**Problem 13-1**

Determine the gravitational attraction between two spheres which are just touching each other. Each sphere has a mass \( M \) and radius \( r \).

**Given:**

\[
r = 200 \text{ mm} \quad M = 10 \text{ kg} \quad G = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \quad \text{nN} = 1 \times 10^{-9} \text{ N}
\]

**Solution:**

\[
F = \frac{GM^2}{(2r)^2} \quad F = 41.7 \text{ nN}
\]

**Problem 13-2**

By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., \( s \propto t^2 \) by determining the time \( t_B \), \( t_C \), and \( t_D \) needed for a block of mass \( m \) to slide from rest at \( A \) to points \( B \), \( C \), and \( D \), respectively. Neglect the effects of friction.

**Given:**

\[
s_B = 2 \text{ m} \quad s_C = 4 \text{ m} \quad s_D = 9 \text{ m} \quad \theta = 20 \text{ deg} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}
\]

**Solution:**

\[
W \sin(\theta) = \left( \frac{W}{g} \right) a
\]

\[
a = g \sin(\theta) \quad a = 3.355 \frac{\text{m}}{\text{s}^2}
\]

\[
s = \frac{1}{2} at^2
\]

\[
t_B = \sqrt{\frac{2s_B}{a}} \quad t_B = 1.09 \text{ s}
\]
\[ t_C = \sqrt{\frac{2s_C}{a}} \quad t_C = 1.54 \text{ s} \]

\[ t_D = \sqrt{\frac{2s_D}{a}} \quad t_D = 2.32 \text{ s} \]

**Problem 13-3**

A bar \( B \) of mass \( M_J \), originally at rest, is being towed over a series of small rollers. Determine the force in the cable at time \( t \) if the motor \( M \) is drawing in the cable for a short time at a rate \( \nu = kt^2 \). How far does the bar move in time \( t \)? Neglect the mass of the cable, pulley, and the rollers.

Given:

\[ kN = 10^3 \text{ N} \]

\[ M_J = 300 \text{ kg} \]

\[ t = 5 \text{ s} \]

\[ k = 0.4 \frac{\text{m}}{\text{s}^3} \]

Solution:

\[ \nu = kt^2 \quad \nu = 10 \frac{\text{m}}{\text{s}} \]

\[ a = 2kt \quad a = 4 \frac{\text{m}}{\text{s}^2} \]

\[ T = M_J a \quad T = 1.2 \text{ kN} \]

\[ d = \int_{0}^{t} k \nu^2 \, dt \quad d = 16.7 \text{ m} \]

*Problem 13-4*

A crate having a mass \( M \) falls horizontally off the back of a truck which is traveling with speed \( \nu \). Determine the coefficient of kinetic friction between the road and the crate if the crate slides a distance \( d \) on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is \( \nu \).
Given:

\[ M = 60 \text{ kg} \]
\[ d = 45 \text{ m} \]
\[ v = 80 \text{ km/hr} \]
\[ g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution:

\[ N_C - Mg = 0 \quad N_C = Mg \]
\[ \mu_k N_C = Ma \quad a = \mu_k g \]
\[ \frac{v^2}{2} = ad = \mu_k gd \]
\[ \mu_k = \frac{v^2}{2gd} \quad \mu_k = 0.559 \]

**Problem 13-5**

The crane lifts a bin of mass \( M \) with an initial acceleration \( a \). Determine the force in each of the supporting cables due to this motion.

Given:

\[ M = 700 \text{ kg} \quad b = 3 \quad \text{kN} = 10^3 \text{ N} \]
\[ a = 3 \frac{\text{m}}{\text{s}^2} \quad c = 4 \]

Solution:

\[ 2T \left( \frac{c}{\sqrt{b^2 + c^2}} \right) - Mg = Ma \]
\[ T = M(a + g) \left( \frac{\sqrt{b^2 + c^2}}{2c} \right) \quad T = 5.60 \text{ kN} \]
Problem 13-6

The baggage truck \( A \) has mass \( m_t \) and is used to pull the two cars, each with mass \( m_c \). The tractive force on the truck is \( F \). Determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at \( C \) suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

Given:
- \( m_t = 800 \text{ kg} \)
- \( m_c = 300 \text{ kg} \)
- \( F = 480 \text{ N} \)

Solution:
\[ \sum F_x = ma; \quad F = (m_t + 2m_c)a \]
\[ a = \frac{F}{m_t + 2m_c}; \quad a = 0.343 \frac{\text{m}}{\text{s}^2} \]

\[ \sum F_x = ma; \quad F = (m_t + m_c)a_{Fail} \]
\[ a_{Fail} = \frac{F}{m_t + m_c}; \quad a_{Fail} = 0.436 \frac{\text{m}}{\text{s}^2} \]

Problem 13-7

The fuel assembly of mass \( M \) for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that \( s = 0 \) and \( v = 0 \) when \( t = 0 \) and \( s = s_1 \) when \( t = t_1 \). Determine the tension in the cable at \( A \) during the motion.
Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[ M = 500 \text{ kg} \]
\[ s_1 = 2.5 \text{ m} \]
\[ t_1 = 1.5 \text{ s} \]
\[ g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution:

\[ s = \left( \frac{a}{2} \right) t^2 \quad a = \frac{2s_1}{t_1^2} \quad a = 2.222 \frac{\text{m}}{\text{s}^2} \]

\[ 2T - Mg = Ma \quad T = \frac{M(a + g)}{2} \quad T = 3.008 \text{ kN} \]

*Problem 13-8*

The crate of mass \( M \) is suspended from the cable of a crane. Determine the force in the cable at time \( t \) if the crate is moving upward with (a) a constant velocity \( v_1 \) and (b) a speed of \( v = bt^2 + c \).

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[ M = 200 \text{ kg} \]
\[ t = 2 \text{ s} \]
\[ v_1 = 2 \frac{\text{m}}{\text{s}} \]
\[ b = 0.2 \frac{\text{m}}{\text{s}^2} \]
\[ c = 2 \frac{\text{m}}{\text{s}} \]

Solution:

(a) \[ a = 0 \frac{\text{m}}{\text{s}^2} \quad T_a - Mg = Ma \quad T_a = M(g + a) \quad T_a = 1.962 \text{ kN} \]
Problem 13-9

The elevator $E$ has a mass $M_E$, and the counterweight at $A$ has a mass $M_A$. If the motor supplies a constant force $F$ on the cable at $B$, determine the speed of the elevator at time $t$ starting from rest. Neglect the mass of the pulleys and cable.

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

- $M_E = 500 \text{ kg}$
- $M_A = 150 \text{ kg}$
- $F = 5 \text{ kN}$
- $t = 3 \text{ s}$

Solution:

Guesses

- $T = 1 \text{ kN}$
- $a = 1 \frac{\text{m}}{\text{s}^2}$
- $v = 1 \frac{\text{m}}{\text{s}}$

Given

\[ T - M_A g = -M_A a \]
\[ F + T - M_E g = M_E a \]
\[ v = a t \]

\[
\begin{pmatrix}
T \\
\vdashline
a \\
\vdashline
v
\end{pmatrix}
= \text{Find}(T, a, v)
\]

\[
T = 1.11 \text{ kN} \quad a = 2.41 \frac{\text{m}}{\text{s}^2} \quad v = 7.23 \frac{\text{m}}{\text{s}}
\]
Problem 13-10

The elevator $E$ has a mass $M_E$ and the counterweight at $A$ has a mass $M_A$. If the elevator attains a speed $v$ after it rises a distance $h$, determine the constant force developed in the cable at $B$. Neglect the mass of the pulleys and cable.

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[
M_E = 500 \text{ kg} \\
M_A = 150 \text{ kg} \\
v = 10 \frac{\text{m}}{\text{s}} \\
h = 40 \text{ m}
\]

Solution:

Guesses

\[ T = 1 \text{ kN} \]
\[ F = 1 \text{ kN} \]
\[ a = 1 \frac{\text{m}}{\text{s}^2} \]

Given

\[
T - M_Ag = -M_Aa \\
F + T - M_Eg = M_Ea \\
v^2 = 2ah
\]

\[
\begin{pmatrix} F \\ T \\ a \end{pmatrix} = \text{Find}(F, T, a) \\
a = 1.250 \frac{\text{m}}{\text{s}^2} \\
T = 1.28 \text{ kN} \\
F = 4.25 \text{ kN}
\]

Problem 13-11

The water-park ride consists of a sled of weight $W$ which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is $F_{r1}$ and in the pool for a short distance is $F_{r2}$, determine how fast the sled is traveling when $s = s_2$. 

Given:
\[ W = 800 \text{ lb} \]
\[ F_{r1} = 30 \text{ lb} \]
\[ F_{r2} = 80 \text{ lb} \]
\[ s_2 = 5 \text{ ft} \]
\[ a = 100 \text{ ft} \]
\[ b = 100 \text{ ft} \]
\[ g = \frac{32.2}{s^2} \text{ ft/s}^2 \]

Solution:
\[ \theta = \arctan\left(\frac{b}{a}\right) \]

On the incline
\[ W \sin(\theta) - F_{r1} = \left(\frac{W}{g}\right) a_1 \]
\[ a_1 = g \left(\frac{W \sin(\theta) - F_{r1}}{W}\right) \]
\[ a_1 = \frac{21.561}{s^2} \text{ ft/s}^2 \]
\[ v_1^2 = 2a_1 \sqrt{a^2 + b^2} \]
\[ v_1 = \sqrt{2a_1 \sqrt{a^2 + b^2}} \]
\[ v_1 = \frac{78.093}{s} \text{ ft/s} \]

In the water
\[ F_{r2} = \left(\frac{W}{g}\right) a_2 \]
\[ a_2 = \frac{gF_{r2}}{W} \]
\[ a_2 = \frac{3.22}{s^2} \text{ ft/s}^2 \]
\[ \frac{v_2^2}{2} - \frac{v_1^2}{2} = -a_2 s_2 \]
\[ v_2 = \sqrt{v_1^2 - 2a_2 s_2} \]
\[ v_2 = \frac{77.886}{s} \text{ ft/s} \]

*Problem 13-12*

A car of mass \( m \) is traveling at a slow velocity \( v_0 \). If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., \( F_D = kv \) determine the distance and the time the car will travel before its velocity becomes \( 0.5v_0 \). Assume no other frictional forces act on the car.

Solution:
\[ -F_D = ma \]
\[ -kv = ma \]
Find time \[ a = \frac{d}{dt}v = \frac{-k}{m}v \]

\[ \frac{-k}{m} \int_0^t 1 \, dt = \int_{0.5v_0}^{1} \frac{1}{v} \, dv \]

\[ t = \frac{m}{k} \ln \left( \frac{v_0}{0.5v_0} \right) \]

\[ t = \frac{m}{k} \ln(2) \]

\[ t = 0.693 \frac{m}{k} \]

Find distance \[ a = \frac{d}{dx}v = \frac{-k}{m}v \]

\[ \int_0^x k \, dx = \int_{0.5v_0}^{v_0} m \, dv \]

\[ x = \frac{m}{k} \left( 0.5v_0 \right) \]

\[ x = 0.5 \frac{mv_0}{k} \]

Problem 13-13

Determine the normal force the crate A of mass M exerts on the smooth cart if the cart is given an acceleration a down the plane. Also, what is the acceleration of the crate?

Given:

\( M = 10 \text{ kg} \)

\( a = 2 \frac{m}{s^2} \)

\( \theta = 30 \text{ deg} \)

Solution:

\[ N - Mg = -M(a \sin(\theta)) \]

\[ N = M[g - (a \sin(\theta))] \]

\[ N = 88.1 \text{ N} \]

\[ a_{\text{crate}} = (a \sin(\theta)) \]

\[ a_{\text{crate}} = 1 \frac{m}{s^2} \]

Problem 13-14

Each of the two blocks has a mass m. The coefficient of kinetic friction at all surfaces of contact is \( \mu \). If a horizontal force P moves the bottom block, determine the acceleration of the bottom block in each case.
Solution:

(a) Block $A$:
\[ \Sigma F_x = ma_A; \quad P - 3\mu mg = ma_A \]
\[ a_A = \frac{P}{m} - 3\mu g \]

(b) \[ S_B + S_A = L \]
\[ a_A = -a_B \]

Block $A$:
\[ \Sigma F_x = ma_A; \quad P - T - 3\mu mg = ma_A \]
Block $B$:
\[ \Sigma F_x = ma_B; \quad \mu mg - T = ma_B \]

Solving simultaneously:
\[ a_A = \frac{P}{2m} - 2\mu g \]

Problem 13-15

The driver attempts to tow the crate using a rope that has a tensile strength $T_{\text{max}}$. If the crate is originally at rest and has weight $W$, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$. 


Given:

\[ T_{\text{max}} = 200 \text{ lb} \]
\[ W = 500 \text{ lb} \]
\[ \mu_s = 0.4 \]
\[ \mu_k = 0.3 \]
\[ g = 32.2 \frac{\text{ft}}{\text{s}^2} \]
\[ \theta = 30 \text{ deg} \]

Solution:

Equilibrium: In order to slide the crate, the towing force must overcome static friction.

Initial guesses \( F_N = 100 \text{ lb} \) \( T = 50 \text{ lb} \)

Given \( T \cos(\theta) - \mu_s F_N = 0 \) \( F_N + T \sin(\theta) - W = 0 \) \( \begin{bmatrix} F_N \\ T \end{bmatrix} = \text{Find}(F_N, T) \)

If \( T = 187.613 \text{ lb} > T_{\text{max}} = 200 \text{ lb} \) then the truck will not be able to pull the create without breaking the rope.

If \( T = 187.613 \text{ lb} < T_{\text{max}} = 200 \text{ lb} \) then the truck will be able to pull the create without breaking the rope and we will now calculate the acceleration for this case.

Initial guesses \( F_N = 100 \text{ lb} \) \( a = 1 \frac{\text{ft}}{\text{s}^2} \)

Require \( T = T_{\text{max}} \)

Given \( T \cos(\theta) - \mu_k F_N = \frac{W}{g} a \) \( F_N + T \sin(\theta) - W = 0 \) \( \begin{bmatrix} F_N \\ a \end{bmatrix} = \text{Find}(F_N, a) \)

\[ a = 3.426 \frac{\text{ft}}{\text{s}^2} \]

*Problem 13-16*

An engine of mass \( M_1 \) is suspended from a spreader beam of mass \( M_2 \) and hoisted by a crane which gives it an acceleration \( a \) when it has a velocity \( v \). Determine the force in chains \( AC \) and \( AD \) during the lift.
Units Used:

\[ 1 \text{ Mg} = 10^3 \text{ kg} \quad 1 \text{kN} = 10^3 \text{ N} \]

Given:

\[ M_1 = 3.5 \text{ Mg} \]
\[ M_2 = 500 \text{ kg} \]
\[ a = 4 \frac{\text{m}}{\text{s}^2} \]
\[ v = 2 \frac{\text{m}}{\text{s}} \]
\[ \theta = 60 \text{ deg} \]

Solution:

Guesses \( T = 1 \text{ N} \quad T' = 1 \text{ N} \)

Given

\[ 2T \sin(\theta) - (M_1 + M_2)g = (M_1 + M_2)a \]
\[ 2T' - M_1g = M_1a \]

\[
\begin{pmatrix}
T \\
T'
\end{pmatrix}
= \text{Find}(T, T')
\begin{pmatrix}
T_{AC} \\
T_{AD}
\end{pmatrix}
= \begin{pmatrix}
T \\
T'
\end{pmatrix}
\]

\[
\begin{pmatrix}
T_{AC} \\
T_{AD}
\end{pmatrix}
= \begin{pmatrix}
31.9 \\
24.2
\end{pmatrix} \text{kN}
\]

Problem 13-17

The bullet of mass \( m \) is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of \( F = F_0 \sin(\pi t / t_0) \) on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet’s maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.
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Solution:

\[ F_0 \sin \left( \frac{\pi t}{t_0} \right) = ma \quad a = \frac{dv}{dt} = \frac{F_0}{m} \sin \left( \frac{\pi t}{t_0} \right) \]

\[ \int_0^v 1 \, dv = \int_0^t \frac{F_0}{m} \sin \left( \frac{\pi t}{t_0} \right) \, dt \]

\[ v = \frac{F_0 t_0}{\pi m} \left( 1 - \cos \left( \frac{\pi t}{t_0} \right) \right) \]

\( v_{\text{max}} \) occurs when \( \cos \left( \frac{\pi t}{t_0} \right) = -1 \), or \( t = t_0 \)

\[ v_{\text{max}} = \frac{2F_0 t_0}{\pi m} \]

\[ \int_0^s 1 \, ds = \int_0^t \left( \frac{F_0 t_0}{\pi m} \right) \left( 1 - \cos \left( \frac{\pi t}{t_0} \right) \right) \, dt \]

\[ s = \frac{F_0 t_0}{\pi m} \left( t - \frac{t_0}{\pi} \sin \left( \frac{\pi t}{t_0} \right) \right) \]

Problem 13-18

The cylinder of weight \( W \) at \( A \) is hoisted using the motor and the pulley system shown. If the speed of point \( B \) on the cable is increased at a constant rate from zero to \( v_B \) in time \( t \), determine the tension in the cable at \( B \) to cause the motion.

Given:

\[ W = 400 \, \text{lb} \]

\[ v_B = 10 \, \text{ft/s} \]

\[ t = 5 \, \text{s} \]

Solution:

\[ 2s_A + s_B = 1 \]
Problem 13-19

A suitcase of weight $W$ slides from rest a distance $d$ down the smooth ramp. Determine the point where it strikes the ground at $C$. How long does it take to go from $A$ to $C$?

Given:
- $W = 40 \text{ lb}$
- $\theta = 30 \text{ deg}$
- $d = 20 \text{ ft}$
- $g = 32.2 \frac{\text{ft}}{\text{s}^2}$
- $h = 4 \text{ ft}$

Solution:
- $W \sin(\theta) = \left(\frac{W}{g}\right)a$
- $a = g \sin(\theta)$
- $a = 16.1 \frac{\text{ft}}{\text{s}^2}$
- $v_B = \sqrt{2ad}$
- $v_B = 25.377 \frac{\text{ft}}{\text{s}}$
- $t_{AB} = \frac{v_B}{a}$
- $t_{AB} = 1.576 \text{ s}$

Guesses
- $t_{BC} = 1 \text{ s}$
- $R = 1 \text{ ft}$

Given
- $-\frac{g}{2}t_{BC}^2 - v_B \sin(\theta)t_{BC} + h = 0$
- $R = v_B \cos(\theta)t_{BC}$

Find $t_{BC}, R$
- $t_{BC} = 0.241 \text{ s}$
- $R = 5.304 \text{ ft}$
- $t_{AB} + t_{BC} = 1.818 \text{ s}$
*Problem 13-20*

A suitcase of weight \( W \) slides from rest a distance \( d \) down the rough ramp. The coefficient of kinetic friction along ramp \( AB \) is \( \mu_k \). The suitcase has an initial velocity down the ramp \( v_0 \).

Determine the point where it strikes the ground at \( C \). How long does it take to go from \( A \) to \( C \)?

Given:

\[
\begin{align*}
W &= 40 \text{ lb} \\
d &= 20 \text{ ft} \\
h &= 4 \text{ ft} \\
\mu_k &= 0.2 \\
\theta &= 30 \text{ deg} \\
v_0 &= 10 \text{ ft/s} \\
g &= 32.2 \text{ ft/s}^2
\end{align*}
\]

Solution:

\[
\begin{align*}
F_N - W \cos(\theta) &= 0 \\
W \sin(\theta) - \mu_k W \cos(\theta) &= \left( \frac{W}{g} \right) a \\
a &= g(\sin(\theta) - \mu_k \cos(\theta)) \\
v_B &= \sqrt{2ad + v_0^2} \\
t_{AB} &= \frac{v_B - v_0}{a}
\end{align*}
\]

\( a = 10.523 \text{ ft/s}^2 \)

\[
\begin{align*}
v_B &= 22.823 \text{ ft/s} \\
t_{AB} &= 1.219 \text{ s}
\end{align*}
\]

Guesses

\( t_{BC} = 1 \text{ s} \quad R = 1 \text{ ft} \)

Given

\[
\left( \frac{-g}{2} \right) t_{BC}^2 - v_B \sin(\theta)t_{BC} + h = 0 \\
R = v_B \cos(\theta)t_{BC}
\]

\[
\left( \frac{t_{BC}}{R} \right) = \text{Find}(t_{BC}, R) \\
t_{BC} = 0.257 \text{ s} \\
R = 5.084 \text{ ft} \\
t_{AB} + t_{BC} = 1.476 \text{ s}
\]
Problem 13-21

The winding drum $D$ is drawing in the cable at an accelerated rate $a$. Determine the cable tension if the suspended crate has mass $M$.

Units Used:

$kN = 1000 N$

Given:

\[ a = 5 \frac{m}{s^2} \]

\[ M = 800 kg \]

\[ g = 9.81 \frac{m}{s^2} \]

Solution:

\[ L = s_A + 2s_B \quad a_B = \frac{-a}{2} \quad a_B = -2.5 \frac{m}{s^2} \]

\[ 2T - Mg = -Ma_B \quad T = \frac{M(g - a_B)}{2} \quad T = 4.924 kN \]

Problem 13-22

At a given instant block $A$ of weight $W_A$ is moving downward with a speed $v_I$. Determine its speed at the later time $t$. Block $B$ has weight $W_B$, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k$. Neglect the mass of the pulleys and cord.

Given:

\[ W_A = 5 lb \quad v_I = 4 \frac{ft}{s} \]

\[ W_B = 6 lb \quad t = 2 s \]

\[ \mu_k = 0.3 \]

Solution:

\[ 2s_B + s_A = L \]

Guesses

\[ a_A = 1 \frac{ft}{s^2} \quad a_B = 1 \frac{ft}{s^2} \]

\[ T = 1 lb \quad F_N = 1 lb \]

Given \[ F_N - W_B = 0 \quad 2a_B + a_A = 0 \]
\[ 2T - \mu_k F_N = \left( \frac{-W_B}{g} \right) a_B \quad T - W_A = \left( \frac{-W_A}{g} \right) a_A \]

\[
\begin{bmatrix}
F_N \\
T \\
a_A \\
a_B \\
\end{bmatrix} = \text{Find} \begin{bmatrix} F_N, T, a_A, a_B \end{bmatrix} \quad \begin{bmatrix} F_N \\
T \\
a_A \\
a_B \\
\end{bmatrix} = \begin{bmatrix} 6.000 \\
1.846 \\
6.000 \\
1.846 \\
\end{bmatrix} \text{lb} \quad \begin{bmatrix} a_A \\
a_B \\
\end{bmatrix} = \begin{bmatrix} 20.3 \\
-10.2 \\
\end{bmatrix} \text{ ft}^2 \text{s}^{-2}
\]

\[ v_2 = v_1 + a_A t \quad v_2 = 44.6 \text{ ft} \text{s}^{-1} \]

### Problem 13-23

A force \( F \) is applied to the cord. Determine how high the block \( A \) of weight \( W \) rises in time \( t \) starting from rest. Neglect the weight of the pulleys and cord.

**Given:**

\[ F = 15 \text{ lb} \quad t = 2 \text{ s} \quad W = 30 \text{ lb} \quad g = 32.2 \text{ ft} \text{s}^{-2} \]

**Solution:**

\[ 4F - W = \left( \frac{W}{g} \right) a \]

\[ a = \frac{g}{W} (4F - W) \]

\[ a = 32.2 \text{ ft}^2 \text{s}^{-2} \]

\[ d = \frac{1}{2} a t^2 \quad d = 64.4 \text{ ft} \]

### Problem 13-24

At a given instant block \( A \) of weight \( W_A \) is moving downward with speed \( v_{A0} \). Determine its speed at a later time \( t \). Block \( B \) has a weight \( W_B \) and the coefficient of kinetic friction between it and the
horizontal plane is \( \mu_k \). Neglect the mass of the pulleys and cord.

Given:

- \( W_A = 10 \text{ lb} \)
- \( v_{A0} = 6 \frac{\text{ft}}{\text{s}} \)
- \( t = 2 \text{ s} \)
- \( W_B = 4 \text{ lb} \)
- \( \mu_k = 0.2 \)
- \( g = 32.2 \frac{\text{ft}}{\text{s}^2} \)

Solution:

\[ L = s_B + 2s_A \]

Guesses

- \( a_A = 1 \frac{\text{ft}}{\text{s}^2} \)
- \( a_B = 1 \frac{\text{ft}}{\text{s}^2} \)
- \( T = 1 \text{ lb} \)

Given

\[ T - \mu_k W_B = \left( -\frac{W_B}{g} \right) a_B \]

\[ 2T - W_A = \left( -\frac{W_A}{g} \right) a_A \]

\[ 0 = a_B + 2a_A \]

\[
\begin{bmatrix}
T \\
a_A \\
a_B
\end{bmatrix} = \text{Find}(T, a_A, a_B) \quad T = 3.385 \text{ lb} \quad \begin{bmatrix}
a_A \\
a_B
\end{bmatrix} = \begin{bmatrix}
10.403 \\
-20.806
\end{bmatrix} \frac{\text{ft}}{\text{s}^2}
\]

\[ v_A = v_{A0} + a_A t \quad v_A = 26.8 \frac{\text{ft}}{\text{s}} \]

**Problem 13-25**

A freight elevator, including its load, has mass \( M_e \). It is prevented from rotating due to the track and wheels mounted along its sides. If the motor \( M \) develops a constant tension \( T \) in its attached cable, determine the velocity of the elevator when it has moved upward at a distance \( d \) starting from rest. Neglect the mass of the pulleys and cables.
Units Used:
\[ \text{kN} = 10^3 \text{ N} \]

Given:
\[ M_e = 500 \text{ kg} \]
\[ T = 1.50 \text{ kN} \]
\[ d = 3 \text{ m} \]
\[ g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution:
\[ 4T - M_e g = M_e a \]
\[ a = 4 \left( \frac{T}{M_e} \right) - g \]
\[ v = \sqrt{2ad} \]
\[ v = 3.62 \frac{\text{m}}{\text{s}} \]

Problem 13-26

At the instant shown the block \( A \) of weight \( W_A \) is moving down the plane at \( v_0 \) while being attached to the block \( B \) of weight \( W_B \). If the coefficient of kinetic friction is \( \mu_k \), determine the acceleration of \( A \) and the distance \( A \) slides before it stops. Neglect the mass of the pulleys and cables.

Given:
\[ W_A = 100 \text{ lb} \]
\[ W_B = 50 \text{ lb} \]
\[ v_0 = 5 \frac{\text{ft}}{\text{s}} \]
\[ \mu_k = 0.2 \]
\[ a = 3 \]
\[ b = 4 \]

Solution: \[ \theta = \tan\left( \frac{a}{b} \right) \]

Rope constraints
\[ s_A + 2s_C = L_1 \]
\[ s_D + (s_D - s_B) = L_2 \]
\[ s_C + s_D + d = d' \]

Guesses
\[ a_A = \frac{1}{s^2} \quad a_B = \frac{1}{s^2} \]
\[ a_C = \frac{1}{s^2} \quad a_D = \frac{1}{s^2} \]
\[ T_A = 1 \text{ lb} \quad T_B = 1 \text{ lb} \quad N_A = 1 \text{ lb} \]

Given
\[ a_A + 2a_C = 0 \quad 2a_D - a_B = 0 \]
\[ a_C + a_D = 0 \]
\[ T_B - W_B = \left( \frac{W_B}{g} \right) a_B \]
\[ T_A - W_A \sin(\theta) + \mu_k N_A = \left( \frac{-W_A}{g} \right) a_A \]
\[ N_A - W_A \cos(\theta) = 0 \quad 2T_A - 2T_B = 0 \]

\[
\begin{pmatrix}
  a_A \\
  a_B \\
  a_C \\
  a_D \\
  T_A \\
  T_B \\
  N_A
\end{pmatrix} = \text{Find} \begin{pmatrix} a_A, a_B, a_C, a_D, T_A, T_B, N_A \end{pmatrix}
\]
\[
\begin{pmatrix}
  T_A \\
  T_B \\
  N_A
\end{pmatrix} = \begin{pmatrix} 48 \\
  48 \\
  80
\end{pmatrix} \text{ lb}
\begin{pmatrix}
  a_A \\
  a_B \\
  a_C \\
  a_D
\end{pmatrix} = \begin{pmatrix} -1.287 \\
  -1.287 \\
  0.644 \\
  -0.644
\end{pmatrix} \text{ ft}^2/s^2
\]

\[ d_A = \frac{-v_0^2}{2a_A} \quad a_A = -1.287 \text{ ft/s}^2 \quad d_A = 9.71 \text{ ft} \]
Problem 13-27

The safe $S$ has weight $W_s$ and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy $B$ of weight $W_b$, determine his acceleration if in the confusion he doesn’t let go of the rope. Neglect the mass of the pulleys and rope.

Given:

\[ W_s = 200 \text{ lb} \quad W_b = 90 \text{ lb} \quad g = 32.2 \frac{\text{ft}}{\text{s}^2} \]

Solution: \[ L = 2s_s + s_b \]

Initial guesses: \[ a_b = 1 \frac{\text{ft}}{\text{s}^2} \quad a_s = 1 \frac{\text{ft}}{\text{s}^2} \quad T = 1 \text{ lb} \]

Given \[ 0 = 2a_s + a_b \quad 2T - W_s = \left( -\frac{W_s}{g} \right)a_s \quad T - W_b = \left( -\frac{W_b}{g} \right)a_b \]

\[ \begin{bmatrix} a_b \\ a_s \\ T \end{bmatrix} = \text{Find}(a_b, a_s, T) \quad T = 96.429 \text{ lb} \quad a_s = 1.15 \frac{\text{ft}}{\text{s}^2} \quad a_b = -2.3 \frac{\text{ft}}{\text{s}^2} \text{ Negative means up} \]

*Problem 13-28

The mine car of mass $m_{car}$ is hoisted up the incline using the cable and motor $M$. For a short time, the force in the cable is $F = bt^2$. If the car has an initial velocity $v_0$ when $t = 0$, determine its velocity when $t = t_f$. 

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Given:

\[ m_{\text{car}} = 400 \text{ kg} \]
\[ b = 3200 \frac{N}{s^2} \]
\[ v_0 = 2 \frac{m}{s} \]
\[ t_1 = 2 \text{ s} \]
\[ g = 9.81 \frac{m}{s^2} \]
\[ c = 8 \]
\[ d = 15 \]

Solution:

\[ bt^2 - m_{\text{car}}g \left( \frac{c}{\sqrt{c^2 + d^2}} \right) = m_{\text{car}}a \]
\[ a = \left( \frac{b}{m_{\text{car}}} \right)^2 - \frac{gc}{\sqrt{c^2 + d^2}} \]
\[ v_1 = \left( \frac{b}{m_{\text{car}}} \right) \sqrt[3]{3} - \frac{gct_1}{\sqrt{c^2 + d^2}} + v_0 \]

\[ v_1 = 14.1 \frac{m}{s} \]

**Problem 13-29**

The mine car of mass \( m_{\text{car}} \) is hoisted up the incline using the cable and motor \( M \). For a short time, the force in the cable is \( F = bt^2 \). If the car has an initial velocity \( v_0 \) when \( t = 0 \), determine the distance it moves up the plane when \( t = t_1 \).
Given:
\[ m_{car} = 400 \text{ kg} \]
\[ b = \frac{3200 \text{ N}}{s^2} \]
\[ v_0 = \frac{2 \text{ m}}{s} \]
\[ t_1 = 2 \text{ s} \]
\[ g = \frac{9.81 \text{ m}}{s^2} \]
\[ c = 8 \]
\[ d = 15 \]

Solution:
\[ b t^2 - m_{car} g \left( \frac{c}{\sqrt{c^2 + d^2}} \right) = m_{car} a \quad a = \left( \frac{b}{m_{car}} \right) t^2 - \frac{g c}{\sqrt{c^2 + d^2}} \]
\[ v = \left( \frac{b}{m_{car}} \right) t^3 - \frac{g c t}{\sqrt{c^2 + d^2}} + v_0 \]
\[ s_1 = \left( \frac{b}{m_{car}} \right) t_1^4 - \left( \frac{g c}{\sqrt{c^2 + d^2}} \right) t_1^2 + v_0 t_1 \quad s_1 = 5.434 \text{ m} \]

**Problem 13-30**

The tanker has a weight \( W \) and is traveling forward at speed \( v_0 \) in still water when the engines are shut off. If the drag resistance of the water is proportional to the speed of the tanker at any instant and can be approximated by \( F_D = cv \), determine the time needed for the tanker’s speed to become \( v_1 \). Given the initial velocity \( v_0 \) through what distance must the tanker travel before it stops?

Given:
\[ W = 800 \times 10^6 \text{ lb} \]
\[ c = 400 \times 10^3 \text{ lb} \cdot \text{s}^{-1} \cdot \text{ft}^{-1} \]
\[ v_0 = \frac{3 \text{ ft}}{s} \quad v_1 = \frac{1.5 \text{ ft}}{s} \]

Solution:
\[ a(v) = \frac{c g}{W} v \]
Problem 13-31

The spring mechanism is used as a shock absorber for railroad cars. Determine the maximum compression of spring HI if the fixed bumper R of a railroad car of mass \( M \), rolling freely at speed \( v \) strikes the plate P. Bar AB slides along the guide paths CE and DF. The ends of all springs are attached to their respective members and are originally unstretched.

Units Used:

\[ \text{kN} = 10^3 \text{ N} \quad \text{Mg} = 10^3 \text{ kg} \]

Given:

\[ M = 5 \text{ Mg} \quad k = 80 \frac{\text{kN}}{\text{m}} \]
\[ v = 2 \frac{\text{m}}{\text{s}} \quad k' = 160 \frac{\text{kN}}{\text{m}} \]

Solution:

The springs stretch or compress an equal amount \( x \). Thus,

\[ (k' + 2k)x = -Ma \]
\[ a = -\frac{k' + 2k}{M} \cdot x = v^2 \frac{d}{dx} \]

Guess \( d = 1 \text{ m} \)  Given \( \int^0_v v \, dv = -\int^d_0 \left( \frac{k' + 2k}{M} \right) x \, dx \)

\[ d = \text{Find}(d) \]

\[ d = 0.250 \text{ m} \]

*Problem 13-32

The collar C of mass \( m_c \) is free to slide along the smooth shaft AB. Determine the acceleration of collar C if (a) the shaft is fixed from moving, (b) collar A, which is fixed to shaft AB, moves
downward at constant velocity along the vertical rod, and (c) collar $A$ is subjected to downward acceleration $a_A$. In all cases, the collar moves in the plane.

**Given:**

$m_c = 2 \text{ kg}$

$a_A = \frac{2 \text{ m}}{s^2}$

$g = 9.81 \frac{\text{m}}{s^2}$

$\theta = 45 \text{ deg}$

**Solution:**

(a) $m_c g \cos(\theta) = m_c a_d$  
   $a_d = g \cos(\theta)$  
   $a_d = 6.937 \frac{\text{m}}{s^2}$

(b) $m_c g \cos(\theta) = m_c a_b$  
   $a_b = g \cos(\theta)$  
   $a_b = 6.937 \frac{\text{m}}{s^2}$

(c) $m_c (g - a_A) \cos(\theta) = m_c a_{crel}$  
   $a_{crel} = (g - a_A) \cos(\theta)$

$$a_c = a_{crel} \begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \end{pmatrix} + a_A \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_c = \begin{pmatrix} -3.905 \\ -5.905 \end{pmatrix} \frac{\text{m}}{s^2}$$

$|a_c| = 7.08 \frac{\text{m}}{s^2}$

**Problem 13-33**

The collar $C$ of mass $m_c$ is free to slide along the smooth shaft $AB$. Determine the acceleration of collar $C$ if collar $A$ is subjected to an upward acceleration $a$. The collar moves in the plane.

**Given:**

$m_C = 2 \text{ kg}$

$a = \frac{4 \text{ m}}{s^2}$

$g = 9.81 \frac{\text{m}}{s^2}$

$\theta = 45 \text{ deg}$

**Solution:**

The collar accelerates along the rod and the rod accelerates upward.
\[ m_C g \cos(\theta) = m_C [a_{CA} - (a) \cos(\theta)] \quad a_{CA} = (g + a) \cos(\theta) \]

\[
a_C = \begin{bmatrix} -a_{CA} \sin(\theta) \\ -a_{CA} \cos(\theta) + a \end{bmatrix}
\]

\[
\text{Solution:} \quad \begin{align*}
(a) & \quad 2T_a - W = 0 \\
& \quad T_a = \frac{W}{2} \\
& \quad T_a = 40 \text{ lb}
\end{align*}
\]

\[
(b) & \quad 2T_b - W = \left( \frac{W}{g} \right) 2bt \\
& \quad T_b = \frac{1}{2} \left( W + \frac{W}{g} 2bt \right) \\
& \quad T_b = 59.885 \text{ lb}
\]

**Problem 13-35**

The block \( A \) of mass \( m_A \) rests on the plate \( B \) of mass \( m_B \) in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block \( A \) to slide a distance \( s' \) on the plate when the system is released from rest.

**Given:**

\[
\begin{align*}
m_A &= 10 \text{ kg} \\
m_B &= 50 \text{ kg} \\
s' &= 0.5 \text{ m} \\
\mu_{AB} &= 0.2 \\
\mu_{BC} &= 0.1 \\
\theta &= 30 \text{ deg} \\
g &= 9.81 \frac{\text{m}}{\text{s}^2}
\end{align*}
\]
Solution:

\[ s_A + s_B = L \]

Guesses

\[ a_A = 1 \frac{m}{s^2} \quad a_B = 1 \frac{m}{s^2} \]

\[ T = 1 \text{ N} \quad N_A = 1 \text{ N} \quad N_B = 1 \text{ N} \]

Given

\[ a_A + a_B = 0 \]

\[ N_A - m_A g \cos(\theta) = 0 \]

\[ N_B - N_A - m_B g \cos(\theta) = 0 \]

\[ T - \mu_{AB} N_A - m_A g \sin(\theta) = -m_A a_A \]

\[ T + \mu_{AB} N_A + \mu_{BC} N_B - m_B g \sin(\theta) = -m_B a_B \]

\[
\begin{pmatrix}
  a_A \\
  a_B \\
  T \\
  N_A \\
  N_B
\end{pmatrix}
= \text{Find}(a_A, a_B, T, N_A, N_B)
\begin{pmatrix}
  T \\
  N_A \\
  N_B
\end{pmatrix}
= \begin{pmatrix}
  84.58 \\
  84.96 \\
  509.74
\end{pmatrix} \text{ N}
\]

\[
\begin{pmatrix}
  a_A \\
  a_B
\end{pmatrix}
= \begin{pmatrix}
  -1.854 \\
  1.854
\end{pmatrix} \frac{m}{s^2}
\]

\[ a_{BA} = a_B - a_A \quad a_{BA} = 3.708 \frac{m}{s^2} \]

\[ t = \frac{2s'}{a_{BA}} \quad t = 0.519 \text{ s} \]

*Problem 13-36*

Determine the acceleration of block A when the system is released from rest. The coefficient of kinetic friction and the weight of each block are indicated. Neglect the mass of the pulleys and cord.

Given:

\[ W_A = 80 \text{ lb} \]

\[ W_B = 20 \text{ lb} \]

\[ \theta = 60 \text{ deg} \]

\[ \mu_k = 0.2 \]
\[ g = 32.2 \text{ ft} \text{s}^{-2} \]

Solution:
\[ 2s_A + s_B = L \]

Guesses
\[ a_A = 1 \text{ ft} \text{s}^{-2} \quad a_B = 1 \text{ ft} \text{s}^{-2} \]

\[ T = 1 \text{ lb} \quad N_A = 1 \text{ lb} \]

Given
\[ 2T - W_A \sin(\theta) + \mu_k N_A = \left( -\frac{W_A}{g} \right) a_A \]
\[ N_A - W_A \cos(\theta) = 0 \]
\[ T - W_B = \left( -\frac{W_B}{g} \right) a_B \]
\[ 2a_A + a_B = 0 \]

\[
\begin{pmatrix}
a_A \\
a_B \\
T \\
N_A
\end{pmatrix}
= \text{Find}(a_A, a_B, T, N_A)
\]

\[ a_A = 4.28 \text{ ft} \text{s}^{-2} \]

Problem 13-37

The conveyor belt is moving at speed \( v \). If the coefficient of static friction between the conveyor and the package \( B \) of mass \( M \) is \( \mu_s \), determine the shortest time the belt can stop so that the package does not slide on the belt.

Given:
\[ v = 4 \text{ m} \text{s}^{-1} \]
\[ M = 10 \text{ kg} \]
\[ \mu_s = 0.2 \]
\[ g = 9.81 \text{ m} \text{s}^{-2} \]

Solution:
\[ \mu_s M g = Ma \quad a = \mu_s g \quad a = 1.962 \text{ m} \text{s}^{-2} \]
\[ t = \frac{v}{a} \quad t = 2.039 \text{ s} \]
**Problem 13-38**

An electron of mass \( m \) is discharged with an initial horizontal velocity of \( v_0 \). If it is subjected to two fields of force for which \( F_x = F_0 \) and \( F_y = 0.3F_0 \) where \( F_0 \) is constant, determine the equation of the path, and the speed of the electron at any time \( t \).

Solution:

\[
F_0 = ma_x \quad \quad \quad \quad 0.3F_0 = ma_y
\]

\[
a_x = \frac{F_0}{m} \quad \quad \quad \quad a_y = 0.3 \left( \frac{F_0}{m} \right)
\]

\[
v_x = \left( \frac{F_0}{m} \right) t + v_0 \quad \quad \quad \quad v_y = 0.3 \left( \frac{F_0}{m} \right) t
\]

\[
s_x = \frac{F_0}{m} \left( \frac{t^2}{2} \right) + v_0 t \quad \quad \quad \quad s_y = 0.3 \frac{F_0}{m} \left( \frac{t^2}{2} \right) \quad \quad \quad \quad t = \sqrt{\frac{20s_y m}{3F_0}}
\]

Thus

\[
s_x = \frac{10s_y}{3} + v_0 \sqrt{\frac{20s_y m}{3F_0}}
\]

\[
v = \sqrt{v_x^2 + v_y^2} \quad \quad \quad \quad v = \sqrt{\frac{F_0}{m} t + v_0} + \left( \frac{0.3F_0}{m} t \right)^2
\]

*Problem 13-39*

The conveyor belt delivers each crate of mass \( M \) to the ramp at \( A \) such that the crate’s speed is \( v_A \) directed down along the ramp. If the coefficient of kinetic friction between each crate and the ramp is \( \mu_k \), determine the speed at which each crate slides off the ramp at \( B \). Assume that no tipping occurs.

Given:

\( M = 12 \) kg

\( v_A = 2.5 \) m/s

\( d = 3 \) m

\( \mu_k = 0.3 \)

\( \theta = 30 \) deg

\( g = 9.81 \) m/s²
Solution:

\[ N_C - Mg \cos(\theta) = 0 \quad N_C = Mg \cos(\theta) \]
\[ Mg \sin(\theta) - \mu_k N_C = Ma \quad a = g \sin(\theta) - \mu_k \left( \frac{N_C}{M} \right) \]
\[ v_B = \sqrt{v_A^2 + 2ad} \quad v_B = 4.515 \frac{m}{s} \]

*Problem 13-40*

A parachutist having a mass \( m \) opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is \( F_D = kv^2 \), where \( k \) is a constant, determine his velocity when he has fallen for a time \( t \). What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall \( t \to \infty \).

Solution:

\[ k v^2 - mg = -ma \]
\[ a = g - \left( \frac{k}{m} \right) v^2 = \frac{dv}{dt} \]
\[ t = \int_{0}^{v} \frac{1}{g - \left( \frac{k}{m} \right) v^2} \, dv \]
\[ t = \left( \frac{m}{gk} \right) \text{atanh} \left( v \left( \frac{g}{k} \right) \sqrt{\frac{m}{g}} \right) \]
\[ v = \left( \frac{mg}{k} \right) \text{tanh} \left( \frac{g}{k} \sqrt{\frac{m}{g}} \right) \quad \text{When } t \to \infty \]
\[ v = \sqrt{\frac{mg}{k}} \]

**Problem 13-41**

Block \( B \) rests on a smooth surface. If the coefficients of static and kinetic friction between \( A \) and \( B \) are \( \mu_s \) and \( \mu_k \) respectively, determine the acceleration of each block if someone pushes horizontally on block \( A \) with a force of \( (a) F = F_a \) and \( (b) F = F_b \).

Given:

\[ \mu_s = 0.4 \quad F_a = 6 \text{ lb} \]
\[ \mu_k = 0.3 \quad F_b = 50 \text{ lb} \]
\[ W_A = 20 \text{ lb} \quad W_B = 30 \text{ lb} \]
Solution:

Guesses \( F_A = 1 \text{ lb} \) \( F_{\text{max}} = 1 \text{ lb} \)

\[ a_A = 1 \frac{\text{ft}}{\text{s}^2} \quad a_B = 1 \frac{\text{ft}}{\text{s}^2} \]

(a) \( F = F_a \) \quad \text{First assume no slip}

Given \( F - F_A = \left( \frac{W_A}{g} \right) a_A \quad F_A = \left( \frac{W_B}{g} \right) a_B \)

\[
\begin{pmatrix} F_A \\ F_{\text{max}} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{\text{max}}, a_A, a_B) \quad \text{If} \quad F_A = 3.599 \text{ lb} < F_{\text{max}} = 8 \text{ lb} \quad \text{then our assumption is correct and} \quad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 3.86 \\ 3.86 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}
\]

(b) \( F = F_b \) \quad \text{First assume no slip}

Given \( F - F_A = \left( \frac{W_A}{g} \right) a_A \quad F_A = \left( \frac{W_B}{g} \right) a_B \)

\[ a_A = a_B \quad F_{\text{max}} = \mu_s W_A \]

\[
\begin{pmatrix} F_A \\ F_{\text{max}} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{\text{max}}, a_A, a_B) \quad \text{Since} \quad F_A = 30 \text{ lb} > F_{\text{max}} = 8 \text{ lb} \quad \text{then our assumption is not correct.}
\]

Now we know that it slips

Given \( F_A = \mu_k W_A \) \quad \( F - F_A = \left( \frac{W_A}{g} \right) a_A \quad F_A = \left( \frac{W_B}{g} \right) a_B \)

\[
\begin{pmatrix} F_A \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, a_A, a_B) \quad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 70.84 \\ 6.44 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}
\]
Problem 13-42

Blocks \( A \) and \( B \) each have a mass \( M \). Determine the largest horizontal force \( P \) which can be applied to \( B \) so that \( A \) will not move relative to \( B \). All surfaces are smooth.

Solution:
Require \( a_A = a_B = a \)

Block \( A \):

\[ \Sigma F_y = 0; \quad N \cos(\theta) - M g = 0 \]
\[ \Sigma F_x = M a; \quad N \sin(\theta) = M a \]

\( a = g \tan(\theta) \)

Block \( B \):

\[ \Sigma F_x = M a; \quad P - N \sin(\theta) = M a \]

\[ P = 2M g \tan(\theta) \]

Problem 13-43

Blocks \( A \) and \( B \) each have mass \( m \). Determine the largest horizontal force \( P \) which can be applied to \( B \) so that \( A \) will not slip up \( B \). The coefficient of static friction between \( A \) and \( B \) is \( \mu_s \). Neglect any friction between \( B \) and \( C \).

Solution:
Require \( a_A = a_B = a \)

Block \( A \):

\[ \Sigma F_y = 0; \quad N \cos(\theta) - \mu_s N \sin(\theta) - m g = 0 \]
\[ \Sigma F_x = m a; \quad N \sin(\theta) + \mu_s N \cos(\theta) = m a \]

\[ N = \frac{m g}{\cos(\theta) - \mu_s \sin(\theta)} \]

\[ a = g \left( \frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right) \]

Block \( B \):

\[ \Sigma F_x = m a; \quad P - \mu_s N \cos(\theta) - N \sin(\theta) = m a \]
\[ P - \frac{\mu_s m g \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} = mg \left( \frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right) \]
\[ p = 2mg \left( \frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right) \]

**Problem 13-44**

Each of the three plates has mass \( M \). If the coefficients of static and kinetic friction at each surface of contact are \( \mu_s \) and \( \mu_k \) respectively, determine the acceleration of each plate when the three horizontal forces are applied.

Given:
- \( M = 10 \text{ kg} \)
- \( \mu_s = 0.3 \)
- \( \mu_k = 0.2 \)
- \( F_B = 15 \text{ N} \)
- \( F_C = 100 \text{ N} \)
- \( F_D = 18 \text{ N} \)
- \( g = 9.81 \frac{\text{m}}{\text{s}^2} \)

Solution:

Case 1: Assume that no slipping occurs anywhere.

\[ F_{AB\text{max}} = \mu_s(3Mg) \quad F_{BC\text{max}} = \mu_s(2Mg) \quad F_{CD\text{max}} = \mu_s(Mg) \]

Guesses: \( F_{AB} = 1 \text{ N} \quad F_{BC} = 1 \text{ N} \quad F_{CD} = 1 \text{ N} \)

Given:
- \( -F_D + F_{CD} = 0 \)
- \( F_C - F_{CD} - F_{BC} = 0 \)
- \( -F_B - F_{AB} + F_{BC} = 0 \)

\[
\begin{align*}
\begin{bmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{bmatrix} &= \text{Find}(F_{AB}, F_{BC}, F_{CD}) \\
\begin{bmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{bmatrix} &= \begin{bmatrix} 67 \\ 82 \\ 18 \end{bmatrix} \quad \text{N} \\
\begin{bmatrix} F_{AB\text{max}} \\ F_{BC\text{max}} \\ F_{CD\text{max}} \end{bmatrix} &= \begin{bmatrix} 88.29 \\ 58.86 \\ 29.43 \end{bmatrix} \quad \text{N}
\end{align*}
\]

If \( F_{AB} = 67 \text{ N} < F_{AB\text{max}} = 88.29 \text{ N} \) and \( F_{BC} = 82 \text{ N} > F_{BC\text{max}} = 58.86 \text{ N} \) and \( F_{CD} = 18 \text{ N} < F_{CD\text{max}} = 29.43 \text{ N} \) then nothing moves and there is no acceleration.
Case 2: If $F_{AB} = 67 \text{ N} < F_{AB\text{max}} = 88.29 \text{ N}$ and $F_{BC} = 82 \text{ N} > F_{BC\text{max}} = 58.86 \text{ N}$ and $F_{CD} = 18 \text{ N} < F_{CD\text{max}} = 29.43 \text{ N}$ then slipping occurs between $B$ and $C$. We will assume that no slipping occurs at the other 2 surfaces.

Set $F_{BC} = \mu k(2Mg)$ $a_B = 0$ $a_C = a_D = a$

Guesses $F_{AB} = 1 \text{ N}$ $F_{CD} = 1 \text{ N}$ $a = \frac{1 \text{ m}}{s^2}$

Given $-F_D + F_{CD} = Ma$ $F_C - F_{CD} - F_{BC} = Ma$ $-F_B - F_{AB} + F_{BC} = 0$

\[
\begin{bmatrix}
F_{AB} \\
F_{CD} \\
a
\end{bmatrix}
= \text{Find}(F_{AB}, F_{CD}, a)
\begin{bmatrix}
24.24 \\
39.38 \\
aC \\
aD
\end{bmatrix}
\text{N}
\]

If $F_{AB} = 24.24 \text{ N} < F_{AB\text{max}} = 88.29 \text{ N}$ and $F_{CD} = 39.38 \text{ N} > F_{CD\text{max}} = 29.43 \text{ N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 2.138 \frac{\text{m}}{s^2}$, $a_D = 2.138 \frac{\text{m}}{s^2}$

Case 3: If $F_{AB} = 24.24 \text{ N} < F_{AB\text{max}} = 88.29 \text{ N}$ and $F_{CD} = 39.38 \text{ N} > F_{CD\text{max}} = 29.43 \text{ N}$ then slipping occurs between $C$ and $D$ as well as between $B$ and $C$. We will assume that no slipping occurs at the other surface.

Set $F_{BC} = \mu k(2Mg)$ $F_{CD} = \mu k(Mg)$

Guesses $F_{AB} = 1 \text{ N}$ $a_C = 1 \frac{\text{m}}{s^2}$ $a_D = 1 \frac{\text{m}}{s^2}$

Given $-F_D + F_{CD} = MaD$ $F_C - F_{CD} - F_{BC} = MaC$ $-F_B - F_{AB} + F_{BC} = 0$

\[
\begin{bmatrix}
F_{AB} \\
aC \\
aD
\end{bmatrix}
= \text{Find}(F_{AB}, aC, aD)
\begin{bmatrix}
24.24 \\
4.114 \\
0.162
\end{bmatrix}
\text{N}
\]

If $F_{AB} = 24.24 \text{ N} < F_{AB\text{max}} = 88.29 \text{ N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 4.114 \frac{\text{m}}{s^2}$, $a_D = 0.162 \frac{\text{m}}{s^2}$

There are other permutations of this problems depending on the numbers that one chooses.

**Problem 13-45**

Crate $B$ has a mass $m$ and is released from rest when it is on top of cart $A$, which has a mass $3m$. Determine the tension in cord $CD$ needed to hold the cart from moving while $B$ is
sliding down $A$. Neglect friction.

Solution:

Block $B$:

$$N_B - mg \cos(\theta) = 0$$

$$N_B = mg \cos(\theta)$$

Cart:

$$-T + N_B \sin(\theta) = 0$$

$$T = mg \sin(\theta) \cos(\theta)$$

$$T = \left(\frac{mg}{2}\right) \sin(2\theta)$$

Problem 13-46

The tractor is used to lift load $B$ of mass $M$ with the rope of length $2h$, and the boom, and pulley system. If the tractor is traveling to the right at constant speed $v$, determine the tension in the rope when $s_A = d$. When $s_A = 0$, $s_B = 0$

Units used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$M = 150 \text{ kg}$$

$$v = 4 \frac{\text{m}}{\text{s}} \quad h = 12 \text{ m}$$

$$d = 5 \text{ m} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution: $v_A = v \quad s_A = d$

Guesses

$$T = 1 \text{ kN} \quad s_B = 1 \text{ m}$$

$$a_B = 1 \frac{\text{m}}{\text{s}^2} \quad v_B = 1 \frac{\text{m}}{\text{s}}$$

Given

$$h - s_B + \sqrt{s_A^2 + h^2} = 2h$$

$$-v_B + \frac{s_A v_A}{\sqrt{s_A^2 + h^2}} = 0$$
\[-a_B + \frac{v_A^2}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{(s_A^2 + h^2)^{3/2}} = 0\]

\[T - M g = m a_B\]

\[
\begin{pmatrix}
T \\
 s_B \\
 v_B \\
 a_B
\end{pmatrix}
= \text{Find}(T, s_B, v_B, a_B)
\]

\[s_B = 1 \text{ m} \quad a_B = 1.049 \text{ m/s}^2 \quad v_B = 1.538 \text{ m/s} \quad T = 1.629 \text{kN}\]

**Problem 13-47**

The tractor is used to lift load \( B \) of mass \( M \) with the rope of length \( 2h \), and the boom, and pulley system. If the tractor is traveling to the right with an acceleration \( a \) and has speed \( v \) at the instant \( s_A = d \), determine the tension in the rope. When \( s_A = 0 \), \( s_B = 0 \).

Units used: \( \text{kN} = 10^3 \text{ N} \)

Given:

\[d = 5 \text{ m} \quad h = 12 \text{ m}\]

\[M = 150 \text{ kg} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}\]

\[v = 4 \frac{\text{m}}{\text{s}}\]

\[a = 3 \frac{\text{m}}{\text{s}^2}\]

Solution: \( a_A = a \quad v_A = v \quad s_A = d \)

Guesses \( T = 1 \text{kN} \quad s_B = 1 \text{ m} \quad a_B = 1 \frac{\text{m}}{\text{s}^2} \quad v_B = 1 \frac{\text{m}}{\text{s}} \)
Given \( h - s_B + \sqrt{s_A^2 + h^2} = 2h \)

\(-v_B + \frac{s_A v_A}{\sqrt{s_A^2 + h^2}} = 0 \)

\(-a_B + \frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} = 0 \)

\[ T - M g = M a_B \]

\[
\begin{bmatrix}
  T \\
  s_B \\
  v_B \\
  a_B
\end{bmatrix} = \text{Find}(T, s_B, v_B, a_B) \quad s_B = 1 \text{ m}
\]

\[ a_B = 2.203 \, \frac{\text{m}}{\text{s}^2} \quad v_B = 1.538 \, \frac{\text{m}}{\text{s}} \quad T = 1.802 \, \text{kN} \]

*Problem 13-48*

Block \( B \) has a mass \( m \) and is hoisted using the cord and pulley system shown. Determine the magnitude of force \( F \) as a function of the block’s vertical position \( y \) so that when \( F \) is applied the block rises with a constant acceleration \( a_B \). Neglect the mass of the cord and pulleys.

Solution:

\[ 2F \cos(\theta) - mg = ma_B \]

where \( \cos(\theta) = \frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}} \)

\[ 2F \left[ \frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}} \right] - mg = ma_B \]

\[ F = m \left( a_B + g \right) \frac{4y^2 + d^2}{4y} \]

*Problem 13-49*

Block \( A \) has mass \( m_A \) and is attached to a spring having a stiffness \( k \) and unstretched length \( l_0 \). If another block \( B \), having mass \( m_B \) is pressed against \( A \) so that the spring deforms a distance \( d \),
determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?

Solution:

Block \( A \):
\[-k(x - d) - N = m_A a_A\]

Block \( B \):
\[N = m_B a_B\]

Since \( a_A = a_B = a \),
\[a = \frac{k(d - x)}{m_A + m_B}\]

\[N = \frac{km_B(d - x)}{m_A + m_B}\]

Separation occurs when
\[N = 0 \quad \text{or} \quad x = d\]

\[
\int_0^v v \, dv = \int_0^d \frac{k(d - x)}{m_A + m_B} \, dx
\]

\[
\frac{v^2}{2} = \frac{k}{m_A + m_B} \left( dd - \frac{d^2}{2} \right)
\]

\[v = \sqrt{\frac{kd^2}{m_A + m_B}}\]

**Problem 13-50**

Block \( A \) has a mass \( m_A \) and is attached to a spring having a stiffness \( k \) and unstretched length \( l_0 \). If another block \( B \), having a mass \( m_B \) is pressed against \( A \) so that the spring deforms a distance \( d \), show that for separation to occur it is necessary that \( d > \frac{2\mu_k g(m_A + m_B)}{k} \), where \( \mu_k \) is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?

Solution: Block \( A \):

\[-k(x - d) - N - \mu_k m_A g = m_A a_A\]

Block \( B \):
\[N - \mu_k m_B g = m_B a_B\]
Since \( a_A = a_B = a \)

\[
a = \frac{k(d - x)}{m_A + m_B} - \mu_k g
\]

\[
N = \frac{k m_B (d - x)}{m_A + m_B}
\]

\( N = 0 \), then \( x = d \) for separation.

At the moment of separation:

\[
\int_0^v v \, dv = \int_0^d \left[ \frac{k(d - x)}{m_A + m_B} \right] dx - \mu_k g \int_0^d dx
\]

\[
v = \sqrt{\frac{k d^2 - 2 \mu_k g (m_A + m_B) d}{m_A + m_B}}
\]

Require \( v > 0 \), so that

\[
k d^2 - 2 \mu_k g (m_A + m_B) d > 0 \\
\]

\[
d > \frac{2 \mu_k g}{k} \left( m_A + m_B \right) \quad \text{Q.E.D}
\]

**Problem 13-51**

The block \( A \) has mass \( m_A \) and rests on the pan \( B \), which has mass \( m_B \). Both are supported by a spring having a stiffness \( k \) that is attached to the bottom of the pan and to the ground.

Determine the distance \( d \) the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

\[
\begin{align*}
\text{Solution:} \\
\text{For Equilibrium} & \quad k y_{eq} - (m_A + m_B) g = 0 \\
\text{Block:} & \quad -m_A g + N = m_A a
\end{align*}
\]
Block and Pan  
\[ (-m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a \]

Thus,  
\[ -(m_A + m_B)g + k\left(\frac{m_A + m_B}{k}g + y\right) = (m_A + m_B)\left(\frac{-m_Ag + N}{m_A}\right) \]

Set \( y = -d, N = 0 \)  
Thus  
\[ d = y_{eq} = \frac{(m_A + m_B)g}{k} \]

*Problem 13-52*

Determine the mass of the sun, knowing that the distance from the earth to the sun is \( R \). *Hint:* Use Eq. 13-1 to represent the force of gravity acting on the earth.

Given:  
\[ R = 149.6 \times 10^6 \text{ km} \quad G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

Solution:  
\[ v = \frac{s}{t} \quad v = \frac{2\pi R}{1 \text{ yr}} \quad v = 2.98 \times 10^4 \text{ m/s} \]

\[ \sum F_n = ma_n; \quad G\left(\frac{M_eM_s}{R^2}\right) = M_e\left(\frac{v^2}{R}\right) \quad MS = v^2\left(\frac{R}{G}\right) \quad MS = 1.99 \times 10^{30} \text{ kg} \]

Problem 13-53

The helicopter of mass \( M \) is traveling at a constant speed \( v \) along the horizontal curved path while banking at angle \( \theta \). Determine the force acting normal to the blade, i.e., in the \( y' \) direction, and the radius of curvature of the path.
Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[ v = 40 \text{ m/s} \quad M = 1.4 \times 10^3 \text{ kg} \]

\[ \theta = 30 \text{ deg} \quad g = 9.81 \text{ m/s}^2 \]

Solution:

Guesses \( F_N = 1 \text{ kN} \quad \rho = 1 \text{ m} \)

Given

\[ F_N \cos(\theta) - Mg = 0 \]

\[ F_N \sin(\theta) = M \left( \frac{v^2}{\rho} \right) \]

\[ \left( \frac{F_N}{\rho} \right) = \text{Find}(F_N, \rho) \quad F_N = 15.86 \text{ kN} \]

\[ \rho = 282 \text{ m} \]

---

**Problem 13-54**

The helicopter of mass \( M \) is traveling at a constant speed \( v \) along the horizontal curved path having a radius of curvature \( \rho \). Determine the force the blade exerts on the frame and the bank angle \( \theta \).

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[ v = 33 \text{ m/s} \quad M = 1.4 \times 10^3 \text{ kg} \]

\[ \rho = 300 \text{ m} \quad g = 9.81 \text{ m/s}^2 \]
Solution:

Guesses  \( F_N = 1 \text{ kN} \quad \theta = 1 \text{ deg} \)

Given  
\[
F_N \cos(\theta) - Mg = 0 \\
F_N \sin(\theta) = M \left( \frac{v^2}{\rho} \right)
\]

\[
\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta) \quad F_N = 14.64 \text{ kN} \\
\theta = 20 \text{ deg}
\]

Problem 13-55

The plane is traveling at a constant speed \( v \) along the curve \( y = bx^2 + c \). If the pilot has weight \( W \), determine the normal and tangential components of the force the seat exerts on the pilot when the plane is at its lowest point.

Given:

\[
b = 20 \times 10^{-6} \ \frac{1}{\text{ft}} \\
c = 5000 \ \text{ft} \\
W = 180 \ \text{lb} \\
v = 800 \ \frac{\text{ft}}{\text{s}}
\]

Solution:

\[
x = 0 \ \text{ft} \quad y = bx^2 + c \\
y' = 2bx \quad y'' = 2b
\]

\[
\rho = \frac{\sqrt{1 + y'^2}}{y''}
\]

\[
F_n = W - \frac{W}{g} \left( \frac{v^2}{\rho} \right) \\
F_n = W + \frac{W}{g} \left( \frac{v^2}{\rho} \right) \\
F_n = 323 \ \text{lb}
\]

\[
a_t = 0 \\
F_t = \left( \frac{W}{g} \right) a_t \\
F_t = 0
\]
*Problem 13-56*

The jet plane is traveling at a constant speed of \( v \) along the curve \( y = bx^2 + c \). If the pilot has a weight \( W \), determine the normal and tangential components of the force the seat exerts on the pilot when \( y = y_1 \).

Given:

\[
\begin{align*}
  b &= 20 \times 10^{-6} \text{ ft}^{-1} \quad W = 180 \text{ lb} \\
  c &= 5000 \text{ ft} \quad v = 1000 \text{ ft/s} \\
  g &= 32.2 \text{ ft/s}^2 \quad y_1 = 10000 \text{ ft}
\end{align*}
\]

Solution:

\[
\begin{align*}
  y(x) &= bx^2 + c \\
  y'(x) &= 2bx \\
  y''(x) &= 2b \\
  \rho(x) &= \sqrt{\left(1 + y'(x)^2\right)^3} \\
  y''(x) &= \left(1 + y'(x)^2\right)^{3/2} y''(x)^{-1/2}
\end{align*}
\]

Guesses

\[
\begin{align*}
  x_1 &= 1 \text{ ft} \quad F_n = 1 \text{ lb} \\
  \theta &= 1 \text{ deg} \quad F_t = 1 \text{ lb}
\end{align*}
\]

Given

\[
\begin{align*}
  y_1 &= y(x_1) \\
  \tan(\theta) &= y'(x_1)
\end{align*}
\]

\[
F_n - W \cos(\theta) = W \left( \frac{v^2}{\rho(x_1)} \right) \\
F_t - W \sin(\theta) = 0
\]

\[
\begin{pmatrix}
  x_1 \\
  \theta \\
  F_n \\
  F_t
\end{pmatrix} = \text{Find}(x_1, \theta, F_n, F_t) \\
\begin{pmatrix}
  x_1 = 15811 \text{ ft} \\
  \theta = 32.3 \text{ deg}
\end{pmatrix} \\
\begin{pmatrix}
  F_n \\
  F_t
\end{pmatrix} = \begin{pmatrix}
  287.1 \\
  96.2
\end{pmatrix} \text{ lb}
\]
**Problem 13-57**

The wrecking ball of mass \( M \) is suspended from the crane by a cable having a negligible mass. If the ball has speed \( v \) at the instant it is at its lowest point \( \theta \), determine the tension in the cable at this instant. Also, determine the angle \( \theta \) to which the ball swings before it stops.

Units Used:

\[
\text{kN} = 10^3 \text{ N}
\]

Given:

\[
M = 600 \text{ kg}
\]
\[
v = 8 \frac{\text{m}}{\text{s}}
\]
\[
r = 12 \text{ m}
\]
\[
g = 9.81 \frac{\text{m}}{\text{s}^2}
\]

Solution:

At the lowest point

\[
T - Mg = M \left( \frac{v^2}{r} \right)
\]
\[
T = Mg + M \left( \frac{v^2}{r} \right)
\]
\[
T = 9.086 \text{ kN}
\]

At some arbitrary angle

\[
-Mg \sin(\theta) = Ma_t
\]
\[
a_t = -g \sin(\theta) = -\frac{v}{r} \left( \frac{dv}{d\theta} \right)
\]

\[
\int_{v}^{-} v \, dv = \int_{0}^{\theta} rg \sin(\theta) \, d\theta
\]
\[
-\frac{v^2}{2} = rg(\cos(\theta) - 1)
\]
\[
\theta = \arccos \left( 1 - \frac{v^2}{2rg} \right)
\]
\[
\theta = 43.3 \text{ deg}
\]

**Problem 13-58**

Prove that if the block is released from rest at point \( B \) of a smooth path of arbitrary shape, the speed it attains when it reaches point \( A \) is equal to the speed it attains when it falls freely through a distance \( h \); i.e., \( v = \sqrt{2gh} \).
Solution:

\[ \sum F_t = ma_t; \quad (mg)\sin(\theta) = ma_t \quad a_t = g\sin(\theta) \]

\[ vdv = a_t ds = g\sin(\theta) \, ds \quad \text{However} \quad dy = ds \sin(\theta) \]

\[ \int_0^v v \, dv = \int_0^h g \, dy \quad \frac{v^2}{2} = gh \quad v = \sqrt{2gh} \quad \text{Q.E.D} \]

**Problem 13-59**

The sled and rider have a total mass \( M \) and start from rest at \( A(b, 0) \). If the sled descends the smooth slope, which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point \( B \). Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.

Units Used:

\( \text{kN} = 10^3 \text{ N} \)

Given:

\( a = 2 \text{ m} \quad b = 10 \text{ m} \quad c = 5 \text{ m} \)

\( M = 80 \text{ kg} \quad g = 9.81 \text{ m/s}^2 \)

Solution:

\[ v = \sqrt{2gc} \]

\[ y(x) = c \left( \frac{x}{b} \right)^2 - c \]

\[ y'(x) = \frac{2c}{b^2} x \quad y''(x) = \frac{2c}{b^2} \]

\[ \rho(x) = \sqrt{\left(1 + y'(x)^2\right)^3} \]

\[ y''(x) = \frac{\rho(x)}{y''(x)} \]
\[
N_b - Mg = M\left(\frac{v^2}{\rho}\right)
\]
\[
N_b = Mg + M\left(\frac{v^2}{\rho(0 \text{ m})}\right) \quad N_b = 1.57 \text{ kN}
\]

**Problem 13-60**

The sled and rider have a total mass \( M \) and start from rest at \( A(b, 0) \). If the sled descends the smooth slope which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point \( C \). Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.

Units Used:

\[ kN = 10^3 \text{ N} \]

Given:

\[
a = 2 \text{ m} \quad b = 10 \text{ m} \quad c = 5 \text{ m}
\]
\[
M = 80 \text{ kg} \quad g = 9.81 \text{ m/s}^2
\]

Solution:

\[
y(x) = c\left(\frac{x}{b}\right)^2 - c
\]
\[
y'(x) = \frac{2c}{b^2}x \quad y''(x) = \frac{2c}{b^2}
\]
\[
\rho(x) = \sqrt{\left(1 + y'(x)^2\right)^3} \quad y''(x)
\]
\[
v = \sqrt{2g(-y(-a))}
\]
\[
\theta = \tan(y'(-a))
\]
\[
N_C - Mg \cos(\theta) = M\left(\frac{v^2}{\rho}\right) \quad N_C = M\left(g \cos(\theta) + \frac{v^2}{\rho(-a)}\right) \quad N_C = 1.48 \text{ kN}
\]

**Problem 13-61**

At the instant \( \theta = \theta_f \) the boy’s center of mass \( G \) has a downward speed \( v_G \). Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this
The boy has a weight \( W \). Neglect his size and the mass of the seat and cords.

Given:

\[
\begin{align*}
W &= 60 \text{ lb} \\
\theta_1 &= 60 \text{ deg} \\
l &= 10 \text{ ft} \\
v_G &= 15 \frac{\text{ft}}{\text{s}} \\
g &= 32.2 \frac{\text{ft}}{\text{s}^2}
\end{align*}
\]

Solution:

\[
\begin{align*}
W \cos(\theta_1) &= \left(\frac{W}{g}\right) a_t \\
a_t &= g \cos(\theta_1) \\
a_t &= 16.1 \frac{\text{ft}}{\text{s}^2}
\end{align*}
\]

\[
2T - W \sin(\theta_1) = \frac{W v_G^2}{l} + W \sin(\theta_1)
\]

\[
T = \frac{1}{2} \left[ \frac{W v_G^2}{l} + W \sin(\theta_1) \right]
\]

\( T = 46.9 \text{ lb} \)

**Problem 13-62**

At the instant \( \theta = \theta_1 \), the boy’s center of mass \( G \) is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when \( \theta = \theta_2 \). The boy has a weight \( W \). Neglect his size and the mass of the seat and cords.

Given:

\[
\begin{align*}
W &= 60 \text{ lb} \\
g &= 32.2 \frac{\text{ft}}{\text{s}^2} \\
\theta_1 &= 60 \text{ deg} \\
\theta_2 &= 90 \text{ deg} \\
l &= 10 \text{ ft}
\end{align*}
\]
Solution:

\[ W \cos(\theta) = \left( \frac{W}{g} \right) a_t \quad a_t = g \cos(\theta) \]

\[ v_2 = \sqrt{\frac{2gl}{\theta_2}} \cos(\theta) \, d\theta \]

\[ v_2 = 9.29 \text{ ft/s} \]

\[ 2T - W \sin(\theta_2) = W \left( \frac{v_2^2}{l} \right) \]

\[ T = \frac{W}{2} \left( \sin(\theta_2) + \frac{v_2^2}{gl} \right) \quad T = 38.0 \text{ lb} \]

Problem 13-63

If the crest of the hill has a radius of curvature \( \rho \), determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has weight \( W \).

Given:

\[ \rho = 200 \text{ ft} \]

\[ W = 3500 \text{ lb} \]

\[ g = 9.815 \frac{\text{m}}{\text{s}^2} \]

Solution: Limiting case is \( N = 0 \).

\[ \sum F_n = ma_n; \quad W = \frac{W}{g} \left( \frac{v^2}{\rho} \right) \quad v = \sqrt{g\rho} \quad v = 80.25 \text{ ft/s} \]

*Problem 13-64

The airplane, traveling at constant speed \( v \) is executing a horizontal turn. If the plane is banked at angle \( \theta \) when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature \( \rho \) of the turn. Also, what is the normal force of the seat on the pilot if he has mass \( M \)?

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]
Given:

\[ v = 50 \frac{m}{s} \]
\[ \theta = 15 \text{ deg} \]
\[ M = 70 \text{ kg} \]
\[ g = 9.815 \frac{m}{s^2} \]

Solution:

\[ + \sum F_b = ma_b; \quad N_p \sin(\theta) - Mg = 0 \]
\[ N_p = M \left( \frac{g}{\sin(\theta)} \right) \quad N_p = 2.654 \text{kN} \]

\[ \sum F_n = ma_n; \quad N_p \cos(\theta) = M \left( \frac{v^2}{\rho} \right) \quad \rho = 68.3 \text{ m} \]

Problem 13-65

The man has weight \( W \) and lies against the cushion for which the coefficient of static friction is \( \mu_s \). Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the \( z \) axis, he has constant speed \( v \). Neglect the size of the man.

Given:

\[ W = 150 \text{ lb} \]
\[ \mu_s = 0.5 \]
\[ v = 20 \frac{\text{ft}}{s} \]
\[ \theta = 60 \text{ deg} \]
\[ d = 8 \text{ ft} \]

Solution: Assume no slipping occurs

Guesses \( F_N = 1 \text{ lb} \quad F = 1 \text{ lb} \)

Given

\[-F_N \sin(\theta) + F \cos(\theta) = -\frac{W}{g} \left( \frac{v^2}{d} \right) \]
\[ F_N \cos(\theta) - W + F \sin(\theta) = 0 \]

\[ \begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F) \]

\[ \begin{pmatrix} 276.714 \\ 13.444 \end{pmatrix} \text{ lb} \quad F_{\text{max}} = \mu_s F_N \quad F_{\text{max}} = 138.357 \text{ lb} \]

Since \( F = 13.444 \text{ lb} < F_{\text{max}} = 138.357 \text{ lb} \) then our assumption is correct and there is no slipping.
Problem 13-66

The man has weight $W$ and lies against the cushion for which the coefficient of static friction is $\mu_s$. If he rotates about the $z$ axis with a constant speed $v$, determine the smallest angle $\theta$ of the cushion at which he will begin to slip off.

Given:
- $W = 150$ lb
- $\mu_s = 0.5$
- $v = 30 \text{ ft/s}$
- $d = 8$ ft

Solution: Assume verge of slipping

Guesses
- $F_N = 1$ lb
- $\theta = 20$ deg

Given

\[ -F_N \sin(\theta) - \mu_s F_N \cos(\theta) = -\frac{W}{g} \left(\frac{v^2}{d}\right) \]
\[ F_N \cos(\theta) - W - \mu_s F_N \sin(\theta) = 0 \]

\[
\begin{pmatrix}
F_N \\
\theta
\end{pmatrix} = \text{Find}(F_N, \theta)
\]

\[
F_N = 487.563 \text{ lb} \quad \theta = 47.463 \text{ deg}
\]

Problem 13-67

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at angle $\theta$ from the vertical. Each chair including its passenger has a mass $m_c$. Also, what are the components of force in the $n$, $t$, and $b$ directions which the chair exerts on a passenger of mass $m_p$ during the motion?

Given:
- $\theta = 30 \text{ deg}$
- $d = 4$ m
- $m_c = 80$ kg
- $b = 6$ m
- $m_p = 50$ kg
- $g = 9.81 \text{ m/s}^2$

Solution:

The initial guesses:

\[
T = 100 \text{ N} \quad v = 10 \text{ m/s}
\]
Given

\[ T \sin(\theta) = m_c \left( \frac{v^2}{d + b \sin(\theta)} \right) \]

\[ T \cos(\theta) - m_c g = 0 \]

\[ \begin{bmatrix} T \\ Tc \end{bmatrix} = \text{Find}(T, v) \quad T = 906.209 \text{ N} \quad v = 6.30 \frac{\text{m}}{\text{s}} \]

\[ \sum F_n = m_a; \quad F_n = \frac{m_p v^2}{d + b \sin(\theta)} \quad F_n = 283 \text{ N} \]

\[ \sum F_t = m_a; \quad F_t = 0 \text{ N} \quad F_t = 0 \]

\[ \sum F_b = m_a; \quad F_b - m_pg = 0 \quad F_b = m_p g \quad F_b = 491 \text{ N} \]

*Problem 13-68*

The snowmobile of mass \( M \) with passenger is traveling down the hill at a constant speed \( v \). Determine the resultant normal force and the resultant frictional force exerted on the tracks at the instant it reaches point \( A \). Neglect the size of the snowmobile.

Units Used:

\( \text{kN} = 10^3 \text{N} \)

Given:

\( M = 200 \text{ kg} \)

\( v = 6 \frac{\text{m}}{\text{s}} \)

\( a = 5 \text{ m} \)

\( b = 10 \text{ m} \)

\( g = 9.81 \frac{\text{m}}{\text{s}^2} \)

Solution:

\[ y(x) = -a \left( \frac{x}{b} \right)^3 \quad y''(x) = -3 \left( \frac{a}{b^3} \right) x^2 \]

\[ y''''(x) = -6 \left( \frac{a}{b^3} \right) x \quad \rho(x) = \frac{\sqrt{1 + y'(x)^2}}{y''''(x)} \]
\[ \theta = \tan(y'(b)) \]

Guesses \( N_S = 1 \text{ N} \) \( F = 1 \text{ N} \)

Given \( N_S - M g \cos(\theta) = M \left( \frac{v^2}{\rho(b)} \right) \)

\[ F - M g \sin(\theta) = 0 \]

\[ \begin{pmatrix} N_S \\ F \end{pmatrix} = \text{Find}(N_S, F) \quad \begin{pmatrix} N_S \\ F \end{pmatrix} = \begin{pmatrix} 0.72 \\ -1.632 \end{pmatrix} \text{ kN} \]

**Problem 13-69**

The snowmobile of mass \( M \) with passenger is traveling down the hill such that when it is at point \( A \), it is traveling at speed \( v \) and increasing its speed at \( v' \). Determine the resultant normal force and the resultant frictional force exerted on the tracks at this instant. Neglect the size of the snowmobile.

**Units Used:**

\[ \text{kJN} = 10^3 \text{ N} \]

**Given:**

\[ M = 200 \text{ kg} \quad a = 5 \text{ m} \]

\[ v = 4 \frac{\text{m}}{\text{s}} \quad b = 10 \text{ m} \]

\[ g = 9.81 \frac{\text{m}}{\text{s}^2} \quad v' = 2 \frac{\text{m}}{\text{s}^2} \]

**Solution:**

\[ y(x) = -a \left( \frac{x}{b} \right)^3 \quad y'(x) = -3 \left( \frac{a}{b^3} \right) x^2 \]

\[ y''(x) = -6 \left( \frac{a}{b^3} \right) x \quad \rho(x) = \frac{\sqrt{1 + y'(x)^2}}{y''(x)} \]

\[ \theta = \tan(y'(b)) \]

Guesses \( N_S = 1 \text{ N} \) \( F = 1 \text{ N} \)

Given \( N_S - M g \cos(\theta) = M \left( \frac{v^2}{\rho(b)} \right) \)
\[ F - M g \sin(\theta) = M v' \]

\[
\begin{pmatrix}
N_S \\
F
\end{pmatrix} = \text{Find}(N_S, F) \quad \begin{pmatrix}
N_S \\
F
\end{pmatrix} = \begin{pmatrix}
0.924 \\
-1.232
\end{pmatrix} \text{kN}
\]

**Problem 13-70**

A collar having a mass \( M \) and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is \( \mu_k \). If the collar is given a speed \( v_i \) and then released at \( \theta = 0 \) deg, determine how far, \( d \), it slides on the rod before coming to rest.

Given:

- \( M = 0.75 \text{ kg} \)
- \( r = 100 \text{ mm} \)
- \( \mu_k = 0.3 \)
- \( g = 9.81 \frac{\text{m}}{\text{s}^2} \)
- \( v_i = 4 \frac{\text{m}}{\text{s}} \)

Solution:

\[
N_{Cz} - Mg = 0
\]

\[
N_{Ch} = M \left( \frac{v^2}{r} \right)
\]

\[
N_C = \sqrt{N_{Cz}^2 + N_{Ch}^2}
\]

\[
F_C = \mu_k N_C = -Ma_t
\]

\[
a_t(v) = -\mu_k \sqrt{\frac{2}{g} + \frac{v^4}{r^2}}
\]

\[
d = \int_{v_i}^{0} \frac{v}{a_t(v)} \, dv
\]

\[ d = 0.581 \text{ m} \]

**Problem 13-71**

The roller coaster car and passenger have a total weight \( W \) and starting from rest at \( A \) travel down the track that has the shape shown. Determine the normal force of the tracks on the car when the car is at point \( B \), it has a velocity of \( v \). Neglect friction and the size of the car and passenger.

Given:

- \( W = 600 \text{ lb} \)
\[ v = 15 \text{ ft/s} \]
\[ a = 20 \text{ ft} \]
\[ b = 40 \text{ ft} \]

Solution:
\[ y(x) = b \cos \left( \frac{\pi x}{2a} \right) \]
\[ y'(x) = \frac{d}{dx} y(x) \]
\[ y''(x) = \frac{d}{dx} y'(x) \]
\[ \rho(x) = \frac{\sqrt{1 + y'(x)^2}}{y''(x)} \]

At \( B \)
\[ \theta = \text{atan}(y'(a)) \]

\[ F_N - W \cos(\theta) = \frac{W v^2}{g} \]

\[ F_N = W \cos(\theta) + \frac{W v^2}{g \rho(a)} \]

\[ F_N = 182.0 \text{ lb} \]

*Problem 13-72*

The smooth block \( B \), having mass \( M \), is attached to the vertex \( A \) of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the \( z \) axis such that the block attains speed \( v \). At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.
Given: \( M = 0.2 \text{ kg} \quad v = 0.5 \text{ m/s} \quad a = 300 \text{ mm} \quad b = 400 \text{ mm} \)
\( c = 200 \text{ mm} \quad g = 9.81 \text{ m/s}^2 \)

Solution:

Guesses \( T = 1 \text{ N} \quad N_B = 1 \text{ N} \)

Set \( \theta = \arctan\left(\frac{a}{b}\right) \quad \theta = 36.87 \text{ deg} \)

\( \rho = \left(\frac{c}{\sqrt{a^2 + b^2}}\right)a \quad \rho = 120 \text{ mm} \)

Given \( T \sin(\theta) - N_B \cos(\theta) = M\left(\frac{v^2}{\rho}\right) \quad T \cos(\theta) + N_B \sin(\theta) - Mg = 0 \)

\( \begin{pmatrix} T \\ N_B \end{pmatrix} = \text{Find}(T, N_B) \quad \begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 1.82 \\ 0.844 \end{pmatrix} \text{ N} \)

**Problem 13-73**

The pendulum bob \( B \) of mass \( M \) is released from rest when \( \theta = 0^\circ \). Determine the initial tension in the cord and also at the instant the bob reaches point \( D, \theta = \theta_f \). Neglect the size of the bob.

Given:
\( M = 5 \text{ kg} \quad \theta_f = 45 \text{ deg} \)
\( L = 2 \text{ m} \quad g = 9.81 \text{ m/s}^2 \)

Solution:

Initially, \( v = 0 \) so \( a_n = 0 \quad T = 0 \)

At \( D \) we have
\( Mg \cos(\theta_f) = Ma_t \)
\( a_t = g \cos(\theta_f) \quad a_t = 6.937 \text{ m/s}^2 \)

\( T_D - Mg \sin(\theta_f) = \frac{Mv^2}{L} \)

Now find the velocity \( v \)

Guess \( v = 1 \text{ m/s} \)
Given \[ \int_{\theta_1}^{\theta} v \, dv = \int_{\theta_1}^{\theta} g \cos(\theta) L \, d\theta \]

\[ v = \text{Find}(v) \quad v = 5.268 \, \text{m/s} \]

\[ T_D = M g \sin(\theta_1) + M \left( \frac{v^2}{L} \right) \]

\[ T_D = 104.1 \, \text{N} \]

**Problem 13-74**

A ball having a mass \( M \) and negligible size moves within a smooth vertical circular slot. If it is released from rest at \( \theta_1 \), determine the force of the slot on the ball when the ball arrives at points \( A \) and \( B \).

Given:

\( M = 2 \, \text{kg} \quad \theta = 90 \, \text{deg} \quad \theta_1 = 10 \, \text{deg} \quad r = 0.8 \, \text{m} \quad g = 9.81 \, \text{m/s}^2 \)

Solution:

\( M g \sin(\theta) = M a_t \quad a_t = g \sin(\theta) \)

At \( A \) \quad \theta_A = 90 \, \text{deg}

\[ v_A = \sqrt{2g \int_{\theta_1}^{\theta_A} \sin(\theta) r \, d\theta} \]

\[ N_A - M g \cos(\theta_A) = -M \left( \frac{v_A^2}{r} \right) \]

\[ N_A = M g \cos(\theta_A) - M \left( \frac{v_A^2}{r} \right) \]

\[ N_A = -38.6 \, \text{N} \]

At \( B \) \quad \theta_B = 180 \, \text{deg} - \theta_1

\[ v_B = \sqrt{2g \int_{\theta_1}^{\theta_B} \sin(\theta) r \, d\theta} \]

\[ N_B - M g \cos(\theta_B) = -M \left( \frac{v_B^2}{r} \right) \]

\[ N_B = M g \cos(\theta_B) - M \left( \frac{v_B^2}{r} \right) \]

\[ N_B = -96.6 \, \text{N} \]
**Problem 13-75**

The rotational speed of the disk is controlled by a smooth contact arm $AB$ of mass $M$ which is spring-mounted on the disk. When the disk is at rest, the center of mass $G$ of the arm is located distance $d$ from the center $O$, and the preset compression in the spring is $a$. If the initial gap between $B$ and the contact at $C$ is $b$, determine the (controlling) speed $v_G$ of the arm’s mass center, $G$, which will close the gap. The disk rotates in the horizontal plane. The spring has a stiffness $k$ and its ends are attached to the contact arm at $D$ and to the disk at $E$.

Given:

\[ M = 30 \text{ gm} \quad a = 20 \text{ mm} \quad b = 10 \text{ mm} \quad d = 150 \text{ mm} \quad k = 50 \frac{\text{N}}{\text{m}} \]

Solution:

\[ F_s = k(a + b) \quad F_s = 1.5 \text{ N} \]

\[ a_n = \frac{v_G^2}{d + b} \quad F_s = M \left( \frac{v_G^2}{d + b} \right) \]

\[ v_G = \frac{1}{M} \sqrt{M k (a + b)(d + b)} \quad v_G = 2.83 \frac{\text{m}}{\text{s}} \]

*Problem 13-76*

The spool $S$ of mass $M$ fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s$. If the spool is located a distance $d$ from $A$, determine the maximum constant speed the spool can have so that it does not slip up the rod.

Given:

\[ M = 2 \text{ kg} \quad e = 3 \]
\[ \mu_s = 0.2 \quad f = 4 \]
\[ d = 0.25 \text{ m} \quad g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution:

\[ \rho = d \left( \frac{f}{\sqrt{e^2 + f^2}} \right) \]

Guesses \[ N_S = 1 \text{ N} \quad v = 1 \frac{\text{m}}{\text{s}} \]
Problem 13-77

The box of mass $M$ has a speed $v_0$ when it is at $A$ on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = x_1$. Also, what is the rate of increase in its speed at this instant?

Given:

- $M = 35 \text{ kg}$
- $a = 4 \text{ m}$
- $v_0 = 2 \frac{\text{m}}{\text{s}}$
- $b = \frac{1}{9} \frac{\text{m}}{\text{s}}$
- $x_1 = 3 \text{ m}$

Solution:

$$y(x) = a - bx^2$$

$$y'(x) = -2bx \quad y''(x) = -2b$$

$$\rho(x) = \frac{\sqrt{1 + y'(x)^2}}{y''(x)}$$

$$\theta(x) = \arctan(y'(x))$$

Find the velocity

$$v_1 = \sqrt{v_0^2 + 2g(y(0 \text{ m}) - y(x_1))}$$

$$v_1 = 4.86 \frac{\text{m}}{\text{s}}$$

Guesses

- $F_N = 1 \text{ N}$
- $v' = 1 \frac{\text{m}}{\text{s}}$

Given

$$F_N - Mg \cos(\theta(x_1)) = \frac{v_1^2}{\rho(x_1)} - Mg \sin(\theta(x_1)) = Mv'$$

$$v = 0.969 \frac{\text{m}}{\text{s}}$$

$$N_s = 21.326 \text{ N}$$
\[
\begin{bmatrix}
F_N \\
v'
\end{bmatrix} = \text{Find}(F_N, v') \quad F_N = 179.9 \text{ N} \quad v' = \frac{5.442 \text{ m}}{2 \text{ s}}
\]

**Problem 13-78**

The man has mass \( M \) and sits a distance \( d \) from the center of the rotating platform. Due to the rotation his speed is increased from rest by the rate \( v' \). If the coefficient of static friction between his clothes and the platform is \( \mu_s \), determine the time required to cause him to slip.

Given:
- \( M = 80 \text{ kg} \)
- \( \mu_s = 0.3 \)
- \( d = 3 \text{ m} \)
- \( D = 10 \text{ m} \)
- \( v' = 0.4 \frac{\text{m}}{\text{s}^2} \)
- \( g = 9.81 \frac{\text{m}}{\text{s}^2} \)

Solution:

\[
\mu_s M g = \sqrt{(M v')^2 + \left(\frac{M (v')^2}{d}\right)^2}
\]

\[
t = \text{Find}(t) \quad t = 7.394 \text{ s}
\]

**Problem 13-79**

The collar \( A \), having a mass \( M \), is attached to a spring having a stiffness \( k \). When rod \( BC \) rotates about the vertical axis, the collar slides outward along the smooth rod \( DE \). If the spring is unstretched when \( x = 0 \), determine the constant speed of the collar in order that \( x = x_1 \).

Also, what is the normal force of the rod on the collar? Neglect the size of the collar.

Given:
- \( M = 0.75 \text{ kg} \)
- \( k = 200 \frac{\text{N}}{\text{m}} \)
- \( x_1 = 100 \text{ mm} \)
\[ g = 9.81 \frac{m}{s^2} \]

Solution:

Guesses

\[ Nb = 1 \text{ N} \quad N_t = 1 \text{ N} \quad v = 1 \frac{m}{s} \]

Given

\[ Nb - Mg = 0 \quad N_t = 0 \quad kx_1 = M \left( \frac{v^2}{x_1} \right) \]

\[
\begin{bmatrix}
Nb \\
N_t \\
v
\end{bmatrix}
= \text{Find}(Nb, N_t, v)
\]

\[
\begin{bmatrix}
Nb \\
N_t \\
v
\end{bmatrix}
= \begin{bmatrix}
7.36 \\
0 \\
0
\end{bmatrix} \text{ N}
\]

\[ Nb = 7.36 \text{ N} \quad v = 1.633 \frac{m}{s} \]

*Problem 13-80*

The block has weight \( W \) and it is free to move along the smooth slot in the rotating disk. The spring has stiffness \( k \) and an unstretched length \( \delta \). Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with constant speed \( v \).

Given:

\[ W = 2 \text{ lb} \]
\[ k = 2.5 \frac{\text{lb}}{\text{ft}} \]
\[ \delta = 1.25 \text{ ft} \]
\[ v = 12 \frac{\text{ft}}{s} \]

Solution:

\[ \Sigma F_n = ma_n; \quad F_s = k(\rho - \delta) = \frac{W}{g} \left( \frac{v^2}{\rho} \right) \]

Choosing the positive root,

\[ \rho = \frac{1}{2kg} \left[ kg_\delta + \sqrt{k^2 g^2 \delta^2 + 4kgWv^2} \right] \]

\[ \rho = 2.617 \text{ ft} \]

\[ F_s = k(\rho - \delta) \]

\[ F_s = 3.419 \text{ lb} \]

\[ \Sigma F_t = ma_t; \quad \Sigma F_t = ma_t; \quad F_t = 0 \]

Problem 13-81

If the bicycle and rider have total weight \( W \), determine the resultant normal force acting on the
bicycle when it is at point $A$ while it is freely coasting at speed $v_A$. Also, compute the increase in the bicyclist’s speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.

**Given:**

- $W = 180$ lb
- $d = 5$ ft
- $v_A = 6 \text{ ft/s}$
- $g = 32.2 \text{ ft/s}^2$
- $h = 20$ ft

**Solution:**

$$y(x) = h \cos\left(\frac{\pi x}{h}\right)$$

$$y'(x) = \frac{d}{dx} y(x) \quad y''(x) = \frac{d}{dx} y'(x)$$

At $A$ $x = d$ $\theta = \tan(y'(x))$

$$\rho = \frac{\sqrt{1 + y'(x)^2}}{y''(x)}$$

Guesses $F_N = 1$ lb $v' = 1 \text{ ft/s}^2$

**Given**

$$F_N - W \cos(\theta) = \frac{W v_A^2}{g} \quad -W \sin(\theta) = \frac{W v}{g} v'$$

$$\begin{bmatrix} F_N \\ v' \end{bmatrix} = \text{Find}(F_N, v')$$

$F_N = 69.03$ lb $v' = 29.362 \text{ ft/s}^2$

---

**Problem 13-82**

The packages of weight $W$ ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed $v_1$ in time $t_1$, determine the maximum angle $\theta$ so that none of the packages slip on the inclined surface $AB$ of the belt. The coefficient of static friction between the belt and a package is $\mu_s$. At what angle $\phi$ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of $v_1$? Neglect the size of the packages.
Given:

\[ W = 5 \text{ lb} \quad t_1 = 2 \text{ s} \quad r = 6 \text{ in} \]
\[ v_1 = 2 \frac{\text{ft}}{\text{s}} \quad \mu_s = 0.3 \]

Solution: \[ a = \frac{v_1}{t_1} \]

Guesses

\[ N_1 = 1 \text{ lb} \quad N_2 = 1 \text{ lb} \quad \theta = 1 \text{ deg} \quad \phi = 1 \text{ deg} \]

Given

\[ N_1 - W \cos(\theta) = 0 \quad \mu_s N_1 - W \sin(\theta) = \left( \frac{W}{g} \right) a \]
\[ N_2 - W \cos(\phi) = \left( \frac{W}{r} \right) v_1^2 \quad \mu_s N_2 - W \sin(\phi) = 0 \]

\[
\begin{pmatrix}
N_1 \\
N_2 \\
\theta \\
\phi
\end{pmatrix}
= \text{Find}(N_1, N_2, \theta, \phi)
\]
\[
\begin{pmatrix}
N_1 \\
N_2
\end{pmatrix}
= \begin{pmatrix} 4.83 \\ 3.637 \end{pmatrix} \text{ lb}
\]
\[
\begin{pmatrix}
\theta \\
\phi
\end{pmatrix}
= \begin{pmatrix} 14.99 \\ 12.61 \end{pmatrix} \text{ deg}
\]

**Problem 13-83**

A particle having mass \( M \) moves along a path defined by the equations \( r = a + bt, \theta = c t^2 + d \) and \( z = e + ft^3 \). Determine the \( r, \theta, \text{ and } z \) components of force which the path exerts on the particle when \( t = t_1 \).

Given:

\[ M = 1.5 \text{ kg} \quad a = 4 \text{ m} \quad b = 3 \frac{\text{m}}{\text{s}} \]
\[ c = 1 \frac{\text{rad}}{\text{s}^2} \quad d = 2 \text{ rad} \quad e = 6 \text{ m} \]
\[ f = -1 \frac{\text{m}}{\text{s}^3} \quad t_1 = 2 \text{ s} \quad g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution: \( t = t_1 \)

\[ r = a + b t \quad r' = b \quad r'' = 0 \frac{\text{m}}{\text{s}^2} \]
\[ \theta = c t^2 + d \quad \theta' = 2 c t \quad \theta'' = 2 c \]
\[ z = e + f t^3 \quad z' = 3 f t^2 \quad z'' = 6 f t \]
\[ F_r = M(r'' - r\theta'^2) \quad F_r = -240 \text{ N} \]
\[ F_\theta = M(r\theta' + 2r'\theta) \quad F_\theta = 66.0 \text{ N} \]
\[ F_z = Mz'' + Mg \quad F_z = -3.285 \text{ N} \]

*Problem 13-84*

The path of motion of a particle of weight \(W\) in the horizontal plane is described in terms of polar coordinates as \(r = at + b\) and \(\theta = ct^2 + dt\). Determine the magnitude of the unbalanced force acting on the particle when \(t = t_1\).

**Given:**
- \(W = 5 \text{ lb}\)
- \(a = 2 \text{ ft/s}\)
- \(b = 1 \text{ ft}\)
- \(c = 0.5 \text{ rad/s}^2\)
- \(d = -1 \text{ rad/s}\)
- \(t_1 = 2 \text{ s}\)
- \(g = 32.2 \text{ ft/s}^2\)

**Solution:**
- \(t = t_1\)
- \(r = at + b\)
- \(r' = a\)
- \(r'' = 0 \text{ ft/s}^2\)
- \(\theta = ct^2 + dt\)
- \(\theta' = 2c\)
- \(\theta'' = 2c\)
- \(a_r = r'' - r\theta'^2\)
- \(a_r = -5 \text{ ft/s}^2\)
- \(a_\theta = r\theta' + 2r'\theta\)
- \(a_\theta = 9 \text{ ft/s}^2\)
- \(F = \frac{W}{g}\sqrt{a_r^2 + a_\theta^2} \quad F = 1.599 \text{ lb}\)

**Problem 13-85**

The spring-held follower \(AB\) has weight \(W\) and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is \(r\) and \(z = a\sin(2\theta)\). If the cam is rotating at a constant rate \(\theta'\), determine the force at the end \(A\) of the follower when \(\theta = \theta_1\). In this position the spring is compressed \(\delta_j\). Neglect friction at the bearing \(C\).
Problem 13-86

The spring-held follower $AB$ has weight $W$ and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is $r$ and $z = a \sin(2\theta)$. If the cam is rotating at a constant rate of $\dot{\theta}$, determine the maximum and minimum force the follower exerts on the cam if the spring is compressed $\delta_1$ when $\theta = 45^\circ$.

Given:

- $W = 0.75$ lb
- $\delta_1 = 0.4$ ft
- $r = 0.2$ ft
- $\theta_1 = 45$ deg
- $a = 0.1$ ft
- $k = 12 \frac{lb}{ft}$
- $\dot{\theta} = 6 \frac{rad}{s}$
- $g = 9.81 \frac{m}{s^2}$

Solution:

When $\theta = 45$ deg 

$$z = (a) \cos(2\theta)$$

So in other positions the spring is compresses a distance $\delta_1 + z$

$$z = (a) \sin(2\theta)$$

$$z' = 2(a) \cos(2\theta) \dot{\theta}$$

$$z'' = -4(a) \sin(2\theta) \dot{\theta}^2$$

$$F_a - k\delta_1 = \left(\frac{W}{g}\right)z''$$

$$F_a = k\delta_1 + \left(\frac{W}{g}\right)z''$$

$F_a = 4.464$ lb
\[ F_a - k(\delta_1 + z) = \left( \frac{W}{g} \right) z'' \]

\[ F_a = k[\delta_1 + (a)\sin(2\theta)] - \left( \frac{W}{g} \right) 4(a) \sin(2\theta) \theta'^2 \]

The maximum values occur when \( \sin(2\theta) = -1 \) and the minimum occurs when \( \sin(2\theta) = 1 \)

\[ F_{amin} = k(\delta_1 - a) + \left( \frac{W}{g} \right) 4a\theta'^2 \quad F_{amin} = 1.535 \text{ lb} \]

\[ F_{amax} = k(\delta_1 + a) - \left( \frac{W}{g} \right) 4a\theta'^2 \quad F_{amax} = 3.265 \text{ lb} \]

**Problem 13-87**

The spool of mass \( M \) slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is \( \theta' \), which is increasing at \( \theta'' \). At this same instant, the spool is moving outward along the rod at \( r' \) which is increasing at \( r'' \). Determine the radial frictional force and the normal force of the rod on the spool at this instant.

**Given:**

\[
\begin{align*}
M &= 4 \text{ kg} \\
r &= 0.5 \text{ m} \\
\theta' &= 6 \text{ rad/s} \\
r' &= 3 \text{ m/s} \\
\theta'' &= 2 \text{ rad/s}^2 \\
r'' &= 1 \text{ m/s}^2 \\
g &= 9.81 \text{ m/s}^2
\end{align*}
\]

**Solution:**

\[
\begin{align*}
ar &= r'' - r\theta'^2 \\
\theta' &= r\theta' + 2r\theta' \\
F_r &= Ma_r \\
F_{\theta} &= Ma_{\theta} \\
F_z &= M g \\
F_r &= -68.0 \text{ N} \\
\sqrt{F_{\theta}^2 + F_z^2} &= 153.1 \text{ N}
\end{align*}
\]

*Problem 13-88*

The boy of mass \( M \) is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components \( r = r_0, \theta = bt \) and \( z = ct \). Determine the components of force \( F_r, F_\theta \) and \( F_z \) which the slide exerts on him at the instant \( t = t_1 \). Neglect
the size of the boy.

Given:

\[ M = 40 \text{ kg} \]
\[ r_0 = 1.5 \text{ m} \]
\[ b = 0.7 \text{ rad/s} \]
\[ c = -0.5 \text{ m/s} \]
\[ t_1 = 2 \text{ s} \]
\[ g = 9.81 \frac{\text{m}}{s^2} \]

Solution:

\[ r = r_0 \]
\[ r' = 0 \frac{\text{m}}{\text{s}} \]
\[ r'' = 0 \frac{\text{m}}{\text{s}^2} \]
\[ \theta = b t \]
\[ \theta' = b \]
\[ \theta'' = 0 \frac{\text{rad}}{\text{s}^2} \]
\[ z = c t \]
\[ z' = c \]
\[ z'' = 0 \frac{\text{m}}{\text{s}^2} \]

\[ F_r = M \left( r'' - r \theta'^2 \right) \quad F_r = -29.4 \text{ N} \]
\[ F_\theta = M \left( r \theta' + 2r' \theta \right) \quad F_\theta = 0 \]
\[ F_z - Mg = Mz'' \quad F_z = M(g + z'') \quad F_z = 392 \text{ N} \]

**Problem 13-89**

The girl has a mass \( M \). She is seated on the horse of the merry-go-round which undergoes constant rotational motion \( \theta \). If the path of the horse is defined by \( r = r_0, z = b \sin(\theta) \), determine the maximum and minimum force \( F_z \) the horse exerts on her during the motion.
Given:
\[ M = 50 \text{ kg} \]
\[ \theta = 1.5 \frac{\text{rad}}{s} \]
\[ r_0 = 4 \text{ m} \]
\[ b = 0.5 \text{ m} \]

Solution:
\[ z = b \sin(\theta) \]
\[ z' = b \cos(\theta) \theta \]
\[ z'' = -b \sin(\theta) \theta^2 \]
\[ F_z - M g = M z'' \]
\[ F_z = M \left( g - b \sin(\theta) \theta^2 \right) \]
\[ F_{z\text{max}} = M \left( g + b \theta^2 \right) \]
\[ F_{z\text{max}} = 547 \text{ N} \]
\[ F_{z\text{min}} = M \left( g - b \theta^2 \right) \]
\[ F_{z\text{min}} = 434 \text{ N} \]

**Problem 13-90**

The particle of weight \( W \) is guided along the circular path using the slotted arm guide. If the arm has angular velocity \( \theta \) and angular acceleration \( \theta'' \) at the instant \( \theta = \theta_1 \), determine the force of the guide on the particle. Motion occurs in the horizontal plane.

Given:
\[ \theta_1 = 30 \text{ deg} \]
\[ W = 0.5 \text{ lb} \]
\[ a = 0.5 \text{ ft} \]
\[ \theta = 4 \frac{\text{rad}}{s} \]
\[ b = 0.5 \text{ ft} \]
\[ \theta'' = 8 \frac{\text{rad}}{s^2} \]
\[ g = 32.2 \frac{\text{ft}}{s^2} \]

Solution:
\[ \theta = \theta_1 \]
\[ (a) \sin(\theta) = b \sin(\phi) \]
\[ \phi = \arcsin \left( \frac{a}{b} \sin(\theta) \right) \]
\[ \phi = 30 \text{ deg} \]
(a) \cos(\theta) \dot{\theta} = b \cos(\phi) \dot{\phi}' \quad \dot{\phi}' = \frac{(a) \cos(\theta)}{b \cos(\phi)} \dot{\theta} \quad \phi' = 4 \text{ rad/s}

(a) \cos(\theta) \dot{\theta}' - (a) \sin(\theta) \ddot{\theta}^2 = b \cos(\phi) \ddot{\phi}' - b \sin(\phi) \dot{\phi}'^2

\dot{\phi}'' = \frac{(a) \cos(\theta) \ddot{\theta}' - (a) \sin(\theta) \dot{\theta}'^2 + b \sin(\phi) \dot{\phi}'^2}{b \cos(\phi)} \quad \phi'' = 8 \text{ rad/s}^2

r = (a) \cos(\theta) + b \cos(\phi) \quad r' = -(a) \sin(\theta) \dot{\theta} - b \sin(\phi) \dot{\phi}'

r'' = -(a) \sin(\theta) \ddot{\theta}' - (a) \cos(\theta) \dot{\theta}'^2 - b \sin(\phi) \ddot{\phi}' - b \cos(\phi) \dot{\phi}'^2

-F_N \cos(\phi) = M (r'' - r \dot{\theta}'^2) \quad F_N = \frac{-W (r'' - r \dot{\theta}'^2)}{g \cos(\phi)} \quad F_N = 0.569 \text{ lb}

F - F_N \sin(\phi) = \left(\frac{W}{g}\right) (r \dot{\theta}' + 2 r' \dot{\theta}') \quad F = F_N \sin(\phi) + \left(\frac{W}{g}\right) (r \dot{\theta}' + 2 r' \dot{\theta}') \quad F = 0.143 \text{ lb}

**Problem 13-91**

The particle has mass \(M\) and is confined to move along the smooth horizontal slot due to the rotation of the arm \(OA\). Determine the force of the rod on the particle and the normal force of the slot on the particle when \(\theta = \theta_1\). The rod is rotating with a constant angular velocity \(\theta\). Assume the particle contacts only one side of the slot at any instant.

**Given:**

- \(M = 0.5 \text{ kg}\)
- \(\theta_1 = 30 \text{ deg}\)
- \(\theta' = 2 \text{ rad/s}\)
- \(\theta'' = 0 \text{ rad/s}^2\)
- \(h = 0.5 \text{ m}\)
- \(g = 9.81 \text{ m/s}^2\)

**Solution:**

\(\theta = \theta_1\) \quad h = r \cos(\theta) \quad r = \frac{h}{\cos(\theta)} \quad r = 0.577 \text{ m}
*Problem 13-92*

The particle has mass $M$ and is confined to move along the smooth horizontal slot due to the rotation of the arm $OA$. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = \theta_1$. The rod is rotating with angular velocity $\theta'$ and angular acceleration $\theta''$. Assume the particle contacts only one side of the slot at any instant.

Given:

- $M = 0.5 \text{ kg}$
- $\theta_1 = 30 \text{ deg}$
- $\theta = 2 \text{ rad/s}$
- $h = 0.5 \text{ m}$
- $\theta'' = 3 \text{ rad/s}^2$
- $g = 9.81 \text{ m/s}^2$

Solution:

\[
0 = r' \cos(\theta) - r \sin(\theta) \theta' \\
r' = \frac{r \sin(\theta)}{\cos(\theta)} \theta' \\
 r' = 0.667 \text{ m/s}
\]

\[
0 = r'' \cos(\theta) - 2r' \sin(\theta) \theta' - r \cos(\theta) \theta'' - r \sin(\theta) \theta'
\]

\[
F = (F_N - Mg) \sin(\theta) + M(r \theta' + 2r' \theta') \\
F = 1.778 \text{ N}
\]
\[ r'' = 2r' \theta \tan(\theta) + r \theta'^2 + r \tan(\theta) \theta'' \]

\[ F_N - Mg \cos(\theta) = M \left( r'' - r \theta'^2 \right) \]

\[ F_N = Mg + M \left( \frac{r'' - r \theta'^2}{\cos(\theta)} \right) \]

\[ F_N = 6.371 \text{ N} \]

\[ -F + (F_N - Mg) \sin(\theta) = -M (r \theta'' + 2r' \theta) \]

\[ F = (F_N - Mg) \sin(\theta) + M (r \theta'' + 2r' \theta) \]

\[ F = 2.932 \text{ N} \]

---

**Problem 13-93**

A smooth can \( C \), having a mass \( M \), is lifted from a feed at \( A \) to a ramp at \( B \) by a rotating rod. If the rod maintains a constant angular velocity of \( \theta' \), determine the force which the rod exerts on the can at the instant \( \theta = \theta'_1 \). Neglect the effects of friction in the calculation and the size of the can so that \( r = 2b \cos \theta \). The ramp from \( A \) to \( B \) is circular, having a radius of \( b \).

**Given:**

\[ M = 3 \text{ kg} \quad \theta'_1 = 30 \text{ deg} \]

\[ \theta = 0.5 \frac{\text{rad}}{\text{s}} \quad b = 600 \text{ mm} \]

**Solution:**

\[ \theta = \theta'_1 \]

\[ r = 2b \cos(\theta) \]

\[ r' = -2b \sin(\theta) \theta' \]

\[ r'' = -2b \cos(\theta) \theta'^2 \]

**Guesses**

\[ F_N = 1 \text{ N} \quad F = 1 \text{ N} \]

**Given**

\[ F_N \cos(\theta) - Mg \sin(\theta) = M \left( r'' - r \theta'^2 \right) \]

\[ F + F_N \sin(\theta) - Mg \cos(\theta) = M (2r' \theta) \]

\[ \begin{bmatrix} F_N \\ F \end{bmatrix} = \text{Find}(F_N, F) \quad F_N = 15.191 \text{ N} \quad F = 16.99 \text{ N} \]

---

**Problem 13-94**

The collar of weight \( W \) slides along the smooth horizontal spiral rod \( r = b \theta \), where \( \theta \) is in
radians. If its angular rate of rotation $\theta'$ is constant, determine the tangential force $P$ needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$.

Given:

\[
W = 2 \text{ lb} \\
\theta_1 = 90 \text{ deg} \\
\theta' = 4 \text{ rad/s} \\
b = 2 \text{ ft}
\]

Solution:

\[
\theta = \theta_1 \quad r = b\theta \quad r' = b\theta'
\]

\[
\psi = \arctan\left(\frac{r\theta}{r'}\right)
\]

Guesses $N_B = 1 \text{ lb}$ $P = 1 \text{ lb}$

Given

\[
-N_B\sin(\psi) + P\cos(\psi) = \left(\frac{W}{g}\right)(-r\theta^2)
\]

\[
P\sin(\psi) + N_B\cos(\psi) = \left(\frac{W}{g}\right)(2r'\theta)
\]

\[
\begin{bmatrix} N_B \\ P \end{bmatrix} = \text{Find}(N_B, P) \quad \psi = 57.52 \text{ deg} \quad N_B = 4.771 \text{ lb} \quad P = 1.677 \text{ lb}
\]

**Problem 13-95**

The collar of weight $W$ slides along the smooth vertical spiral rod $r = b\theta$, where $\theta$ is in radians. If its angular rate of rotation $\theta'$ is constant, determine the tangential force $P$ needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$. 

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Given:
\[ W = 2 \text{ lb} \]
\[ \theta_1 = 90 \text{ deg} \]
\[ \dot{\theta} = 4 \text{ rad/s} \]
\[ b = 2 \text{ ft} \]

Solution:
\[ \theta = \theta_1 \quad r = b\theta \quad r' = b\theta \]
\[ \psi = \text{atan}\left(\frac{r\theta}{r'}\right) \]

Guesses
\[ N_B = 1 \text{ lb} \quad P = 1 \text{ lb} \]

Given
\[ -N_B \sin(\psi) + P \cos(\psi) - W = \left(\frac{W}{g}\right)(-r\dot{\theta}^2) \]
\[ P \sin(\psi) + N_B \cos(\psi) = \left(\frac{W}{g}\right)(2r'\theta) \]

\[ \begin{pmatrix} N_B \\ P \end{pmatrix} = \text{Find}(N_B, P) \quad \psi = 57.52 \text{ deg} \quad N_B = 3.084 \text{ lb} \quad P = 2.751 \text{ lb} \]

*Problem 13-96*

The forked rod is used to move the smooth particle of weight \( W \) around the horizontal path in the shape of a limacon \( r = a + b \cos \theta \). If \( \dot{\theta} = ct^2 \), determine the force which the rod exerts on the particle at the instant \( t = t_1 \). The fork and path contact the particle on only one side.

Given:
\[ W = 2 \text{ lb} \]
\[ a = 2 \text{ ft} \]
\[ b = 1 \text{ ft} \]
\[ c = 0.5 \text{ rad/s}^2 \]
\[ t_1 = 1 \text{ s} \]
\[ g = 32.2 \text{ ft/s}^2 \]

Solution: \[ t = t_1 \quad \theta = ct^2 \quad \theta' = 2ct \]

Find the angle \( \psi \) using rectangular coordinates. The path is tangent to the velocity therefore.

\[
\begin{align*}
x &= r \cos(\theta) = (a) \cos(\theta) + b \cos(\theta)^2 \\
y &= r \sin(\theta) = (a) \sin(\theta) + \frac{1}{2} b \sin(2\theta) \\
\psi &= \theta - \arctan\left(\frac{y'}{x'}\right) \\
\psi &= 80.541 \text{ deg}
\end{align*}
\]

Now do the dynamics using polar coordinates

\[
\begin{align*}
r &= a + b \cos(\theta) \\
r' &= -b \sin(\theta) \theta \\
r'' &= -b \cos(\theta) \theta^2 - b \sin(\theta) \theta' \\
\end{align*}
\]

Guesses: \( F = 1 \text{ lb} \quad F_N = 1 \text{ lb} \)

Given: \( F - F_N \cos(\psi) = \left(\frac{W}{g}\right)(r\theta^2 + 2r\theta') \quad -F_N \sin(\psi) = \left(\frac{W}{g}\right) (r'' - r\theta^2) \)

\[
\begin{align*}
\begin{bmatrix} F \\ F_N \end{bmatrix} &= \text{Find}(F,F_N) \\
F_N &= 0.267 \text{ lb} \quad F = 0.163 \text{ lb}
\end{align*}
\]

**Problem 13-97**

The smooth particle has mass \( M \). It is attached to an elastic cord extending from \( O \) to \( P \) and due to the slotted arm guide moves along the horizontal circular path \( r = b \sin \theta \). If the cord has stiffness \( k \) and unstretched length \( \delta \) determine the force of the guide on the particle when \( \theta = \theta_1 \). The guide has a constant angular velocity \( \theta' \).

Given:

\[
\begin{align*}
M &= 80 \text{ gm} \\
b &= 0.8 \text{ m} \\
k &= 30 \text{ N/m} \\
\delta &= 0.25 \text{ m} \\
\theta_1 &= 60 \text{ deg}
\end{align*}
\]
Solution: \[ \theta = 60 \text{ deg} \]
\[ r = b \sin(\theta) \]
\[ r' = b \cos(\theta) \theta \]
\[ r'' = b \cos(\theta) \theta' - b \sin(\theta) \theta'^2 \]

Guesses
\[ N_p = 1 \text{ N} \]
\[ F = 1 \text{ N} \]

Given
\[ N_p \sin(\theta) - k(r - \delta) = M(r'' - r\theta^2) \]
\[ F - N_p \cos(\theta) = M(\theta r'' + 2r'\theta) \]

\[ \left( \frac{F}{N_p} \right) = \text{Find}(F, N_p) \]
\[ N_p = 12.14 \text{ N} \]
\[ F = 7.67 \text{ N} \]
Given \[ N_P \sin(\theta) - k(r - \delta) = M(r'' - r\theta'^2) \]

\[ F - N_P \cos(\theta) = M(r\theta'' + 2r'\theta) \]

\[
\begin{pmatrix}
F \\
N_P
\end{pmatrix} = \text{Find}(F, N_P) \quad N_P = 12.214 \text{ N} \quad F = 7.818 \text{ N}
\]

**Problem 13-99**

Determine the normal and frictional driving forces that the partial spiral track exerts on the motorcycle of mass \( M \) at the instant \( \theta, \theta', \) and \( \theta'' \). Neglect the size of the motorcycle.

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\( M = 200 \text{ kg} \)

\( b = 5 \text{ m} \)

\( \theta = \frac{5 \pi}{3} \text{ rad} \)

\( \theta' = 0.4 \text{ rad/s} \)

\( \theta'' = 0.8 \text{ rad/s}^2 \)

Solution:

\( r = b\theta \quad r' = b\theta' \quad r'' = b\theta'' \)

\( \psi = \arctan\left(\frac{r\theta'}{r'}\right) \quad \psi = 79.188 \text{ deg} \)

Guesses \( F_N = 1 \text{ N} \quad F = 1 \text{ N} \)

Given

\[ -F_N \sin(\psi) + F \cos(\psi) - Mg \sin(\theta) = M(r'' - r\theta'^2) \]

\[ F_N \cos(\psi) + F \sin(\psi) - Mg \cos(\theta) = M(r\theta'' + 2r'\theta) \]

\[
\begin{pmatrix}
F_N \\
F
\end{pmatrix} = \text{Find}(F_N, F) \quad \begin{pmatrix}
F_N \\
F
\end{pmatrix} = \begin{pmatrix}
2.74 \\
5.07
\end{pmatrix} \text{kN}
\]
*Problem 13-100*

Using a forched rod, a smooth cylinder $C$ having a mass $M$ is forced to move along the vertical slotted path $r = a \theta$. If the angular position of the arm is $\theta = b t^2$, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant $t$. The cylinder is in contact with only one edge of the rod and slot at any instant.

**Given:**

$M = 0.5 \text{ kg}$

$a = 0.5 \text{ m}$

$b = 0.5 \frac{1}{s^2}$

$t_1 = 2 \text{ s}$

**Solution:**

$t = t_1$

Find the angle $\psi$ using rectangular components. The velocity is parallel to the track therefore

$$x = r \cos(\theta) = (a b t^2) \cos(b t^2)$$

$$x' = (2a b t) \cos(b t^2) - (2a b^2 t^3) \sin(b t^2)$$

$$y = r \sin(\theta) = (a b t^2) \sin(b t^2)$$

$$y' = (2a b t) \sin(b t^2) + (2a b^2 t^3) \cos(b t^2)$$

$$\psi = \tan\left(\frac{y'}{x'}\right) - b t^2 + \pi$$

$$\psi = 63.435 \text{ deg}$$

Now do the dynamics using polar coordinates

$$\theta = b t^2$$

$$\theta' = 2 b t$$

$$\theta'' = 2 b$$

$$r = a \theta$$

$$r' = a \theta$$

$$r'' = a \theta'$$

**Guesses**

$F = 1 \text{ N}$

$N_C = 1 \text{ N}$

**Given**

$$N_C \sin(\psi) - M g \sin(\theta) = M(r'' - r' \theta)^2$$

$$F - N_C \cos(\psi) - M g \cos(\theta) = M(r \theta'' + 2r' \theta)$$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \text{Find}(F, N_C)$$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \begin{pmatrix} 1.814 \\ 3.032 \end{pmatrix} \text{ N}$$

**Problem 13-101**

The ball has mass $M$ and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0$ such that the angular rate of rotation is $\theta_0$. If the attached cord $ABC$ is drawn down through the hole at constant speed $v$, determine the tension the cord exerts on the ball at the instant $r = r_1$. Also, compute the angular velocity of the ball at this instant. Neglect the effects of
friction between the ball and horizontal plane. Hint: First show that the equation of motion in the \( \theta \) direction yields 
\[ a_\theta = r \theta'' + 2r' \theta' = (1/r)(d(r^2 \theta')/dt) = 0. \]
When integrated, \( r^2 \theta' = c \) where the constant \( c \) is determined from the problem data.

**Given:**
- \( M = 2 \) kg
- \( r_0 = 0.5 \) m
- \( \theta_0 = 1 \) rad/s
- \( v = 0.2 \) m/s
- \( r_1 = 0.25 \) m

**Solution:**

\[
\begin{align*}
\Sigma F_\theta &= Ma_\theta \\
0 &= M(r \theta' + 2r' \theta') = M\left[ \frac{1}{r} \frac{d}{dt} \left( r^2 \theta' \right) \right]
\end{align*}
\]

Thus 
\[
c = r_0^2 \theta_0 = r_1^2 \theta_1
\]

\[
\theta_1 = \left( \frac{r_0}{r_1} \right) \theta_0 \quad \theta_1 = 4 \text{ rad/s}
\]

\[
r = r_1 \\
r' = -v \\
r'' = 0 \quad \theta = \theta_1
\]

\[
T = -M \left( r'' - r \theta'^2 \right) \\
T = 8 \text{ N}
\]

---

**Problem 13-102**

The smooth surface of the vertical cam is defined in part by the curve 
\( r = (a \cos \theta + b) \). If the forked rod is rotating with a constant angular velocity \( \theta' \), determine the force the cam and the rod exert on the roller of mass \( M \) at angle \( \theta \). The attached spring has a stiffness \( k \) and an unstretched length \( l \).

**Given:**
- \( a = 0.2 \) m
- \( k = 30 \frac{N}{m} \)
- \( \theta = 30 \) deg
- \( b = 0.3 \) m
- \( l = 0.1 \) m
- \( \theta' = 4 \) rad/s
- \( g = 9.81 \frac{m}{s^2} \)
- \( M = 2 \) kg
- \( \theta'' = 0 \) rad/s

**Solution:**

\[
r = (a) \cos(\theta) + b
\]
\[ r' = -(a) \sin(\theta)\theta \]
\[ r'' = -(a) \cos(\theta) \theta^2 - (a) \sin(\theta)\theta' \]
\[ \psi = \tan\left(\frac{r\theta}{r'}\right) + \pi \]

Guesses \[ F_N = 1 \text{ N} \quad F = 1 \text{ N} \]

Given
\[ F_N \sin(\psi) - Mg \sin(\theta) - k(r - l) = M(r'' - r\theta^2) \]
\[ F - F_N \cos(\psi) - Mg \cos(\theta) = M(r\theta' + 2r\theta') \]
\[ \begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N) \quad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} 10.524 \\ 0.328 \end{pmatrix} \text{ N} \]

Problem 13-103

The collar has mass \( M \) and travels along the smooth horizontal rod defined by the equiangular spiral \( r = ae^{\theta} \). Determine the tangential force \( F \) and the normal force \( N_C \) acting on the collar when \( \theta = \theta_1 \) if the force \( F \) maintains a constant angular motion \( \theta' \).

Given:
\[ M = 2 \text{ kg} \]
\[ a = 1 \text{ m} \]
\[ \theta_1 = 90 \text{ deg} \]
\[ \theta = 2 \text{ rad/s} \]

Solution:
\[ \theta = \theta_1 \quad \theta' = \theta \quad \theta'' = 0 \text{ rad/s}^2 \]

\[ r = ae^{\theta} \quad r' = a\theta e^{\theta} \quad r'' = a\left(\theta' + \theta^2\right)e^{\theta} \]

Find the angle \( \psi \) using rectangular coordinates. The velocity is parallel to the path therefore
\[ x = r\cos(\theta) \quad x' = r'\cos(\theta) - r\theta\sin(\theta) \]
\[ y = r\sin(\theta) \quad y' = r'\sin(\theta) + r\theta\cos(\theta) \]
\[ \psi = \tan\left(\frac{y'}{x'}\right) - \theta + \pi \quad \psi = 112.911 \text{ deg} \]

Now do the dynamics using polar coordinates \quad \text{Guesses} \quad F = 1 \text{ N} \quad N_C = 1 \text{ N}
Given

\[ F \cos(\psi) - N_C \cos(\psi) = M(r'' - r\theta^2) \]

\[ F \sin(\psi) + N_C \sin(\psi) = M(r\theta'' + 2r'\theta') \]

\[
\begin{pmatrix}
F \\
N_C
\end{pmatrix} = \text{Find}(F, N_C)
\]

*Problem 13-104*

The smooth surface of the vertical cam is defined in part by the curve \( r = (a \cos \theta + b) \). The forked rod is rotating with an angular acceleration \( \theta'' \), and at angle \( \theta \) the angular velocity is \( \theta' \). Determine the force the cam and the rod exert on the roller of mass \( M \) at this instant. The attached spring has a stiffness \( k \) and an unstretched length \( l \).

Given:

\[ a = 0.2 \text{ m} \quad k = 100 \text{ N/m} \quad \theta = 45 \text{ deg} \]

\[ b = 0.3 \text{ m} \quad l = 0.1 \text{ m} \quad \theta' = 6 \text{ rad/s} \]

\[ g = 9.81 \text{ m/s}^2 \quad M = 2 \text{ kg} \quad \theta'' = 2 \text{ rad/s}^2 \]

Solution:

\[ r = a \cos(\theta) + b \quad r' = -(a) \sin(\theta) \theta' \]

\[ r'' = -(a) \cos(\theta) \theta'^2 - (a) \sin(\theta) \theta'' \]

\[ \psi = \tan\left(\frac{r\theta'}{r''}\right) + \pi \]

Guesses \( F_N = 1 \text{ N} \quad F = 1 \text{ N} \)

Given

\[ F_N \sin(\psi) - Mg \sin(\theta) - k(r - l) = M(r'' - r\theta^2) \]

\[ F - F_N \cos(\psi) - Mg \cos(\theta) = M(r\theta'' + 2r'\theta') \]

\[
\begin{pmatrix}
F \\
F_N
\end{pmatrix} = \text{Find}(F, F_N)
\]

Problem 13-105

The pilot of an airplane executes a vertical loop which in part follows the path of a “four-leaved rose,” \( r = a \cos 2\theta \). If his speed at \( A \) is a constant \( v_p \), determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at \( A \). His weight is \( W \).
Given:

\[ a = -600 \text{ ft} \quad W = 130 \text{ lb} \]

\[ v_p = 80 \frac{\text{ft}}{\text{s}} \quad g = \frac{32.2}{2} \frac{\text{ft}}{\text{s}^2} \]

Solution:

\[ \theta = 90 \text{ deg} \]

\[ r = (a) \cos(2\theta) \]

Guesses

\[ r' = 1 \frac{\text{ft}}{\text{s}} \quad r'' = 1 \frac{\text{ft}}{\text{s}^2} \quad \theta' = \frac{\text{rad}}{\text{s}} \quad \theta'' = \frac{\text{rad}}{\text{s}^2} \]

Given

Note that \( v_p \) is constant so \( \frac{dv_p}{dt} = 0 \)

\[ r' = -(a) \sin(2\theta) 2\theta \quad r'' = -(a) \sin(2\theta) 2\theta' - (a) \cos(2\theta) 4\theta'^2 \]

\[ v_p = \sqrt{r'^2 + (r\theta')^2} \quad 0 = \frac{r'' r' + r\theta' (r\theta'' + r\theta')} {\sqrt{r'^2 + (r\theta')^2}} \]

\[ \begin{pmatrix} r' \\ r'' \\ \theta' \end{pmatrix} = \text{Find}(r', r'', \theta', \theta'') \quad r' = 0.000 \frac{\text{ft}}{\text{s}} \quad r'' = -42.7 \frac{\text{ft}}{\text{s}^2} \]

\[ \theta = 0.133 \frac{\text{rad}}{\text{s}} \quad \theta' = 1.919 \times 10^{-14} \frac{\text{rad}}{\text{s}^2} \]

\[ -F_N - W = M(r'' - r\theta'^2) \quad F_N = -W - \left(\frac{W}{g}\right)(r'' - r\theta'^2) \quad F_N = 85.3 \text{ lb} \]

**Problem 13-106**

Using air pressure, the ball of mass \( M \) is forced to move through the tube lying in the horizontal plane and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is
\( F \), determine the rate of increase in the ball's speed at the instant \( \theta = \theta_1 \). What direction does it act in?

Given:
\[
M = 0.5 \text{ kg} \quad a = 0.2 \text{ m} \quad b = 0.1
\]
\[
\theta_1 = \frac{\pi}{2} \quad F = 6 \text{ N}
\]

Solution:
\[
\tan(\psi) = \frac{r}{\frac{dr}{d\theta}} = \frac{ae^{b \theta}}{ab e^{b \theta}} = \frac{1}{b}
\]
\[
\psi = \tan^{-1}\left(\frac{1}{b}\right) \quad \psi = 84.289 \text{ deg}
\]
\[
F = Mv' \quad v' = \frac{F}{M} \quad v' = 12 \frac{\text{m}}{\text{s}^2}
\]

Problem 13-107

Using air pressure, the ball of mass \( M \) is forced to move through the tube lying in the vertical plane and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is \( F \), determine the rate of increase in the ball's speed at the instant \( \theta = \theta_1 \). What direction does it act in?

Given:
\[
M = 0.5 \text{ kg} \quad a = 0.2 \text{ m} \quad b = 0.1
\]
\[
F = 6 \text{ N} \quad \theta_1 = \frac{\pi}{2}
\]

Solution:
\[
\tan(\psi) = \frac{r}{\frac{dr}{d\theta}} = \frac{ae^{b \theta}}{ab e^{b \theta}} = \frac{1}{b}
\]
\[
\psi = \tan^{-1}\left(\frac{1}{b}\right) \quad \psi = 84.289 \text{ deg}
\]
\[
F - Mg \cos(\psi) = Mv' \quad v' = \frac{F}{M} - g \cos(\psi) \quad v' = 11.023 \frac{\text{m}}{\text{s}^2}
\]
*Problem 13-108*

The arm is rotating at the rate $\theta$ when the angular acceleration is $\theta''$ and the angle is $\theta_0$. Determine the normal force it must exert on the particle of mass $M$ if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r=\theta b$.

Given:

- $\theta = 5 \text{ rad/s}$
- $\theta' = 2 \text{ rad/} s^2$
- $\theta_0 = 90 \text{ deg}$
- $M = 0.5 \text{ kg}$
- $b = 0.2 \text{ m}$

Solution:

\[
\theta = \theta_0 \quad r = \frac{b}{\theta} \quad r' = \left(\frac{-b}{\theta^2}\right) \theta \quad r'' = \left(\frac{-b}{\theta^2}\right) \theta' + \left(\frac{2b}{\theta^3}\right) \theta^2
\]

\[
\tan(\psi) = \frac{r}{d} = \frac{b}{\theta} = -\theta \quad \psi = \arctan(-\theta) \quad \psi = -57.518 \text{ deg}
\]

Guesses

- $N_p = 1 \text{ N}$
- $F = 1 \text{ N}$

Given

\[-N_p \sin(\psi) = M(r'' - r\theta^2) \quad F + N_p \cos(\psi) = M(r\theta' + 2r\theta')\]

\[
\begin{bmatrix}
N_p \\
F
\end{bmatrix} = \text{Find}(N_p, F) \quad \begin{bmatrix}
N_p \\
F
\end{bmatrix} = \begin{bmatrix}
-0.453 \\
-1.656
\end{bmatrix} \text{ N}
\]

**Problem 13-109**

The collar, which has weight $W$, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = a/(1 - \cos \theta)$. If the collar's angular rate is $\theta'$, determine the tangential retarding force $P$ needed to cause the motion and the normal force that the collar exerts on the rod at the instant $\theta = \theta_1$.

Given:

- $W = 3 \text{ lb}$
- $a = 4 \text{ ft}$
- $\theta = 4 \text{ rad/s}$
- $\theta' = 0 \text{ rad/} s^2$
\[
\theta_I = 90 \text{ deg} \quad g = 32.2 \frac{\text{ft}}{s^2} \quad M = \frac{W}{g}
\]

Solution: \[\theta = \theta_I\]

\[
r = \frac{a}{1 - \cos(\theta)} \quad r' = \frac{-a \sin(\theta)}{(1 - \cos(\theta))^2} \cdot \theta
\]

\[
r'' = \frac{-a \sin(\theta)}{(1 - \cos(\theta))^2} \cdot \theta' + \frac{-a \cos(\theta) \cdot \theta^2}{(1 - \cos(\theta))^2} + \frac{2a \sin(\theta)^2 \cdot \theta^2}{(1 - \cos(\theta))^3}
\]

Find the angle \(\psi\) using rectangular coordinates. The velocity is parallel to the path

\[
x = r \cos(\theta) \quad x' = r' \cos(\theta) - r \theta \sin(\theta) \quad y = r \sin(\theta) \quad y' = r' \sin(\theta) + r \theta \cos(\theta)
\]

\[
x'' = r'' \cos(\theta) - 2r' \theta \sin(\theta) - r \theta' \sin(\theta) - r \theta^2 \cos(\theta)
\]

\[
y'' = r'' \sin(\theta) + 2r' \theta \cos(\theta) + r \theta' \sin(\theta) - r \theta^2 \sin(\theta)
\]

\[
\psi = \tan^{-1}\left(\frac{y'}{x'}\right) \quad \psi = 45 \text{ deg} \quad \text{Guesses} \quad P = 1 \text{ lb} \quad H = 1 \text{ lb}
\]

Given \[P \cos(\psi) + H \sin(\psi) = M x'' \quad P \sin(\psi) - H \cos(\psi) = M y''\]

\[
\begin{pmatrix} P \\ H \end{pmatrix} \text{ = Find}(P, H) \quad \begin{pmatrix} P \\ H \end{pmatrix} = \begin{pmatrix} 12.649 \\ 4.216 \end{pmatrix} \text{ lb}
\]

**Problem 13-110**

The tube rotates in the horizontal plane at a constant rate \(\theta\). If a ball \(B\) of mass \(M\) starts at the origin \(O\) with an initial radial velocity \(r'_0\) and moves outward through the tube, determine the radial and transverse components of the ball’s velocity at the instant it leaves the outer end at \(C\).

*Hint:* Show that the equation of motion in the \(r\) direction is \(r'' - r \theta^2 = 0\). The solution is of the form \(r = Ae^{-\theta t} + Be^{\theta t}\). Evaluate the integration constants \(A\) and \(B\), and determine the time \(t\) at \(r_I\).

Proceed to obtain \(v_r\) and \(v_\theta\).
Given:
\[ \theta = 4 \, \frac{\text{rad}}{s} \quad r'0 = 1.5 \, \frac{\text{m}}{s} \]
\[ M = 0.2 \, \text{kg} \quad r_I = 0.5 \, \text{m} \]

Solution:
\[ 0 = M(r'' - r\theta'^2) \]
\[ r(t) = Ae^{\theta't} + Be^{-\theta't} \]
\[ r'(t) = \theta'(Ae^{\theta't} - Be^{-\theta't}) \]

Guess \[ A = 1 \, \text{m} \quad B = 1 \, \text{m} \]
\[ t = 1 \, \text{s} \]

Given \[ 0 = A + B \quad r'0 = \theta(A - B) \quad r_I = Ae^{\theta't} + Be^{-\theta't} \]
\[ \begin{pmatrix} A \\ B \\ t_I \end{pmatrix} = \text{Find}(A, B, t) \quad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.188 \\ -0.188 \end{pmatrix} \, \text{m} \quad t_I = 0.275 \, \text{s} \]
\[ r(t) = Ae^{\theta't} + Be^{-\theta't} \quad r'(t) = \theta'(Ae^{\theta't} - Be^{-\theta't}) \]
\[ v_r = r'(t_I) \quad v_\theta = r(t_I) \theta \]
\[ \begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix} \, \frac{\text{m}}{\text{s}} \]

Problem 13-111
A spool of mass \( M \) slides down along a smooth rod. If the rod has a constant angular rate of rotation \( \theta' \) in the vertical plane, show that the equations of motion for the spool are \( r'' - r\theta'^2 - g\sin\theta = 0 \) and
\[ 2M\theta' r' + N_s - M g \cos \theta = 0 \]
where \( N_s \) is the magnitude of the normal force of the rod on the spool.

Using the methods of differential equations, it can be shown that the solution of the first of these equations is \( r = C_1 e^{-\theta t} + C_2 e^{\theta t} - (g/2\theta^2) \sin(\theta t) \). If \( r, r' \) and \( \theta \) are zero when \( t = 0 \), evaluate the constants \( C_1 \) and \( C_2 \) and determine \( r \) at the instant \( \theta = \theta_f \).
Given:
\[ M = 0.2 \text{ kg} \]
\[ \dot{\theta} = 2 \frac{\text{rad}}{\text{s}} \]
\[ \theta_1 = \frac{\pi}{4} \]
\[ \ddot{\theta} = 0 \frac{\text{rad}}{\text{s}^2} \]
\[ g = 9.81 \frac{\text{m}}{\text{s}^2} \]

Solution:
\[
\sum F_r = Ma_r; \quad M g \sin(\theta) = M \left( \dddot{r} - r \theta'^2 \right)
\]
\[
\sum F_\theta = Ma_\theta; \quad M g \cos(\theta) - N_s = M \left( r \theta' + 2r' \theta' \right)
\]
\[
2M \theta r' + N_s - M g \cos(\theta) = 0
\]

The solution of the differential equation (Eq.[1]) is given by

\[
r = C_1 e^{-\theta t} + C_2 e^{\theta t} - \left( \frac{g}{2 \theta^2} \right) \sin(\theta t)
\]
\[
r' = -\theta C_1 e^{-\theta t} + \theta C_2 e^{\theta t} - \left( \frac{g}{2 \theta} \right) \cos(\theta t)
\]

At \[ t = 0 \quad r = 0 \quad 0 = C_1 + C_2 \quad r' = 0 \quad 0 = -\theta C_1 + \theta C_2 - \frac{g}{2 \theta}
\]

Thus \[ C_1 = -\frac{g}{4 \theta^2} \quad C_2 = \frac{g}{4 \theta^2} \]

\[
r = C_1 e^{-\theta t} + C_2 e^{\theta t} - \left( \frac{g}{2 \theta^2} \right) \sin(\theta t)
\]

\[ r = 0.198 \text{ m} \]

*Problem 13-112*

The rocket is in circular orbit about the earth at altitude \( h \). Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.
Given:

\[ h = 4 \times 10^6 \text{ m} \]

\[ G = 66.73 \times 10^{-12} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]

\[ M_e = 5.976 \times 10^{24} \text{ kg} \]

\[ R_e = 6378 \text{ km} \]

Solution:

Circular orbit:  

\[ v_C = \sqrt{\frac{G M_e}{R_e + h}} \]

\[ v_C = 6.199 \text{ km/s} \]

Parabolic orbit:  

\[ v_e = \sqrt{\frac{2G M_e}{R_e + h}} \]

\[ v_e = 8.766 \text{ km/s} \]

\[ \Delta v = v_e - v_C \]

\[ \Delta v = 2.57 \text{ km/s} \]

**Problem 13-113**


Solution:

From Eq. 13-19,

\[ \frac{1}{r} = C \cos(\theta) + \frac{G M_e}{h^2} \]

For \( \theta = 0 \text{ deg} \) and \( \theta = 180 \text{ deg} \)

\[ \frac{1}{r_p} = C + \frac{G M_e}{h^2} \]

\[ \frac{1}{r_a} = -C + \frac{G M_e}{h^2} \]

Eliminating \( C \), From Eqs. 13-28 and 13-29,

\[ \frac{2a}{b^2} = \frac{2G M_e}{h^2} \]

From Eq. 13-31,

\[ T = \frac{\pi}{h} (2a)(b) \]

Thus,

\[ b^2 = \frac{I^2 h^2}{4 \pi^2 a^2} \]

\[ \frac{4 \pi^2 a^2}{I^2 h^2} = \frac{G M_e}{h^2} \]

\[ T^2 = \left( \frac{4 \pi^2}{G M_e} \right) a^2 \]

**Problem 13-114**

A satellite is to be placed into an elliptical orbit about the earth such that at the perigee of its orbit it has an altitude \( h_p \), and at apogee its altitude is \( h_a \). Determine its required launch velocity.
tangent to the earth’s surface at perigee and the period of its orbit.

Given:

\[ h_p = 800 \text{ km} \quad G = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \]
\[ h_a = 2400 \text{ km} \quad M_e = 5.976 \times 10^{24} \text{ kg} \]
\[ s_I = 6378 \text{ km} \]

Solution:

\[ r_p = h_p + s_I \quad r_p = 7178 \text{ km} \]
\[ r_a = h_a + s_I \quad r_a = 8778 \text{ km} \]

\[ r_a = \frac{r_p}{2GM_e} \left( 1 - \frac{r_p}{r_a} \right) \]

\[ v_0 = \sqrt{\frac{2}{r_a + r_p} \left( \frac{1}{r_a} - \frac{1}{r_p} \right)} \sqrt{2G(r_a + r_p)r_aGM_e} \]

\[ v_0 = 7.82 \text{ km/s} \]

\[ h = r_p v_0 \]
\[ h = 56.12 \times 10^9 \frac{\text{m}^2}{\text{s}} \]

\[ T = \frac{\pi}{h} \left( r_p + r_a \right) \sqrt{\frac{r_p}{r_a}} \]
\[ T = 1.97 \text{ hr} \]

Problem 13-115

The rocket is traveling in free flight along an elliptical trajectory. The planet has a mass \( k \) times that of the earth’s. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point \( A \).

Units Used:

\[ \text{Mm} = 10^3 \text{ km} \]

Given:

\[ k = 0.60 \]
\[ a = 6.40 \text{ Mm} \]
\[ b = 16 \text{ Mm} \]
\[ r = 3.20 \text{ Mm} \]
Engineering Mechanics - Dynamics

\[ G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

\[ M_e = 5.976 \times 10^{24} \text{ kg} \]

Solution:  \( \text{Central - Force Motion: Substitute Eq 13-27} \)

\[ r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \]

\[ b = \frac{a}{\frac{2GM}{a v_0^2} - 1} = \frac{a}{b} = \left( \frac{2GM}{a v_0^2} - 1 \right) \]

\[ \left( 1 + \frac{a}{b} \right) = \frac{2GkM_e}{a v_p^2} \]

\[ v_p = \sqrt{\frac{2GkM_e b}{(a + b)a}} \]

\[ v_p = 7.308 \text{ km/s} \]

*Problem 13-116*

An elliptical path of a satellite has an eccentricity \( e \). If it has speed \( v_p \) when it is at perigee, \( P \), determine its speed when it arrives at apogee, \( A \). Also, how far is it from the earth's surface when it is at \( A \)?

Units Used:

\( \text{Mm} = 10^3 \text{ km} \)

Given:

\[ e = 0.130 \]

\[ v_p = 15 \text{ Mm/hr} \]

\[ G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

\[ M_e = 5.976 \times 10^{24} \text{ kg} \]

\[ R_e = 6.378 \times 10^6 \text{ m} \]

Solution:

\[ v_0 = v_p \quad e = \left( \frac{r_0 v_0^2}{GM_e} - 1 \right) \quad r_0 = \frac{(e + 1)GM_e}{v_0^2} \quad r_0 = 25.956 \text{ Mm} \]
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Chapter 13

Problem 13-117

A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth’s surface. If this requires the period \( T \) (approximately), determine the radius of the orbit and the satellite’s velocity.

Units Used: \( \text{Mm} = 10^3 \text{ km} \)

Given: \( T = 24 \text{ hr} \) \( G = 66.73 \times 10^{-12} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2 \) \( M_e = 5.976 \times 10^{24} \text{ kg} \)

Solution:

\[
\frac{G M_e M_s}{r^2} = \frac{M_s v^2}{r} \quad \frac{G M_e}{r} = \left( \frac{2 \pi r}{T} \right)^2
\]

\[
r = \frac{1}{2 \pi} 2^{\frac{1}{3}} \left( G M_e T^2 \right)^{\frac{1}{3}} \]

\[
\frac{1}{r} = 42.2 \text{ Mm}
\]

\[
v = \frac{2 \pi r}{T} \quad v = 3.07 \text{ km/s}
\]

Problem 13-118

The rocket is traveling in free flight along an elliptical trajectory \( A'A \). The planet has no atmosphere, and its mass is \( c \) times that of the earth’s. If the rocket has the apogee and perigee shown, determine the rocket’s velocity when it is at point \( A \).

Given:

\( a = 4000 \text{ mi} \)
\[ b = 10000 \text{ mi} \]
\[ c = 0.6 \]
\[ r = 2000 \text{ mi} \]
\[ G = 34.4 \times 10^{-9} \text{ lbf-ft}^2 \text{ slug}^{-2} \]
\[ M_e = 409 \times 10^{21} \text{ slug} \]

Solution:
\[ r_0 = a \quad OA' = b \quad M_p = M_e c \]
\[ OA' = \frac{r_0}{2 \left( \frac{GM_p}{r_0 v_0^2} \right)^2 - 1} \]
\[ v_0 = \sqrt{\frac{2GM_p}{r_0 \left( \frac{r_0}{OA'} + 1 \right)}} \]
\[ v_0 = 23.9 \times 10^3 \text{ ft/s} \]

**Problem 13-119**

The rocket is traveling in free flight along an elliptical trajectory \( A' A \). If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at \( A' \) so that the landing occurs at \( B \). How long does it take for the rocket to land, in going from \( A' \) to \( B \)? The planet has no atmosphere, and its mass is 0.6 times that of the earth’s.

**Units Used:**

\[ \text{Mm} = 10^3 \text{ km} \]

**Given:**

\[ a = 4000 \text{ mi} \quad r = 2000 \text{ mi} \]
\[ b = 10000 \text{ mi} \quad M_e = 409 \times 10^{21} \text{ slug} \]
\[ c = 0.6 \]
\[ G = 34.4 \times 10^{-9} \text{ lbf-ft}^2 \text{ slug}^{-2} \]

**Solution:**

\[ M_p = M_e c \quad OA' = b \quad OB = r \quad OA' = \frac{OB}{2 \left( \frac{GM_p}{OB v_0^2} \right)^2 - 1} \]
\[ v_0 = \sqrt{\frac{2}{OA'OB + OB^2}} \sqrt{OB(3 + OB)OA'GM_p} \quad \text{v}_0 = 36.5 \times 10^3 \text{ ft/s} \]  

\[ v_{A'} = \frac{OBv_0}{OA'} \quad h = OBv_0 \quad h = 385.5 \times 10^9 \text{ ft}^2/\text{s} \]

Thus,

\[ T = \frac{\pi(OB + OA')}{h} \sqrt{OBOA'} \quad T = 12.19 \times 10^3 \text{ s} \quad \frac{T}{2} = 1.69 \text{ hr} \]

*Problem 13-120*

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13-25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit a distance \( d \) from the earth’s surface.

Given: \( d = 800 \text{ km} \quad G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \quad M_e = 5.976 \times 10^{24} \text{ kg} \)

\( r_e = 6378 \text{ km} \)

Solution:

\[ v = \sqrt{\frac{GM_e}{d + r_e}} \quad v = 7.454 \text{ km/s} \]

**Problem 13-121**

The rocket is traveling in free flight along an elliptical trajectory \( A'A \). The planet has no atmosphere, and its mass is \( k \) times that of the earth’s. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point \( A \).

Units used:

\( \text{Mm} = 10^3 \text{ km} \)

Given:

\( k = 0.70 \)

\( a = 6 \text{ Mm} \)

\( b = 9 \text{ Mm} \)

\( r = 3 \text{ Mm} \)

\( M_e = 5.976 \times 10^{24} \text{ kg} \)
"Problem 13-122"

The rocket is traveling in free flight along an elliptical trajectory \( A'A \). The planet has no atmosphere, and its mass is \( k \) times that of the earth’s. The rocket has an apoapsis and periapsis as shown in the figure. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at \( A' \) so that it strikes the planet at \( B \). How long does it take for the rocket to land, going from \( A' \) to \( B \) along an elliptical path?

Units used:

\( \text{Mm} = 10^3 \text{ km} \)

Given:

- \( k = 0.70 \)
- \( a = 6 \text{ Mm} \)
- \( b = 9 \text{ Mm} \)
- \( r = 3 \text{ Mm} \)
- \( M_e = 5.976 \times 10^{24} \text{ kg} \)
- \( G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \)

Solution:

Central Force motion:

\[
\begin{align*}
\frac{r_a}{r_0} &= \frac{\frac{r_0}{2GM}}{\frac{r_0 v_0^2}{2}} - 1 \\
\frac{b}{a v_p} &= \frac{2G(kM_e)}{a v_p^2} - 1 \\
\frac{v_p}{v_p} &= \sqrt{\frac{2GkM_e}{a(a + b)}} \\
\end{align*}
\]

\[
\begin{align*}
ra v_a &= r_p v_p \\
v_a &= \left(\frac{r}{b}\right) v_p \\
v_a &= 3.938 \text{ km s}^{-1}
\end{align*}
\]
Eq. 13-20 gives \( h = v_pr \) \( h = 35.44 \times 10^9 \text{ m}^2/\text{s} \)

Thus, applying Eq. 13-31 we have \( T = \frac{\pi}{h}(r + b)\sqrt{rb} \) \( T = 5.527 \times 10^3 \text{ s} \)

The time required for the rocket to go from \( A' \) to \( B \) (half the orbit) is given by

\[
 t = \frac{T}{2} \quad t = 46.1 \text{ min}
\]

**Problem 13-123**

A satellite \( S \) travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which the eccentricity is \( e \). Determine the sudden change in speed that must occur at \( A \) so that the rocket can enter the satellite’s orbit while in free flight along the blue elliptical trajectory. When it arrives at \( B \), determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.

Units used:

\( \text{Mm} = 10^3 \text{ km} \)

Given:

\( e = 0.58 \)

\( a = 10 \text{ Mm} \)

\( b = 120 \text{ Mm} \)

\( M_e = 5.976 \times 10^{24} \text{ kg} \)

\( G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \)

Solution:

Central - Force motion:

\[
 C = \frac{1}{r_0} \left( 1 - \frac{GM_e}{r_0 v_0^2} \right) \quad h = r_0 v_0 \quad e = \frac{Ch^2}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1
\]

\[
 v_0 = \sqrt{\frac{(1 + e)GM_e}{r_0}} \quad r_a = \frac{r_0}{2GM_e} - 1 = \frac{r_0}{2 \left( \frac{1}{1 + e} \right) - 1}
\]

\[
 r_0 = r_a \left( \frac{1 - e}{1 + e} \right) \quad r_0 = b \left( \frac{1 - e}{1 + e} \right) \quad r_0 = 31.90 \times 10^6 \text{ m}
\]

Substitute \( r_{pl} = r_0 \) \( v_{pl} = \sqrt{\frac{(1 + e)(G)(M_e)}{r_{pl}}} \) \( v_{pl} = 4.444 \times 10^3 \text{ m/s} \)
\[ \nu_{al} = \left( \frac{r_{pl}}{b} \right) v_{pl} \quad \nu_{al} = 1.181 \times 10^3 \, \text{m/s} \]

When the rocket travels along the second elliptical orbit, from Eq.[4], we have

\[ a = \frac{1 - e'}{1 + e'} b \quad e' = \frac{-a + b}{b + a} \quad e' = 0.8462 \]

Substitute

\[ r_0 = r_{p2} = a \quad r_{p2} = a \quad v_{p2} = \sqrt{\frac{(1 + e')(G)(M_e)}{r_{p2}}} \quad v_{p2} = 8.58 \times 10^3 \, \text{m/s} \]

Applying Eq. 13-20, we have

\[ v_{a2} = \frac{r_{p2}}{b} v_{p2} \quad v_{a2} = 715.021 \, \text{m/s} \]

For the rocket to enter into orbit two from orbit one at \( A \), its speed must be decreased by

\[ \Delta v = v_{a1} - v_{a2} \quad \Delta v = 466 \, \text{m/s} \]

If the rocket travels in a circular free-flight trajectory, its speed is given by Eq. 13-25

\[ v_c = \sqrt{\frac{GM_e}{a}} \quad v_c = 6.315 \times 10^3 \, \text{m/s} \]

The speed for which the rocket must be decreased in order to have a circular orbit is

\[ \Delta v = v_{p2} - v_c \quad \Delta v = 2.27 \, \text{km/s} \]

*Problem 13-124*

An asteroid is in an elliptical orbit about the sun such that its perihelion distance is \( d \). If the eccentricity of the orbit is \( e \), determine the aphelion distance of the orbit.

Given: \( d = 9.30 \times 10^9 \, \text{km} \) \quad \( e = 0.073 \)

Solution:

\[ r_p = d \quad r_0 = d \]

\[ e = \frac{Ch^2}{GM_s} = \frac{1}{r_0} \left( 1 - \frac{GM_s}{r_0 v_0^2} \right) \left( \frac{r_0^2 v_0^2}{GM_s} \right) \quad e = \left( \frac{r_0 v_0^2}{GM_s} - 1 \right) \]

\[ \frac{GM_s}{r_0 v_0^2} = \frac{1}{e + 1} \quad r_A = \frac{r_0}{2} \quad e + 1 \quad r_A = \frac{r_0(e + 1)}{1 - e} \quad r_A = 10.76 \times 10^9 \, \text{km} \]

**Problem 13-125**

A satellite is in an elliptical orbit around the earth with eccentricity \( e \). If its perigee is \( h_p \), determine its velocity at this point and also the distance \( OB \) when it is at point \( B \), located at angle \( \theta \) from perigee as shown.
Units Used: \( \text{Mm} = 10^3 \text{ km} \)

Given:

\( e = 0.156 \)
\( \theta = 135 \text{ deg} \)
\( h_p = 5 \text{ Mm} \)
\( G = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \)
\( M_e = 5.976 \times 10^{24} \text{ kg} \)

Solution:

\[
e = \frac{C \cdot h_p^2}{G M_e} = \frac{1}{h_p} \left( \frac{1 - \frac{G M_e}{h_p v_0^2}}{GM_e} \right) h_p v_0^2 = e + 1
\]

\[
v_0 = \frac{1}{h_p} \sqrt{h_p G M_e (e + 1)} \quad v_0 = 9.6 \text{ km/s}
\]

\[
\frac{1}{r} = \frac{1}{h_p} \left( 1 - \frac{G M_e}{h_p v_0^2} \right) \cos(\theta) + \frac{G M_e}{h_p v_0^2}
\]

\[
\frac{1}{r} = \frac{1}{h_p} \left( 1 - \frac{1}{e + 1} \right) \cos(\theta) + \frac{1}{h_p} \left( \frac{1}{e + 1} \right)
\]

\[
 r = h_p \left( \frac{e + 1}{e \cos(\theta) + 1} \right) \quad r = 6.5 \text{ Mm}
\]
Problem 13-126

The rocket is traveling in a free-flight elliptical orbit about the earth such that the eccentricity is $e$ and its perigee is a distance $d$ as shown. Determine its speed when it is at point $B$. Also determine the sudden decrease in speed the rocket must experience at $A$ in order to travel in a circular orbit about the earth.

Given:

$$e = 0.76$$
$$d = 9 \times 10^6 \text{ m}$$
$$G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$
$$M_e = 5.976 \times 10^{24} \text{ kg}$$

Solution:

Central - Force motion:

$$C = \frac{1}{r_0} \left( 1 - \frac{GM_e}{r_0 v_0^2} \right)$$
$$h = r_0 v_0$$

$$e = \frac{c h^2}{GM_e} = \frac{r_0 v_0^2}{GM_e} - 1$$
$$\frac{1}{1 + e} = \frac{GM_e}{r_0 v_0^2}$$
$$v_0 = \sqrt{\frac{(1 + e)GM_e}{r_0}}$$

$$r_a = \left( \frac{1 + e}{1 - e} \right) d$$
$$r_a = 66 \times 10^6 \text{ m}$$
$$r_p = d$$

$$v_p = \sqrt{\frac{(1 + e)GM_e}{d}}$$
$$v_p = 8.831 \text{ km/s}$$
$$v_a = \left( \frac{d}{r_a} \right) v_p$$
$$v_a = 1.2 \text{ km/s}$$

If the rockets in a circular free-flight trajectory, its speed is given by eq.13-25

$$v_c = \sqrt{\frac{GM_e}{d}}$$
$$v_c = 6656.48 \text{ m/s}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_p - v_c$$
$$\Delta v = 2.17 \text{ km/s}$$
Problem 13-127

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are \( r_p \) and \( a_p \), respectively, determine (a) the speed of the rocket at point \( A' \), (b) the required speed it must attain at \( A \) just after braking so that it undergoes a free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is \( a \) times the mass of the earth.

Units Used:
\[ \text{Mm} = 10^3 \text{ km} \]

Given:
\[
\begin{align*}
    a &= 0.816 & a_p &= 26 \text{ Mm} \\
    f &= 8 \text{ Mm} & r_p &= 8 \text{ Mm} \\
    G &= 6.673 \times 10^{-12} \text{ m}^3 \text{ kg}^{-2} \text{ s}^2 \\
    M_e &= 5.976 \times 10^{24} \text{ kg} \\
\end{align*}
\]

Solution:
\[
\begin{align*}
    M_v &= a M_e & M_v &= 4.876 \times 10^{24} \text{ kg} \\
    OA' &= \frac{OA}{2\left(\frac{GM_p}{OA v_A^2}\right)} - 1 & a_p &= \frac{r_p}{2GM_v} - 1 \\
    v_A &= \left(\frac{1}{a_p r_p + r_p^2}\right)^\frac{1}{2}\left(2\sqrt{r_p(a_p + r_p) a_p G M_v}\right) & v_A &= 7.89 \frac{\text{km}}{\text{s}} \\
    v'_A &= \frac{r_p v_A}{a_p} & v'_A &= 2.43 \frac{\text{km}}{\text{s}} \\
    v''_A &= \frac{GM_v}{r_p} & v''_A &= 6.38 \frac{\text{km}}{\text{s}} \\
    \text{Circular Orbit: } T_c &= \frac{2\pi r_p}{v''_A} & T_c &= 2.19 \text{ hr} \\
    \text{Elliptic Orbit: } T_e &= \frac{\pi}{r_p v_A} (r_p + a_p) \sqrt{r_p a_p} & T_e &= 6.78 \text{ hr}
\end{align*}
\]