v_\theta = r(t_1)\,\theta'(t_1)$ $\quad v_\theta = 4.87\ \frac{m}{s}$

$a_r = r''(t_1) - r(t_1)\,\theta'(t_1)^2$ $\quad a_r = -89.4\ \frac{m}{s^2}$

$a_\theta = r(t_1)\,\theta''(t_1) + 2\,r'(t_1)\,\theta'(t_1)$ $\quad a_\theta = -53.7\ \frac{m}{s^2}$

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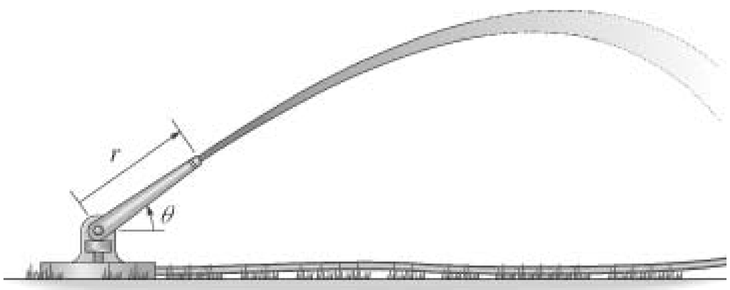
*__Problem 12-152__

At the instant shown, the watersprinkler is rotating with an angular speed θ' and an angular acceleration θ''. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate r', determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, r.

Given:

$\theta' = 2\ \frac{rad}{s}$ $\qquad \theta'' = 3\ \frac{rad}{s^2}$

$r' = 3\ \frac{m}{s}$ $\qquad r = 0.2\ m$



Solution:

$v = \sqrt{r'^2 + (r\theta')^2}$ $\quad v = 3.027\ \frac{m}{s}$

$a = \sqrt{\left(-r\,\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2}$ $\quad a = 12.625\ \frac{m}{s^2}$

Problem 12–153

The boy slides down the slide at a constant speed v. If the slide is in the form of a helix, defined by the equations r = constant and $z = -(h\theta)/(2\pi)$, determine the boy's angular velocity about the z axis, θ' and the magnitude of his acceleration.

Given:

$v = 2\ \frac{m}{s}$

$r = 1.5\ m$

$h = 2\ m$

Solution:

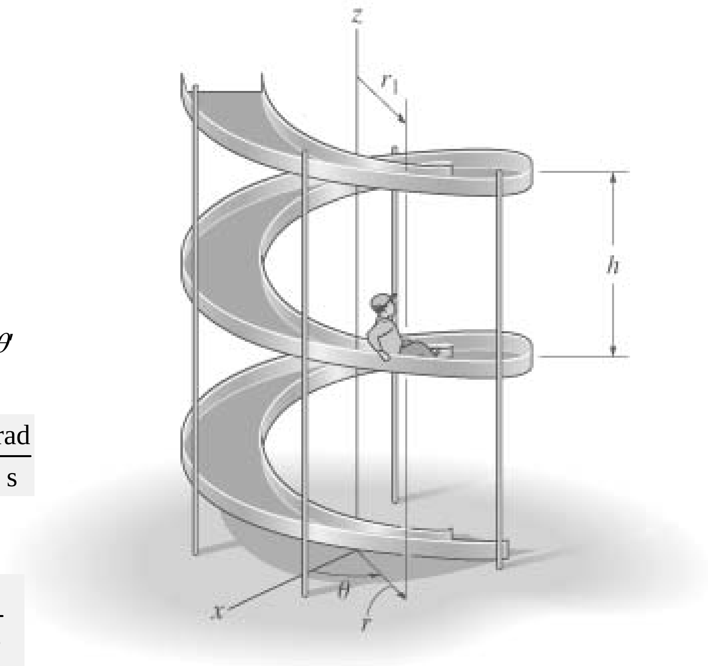
$$z = \frac{h}{2\pi}\theta$$

$$z' = \frac{h}{2\pi}\theta'$$

$$v = \sqrt{z'^2 + (r\theta')^2} = \sqrt{\left(\frac{h}{2\pi}\right)^2 + r^2}\,\theta'$$

$$\theta' = \frac{v}{\sqrt{\left(\frac{h}{2\pi}\right)^2 + r^2}} \qquad \theta' = 1.304\ \frac{\text{rad}}{\text{s}}$$

$$a = \left|-r\theta'^2\right| \qquad a = 2.55\ \frac{\text{m}}{\text{s}^2}$$



Problem 12–154

A cameraman standing at *A* is following the movement of a race car, *B*, which is traveling along a straight track at a constant speed *v*. Determine the angular rate at which he must turn in order to keep the camera directed on the car at the instant $\theta = \theta_1$.

Given:

$$v = 80\ \frac{\text{ft}}{\text{s}} \qquad \theta_1 = 60\ \text{deg} \qquad a = 100\ \text{ft}$$

Solution: $\theta = \theta_1$

$$a = r\sin(\theta)$$

$$0 = r'\sin(\theta) + r\theta'\cos(\theta)$$

$$x = r\cos(\theta)$$

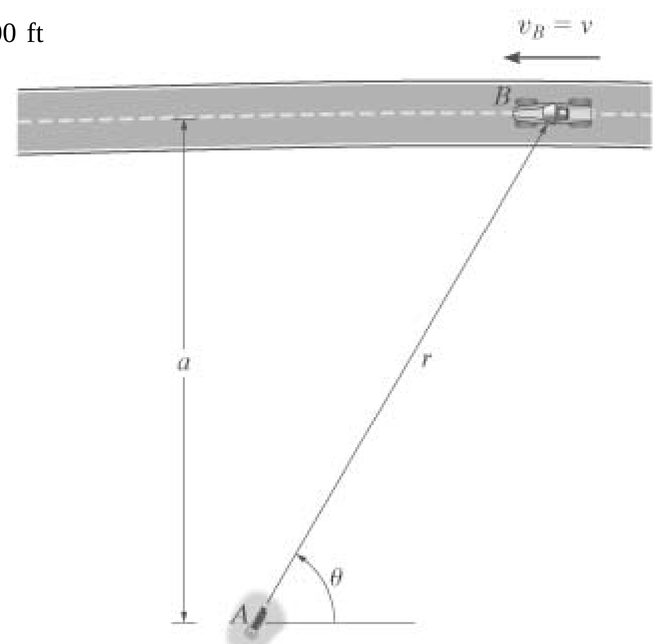
$$v_x = -v = r'\cos(\theta) - r\theta'\sin(\theta)$$

Guess

$$r = 1\ \text{ft} \qquad r' = 1\ \frac{\text{ft}}{\text{s}} \qquad \theta' = 1\ \frac{\text{rad}}{\text{s}}$$

Given

$$a = r\sin(\theta)$$



$$0 = r' \sin(\theta) + r\theta' \cos(\theta)$$

$$-v = r' \cos(\theta) - r\theta' \sin(\theta)$$

$$\begin{pmatrix} r \\ r' \\ \theta' \end{pmatrix} = \text{Find}(r, r', \theta')$$

$r = 115.47\ \text{ft}$ $\quad r' = -40\ \dfrac{\text{ft}}{\text{s}}$ $\quad \theta' = 0.6\ \dfrac{\text{rad}}{\text{s}}$

Problem 12-155

For a short distance the train travels along a track having the shape of a spiral, $r = a/\theta$. If it maintains a constant speed v, determine the radial and transverse components of its velocity when $\theta = \theta_1$.

Given: $a = 1000\ \text{m}$ $\quad v = 20\ \dfrac{\text{m}}{\text{s}}$ $\quad \theta_1 = 9\ \dfrac{\pi}{4}\ \text{rad}$

Solution: $\theta = \theta_1$

$$r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta' \qquad v^2 = r'^2 + r^2\theta'^2 = \left(\frac{a^2}{\theta^4} + \frac{a^2}{\theta^2}\right)\theta'^2$$

$$\theta' = \frac{v\theta^2}{a\sqrt{1+\theta^2}} \qquad r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta'$$

$v_r = r'$ $\quad v_r = -2.802\ \dfrac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $\quad v_\theta = 19.803\ \dfrac{\text{m}}{\text{s}}$

***Problem 12-156**

For a short distance the train travels along a track having the shape of a spiral, $r = a / \theta$. If the angular rate θ' is constant, determine the radial and transverse components of its velocity and acceleration when $\theta = \theta_1$.

Given: $a = 1000\ \text{m}$ $\quad \theta' = 0.2\ \dfrac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 9\dfrac{\pi}{4}$

Solution: $\theta = \theta_1$

$$r = \frac{a}{\theta} \qquad r' = \frac{-a}{\theta^2}\theta' \qquad r'' = \frac{2a}{\theta^3}\theta'^2$$

$v_r = r'$ $\quad v_r = -4.003\ \frac{m}{s}$

$v_\theta = r\theta'$ $\quad v_\theta = 28.3\ \frac{m}{s}$

$a_r = r'' - r\theta'^2$ $\quad a_r = -5.432\ \frac{m}{s^2}$

$a_\theta = 2r'\theta'$ $\quad a_\theta = -1.601\ \frac{m}{s^2}$

Problem 12-157

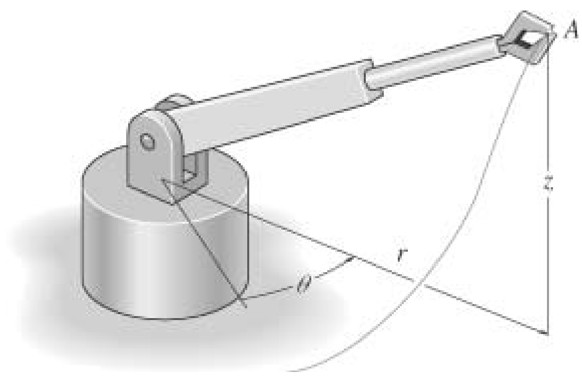
The arm of the robot has a variable length so that *r* remains constant and its grip. *A* moves along the path $z = a\ \sin b\theta$. If $\theta = ct$, determine the magnitudes of the grip's velocity and acceleration when $t = t_1$.

Given:

$r = 3\ \text{ft}$ $\quad c = 0.5\ \frac{\text{rad}}{\text{s}}$

$a = 3\ \text{ft}$ $\quad t_1 = 3\ \text{s}$

$b = 4$



Solution: $\quad t = t_1$

$\theta = ct \qquad r = r \qquad z = a\sin(bct)$

$\theta' = c \qquad r' = 0\frac{\text{ft}}{\text{s}} \qquad z' = abc\cos(bct)$

$\theta'' = 0\frac{\text{rad}}{\text{s}^2} \qquad r'' = 0\frac{\text{ft}}{\text{s}^2} \qquad z'' = -ab^2c^2\sin(bct)$

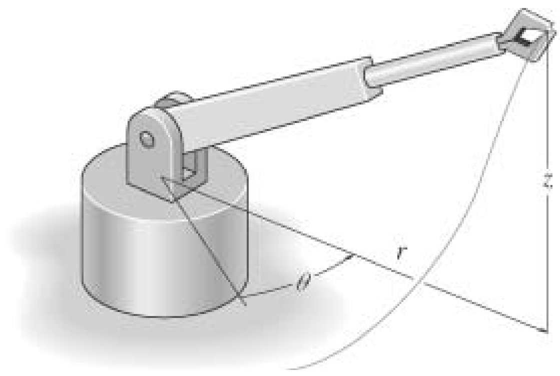
$v = \sqrt{r'^2 + (r\theta')^2 + z'^2}$ $\quad v = 5.953\frac{\text{ft}}{\text{s}}$

$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2 + z''^2}$ $\quad a = 3.436\frac{\text{ft}}{\text{s}^2}$

Problem 12-158

For a short time the arm of the robot is extending so that r' remains constant, $z = bt^2$ and $\theta = ct$. Determine the magnitudes of the velocity and acceleration of the grip *A* when $t = t_1$ and $r = r_1$.

Given:



$r' = 1.5 \frac{ft}{s}$

$b = 4 \frac{ft}{s^2}$

$c = 0.5 \frac{rad}{s}$

$t_1 = 3\ s$

$r_1 = 3\ ft$

Solution: $t = t_1$

$r = r_1$ $\theta = ct$ $z = bt^2$

$\theta' = c$ $z' = 2bt$ $z'' = 2b$

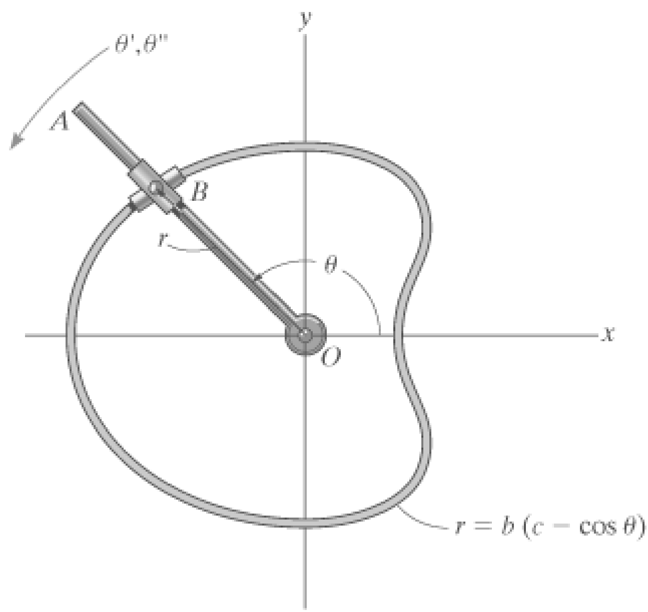
$v = \sqrt{r'^2 + (r\theta')^2 + z'^2}$ $v = 24.1 \frac{ft}{s}$

$a = \sqrt{\left(-r\theta'^2\right)^2 + (2r'\theta')^2 + z''^2}$ $a = 8.174 \frac{ft}{s^2}$

Problem 12–159

The rod *OA* rotates counterclockwise with a constant angular velocity of θ'. Two pin-connected slider blocks, located at *B*, move freely on *OA* and the curved rod whose shape is a limaçon described by the equation $r = b(c - \cos(\theta))$. Determine the speed of the slider blocks at the instant $\theta = \theta_1$.

Given:



$\theta' = 5 \frac{rad}{s}$

$b = 100\ mm$

$c = 2$

$\theta_1 = 120\ deg$

Solution:

$\theta = \theta_1$

$r = b(c - \cos(\theta))$

$r' = b\sin(\theta)\theta'$

$v = \sqrt{r'^2 + (r\theta')^2}$ $v = 1.323 \frac{m}{s}$

***Problem 12–160**

The rod OA rotates counterclockwise with a constant angular velocity of θ'. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = b(c - \cos(\theta))$. Determine the acceleration of the slider blocks at the instant $\theta = \theta_1$.

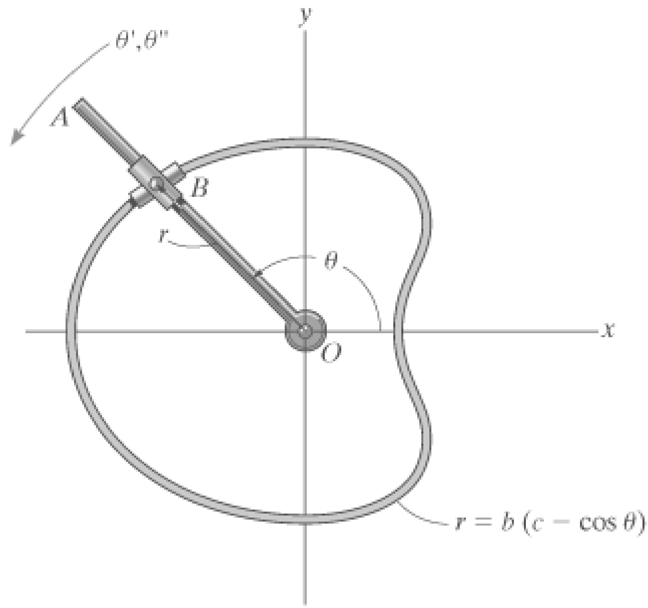
Given:

$\theta' = 5 \frac{rad}{s}$

$b = 100 \text{ mm}$

$c = 2$

$\theta_1 = 120 \text{ deg}$



Solution:

$\theta = \theta_1$

$r = b(c - \cos(\theta))$

$r' = b\sin(\theta)\theta'$

$r'' = b\cos(\theta)\,\theta'^2$

$a = \sqrt{(r'' - r\theta'^2)^2 + (2r'\theta')^2}$ $a = 8.66 \frac{m}{s^2}$

Problem 12-161

The searchlight on the boat anchored a distance d from shore is turned on the automobile, which is traveling along the straight road at a constant speed v. Determine the angular rate of rotation of the light when the automobile is $r = r_1$ from the boat.

Given:

$d = 2000 \text{ ft}$

$v = 80 \frac{ft}{s}$

$r_1 = 3000 \text{ ft}$

Solution:

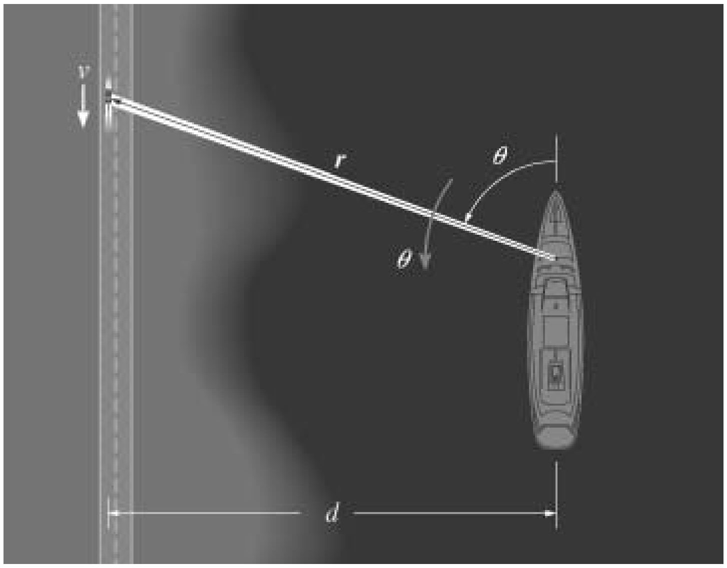
$r = r_1$

$\theta = \operatorname{asin}\left(\frac{d}{r}\right)$

$\theta = 41.81 \text{ deg}$

$\theta' = \frac{v \sin(\theta)}{r}$

$\theta' = 0.0178 \frac{\text{rad}}{\text{s}}$



Problem 12-162

The searchlight on the boat anchored a distance d from shore is turned on the automobile, which is traveling along the straight road at speed v and acceleration a. Determine the required angular acceleration θ'' of the light when the automobile is $r = r_1$ from the boat.

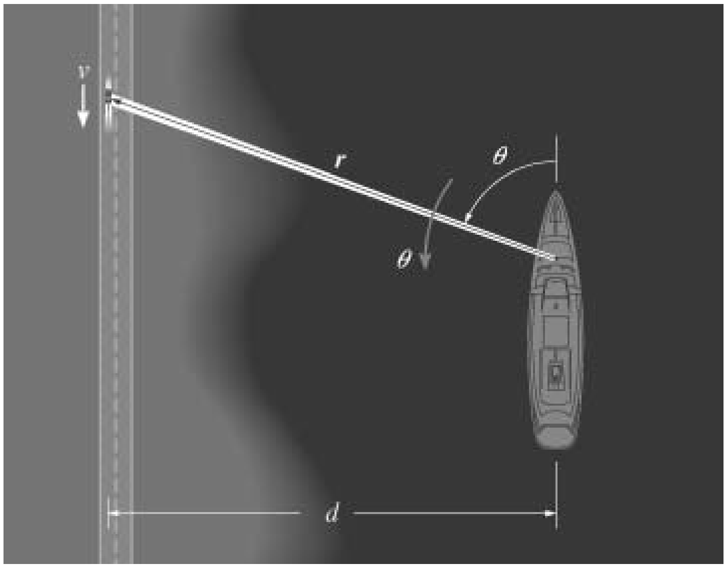
Given:

$d = 2000 \text{ ft}$

$v = 80 \frac{\text{ft}}{\text{s}}$

$a = 15 \frac{\text{ft}}{\text{s}^2}$

$r_1 = 3000 \text{ ft}$



Solution:

$r = r_1$

$\theta = \operatorname{asin}\left(\frac{d}{r}\right) \quad \theta = 41.81 \text{ deg}$

$\theta' = \frac{v \sin(\theta)}{r} \quad \theta' = 0.0178 \frac{\text{rad}}{\text{s}}$

$r' = -v \cos(\theta) \quad r' = -59.628 \frac{\text{ft}}{\text{s}}$

$$\theta'' = \frac{a\sin(\theta) - 2r'\theta'}{r}$$

$$\theta'' = 0.00404\ \frac{\text{rad}}{\text{s}^2}$$

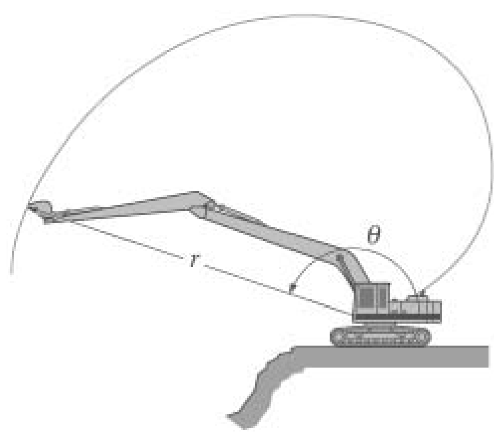
Problem 12–163

For a short time the bucket of the backhoe traces the path of the cardioid $r = a(1 - \cos\theta)$. Determine the magnitudes of the velocity and acceleration of the bucket at $\theta = \theta_1$ if the boom is rotating with an angular velocity θ' and an angular acceleration θ'' at the instant shown.

Given:

$a = 25\ \text{ft}$ $\qquad \theta' = 2\ \frac{\text{rad}}{\text{s}}$

$\theta_1 = 120\ \text{deg}$ $\qquad \theta'' = 0.2\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\theta = \theta_1$$

$$r = a(1 - \cos(\theta)) \qquad r' = a\sin(\theta)\theta'$$

$$r'' = a\sin(\theta)\theta'' + a\cos(\theta)\,\theta'^2$$

$$v = \sqrt{r'^2 + (r\theta')^2} \qquad v = 86.6\ \frac{\text{ft}}{\text{s}}$$

$$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2} \qquad a = 266\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 12-164**

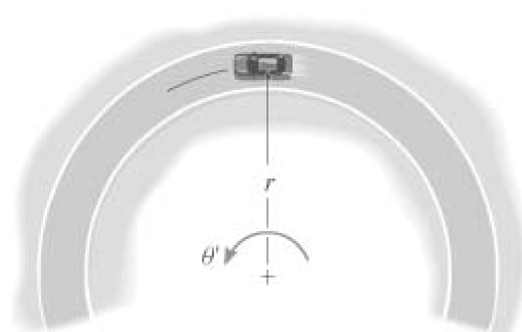
A car is traveling along the circular curve having a radius r. At the instance shown, its angular rate of rotation is θ', which is decreasing at the rate θ''. Determine the radial and transverse components of the car's velocity and acceleration at this instant.

Given:

$r = 400\ \text{ft}$

$\theta' = 0.025\ \frac{\text{rad}}{\text{s}}$

$\theta'' = -0.008\ \frac{\text{rad}}{\text{s}^2}$



Solution:

$v_r = r\theta'$ $\quad v_r = 3.048\ \frac{\text{m}}{\text{s}}$

$v_\theta = 0$

$a_r = r\theta''$ $\quad a_r = -0.975\ \frac{\text{m}}{\text{s}^2}$

$a_\theta = r\theta'^2$ $\quad a_\theta = 0.076\ \frac{\text{m}}{\text{s}^2}$

Problem 12–165

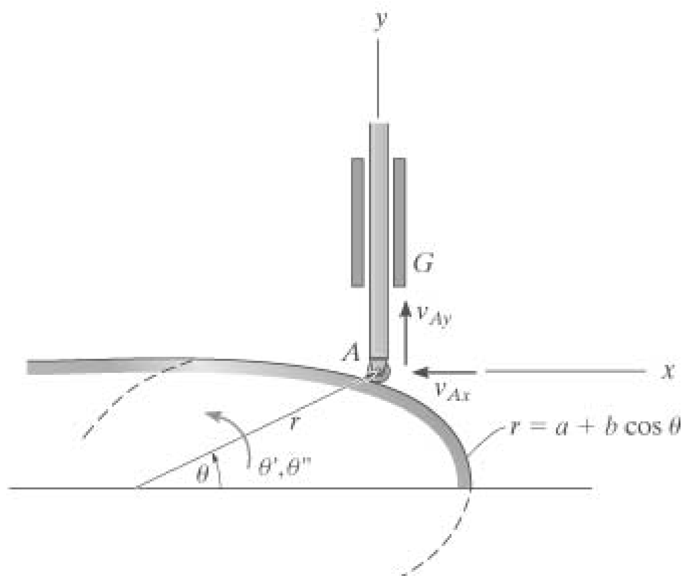
The mechanism of a machine is constructed so that for a short time the roller at *A* follows the surface of the cam described by the equation $r = a + b\cos\theta$. If θ' and θ'' are given, determine the magnitudes of the roller's velocity and acceleration at the instant $\theta = \theta_1$. Neglect the size of the roller. Also determine the velocity components v_{Ax} and v_{Ay} of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides move horizontally to the left.

Given:

$\theta' = 0.5\ \frac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 30\ \text{deg}$

$a = 0.3\ \text{m}$

$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 0.2\ \text{m}$



Solution:

$\theta = \theta_1$

$r = a + b\cos(\theta)$

$r' = -b\sin(\theta)\theta'$

$r'' = -b\sin(\theta)\theta'' - b\cos(\theta)\,\theta'^2$

$v = \sqrt{r'^2 + (r\theta')^2}$ $\quad v = 0.242\ \frac{\text{m}}{\text{s}}$

$a = \sqrt{\left(r'' - r\theta'^2\right)^2 + \left(r\theta'' + 2r'\theta'\right)^2}$ $\quad a = 0.169\ \frac{\text{m}}{\text{s}^2}$

$v_{Ax} = -r'\cos(\theta) + r\theta'\sin(\theta)$ $v_{Ax} = 0.162\ \frac{m}{s}$

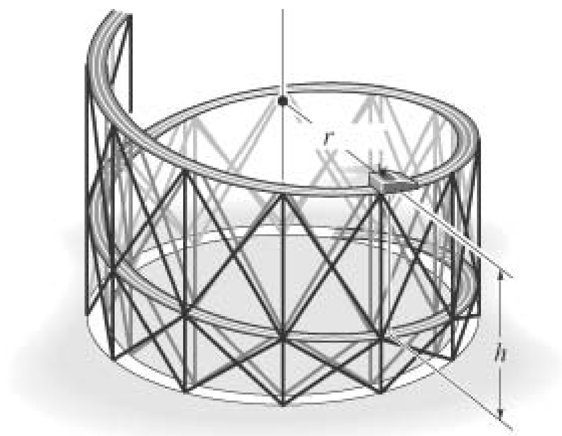
$v_{Ay} = r'\sin(\theta) + r\theta'\cos(\theta)$ $v_{Ay} = 0.18\ \frac{m}{s}$

Problem 12-166

The roller coaster is traveling down along the spiral ramp with a constant speed v. If the track descends a distance h for every full revolution, determine the magnitude of the roller coaster's acceleration as it moves along the track, r of radius. *Hint*: For part of the solution, note that the tangent to the ramp at any point is at an angle $\phi = \tan^{-1}(h/2\pi r)$ from the horizontal. Use this to determine the velocity components v_θ and v_z which in turn are used to determine θ and z.

Given:

$v = 6\ \frac{m}{s}$ $h = 10\ m$ $r = 5\ m$



Solution:

$\phi = \operatorname{atan}\left(\frac{h}{2\pi r}\right)$ $\phi = 17.657\ deg$

$\theta' = \frac{v\cos(\phi)}{r}$ $a = \left|-r\theta'^2\right|$

$a = 6.538\ \frac{m}{s^2}$

Problem 12-167

A cameraman standing at A is following the movement of a race car, B, which is traveling around a curved track at constant speed v_B. Determine the angular rate at which the man must turn in order to keep the camera directed on the car at the instant $\theta = \theta_1$.

Given:

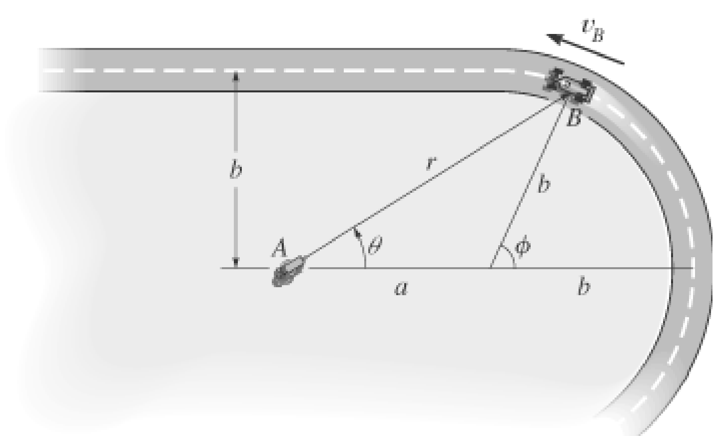
$v_B = 30\ \frac{m}{s}$

$\theta_1 = 30\ deg$

$a = 20\ m$

$b = 20\ m$

$\theta = \theta_1$



Solution:

Guess

$r = 1\ \text{m} \qquad r' = 1\ \frac{\text{m}}{\text{s}} \qquad \theta' = 1\ \frac{\text{rad}}{\text{s}} \qquad \phi = 20\ \text{deg} \qquad \phi' = 2\ \frac{\text{rad}}{\text{s}}$

Given $r\sin(\theta) = b\sin(\phi)$

$r'\sin(\theta) + r\cos(\theta)\theta' = b\cos(\phi)\phi'$

$r\cos(\theta) = a + b\cos(\phi)$

$r'\cos(\theta) - r\sin(\theta)\theta' = -b\sin(\phi)\phi'$

$v_B = b\phi'$

$\begin{pmatrix} r \\ r' \\ \theta' \\ \phi \\ \phi' \end{pmatrix} = \text{Find}(r, r', \theta', \phi, \phi') \qquad r = 34.641\ \text{m} \qquad r' = -15\ \frac{\text{m}}{\text{s}}$

$\phi = 60\,\text{deg} \qquad \phi' = 1.5\,\frac{\text{rad}}{\text{s}}$

$\theta' = 0.75\,\frac{\text{rad}}{\text{s}}$

---

*__Problem 12-168__

The pin follows the path described by the equation $r = a + b\cos\theta$. At the instant $\theta = \theta_1$. the angular velocity and angular acceleration are θ' and θ''. Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.

Given:

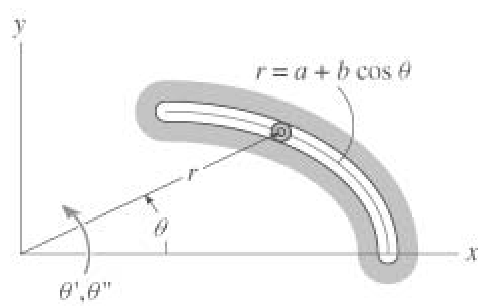
$a = 0.2\ \text{m}$

$b = 0.15\ \text{m}$

$\theta_1 = 30\ \text{deg}$

$\theta' = 0.7\ \frac{\text{rad}}{\text{s}}$

$\theta'' = 0.5\,\frac{\text{rad}}{\text{s}^2}$



Solution: $\theta = \theta_1$

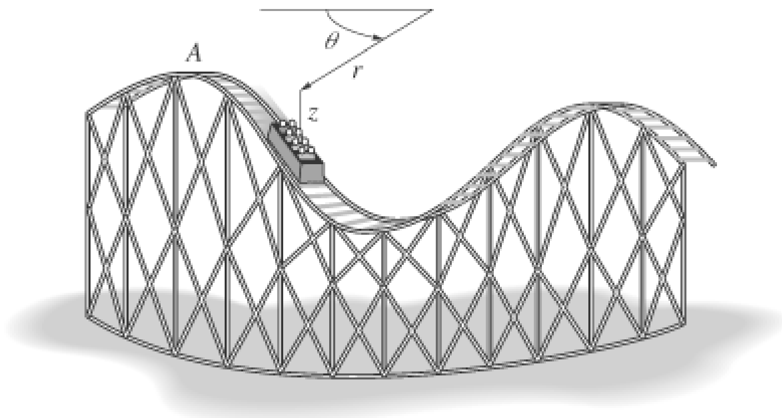
$r = a + b\cos(\theta) \qquad r' = -b\sin(\theta)\theta' \qquad r'' = -b\cos(\theta)\,\theta'^2 - b\sin(\theta)\theta''$

$v = \sqrt{r'^2 + (r\theta')^2}$ $\qquad$ $v = 0.237\ \frac{\text{m}}{\text{s}}$

$a = \sqrt{(r'' - r\theta'^2)^2 + (r\theta'' + 2r'\theta')^2}$ $\qquad$ $a = 0.278\ \frac{\text{m}}{\text{s}^2}$

Problem 12-169

For a short time the position of the roller-coaster car along its path is defined by the equations $r = r_0$, $\theta = at$, and $z = b\cos\theta$. Determine the magnitude of the car's velocity and acceleration when $t = t_1$.



Given:

$r_0 = 25\ \text{m}$

$a = 0.3\ \frac{\text{rad}}{\text{s}}$

$b = -8\ \text{m}$

$t_1 = 4\ \text{s}$

Solution: $\quad t = t_1$

$r = r_0 \qquad \theta = a\,t \qquad z = b\cos(\theta)$

$\theta' = a \qquad z' = -b\sin(\theta)\theta'$

$z'' = -b\cos(\theta)\,\theta'^2$

$v = \sqrt{(r\theta')^2 + z'^2}$ $\qquad$ $v = 7.826\ \frac{\text{m}}{\text{s}}$

$a = \sqrt{(-r\,\theta'^2)^2 + z''^2}$ $\qquad$ $a = 2.265\ \frac{\text{m}}{\text{s}^2}$

Problem 12-170

The small washer is sliding down the cord *OA*. When it is at the midpoint, its speed is *v* and its acceleration is *a'*. Express the velocity and acceleration of the washer at this point in terms of its cylindrical components.

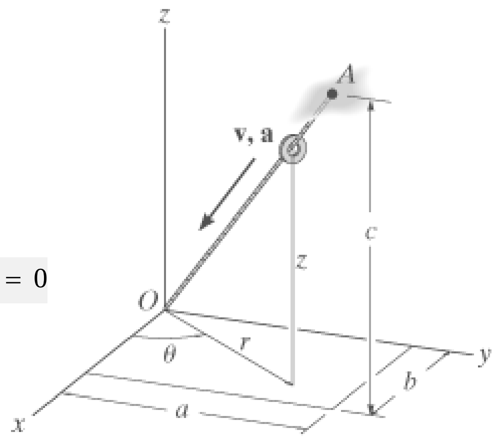
Given:

$v = 200\ \frac{\text{mm}}{\text{s}} \qquad a' = 10\ \frac{\text{mm}}{\text{s}^2}$

$a = 400\ \text{mm}$

$b = 300\ \text{mm}$

$c = 700\ \text{mm}$



Solution:

$v_r = \dfrac{-v\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + c^2}}$ $\quad v_r = -0.116\ \dfrac{\text{m}}{\text{s}}$ $\quad v_\theta = 0$

$v_z = \dfrac{-v\,c}{\sqrt{a^2 + b^2 + c^2}}$ $\quad v_z = -0.163\ \dfrac{\text{m}}{\text{s}}$

$a_r = -a\cos(\alpha)$

$a_r = \dfrac{-a'\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + c^2}}$ $\quad a_r = -5.812 \times 10^{-3}\ \dfrac{\text{m}}{\text{s}^2}$ $\quad a_\theta = 0$

$a_z = \dfrac{-v\,c}{\sqrt{a^2 + b^2 + c^2}}$ $\quad a_z = -0.163\ \dfrac{\text{m}}{\text{s}}$

Problem 12–171

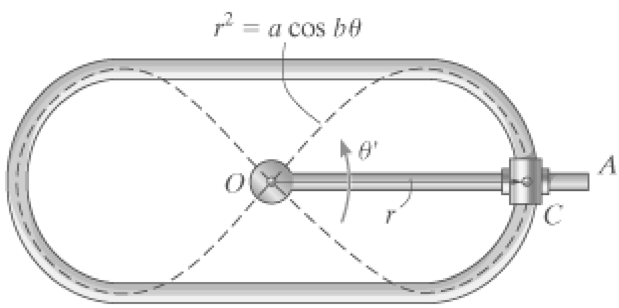
A double collar C is pin-connected together such that one collar slides over a fixed rod and the other slides over a rotating rod. If the geometry of the fixed rod for a short distance can be defined by a lemniscate, $r^2 = (a \cos b\theta)$, determine the collar's radial and transverse components of velocity and acceleration at the instant $\theta = 0°$ as shown. Rod OA is rotating at a constant rate of θ'.

Given:

$a = 4\ \text{ft}^2$

$b = 2$

$\theta' = 6\ \dfrac{\text{rad}}{\text{s}}$



Solution:

$\theta = 0\ \text{deg}$ $\qquad r = \sqrt{a\cos(b\theta)}$

$r^2 = a\cos(b\theta)$

$2r\,r' = -a\,b\sin(b\theta)\theta'$ $\qquad r' = \dfrac{-a\,b\sin(b\theta)\theta'}{2r}$

$$2r\,r'' + 2r'^2 = -a\,b^2\cos(b\theta)\,\theta'^2 \qquad r'' = \frac{-a\,b^2\cos(b\theta)\,\theta'^2 - 2r'^2}{2r}$$

$v_r = r'$ $\quad v_r = 0\ \frac{\text{m}}{\text{s}}$

$v_\theta = r\theta'$ $\quad v_\theta = 12\frac{\text{ft}}{\text{s}}$

$a_r = r'' - r\theta'^2$ $\quad a_r = -216\frac{\text{ft}}{\text{s}^2}$

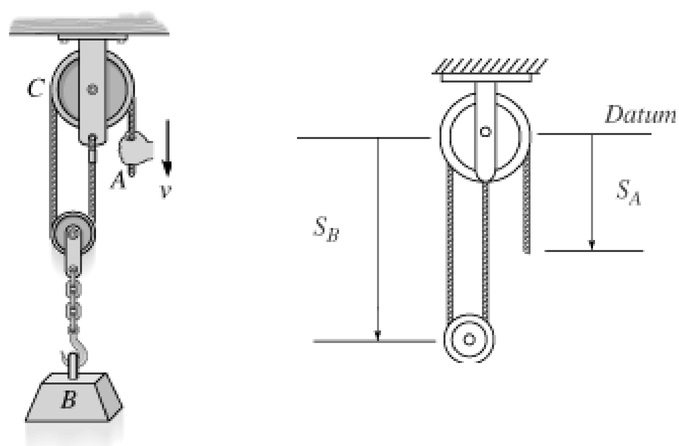
$a_\theta = 2r'\theta'$ $\quad a_\theta = 0\frac{\text{ft}}{\text{s}^2}$

---

*__Problem 12-172__

If the end of the cable at *A* is pulled down with speed *v*, determine the speed at which block *B* rises.

Given: $v = 2\ \frac{\text{m}}{\text{s}}$



Solution:

$v_A = v$

$L = 2s_B + s_A$

$0 = 2v_B + v_A$

$v_B = \frac{-v_A}{2}$

$v_B = -1\ \frac{\text{m}}{\text{s}}$

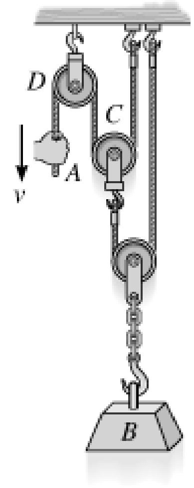
Problem 12-173

If the end of the cable at *A* is pulled down with speed *v*, determine the speed at which block *B* rises.

Given:

$v = 2\ \frac{\text{m}}{\text{s}}$

Solution:



$v_A = v$

$L_1 = s_A + 2s_C$

$0 = v_A + 2v_C \qquad v_C = \frac{-v_A}{2}$

$L_2 = (s_B - s_C) + s_B \quad 0 = 2v_B - v_C$

$v_B = \frac{v_C}{2} \qquad v_B = -0.5\ \frac{\text{m}}{\text{s}}$

Problem 12-174

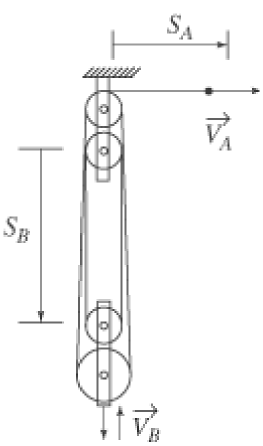
Determine the constant speed at which the cable at *A* must be drawn in by the motor in order to hoist the load at *B* a distance *d* in a time *t*.

Given:

$d = 15\ \text{ft}$

$t = 5\ \text{s}$

Solution:



$L = 4s_B + s_A$

$0 = 4v_B + v_A$

$v_A = -4v_B$

$v_A = -4\left(\frac{-d}{t}\right)$

$v_A = 12\ \frac{\text{ft}}{\text{s}}$

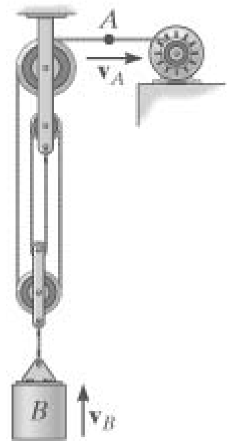
Problem 12-175

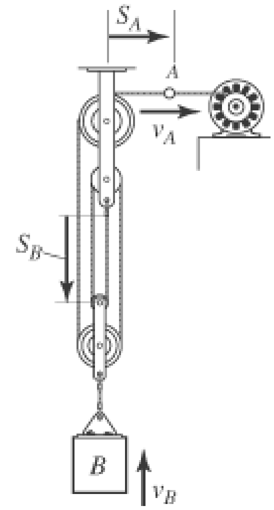
Determine the time needed for the load at *B* to attain speed *v*, starting from rest, if the cable is drawn into the motor with acceleration *a*.

Given:

$v = -8\ \frac{\text{m}}{\text{s}}$

$a = 0.2\ \frac{\text{m}}{\text{s}^2}$





Solution:

$v_B = v$

$L = 4s_B + s_A$

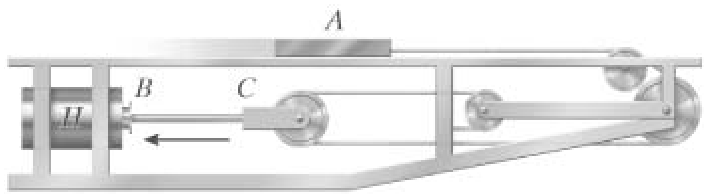
$0 = 4v_B + v_A$

$v_B = \frac{-v_A}{4} = \frac{-1}{4} a\, t$

$t = \frac{-4v_B}{a}$ $\quad$ $t = 160\ \text{s}$

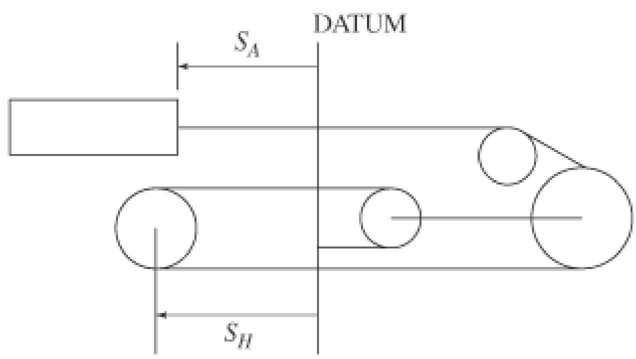
***Problem 12–176**

If the hydraulic cylinder at H draws rod BC in by a distance d, determine how far the slider at A moves.



Given:

$d = 8\ \text{in}$



Solution:

$\Delta s_H = d$

$L = s_A + 2s_H$ $\quad$ $0 = \Delta s_A + 2\Delta s_H$

$\Delta s_A = -2\Delta s_H$ $\quad$ $\Delta s_A = -16\,\text{in}$

Problem 12-177

The crate is being lifted up the inclined plane using the motor M and the rope and pulley arrangement shown. Determine the speed at which the cable must be taken up by the motor in order to move the crate up the plane with constant speed v.

Given:

$v = 4\ \frac{\text{ft}}{\text{s}}$

Solution:

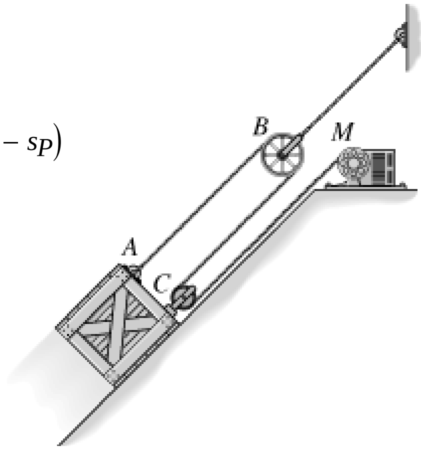
$v_A = v$

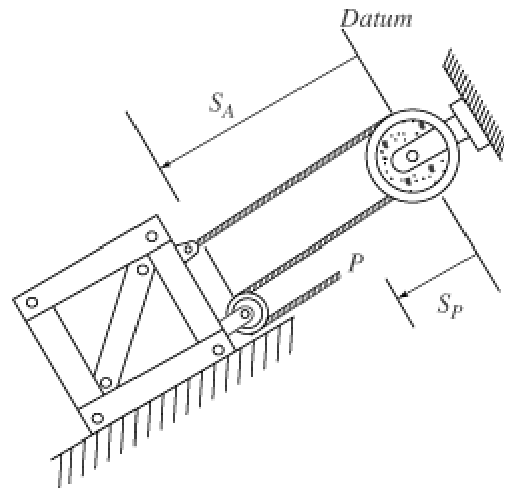
$L = 2s_A + (s_A - s_P)$

$0 = 3v_A - v_P$

$v_P = 3v_A$

$v_P = 12\,\frac{\text{ft}}{\text{s}}$



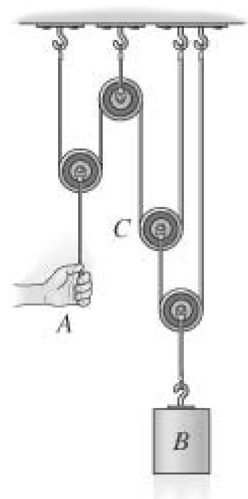


Problem 12-178

Determine the displacement of the block at B if A is pulled down a distance d.

Given:

$d = 4\text{ ft}$



Solution:

$\Delta s_A = d$

$L_1 = 2s_A + 2s_C \qquad L_2 = (s_B - s_C) + s_B$

$0 = 2\Delta s_A + 2\Delta s_C \qquad 0 = 2\Delta s_B - \Delta s_C$

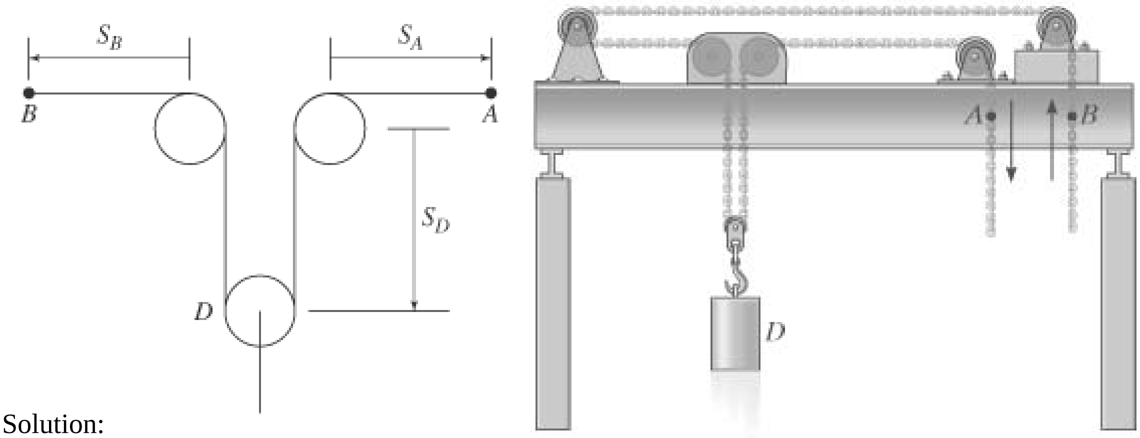
$\Delta s_C = -\Delta s_A \qquad \Delta s_B = \frac{\Delta s_C}{2} \qquad \Delta s_B = -2\text{ ft}$

Problem 12–179

The hoist is used to lift the load at D. If the end A of the chain is travelling downward at v_A and the end B is travelling upward at v_B, determine the velocity of the load at D.

Given:

$v_A = 5\,\frac{\text{ft}}{\text{s}} \qquad v_B = 2\,\frac{\text{ft}}{\text{s}}$



Solution:

$L = s_B + s_A + 2s_D \qquad 0 = -v_B + v_A + 2v_D$

$v_D = \dfrac{v_B - v_A}{2} \qquad v_D = -1.5\,\dfrac{\text{ft}}{\text{s}}$ Positive means down, Negative means up

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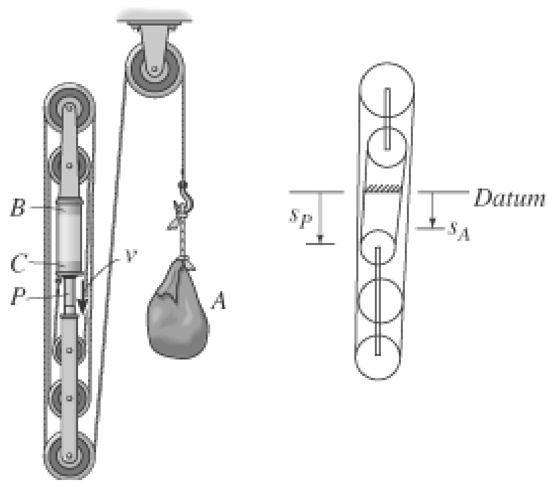
***Problem 12-180**

The pulley arrangement shown is designed for hoisting materials. If *BC remains fixed* while the plunger *P* is pushed downward with speed *v*, determine the speed of the load at *A*.



Given:

$v = 4\,\dfrac{\text{ft}}{\text{s}}$

Solution:

$v_P = v$

$L = 6s_P + s_A \qquad 0 = 6v_P + v_A$

$v_A = -6v_P \qquad v_A = -24\,\dfrac{\text{ft}}{\text{s}}$

Problem 12-181

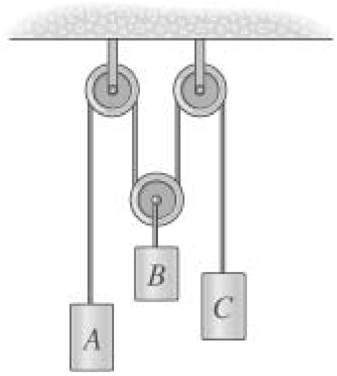
If block *A* is moving downward with speed v_A while *C* is moving up at speed v_C, determine the speed of block *B*.

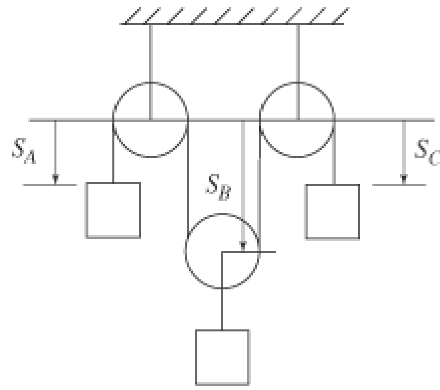
Given:

$v_A = 4\,\dfrac{\text{ft}}{\text{s}}$

$$v_C = -2\ \frac{\text{ft}}{\text{s}}$$

Solution:





$$S_A + 2S_B + S_C = L$$

Taking time derivative:

$$v_A + 2v_B + v_C = 0$$

$$v_B = \frac{-(v_C + v_A)}{2}$$

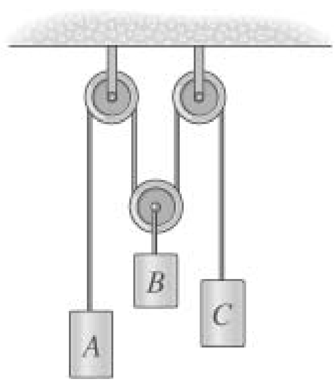
$$v_B = -1\,\frac{\text{ft}}{\text{s}}$$ Positive means down, negative means up.

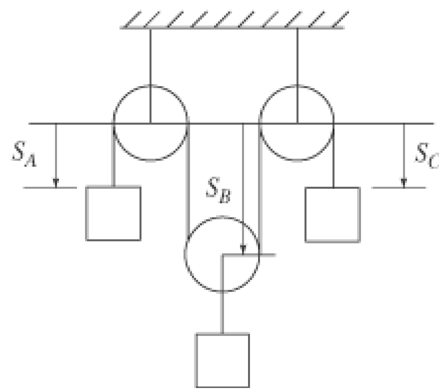
Problem 12-182

If block A is moving downward at speed v_A while block C is moving down at speed v_C, determine the relative velocity of block B with respect to C.

Given:

$$v_A = 6\ \frac{\text{ft}}{\text{s}} \quad v_C = 18\ \frac{\text{ft}}{\text{s}}$$





Solution:

$$S_A + 2S_B + S_C = L$$

Taking time derivative

$$v_A + 2v_B + v_C = 0$$

$$v_B = \frac{-(v_A + v_C)}{2} \qquad v_B = -12\,\frac{\text{ft}}{\text{s}}$$

$$v_{BC} = v_B - v_C \qquad v_{BC} = -30\,\frac{\text{ft}}{\text{s}}$$ Positive means down, negative means up

---

Problem 12–183

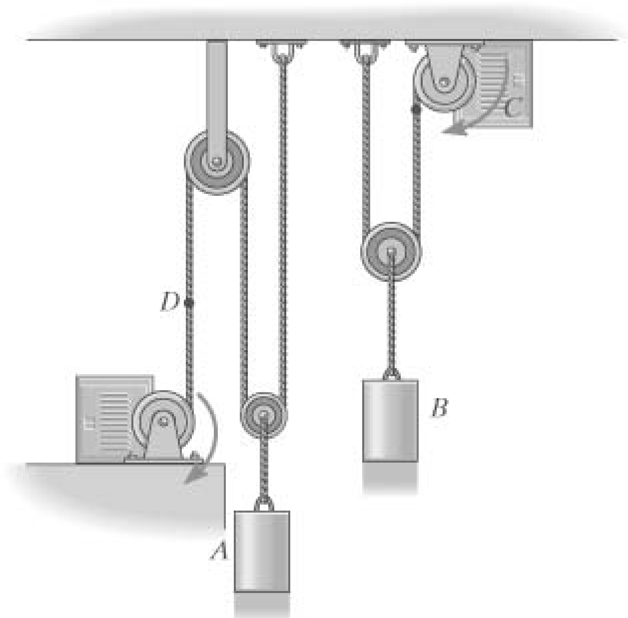
The motor draws in the cable at C with a constant velocity v_C. The motor draws in the cable at D with a constant acceleration of a_D. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise a distance h, and (b) the relative velocity of block A with respect to block B when this occurs.

Given:

$v_C = -4 \frac{m}{s}$

$a_D = 8 \frac{m}{s^2}$

$h = 3\ m$



Solution:

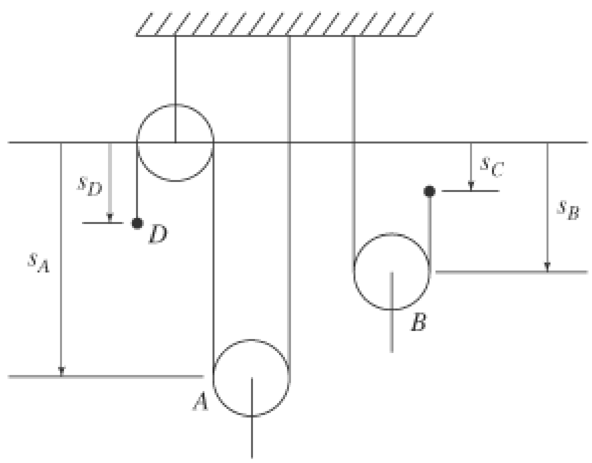
$L_1 = s_D + 2s_A$

$0 = v_D + 2v_A$

$0 = a_D + 2a_A$

$L_2 = s_B + (s_B - s_C)$

$0 = 2v_B - v_C \qquad 0 = 2a_B - a_C$



$a_A = \frac{-a_D}{2}$

$v_A = a_A t$

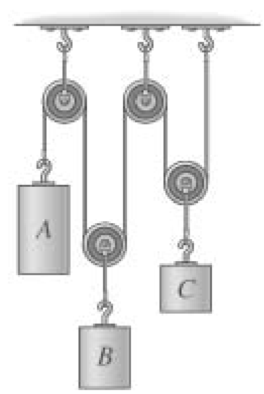
$s_A = -h = a_A\left(\frac{t^2}{2}\right)$

$t = \sqrt{\frac{-2h}{a_A}} \qquad t = 1.225\ s$

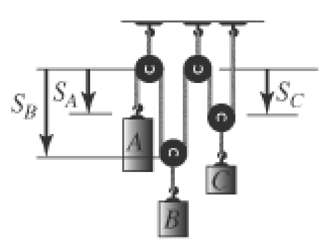
$v_A = a_A t \qquad v_B = \frac{1}{2}v_C \qquad v_{AB} = v_A - v_B \qquad v_{AB} = -2.90 \frac{m}{s}$

***Problem 12-184**

If block A of the pulley system is moving downward with speed v_A while block C is moving up at v_C determine the speed of block B.



Given:



$v_A = 4 \frac{ft}{s}$

$v_C = -2 \frac{ft}{s}$

Solution:

$$S_A + 2S_B + 2S_C = L$$

$$v_A + 2v_B + 2v_C = 0 \qquad v_B = \frac{-2v_C - v_A}{2} \qquad v_B = 0\ \frac{\text{m}}{\text{s}}$$

Problem 12–185

If the point A on the cable is moving upwards at v_A, determine the speed of block B.

Given: $v_A = -14\ \frac{\text{m}}{\text{s}}$

Solution:

$$L_1 = (s_D - s_A) + (s_D - s_E)$$

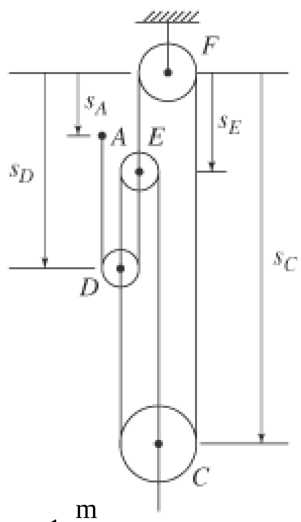
$$0 = 2v_D - v_A - v_E$$

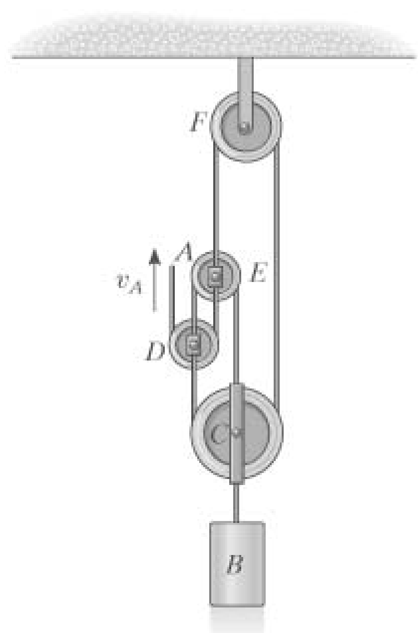
$$L_2 = (s_D - s_E) + (s_C - s_E)$$

$$0 = v_D + v_C - 2v_E$$

$$L_3 = (s_C - s_D) + s_C + s_E$$

$$0 = 2v_C - v_D + v_E$$





Guesses

$$v_C = 1\ \frac{\text{m}}{\text{s}} \qquad v_D = 1\ \frac{\text{m}}{\text{s}} \qquad v_E = 1\ \frac{\text{m}}{\text{s}}$$

Given $\quad 0 = 2v_D - v_A - v_E$

$$0 = v_D + v_C - 2v_E$$

$$0 = 2v_C - v_D + v_E$$

$$\begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \text{Find}(v_C, v_D, v_E) \qquad \begin{pmatrix} v_C \\ v_D \\ v_E \end{pmatrix} = \begin{pmatrix} -2 \\ -10 \\ -6 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$v_B = v_C \qquad v_B = -2\ \frac{\text{m}}{\text{s}}$$

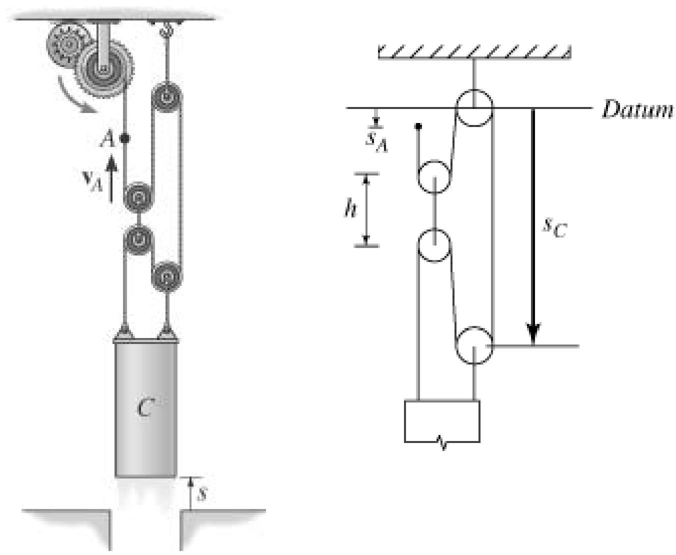
Positive means down,
Negative means up

Problem 12-186

The cylinder C is being lifted using the cable and pulley system shown. If point A on the cable is being drawn toward the drum with speed of v_A, determine the speed of the cylinder.

Given:

$$v_A = -2\ \frac{\text{m}}{\text{s}}$$



Solution:

$$L = 2s_C + (s_C - s_A)$$

$$0 = 3v_C - v_A$$

$$v_C = \frac{v_A}{3}$$

$$v_C = -0.667\ \frac{\text{m}}{\text{s}}$$

Positive means down, negative means up.

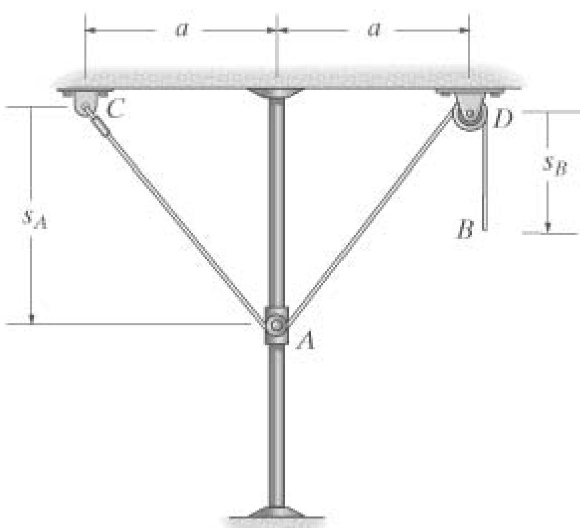
Problem 12–187

The cord is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. Determine the velocity and acceleration of the end of the cord at B if at the instant $s_A = b$ the collar is moving upwards at speed v, which is decreasing at rate a.

Given:

$$a = 3\ \text{ft} \qquad v_A = -5\ \frac{\text{ft}}{\text{s}}$$

$$b = 4\ \text{ft} \qquad a_A = 2\ \frac{\text{ft}}{\text{s}^2}$$



Solution:

$$L = 2\sqrt{a^2 + s_A^2} + s_B \qquad s_A = b$$

Guesses $\quad v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $$0 = \frac{2s_A v_A}{\sqrt{a^2 + s_A^2}} + v_B$$

$$0 = \frac{2s_A a_A + 2v_A^2}{\sqrt{a^2 + s_A^2}} - \frac{2s_A^2 v_A^2}{\sqrt{(a^2 + s_A^2)^3}} + a_B$$

$$\begin{pmatrix} v_B \\ a_B \end{pmatrix} = \text{Find}(v_B, a_B) \qquad v_B = 8\,\frac{\text{ft}}{\text{s}} \qquad a_B = -6.8\,\frac{\text{ft}}{\text{s}^2}$$

---

*__Problem 12–188__

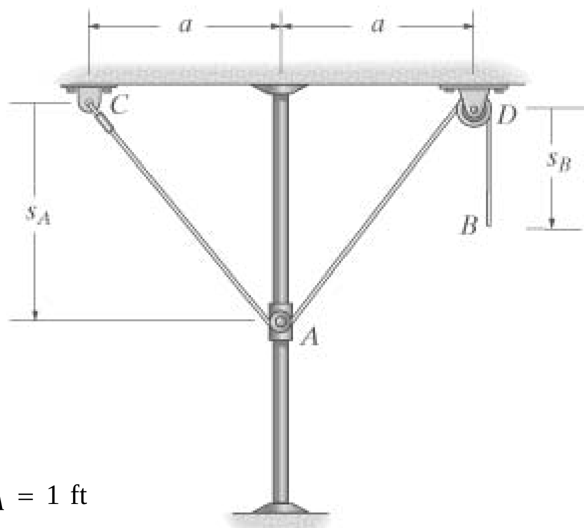
The cord of length L is attached to the pin at C and passes over the two pulleys at A and D. The pulley at A is attached to the smooth collar that travels along the vertical rod. When $s_B = b$, the end of the cord at B is pulled downwards with a velocity v_B and is given an acceleration a_B. Determine the velocity and acceleration of the collar A at this instant.

Given:

$L = 16$ ft

$a = 3$ ft $\qquad v_B = 4\,\frac{\text{ft}}{\text{s}}$

$b = 6$ ft $\qquad a_B = 3\,\frac{\text{ft}}{\text{s}^2}$



Solution: $\qquad s_B = b$

Guesses

$v_A = 1\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1\,\frac{\text{ft}}{\text{s}^2} \qquad s_A = 1$ ft

Given $\qquad L = 2\sqrt{a^2 + s_A^2} + s_B$

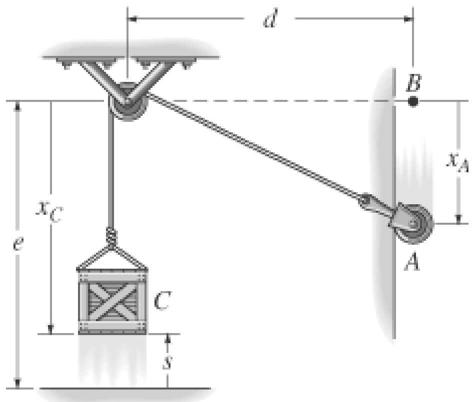
$$0 = \frac{2s_A v_A}{\sqrt{a^2 + s_A^2}} + v_B$$

$$0 = \frac{2s_A a_A + 2v_A^2}{\sqrt{a^2 + s_A^2}} - \frac{2s_A^2 v_A^2}{\sqrt{(a^2 + s_A^2)^3}} + a_B$$

$$\begin{pmatrix} s_A \\ v_A \\ a_A \end{pmatrix} = \text{Find}(s_A, v_A, a_A) \qquad s_A = 4\,\text{ft} \qquad v_A = -2.50\,\frac{\text{ft}}{\text{s}} \qquad a_A = -2.44\,\frac{\text{ft}}{\text{s}^2}$$

Problem 12-189

The crate C is being lifted by moving the roller at A downward with constant speed v_A along the guide. Determine the velocity and acceleration of the crate at the instant $s = s_1$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



Given:

$$v_A = 2\ \frac{\text{m}}{\text{s}}$$

$$s_1 = 1\ \text{m}$$

$$d = 4\ \text{m}$$

$$e = 4\ \text{m}$$

Solution:

$$x_C = e - s_1 \qquad L = d + e$$

Guesses $v_C = 1\ \frac{\text{m}}{\text{s}}$ $\quad a_C = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad x_A = 1\ \text{m}$

Given $L = x_C + \sqrt{x_A^2 + d^2}$ $\qquad 0 = v_C + \frac{x_A v_A}{\sqrt{x_A^2 + d^2}}$

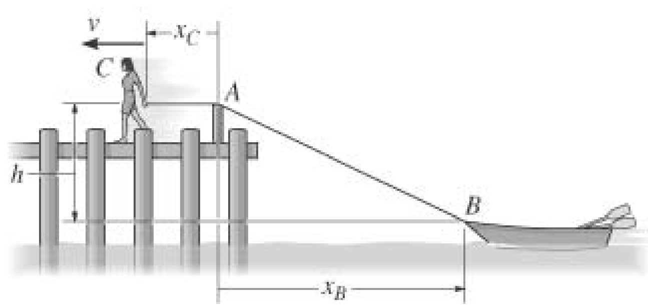
$$0 = a_C - \frac{x_A^2 v_A^2}{\sqrt{\left(x_A^2 + d^2\right)^3}} + \frac{v_A^2}{\sqrt{x_A^2 + d^2}}$$

$$\begin{pmatrix} x_A \\ v_C \\ a_C \end{pmatrix} = \text{Find}(x_A, v_C, a_C) \qquad x_A = 3\ \text{m} \qquad v_C = -1.2\ \frac{\text{m}}{\text{s}} \qquad a_C = -0.512\ \frac{\text{m}}{\text{s}^2}$$

Problem 12-190

The girl at C stands near the edge of the pier and pulls in the rope *horizontally* at constant speed v_C. Determine how fast the boat approaches the pier at the instant the rope length AB is d.

Given:



$$v_C = 6\ \frac{\text{ft}}{\text{s}}$$

$$h = 8\ \text{ft}$$

$$d = 50\ \text{ft}$$

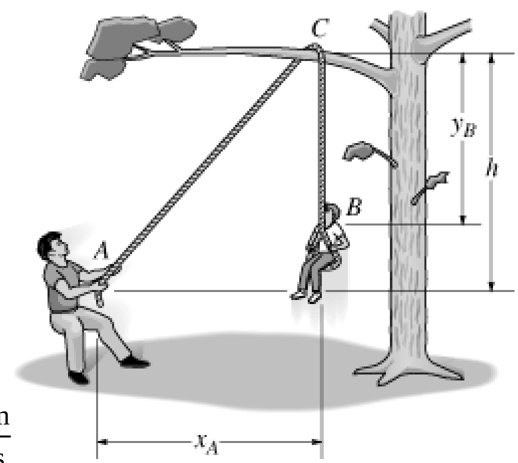
Solution: $x_B = \sqrt{d^2 - h^2}$

$L = x_C + \sqrt{h^2 + x_B^2}$ $\quad 0 = v_C + \dfrac{x_B v_B}{\sqrt{h^2 + x_B^2}}$

$v_B = -v_C \left(\dfrac{\sqrt{h^2 + x_B^2}}{x_B} \right)$ $\quad v_B = -6.078 \dfrac{\text{ft}}{\text{s}}$ Positive means to the right, negative to the left.

Problem 12-191

The man pulls the boy up to the tree limb C by walking backward. If he starts from rest when $x_A = 0$ and moves backward with constant acceleration a_A, determine the speed of the boy at the instant $y_B = y_{B1}$. Neglect the size of the limb. When $x_A = 0$, $y_B = h$ so that A and B are coincident, i.e., the rope is $2h$ long.



Given:

$a_A = 0.2 \dfrac{\text{m}}{\text{s}^2}$

$y_{B1} = 4 \text{ m}$

$h = 8 \text{ m}$

Solution: $y_B = y_{B1}$

Guesses

$x_A = 1 \text{ m}$ $\quad v_A = 1 \dfrac{\text{m}}{\text{s}}$ $\quad v_B = 1 \dfrac{\text{m}}{\text{s}}$

Given $\quad 2h = \sqrt{x_A^2 + h^2} + y_B$ $\quad 0 = \dfrac{x_A v_A}{\sqrt{x_A^2 + h^2}} + v_B$ $\quad v_A^2 = 2 a_A x_A$

$\begin{pmatrix} x_A \\ v_A \\ v_B \end{pmatrix} = \text{Find}(x_A, v_A, v_B)$ $\quad x_A = 8.944 \text{ m}$ $\quad v_A = 1.891 \dfrac{\text{m}}{\text{s}}$ $\quad v_B = -1.41 \dfrac{\text{m}}{\text{s}}$

Positive means down, negative means up

***Problem 12-192**

Collars A and B are connected to the cord that passes over the small pulley at C. When A is located at D, B is a distance d_1 to the left of D. If A moves at a constant speed v_A, to the right, determine the speed of B when A is distance d_2 to the right of D.

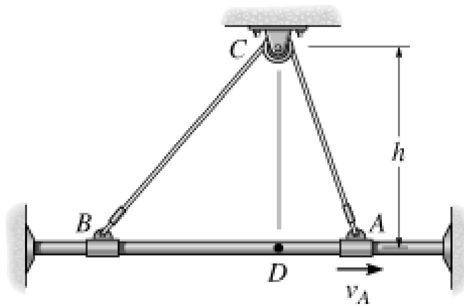
Given:

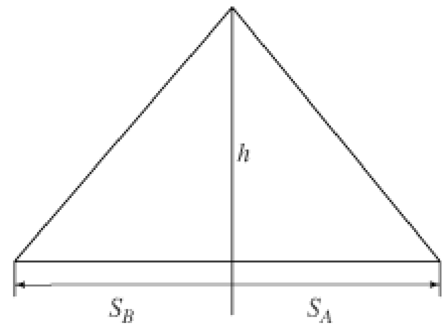
$h = 10\ \text{ft}$

$d_1 = 24\ \text{ft}$

$d_2 = 4\ \text{ft}$

$v_A = 2\ \dfrac{\text{ft}}{\text{s}}$





Solution:

$$L = \sqrt{h^2 + d_1^{\,2}} + h \qquad s_A = d_2$$

$$\sqrt{s_B^{\,2} + h^2} = L - \sqrt{s_A^{\,2} + h^2} \qquad s_B = \sqrt{\left(L - \sqrt{s_A^{\,2} + h^2}\right)^2 - h^2} \qquad s_B = 23.163\ \text{ft}$$

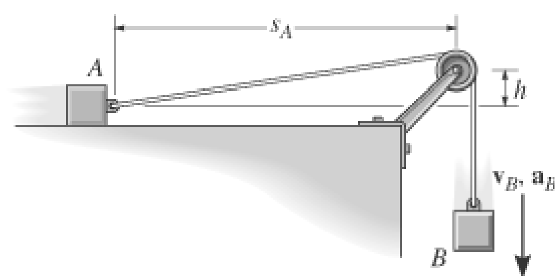
$$\frac{s_B v_B}{\sqrt{s_B^{\,2} + h^2}} = \frac{-s_A v_A}{\sqrt{s_A^{\,2} + h^2}} \qquad v_B = \frac{-s_A v_A \sqrt{s_B^{\,2} + h^2}}{s_B \sqrt{s_A^{\,2} + h^2}} \qquad v_B = -0.809\ \frac{\text{ft}}{\text{s}}$$

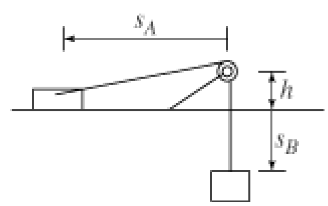
Positive means to the left,
negative to the right.

Problem 12-193

If block B is moving down with a velocity v_B and has an acceleration a_B, determine the velocity and acceleration of block A in terms of the parameters shown.

Solution:





$$L = s_B + \sqrt{s_A^{\,2} + h^2}$$

$$0 = v_B + \frac{s_A v_A}{\sqrt{s_A^{\,2} + h^2}}$$

$$v_A = \frac{-v_B \sqrt{s_A^{\,2} + h^2}}{s_A}$$

$$0 = a_B - \frac{s_A^{\,2} v_A^{\,2}}{\left(s_A^{\,2} + h^2\right)^{\frac{3}{2}}} + \frac{v_A^{\,2} + s_A a_A}{\sqrt{s_A^{\,2} + h^2}}$$

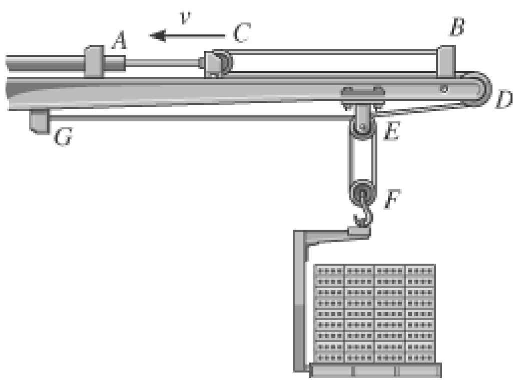
$$a_A = \frac{s_A v_A^2}{s_A^2 + h^2} - a_B\frac{\sqrt{s_A^2 + h^2}}{s_A} - \frac{v_A^2}{s_A} \qquad a_A = \frac{-a_B\sqrt{s_A^2 + h^2}}{s_A} - \frac{v_B^2 h^2}{s_A^3}$$

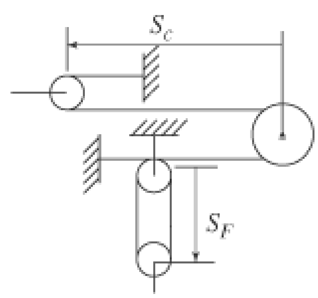
Problem 12-194

Vertical motion of the load is produced by movement of the piston at A on the boom. Determine the distance the piston or pulley at C must move to the left in order to lift the load a distance h. The cable is attached at B, passes over the pulley at C, then D, E, F, and again around E, and is attached at G.

Given:

$h = 2$ ft





Solution:

$\Delta s_F = -h$

$L = 2s_C + 2s_F$

$2\Delta s_C = -2\Delta s_F$

$\Delta s_C = -\Delta s_F \qquad \Delta s_C = 2\,\text{ft}$

Problem 12-195

The motion of the collar at A is controlled by a motor at B such that when the collar is at s_A, it is moving upwards at v_A and slowing down at a_A. Determine the velocity and acceleration of the cable as it is drawn into the motor B at this instant.

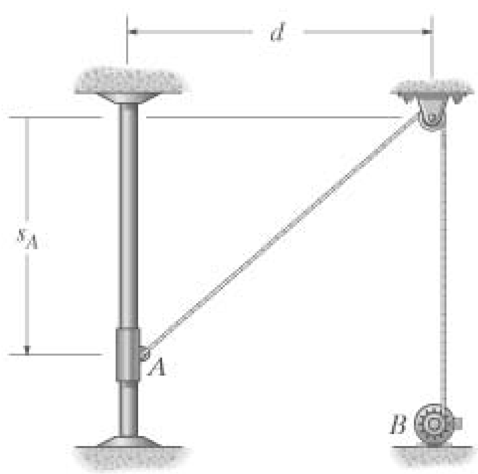
Given:

$d = 4$ ft

$s_A = 3$ ft

$v_A = -2\ \frac{\text{ft}}{\text{s}}$

$a_A = 1\ \frac{\text{ft}}{\text{s}^2}$



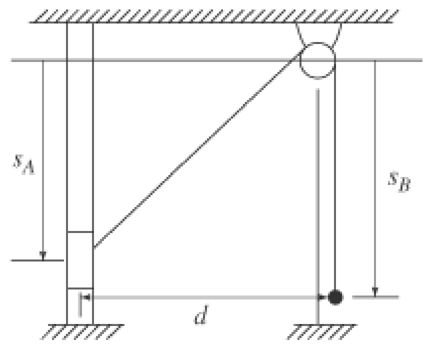
Solution: $L = \sqrt{s_A^2 + d^2} + s_B$

Guesses $v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

$$v_B = -\frac{s_A v_A}{\sqrt{s_A^2 + d^2}}$$

$$a_B = -\frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + d^2}} + \frac{s_A^2 v_A^2}{\sqrt{\left(s_A^2 + d^2\right)^3}}$$

$v_B = 1.2\ \frac{\text{ft}}{\text{s}}$ $\qquad a_B = -1.112\ \frac{\text{ft}}{\text{s}^2}$



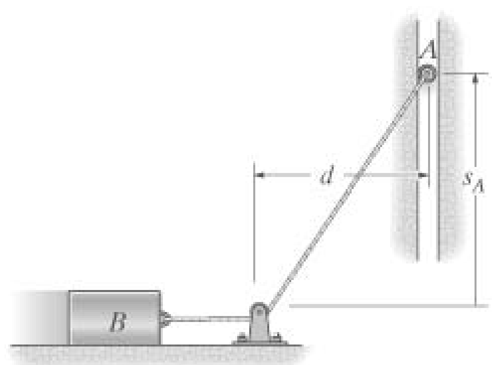
***Problem 12-196**

The roller at A is moving upward with a velocity v_A and has an acceleration a_A at s_A. Determine the velocity and acceleration of block B at this instant.

Given:

$s_A = 4\ \text{ft}$ $\qquad a_A = 4\ \frac{\text{ft}}{\text{s}^2}$

$v_A = 3\ \frac{\text{ft}}{\text{s}}$ $\qquad d = 3\ \text{ft}$



Solution:

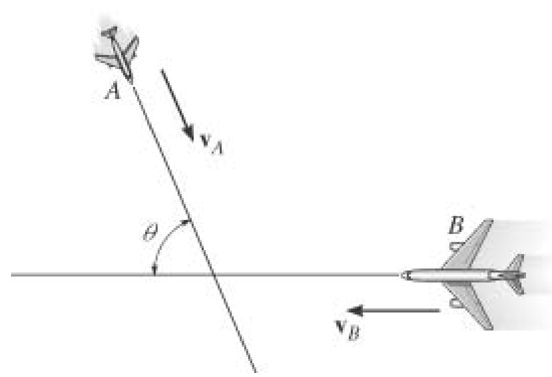
$$l = s_B + \sqrt{s_A^2 + d^2} \qquad 0 = v_B + \frac{s_A v_A}{\sqrt{s_A^2 + d^2}}$$

$$v_B = \frac{-s_A v_A}{\sqrt{s_A^2 + d^2}}$$ $\qquad v_B = -2.4\ \frac{\text{ft}}{\text{s}}$

$$a_B = \frac{-v_A^2 - s_A a_A}{\sqrt{s_A^2 + d^2}} + \frac{s_A^2 v_A^2}{\sqrt{\left(s_A^2 + d^2\right)^3}}$$ $\qquad a_B = -3.848\ \frac{\text{ft}}{\text{s}^2}$

Problem 12-197

Two planes, A and B, are flying at the same altitude. If their velocities are v_A and v_B such that the angle between their straight-line courses is θ, determine the velocity of plane B with respect to plane A.



Given:

$v_A = 600\ \frac{\text{km}}{\text{hr}}$

$v_B = 500\ \frac{\text{km}}{\text{hr}}$

$\theta = 75\ \text{deg}$

Solution:

$\mathbf{v_{Av}} = v_A\begin{pmatrix}\cos(\theta)\\ -\sin(\theta)\end{pmatrix}$ $\qquad \mathbf{v_{Av}} = \begin{pmatrix}155.291\\ -579.555\end{pmatrix}\frac{\text{km}}{\text{hr}}$

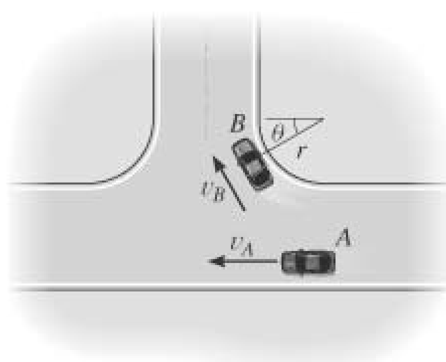
$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-1\\ 0\end{pmatrix}$ $\qquad \mathbf{v_{Bv}} = \begin{pmatrix}-500\\ 0\end{pmatrix}\frac{\text{km}}{\text{hr}}$

$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}}$ $\qquad \mathbf{v_{BA}} = \begin{pmatrix}-655\\ 580\end{pmatrix}\frac{\text{km}}{\text{hr}}$ $\qquad |\mathbf{v_{BA}}| = 875\ \frac{\text{km}}{\text{hr}}$

Problem 12-198

At the instant shown, cars A and B are traveling at speeds v_A and v_B respectively. If B is increasing its speed at v'_A, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.

Given:



$v_A = 30\ \frac{\text{mi}}{\text{hr}}$

$v_B = 20\ \frac{\text{mi}}{\text{hr}}$

$v'_A = 0\ \frac{\text{mi}}{\text{hr}^2}$

$v'_B = 1200\ \frac{\text{mi}}{\text{hr}^2}$

$\theta = 30\ \text{deg}$

$r = 0.3\ \text{mi}$

Solution:

$\mathbf{v_{Av}} = v_A\begin{pmatrix}-1\\ 0\end{pmatrix}$ $\qquad \mathbf{v_{Av}} = \begin{pmatrix}-30\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}}$

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix}-10\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}} \qquad \mathbf{v_{BA}} = \begin{pmatrix}20\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}} \qquad \left|\mathbf{v_{BA}}\right| = 26.5\,\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{a_{Av}} = \begin{pmatrix}-v'_A\\ 0\end{pmatrix} \qquad \mathbf{a_{Av}} = \begin{pmatrix}0\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}^2}$$

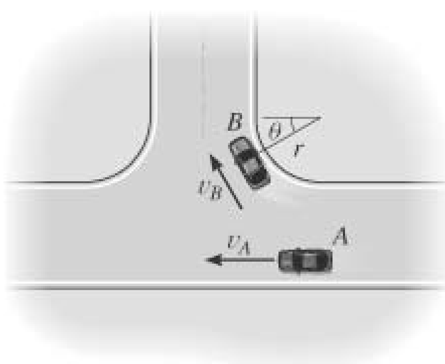
$$\mathbf{a_{Bv}} = v'_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} + \frac{v_B^{\,2}}{r}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix}554.701\\ 1.706\times 10^3\end{pmatrix}\frac{\text{mi}}{\text{hr}^2}$$

$$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}} \qquad \mathbf{a_{BA}} = \begin{pmatrix}555\\ 1706\end{pmatrix}\frac{\text{mi}}{\text{hr}^2} \qquad \left|\mathbf{a_{BA}}\right| = 1794\,\frac{\text{mi}}{\text{hr}^2}$$

Problem 12-199

At the instant shown, cars *A* and *B* are traveling at speeds v_A and v_B respectively. If *A* is increasing its speed at v'_A whereas the speed of *B* is decreasing at v'_B, determine the velocity and acceleration of *B* with respect to *A*.

Given:



$$v_A = 30\,\frac{\text{mi}}{\text{hr}}$$

$$v_B = 20\,\frac{\text{mi}}{\text{hr}}$$

$$v'_A = 400\,\frac{\text{mi}}{\text{hr}^2}$$

$$v'_B = -800\,\frac{\text{mi}}{\text{hr}^2}$$

$$\theta = 30\text{ deg}$$

$$r = 0.3\text{ mi}$$

Solution:

$$\mathbf{v_{Av}} = v_A\begin{pmatrix}-1\\ 0\end{pmatrix} \qquad \mathbf{v_{Av}} = \begin{pmatrix}-30\\ 0\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$$\mathbf{v_{Bv}} = v_B\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix}-10\\ 17.321\end{pmatrix}\frac{\text{mi}}{\text{hr}}$$

$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}}$ $\quad \mathbf{v_{BA}} = \begin{pmatrix} 20 \\ 17.321 \end{pmatrix} \frac{mi}{hr}$ $\quad |\mathbf{v_{BA}}| = 26.458 \frac{mi}{hr}$

$\mathbf{a_{Av}} = \begin{pmatrix} -v'_A \\ 0 \end{pmatrix}$ $\quad \mathbf{a_{Av}} = \begin{pmatrix} -400 \\ 0 \end{pmatrix} \frac{mi}{hr^2}$

$\mathbf{a_{Bv}} = v'_B \begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} + \frac{v_B^2}{r} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ $\quad \mathbf{a_{Bv}} = \begin{pmatrix} 1.555 \times 10^3 \\ -26.154 \end{pmatrix} \frac{mi}{hr^2}$

$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}}$ $\quad \mathbf{a_{BA}} = \begin{pmatrix} 1955 \\ -26 \end{pmatrix} \frac{mi}{hr^2}$ $\quad |\mathbf{a_{BA}}| = 1955 \frac{mi}{hr^2}$

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*__Problem 12-200__

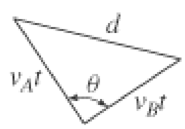
Two boats leave the shore at the same time and travel in the directions shown with the given speeds. Determine the speed of boat *A* with respect to boat *B*. How long after leaving the shore will the boats be at a distance *d* apart?

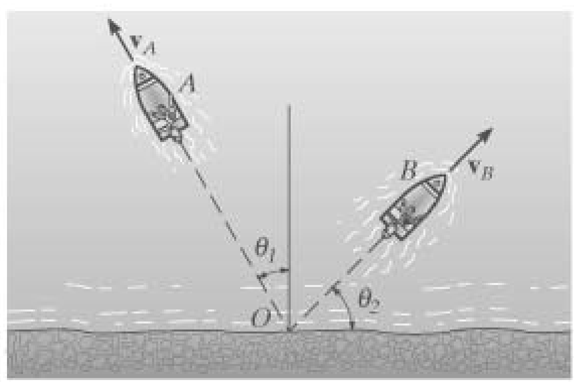
Given:

$v_A = 20 \frac{ft}{s}$ $\quad \theta_1 = 30 \text{ deg}$

$v_B = 15 \frac{ft}{s}$ $\quad \theta_2 = 45 \text{ deg}$

$d = 800 \text{ ft}$





Solution:

$\mathbf{v_{Av}} = v_A \begin{pmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \end{pmatrix}$ $\quad \mathbf{v_{Bv}} = v_B \begin{pmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{pmatrix}$

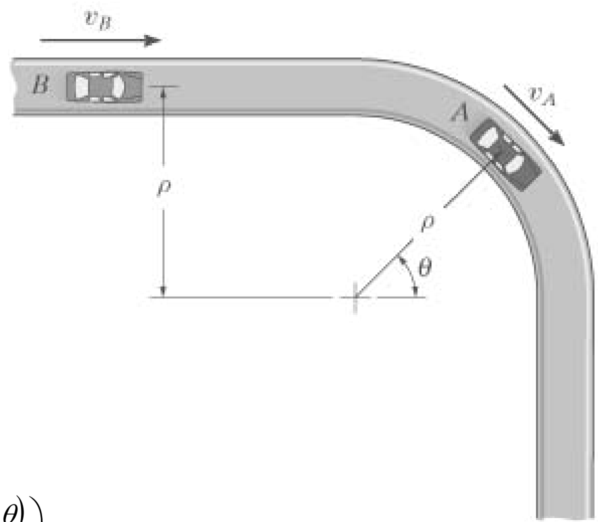
$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}}$ $\quad \mathbf{v_{AB}} = \begin{pmatrix} -20.607 \\ 6.714 \end{pmatrix} \frac{ft}{s}$ $\quad t = \frac{d}{|\mathbf{v_{AB}}|}$

$|\mathbf{v_{AB}}| = 21.673 \frac{ft}{s}$ $\quad t = 36.913 \text{ s}$

Problem 12–201

At the instant shown, the car at A is traveling at v_A around the curve while increasing its speed at v'_A. The car at B is traveling at v_B along the straightaway and increasing its speed at v'_B. Determine the relative velocity and relative acceleration of A with respect to B at this instant.

Given:



$v_A = 10 \frac{m}{s}$ $\quad v_B = 18.5 \frac{m}{s}$

$v'_A = 5 \frac{m}{s^2}$ $\quad v'_B = 2 \frac{m}{s^2}$

$\theta = 45 \text{ deg}$ $\quad \rho = 100 \text{ m}$

Solution:

$$\mathbf{v_{Av}} = v_A \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix}$$

$$\mathbf{a_{Av}} = v'_A \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix} + \frac{v_A^2}{\rho} \begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \end{pmatrix}$$

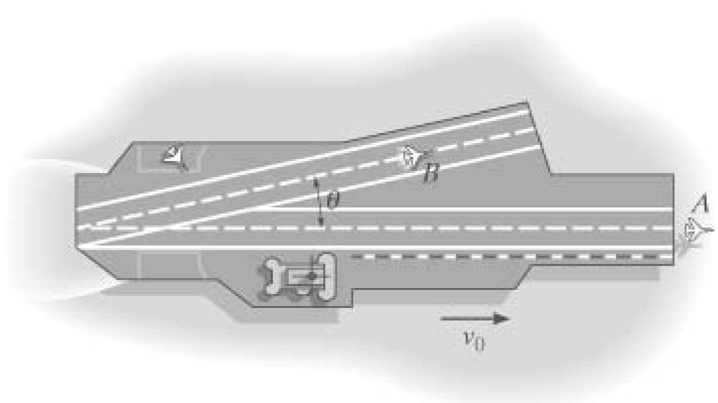
$$\mathbf{v_{Bv}} = \begin{pmatrix} v_B \\ 0 \end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix} v'_B \\ 0 \end{pmatrix}$$

$$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -11.43 \\ -7.07 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_{AB}} = \mathbf{a_{Av}} - \mathbf{a_{Bv}} \qquad \mathbf{a_{AB}} = \begin{pmatrix} 0.828 \\ -4.243 \end{pmatrix} \frac{m}{s^2}$$

Problem 12–202

An aircraft carrier is traveling forward with a velocity v_0. At the instant shown, the plane at A has just taken off and has attained a forward horizontal air speed v_A, measured from still water. If the plane at B is traveling along the runway of the carrier at v_B in the direction shown measured relative to the carrier, determine the velocity of A with respect to B.



Given:

$v_0 = 50\ \frac{\text{km}}{\text{hr}} \qquad v_A = 200\ \frac{\text{km}}{\text{hr}}$

$\theta = 15\ \text{deg} \qquad v_B = 175\ \frac{\text{km}}{\text{hr}}$

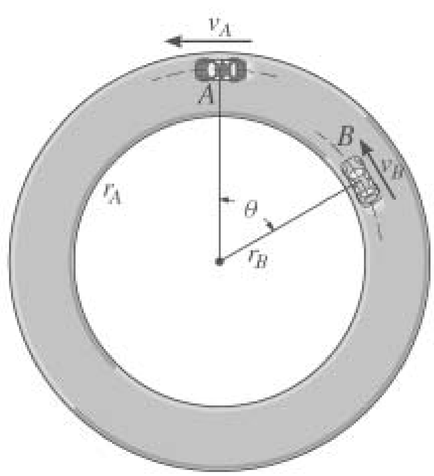
Solution:

$\mathbf{v_A} = \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} v_0 \\ 0 \end{pmatrix} + v_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$

$\mathbf{v_{AB}} = \mathbf{v_A} - \mathbf{v_B} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -19.04 \\ -45.29 \end{pmatrix} \frac{\text{km}}{\text{hr}} \qquad |\mathbf{v_{AB}}| = 49.1\ \frac{\text{km}}{\text{hr}}$

Problem 12-203

Cars A and B are traveling around the circular race track. At the instant shown, A has speed v_A and is increasing its speed at the rate of v'_A, whereas B has speed v_B and is decreasing its speed at v'_B. Determine the relative velocity and relative acceleration of car A with respect to car B at this instant.



Given: $\theta = 60\ \text{deg}$

$r_A = 300\ \text{ft} \qquad r_B = 250\ \text{ft}$

$v_A = 90\ \frac{\text{ft}}{\text{s}} \qquad v_B = 105\ \frac{\text{ft}}{\text{s}}$

$v'_A = 15\ \frac{\text{ft}}{\text{s}^2} \qquad v'_B = -25\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\mathbf{v_{Av}} = v_A \begin{pmatrix} -1 \\ 0 \end{pmatrix} \qquad \mathbf{v_{Av}} = \begin{pmatrix} -90 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$

$\mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -52.5 \\ 90.933 \end{pmatrix} \frac{\text{ft}}{\text{s}}$

$\mathbf{v_{AB}} = \mathbf{v_{Av}} - \mathbf{v_{Bv}} \qquad \mathbf{v_{AB}} = \begin{pmatrix} -37.5 \\ -90.9 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_{AB}}| = 98.4\ \frac{\text{ft}}{\text{s}}$

$\mathbf{a_A} = v'_A \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \frac{v_A^2}{r_A} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \mathbf{a_A} = \begin{pmatrix} -15 \\ -27 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$

$$\mathbf{a_B} = v'_B\begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_B{}^2}{r_B}\begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \end{pmatrix} \qquad \mathbf{a_B} = \begin{pmatrix} -25.692 \\ -43.701 \end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

$$\mathbf{a_{AB}} = \mathbf{a_A} - \mathbf{a_B} \qquad \mathbf{a_{AB}} = \begin{pmatrix} 10.692 \\ 16.701 \end{pmatrix}\frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{AB}}\right| = 19.83\,\frac{\text{ft}}{\text{s}^2}$$

***Problem 12–204**

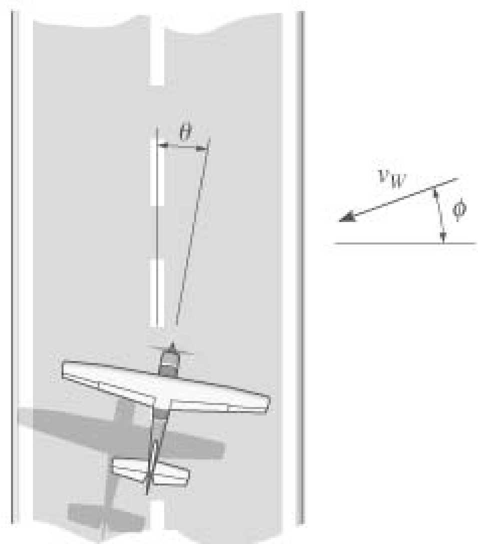
The airplane has a speed relative to the wind of v_A. If the speed of the wind relative to the ground is v_W, determine the angle θ at which the plane must be directed in order to travel in the direction of the runway. Also, what is its speed relative to the runway?

Given:

$$v_A = 100\,\frac{\text{mi}}{\text{hr}}$$

$$v_W = 10\,\frac{\text{mi}}{\text{hr}}$$

$$\phi = 20\text{ deg}$$



Solution:

Guesses $\theta = 1\text{ deg}$

$$v_{Ag} = 1\,\frac{\text{mi}}{\text{hr}}$$

Given $$\begin{pmatrix} 0 \\ v_{Ag} \end{pmatrix} = v_A\begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix} + v_W\begin{pmatrix} -\cos(\phi) \\ -\sin(\phi) \end{pmatrix}$$

$$\begin{pmatrix} \theta \\ v_{Ag} \end{pmatrix} = \text{Find}(\theta, v_{Ag}) \qquad \theta = 5.39\text{ deg} \qquad v_{Ag} = 96.1\,\frac{\text{mi}}{\text{hr}}$$

Problem 12–205

At the instant shown car A is traveling with a velocity v_A and has an acceleration a_A along the highway. At the same instant B is traveling on the trumpet interchange curve with a speed v_B which is decreasing at v'_B. Determine the relative velocity and relative acceleration of B with respect to A at this instant.

Given:

$v_A = 30 \frac{m}{s}$

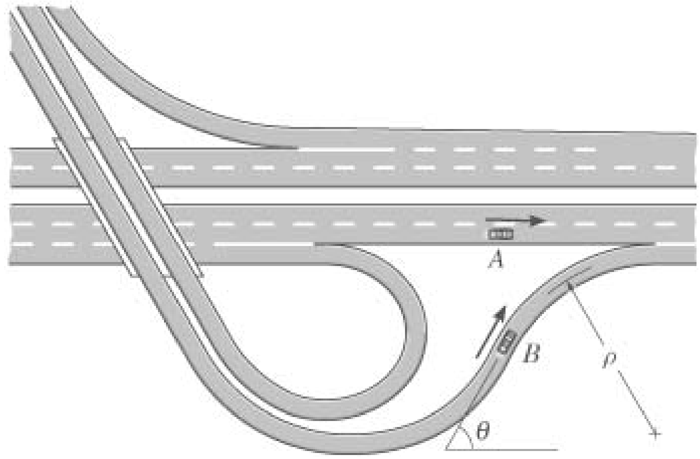
$v_B = 15 \frac{m}{s}$

$a_A = 2 \frac{m}{s^2}$

$v'_B = -0.8 \frac{m}{s^2}$

$\rho = 250\ m$

$\theta = 60\ deg$



Solution:

$\mathbf{v_{Av}} = \begin{pmatrix} v_A \\ 0 \end{pmatrix}$ $\qquad$ $\mathbf{a_{Av}} = \begin{pmatrix} a_A \\ 0 \end{pmatrix}$

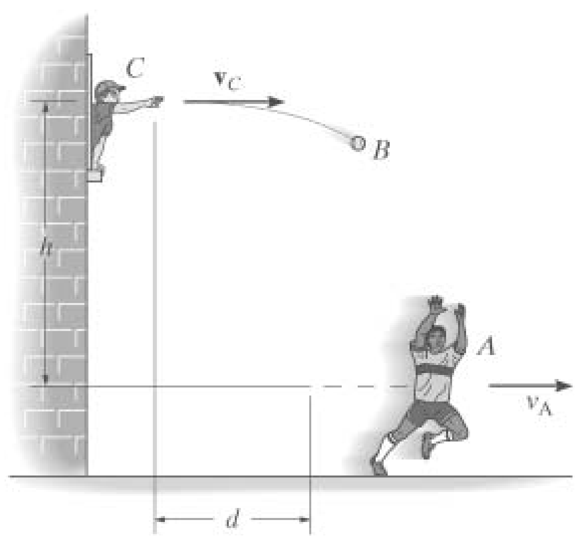
$\mathbf{v_{Bv}} = v_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$ $\qquad$ $\mathbf{a_{Bv}} = v'_B \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_B^2}{\rho} \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \end{pmatrix}$

$\mathbf{v_{BA}} = \mathbf{v_{Bv}} - \mathbf{v_{Av}}$ $\qquad$ $\mathbf{v_{BA}} = \begin{pmatrix} -22.5 \\ 12.99 \end{pmatrix} \frac{m}{s}$ $\qquad$ $|\mathbf{v_{BA}}| = 26.0 \frac{m}{s}$

$\mathbf{a_{BA}} = \mathbf{a_{Bv}} - \mathbf{a_{Av}}$ $\qquad$ $\mathbf{a_{BA}} = \begin{pmatrix} -1.621 \\ -1.143 \end{pmatrix} \frac{m}{s^2}$ $\qquad$ $|\mathbf{a_{BA}}| = 1.983 \frac{m}{s^2}$

Problem 12–206

The boy A is moving in a straight line away from the building at a constant speed v_A. The boy C throws the ball B horizontally when A is at d. At what speed must C throw the ball so that A can catch it? Also determine the relative speed of the ball with respect to boy A at the instant the catch is made.



Given:

$v_A = 4 \frac{ft}{s}$

$d = 10\ ft$

$h = 20\ ft$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v_C = 1\ \frac{\text{ft}}{\text{s}}$

$t = 1\ \text{s}$

Given $h - \frac{1}{2} g t^2 = 0$

$v_C t = d + v_A t$

$\begin{pmatrix} t \\ v_C \end{pmatrix} = \text{Find}(t, v_C)$ $t = 1.115\ \text{s}$ $v_C = 12.97\ \frac{\text{ft}}{\text{s}}$

$\mathbf{v_{BA}} = \begin{pmatrix} v_C \\ -g t \end{pmatrix} - \begin{pmatrix} v_A \\ 0 \end{pmatrix}$ $\mathbf{v_{BA}} = \begin{pmatrix} 8.972 \\ -35.889 \end{pmatrix} \frac{\text{ft}}{\text{s}}$ $|\mathbf{v_{BA}}| = 37.0\ \frac{\text{ft}}{\text{s}}$

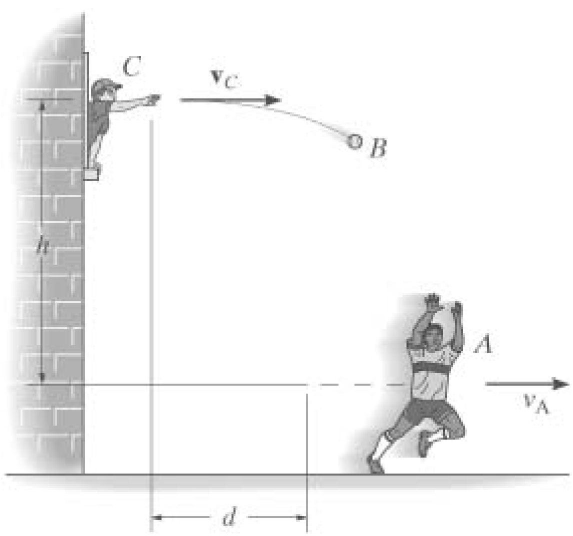
Problem 12–207

The boy A is moving in a straight line away from the building at a constant speed v_A. At what horizontal distance d must he be from C in order to make the catch if the ball is thrown with a horizontal velocity v_C? Also determine the relative speed of the ball with respect to the boy A at the instant the catch is made.

Given:

$v_A = 4\ \frac{\text{ft}}{\text{s}}$ $h = 20\ \text{ft}$

$v_C = 10\ \frac{\text{ft}}{\text{s}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $d = 1\ \text{ft}$ $t = 1\ \text{s}$

Given $h - \frac{1}{2} g t^2 = 0$

$v_C t = d + v_A t$

$\begin{pmatrix} t \\ d \end{pmatrix} = \text{Find}(t, d)$ $t = 1.115\ \text{s}$ $d = 6.69\ \text{ft}$

$$\mathbf{v_{BA}} = \begin{pmatrix} v_C \\ -g\,t \end{pmatrix} - \begin{pmatrix} v_A \\ 0 \end{pmatrix} \qquad \mathbf{v_{BA}} = \begin{pmatrix} 6 \\ -35.889 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_{BA}}\right| = 36.4\,\frac{\text{ft}}{\text{s}}$$

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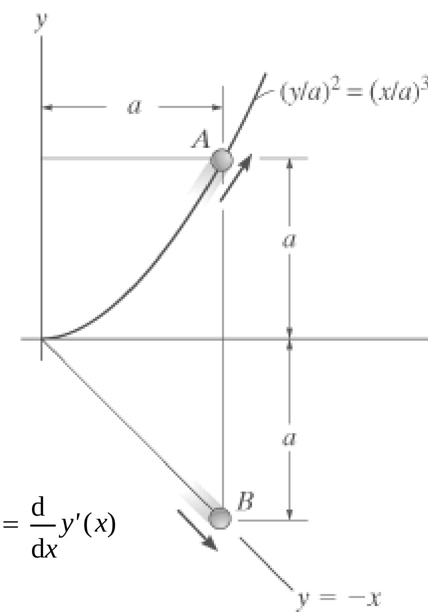
*__Problem 12–208__

At a given instant, two particles *A* and *B* are moving with a speed of v_0 along the paths shown. If *B* is decelerating at v'_B and the speed of *A* is increasing at v'_A, determine the acceleration of *A* with respect to *B* at this instant.

Given:

$$v_0 = 8\,\frac{\text{m}}{\text{s}} \qquad v'_A = 5\,\frac{\text{m}}{\text{s}^2}$$

$$a = 1\ \text{m} \qquad v'_B = -6\,\frac{\text{m}}{\text{s}^2}$$



Solution:

$$y(x) = a\left(\frac{x}{a}\right)^{\frac{3}{2}} \qquad y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

$$\rho = \frac{\sqrt{\left(1 + y'(a)^2\right)^3}}{y''(a)} \qquad \theta = \operatorname{atan}(y'(a)) \qquad \rho = 7.812\ \text{m}$$

$$\mathbf{a_A} = v'_A\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} + \frac{v_0^2}{\rho}\begin{pmatrix} -\sin(\theta) \\ \cos(\theta) \end{pmatrix} \qquad \mathbf{a_B} = \frac{v'_B}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\mathbf{a_{AB}} = \mathbf{a_A} - \mathbf{a_B} \qquad \mathbf{a_{AB}} = \begin{pmatrix} 0.2 \\ 4.46 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_{AB}}\right| = 4.47\,\frac{\text{m}}{\text{s}^2}$$
