Problem 13-1

Determine the gravitational attraction between two spheres which are just touching each other. Each sphere has a mass *M* and radius *r*.

Given:

$r = 200 \text{ mm} \qquad M = 10 \text{ kg} \qquad G = 66.73 \times 10^{-12} \dfrac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \qquad \text{nN} = 1 \times 10^{-9} \text{ N}$

Solution:

$$F = \frac{G M^2}{(2r)^2} \qquad \boxed{F = 41.7 \text{ nN}}$$

Problem 13-2

By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., $s \propto t^2$ by determining the time t_B, t_C, and t_D needed for a block of mass *m* to slide from rest at *A* to points *B*, *C*, and *D*, respectively. Neglect the effects of friction.

Given:

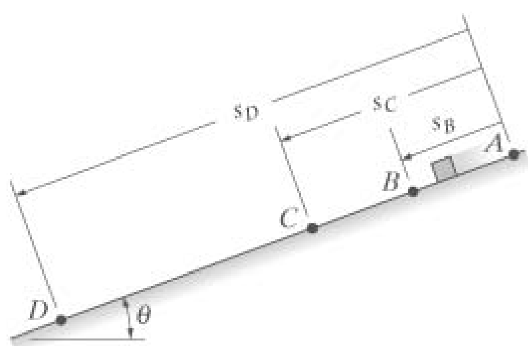
$s_B = 2 \text{ m}$

$s_C = 4 \text{ m}$

$s_D = 9 \text{ m}$

$\theta = 20 \text{ deg}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



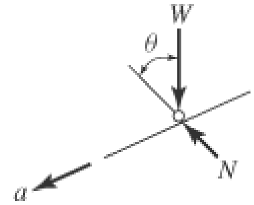
Solution:

$$W \sin(\theta) = \left(\frac{W}{g}\right) a$$

$$a = g \sin(\theta) \qquad a = 3.355 \frac{\text{m}}{\text{s}^2}$$

$$s = \frac{1}{2} a t^2$$

$$t_B = \sqrt{\frac{2 s_B}{a}} \qquad \boxed{t_B = 1.09 \text{ s}}$$

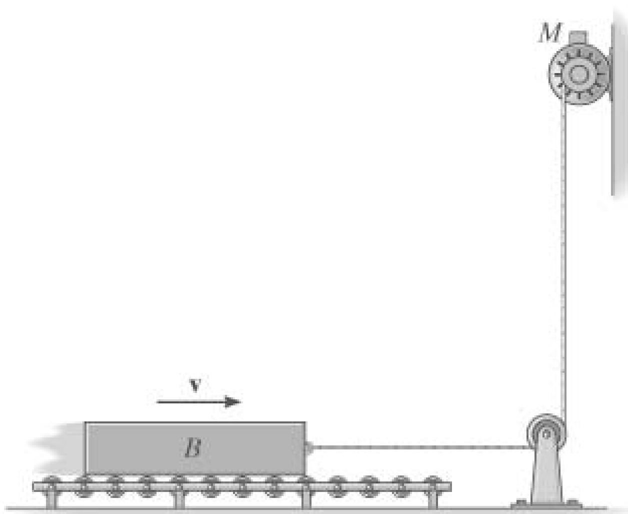


$$t_C = \sqrt{\frac{2s_C}{a}} \qquad t_C = 1.54\ \text{s}$$

$$t_D = \sqrt{\frac{2s_D}{a}} \qquad t_D = 2.32\ \text{s}$$

Problem 13-3

A bar *B* of mass M_1, originally at rest, is being towed over a series of small rollers. Determine the force in the cable at time *t* if the motor *M* is drawing in the cable for a short time at a rate $v = kt^2$. How far does the bar move in time *t*? Neglect the mass of the cable, pulley, and the rollers.



Given:

$$\text{kN} = 10^3\ \text{N}$$

$$M_1 = 300\ \text{kg}$$

$$t = 5\ \text{s}$$

$$k = 0.4\ \frac{\text{m}}{\text{s}^3}$$

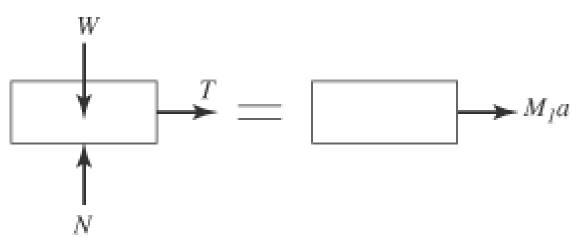
Solution:

$$v = kt^2 \qquad v = 10\ \frac{\text{m}}{\text{s}}$$

$$a = 2kt \qquad a = 4\ \frac{\text{m}}{\text{s}^2}$$

$$T = M_1 a \qquad T = 1.2\ \text{kN}$$

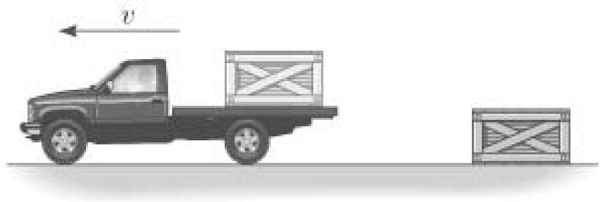
$$d = \int_0^t kt^2\ \text{d}t \qquad d = 16.7\ \text{m}$$



***Problem 13-4**

A crate having a mass *M* falls horizontally off the back of a truck which is traveling with speed *v*. Determine the coefficient of kinetic friction between the road and the crate if the crate slides a distance *d* on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is *v*.

Given:



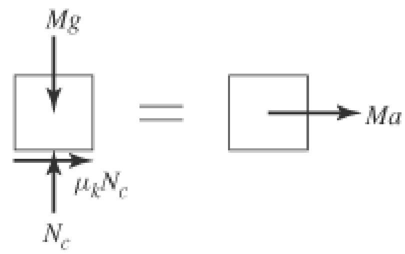
$M = 60\ \text{kg}$

$d = 45\ \text{m}$

$v = 80\ \frac{\text{km}}{\text{hr}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:



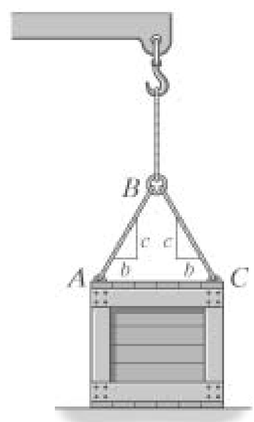
$N_C - Mg = 0 \qquad N_C = Mg$

$\mu_k N_C = Ma \qquad a = \mu_k g$

$\frac{v^2}{2} = ad = \mu_k g d$

$\mu_k = \frac{v^2}{2gd} \qquad \mu_k = 0.559$

Problem 13-5



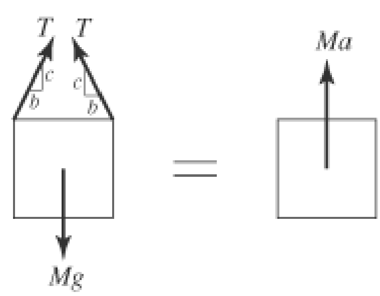
The crane lifts a bin of mass M with an initial acceleration a. Determine the force in each of the supporting cables due to this motion.

Given:

$M = 700\ \text{kg} \quad b = 3 \quad \text{kN} = 10^3\ \text{N}$

$a = 3\ \frac{\text{m}}{\text{s}^2} \quad c = 4$

Solution:



$$2T\left(\frac{c}{\sqrt{b^2 + c^2}}\right) - Mg = Ma$$

$$T = M(a+g)\left(\frac{\sqrt{b^2 + c^2}}{2c}\right) \qquad T = 5.60\ \text{kN}$$

Problem 13-6

The baggage truck A has mass m_t and is used to pull the two cars, each with mass m_c. The tractive force on the truck is F. Determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at C suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

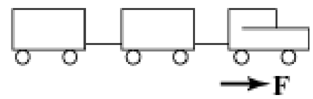
Given:

$m_t = 800\ \text{kg}$

$m_C = 300\ \text{kg}$

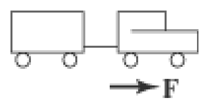
$F = 480\ \text{N}$

Solution:



$\overset{+}{\rightarrow}\ \Sigma\ F_x = ma_x;$ $\quad F = (m_t + 2m_C)a$

$$a = \frac{F}{m_t + 2\ m_C} \qquad \boxed{a = 0.343\ \frac{\text{m}}{\text{s}^2}}$$



$\overset{+}{\rightarrow}\ \Sigma\ F_x = ma_x;$ $\quad F = (m_t + m_C)a_{Fail}$

$$a_{Fail} = \frac{F}{m_t + m_C} \qquad \boxed{a_{Fail} = 0.436\ \frac{\text{m}}{\text{s}^2}}$$

Problem 13-7

The fuel assembly of mass M for a nuclear reactor is being lifted out from the core of the nuclear reactor using the pulley system shown. It is hoisted upward with a constant acceleration such that $s = 0$ and $v = 0$ when $t = 0$ and $s = s_1$ when $t = t_1$. Determine the tension in the cable at A during the motion.

Units Used:

$$kN = 10^3\ N$$

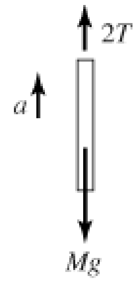
Given:

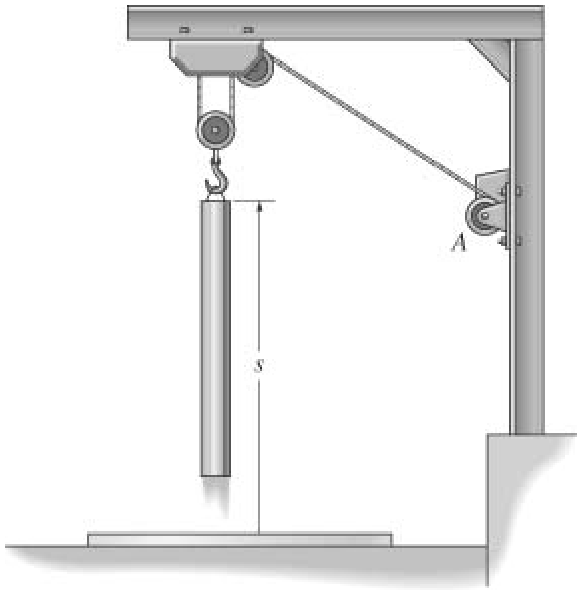
$$M = 500\ kg$$

$$s_1 = 2.5\ m$$

$$t_1 = 1.5\ s$$

$$g = 9.81\ \frac{m}{s^2}$$





Solution:

$$s = \left(\frac{a}{2}\right)t^2 \qquad a = \frac{2s_1}{t_1^2} \qquad a = 2.222\ \frac{m}{s^2}$$

$$2T - Mg = Ma \qquad T = \frac{M(a+g)}{2} \qquad T = 3.008\,\text{kN}$$

***Problem 13-8**

The crate of mass M is suspended from the cable of a crane. Determine the force in the cable at time t if the crate is moving upward with (a) a constant velocity v_1 and (b) a speed of $v = bt^2 + c$.

Units Used:

$$kN = 10^3\ N$$

Given:

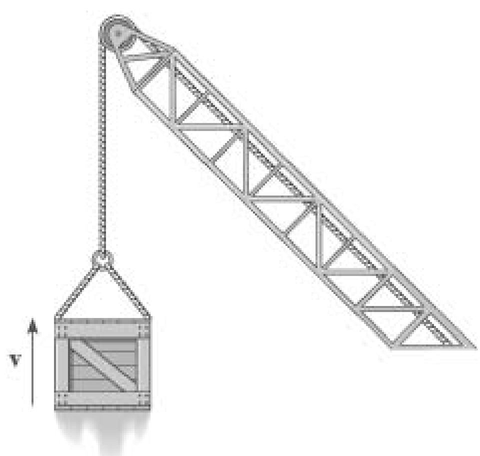
$$M = 200\ kg$$

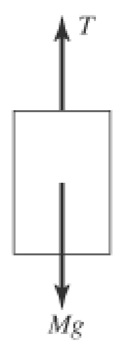
$$t = 2\ s$$

$$v_1 = 2\ \frac{m}{s}$$

$$b = 0.2\ \frac{m}{s^3}$$

$$c = 2\ \frac{m}{s}$$





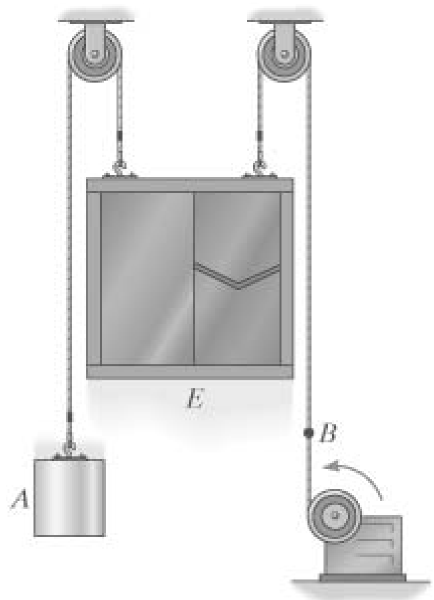
Solution:

$$(a) \qquad a = 0\ \frac{m}{s^2} \qquad T_a - Mg = Ma \qquad T_a = M(g+a) \qquad T_a = 1.962\,\text{kN}$$

(b) $v = bt^2 + c$ $a = 2bt$ $T_b = M(g + a)$ $T_b = 2.12\ \text{kN}$

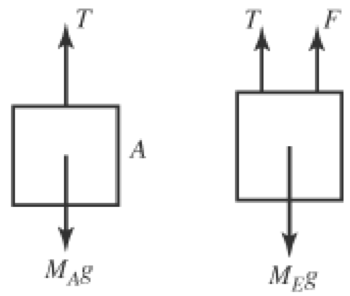
Problem 13-9

The elevator E has a mass M_E, and the counterweight at A has a mass M_A. If the motor supplies a constant force F on the cable at B, determine the speed of the elevator at time t starting from rest. Neglect the mass of the pulleys and cable.



Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$M_E = 500\ \text{kg}$

$M_A = 150\ \text{kg}$

$F = 5\ \text{kN}$

$t = 3\ \text{s}$

Solution:

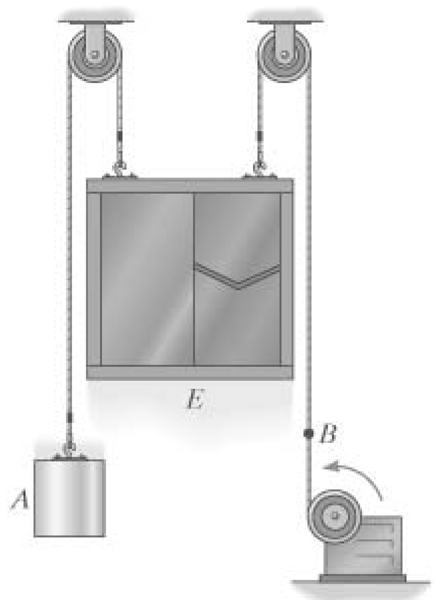
Guesses $T = 1\ \text{kN}$ $a = 1\ \frac{\text{m}}{\text{s}^2}$ $v = 1\ \frac{\text{m}}{\text{s}}$

Given $T - M_A g = -M_A a$ $F + T - M_E g = M_E a$ $v = at$

$\begin{pmatrix} T \\ a \\ v \end{pmatrix} = \text{Find}(T, a, v)$ $T = 1.11\ \text{kN}$ $a = 2.41\ \frac{\text{m}}{\text{s}^2}$ $v = 7.23\ \frac{\text{m}}{\text{s}}$

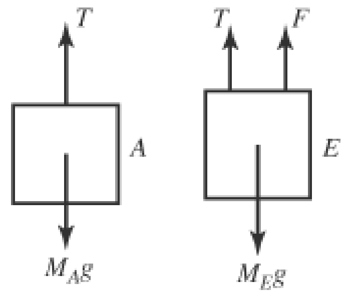
Problem 13-10

The elevator E has a mass M_E and the counterweight at A has a mass M_A. If the elevator attains a speed v after it rises a distance h, determine the constant force developed in the cable at B. Neglect the mass of the pulleys and cable.



Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$M_E = 500\ \text{kg}$

$M_A = 150\ \text{kg}$

$v = 10\ \dfrac{\text{m}}{\text{s}}$

$h = 40\ \text{m}$

Solution:

Guesses $\quad T = 1\ \text{kN} \quad F = 1\ \text{kN} \quad a = 1\ \dfrac{\text{m}}{\text{s}^2}$

Given $\quad T - M_A g = -M_A a \qquad F + T - M_E g = M_E a \qquad v^2 = 2ah$

$$\begin{pmatrix} F \\ T \\ a \end{pmatrix} = \text{Find}(F, T, a) \qquad a = 1.250\ \frac{\text{m}}{\text{s}^2} \qquad T = 1.28\ \text{kN} \qquad F = 4.25\ \text{kN}$$

Problem 13-11

The water-park ride consists of a sled of weight W which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is F_{r1} and in the pool for a short distance is F_{r2}, determine how fast the sled is traveling when $s = s_2$.

Given:

$W = 800 \text{ lb}$

$F_{r1} = 30 \text{ lb}$

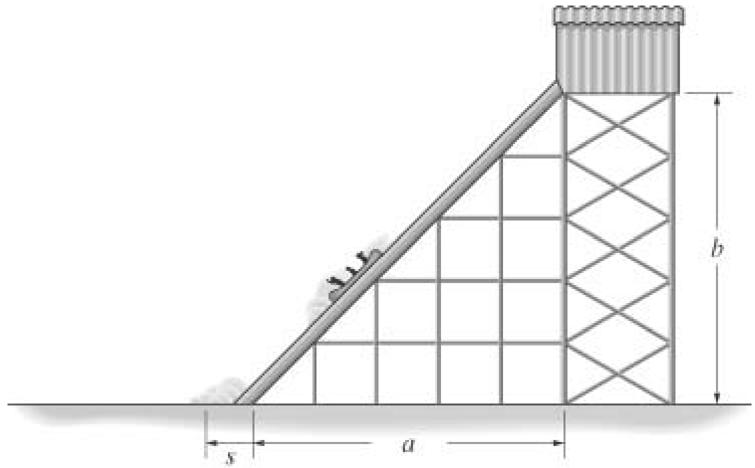
$F_{r2} = 80 \text{ lb}$

$s_2 = 5 \text{ ft}$

$a = 100 \text{ ft}$

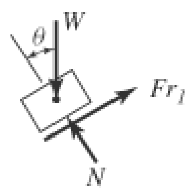
$b = 100 \text{ ft}$

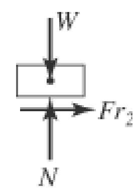
$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$\theta = \text{atan}\left(\frac{b}{a}\right)$





On the incline

$W\sin(\theta) - F_{r1} = \left(\frac{W}{g}\right)a_1$ $\quad a_1 = g\left(\frac{W\sin(\theta) - F_{r1}}{W}\right)$ $\quad a_1 = 21.561 \frac{\text{ft}}{\text{s}^2}$

$v_1^{\,2} = 2a_1\sqrt{a^2 + b^2}$ $\quad v_1 = \sqrt{2a_1\sqrt{a^2 + b^2}}$ $\quad v_1 = 78.093 \frac{\text{ft}}{\text{s}}$

In the water

$F_{r2} = \left(\frac{W}{g}\right)a_2$ $\quad a_2 = \frac{g F_{r2}}{W}$ $\quad a_2 = 3.22 \frac{\text{ft}}{\text{s}^2}$

$\frac{v_2^{\,2}}{2} - \frac{v_1^{\,2}}{2} = -a_2 s_2$ $\quad v_2 = \sqrt{v_1^{\,2} - 2a_2 s_2}$ $\quad v_2 = 77.886 \frac{\text{ft}}{\text{s}}$

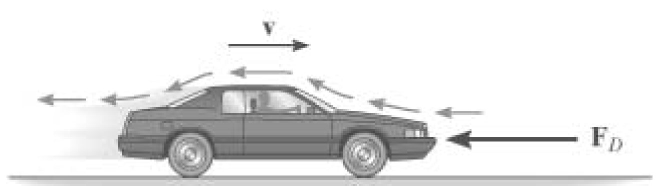
***Problem 13-12**

A car of mass m is traveling at a slow velocity v_0. If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., $F_D = kv$ determine the distance and the time the car will travel before its velocity becomes 0.5 v_0. Assume no other frictional forces act on the car.

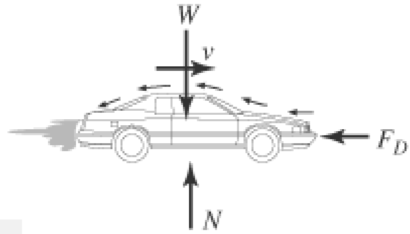
Solution:

$-F_D = m a$

$-k v = m a$



Find time $a = \frac{d}{dt}v = \frac{-k}{m}v$



$$\frac{-k}{m}\int_0^t 1\,dt = \int_{v_0}^{0.5v_0} \frac{1}{v}\,dv$$

$$t = \frac{m}{k}\ln\left(\frac{v_0}{0.5v_0}\right) \qquad t = \frac{m}{k}\ln(2) \qquad t = 0.693\frac{m}{k}$$

Find distance $a = v\frac{d}{dx}v = \frac{-k}{m}v$

$$-\int_0^x k\,dx = \int_{v_0}^{0.5v_0} m\,dv \qquad x = \frac{m}{k}(0.5v_0) \qquad x = 0.5\frac{m v_0}{k}$$

Problem 13-13

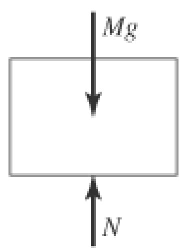
Determine the normal force the crate *A* of mass *M* exerts on the smooth cart if the cart is given an acceleration *a* down the plane. Also, what is the acceleration of the crate?

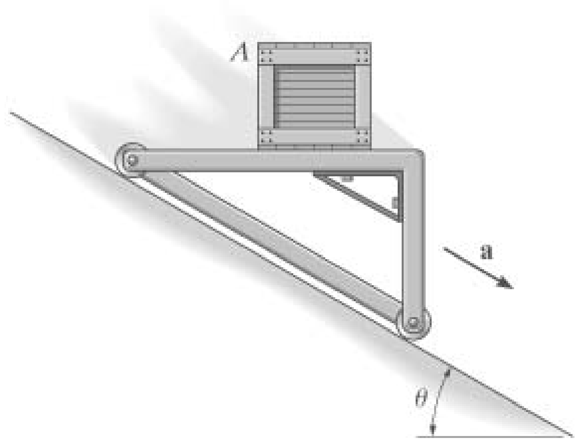
Given:

$M = 10$ kg

$a = 2\ \frac{m}{s^2}$

$\theta = 30$ deg





Solution:

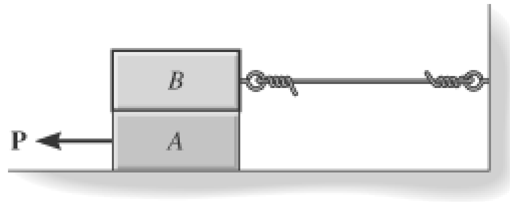
$$N - Mg = -M(a)\sin(\theta)$$

$$N = M[g - (a)\sin(\theta)] \qquad N = 88.1\ \text{N}$$

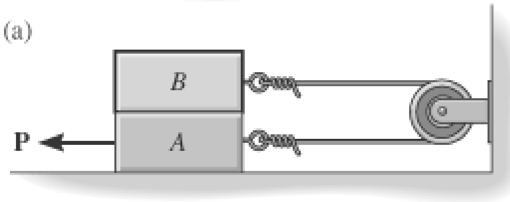
$$a_{crate} = (a)\sin(\theta) \qquad a_{crate} = 1\ \frac{m}{s^2}$$

Problem 13-14

Each of the two blocks has a mass *m*. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the bottom block, determine the acceleration of the bottom block in each case.



(a)



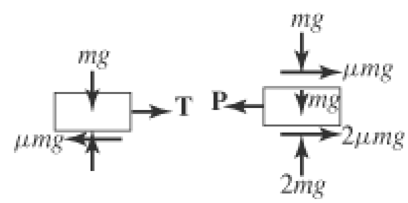
(b)

Solution:

(a) Block A:

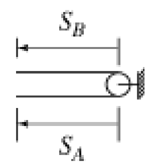
$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $P - 3\mu m g = m a_A$

$$a_A = \frac{P}{m} - 3\mu g$$



(b) $S_B + S_A = L$

$a_A = -a_B$



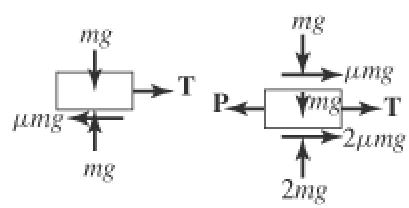
Block A:

$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $P - T - 3\mu m g = m a_A$

Block B:

$\overset{+}{\leftarrow}$ $\Sigma F_x = ma_x;$ $\mu m g - T = m a_B$

Solving simultaenously $$a_A = \frac{P}{2m} - 2\mu g$$



Problem 13-15

The driver attempts to tow the crate using a rope that has a tensile strength T_{max}. If the crate is originally at rest and has weight W, determine the greatest acceleration it can have if the coefficient of static friction between the crate and the road is μ_s and the coefficient of kinetic friction is μ_k.

Given:

$T_{max} = 200 \text{ lb}$

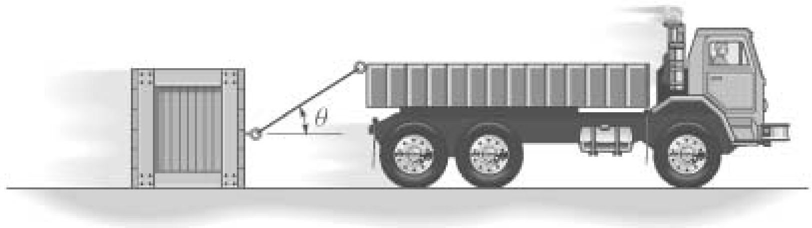
$W = 500 \text{ lb}$

$\mu_s = 0.4$

$\mu_k = 0.3$

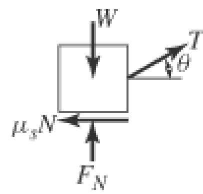
$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

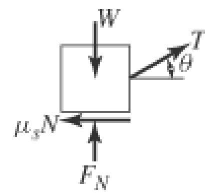
$\theta = 30 \text{ deg}$



Solution:

Equilibrium : In order to slide the crate, the towing force must overcome static friction.





Initial guesses $F_N = 100 \text{ lb}$ $T = 50 \text{ lb}$

Given $T\cos(\theta) - \mu_s F_N = 0$ $F_N + T\sin(\theta) - W = 0$ $\begin{pmatrix} F_N \\ T \end{pmatrix} = \text{Find}(F_N, T)$

If $T = 187.613\,\text{lb} > T_{max} = 200\,\text{lb}$ then the truck will not be able to pull the create without breaking the rope.

If $T = 187.613\,\text{lb} < T_{max} = 200\,\text{lb}$ then the truck will be able to pull the create without breaking the rope and we will now calculate the acceleration for this case.

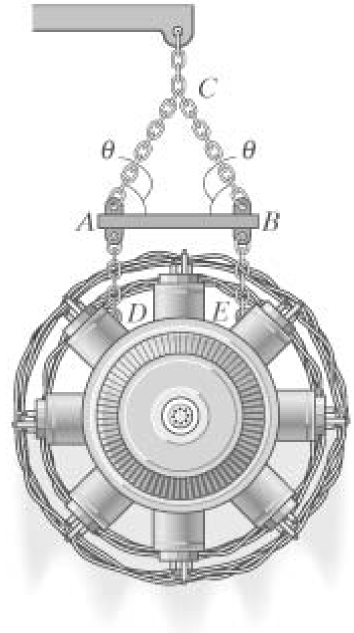
Initial guesses $F_N = 100 \text{ lb}$ $a = 1 \dfrac{\text{ft}}{\text{s}^2}$ Require $T = T_{max}$

Given $T\cos(\theta) - \mu_k F_N = \dfrac{W}{g} a$ $F_N + T\sin(\theta) - W = 0$ $\begin{pmatrix} F_N \\ a \end{pmatrix} = \text{Find}(F_N, a)$

$a = 3.426 \dfrac{\text{ft}}{\text{s}^2}$

***Problem 13-16**

An engine of mass M_1 is suspended from a spreader beam of mass M_2 and hoisted by a crane which gives it an acceleration a when it has a velocity v. Determine the force in chains AC and AD during the lift.



Units Used:

$$\mathrm{Mg} = 10^3\ \mathrm{kg} \quad \mathrm{kN} = 10^3\ \mathrm{N}$$

Given:

$$M_1 = 3.5\ \mathrm{Mg}$$

$$M_2 = 500\ \mathrm{kg}$$

$$a = 4\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$v = 2\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\theta = 60\ \mathrm{deg}$$

Solution:

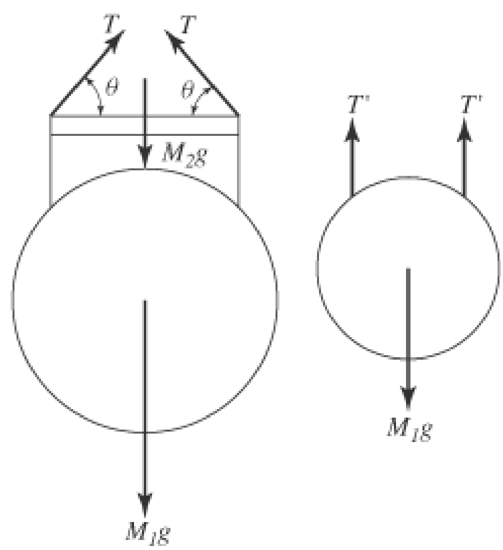
Guesses $T = 1\ \mathrm{N}$ $T' = 1\ \mathrm{N}$

Given

$$2T\sin(\theta) - (M_1 + M_2)g = (M_1 + M_2)a$$

$$2T' - M_1 g = M_1 a$$

$$\begin{pmatrix} T \\ T' \end{pmatrix} = \mathrm{Find}(T, T') \qquad \begin{pmatrix} T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} T \\ T' \end{pmatrix}$$



$$\begin{pmatrix} T_{AC} \\ T_{AD} \end{pmatrix} = \begin{pmatrix} 31.9 \\ 24.2 \end{pmatrix} \mathrm{kN}$$

Problem 13-17

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0\sin(\pi t / t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.

Solution:

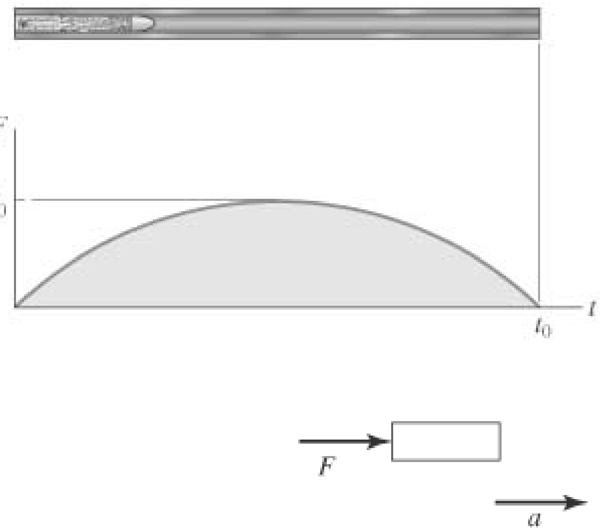
$$F_0 \sin\left(\pi \frac{t}{t_0}\right) = m\,a \qquad a = \frac{dv}{dt} = \frac{F_0}{m}\sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v 1\,dv = \int_0^t \frac{F_0}{m}\sin\left(\frac{\pi t}{t_0}\right)dt$$

$$v = \frac{F_0 t_0}{\pi m}\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$

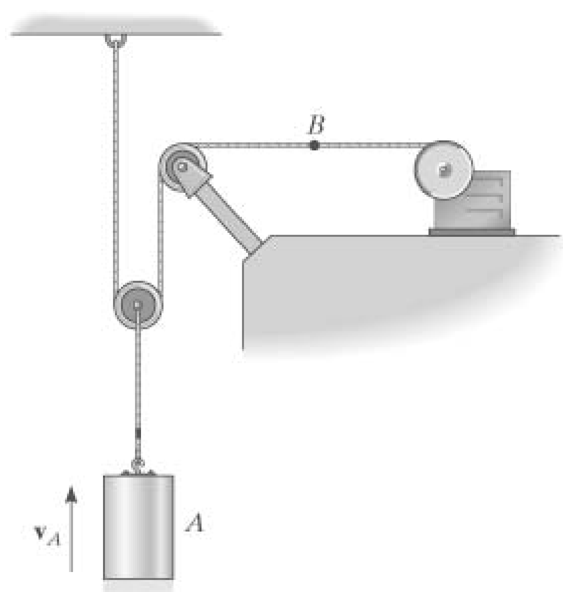
$$v_{max} = \frac{2F_0 t_0}{\pi m}$$



$$\int_0^s 1\,ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt \qquad s = \frac{F_0 t_0}{\pi m}\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$$

Problem 13-18

The cylinder of weight W at A is hoisted using the motor and the pulley system shown. If the speed of point B on the cable is increased at a constant rate from zero to v_B in time t, determine the tension in the cable at B to cause the motion.



Given:

$W = 400$ lb

$v_B = 10 \frac{\text{ft}}{\text{s}}$

$t = 5$ s

Solution:

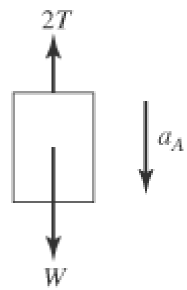
$2s_A + s_B = l$

$$a_B = \frac{v_B}{t}$$

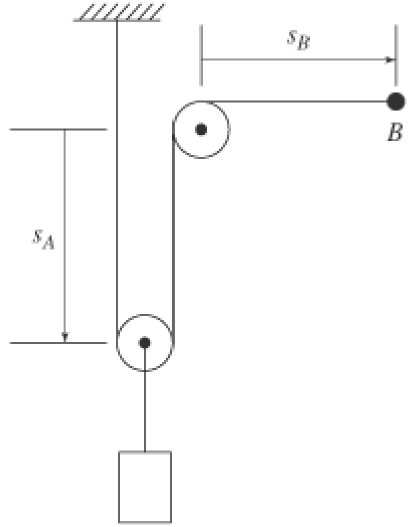
$$a_A = \frac{-a_B}{2}$$

$$2T - W = -\frac{W}{g}a_A$$

$$T = \frac{W}{2}\left(1 - \frac{a_A}{g}\right)$$



$T = 206\,\text{lb}$



Problem 13-19

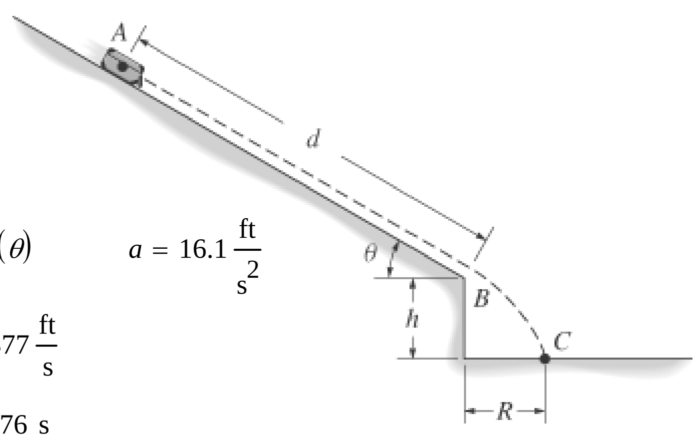
A suitcase of weight *W* slides from rest a distance *d* down the smooth ramp. Determine the point where it strikes the ground at *C*. How long does it take to go from *A* to *C*?

Given:

$W = 40\ \text{lb}$ $\theta = 30\ \text{deg}$

$d = 20\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$h = 4\ \text{ft}$



Solution:

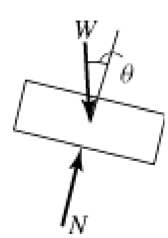
$W\sin(\theta) = \left(\dfrac{W}{g}\right)a$ $a = g\sin(\theta)$ $a = 16.1\ \dfrac{\text{ft}}{\text{s}^2}$

$v_B = \sqrt{2ad}$ $v_B = 25.377\ \dfrac{\text{ft}}{\text{s}}$

$t_{AB} = \dfrac{v_B}{a}$ $t_{AB} = 1.576\ \text{s}$

Guesses $t_{BC} = 1\ \text{s}$ $R = 1\ \text{ft}$

Given $\left(\dfrac{-g}{2}\right)t_{BC}^{\ 2} - v_B\sin(\theta)t_{BC} + h = 0$ $R = v_B\cos(\theta)t_{BC}$



$\begin{pmatrix} t_{BC} \\ R \end{pmatrix} = \text{Find}(t_{BC}, R)$ $t_{BC} = 0.241\ \text{s}$

$R = 5.304\,\text{ft}$ $t_{AB} + t_{BC} = 1.818\ \text{s}$

***Problem 13-20**

A suitcase of weight W slides from rest a distance d down the rough ramp. The coefficient of kinetic friction along ramp AB is μ_k. The suitcase has an initial velocity down the ramp v_0. Determine the point where it strikes the ground at C. How long does it take to go from A to C?

Given:

$W = 40\ \text{lb}$

$d = 20\ \text{ft}$

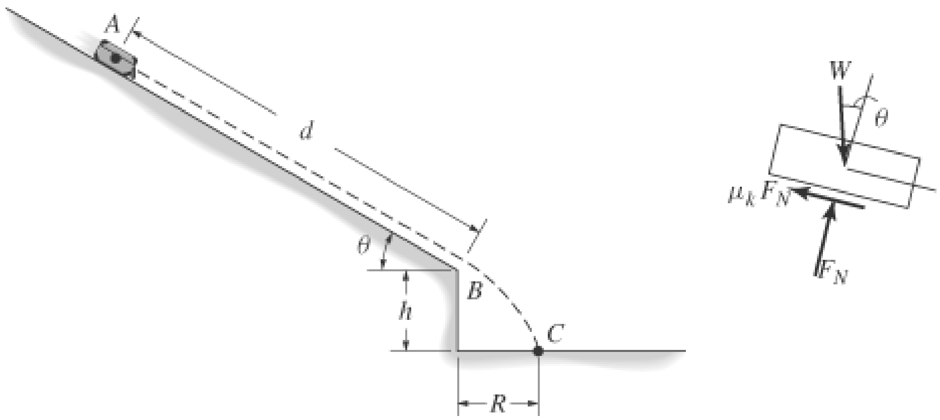
$h = 4\ \text{ft}$

$\mu_k = 0.2$

$\theta = 30\ \text{deg}$

$v_0 = 10\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$F_N - W\cos(\theta) = 0 \qquad F_N = W\cos(\theta)$

$W\sin(\theta) - \mu_k W\cos(\theta) = \left(\frac{W}{g}\right)a$

$a = g\left(\sin(\theta) - \mu_k\cos(\theta)\right) \qquad a = 10.523\ \frac{\text{ft}}{\text{s}^2}$

$v_B = \sqrt{2ad + v_0^2} \qquad v_B = 22.823\ \frac{\text{ft}}{\text{s}}$

$t_{AB} = \frac{v_B - v_0}{a} \qquad t_{AB} = 1.219\ \text{s}$

Guesses $\quad t_{BC} = 1\ \text{s} \qquad R = 1\ \text{ft}$

Given $\quad \left(\frac{-g}{2}\right)t_{BC}^2 - v_B\sin(\theta)t_{BC} + h = 0 \qquad R = v_B\cos(\theta)t_{BC}$

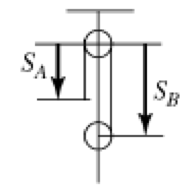
$\begin{pmatrix} t_{BC} \\ R \end{pmatrix} = \text{Find}(t_{BC}, R) \qquad t_{BC} = 0.257\ \text{s} \qquad R = 5.084\ \text{ft} \qquad t_{AB} + t_{BC} = 1.476\ \text{s}$

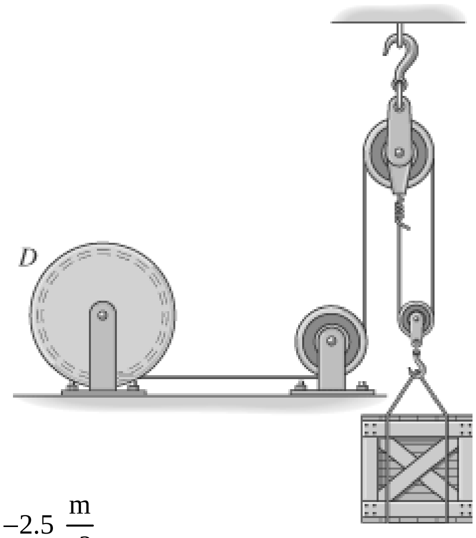
Problem 13-21

The winding drum D is drawing in the cable at an accelerated rate a. Determine the cable tension if the suspended crate has mass M.

Units Used:

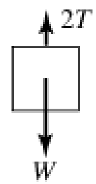
$\mathrm{kN} = 1000\ \mathrm{N}$





Given:

$a = 5\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$



$M = 800\ \mathrm{kg}$

$g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

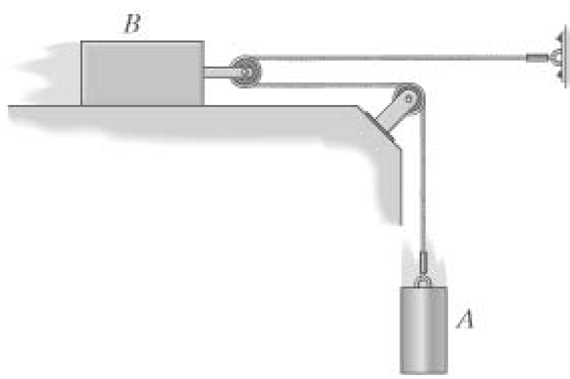
Solution:

$L = s_A + 2s_B \qquad a_B = \dfrac{-a}{2} \qquad a_B = -2.5\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

$2T - Mg = -M a_B \qquad T = \dfrac{M(g - a_B)}{2} \qquad T = 4.924\ \mathrm{kN}$

Problem 13-22

At a given instant block A of weight W_A is moving downward with a speed v_1. Determine its speed at the later time t. Block B has weight W_B, and the coefficient of kinetic friction between it and the horizontal plane is μ_k. Neglect the mass of the pulleys and cord.



Given:

$W_A = 5\ \mathrm{lb} \qquad v_1 = 4\ \dfrac{\mathrm{ft}}{\mathrm{s}}$

$W_B = 6\ \mathrm{lb} \qquad t = 2\ \mathrm{s}$

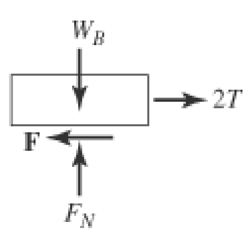
$\mu_k = 0.3$

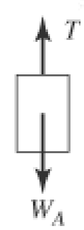
Solution: $\quad 2s_B + s_A = L$

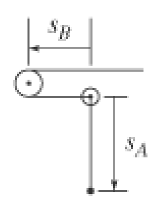
Guesses

$a_A = 1\ \dfrac{\mathrm{ft}}{\mathrm{s}^2} \qquad a_B = 1\ \dfrac{\mathrm{ft}}{\mathrm{s}^2}$

$T = 1\ \mathrm{lb} \qquad F_N = 1\ \mathrm{lb}$







Given $\quad F_N - W_B = 0 \qquad 2a_B + a_A = 0$

$$2T - \mu_k F_N = \left(\frac{-W_B}{g}\right) a_B \qquad T - W_A = \left(\frac{-W_A}{g}\right) a_A$$

$$\begin{pmatrix} F_N \\ T \\ a_A \\ a_B \end{pmatrix} = \mathrm{Find}(F_N, T, a_A, a_B) \qquad \begin{pmatrix} F_N \\ T \end{pmatrix} = \begin{pmatrix} 6.000 \\ 1.846 \end{pmatrix} \mathrm{lb} \qquad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 20.3 \\ -10.2 \end{pmatrix} \frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$v_2 = v_1 + a_A t \qquad v_2 = 44.6 \frac{\mathrm{ft}}{\mathrm{s}}$$

Problem 13-23

A force F is applied to the cord. Determine how high the block A of weight W rises in time t starting from rest. Neglect the weight of the pulleys and cord.

Given:

$F = 15$ lb $\quad t = 2$ s

$W = 30$ lb $\quad g = 32.2 \frac{\mathrm{ft}}{\mathrm{s}^2}$

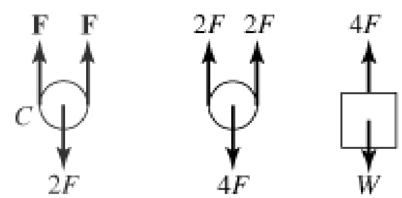
Solution:

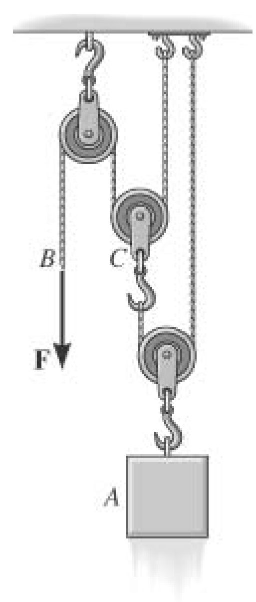
$$4F - W = \left(\frac{W}{g}\right) a$$

$$a = \frac{g}{W}(4F - W)$$

$$a = 32.2 \frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$d = \frac{1}{2} a t^2 \qquad d = 64.4 \mathrm{ft}$$





***Problem 13-24**

At a given instant block A of weight W_A is moving downward with speed v_{A0}. Determine its speed at a later time t. Block B has a weight W_B and the coefficient of kinetic friction between it and the horizontal plane is μ_k. Neglect the mass of the pulleys and cord.

Given:

$W_A = 10 \text{ lb}$

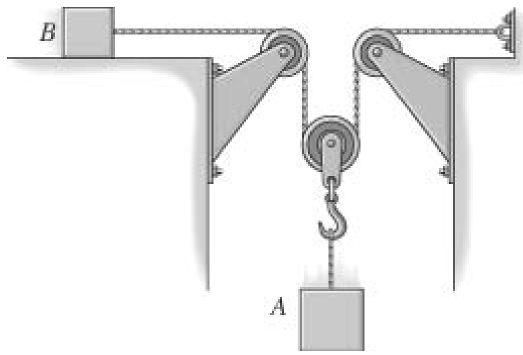
$v_{A0} = 6 \frac{\text{ft}}{\text{s}}$

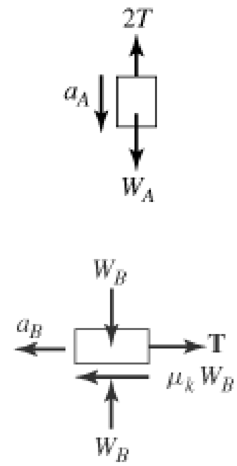
$t = 2 \text{ s}$

$W_B = 4 \text{ lb}$

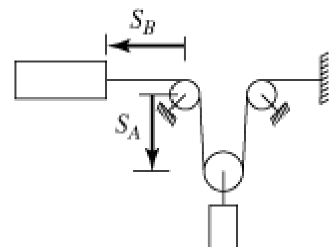
$\mu_k = 0.2$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution: $L = s_B + 2s_A$



Guesses $a_A = 1 \frac{\text{ft}}{\text{s}^2}$ $a_B = 1 \frac{\text{ft}}{\text{s}^2}$ $T = 1 \text{ lb}$

Given $T - \mu_k W_B = \left(\frac{-W_B}{g}\right) a_B$

$$2T - W_A = \left(\frac{-W_A}{g}\right) a_A$$

$$0 = a_B + 2a_A$$

$$\begin{pmatrix} T \\ a_A \\ a_B \end{pmatrix} = \text{Find}(T, a_A, a_B) \qquad T = 3.385 \text{ lb} \qquad \begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 10.403 \\ -20.806 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$v_A = v_{A0} + a_A t \qquad v_A = 26.8 \frac{\text{ft}}{\text{s}}$$

Problem 13-25

A freight elevator, including its load, has mass M_e. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension T in its attached cable, determine the velocity of the elevator when it has moved upward at a distance d starting from rest. Neglect the mass of the pulleys and cables.

Units Used:

$$kN = 10^3\ N$$

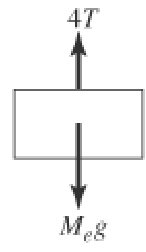
Given:

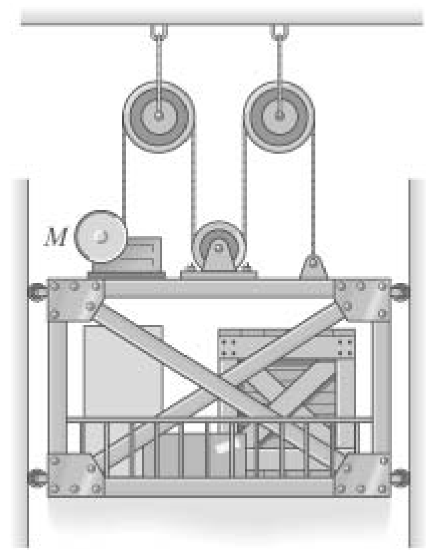
$$M_e = 500\ kg$$

$$T = 1.50\ kN$$

$$d = 3\ m$$

$$g = 9.81\ \frac{m}{s^2}$$





Solution:

$$4T - M_e g = M_e a \qquad a = 4\left(\frac{T}{M_e}\right) - g$$

$$v = \sqrt{2 a d} \qquad v = 3.62\ \frac{m}{s}$$

Problem 13-26

At the instant shown the block A of weight W_A is moving down the plane at v_0 while being attached to the block B of weight W_B. If the coefficient of kinetic friction is μ_k, determine the acceleration of A and the distance A slides before it stops. Neglect the mass of the pulleys and cables.

Given:

$$W_A = 100\ lb$$

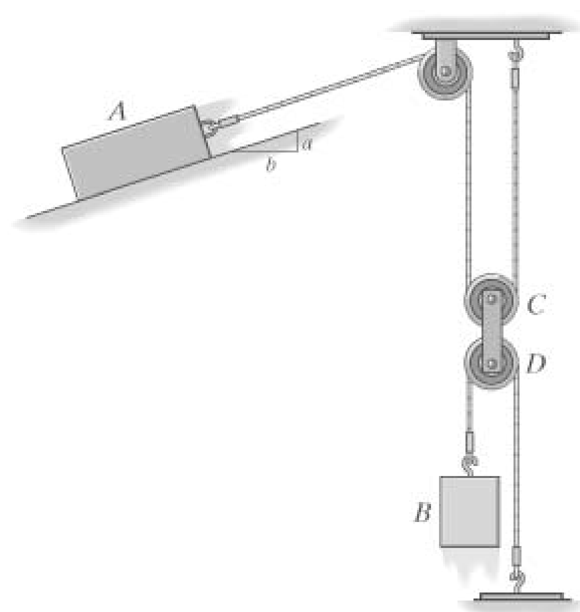
$$W_B = 50\ lb$$

$$v_0 = 5\ \frac{ft}{s}$$

$$\mu_k = 0.2$$

$$a = 3$$

$$b = 4$$



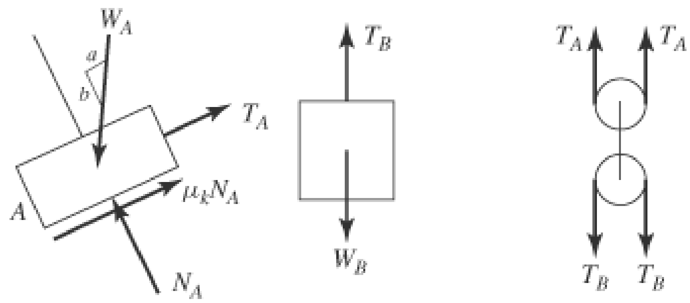
Solution: $\theta = \text{atan}\left(\frac{a}{b}\right)$

Rope constraints

$$s_A + 2s_C = L_1$$

$$s_D + (s_D - s_B) = L_2$$

$$s_C + s_D + d = d'$$

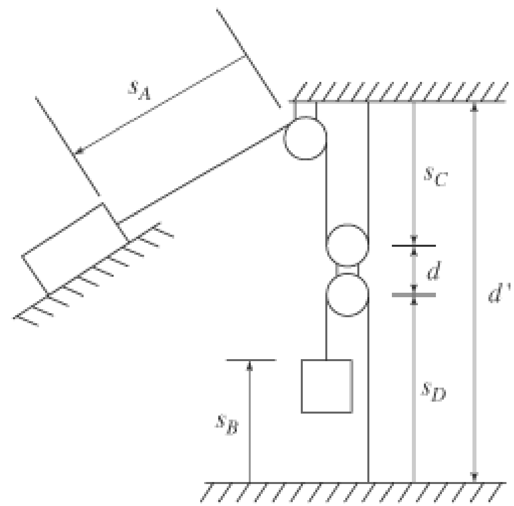


Guesses

$$a_A = 1\ \frac{\text{ft}}{\text{s}^2} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$a_C = 1\ \frac{\text{ft}}{\text{s}^2} \qquad a_D = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$T_A = 1\ \text{lb} \qquad T_B = 1\ \text{lb} \qquad N_A = 1\ \text{lb}$$



Given

$$a_A + 2a_C = 0 \qquad 2a_D - a_B = 0$$

$$a_C + a_D = 0$$

$$T_B - W_B = \left(\frac{W_B}{g}\right)a_B$$

$$T_A - W_A\sin(\theta) + \mu_k N_A = \left(\frac{-W_A}{g}\right)a_A$$

$$N_A - W_A\cos(\theta) = 0 \qquad 2T_A - 2T_B = 0$$

$$\begin{pmatrix} a_A \\ a_B \\ a_C \\ a_D \\ T_A \\ T_B \\ N_A \end{pmatrix} = \text{Find}(a_A, a_B, a_C, a_D, T_A, T_B, N_A) \qquad \begin{pmatrix} T_A \\ T_B \\ N_A \end{pmatrix} = \begin{pmatrix} 48 \\ 48 \\ 80 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} a_A \\ a_B \\ a_C \\ a_D \end{pmatrix} = \begin{pmatrix} -1.287 \\ -1.287 \\ 0.644 \\ -0.644 \end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

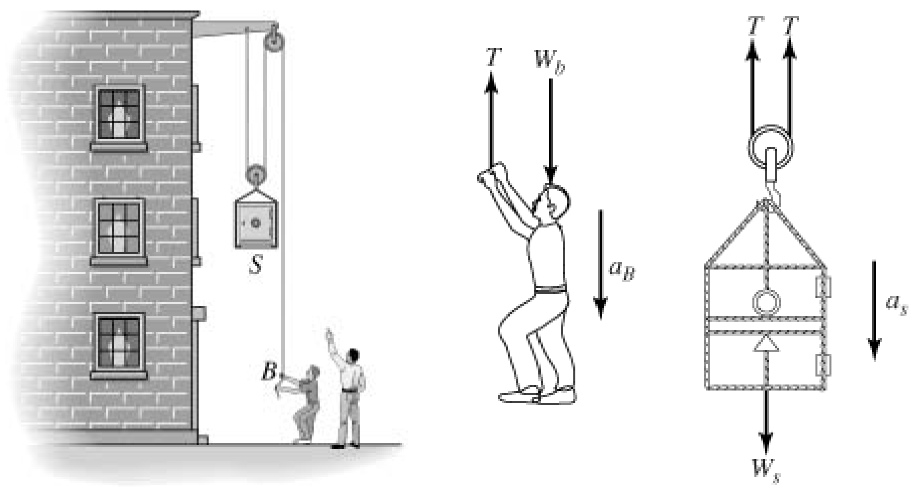
$$d_A = \frac{-v_0^2}{2a_A} \qquad a_A = -1.287\ \frac{\text{ft}}{\text{s}^2} \qquad d_A = 9.71\ \text{ft}$$

Problem 13-27

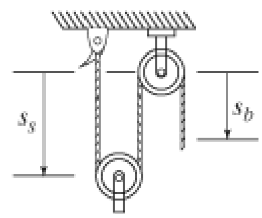
The safe S has weight W_s and is supported by the rope and pulley arrangement shown. If the end of the rope is given to a boy B of weight W_b, determine his acceleration if in the confusion he doesn't let go of the rope. Neglect the mass of the pulleys and rope.

Given:

$$W_s = 200 \text{ lb} \quad W_b = 90 \text{ lb} \quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$



Solution: $L = 2s_s + s_b$



Initial guesses: $a_b = 1 \frac{\text{ft}}{\text{s}^2}$ $\quad a_s = 1 \frac{\text{ft}}{\text{s}^2}$ $\quad T = 1 \text{ lb}$

Given $\quad 0 = 2a_s + a_b \quad 2T - W_s = \left(\frac{-W_s}{g}\right)a_s \quad T - W_b = \left(\frac{-W_b}{g}\right)a_b$

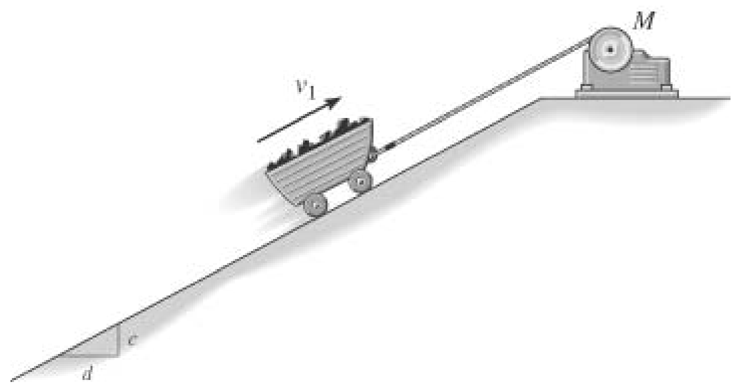
$$\begin{pmatrix} a_b \\ a_s \\ T \end{pmatrix} = \text{Find}(a_b, a_s, T) \quad T = 96.429 \text{ lb} \quad a_s = 1.15 \frac{\text{ft}}{\text{s}^2} \quad a_b = -2.3 \frac{\text{ft}}{\text{s}^2}$$

Negative means up

***Problem 13-28**

The mine car of mass m_{car} is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = bt^2$. If the car has an initial velocity v_0 when $t = 0$, determine its velocity when $t = t_1$.

Given:



$m_{car} = 400\ \text{kg}$

$b = 3200\ \frac{\text{N}}{\text{s}^2}$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$

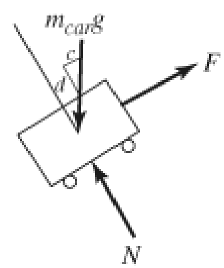
$t_1 = 2\ \text{s}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$c = 8$

$d = 15$

Solution:



$$b\,t^2 - m_{car}g\left(\frac{c}{\sqrt{c^2 + d^2}}\right) = m_{car}a$$

$$a = \left(\frac{b}{m_{car}}\right)t^2 - \frac{g\,c}{\sqrt{c^2 + d^2}}$$

$$v_1 = \left(\frac{b}{m_{car}}\right)\frac{t_1^3}{3} - \frac{g\,c\,t_1}{\sqrt{c^2 + d^2}} + v_0 \qquad v_1 = 14.1\ \frac{\text{m}}{\text{s}}$$

Problem 13-29

The mine car of mass m_{car} is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = bt^2$. If the car has an initial velocity v_0 when $t = 0$, determine the distance it moves up the plane when $t = t_1$.

Given:

$m_{car} = 400 \text{ kg}$

$b = 3200 \dfrac{\text{N}}{\text{s}^2}$

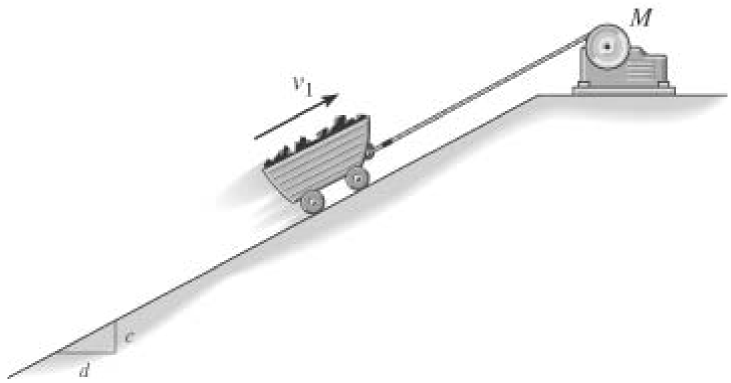
$v_0 = 2 \dfrac{\text{m}}{\text{s}}$

$t_1 = 2 \text{ s}$

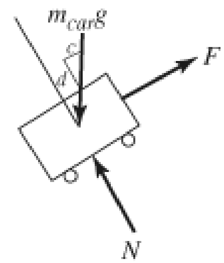
$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$c = 8$

$d = 15$



Solution:



$$b t^2 - m_{car} g \left(\frac{c}{\sqrt{c^2 + d^2}} \right) = m_{car} a \qquad a = \left(\frac{b}{m_{car}} \right) t^2 - \frac{g c}{\sqrt{c^2 + d^2}}$$

$$v = \left(\frac{b}{m_{car}} \right) \frac{t^3}{3} - \frac{g c t}{\sqrt{c^2 + d^2}} + v_0$$

$$s_1 = \left(\frac{b}{m_{car}} \right) \frac{t_1^4}{12} - \left(\frac{g c}{\sqrt{c^2 + d^2}} \right) \frac{t_1^2}{2} + v_0 t_1 \qquad s_1 = 5.434 \text{ m}$$

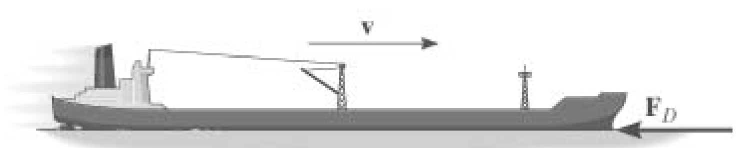
Problem 13-30

The tanker has a weight W and is traveling forward at speed v_0 in still water when the engines are shut off. If the drag resistance of the water is proportional to the speed of the tanker at any instant and can be approximated by $F_D = cv$, determine the time needed for the tanker's speed to become v_1. Given the initial velocity v_0 through what distance must the tanker travel before it stops?

Given:

$W = 800 \times 10^6 \text{ lb}$

$c = 400 \times 10^3 \text{ lb} \cdot \dfrac{\text{s}}{\text{ft}}$



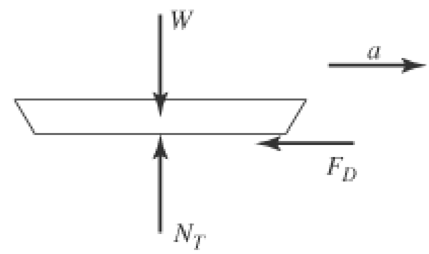
$v_0 = 3 \dfrac{\text{ft}}{\text{s}} \qquad v_1 = 1.5 \dfrac{\text{ft}}{\text{s}}$

Solution:

$$a(v) = \frac{-c g}{W} v$$

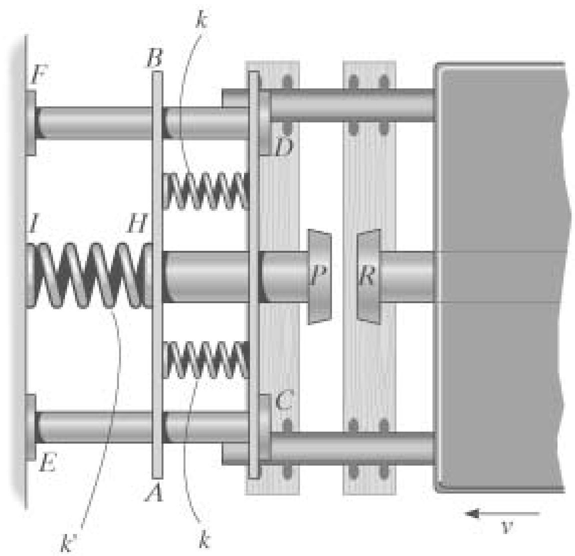
$$t = \int_{v_0}^{v_1} \frac{1}{a(v)}\, dv \qquad t = 43.1\ \text{s}$$

$$d = \int_{v_0}^{0} \frac{v}{a(v)}\, dv \qquad d = 186.4\ \text{ft}$$



Problem 13-31

The spring mechanism is used as a shock absorber for railroad cars. Determine the maximum compression of spring *HI* if the fixed bumper *R* of a railroad car of mass *M*, rolling freely at speed *v* strikes the plate *P*. Bar *AB* slides along the guide paths *CE* and *DF*. The ends of all springs are attached to their respective members and are originally unstretched.



Units Used:

$$\text{kN} = 10^3\ \text{N} \qquad \text{Mg} = 10^3\ \text{kg}$$

Given:

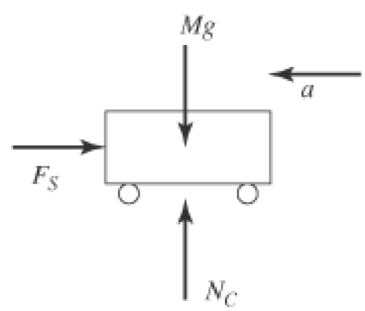
$$M = 5\ \text{Mg} \qquad k = 80\ \frac{\text{kN}}{\text{m}}$$

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad k' = 160\ \frac{\text{kN}}{\text{m}}$$

Solution:

The springs stretch or compress an equal amount *x*. Thus,

$$(k' + 2k)x = -M a \qquad a = -\frac{k' + 2k}{M} x = v\frac{d}{dx}v$$

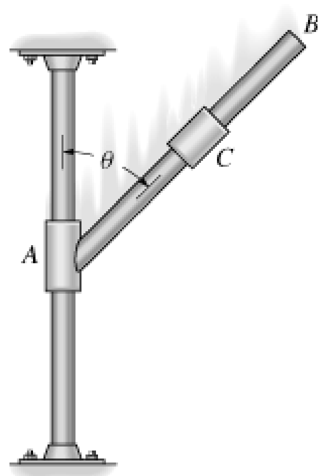


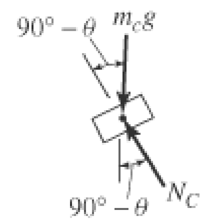
Guess $d = 1\ \text{m}$ Given $\int_{v}^{0} v\, dv = -\int_{0}^{d} \left(\frac{k' + 2k}{M}\right) x\, dx$ $d = \text{Find}(d)$

$$d = 0.250\ \text{m}$$

***Problem 13-32**

The collar *C* of mass m_c is free to slide along the smooth shaft *AB*. Determine the acceleration of collar *C* if (a) the shaft is fixed from moving, (b) collar *A*, which is fixed to shaft *AB*, moves downward at constant velocity along the vertical rod, and (c) collar A is subjected to downward acceleration a_A. In all cases, the collar moves in the plane.





Given:

$m_c = 2\ \text{kg}$

$a_A = 2\ \dfrac{\text{m}}{\text{s}^2}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\theta = 45\ \text{deg}$

Solution:

(a) $m_c\, g \cos(\theta) = m_c\, a_a \qquad a_a = g\cos(\theta) \qquad a_a = 6.937\ \dfrac{\text{m}}{\text{s}^2}$

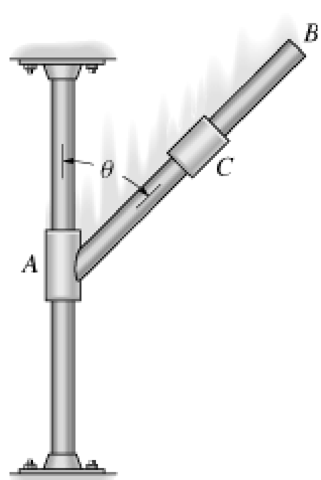
(b) $m_c\, g \cos(\theta) = m_c\, a_b \qquad a_b = g\cos(\theta) \qquad a_b = 6.937\ \dfrac{\text{m}}{\text{s}^2}$

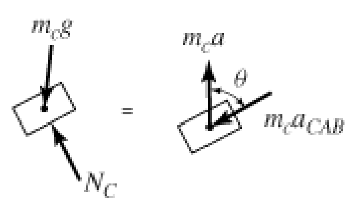
(c) $m_c(g - a_A)\cos(\theta) = m_c\, a_{crel} \qquad a_{crel} = (g - a_A)\cos(\theta)$

$$\mathbf{a_c} = a_{crel}\begin{pmatrix} -\sin(\theta) \\ -\cos(\theta) \end{pmatrix} + a_A\begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \mathbf{a_c} = \begin{pmatrix} -3.905 \\ -5.905 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_c}| = 7.08\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-33

The collar C of mass m_c is free to slide along the smooth shaft AB. Determine the acceleration of collar C if collar A is subjected to an upward acceleration a. The collar moves in the plane.





Given:

$m_C = 2\ \text{kg}$

$a = 4\ \dfrac{\text{m}}{\text{s}^2}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\theta = 45\ \text{deg}$

Solution:

The collar accelerates along the rod and the rod accelerates upward.

$$m_C g\cos(\theta) = m_C\left[a_{CA} - (a)\cos(\theta)\right] \qquad a_{CA} = (g+a)\cos(\theta)$$

$$\mathbf{a_C} = \begin{pmatrix} -a_{CA}\sin(\theta) \\ -a_{CA}\cos(\theta) + a \end{pmatrix} \qquad \mathbf{a_C} = \begin{pmatrix} -6.905 \\ -2.905 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_C}\right| = 7.491\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-34

The boy has weight W and hangs uniformly from the bar. Determine the force in each of his arms at time $t = t_1$ if the bar is moving upward with (a) a constant velocity v_0 and (b) a speed $v = bt^2$

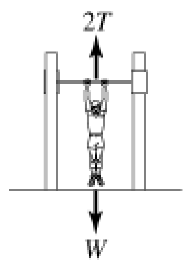
Given:

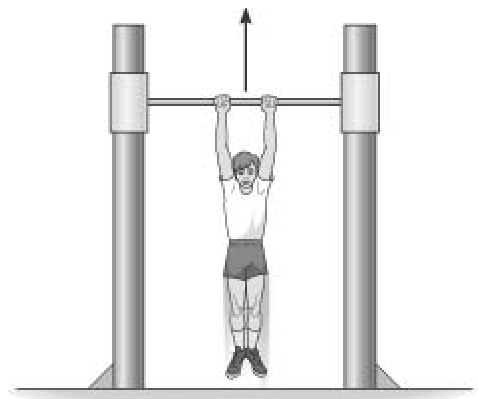
$W = 80\ \text{lb}$

$t_1 = 2\ \text{s}$

$v_0 = 3\ \frac{\text{ft}}{\text{s}}$

$b = 4\ \frac{\text{ft}}{\text{s}^3}$





Solution: (a) $2T_a - W = 0 \qquad T_a = \frac{W}{2} \qquad T_a = 40\,\text{lb}$

(b) $2T_b - W = \left(\frac{W}{g}\right)2bt \qquad T_b = \frac{1}{2}\left(W + \frac{W}{g}2bt_1\right) \qquad T_b = 59.885\,\text{lb}$

Problem 13-35

The block A of mass m_A rests on the plate B of mass m_B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide a distance s' *on the plate* when the system is released from rest.

Given:

$m_A = 10\ \text{kg}$

$m_B = 50\ \text{kg}$

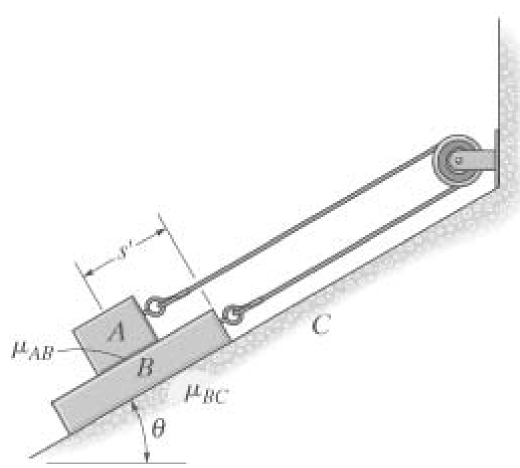
$s' = 0.5\ \text{m}$

$\mu_{AB} = 0.2$

$\mu_{BC} = 0.1$

$\theta = 30\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



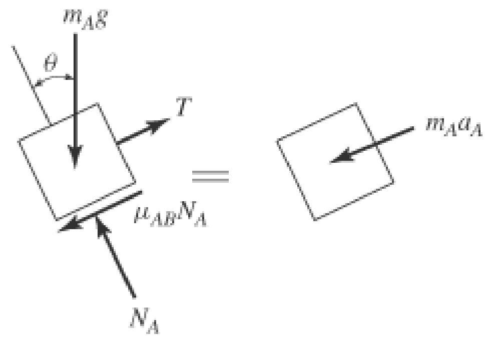
Solution:

$s_A + s_B = L$

Guesses

$a_A = 1\ \frac{m}{s^2}$ $\quad a_B = 1\ \frac{m}{s^2}$

$T = 1\ N$ $\quad N_A = 1\ N$ $\quad N_B = 1\ N$



Given

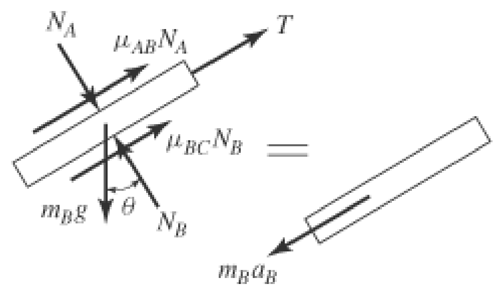
$a_A + a_B = 0$

$N_A - m_A g\cos(\theta) = 0$

$N_B - N_A - m_B g\cos(\theta) = 0$

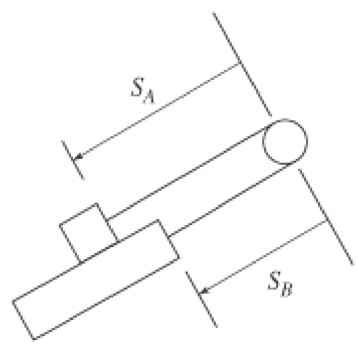
$T - \mu_{AB} N_A - m_A g\sin(\theta) = -m_A a_A$

$T + \mu_{AB} N_A + \mu_{BC} N_B - m_B g\sin(\theta) = -m_B a_B$



$$\begin{pmatrix} a_A \\ a_B \\ T \\ N_A \\ N_B \end{pmatrix} = \mathrm{Find}(a_A, a_B, T, N_A, N_B) \qquad \begin{pmatrix} T \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 84.58 \\ 84.96 \\ 509.74 \end{pmatrix} N$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -1.854 \\ 1.854 \end{pmatrix} \frac{m}{s^2}$$



$a_{BA} = a_B - a_A$ $\quad a_{BA} = 3.708\ \frac{m}{s^2}$ $\quad t = \sqrt{\frac{2s'}{a_{BA}}}$ $\quad t = 0.519\ s$

***Problem 13-36**

Determine the acceleration of block *A* when the system is released from rest. The coefficient of kinetic friction and the weight of each block are indicated. Neglect the mass of the pulleys and cord.

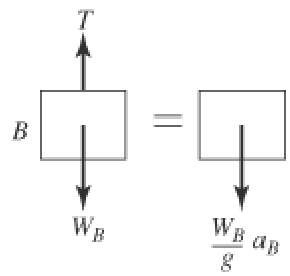
Given:

$W_A = 80\ lb$

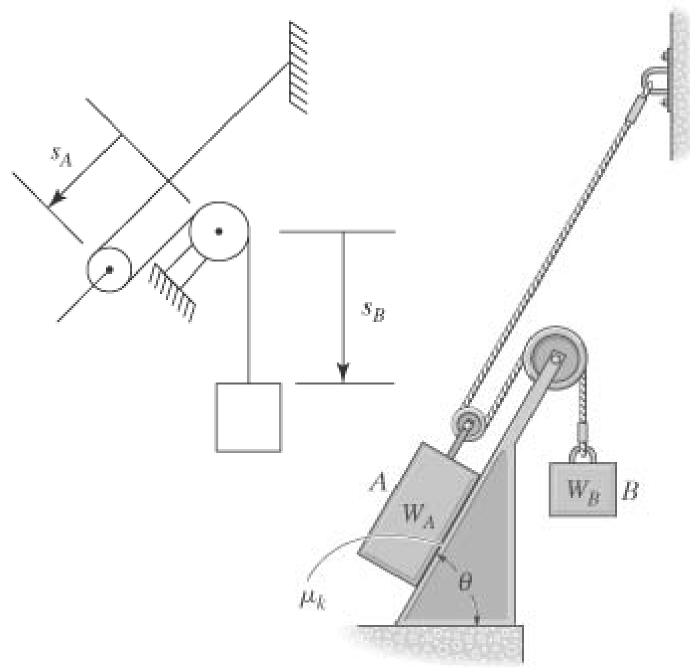
$W_B = 20\ lb$

$\theta = 60\ deg$

$\mu_k = 0.2$



$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$



Solution: $2s_A + s_B = L$

Guesses

$$a_A = 1\ \frac{\text{ft}}{\text{s}^2} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$T = 1\ \text{lb} \qquad N_A = 1\ \text{lb}$$

Given

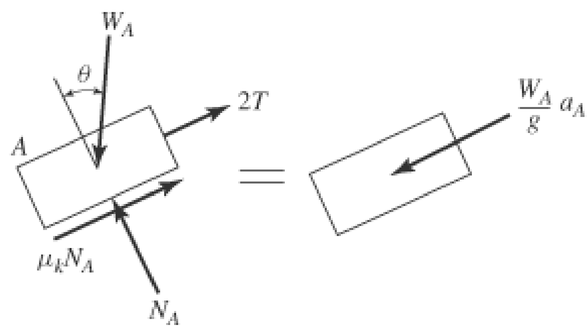
$$2T - W_A \sin(\theta) + \mu_k N_A = \left(\frac{-W_A}{g}\right) a_A$$

$$N_A - W_A \cos(\theta) = 0$$

$$T - W_B = \left(\frac{-W_B}{g}\right) a_B$$

$$2a_A + a_B = 0$$

$$\begin{pmatrix} a_A \\ a_B \\ T \\ N_A \end{pmatrix} = \text{Find}(a_A, a_B, T, N_A) \qquad a_A = 4.28\ \frac{\text{ft}}{\text{s}^2}$$



Problem 13-37

The conveyor belt is moving at speed v. If the coefficient of static friction between the conveyor and the package B of mass M is μ_s, determine the shortest time the belt can stop so that the package does not slide on the belt.

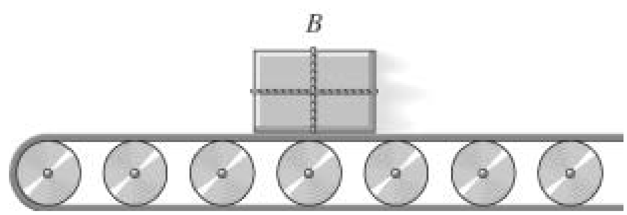
Given:

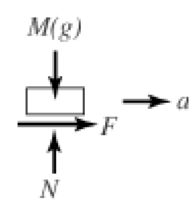
$$v = 4\ \frac{\text{m}}{\text{s}}$$

$$M = 10\ \text{kg}$$

$$\mu_s = 0.2$$

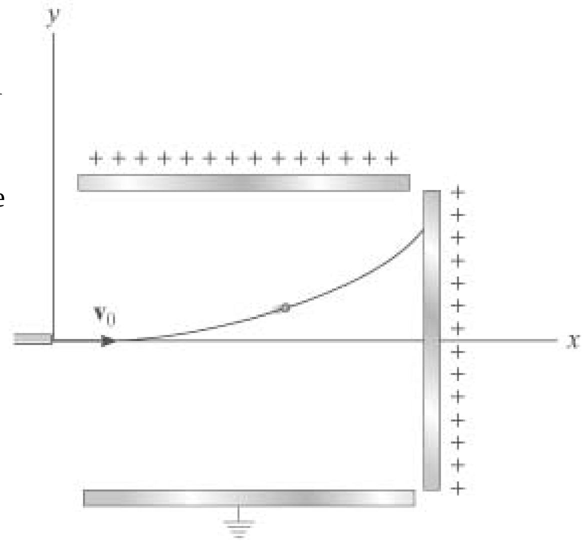
$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$





Solution: $\mu_s M g = M a \qquad a = \mu_s g \qquad a = 1.962\ \frac{\text{m}}{\text{s}^2} \qquad t = \frac{v}{a} \qquad t = 2.039\ \text{s}$

Problem 13-38



An electron of mass m is discharged with an initial horizontal velocity of v_0. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$ where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.

Solution:

$$F_0 = m a_x \qquad 0.3F_0 = m a_y$$

$$a_x = \frac{F_0}{m} \qquad a_y = 0.3\left(\frac{F_0}{m}\right)$$

$$v_x = \left(\frac{F_0}{m}\right)t + v_0 \qquad v_y = 0.3\left(\frac{F_0}{m}\right)t$$

$$s_x = \frac{F_0}{m}\left(\frac{t^2}{2}\right) + v_0 t \qquad s_y = 0.3\frac{F_0}{m}\left(\frac{t^2}{2}\right) \qquad t = \sqrt{\frac{20 s_y m}{3F_0}}$$

Thus $$s_x = \frac{10 s_y}{3} + v_0\sqrt{\frac{20 s_y m}{3F_0}}$$

$$v = \sqrt{v_x^2 + v_y^2} \qquad v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

***Problem 13-39**

The conveyor belt delivers each crate of mass M to the ramp at A such that the crate's speed is v_A directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is μ_k, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs.

Given:

$$M = 12 \text{ kg}$$

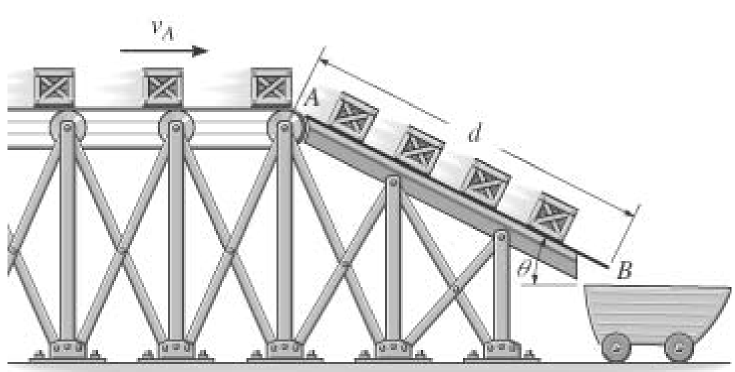
$$v_A = 2.5 \frac{\text{m}}{\text{s}}$$

$$d = 3 \text{ m}$$

$$\mu_k = 0.3$$

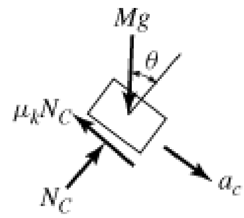
$$\theta = 30 \text{ deg}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$



Solution:

$N_C - M g \cos(\theta) = 0$ $\quad N_C = M g \cos(\theta)$



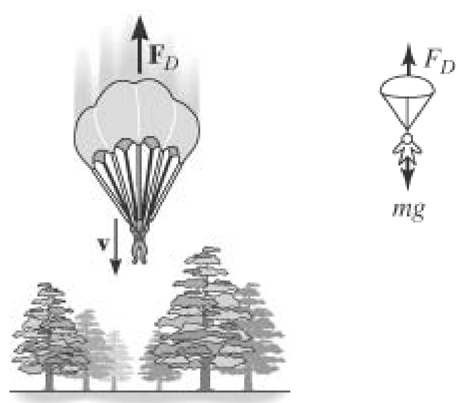
$M g \sin(\theta) - \mu_k N_C = M a$ $\quad a = g \sin(\theta) - \mu_k \left(\frac{N_C}{M}\right)$ $\quad a = 2.356 \frac{\text{m}}{\text{s}^2}$

$v_B = \sqrt{v_A^{\,2} + 2 a d}$ $\quad v_B = 4.515 \frac{\text{m}}{\text{s}}$

***Problem 13-40**

A parachutist having a mass *m* opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where *k* is a constant, determine his velocity when he has fallen for a time *t*. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.

Solution:



$$k v^2 - m g = -m a$$

$$a = g - \left(\frac{k}{m}\right) v^2 = \frac{dv}{dt}$$

$$t = \int_0^v \frac{1}{g - \left(\frac{k}{m}\right) v^2} \, dv$$

$$t = \left(\sqrt{\frac{m}{g k}}\right) \operatorname{atanh}\left(v \sqrt{\frac{k}{g m}}\right)$$

$$v = \left(\sqrt{\frac{mg}{k}}\right) \tanh\left(\sqrt{\frac{g k}{m}}\, t\right)$$

When $t \rightarrow \infty$ $\quad v = \sqrt{m\left(\frac{g}{k}\right)}$

Problem 13-41

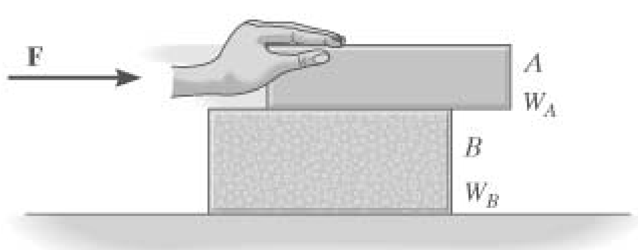
Block *B* rests on a smooth surface. If the coefficients of static and kinetic friction between *A* and *B* are μ_s and μ_k respectively, determine the acceleration of each block if someone pushes horizontally on block *A* with a force of (*a*) $F = F_a$ and (*b*) $F = F_b$.

Given:

$\mu_s = 0.4$ $\quad F_a = 6 \text{ lb}$

$\mu_k = 0.3$ $\quad F_b = 50 \text{ lb}$

$W_A = 20 \text{ lb}$ $\quad W_B = 30 \text{ lb}$

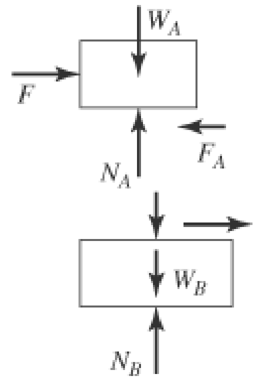


$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $F_A = 1\ \text{lb}$ $F_{max} = 1\ \text{lb}$

$a_A = 1\ \frac{\text{ft}}{\text{s}^2}$ $a_B = 1\ \frac{\text{ft}}{\text{s}^2}$



(a) $F = F_a$ First assume no slip

Given $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

$a_A = a_B$ $F_{max} = \mu_s W_A$

$$\begin{pmatrix} F_A \\ F_{max} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{max}, a_A, a_B)$$

If $F_A = 3.599\ \text{lb} < F_{max} = 8\ \text{lb}$ then our assumption is correct and $\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 3.86 \\ 3.86 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$

(b) $F = F_b$ First assume no slip

Given $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

$a_A = a_B$ $F_{max} = \mu_s W_A$

$$\begin{pmatrix} F_A \\ F_{max} \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, F_{max}, a_A, a_B)$$

Since $F_A = 30\ \text{lb} > F_{max} = 8\ \text{lb}$ then our assumption is not correct.

Now we know that it slips

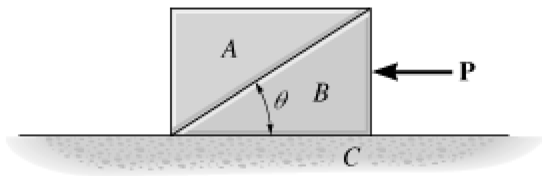
Given $F_A = \mu_k W_A$ $F - F_A = \left(\frac{W_A}{g}\right)a_A$ $F_A = \left(\frac{W_B}{g}\right)a_B$

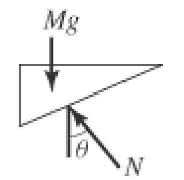
$$\begin{pmatrix} F_A \\ a_A \\ a_B \end{pmatrix} = \text{Find}(F_A, a_A, a_B)$$

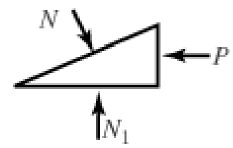
$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} 70.84 \\ 6.44 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 13-42

Blocks A and B each have a mass M. Determine the largest horizontal force P which can be applied to B so that A will not move relative to B. All surfaces are smooth.







Solution:

Require $a_A = a_B = a$

Block A:

$+\uparrow \ \Sigma F_y = 0;$ $\quad N\cos(\theta) - Mg = 0$

$\overset{+}{\leftarrow} \ \Sigma F_x = Ma_x;$ $\quad N\sin(\theta) = Ma$

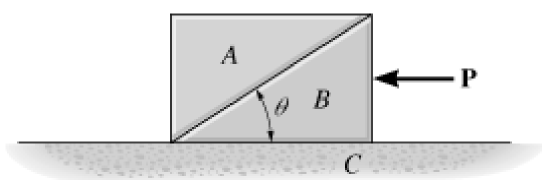
$a = g\tan(\theta)$

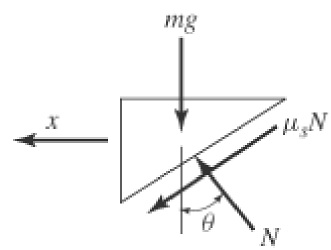
Block B:

$\overset{+}{\leftarrow} \ \Sigma F_x = Ma_x;$ $\quad P - N\sin(\theta) = Ma \qquad P = 2Mg\tan(\theta)$

Problem 13-43

Blocks A and B each have mass m. Determine the largest horizontal force P which can be applied to B so that A will not slip up B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.





Solution:

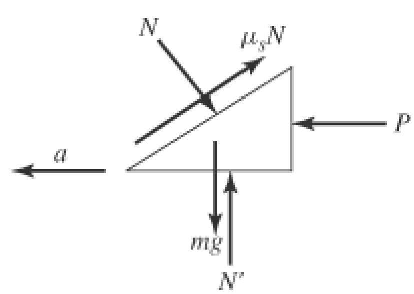
Require

$a_A = a_B = a$

Block A:

$\Sigma F_y = 0;$ $\quad N\cos(\theta) - \mu_s N\sin(\theta) - mg = 0$

$\Sigma F_x = ma_x;$ $\quad N\sin(\theta) + \mu_s N\cos(\theta) = ma$



$$N = \frac{mg}{\cos(\theta) - \mu_s\sin(\theta)}$$

$$a = g\left(\frac{\sin(\theta) + \mu_s\cos(\theta)}{\cos(\theta) - \mu_s\sin(\theta)}\right)$$

Block B:

$\Sigma F_x = ma_x;$ $\quad P - \mu_s N\cos(\theta) - N\sin(\theta) = ma$

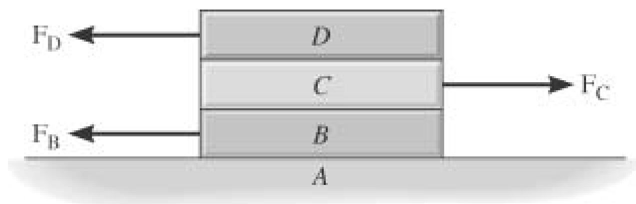
$$P - \frac{\mu_s m g \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} = m g \left(\frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right)$$

$$p = 2 m g \left(\frac{\sin(\theta) + \mu_s \cos(\theta)}{\cos(\theta) - \mu_s \sin(\theta)} \right)$$

***Problem 13-44**

Each of the three plates has mass M. If the coefficients of static and kinetic friction at each surface of contact are μ_s and μ_k respectively, determine the acceleration of each plate when the three horizontal forces are applied.

Given:



$M = 10\ \text{kg}$

$\mu_s = 0.3$

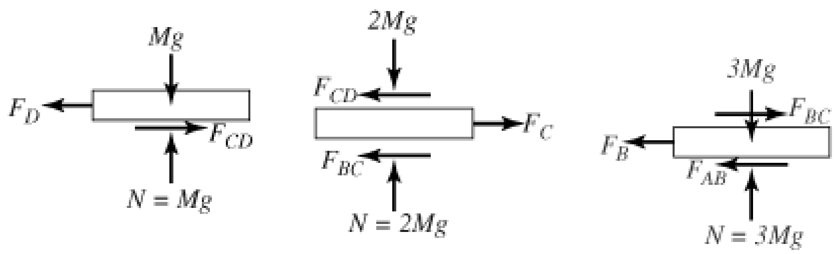
$\mu_k = 0.2$

$F_B = 15\ \text{N}$

$F_C = 100\ \text{N}$

$F_D = 18\ \text{N}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

Case 1: Assume that no slipping occurs anywhere.

$F_{ABmax} = \mu_s(3Mg)$ $\quad F_{BCmax} = \mu_s(2Mg)$ $\quad F_{CDmax} = \mu_s(Mg)$

Guesses $\quad F_{AB} = 1\ \text{N} \quad F_{BC} = 1\ \text{N} \quad F_{CD} = 1\ \text{N}$

Given $\quad -F_D + F_{CD} = 0 \quad F_C - F_{CD} - F_{BC} = 0 \quad -F_B - F_{AB} + F_{BC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{pmatrix} = \text{Find}(F_{AB}, F_{BC}, F_{CD}) \qquad \begin{pmatrix} F_{AB} \\ F_{BC} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 67 \\ 82 \\ 18 \end{pmatrix} \text{N} \qquad \begin{pmatrix} F_{ABmax} \\ F_{BCmax} \\ F_{CDmax} \end{pmatrix} = \begin{pmatrix} 88.29 \\ 58.86 \\ 29.43 \end{pmatrix} \text{N}$$

If $F_{AB} = 67\,\text{N}$ $<$ $F_{ABmax} = 88.29\,\text{N}$ and $F_{BC} = 82\,\text{N}$ $>$ $F_{BCmax} = 58.86\,\text{N}$ and $F_{CD} = 18\,\text{N}$ $<$ $F_{CDmax} = 29.43\,\text{N}$ then nothing moves and there is no acceleration.

Case 2: If $F_{AB} = 67\,\text{N} \quad < \quad F_{ABmax} = 88.29\,\text{N}$ and $F_{BC} = 82\,\text{N} \quad > \quad F_{BCmax} = 58.86\,\text{N}$ and $F_{CD} = 18\,\text{N} \quad < \quad F_{CDmax} = 29.43\,\text{N}$ then slipping occurs between B and C. We will assume that no slipping occurs at the other 2 surfaces.

Set $\quad F_{BC} = \mu_k(2Mg) \qquad a_B = 0 \qquad a_C = a_D = a$

Guesses $\quad F_{AB} = 1\ \text{N} \qquad F_{CD} = 1\ \text{N} \qquad a = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad -F_D + F_{CD} = M\,a \qquad F_C - F_{CD} - F_{BC} = M\,a \qquad -F_B - F_{AB} + F_{BC} = 0$

$$\begin{pmatrix} F_{AB} \\ F_{CD} \\ a \end{pmatrix} = \text{Find}(F_{AB}, F_{CD}, a) \qquad \begin{pmatrix} F_{AB} \\ F_{CD} \end{pmatrix} = \begin{pmatrix} 24.24 \\ 39.38 \end{pmatrix}\text{N} \qquad a = 2.138\ \frac{\text{m}}{\text{s}^2}$$

$$a_C = a \qquad a_D = a$$

If $F_{AB} = 24.24\,\text{N} \quad < \quad F_{ABmax} = 88.29\,\text{N}$ and $F_{CD} = 39.38\,\text{N} \quad > \quad F_{CDmax} = 29.43\,\text{N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 2.138\,\frac{\text{m}}{\text{s}^2}$, $a_D = 2.138\,\frac{\text{m}}{\text{s}^2}$

Case 3: If $F_{AB} = 24.24\,\text{N} \quad < \quad F_{ABmax} = 88.29\,\text{N}$ and $F_{CD} = 39.38\,\text{N} \ > \ F_{CDmax} = 29.43\,\text{N}$ then slipping occurs between C and D as well as between B and C. We will assume that no slipping occurs at the other surface.

Set $\quad F_{BC} = \mu_k(2Mg) \qquad F_{CD} = \mu_k(Mg)$

Guesses $\quad F_{AB} = 1\ \text{N} \qquad a_C = 1\ \frac{\text{m}}{\text{s}^2} \qquad a_D = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad -F_D + F_{CD} = M\,a_D \qquad F_C - F_{CD} - F_{BC} = M\,a_C \qquad -F_B - F_{AB} + F_{BC} = 0$

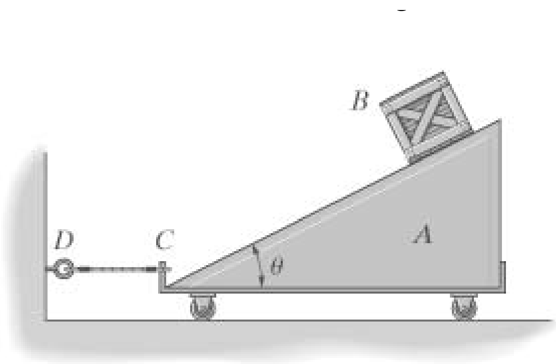
$$\begin{pmatrix} F_{AB} \\ a_C \\ a_D \end{pmatrix} = \text{Find}(F_{AB}, a_C, a_D) \qquad F_{AB} = 24.24\ \text{N} \qquad \begin{pmatrix} a_C \\ a_D \end{pmatrix} = \begin{pmatrix} 4.114 \\ 0.162 \end{pmatrix}\frac{\text{m}}{\text{s}^2}$$

If $F_{AB} = 24.24\,\text{N} \quad < \quad F_{ABmax} = 88.29\,\text{N}$ then we have the correct answer and the accelerations are $a_B = 0$, $a_C = 4.114\,\frac{\text{m}}{\text{s}^2}$, $a_D = 0.162\,\frac{\text{m}}{\text{s}^2}$

There are other permutations of this problems depending on the numbers that one chooses.

Problem 13-45

Crate B has a mass m and is released from rest when it is on top of cart A, which has a mass $3m$. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. Neglect friction.



Solution:

Block B:

$$N_B - m\,g\cos(\theta) = 0$$

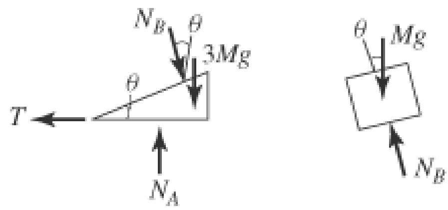
$$N_B = m\,g\cos(\theta)$$

Cart:

$$-T + N_B\sin(\theta) = 0$$

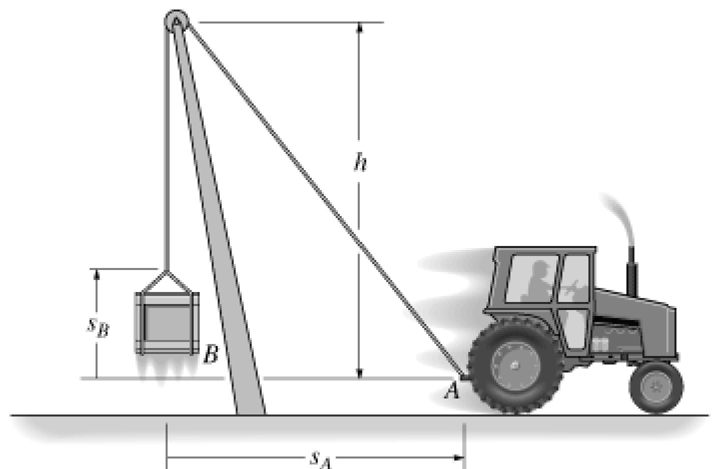
$$T = m\,g\sin(\theta)\cos(\theta)$$

$$T = \left(\frac{mg}{2}\right)\sin(2\theta)$$



Problem 13-46

The tractor is used to lift load B of mass M with the rope of length $2h$, and the boom, and pulley system. If the tractor is traveling to the right at constant speed v, determine the tension in the rope when $s_A = d$. When $s_A = 0$, $s_B = 0$



Units used:

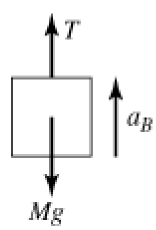
$$\text{kN} = 10^3\ \text{N}$$

Given:

$$M = 150\ \text{kg}$$

$$v = 4\ \frac{\text{m}}{\text{s}} \qquad h = 12\ \text{m}$$

$$d = 5\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$



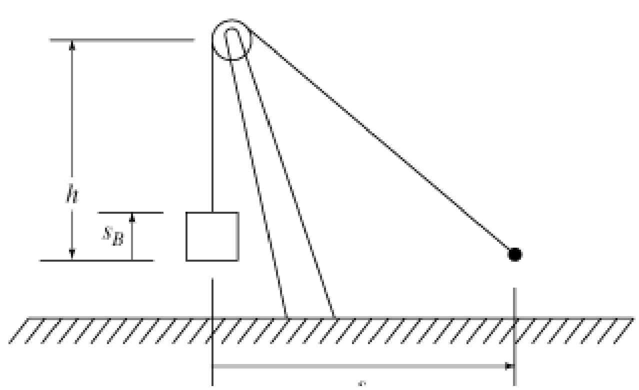
Solution: $\quad v_A = v \qquad s_A = d$

Guesses $\quad T = 1\ \text{kN} \qquad s_B = 1\ \text{m}$

$$a_B = 1\ \frac{\text{m}}{\text{s}^2} \qquad v_B = 1\ \frac{\text{m}}{\text{s}}$$

Given $\quad h - s_B + \sqrt{s_A^{\,2} + h^2} = 2h$

$$-v_B + \frac{s_A v_A}{\sqrt{s_A^{\,2} + h^2}} = 0$$



$$-a_B + \frac{v_A^2}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} = 0 \qquad T - Mg = M a_B$$

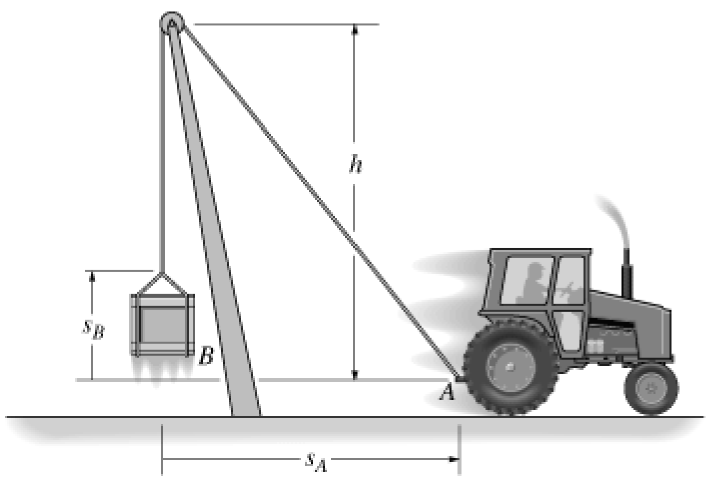
$$\begin{pmatrix} T \\ s_B \\ v_B \\ a_B \end{pmatrix} = \text{Find}(T, s_B, v_B, a_B) \qquad s_B = 1 \text{ m} \qquad a_B = 1.049 \frac{\text{m}}{\text{s}^2} \qquad T = 1.629\,\text{kN}$$

$$v_B = 1.538 \frac{\text{m}}{\text{s}}$$

Problem 13-47

The tractor is used to lift load *B* of mass *M* with the rope of length *2h*, and the boom, and pulley system. If the tractor is traveling to the right with an acceleration *a* and has speed *v* at the instant $s_A = d$, determine the tension in the rope. When $s_A = 0$, $s_B = 0$.

Units used: $\text{kN} = 10^3 \text{ N}$



Given:

$d = 5 \text{ m}$ $\qquad h = 12 \text{ m}$

$M = 150 \text{ kg}$ $\qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$v = 4 \frac{\text{m}}{\text{s}}$

$a = 3 \frac{\text{m}}{\text{s}^2}$

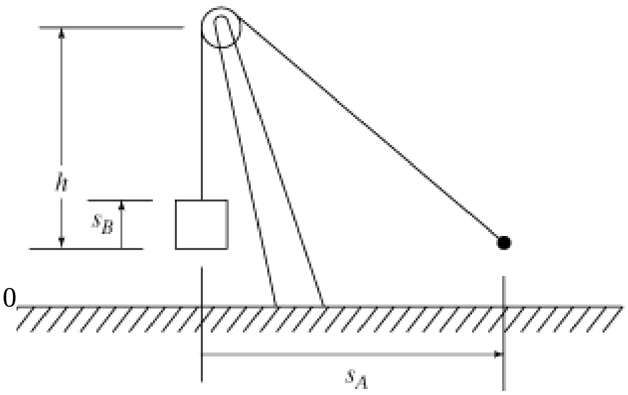
Solution: $a_A = a \qquad v_A = v \qquad s_A = d$

Guesses $\quad T = 1 \text{ kN} \qquad s_B = 1 \text{ m} \qquad a_B = 1 \frac{\text{m}}{\text{s}^2} \qquad v_B = 1 \frac{\text{m}}{\text{s}}$

Given $h - s_B + \sqrt{s_A^2 + h^2} = 2h$

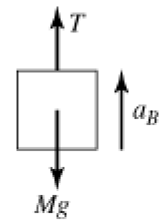
$$-v_B + \frac{s_A v_A}{\sqrt{s_A^2 + h^2}} = 0$$

$$-a_B + \frac{v_A^2 + s_A a_A}{\sqrt{s_A^2 + h^2}} - \frac{s_A^2 v_A^2}{\left(s_A^2 + h^2\right)^{\frac{3}{2}}} = 0$$



$$T - Mg = Ma_B$$

$$\begin{pmatrix} T \\ s_B \\ v_B \\ a_B \end{pmatrix} = \mathrm{Find}(T, s_B, v_B, a_B) \qquad s_B = 1 \text{ m}$$



$$a_B = 2.203 \frac{\text{m}}{\text{s}^2} \qquad v_B = 1.538 \frac{\text{m}}{\text{s}} \qquad T = 1.802 \text{ kN}$$

***Problem 13-48**

Block B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force **F** as a function of the block's vertical position y so that when **F** is applied the block rises with a constant acceleration a_B. Neglect the mass of the cord and pulleys.

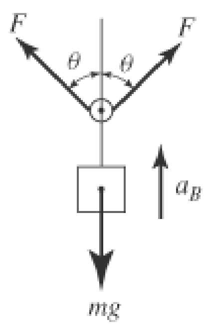
Solution:

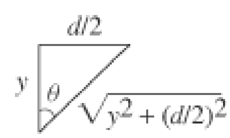
$$2F\cos(\theta) - mg = ma_B$$

where $\cos(\theta) = \dfrac{y}{\sqrt{y^2 + \left(\dfrac{d}{2}\right)^2}}$

$$2F\left[\frac{y}{\sqrt{y^2 + \left(\dfrac{d}{2}\right)^2}}\right] - mg = ma_B$$

$$F = m\frac{(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$



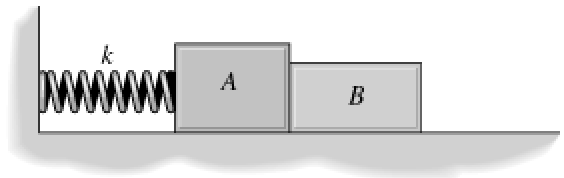


Problem 13-49

Block A has mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having mass m_B is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?

Solution:

Block A: $\quad -k(x-d) - N = m_A a_A$

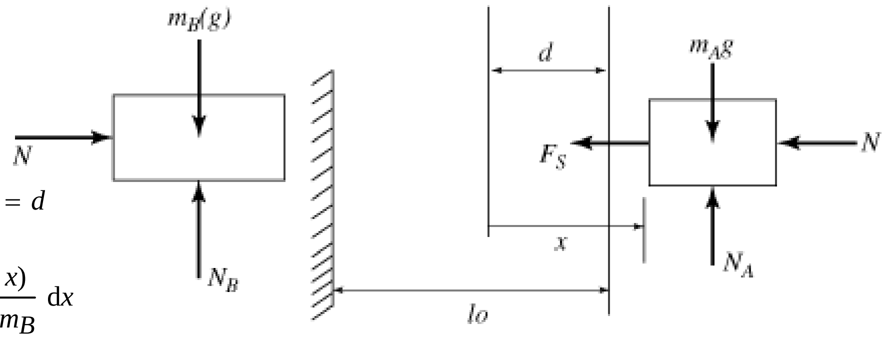


Block B: $\quad N = m_B a_B$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x)}{m_A + m_B}$$

$$N = \frac{k m_B(d-x)}{m_A + m_B}$$



Separation occurs when

$N = 0 \quad$ or $\quad x = d$

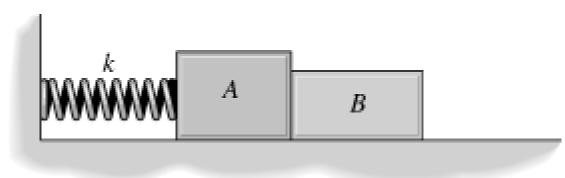
$$\int_0^v v\, dv = \int_0^d \frac{k(d-x)}{m_A + m_B}\, dx$$

$$\frac{v^2}{2} = \frac{k}{m_A + m_B}\left(d\,d - \frac{d^2}{2}\right) \qquad \boxed{v = \sqrt{\frac{kd^2}{m_A + m_B}}}$$

Problem 13-50

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A+m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?

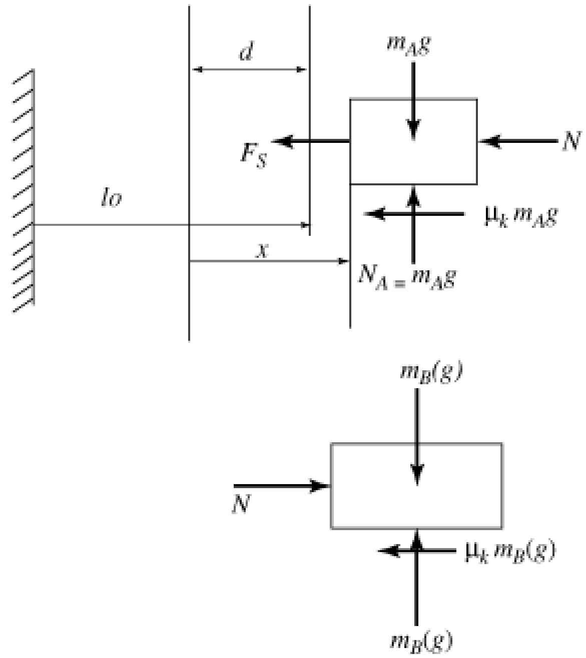
Solution: Block A:



$$-k(x-d) - N - \mu_k m_A g = m_A a_A$$

Block B: $\quad N - \mu_k m_B g = m_B a_B$

Since $a_A = a_B = a$



$$a = \frac{k(d-x)}{m_A + m_B} - \mu_k g$$

$$N = \frac{k m_B (d-x)}{m_A + m_B}$$

$N = 0$, then $x = d$ for separation.

At the moment of separation:

$$\int_0^v v\,\mathrm{d}v = \int_0^d \left[\frac{k(d-x)}{m_A + m_B}dx - \mu_k g\right]\mathrm{d}x$$

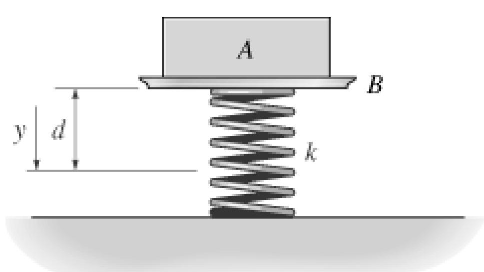
$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{m_A + m_B}}$$

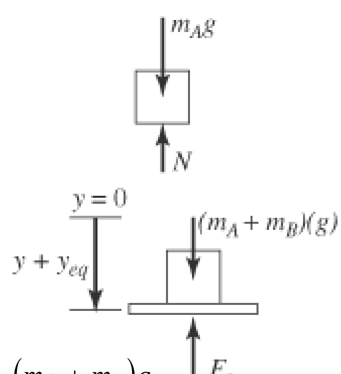
Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0 \qquad d > \frac{2\mu_k g}{k}(m_A + m_B) \qquad \text{Q.E.D}$$

Problem 13-51

The block A has mass m_A and rests on the pan B, which has mass m_B Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.





Solution:

For Equilibrium $\quad k y_{eq} - (m_A + m_B)g = 0 \qquad y_{eq} = \dfrac{(m_A + m_B)g}{k}$

Block: $\quad -m_A g + N = m_A a$

Block and Pan $\left(-m_A + m_B\right)g + k\left(y_{eq} + y\right) = \left(m_A + m_B\right)a$

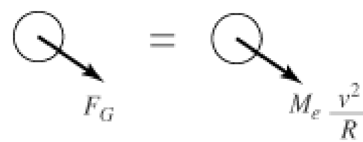
Thus, $-\left(m_A + m_B\right)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = \left(m_A + m_B\right)\left(\frac{-m_A g + N}{m_A}\right)$

Set $y = -d$, $N = 0$ Thus $d = y_{eq} = \frac{\left(m_A + m_B\right)g}{k}$

***Problem 13-52**

Determine the mass of the sun, knowing that the distance from the earth to the sun is R. *Hint:* Use Eq. 13-1 to represent the force of gravity acting on the earth.

Given: $R = 149.6 \times 10^6\ \text{km}$ $G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$

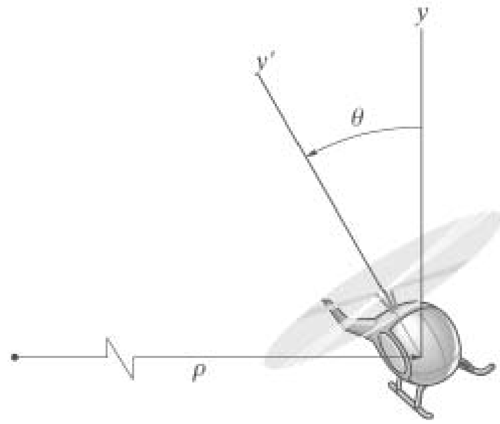


Solution: $v = \frac{s}{t}$ $v = \frac{2\pi R}{1\ \text{yr}}$ $v = 2.98 \times 10^4\ \frac{\text{m}}{\text{s}}$

$\Sigma\ F_n = ma_n;$ $G\left(\frac{M_e M_s}{R^2}\right) = M_e\left(\frac{v^2}{R}\right)$ $M_s = v^2\left(\frac{R}{G}\right)$ $M_s = 1.99 \times 10^{30}\ \text{kg}$

Problem 13-53

The helicopter of mass M is traveling at a constant speed v along the horizontal curved path while banking at angle θ. Determine the force acting normal to the blade, i.e., in the y' direction, and the radius of curvature of the path.



Units Used:

$$\mathrm{kN} = 10^3\ \mathrm{N}$$

Given:

$$v = 40\ \frac{\mathrm{m}}{\mathrm{s}} \qquad M = 1.4 \times 10^3\ \mathrm{kg}$$

$$\theta = 30\ \mathrm{deg} \qquad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Solution:

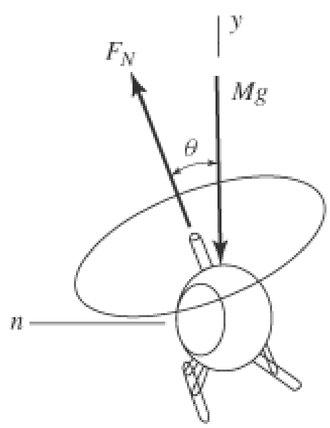
Guesses $\quad F_N = 1\ \mathrm{kN} \qquad \rho = 1\ \mathrm{m}$

Given $\quad F_N\cos(\theta) - M g = 0$

$$F_N \sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

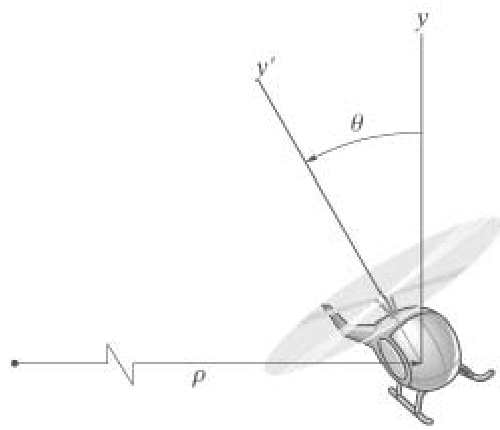
$$\begin{pmatrix} F_N \\ \rho \end{pmatrix} = \mathrm{Find}(F_N, \rho) \qquad F_N = 15.86\,\mathrm{kN}$$

$$\rho = 282\ \mathrm{m}$$



Problem 13-54

The helicopter of mass M is traveling at a constant speed v along the horizontal curved path having a radius of curvature ρ. Determine the force the blade exerts on the frame and the bank angle θ.



Units Used:

$$\mathrm{kN} = 10^3\ \mathrm{N}$$

Given:

$$v = 33\ \frac{\mathrm{m}}{\mathrm{s}} \qquad M = 1.4 \times 10^3\ \mathrm{kg}$$

$$\rho = 300\ \mathrm{m} \qquad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Solution:

Guesses $F_N = 1\ \text{kN}$ $\theta = 1\ \text{deg}$

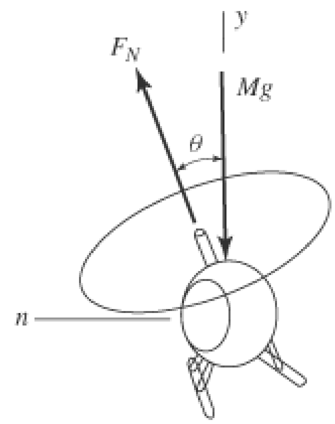
Given $F_N \cos(\theta) - M g = 0$

$$F_N \sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta)$$

$F_N = 14.64\,\text{kN}$

$\theta = 20\,\text{deg}$



Problem 13-55

The plane is traveling at a constant speed v along the curve $y = bx^2 + c$. If the pilot has weight W, determine the normal and tangential components of the force the seat exerts on the pilot when the plane is at its lowest point.

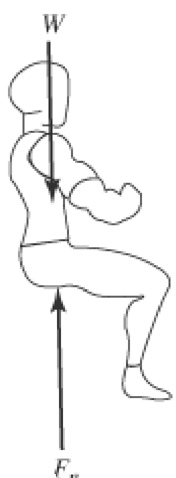
Given:

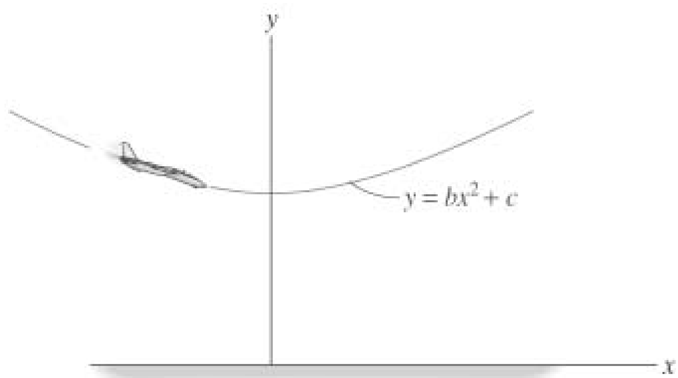
$$b = 20 \times 10^{-6}\ \frac{1}{\text{ft}}$$

$$c = 5000\ \text{ft}$$

$$W = 180\ \text{lb}$$

$$v = 800\ \frac{\text{ft}}{\text{s}}$$





Solution:

$x = 0\ \text{ft}$ $y = b x^2 + c$

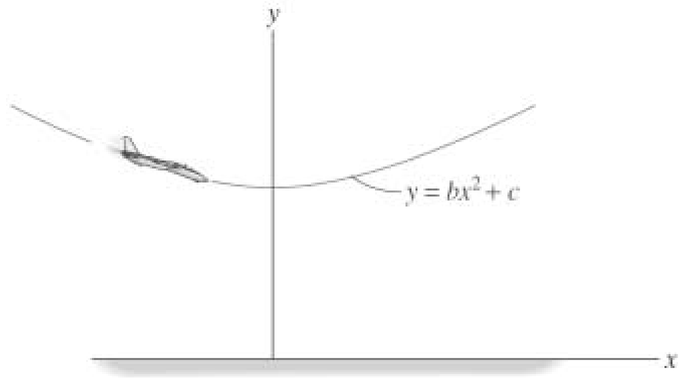
$y' = 2b x$ $y'' = 2b$

$$\rho = \frac{\sqrt{\left(1 + y'^2\right)^3}}{y''}$$

$$F_n - W = \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad F_n = W + \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad F_n = 323\,\text{lb}$$

$$a_t = 0 \qquad F_t = \left(\frac{W}{g}\right) a_t \qquad F_t = 0$$

***Problem 13-56**



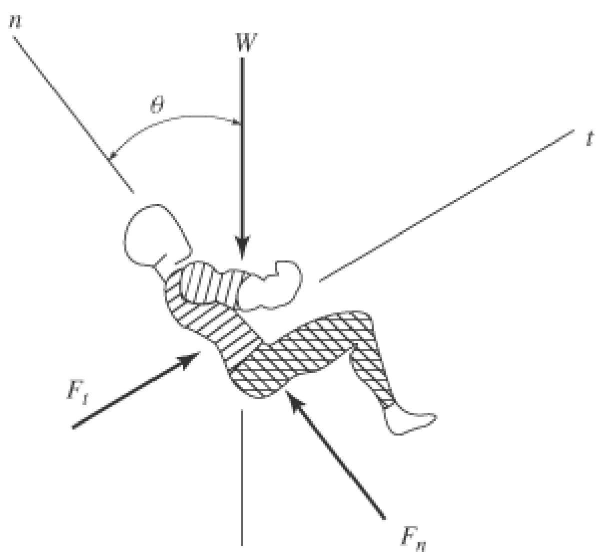
The jet plane is traveling at a constant speed of v along the curve $y = bx^2 + c$. If the pilot has a weight W, determine the normal and tangential components of the force the seat exerts on the pilot when $y = y_1$.

Given:

$$b = 20 \times 10^{-6} \frac{1}{\text{ft}} \qquad W = 180 \text{ lb}$$

$$c = 5000 \text{ ft} \qquad v = 1000 \frac{\text{ft}}{\text{s}}$$

$$g = 32.2 \frac{\text{ft}}{\text{s}^2} \qquad y_1 = 10000 \text{ ft}$$



Solution:

$$y(x) = bx^2 + c$$

$$y'(x) = 2bx$$

$$y''(x) = 2b$$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

Guesses $\quad x_1 = 1 \text{ ft} \quad F_n = 1 \text{ lb}$

$\theta = 1 \text{ deg} \quad F_t = 1 \text{ lb}$

Given $\quad y_1 = y(x_1) \qquad \tan(\theta) = y'(x_1)$

$$F_n - W\cos(\theta) = \frac{W}{g}\left(\frac{v^2}{\rho(x_1)}\right) \qquad F_t - W\sin(\theta) = 0$$

$$\begin{pmatrix} x_1 \\ \theta \\ F_n \\ F_t \end{pmatrix} = \text{Find}(x_1, \theta, F_n, F_t) \qquad x_1 = 15811 \text{ ft} \qquad \theta = 32.3 \text{ deg} \qquad \begin{pmatrix} F_n \\ F_t \end{pmatrix} = \begin{pmatrix} 287.1 \\ 96.2 \end{pmatrix} \text{lb}$$

Problem 13-57

The wrecking ball of mass M is suspended from the crane by a cable having a negligible mass. If the ball has speed v at the instant it is at its lowest point θ, determine the tension in the cable at this instant. Also, determine the angle θ to which the ball swings before it stops.

Units Used:

$$kN = 10^3\ N$$

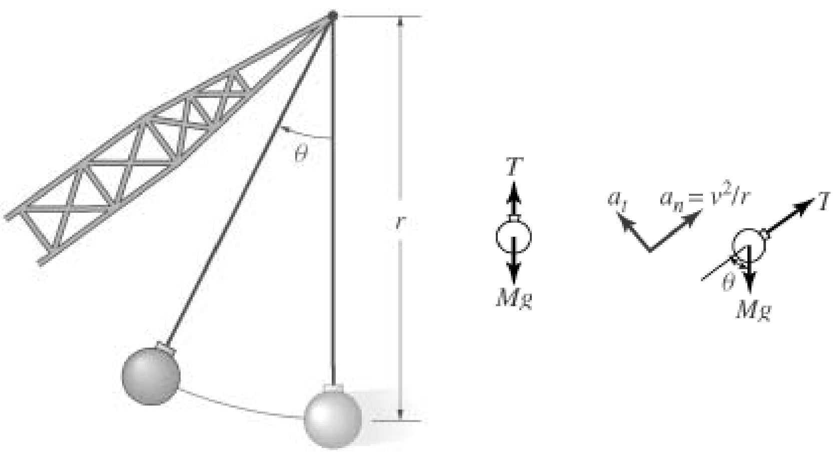
Given:

$$M = 600\ kg$$

$$v = 8\ \frac{m}{s}$$

$$r = 12\ m$$

$$g = 9.81\ \frac{m}{s^2}$$



Solution:

At the lowest point

$$T - Mg = M\left(\frac{v^2}{r}\right) \qquad T = Mg + M\left(\frac{v^2}{r}\right) \qquad T = 9.086\ \text{kN}$$

At some arbitrary angle

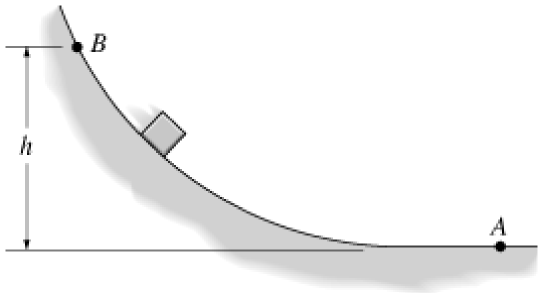
$$-Mg\sin(\theta) = Ma_t \qquad a_t = -g\sin(\theta) = \frac{v}{r}\left(\frac{dv}{d\theta}\right)$$

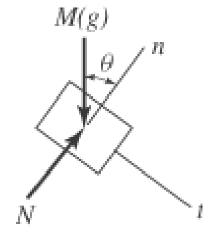
$$\int_v^0 v\ dv = -\int_0^\theta rg\sin(\theta)\ d\theta$$

$$\frac{-v^2}{2} = rg(\cos(\theta) - 1) \qquad \theta = \operatorname{acos}\left(1 - \frac{v^2}{2rg}\right) \qquad \theta = 43.3\ \text{deg}$$

Problem 13-58

Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.





Solution:

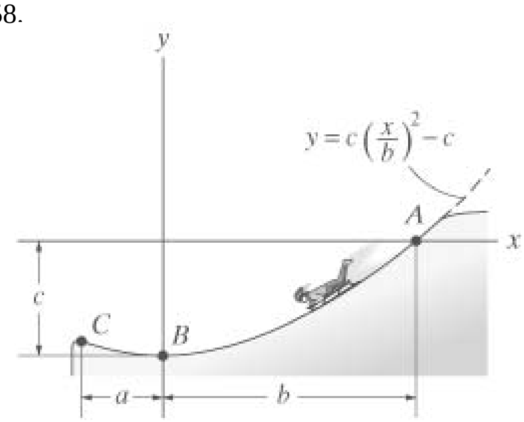
$\searrow^{+}\ \Sigma F_t = ma_t;$ $\quad (mg)\sin(\theta) = m\,a_t \qquad a_t = g\sin(\theta)$

$v\,dv = a_t ds = g\sin(\theta)\,ds$ $\qquad$ However $dy = ds\,\sin(\theta)$ $\qquad dy = ds\,\sin(\theta)$

$\int_0^v v\,dv = \int_0^h g\,dy \qquad \frac{v^2}{2} = gh \qquad v = \sqrt{2gh}$ $\qquad$ Q.E.D

Problem 13-59

The sled and rider have a total mass M and start from rest at $A(b, 0)$. If the sled descends the smooth slope, which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point B. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.



Units Used:

$\mathrm{kN} = 10^3\ \mathrm{N}$

Given:

$a = 2\ \mathrm{m} \qquad b = 10\ \mathrm{m} \qquad c = 5\ \mathrm{m}$

$M = 80\ \mathrm{kg} \qquad g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$

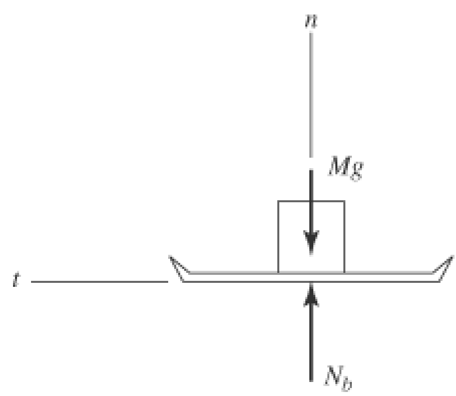
Solution:

$v = \sqrt{2gc}$

$y(x) = c\left(\frac{x}{b}\right)^2 - c$

$y'(x) = \left(\frac{2c}{b^2}\right)x \qquad y''(x) = \frac{2c}{b^2}$

$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$



$$N_b - M g = M\left(\frac{v^2}{\rho}\right)$$

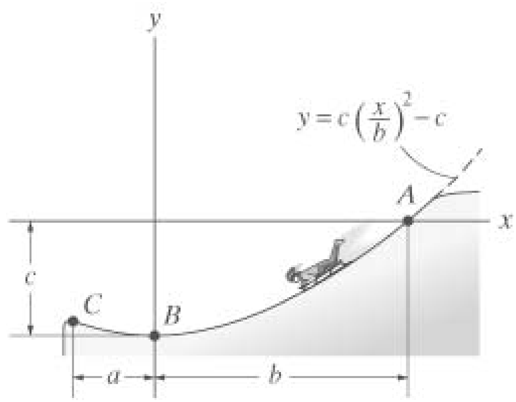
$$N_b = M g + M\left(\frac{v^2}{\rho(0\ \text{m})}\right) \qquad \boxed{N_b = 1.57\,\text{kN}}$$

***Problem 13-60**

The sled and rider have a total mass M and start from rest at $A(b, 0)$. If the sled descends the smooth slope which may be approximated by a parabola, determine the normal force that the ground exerts on the sled at the instant it arrives at point C. Neglect the size of the sled and rider. *Hint:* Use the result of Prob. 13–58.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$



Given:

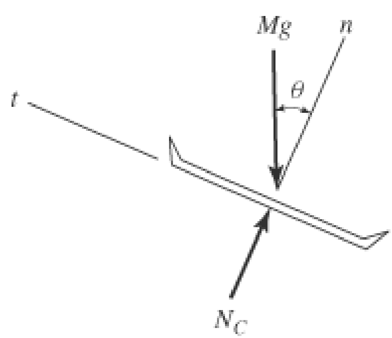
$a = 2\ \text{m} \qquad b = 10\ \text{m} \qquad c = 5\ \text{m}$

$M = 80\ \text{kg} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$y(x) = c\left(\frac{x}{b}\right)^2 - c$$

$$y'(x) = \left(\frac{2c}{b^2}\right)x \qquad y''(x) = \frac{2c}{b^2}$$



$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$v = \sqrt{2g(-y(-a))}$$

$$\theta = \text{atan}(y'(-a))$$

$$N_C - M g\cos(\theta) = M\left(\frac{v^2}{\rho}\right) \qquad N_C = M\left(g\cos(\theta) + \frac{v^2}{\rho(-a)}\right) \qquad \boxed{N_C = 1.48\,\text{kN}}$$

Problem 13-61

At the instant $\theta = \theta_1$ the boy's center of mass G has a downward speed v_G. Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant.The boy has a weight W. Neglect his size and the mass of the seat and cords.

Given:

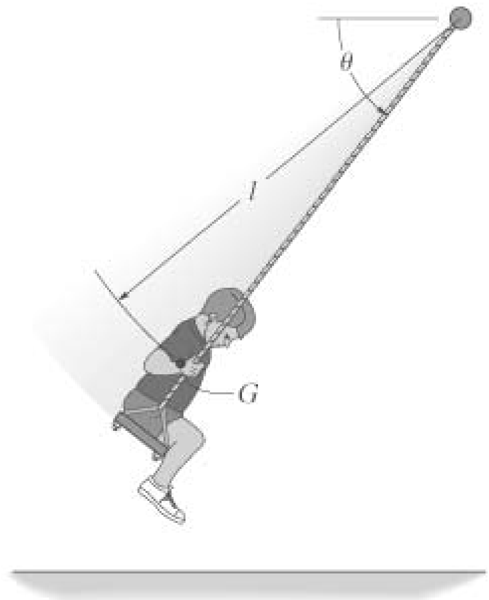
$W = 60\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$l = 10\ \text{ft}$

$v_G = 15\ \frac{\text{ft}}{\text{s}}$

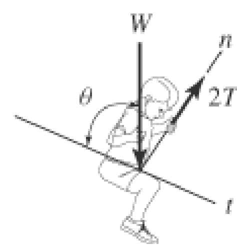
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$W\cos(\theta_1) = \left(\frac{W}{g}\right)a_t$$

$$a_t = g\cos(\theta_1) \qquad a_t = 16.1\ \frac{\text{ft}}{\text{s}^2}$$

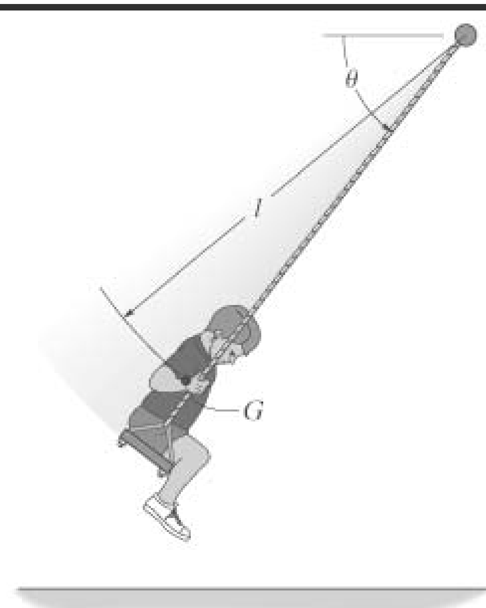


$$2T - W\sin(\theta_1) = \frac{W}{g}\left(\frac{v^2}{l}\right)$$

$$T = \frac{1}{2}\left[\frac{W}{g}\left(\frac{v_G^{\,2}}{l}\right) + W\sin(\theta_1)\right] \qquad T = 46.9\ \text{lb}$$

Problem 13-62

At the instant $\theta = \theta_1$ the boy's center of mass G is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = \theta_2$. The boy has a weight W. Neglect his size and the mass of the seat and cords.



Given:

$W = 60\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta_1 = 60\ \text{deg}$

$\theta_2 = 90\ \text{deg}$

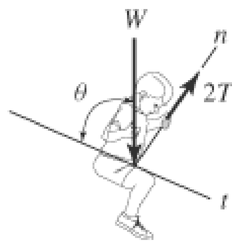
$l = 10\ \text{ft}$

Solution:

$$W\cos(\theta) = \left(\frac{W}{g}\right)a_t \qquad a_t = g\cos(\theta)$$

$$v_2 = \sqrt{2gl\int_{\theta_1}^{\theta_2}\cos(\theta)\,d\theta}$$

$$v_2 = 9.29\,\frac{\text{ft}}{\text{s}}$$



$$2T - W\sin(\theta_2) = \frac{W}{g}\left(\frac{v_2^{\,2}}{l}\right)$$

$$T = \frac{W}{2}\left(\sin(\theta_2) + \frac{v_2^{\,2}}{gl}\right) \qquad T = 38.0\,\text{lb}$$

Problem 13-63

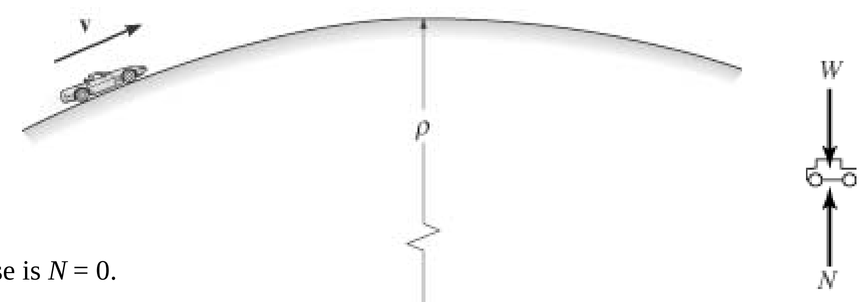
If the crest of the hill has a radius of curvature ρ, determine the maximum constant speed at which the car can travel over it without leaving the surface of the road. Neglect the size of the car in the calculation. The car has weight W.

Given:

$\rho = 200\ \text{ft}$

$W = 3500\ \text{lb}$

$g = 9.815\ \dfrac{\text{m}}{\text{s}^2}$



Solution: Limiting case is $N = 0$.

$$\downarrow \Sigma F_n = ma_n; \qquad W = \frac{W}{g}\left(\frac{v^2}{\rho}\right) \qquad v = \sqrt{g\rho} \qquad v = 80.25\,\frac{\text{ft}}{\text{s}}$$

***Problem 13-64**

The airplane, traveling at constant speed v is executing a horizontal turn. If the plane is banked at angle θ when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has mass M?

Units Used:

$\text{kN} = 10^3\ \text{N}$

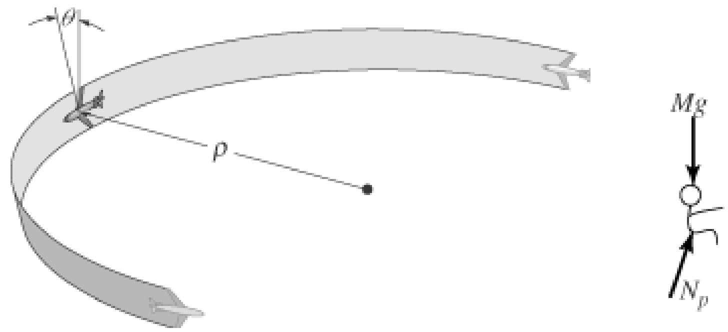
Given:

$v = 50 \frac{m}{s}$

$\theta = 15 \text{ deg}$

$M = 70 \text{ kg}$

$g = 9.815 \frac{m}{s^2}$



Solution:

$+\uparrow \Sigma F_b = ma_b;$ $\quad N_p \sin(\theta) - Mg = 0 \qquad N_p = M\left(\frac{g}{\sin(\theta)}\right) \qquad N_p = 2.654 \text{ kN}$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad N_P \cos(\theta) = M\left(\frac{v^2}{\rho}\right) \qquad \rho = M\left(\frac{v^2}{N_p \cos(\theta)}\right) \qquad \rho = 68.3 \text{ m}$

Problem 13-65

The man has weight W and lies against the cushion for which the coefficient of static friction is μ_s. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the z axis, he has constant speed v. Neglect the size of the man.

Given:

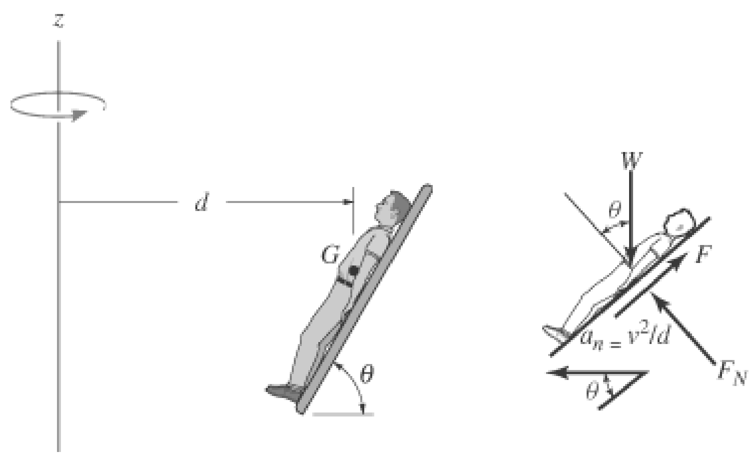
$W = 150 \text{ lb}$

$\mu_s = 0.5$

$v = 20 \frac{ft}{s}$

$\theta = 60 \text{ deg}$

$d = 8 \text{ ft}$



Solution: Assume no slipping occurs Guesses $F_N = 1 \text{ lb}$ $\quad F = 1 \text{ lb}$

Given $\quad -F_N \sin(\theta) + F\cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{d}\right) \qquad F_N \cos(\theta) - W + F\sin(\theta) = 0$

$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F) \qquad \begin{pmatrix} F_N \\ F \end{pmatrix} = \begin{pmatrix} 276.714 \\ 13.444 \end{pmatrix} \text{lb} \qquad F_{max} = \mu_s F_N \qquad F_{max} = 138.357 \text{ lb}$

Since $F = 13.444 \text{ lb} < F_{max} = 138.357 \text{ lb}$ then our assumption is correct and there is no slipping.

Problem 13-66

The man has weight W and lies against the cushion for which the coefficient of static friction is μ_s. If he rotates about the z axis with a constant speed v, determine the smallest angle θ of the cushion at which he will begin to slip off.

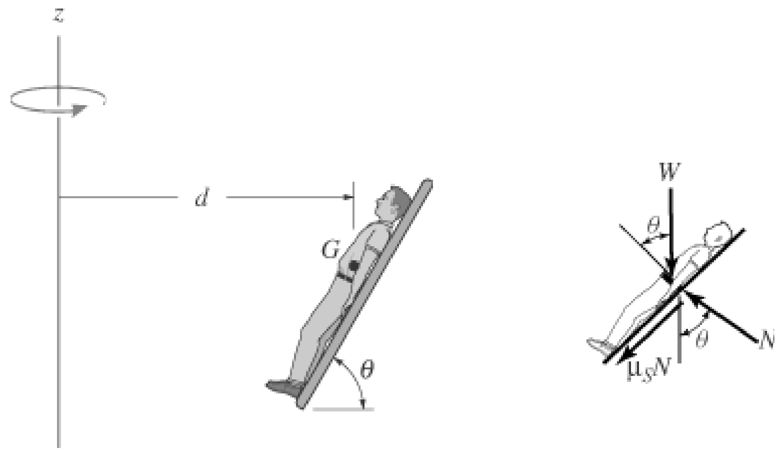
Given:

$W = 150 \text{ lb}$

$\mu_s = 0.5$

$v = 30 \frac{\text{ft}}{\text{s}}$

$d = 8 \text{ ft}$



Solution: Assume verge of slipping Guesses $F_N = 1 \text{ lb}$ $\theta = 20 \text{ deg}$

Given $-F_N \sin(\theta) - \mu_s F_N \cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{d}\right)$ $F_N \cos(\theta) - W - \mu_s F_N \sin(\theta) = 0$

$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta)$ $F_N = 487.563 \text{ lb}$ $\theta = 47.463 \text{ deg}$

Problem 13-67

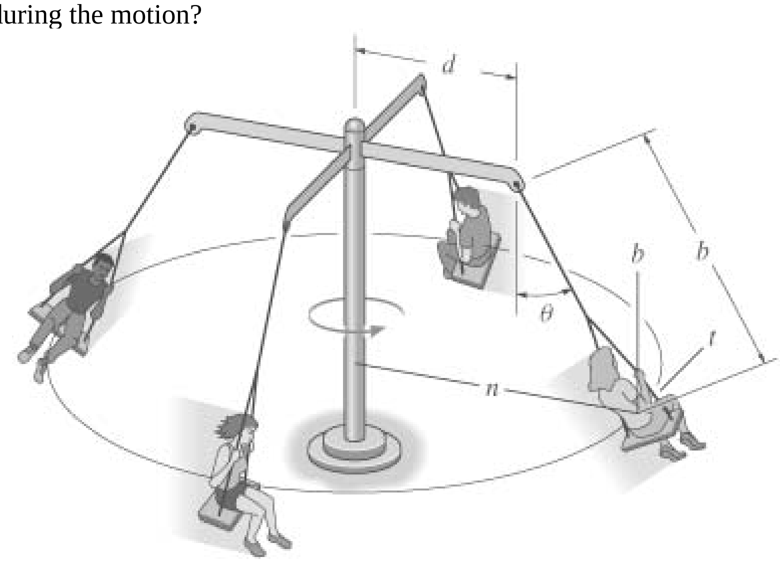
Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at angle q from the vertical. Each chair including its passenger has a mass m_c. Also, what are the components of force in the n, t, and b directions which the chair exerts on a passenger of mass m_p during the motion?

Given:

$\theta = 30 \text{ deg}$ $d = 4 \text{ m}$

$m_c = 80 \text{ kg}$ $b = 6 \text{ m}$

$m_p = 50\text{kg}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution:

The initial guesses:

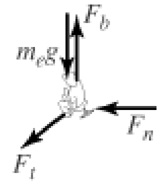
$T = 100 \text{ N}$ $v = 10 \frac{\text{m}}{\text{s}}$

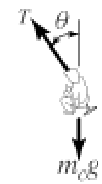
Given

$$T\sin(\theta) = m_c\left(\frac{v^2}{d + b\sin(\theta)}\right)$$

$$T\cos(\theta) - m_c g = 0$$

$$\begin{pmatrix} T \\ v \end{pmatrix} = \text{Find}(T, v) \qquad T = 906.209\ \text{N} \qquad \boxed{v = 6.30\ \frac{\text{m}}{\text{s}}}$$





$\Sigma F_n = m a_n;\qquad F_n = \dfrac{m_p v^2}{d + b\sin(\theta)}\qquad \boxed{F_n = 283\ \text{N}}$

$\Sigma F_t = m a_t;\qquad F_t = 0\ \text{N}\qquad \boxed{F_t = 0}$

$\Sigma F_b = m a_b;\qquad F_b - m_p g = 0 \quad F_b = m_p\, g\qquad \boxed{F_b = 491\ \text{N}}$

***Problem 13-68**

The snowmobile of mass M with passenger is traveling down the hill at a constant speed v. Determine the resultant normal force and the resultant frictional force exerted on the tracks at the instant it reaches point A. Neglect the size of the snowmobile.

Units Used:

$$\text{kN} = 10^3\text{N}$$

Given:

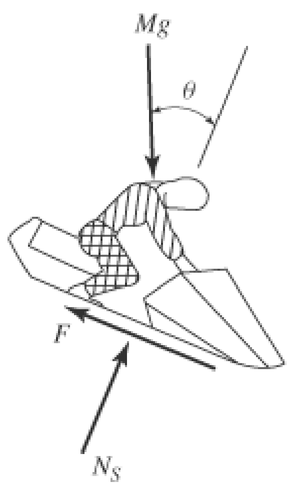
$$M = 200\ \text{kg}$$

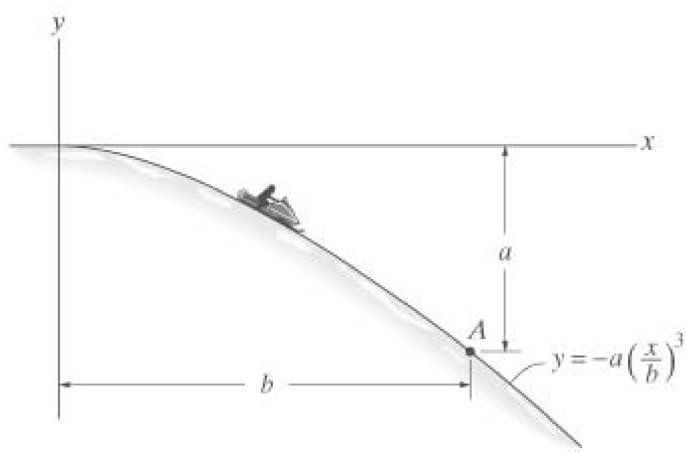
$$v = 6\ \frac{\text{m}}{\text{s}}$$

$$a = 5\ \text{m}$$

$$b = 10\ \text{m}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$





Solution:

$$y(x) = -a\left(\frac{x}{b}\right)^3 \qquad y'(x) = -3\left(\frac{a}{b^3}\right)x^2$$

$$y''(x) = -6\left(\frac{a}{b^3}\right)x \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$\theta = \operatorname{atan}(y'(b))$

Guesses $N_S = 1\ \text{N}$ $F = 1\ \text{N}$

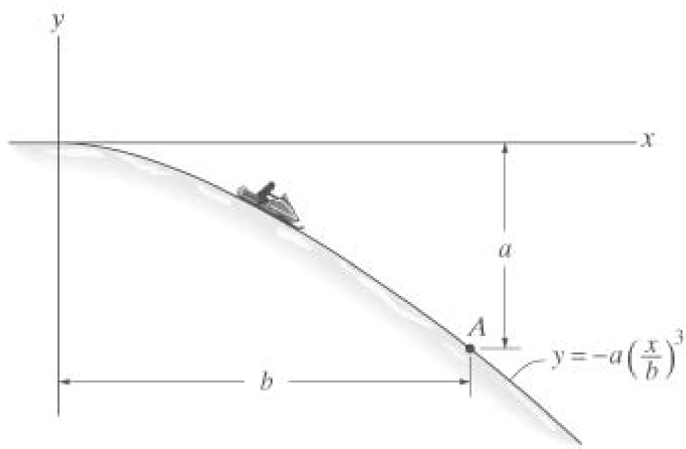
Given $N_S - M g \cos(\theta) = M\left(\dfrac{v^2}{\rho(b)}\right)$

$F - M g \sin(\theta) = 0$

$\begin{pmatrix} N_S \\ F \end{pmatrix} = \text{Find}(N_S, F)$ $\begin{pmatrix} N_S \\ F \end{pmatrix} = \begin{pmatrix} 0.72 \\ -1.632 \end{pmatrix} \text{kN}$

Problem 13-69

The snowmobile of mass M with passenger is traveling down the hill such that when it is at point A, it is traveling at speed v and increasing its speed at v'. Determine the resultant normal force and the resultant frictional force exerted on the tracks at this instant. Neglect the size of the snowmobile.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 200\ \text{kg}$ $a = 5\ \text{m}$

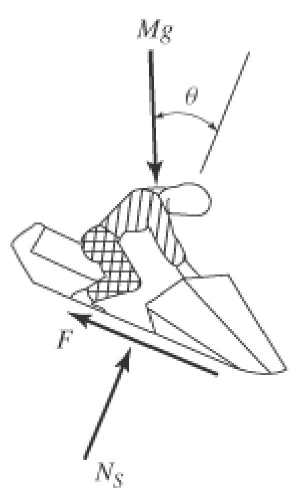
$v = 4\ \dfrac{\text{m}}{\text{s}}$ $b = 10\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $v' = 2\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$y(x) = -a\left(\dfrac{x}{b}\right)^3$ $y'(x) = -3\left(\dfrac{a}{b^3}\right)x^2$

$y''(x) = -6\left(\dfrac{a}{b^3}\right)x$ $\rho(x) = \dfrac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$



$\theta = \operatorname{atan}(y'(b))$

Guesses $N_S = 1\ \text{N}$ $F = 1\ \text{N}$

Given $N_S - M g \cos(\theta) = M\dfrac{v^2}{\rho(b)}$

$$F - M g \sin(\theta) = M v'$$

$$\begin{pmatrix} N_S \\ F \end{pmatrix} = \text{Find}(N_S, F) \qquad \begin{pmatrix} N_S \\ F \end{pmatrix} = \begin{pmatrix} 0.924 \\ -1.232 \end{pmatrix} \text{kN}$$

Problem 13-70

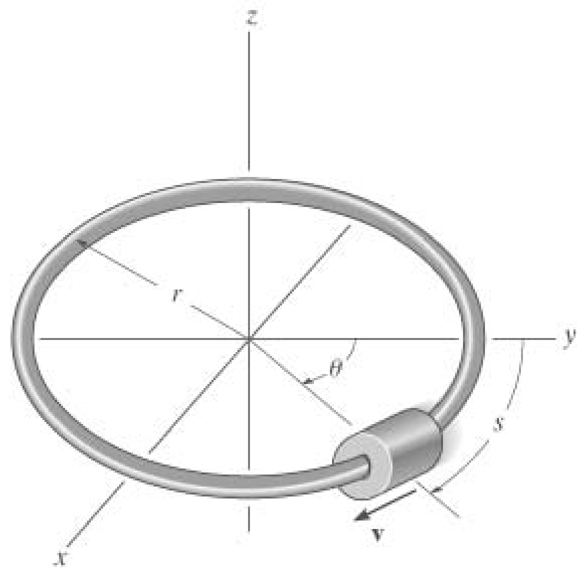
A collar having a mass M and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is μ_k. If the collar is given a speed v_1 and then released at $\theta = 0$ deg, determine how far, d, it slides on the rod before coming to rest.

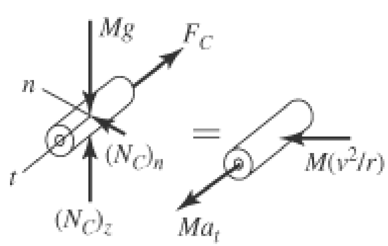
Given:

$M = 0.75$ kg $\quad r = 100$ mm

$\mu_k = 0.3 \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$v_1 = 4 \frac{\text{m}}{\text{s}}$





Solution:

$$N_{Cz} - M g = 0$$

$$N_{Cn} = M\left(\frac{v^2}{r}\right)$$

$$N_C = \sqrt{N_{Cz}^2 + N_{Cn}^2}$$

$$F_C = \mu_k N_C = -M a_t$$

$$a_t(v) = -\mu_k \sqrt{g^2 + \frac{v^4}{r^2}}$$

$$d = \int_{v_1}^{0} \frac{v}{a_t(v)}\, dv \qquad d = 0.581 \text{ m}$$

Problem 13-71

The roller coaster car and passenger have a total weight W and starting from rest at A travel down the track that has the shape shown. Determine the normal force of the tracks on the car when the car is at point B, it has a velocity of v. Neglect friction and the size of the car and passenger.

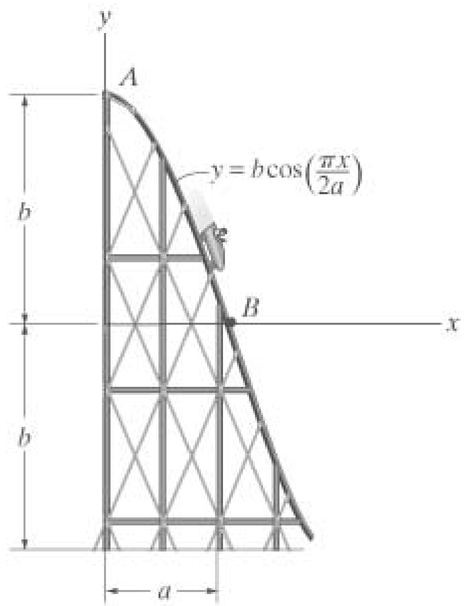
Given:

$W = 600$ lb

$$v = 15\ \frac{\text{ft}}{\text{s}}$$

$$a = 20\ \text{ft}$$

$$b = 40\ \text{ft}$$



Solution:

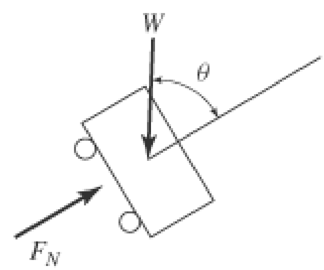
$$y(x) = b\cos\left(\frac{\pi x}{2a}\right) \qquad y'(x) = \frac{d}{dx}y(x)$$

$$y''(x) = \frac{d}{dx}y'(x) \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

At B $\qquad \theta = \operatorname{atan}(y'(a))$

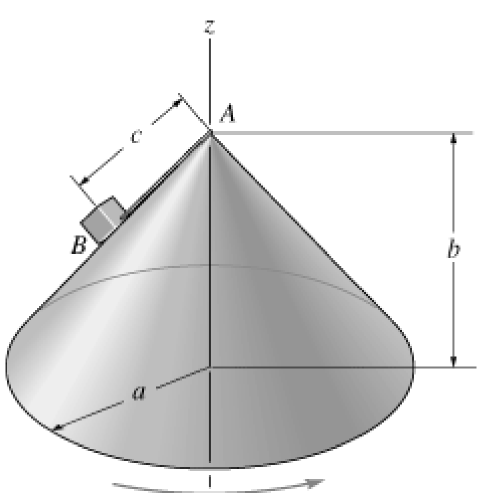
$$F_N - W\cos(\theta) = \frac{W}{g}\left(\frac{v^2}{\rho}\right)$$

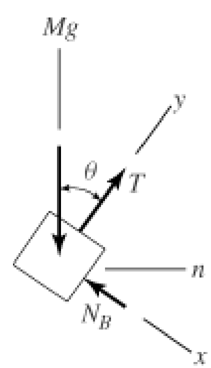
$$F_N = W\cos(\theta) + \frac{W}{g}\left(\frac{v^2}{\rho(a)}\right) \qquad F_N = 182.0\,\text{lb}$$

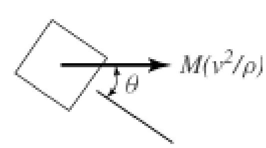


***Problem 13-72**

The smooth block *B*, having mass *M*, is attached to the vertex *A* of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the *z* axis such that the block attains speed *v*. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block.







Given: $M = 0.2\ \text{kg}$ $v = 0.5\ \frac{\text{m}}{\text{s}}$ $a = 300\ \text{mm}$ $b = 400\ \text{mm}$

$c = 200\ \text{mm}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guesses $T = 1\ \text{N}$ $N_B = 1\ \text{N}$

Set $\theta = \text{atan}\left(\frac{a}{b}\right)$ $\theta = 36.87\,\text{deg}$

$$\rho = \left(\frac{c}{\sqrt{a^2 + b^2}}\right)a \qquad \rho = 120\,\text{mm}$$

Given $T\sin(\theta) - N_B\cos(\theta) = M\left(\frac{v^2}{\rho}\right)$ $T\cos(\theta) + N_B\sin(\theta) - Mg = 0$

$$\begin{pmatrix} T \\ N_B \end{pmatrix} = \text{Find}(T, N_B) \qquad \begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 1.82 \\ 0.844 \end{pmatrix} \text{N}$$

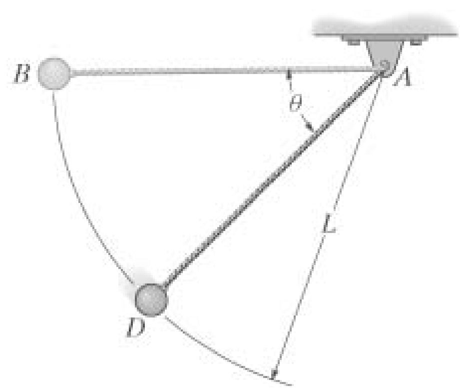
Problem 13-73

The pendulum bob B of mass M is released from rest when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches point D, $\theta = \theta_1$. Neglect the size of the bob.

Given:

$M = 5\ \text{kg}$ $\theta_1 = 45\ \text{deg}$

$L = 2\ \text{m}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



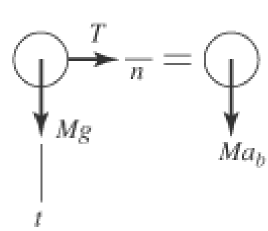
Solution:

Initially, $v = 0$ so $a_n = 0$ $T = 0$

At D we have

$$Mg\cos(\theta_1) = Ma_t$$

$$a_t = g\cos(\theta_1) \qquad a_t = 6.937\ \frac{\text{m}}{\text{s}^2}$$

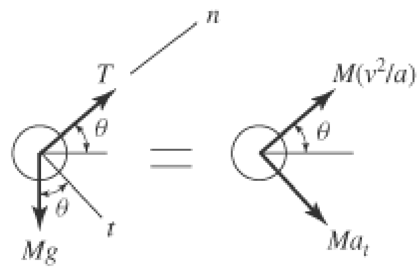


$$T_D - Mg\sin(\theta_1) = \frac{Mv^2}{L}$$

Now find the velocity v

Guess $v = 1\ \frac{\text{m}}{\text{s}}$

Given $\displaystyle \int_0^v v\,dv = \int_0^{\theta_1} g\cos(\theta) L\, d\theta$



$v = \text{Find}(v) \qquad v = 5.268\ \dfrac{\text{m}}{\text{s}}$

$T_D = M g \sin(\theta_1) + M\left(\dfrac{v^2}{L}\right) \qquad T_D = 104.1\ \text{N}$

Problem 13-74

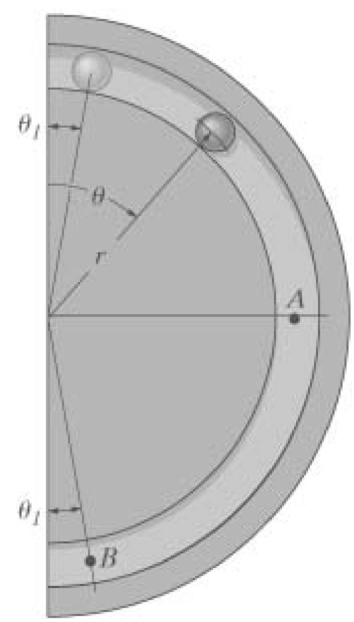
A ball having a mass M and negligible size moves within a smooth vertical circular slot. If it is released from rest at θ_1, determine the force of the slot on the ball when the ball arrives at points A and B.

Given:

$M = 2\ \text{kg} \qquad \theta = 90\ \text{deg} \qquad \theta_1 = 10\ \text{deg} \qquad r = 0.8\ \text{m} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$M g \sin(\theta) = M a_t \qquad a_t = g\sin(\theta)$



At A $\qquad \theta_A = 90\ \text{deg}$

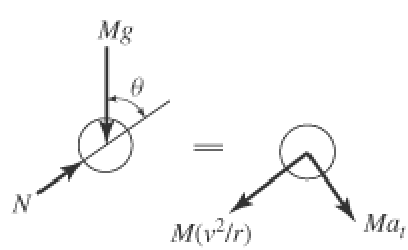
$$v_A = \sqrt{2g\left(\int_{\theta_1}^{\theta_A} \sin(\theta)\, r\, d\theta\right)}$$

$$N_A - M g\cos(\theta_A) = -M\left(\frac{v_A^2}{r}\right)$$

$$N_A = M g\cos(\theta_A) - M\left(\frac{v_A^2}{r}\right) \qquad N_A = -38.6\ \text{N}$$

At B $\qquad \theta_B = 180\ \text{deg} - \theta_1$

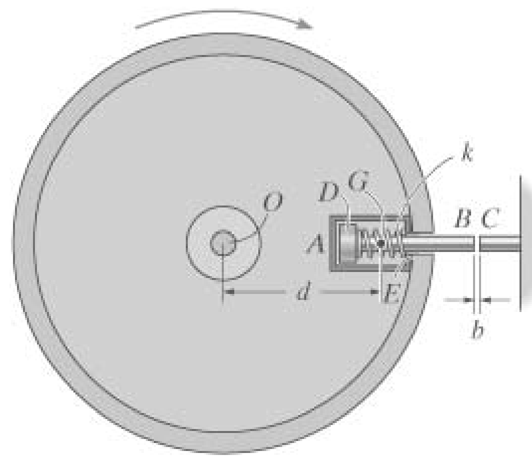
$$v_B = \sqrt{2g\left(\int_{\theta_1}^{\theta_B} \sin(\theta)\, r\, d\theta\right)}$$



$$N_B - M g\cos(\theta_B) = -M\left(\frac{v_B^2}{r}\right)$$

$$N_B = M g\cos(\theta_B) - M\left(\frac{v_B^2}{r}\right) \qquad N_B = -96.6\ \text{N}$$

Problem 13-75



The rotational speed of the disk is controlled by a smooth contact arm AB of mass M which is spring-mounted on the disk.When the disk is *at rest*, the center of mass G of the arm is located distance d from the center O, and the preset compression in the spring is a. If the initial gap between B and the contact at C is b, determine the (controlling) speed v_G of the arm's mass center, G, which will close the gap. The disk rotates in the horizontal plane. The spring has a stiffness k and its ends are attached to the contact arm at D and to the disk at E.

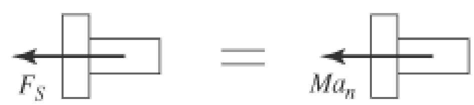
Given:

$$M = 30 \text{ gm} \qquad a = 20 \text{ mm} \qquad b = 10 \text{ mm} \qquad d = 150 \text{ mm} \qquad k = 50 \ \frac{\text{N}}{\text{m}}$$

Solution:

$$F_S = k(a+b) \qquad F_S = 1.5 \text{ N}$$

$$a_n = \frac{v_G^2}{d+b} \qquad F_S = M\left(\frac{v_G^2}{d+b}\right)$$



$$v_G = \frac{1}{M}\sqrt{M\,k\,(a+b)(d+b)} \qquad v_G = 2.83 \ \frac{\text{m}}{\text{s}}$$

***Problem 13-76**

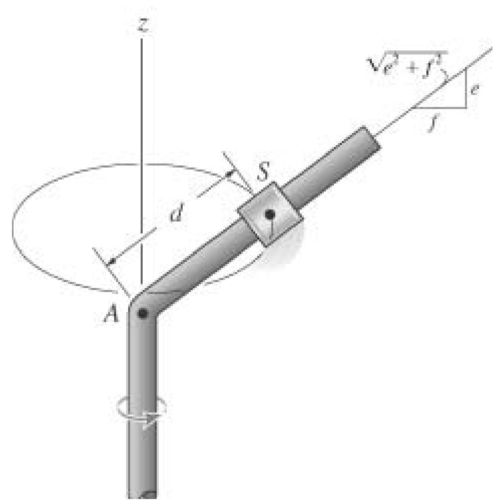
The spool S of mass M fits loosely on the inclined rod for which the coefficient of static friction is μ_s. If the spool is located a distance d from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.

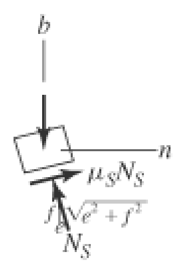
Given:

$$M = 2 \text{ kg} \qquad e = 3$$

$$\mu_S = 0.2 \qquad f = 4$$

$$d = 0.25 \text{ m} \qquad g = 9.81 \ \frac{\text{m}}{\text{s}^2}$$





Solution:

$$\rho = d\left(\frac{f}{\sqrt{e^2+f^2}}\right)$$

Guesses $\qquad N_S = 1 \text{ N} \qquad v = 1 \ \frac{\text{m}}{\text{s}}$

Given $$N_S\left(\frac{e}{\sqrt{e^2+f^2}}\right)-\mu_S N_S\left(\frac{f}{\sqrt{e^2+f^2}}\right)=M\left(\frac{v^2}{\rho}\right)$$

$$N_S\left(\frac{f}{\sqrt{e^2+f^2}}\right)+\mu_S N_S\left(\frac{e}{\sqrt{e^2+f^2}}\right)-Mg=0$$

$$\begin{pmatrix} N_S \\ v \end{pmatrix}=\text{Find}(N_S, v) \qquad N_S = 21.326\ \text{N} \qquad v = 0.969\ \frac{\text{m}}{\text{s}}$$

Problem 13-77

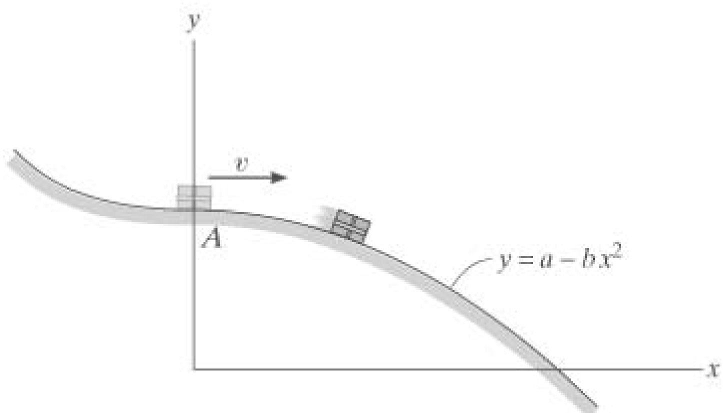
The box of mass M has a speed v_0 when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = x_1$. Also, what is the rate of increase in its speed at this instant?

Given:

$M = 35\ \text{kg}$ $\quad a = 4\ \text{m}$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$ $\quad b = \frac{1}{9}\ \frac{1}{\text{m}}$

$x_1 = 3\ \text{m}$



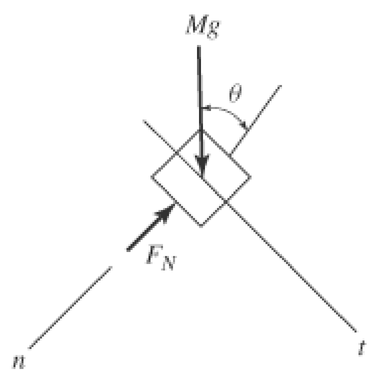
Solution:

$$y(x) = a - bx^2$$

$$y'(x) = -2bx \qquad y''(x) = -2b$$

$$\rho(x) = \frac{\sqrt{\left(1+y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \text{atan}(y'(x))$$



Find the velocity

$$v_1 = \sqrt{v_0^2 + 2g\left(y(0\ \text{m}) - y(x_1)\right)} \qquad v_1 = 4.86\ \frac{\text{m}}{\text{s}}$$

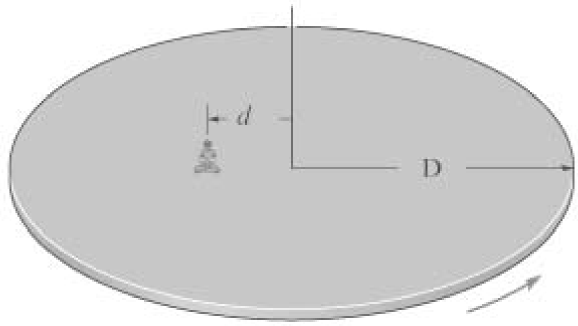
Guesses $\quad F_N = 1\ \text{N} \qquad v' = 1\ \frac{\text{m}}{\text{s}^2}$

Given $$F_N - Mg\cos(\theta(x_1)) = M\left(\frac{v_1^2}{\rho(x_1)}\right) \qquad -Mg\sin(\theta(x_1)) = Mv'$$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \text{Find}(F_N, v') \qquad F_N = 179.9\ \text{N} \qquad v' = 5.442\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-78

The man has mass M and sits a distance d from the center of the rotating platform. Due to the rotation his speed is increased from rest by the rate v'. If the coefficient of static friction between his clothes and the platform is μ_s, determine the time required to cause him to slip.

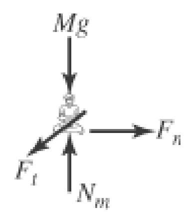


Given:

$M = 80\ \text{kg} \qquad \mu_s = 0.3$

$d = 3\ \text{m} \qquad D = 10\ \text{m}$

$v' = 0.4\ \frac{\text{m}}{\text{s}^2} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$



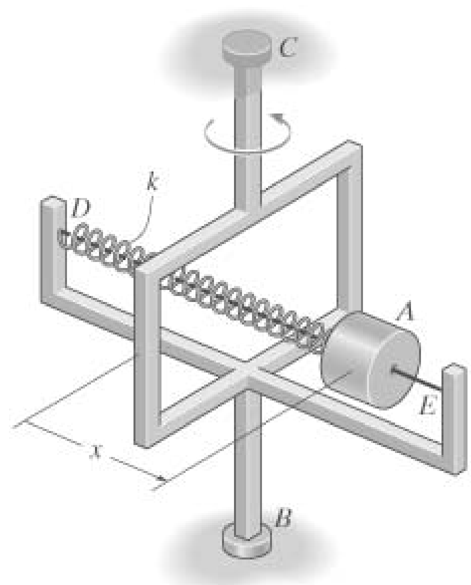
Solution: Guess $t = 1\ \text{s}$

Given $$\mu_s M g = \sqrt{(M v')^2 + \left[M \frac{(v' t)^2}{d}\right]^2}$$

$t = \text{Find}(t) \qquad t = 7.394\ \text{s}$

Problem 13-79

The collar A, having a mass M, is attached to a spring having a stiffness k. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $x = 0$, determine the constant speed of the collar in order that $x = x_1$. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



Given:

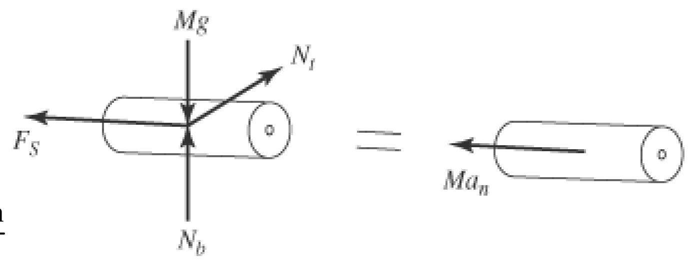
$M = 0.75\ \text{kg}$

$k = 200\ \frac{\text{N}}{\text{m}}$

$x_1 = 100\ \text{mm}$

$g = 9.81\ \frac{m}{s^2}$

Solution:



Guesses

$N_b = 1\ N \quad N_t = 1\ N \quad v = 1\ \frac{m}{s}$

Given $\quad N_b - Mg = 0 \quad N_t = 0 \quad kx_1 = M\left(\frac{v^2}{x_1}\right)$

$$\begin{pmatrix} N_b \\ N_t \\ v \end{pmatrix} = \text{Find}(N_b, N_t, v) \qquad \begin{pmatrix} N_b \\ N_t \end{pmatrix} = \begin{pmatrix} 7.36 \\ 0 \end{pmatrix} N \qquad \left|\begin{pmatrix} N_b \\ N_t \end{pmatrix}\right| = 7.36\ N \qquad v = 1.633\ \frac{m}{s}$$

***Problem 13-80**

The block has weight W and it is free to move along the smooth slot in the rotating disk. The spring has stiffness k and an unstretched length δ. Determine the force of the spring on the block and the tangential component of force which the slot exerts on the side of the block, when the block is at rest with respect to the disk and is traveling with constant speed v.

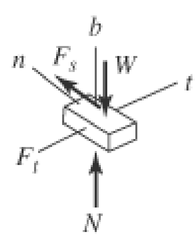
Given:

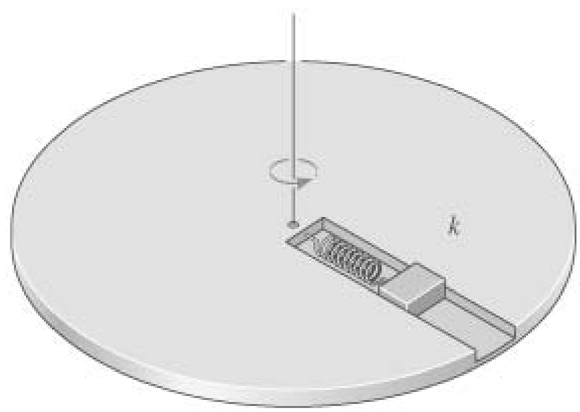
$W = 2\ lb$

$k = 2.5\ \frac{lb}{ft}$

$\delta = 1.25\ ft$

$v = 12\ \frac{ft}{s}$





Solution:

$\Sigma F_n = ma_n; \qquad F_s = k(\rho - \delta) = \frac{W}{g}\left(\frac{v^2}{\rho}\right)$

Choosing the positive root,

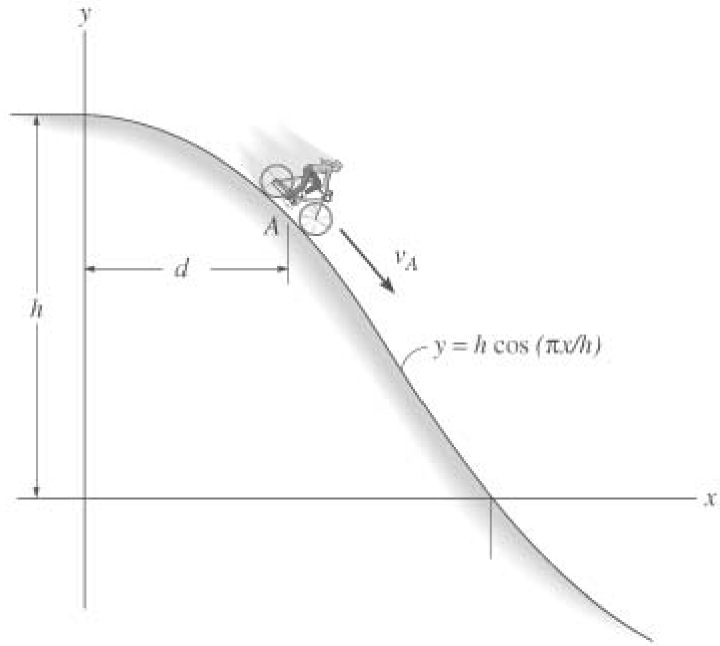
$$\rho = \frac{1}{2kg}\left[kg\delta + \left(\sqrt{k^2 g^2 \delta^2 + 4kgWv^2}\right)\right] \qquad \rho = 2.617\ ft$$

$F_s = k(\rho - \delta) \qquad F_s = 3.419\ lb$

$\Sigma F_t = ma_t; \qquad \Sigma F_t = ma_t; \qquad F_t = 0$

Problem 13-81

If the bicycle and rider have total weight W, determine the resultant normal force acting on the bicycle when it is at point A while it is freely coasting at speed v_A. Also, compute the increase in the bicyclist's speed at this point. Neglect the resistance due to the wind and the size of the bicycle and rider.



Given:

$W = 180\ \text{lb}$ $\quad d = 5\ \text{ft}$

$v_A = 6\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$h = 20\ \text{ft}$

Solution:

$$y(x) = h\cos\left(\pi\frac{x}{h}\right)$$

$$y'(x) = \frac{d}{dx}y(x) \qquad y''(x) = \frac{d}{dx}y'(x)$$

At A $\quad x = d \qquad \theta = \operatorname{atan}(y'(x))$

$$\rho = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

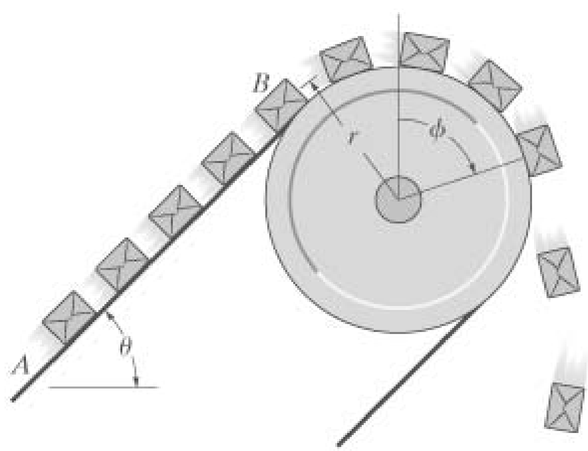
Guesses $\quad F_N = 1\ \text{lb} \qquad v' = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad F_N - W\cos(\theta) = \frac{W}{g}\left(\frac{v_A^2}{\rho}\right) \qquad -W\sin(\theta) = \left(\frac{W}{g}\right)v'$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \operatorname{Find}(F_N, v') \qquad F_N = 69.03\ \text{lb} \qquad v' = 29.362\ \frac{\text{ft}}{\text{s}^2}$$

Problem 13-82

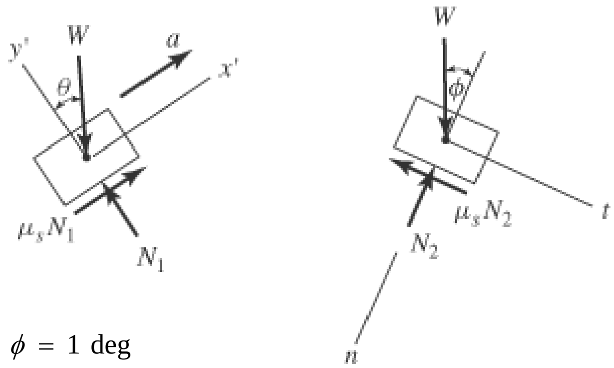
The packages of weight W ride on the surface of the conveyor belt. If the belt starts from rest and increases to a constant speed v_1 in time t_1, determine the maximum angle θ so that none of the packages slip on the inclined surface AB of the belt. The coefficient of static friction between the belt and a package is μ_s. At what angle ϕ do the packages first begin to slip off the surface of the belt after the belt is moving at its constant speed of v_1? Neglect the size of the packages.



Given:

$W = 5\ \text{lb} \qquad t_1 = 2\ \text{s} \qquad r = 6\ \text{in}$

$v_1 = 2\ \frac{\text{ft}}{\text{s}} \qquad \mu_s = 0.3$



Solution: $\quad a = \frac{v_1}{t_1}$

Guesses

$N_1 = 1\ \text{lb} \qquad N_2 = 1\ \text{lb} \qquad \theta = 1\ \text{deg} \qquad \phi = 1\ \text{deg}$

Given

$N_1 - W\cos(\theta) = 0 \qquad \mu_s N_1 - W\sin(\theta) = \left(\frac{W}{g}\right)a$

$N_2 - W\cos(\phi) = \frac{-W}{g}\left(\frac{v_1^{\,2}}{r}\right) \qquad \mu_s N_2 - W\sin(\phi) = 0$

$$\begin{pmatrix} N_1 \\ N_2 \\ \theta \\ \phi \end{pmatrix} = \text{Find}(N_1, N_2, \theta, \phi) \qquad \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} 4.83 \\ 3.637 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} \theta \\ \phi \end{pmatrix} = \begin{pmatrix} 14.99 \\ 12.61 \end{pmatrix}\text{deg}$$

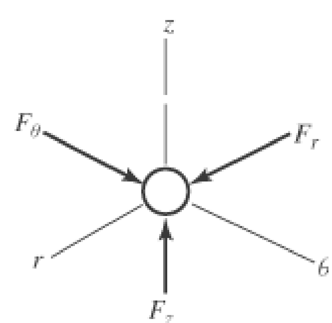
Problem 13-83

A particle having mass M moves along a path defined by the equations $r = a + bt$, $\theta = ct^2 + d$ and $z = e + ft^3$. Determine the r, θ, and z components of force which the path exerts on the particle when $t = t_1$.

Given: $\quad M = 1.5\ \text{kg} \qquad a = 4\ \text{m} \qquad b = 3\ \frac{\text{m}}{\text{s}}$

$c = 1\ \frac{\text{rad}}{\text{s}^2} \qquad d = 2\ \text{rad} \qquad e = 6\ \text{m}$

$f = -1\ \frac{\text{m}}{\text{s}^3} \qquad t_1 = 2\ \text{s} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: $\quad t = t_1$

$r = a + bt \qquad r' = b \qquad r'' = 0\ \frac{\text{m}}{\text{s}^2}$

$\theta = ct^2 + d \qquad \theta' = 2ct \qquad \theta'' = 2c$

$z = e + ft^3 \qquad z' = 3ft^2 \qquad z'' = 6ft$

$F_r = M\left(r'' - r\theta'^2\right)$ $\qquad F_r = -240\ \text{N}$

$F_\theta = M\left(r\theta'' + 2r'\theta'\right)$ $\qquad F_\theta = 66.0\ \text{N}$

$F_z = Mz'' + Mg$ $\qquad F_z = -3.285\ \text{N}$

***Problem 13-84**

The path of motion of a particle of weight W in the horizontal plane is described in terms of polar coordinates as $r = at + b$ and $\theta = ct^2 + dt$. Determine the magnitude of the unbalanced force acting on the particle when $t = t_1$.

Given: $W = 5\ \text{lb}$ $\quad a = 2\ \frac{\text{ft}}{\text{s}}$ $\quad b = 1\ \text{ft}$ $\quad c = 0.5\ \frac{\text{rad}}{\text{s}^2}$

$d = -1\frac{\text{rad}}{\text{s}}$ $\quad t_1 = 2\ \text{s}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: $t = t_1$

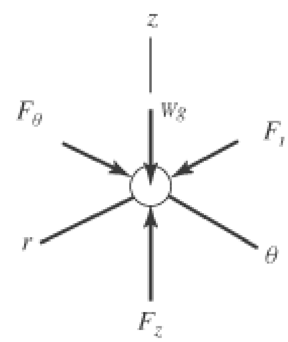
$r = at + b$ $\qquad r' = a$ $\qquad r'' = 0\ \frac{\text{ft}}{\text{s}^2}$

$\theta = ct^2 + dt$ $\qquad \theta' = 2ct + d$ $\qquad \theta'' = 2c$

$a_r = r'' - r\theta'^2$ $\qquad a_r = -5\frac{\text{ft}}{\text{s}^2}$

$a_\theta = r\theta'' + 2r'\theta'$ $\qquad a_\theta = 9\frac{\text{ft}}{\text{s}^2}$

$F = \frac{W}{g}\sqrt{a_r^2 + a_\theta^2}$ $\qquad F = 1.599\ \text{lb}$



Problem 13-85

The spring-held follower AB has weight W and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is r and $z = a\sin(2\theta)$. If the cam is rotating at a constant rate θ', determine the force at the end A of the follower when $\theta = \theta_1$. In this position the spring is compressed δ_1. Neglect friction at the bearing C.

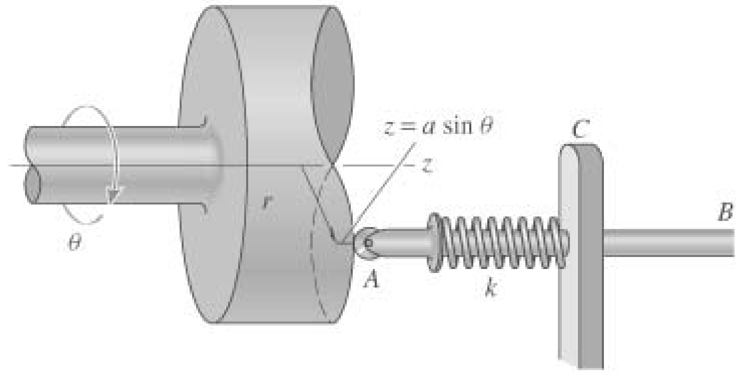
Given:

$W = 0.75 \text{ lb}$ $\quad \delta_1 = 0.4 \text{ ft}$

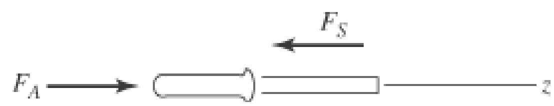
$r = 0.2 \text{ ft}$ $\quad \theta_1 = 45 \text{ deg}$

$a = 0.1 \text{ ft}$ $\quad k = 12 \dfrac{\text{lb}}{\text{ft}}$

$\theta' = 6 \dfrac{\text{rad}}{\text{s}}$ $\quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution: $\quad \theta = \theta_1 \quad z = (a)\sin(2\theta)$



$z' = 2(a)\cos(2\theta)\theta' \qquad z'' = -4(a)\sin(2\theta)\,\theta'^2$

$F_a - k\delta_1 = \left(\dfrac{W}{g}\right)z'' \qquad F_a = k\delta_1 + \left(\dfrac{W}{g}\right)z'' \qquad F_a = 4.464\,\text{lb}$

Problem 13-86

The spring-held follower *AB* has weight *W* and moves back and forth as its end rolls on the contoured surface of the cam, where the radius is *r* and $z = a\sin(2\theta)$. If the cam is rotating at a constant rate of θ', determine the maximum and minimum force the follower exerts on the cam if the spring is compressed δ_1 when $\theta = 45°$.

Given:

$W = 0.75 \text{ lb}$

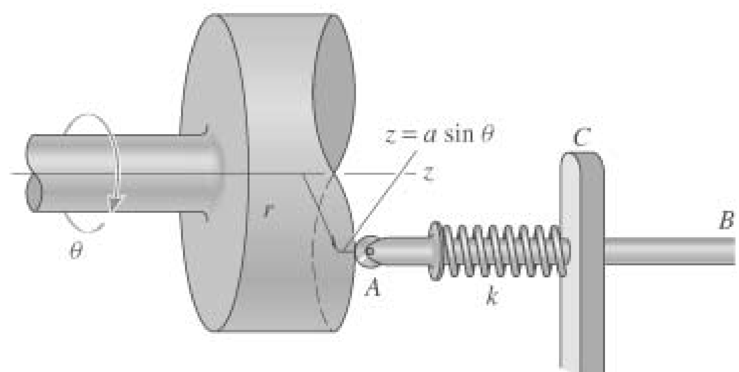
$r = 0.2 \text{ ft}$

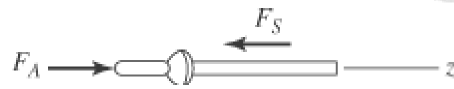
$a = 0.1 \text{ ft}$

$\theta' = 6 \dfrac{\text{rad}}{\text{s}}$

$\delta_1 = 0.2 \text{ ft}$

$k = 12 \dfrac{\text{lb}}{\text{ft}} \qquad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$





Solution: When $\quad \theta = 45 \text{ deg} \quad z = (a)\cos(2\theta) \quad z = 0 \text{ m}$

So in other positions the spring is compresses a distance $\quad \delta_1 + z$

$z = (a)\sin(2\theta) \qquad z' = 2(a)\cos(2\theta)\theta' \qquad z'' = -4(a)\sin(2\theta)\,\theta'^2$

$$F_a - k(\delta_1 + z) = \left(\frac{W}{g}\right)z'' \qquad F_a = k[\delta_1 + (a)\sin(2\theta)] - \left(\frac{W}{g}\right)4(a)\sin(2\theta)\,\theta'^2$$

The maximum values occurs when $\sin(2\theta) = -1$ and the minimum occurs when $\sin(2\theta) = 1$

$$F_{amin} = k(\delta_1 - a) + \left(\frac{W}{g}\right)4a\theta'^2 \qquad F_{amin} = 1.535\ \text{lb}$$

$$F_{amax} = k(\delta_1 + a) - \left(\frac{W}{g}\right)4a\theta'^2 \qquad F_{amax} = 3.265\ \text{lb}$$

Problem 13-87

The spool of mass M slides along the rotating rod. At the instant shown, the angular rate of rotation of the rod is θ', which is increasing at θ''. At this same instant, the spool is moving outward along the rod at r' which is increasing at r'' at r. Determine the radial frictional force and the normal force of the rod on the spool at this instant.

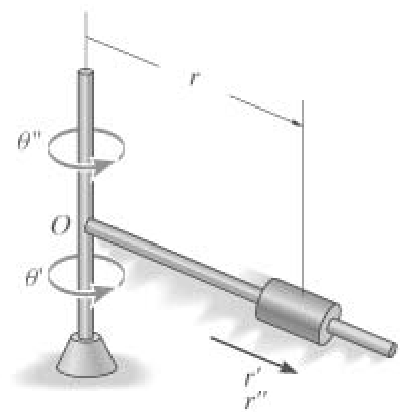
Given:

$M = 4\ \text{kg}$ $\qquad r = 0.5\ \text{m}$

$\theta' = 6\ \frac{\text{rad}}{\text{s}}$ $\qquad r' = 3\ \frac{\text{m}}{\text{s}}$

$\theta'' = 2\ \frac{\text{rad}}{\text{s}^2}$ $\qquad r'' = 1\ \frac{\text{m}}{\text{s}^2}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



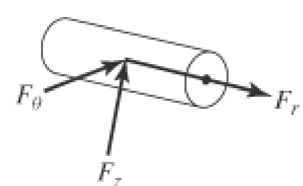
Solution:

$a_r = r'' - r\theta'^2$ $\qquad a_\theta = r\theta'' + 2r'\theta'$

$F_r = M a_r$ $\qquad F_\theta = M a_\theta$

$F_z = M g$

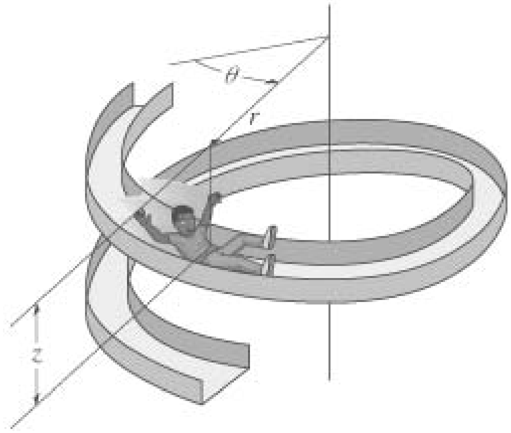
$F_r = -68.0\ \text{N}$ $\qquad \sqrt{F_\theta^{\,2} + F_z^{\,2}} = 153.1\ \text{N}$



***Problem 13-88**

The boy of mass M is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = r_0$, $\theta = bt$ and $z = ct$. Determine the components of force F_r, F_θ and F_z which the slide exerts on him at the instant $t = t_1$. Neglect the size of the boy.

Given:



$M = 40\ \text{kg}$

$r_0 = 1.5\ \text{m}$

$b = 0.7\ \dfrac{\text{rad}}{\text{s}}$

$c = -0.5\ \dfrac{\text{m}}{\text{s}}$

$t_1 = 2\ \text{s}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$r = r_0 \qquad r' = 0\ \dfrac{\text{m}}{\text{s}} \qquad r'' = 0\ \dfrac{\text{m}}{\text{s}^2}$

$\theta = bt \qquad \theta' = b \qquad \theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

$z = ct \qquad z' = c \qquad z'' = 0\ \dfrac{\text{m}}{\text{s}^2}$

$F_r = M\left(r'' - r\theta'^2\right)$ $\qquad F_r = -29.4\ \text{N}$

$F_\theta = M\left(r\theta'' + 2r'\theta'\right)$ $\qquad F_\theta = 0$

$F_z - Mg = Mz'' \qquad F_z = M(g + z'')$ $\qquad F_z = 392\ \text{N}$

Problem 13-89

The girl has a mass M. She is seated on the horse of the merry-go-round which undergoes constant rotational motion θ'. If the path of the horse is defined by $r = r_0$, $z = b\sin(\theta)$, determine the maximum and minimum force F_z the horse exerts on her during the motion.

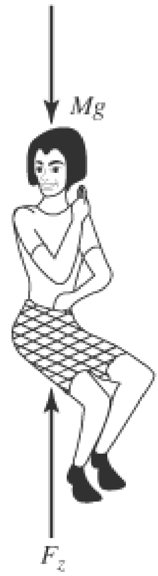
Given:

$M = 50\ \text{kg}$

$\theta' = 1.5\ \dfrac{\text{rad}}{\text{s}}$

$r_0 = 4\ \text{m}$

$b = 0.5\ \text{m}$



Solution:

$z = b\sin(\theta)$

$z' = b\cos(\theta)\theta'$

$z'' = -b\sin(\theta)\,\theta'^2$

$F_z - Mg = Mz''$

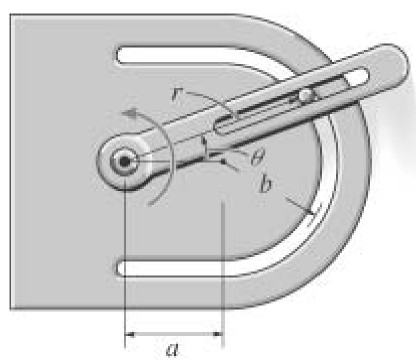
$F_z = M\left(g - b\sin(\theta)\,\theta'^2\right)$

$F_{zmax} = M\left(g + b\theta'^2\right)$ $\quad F_{zmax} = 547\ \text{N}$

$F_{zmin} = M\left(g - b\theta'^2\right)$ $\quad F_{zmin} = 434\ \text{N}$

Problem 13-90

The particle of weight W is guided along the circular path using the slotted arm guide. If the arm has angular velocity θ' and angular acceleration θ'' at the instant $\theta = \theta_1$, determine the force of the guide on the particle. Motion occurs in the *horizontal plane*.

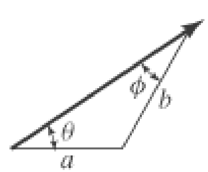


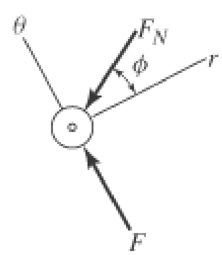
Given: $\theta_1 = 30\ \text{deg}$

$W = 0.5\ \text{lb}$ $\quad a = 0.5\ \text{ft}$

$\theta' = 4\ \dfrac{\text{rad}}{\text{s}}$ $\quad b = 0.5\ \text{ft}$

$\theta'' = 8\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution: $\theta = \theta_1$

$(a)\sin(\theta) = b\sin(\phi)$ $\quad \phi = \operatorname{asin}\left(\dfrac{a}{b}\sin(\theta)\right)$ $\quad \phi = 30\,\text{deg}$

$$(a)\cos(\theta)\theta' = b\cos(\phi)\phi' \qquad \phi' = \left[\frac{(a)\cos(\theta)}{b\cos(\phi)}\right]\theta' \qquad \phi' = 4\frac{\text{rad}}{\text{s}}$$

$$(a)\cos(\theta)\theta'' - (a)\sin(\theta)\,\theta'^2 = b\cos(\phi)\phi'' - b\sin(\phi)\,\phi'^2$$

$$\phi'' = \frac{(a)\cos(\theta)\theta'' - (a)\sin(\theta)\,\theta'^2 + b\sin(\phi)\,\phi'^2}{b\cos(\phi)} \qquad \phi'' = 8\frac{\text{rad}}{\text{s}^2}$$

$$r = (a)\cos(\theta) + b\cos(\phi) \qquad r' = -(a)\sin(\theta)\theta' - b\sin(\phi)\phi'$$

$$r'' = -(a)\sin(\theta)\theta'' - (a)\cos(\theta)\,\theta'^2 - b\sin(\phi)\phi'' - b\cos(\phi)\,\phi'^2$$

$$-F_N\cos(\phi) = M\left(r'' - r\theta'^2\right) \qquad F_N = \frac{-W\left(r'' - r\theta'^2\right)}{g\cos(\phi)} \qquad F_N = 0.569\ \text{lb}$$

$$F - F_N\sin(\phi) = \left(\frac{W}{g}\right)\left(r\theta'' + 2r'\theta'\right) \qquad F = F_N\sin(\phi) + \left(\frac{W}{g}\right)\left(r\theta'' + 2r'\theta'\right) \qquad F = 0.143\ \text{lb}$$

Problem 13-91

The particle has mass M and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = \theta_1$. The rod is rotating with a constant angular velocity θ'. Assume the particle contacts only one side of the slot at any instant.

Given:

$M = 0.5\ \text{kg}$

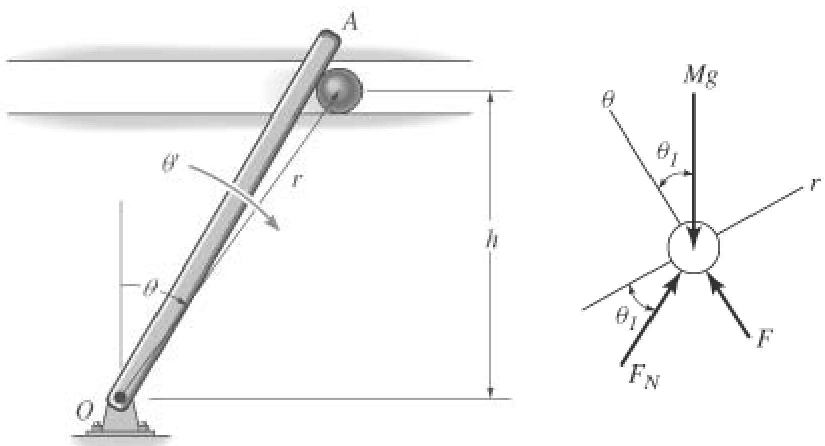
$\theta_1 = 30\ \text{deg}$

$\theta' = 2\ \frac{\text{rad}}{\text{s}}$

$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$

$h = 0.5\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

$$\theta = \theta_1 \qquad h = r\cos(\theta) \qquad r = \frac{h}{\cos(\theta)} \qquad r = 0.577\ \text{m}$$

$$0 = r'\cos(\theta) - r\sin(\theta)\theta' \qquad r' = \left(\frac{r\sin(\theta)}{\cos(\theta)}\right)\theta' \qquad r' = 0.667\ \frac{\text{m}}{\text{s}}$$

$$0 = r''\cos(\theta) - 2r'\sin(\theta)\theta' - r\cos(\theta)\,\theta'^2 - r\sin(\theta)\theta''$$

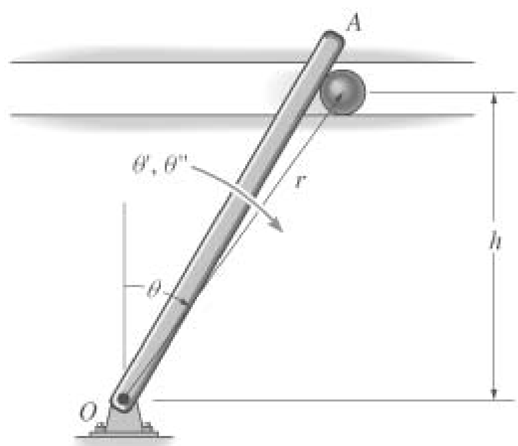
$$r'' = 2r'\theta'\tan(\theta) + r\theta'^2 + r\tan(\theta)\theta'' \qquad r'' = 3.849\ \frac{\text{m}}{\text{s}^2}$$

$$(F_N - Mg)\cos(\theta) = M\left(r'' - r\theta'^2\right) \qquad F_N = Mg + M\left(\frac{r'' - r\theta'^2}{\cos(\theta)}\right) \qquad F_N = 5.794\ \text{N}$$

$$-F + (F_N - Mg)\sin(\theta) = -M(r\theta'' + 2r'\theta')$$

$$F = (F_N - Mg)\sin(\theta) + M(r\theta'' + 2r'\theta') \qquad F = 1.778\ \text{N}$$

***Problem 13-92**



The particle has mass M and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = \theta_1$. The rod is rotating with angular velocity θ' and angular acceleration θ''. Assume the particle contacts only one side of the slot at any instant.

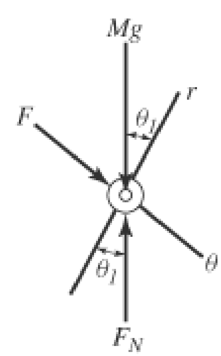
Given:

$$M = 0.5\ \text{kg}$$

$$\theta_1 = 30\ \text{deg}$$

$$\theta' = 2\ \frac{\text{rad}}{\text{s}} \qquad h = 0.5\ \text{m}$$

$$\theta'' = 3\ \frac{\text{rad}}{\text{s}^2} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$



Solution:

$$\theta = \theta_1 \qquad h = r\cos(\theta) \qquad r = \frac{h}{\cos(\theta)} \qquad r = 0.577\ \text{m}$$

$$0 = r'\cos(\theta) - r\sin(\theta)\theta' \qquad r' = \left(\frac{r\sin(\theta)}{\cos(\theta)}\right)\theta' \qquad r' = 0.667\ \frac{\text{m}}{\text{s}}$$

$$0 = r''\cos(\theta) - 2r'\sin(\theta)\theta' - r\cos(\theta)\,\theta'^2 - r\sin(\theta)\theta''$$

$r'' = 2r'\theta' \tan(\theta) + r\theta'^2 + r\tan(\theta)\theta''$ $\qquad r'' = 4.849\ \frac{\text{m}}{\text{s}^2}$

$(F_N - Mg)\cos(\theta) = M(r'' - r\theta'^2)$ $\qquad F_N = Mg + M\left(\frac{r'' - r\theta'^2}{\cos(\theta)}\right)$ $\qquad F_N = 6.371\ \text{N}$

$-F + (F_N - Mg)\sin(\theta) = -M(r\theta'' + 2r'\theta')$

$F = (F_N - Mg)\sin(\theta) + M(r\theta'' + 2r'\theta')$ $\qquad F = 2.932\ \text{N}$

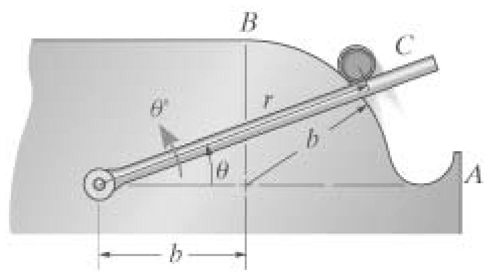
Problem 13-93

A smooth can *C*, having a mass *M*, is lifted from a feed at *A* to a ramp at *B* by a rotating rod. If the rod maintains a constant angular velocity of θ', determine the force which the rod exerts on the can at the instant $\theta = \theta_1$. Neglect the effects of friction in the calculation and the size of the can so that $r = 2b\cos\theta$. The ramp from *A* to *B* is circular, having a radius of *b*.

Given:

$M = 3\ \text{kg}$ $\qquad \theta_1 = 30\ \text{deg}$

$\theta' = 0.5\ \frac{\text{rad}}{\text{s}}$ $\qquad b = 600\ \text{mm}$



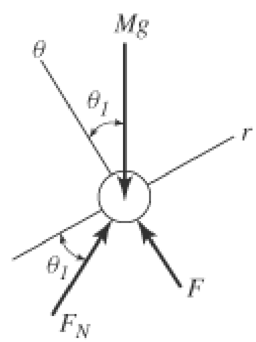
Solution:

$\theta = \theta_1$

$r = 2b\cos(\theta)$

$r' = -2b\sin(\theta)\theta'$

$r'' = -2b\cos(\theta)\,\theta'^2$



Guesses $\qquad F_N = 1\ \text{N} \qquad F = 1\ \text{N}$

Given

$F_N\cos(\theta) - Mg\sin(\theta) = M(r'' - r\theta'^2)$

$F + F_N\sin(\theta) - Mg\cos(\theta) = M(2r'\theta')$

$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F)$ $\qquad F_N = 15.191\ \text{N}$ $\qquad F = 16.99\ \text{N}$

Problem 13-94

The collar of weight *W* slides along the smooth *horizontal* spiral rod $r = b\theta$, where θ is in radians. If its angular rate of rotation θ' is constant, determine the tangential force P needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$.

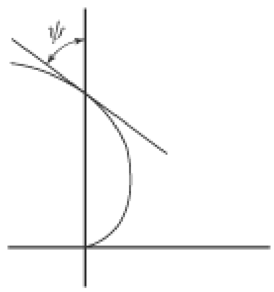
Given:

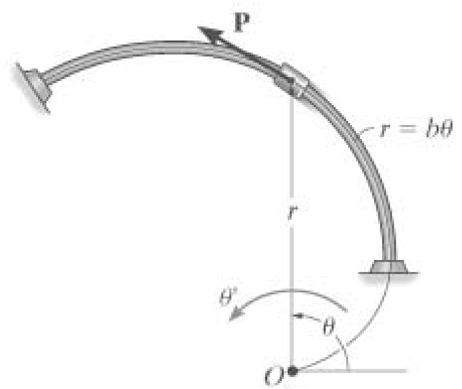
$W = 2 \text{ lb}$

$\theta_1 = 90 \text{ deg}$

$\theta' = 4 \dfrac{\text{rad}}{\text{s}}$

$b = 2 \text{ ft}$



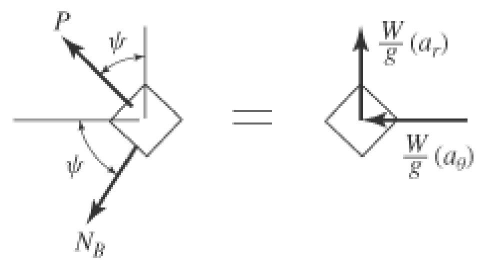


Solution:

$\theta = \theta_1 \qquad r = b\theta \qquad r' = b\theta'$

$\psi = \operatorname{atan}\left(\dfrac{r\theta'}{r'}\right)$

Guesses $\quad N_B = 1 \text{ lb} \qquad P = 1 \text{ lb}$



Given

$-N_B \sin(\psi) + P\cos(\psi) = \left(\dfrac{W}{g}\right)\left(-r\,\theta'^2\right)$

$P\sin(\psi) + N_B\cos(\psi) = \left(\dfrac{W}{g}\right)\left(2r'\theta'\right)$

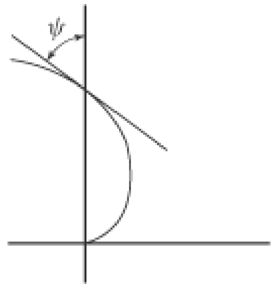
$\begin{pmatrix} N_B \\ P \end{pmatrix} = \operatorname{Find}(N_B, P) \qquad \psi = 57.52 \text{ deg} \qquad N_B = 4.771 \text{ lb} \qquad P = 1.677 \text{ lb}$

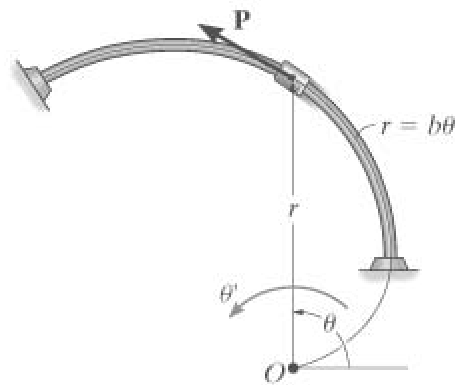
Problem 13-95

The collar of weight W slides along the smooth *vertical* spiral rod $r = b\theta$, where θ is in radians. If its angular rate of rotation θ' is constant, determine the tangential force P needed to cause the motion and the normal force that the rod exerts on the spool at the instant $\theta = \theta_1$.

Given:

$W = 2\ \text{lb}$





$\theta_1 = 90\ \text{deg}$

$\theta' = 4\ \dfrac{\text{rad}}{\text{s}}$

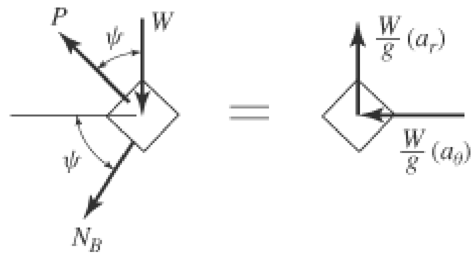
$b = 2\ \text{ft}$

Solution:

$\theta = \theta_1 \qquad r = b\theta \qquad r' = b\theta'$

$\psi = \text{atan}\left(\dfrac{r\theta'}{r'}\right)$

Guesses $\quad N_B = 1\ \text{lb} \qquad P = 1\ \text{lb}$



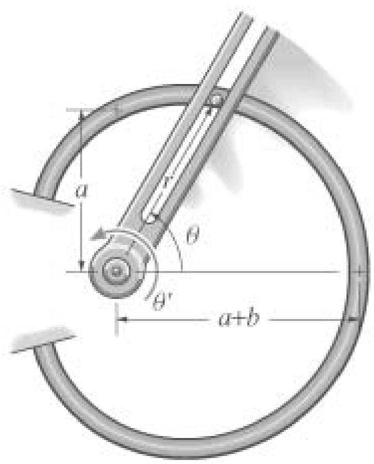
Given

$$-N_B \sin(\psi) + P\cos(\psi) - W = \left(\frac{W}{g}\right)\left(-r\,\theta'^2\right)$$

$$P\sin(\psi) + N_B\cos(\psi) = \left(\frac{W}{g}\right)\left(2r'\theta'\right)$$

$\begin{pmatrix} N_B \\ P \end{pmatrix} = \text{Find}(N_B, P) \qquad \psi = 57.52\ \text{deg} \qquad N_B = 3.084\ \text{lb} \qquad P = 2.751\ \text{lb}$

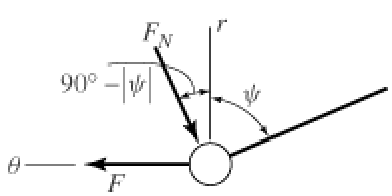
***Problem 13-96**



The forked rod is used to move the smooth particle of weight W around the horizontal path in the shape of a limacon $r = a + b\cos\theta$. If $\theta = ct^2$, determine the force which the rod exerts on the particle at the instant $t = t_1$. The fork and path contact the particle on only one side.

Given:

$W = 2\ \text{lb}$



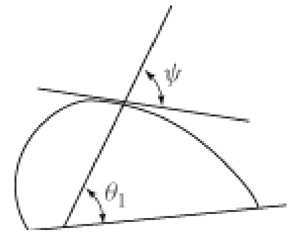
$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

$c = 0.5\ \dfrac{\text{rad}}{\text{s}^2}$

$t_1 = 1\ \text{s}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $t = t_1 \quad \theta = ct^2 \quad \theta' = 2ct \quad \theta'' = 2c$

Find the angel ψ using rectangular coordinates. The path is tangent to the velocity therefore.

$$x = r\cos(\theta) = (a)\cos(\theta) + b\cos(\theta)^2 \qquad x' = \left[-(a)\sin(\theta) - 2b\cos(\theta)\sin(\theta)\right]\theta'$$

$$y = r\sin(\theta) = (a)\sin(\theta) + \frac{1}{2}b\sin(2\theta) \qquad y' = \left[(a)\cos(\theta) + b\cos(2\theta)\right]\theta'$$

$$\psi = \theta - \operatorname{atan}\left(\frac{y'}{x'}\right) \qquad \psi = 80.541\ \text{deg}$$

Now do the dynamics using polar coordinates

$$r = a + b\cos(\theta) \qquad r' = -b\sin(\theta)\theta' \qquad r'' = -b\cos(\theta)\theta'^2 - b\sin(\theta)\theta''$$

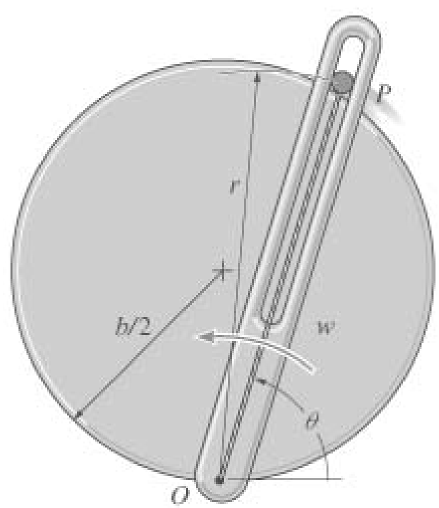
Guesses $F = 1\ \text{lb} \quad F_N = 1\ \text{lb}$

Given $F - F_N\cos(\psi) = \left(\frac{W}{g}\right)(r\theta'' + 2r'\theta') \qquad -F_N\sin(\psi) = \left(\frac{W}{g}\right)(r'' - r\theta'^2)$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N) \qquad F_N = 0.267\ \text{lb} \qquad F = 0.163\ \text{lb}$$

Problem 13-97

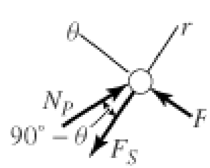
The smooth particle has mass M. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = b\sin\theta$. If the cord has stiffness k and unstretched length δ determine the force of the guide on the particle when $\theta = \theta_1$. The guide has a constant angular velocity θ'.



Given:

$M = 80\ \text{gm}$

$b = 0.8\ \text{m}$



$k = 30\ \frac{\text{N}}{\text{m}}$

$\delta = 0.25\ \text{m}$

$\theta_1 = 60\ \text{deg}$

$$\theta' = 5\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$$

Solution: $\theta = \theta_1$ $\quad r = b\sin(\theta)$ $\quad r' = b\cos(\theta)\theta'$ $\quad r'' = b\cos(\theta)\theta'' - b\sin(\theta)\,\theta'^2$

Guesses $\quad N_P = 1\ \text{N}$ $\quad F = 1\ \text{N}$

Given $\quad N_P\sin(\theta) - k(r-\delta) = M\left(r'' - r\theta'^2\right)$

$$F - N_P\cos(\theta) = M(r\theta'' + 2r'\theta')$$

$$\begin{pmatrix} F \\ N_P \end{pmatrix} = \text{Find}(F, N_P) \qquad N_P = 12.14\ \text{N} \qquad F = 7.67\ \text{N}$$

Problem 13-98

The smooth particle has mass M. It is attached to an elastic cord extending from O to P and due to the slotted arm guide moves along the *horizontal* circular path $r = b\sin\theta$. If the cord has stiffness k and unstretched length δ determine the force of the guide on the particle when $\theta = \theta_1$. The guide has angular velocity θ' and angular acceleration θ'' at this instant.

Given:

$M = 80\ \text{gm}$

$b = 0.8\ \text{m}$

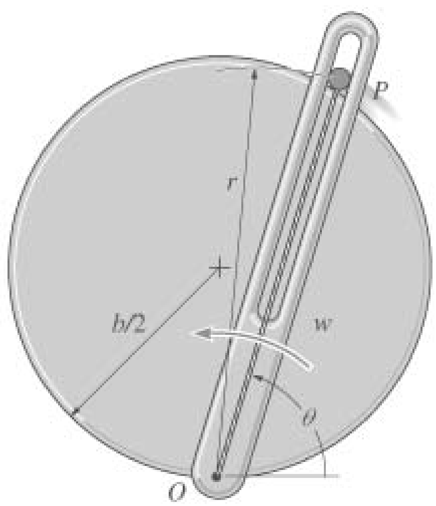
$k = 30\ \frac{\text{N}}{\text{m}}$

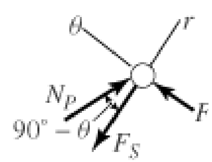
$\delta = 0.25\ \text{m}$

$\theta_1 = 60\ \text{deg}$

$\theta' = 5\ \frac{\text{rad}}{\text{s}}$

$\theta'' = 2\ \frac{\text{rad}}{\text{s}^2}$





Solution: $\theta = \theta_1$ $\quad r = b\sin(\theta)$ $\quad r' = b\cos(\theta)\theta'$ $\quad r'' = b\cos(\theta)\theta'' - b\sin(\theta)\,\theta'^2$

Guesses $\quad N_P = 1\ \text{N}$ $\quad F = 1\ \text{N}$

Given $N_P \sin(\theta) - k(r-\delta) = M\left(r'' - r\theta^2\right)$

$F - N_P \cos(\theta) = M(r\theta' + 2r'\theta)$

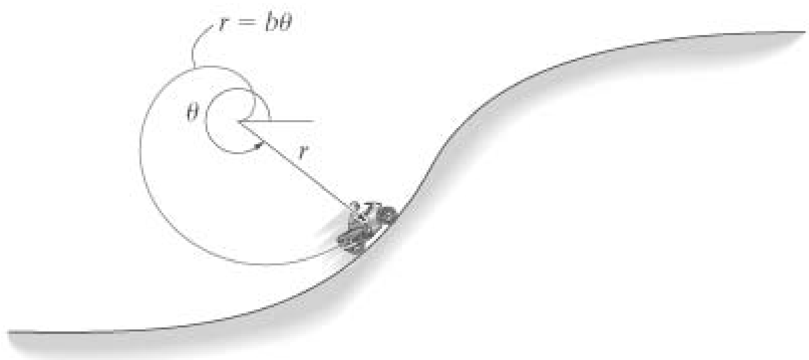
$\begin{pmatrix} F \\ N_P \end{pmatrix} = \text{Find}(F, N_P)$ $\quad N_P = 12.214\ \text{N}$ $\quad F = 7.818\ \text{N}$

Problem 13-99

Determine the normal and frictional driving forces that the partial spiral track exerts on the motorcycle of mass M at the instant θ, θ, and θ'. Neglect the size of the motorcycle.

Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

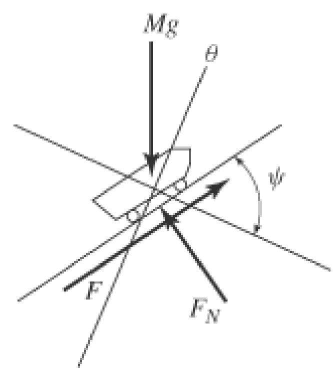
$M = 200\ \text{kg}$

$b = 5\ \text{m}$

$\theta = \dfrac{5\ \pi}{3}\ \text{rad}$

$\theta = 0.4\ \dfrac{\text{rad}}{\text{s}}$

$\theta' = 0.8\ \dfrac{\text{rad}}{\text{s}^2}$



Solution:

$r = b\theta \qquad r' = b\theta \qquad r'' = b\theta'$

$\psi = \text{atan}\left(\dfrac{r\theta}{r'}\right) \qquad \psi = 79.188\ \text{deg}$

Guesses $F_N = 1\ \text{N} \qquad F = 1\ \text{N}$

Given $-F_N \sin(\psi) + F\cos(\psi) - M\,g \sin(\theta) = M\left(r'' - r\theta^2\right)$

$F_N \cos(\psi) + F\sin(\psi) - M\,g\cos(\theta) = M(r\theta' + 2r'\theta)$

$\begin{pmatrix} F_N \\ F \end{pmatrix} = \text{Find}(F_N, F) \qquad \begin{pmatrix} F_N \\ F \end{pmatrix} = \begin{pmatrix} 2.74 \\ 5.07 \end{pmatrix} \text{kN}$

***Problem 13-100**

Using a forked rod, a smooth cylinder C having a mass M is forced to move along the vertical slotted path $r = a\theta$. If the angular position of the arm is $\theta = bt^2$, determine the force of the rod on the cylinder and the normal force of the slot on the cylinder at the instant t. The cylinder is in contact with only one edge of the rod and slot at any instant.

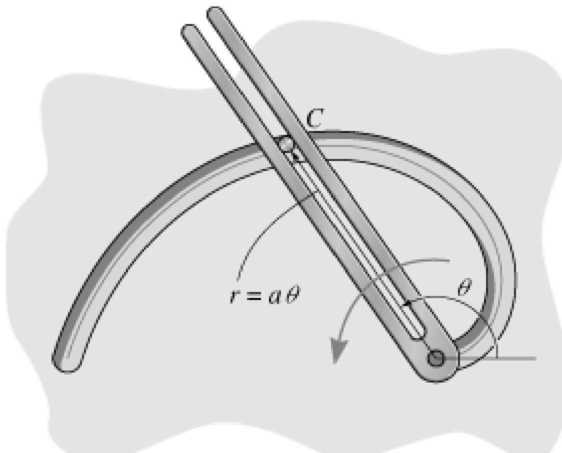
Given:

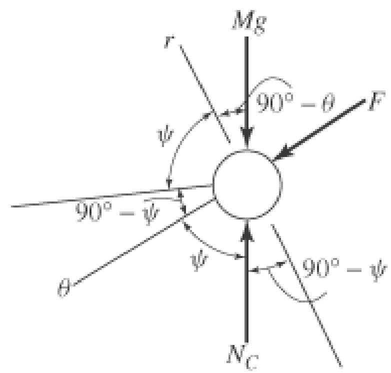
$M = 0.5\ \text{kg}$

$a = 0.5\ \text{m}$

$b = 0.5\ \dfrac{1}{s^2}$

$t_1 = 2\ \text{s}$





Solution:

$t = t_1$

Find the angle ψ using rectangular components. The velocity is parallel to the track therefore

$$x = r\cos(\theta) = \left(a\,b\,t^2\right)\cos\left(b\,t^2\right) \qquad x' = (2a\,b\,t)\cos\left(b\,t^2\right) - \left(2a\,b^2\,t^3\right)\sin\left(b\,t^2\right)$$

$$y = r\sin(\theta) = \left(a\,b\,t^2\right)\sin\left(b\,t^2\right) \qquad y' = (2a\,b\,t)\sin\left(b\,t^2\right) + \left(2a\,b^2\,t^3\right)\cos\left(b\,t^2\right)$$

$$\psi = \operatorname{atan}\left(\frac{y'}{x'}\right) - b\,t^2 + \pi \qquad \psi = 63.435\,\text{deg}$$

Now do the dynamics using polar coordinates

$$\theta = b\,t^2 \qquad \theta' = 2b\,t \qquad \theta'' = 2b \qquad r = a\theta \qquad r' = a\theta' \qquad r'' = a\theta''$$

Guesses $\quad F = 1\ \text{N} \quad N_C = 1\ \text{N}$

Given $\quad N_C\sin(\psi) - M\,g\sin(\theta) = M\left(r'' - r\theta'^2\right)$

$$F - N_C\cos(\psi) - M\,g\cos(\theta) = M\left(r\theta'' + 2r'\theta'\right)$$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \operatorname{Find}(F, N_C) \qquad \begin{pmatrix} F \\ N_C \end{pmatrix} = \begin{pmatrix} 1.814 \\ 3.032 \end{pmatrix}\ \text{N}$$

Problem 13-101

The ball has mass M and a negligible size. It is originally traveling around the horizontal circular path of radius r_0 such that the angular rate of rotation is θ'_0. If the attached cord ABC is drawn down through the hole at constant speed v, determine the tension the cord exerts on the ball at the instant $r = r_1$. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\theta'' + 2r'\theta' = (1/r)(d(r^2\theta')/dt) = 0$. When integrated, $r^2\theta' = c$ where the constant c is determined from the problem data.

Given:

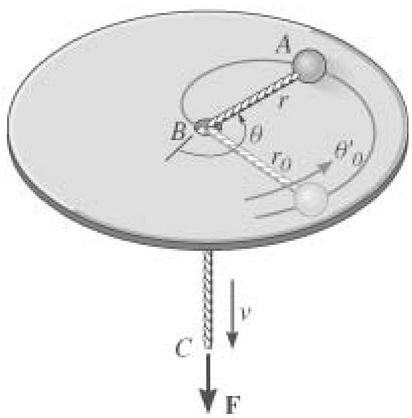
$M = 2\ \text{kg}$

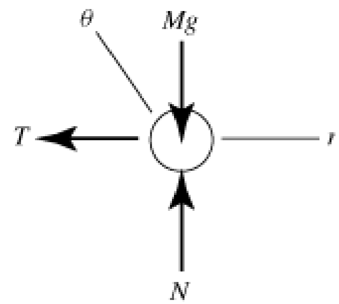
$r_0 = 0.5\ \text{m}$

$\theta'_0 = 1\ \dfrac{\text{rad}}{\text{s}}$

$v = 0.2\ \dfrac{\text{m}}{\text{s}}$

$r_1 = 0.25\ \text{m}$





Solution:

$$\Sigma F_\theta = Ma_\theta; \quad 0 = M(r\theta'' + 2r'\theta') = M\left[\frac{1}{r}\frac{d}{dt}(r^2\theta')\right]$$

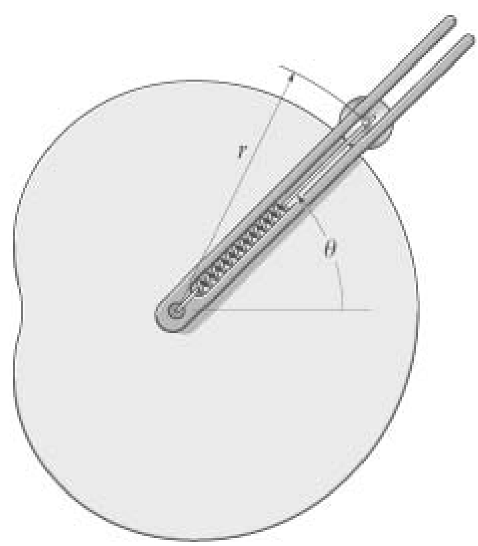
Thus $\quad c = r_0^{\,2}\theta'_0 = r_1^{\,2}\theta'_1 \qquad \theta'_1 = \left(\dfrac{r_0}{r_1}\right)^2\theta'_0 \qquad \theta'_1 = 4\ \dfrac{\text{rad}}{\text{s}}$

$r = r_1 \qquad r' = -v \qquad r'' = 0\ \dfrac{\text{m}}{\text{s}^2} \qquad \theta' = \theta'_1$

$T = -M(r'' - r\theta'^2) \qquad T = 8\ \text{N}$

Problem 13-102

The smooth surface of the vertical cam is defined in part by the curve $r = (a\cos\theta + b)$. If the forked rod is rotating with a constant angular velocity θ', determine the force the cam and the rod exert on the roller of mass M at angle θ. The attached spring has a stiffness k and an unstretched length l.



Given:

$a = 0.2\ \text{m} \qquad k = 30\ \dfrac{\text{N}}{\text{m}} \qquad \theta = 30\ \text{deg}$

$b = 0.3\ \text{m} \qquad l = 0.1\ \text{m} \qquad \theta' = 4\ \dfrac{\text{rad}}{\text{s}}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2} \qquad M = 2\ \text{kg} \qquad \theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

Solution:

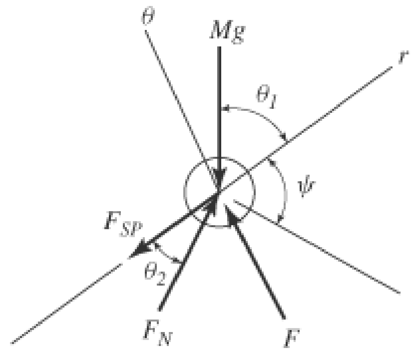
$r = (a)\cos(\theta) + b$

$$r' = -(a)\sin(\theta)\theta'$$

$$r'' = -(a)\cos(\theta)\,\theta'^2 - (a)\sin(\theta)\theta''$$

$$\psi = \operatorname{atan}\left(\frac{r\theta'}{r'}\right) + \pi$$

Guesses $F_N = 1\text{ N}$ $F = 1\text{ N}$



Given $F_N\sin(\psi) - Mg\sin(\theta) - k(r-l) = M\left(r'' - r\theta'^2\right)$

$$F - F_N\cos(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \operatorname{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} 10.524 \\ 0.328 \end{pmatrix}\text{ N}$$

Problem 13-103

The collar has mass M and travels along the smooth horizontal rod defined by the equiangular spiral $r = ae^{\theta}$. Determine the tangential force F and the normal force N_C acting on the collar when $\theta = \theta_1$ if the force F maintains a constant angular motion θ'.

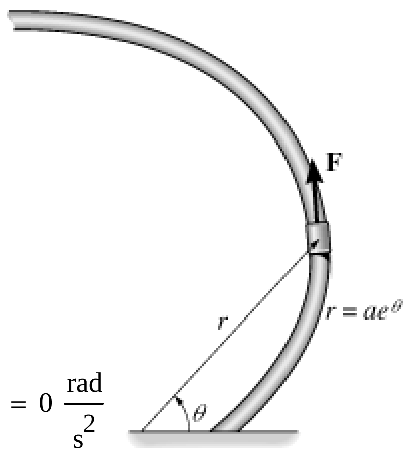
Given:

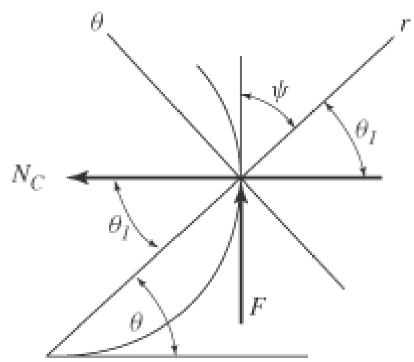
$M = 2\text{ kg}$

$a = 1\text{ m}$

$\theta_1 = 90\text{ deg}$

$\theta' = 2\,\frac{\text{rad}}{\text{s}}$





Solution:

$$\theta = \theta_1 \quad \theta' = \theta' \quad \theta'' = 0\,\frac{\text{rad}}{\text{s}^2}$$

$$r = ae^{\theta} \qquad r' = a\theta' e^{\theta} \qquad r'' = a\left(\theta'' + \theta'^2\right)e^{\theta}$$

Find the angle ψ using rectangular coordinates. The velocity is parallel to the path therefore

$$x = r\cos(\theta) \qquad x' = r'\cos(\theta) - r\theta'\sin(\theta)$$

$$y = r\sin(\theta) \qquad y' = r'\sin(\theta) + r\theta'\cos(\theta)$$

$$\psi = \operatorname{atan}\left(\frac{y'}{x'}\right) - \theta + \pi \qquad \psi = 112.911\,\text{deg}$$

Now do the dynamics using polar coordinates Guesses $F = 1\text{ N}$ $N_C = 1\text{ N}$

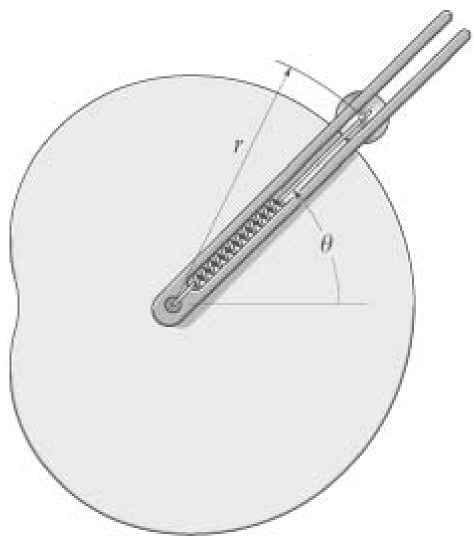
Given $\quad F\cos(\psi) - N_C\cos(\psi) = M\left(r'' - r\theta'^2\right)$

$$F\sin(\psi) + N_C\sin(\psi) = M(r\theta'' + 2r'\theta')$$

$$\begin{pmatrix} F \\ N_C \end{pmatrix} = \text{Find}(F, N_C) \qquad \begin{pmatrix} F \\ N_C \end{pmatrix} = \begin{pmatrix} 10.2 \\ -13.7 \end{pmatrix} \text{ N}$$

***Problem 13-104**

The smooth surface of the vertical cam is defined in part by the curve $r = (a\cos\theta + b)$. The forked rod is rotating with an angular acceleration θ'', and at angle θ the angular velocity is θ'. Determine the force the cam and the rod exert on the roller of mass M at this instant. The attached spring has a stiffness k and an unstretched length l.



Given:

$a = 0.2\ \text{m} \qquad k = 100\ \dfrac{\text{N}}{\text{m}} \qquad \theta = 45\ \text{deg}$

$b = 0.3\ \text{m} \qquad l = 0.1\ \text{m} \qquad \theta' = 6\ \dfrac{\text{rad}}{\text{s}}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2} \qquad M = 2\ \text{kg} \qquad \theta'' = 2\ \dfrac{\text{rad}}{\text{s}^2}$

Solution:

$r = a\cos(\theta) + b \qquad r' = -(a)\sin(\theta)\theta'$

$r'' = -(a)\cos(\theta)\,\theta'^2 - (a)\sin(\theta)\theta''$

$\psi = \text{atan}\left(\dfrac{r\theta'}{r'}\right) + \pi$

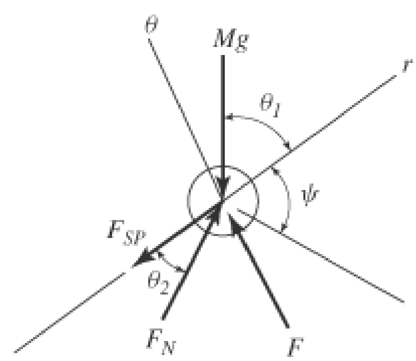
Guesses $\quad F_N = 1\ \text{N} \qquad F = 1\ \text{N}$

Given

$F_N\sin(\psi) - Mg\sin(\theta) - k(r - l) = M\left(r'' - r\theta'^2\right)$

$F - F_N\cos(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$

$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} -6.483 \\ 5.76 \end{pmatrix} \text{ N}$$



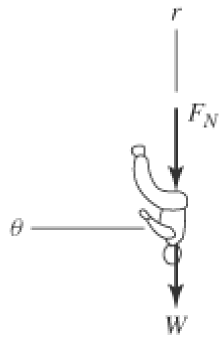
Problem 13-105

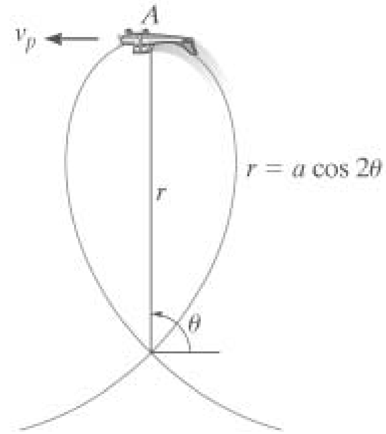
The pilot of an airplane executes a vertical loop which in part follows the path of a “four-leaved rose,” $r = a\cos 2\theta$. If his speed at A is a constant v_p, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. His weight is W.

Given:

$a = -600\ \text{ft} \qquad W = 130\ \text{lb}$

$v_p = 80\ \frac{\text{ft}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$\theta = 90\ \text{deg}$

$r = (a)\cos(2\theta)$

Guesses

$r' = 1\ \frac{\text{ft}}{\text{s}} \quad r'' = 1\ \frac{\text{ft}}{\text{s}^2} \quad \theta' = 1\ \frac{\text{rad}}{\text{s}} \quad \theta'' = 1\ \frac{\text{rad}}{\text{s}^2}$

Given Note that v_p is constant so dv_p/dt = 0

$r' = -(a)\sin(2\theta)\,2\theta' \qquad r'' = -(a)\sin(2\theta)\,2\theta'' - (a)\cos(2\theta)\,4\theta'^2$

$$v_p = \sqrt{r'^2 + (r\theta')^2} \qquad 0 = \frac{r'r'' + r\theta'(r\theta'' + r'\theta')}{\sqrt{r'^2 + (r\theta')^2}}$$

$$\begin{pmatrix} r' \\ r'' \\ \theta' \\ \theta'' \end{pmatrix} = \text{Find}(r', r'', \theta', \theta'') \qquad r' = 0.000\,\frac{\text{ft}}{\text{s}} \qquad r'' = -42.7\,\frac{\text{ft}}{\text{s}^2}$$

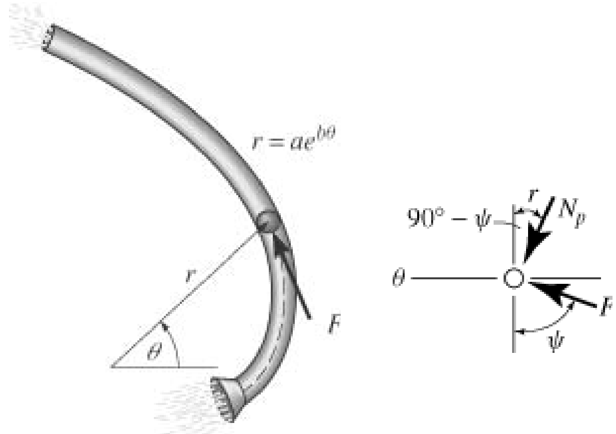
$$\theta' = 0.133\,\frac{\text{rad}}{\text{s}} \qquad \theta'' = 1.919 \times 10^{-14}\,\frac{\text{rad}}{\text{s}^2}$$

$$-F_N - W = M(r'' - r\theta'^2) \qquad F_N = -W - \left(\frac{W}{g}\right)(r'' - r\theta'^2) \qquad F_N = 85.3\,\text{lb}$$

Problem 13-106

Using air pressure, the ball of mass M is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is

F, determine the rate of increase in the ball's speed at the instant $\theta = \theta_1$ .What direction does it act in?



Given:

$$M = 0.5\ \text{kg} \quad a = 0.2\ \text{m} \quad b = 0.1$$

$$\theta_1 = \frac{\pi}{2} \quad F = 6\ \text{N}$$

Solution:

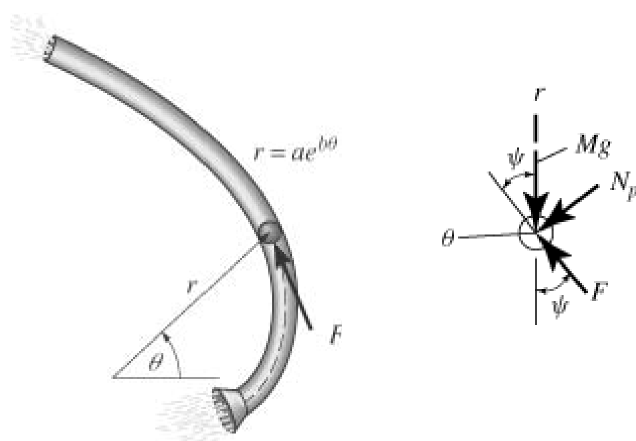
$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b}$$

$$\psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.289\ \text{deg}$$

$$F = M v' \qquad v' = \frac{F}{M} \qquad v' = 12\ \frac{\text{m}}{\text{s}^2}$$

Problem 13-107

Using air pressure, the ball of mass M is forced to move through the tube lying in the *vertical plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is F, determine the rate of increase in the ball's speed at the instant $\theta = \theta_1$. What direction does it act in?



Given:

$$M = 0.5\ \text{kg} \quad a = 0.2\ \text{m} \quad b = 0.1$$

$$F = 6\ \text{N} \quad \theta_1 = \frac{\pi}{2}$$

Solution:

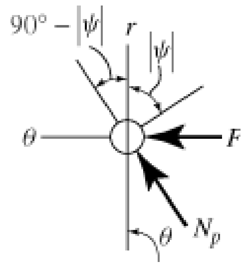
$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b}$$

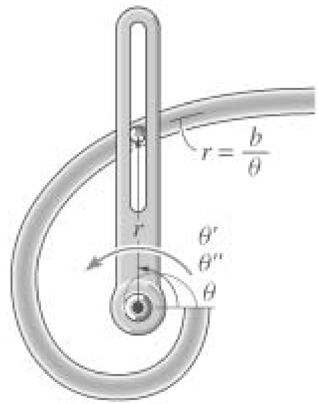
$$\psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.289\ \text{deg}$$

$$F - M g \cos(\psi) = M v' \qquad v' = \frac{F}{M} - g\cos(\psi) \qquad v' = 11.023\ \frac{\text{m}}{\text{s}^2}$$

***Problem 13-108**

The arm is rotating at the rate θ' when the angular acceleration is θ'' and the angle is θ_0. Determine the normal force it must exert on the particle of mass M if the particle is confined to move along the slotted path defined by the *horizontal* hyperbolic spiral $r\theta = b$.





Given:

$$\theta' = 5\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta_0 = 90\ \text{deg}$$

$$M = 0.5\ \text{kg}$$

$$b = 0.2\ \text{m}$$

Solution: $\theta = \theta_0 \qquad r = \dfrac{b}{\theta} \qquad r' = \left(\dfrac{-b}{\theta^2}\right)\theta' \qquad r'' = \left(\dfrac{-b}{\theta^2}\right)\theta'' + \left(\dfrac{2b}{\theta^3}\right)\theta'^2$

$$\tan(\psi) = \frac{r}{\dfrac{d}{d\theta}r} = \frac{\dfrac{b}{\theta}}{\dfrac{-b}{\theta^2}} = -\theta \qquad \psi = \operatorname{atan}(-\theta) \qquad \psi = -57.518\ \text{deg}$$

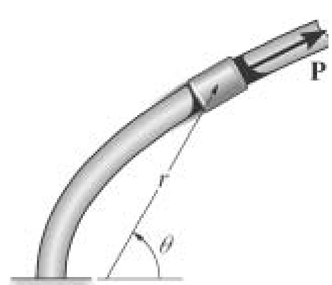
Guesses $\qquad N_P = 1\ \text{N} \qquad F = 1\ \text{N}$

Given $\qquad -N_P\sin(\psi) = M\left(r'' - r\theta'^2\right) \qquad F + N_P\cos(\psi) = M\left(r\theta'' + 2r'\theta'\right)$

$$\begin{pmatrix} N_P \\ F \end{pmatrix} = \text{Find}(N_P, F) \qquad \begin{pmatrix} N_P \\ F \end{pmatrix} = \begin{pmatrix} -0.453 \\ -1.656 \end{pmatrix}\ \text{N}$$

Problem 13-109

The collar, which has weight W, slides along the smooth rod lying in the *horizontal plane* and having the shape of a parabola $r = a/(1 - \cos\theta)$. If the collar's angular rate is θ', determine the tangential retarding force P needed to cause the motion and the normal force that the collar exerts on the rod at the instatnt $\theta = \theta_1$.



Given:

$$W = 3\ \text{lb}$$

$$a = 4\ \text{ft}$$

$$\theta' = 4\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0\ \frac{\text{rad}}{\text{s}^2}$$

$\theta_1 = 90\ \text{deg} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad M = \frac{W}{g}$

Solution: $\theta = \theta_1$

$$r = \frac{a}{1 - \cos(\theta)} \qquad r' = \frac{-a\sin(\theta)}{(1-\cos(\theta))^2}\theta'$$

$$r'' = \frac{-a\sin(\theta)}{(1-\cos(\theta))^2}\theta'' + \frac{-a\cos(\theta)\,\theta'^2}{(1-\cos(\theta))^2} + \frac{2a\sin(\theta)^2\,\theta'^2}{(1-\cos(\theta))^3}$$

Find the angle ψ using rectangular coordinates. The velocity is parallel to the path

$x = r\cos(\theta) \qquad x' = r'\cos(\theta) - r\theta'\sin(\theta) \qquad y = r\sin(\theta) \qquad y' = r'\sin(\theta) + r\theta'\cos(\theta)$

$$x'' = r''\cos(\theta) - 2r'\theta'\sin(\theta) - r\theta''\sin(\theta) - r\theta'^2\cos(\theta)$$

$$y'' = r''\sin(\theta) + 2r'\theta'\cos(\theta) + r\theta''\sin(\theta) - r\theta'^2\sin(\theta)$$

$\psi = \operatorname{atan}\left(\frac{y'}{x'}\right) \qquad \psi = 45\,\text{deg} \qquad$ Guesses $\quad P = 1\ \text{lb} \qquad H = 1\ \text{lb}$

Given $\quad P\cos(\psi) + H\sin(\psi) = M\,x'' \qquad P\sin(\psi) - H\cos(\psi) = M\,y''$

$$\begin{pmatrix} P \\ H \end{pmatrix} = \text{Find}(P, H) \qquad \begin{pmatrix} P \\ H \end{pmatrix} = \begin{pmatrix} 12.649 \\ 4.216 \end{pmatrix}\text{lb}$$

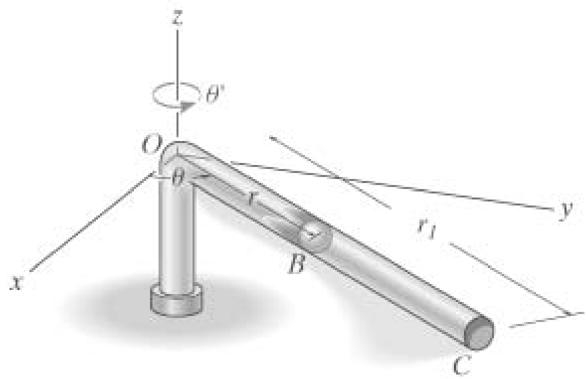
Problem 13-110

The tube rotates in the horizontal plane at a constant rate θ'. If a ball B of mass M starts at the origin O with an initial radial velocity r'_0 and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C.
Hint: Show that the equation of motion in the r direction is $r'' - r\theta'^2 = 0$. The solution is of the form $r = Ae^{-\theta' t} + Be^{\theta' t}$. Evaluate the integration constants A and B, and determine the time t at r_1. Proceed to obtain v_r and v_θ.

Given:

$\theta' = 4\ \frac{\text{rad}}{\text{s}}$ $\quad r'_0 = 1.5\ \frac{\text{m}}{\text{s}}$

$M = 0.2\ \text{kg}$ $\quad r_1 = 0.5\ \text{m}$



Solution:

$0 = M\left(r'' - r\theta'^2\right)$

$r(t) = A e^{\theta' t} + B e^{-\theta' t}$

$r'(t) = \theta'\left(A e^{\theta' t} - B e^{-\theta' t}\right)$

Guess $\quad A = 1\ \text{m} \quad B = 1\ \text{m}$

$t = 1\ \text{s}$

Given $\quad 0 = A + B \quad r'_0 = \theta'(A - B) \quad r_1 = A e^{\theta' t} + B e^{-\theta' t}$

$\begin{pmatrix} A \\ B \\ t_1 \end{pmatrix} = \text{Find}(A, B, t)$ $\quad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.188 \\ -0.188 \end{pmatrix}\text{m}$ $\quad t_1 = 0.275\ \text{s}$

$r(t) = A e^{\theta' t} + B e^{-\theta' t}$ $\quad r'(t) = \theta'\left(A e^{\theta' t} - B e^{-\theta' t}\right)$

$v_r = r'(t_1)$ $\quad v_\theta = r(t_1)\,\theta'$

$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix}\frac{\text{m}}{\text{s}}$

Problem 13-111

A spool of mass M slides down along a smooth rod. If the rod has a constant angular rate of rotation θ' in the vertical plane, show that the equations of motion for the spool are $r'' - r\theta'^2 - g\sin\theta = 0$ and $2M\theta' r' + N_s - Mg\cos\theta = 0$ where N_s is the magnitude of the normal force of the rod on the spool.

Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - (g/2\theta'^2)\sin(\theta' t)$. If r, r' and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 and determine r at the instant $\theta = \theta_1$.

Given:

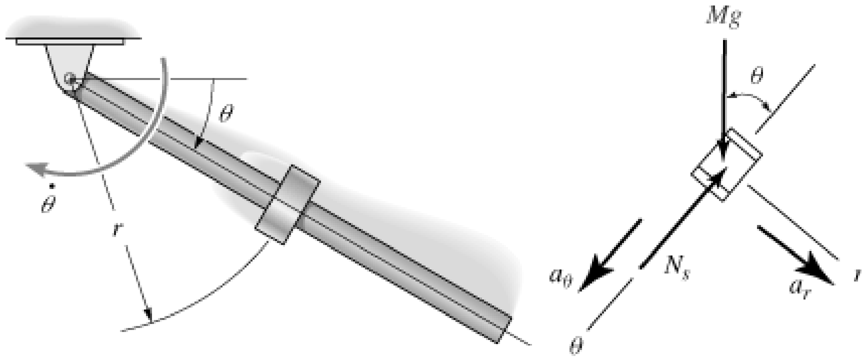
$M = 0.2\ \text{kg}$

$\theta' = 2\ \dfrac{\text{rad}}{\text{s}}$

$\theta_1 = \dfrac{\pi}{4}$

$\theta'' = 0\ \dfrac{\text{rad}}{\text{s}^2}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$\Sigma F_r = Ma_r;$ $\quad M g \sin(\theta) = M\left(r'' - r\theta'^2\right)$ $\quad r'' - r\theta'^2 - g\sin(\theta) = 0$ [1]

$\Sigma F_\theta = Ma_\theta;$ $\quad M g \cos(\theta) - N_s = M\left(r\theta'' + 2r'\theta'\right)$ $\quad 2M\theta' r' + N_s - M g\cos(\theta) = 0$

(Q.E.D)

The solution of the differential equation (Eq.[1] is given by

$$r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - \left(\frac{g}{2\theta'^2}\right)\sin(\theta' t)$$

$$r' = -\theta' C_1 e^{-\theta' t} + \theta' C_2 e^{\theta' t} - \left(\frac{g}{2\theta'}\right)\cos(\theta' t)$$

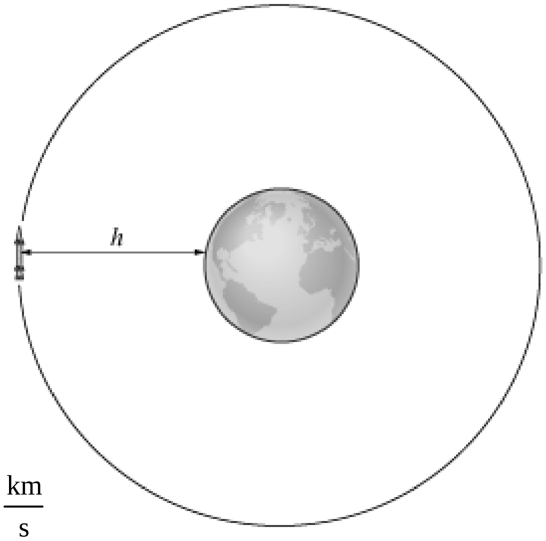
At $\quad t = 0 \quad r = 0 \quad 0 = C_1 + C_2 \quad r' = 0 \quad 0 = -\theta' C_1 + \theta' C_2 - \dfrac{g}{2\theta'}$

Thus $\quad C_1 = \dfrac{-g}{4\theta'^2} \quad C_2 = \dfrac{g}{4\theta'^2} \quad t = \dfrac{\theta_1}{\theta'} \quad t = 0.39\ \text{s}$

$$r = C_1 e^{-\theta' t} + C_2 e^{\theta' t} - \left(\frac{g}{2\theta'^2}\right)\sin(\theta' t) \qquad r = 0.198\ \text{m}$$

***Problem 13-112**

The rocket is in circular orbit about the earth at altitude h. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



Given:

$$h = 4\,10^6 \text{ m}$$

$$G = 66.73 \times 10^{-12} \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

$$M_e = 5.976 \times 10^{24} \text{ kg}$$

$$R_e = 6378 \text{ km}$$

Solution:

Circular orbit: $v_C = \sqrt{\dfrac{G M_e}{R_e + h}}$ $\qquad v_C = 6.199 \dfrac{\text{km}}{\text{s}}$

Parabolic orbit: $v_e = \sqrt{\dfrac{2G M_e}{R_e + h}}$ $\qquad v_e = 8.766 \dfrac{\text{km}}{\text{s}}$

$\Delta v = v_e - v_C$ $\qquad \Delta v = 2.57 \dfrac{\text{km}}{\text{s}}$

Problem 13-113

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–19, 13–28, 13–29, and 13–31.

Solution:

From Eq. 13-19, $\dfrac{1}{r} = C\cos(\theta) + \dfrac{GM_s}{h^2}$

For $\theta = 0$ deg and $\theta = 180$ deg $\qquad \dfrac{1}{r_\rho} = C + \dfrac{GM_s}{h^2} \qquad \dfrac{1}{r_a} = -C + \dfrac{GM_s}{h^2}$

Eliminating C, From Eqs. 13-28 and 13-29, $\dfrac{2a}{b^2} = \dfrac{2GM_s}{h^2}$

From Eq. 13-31, $T = \dfrac{\pi}{h}(2a)(b)$

Thus, $b^2 = \dfrac{T^2 h^2}{4\pi^2 a^2} \qquad \dfrac{4\pi^2 a^2}{T^2 h^2} = \dfrac{GM_s}{h^2} \qquad T^2 = \left(\dfrac{4\pi^2}{GM_s}\right)a^2$

Problem 13-114

A satellite is to be placed into an elliptical orbit about the earth such that at the perigee of its orbit it has an *altitude* h_p, and at apogee its *altitude* is h_a. Determine its required launch velocity tangent to the earth's surface at perigee and the period of its orbit.

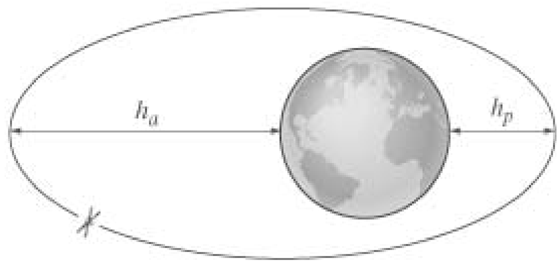
Given:

$$h_p = 800\ \text{km} \qquad G = 66.73 \times 10^{-12}\ \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$$

$$h_a = 2400\ \text{km} \qquad M_e = 5.976 \times 10^{24}\ \text{kg}$$

$$s_1 = 6378\ \text{km}$$

Solution:



$$r_p = h_p + s_1 \qquad r_p = 7178\,\text{km}$$

$$r_a = h_a + s_1 \qquad r_a = 8778\,\text{km}$$

$$r_a = \frac{r_p}{\dfrac{2GM_e}{r_p v_0^2} - 1}$$

$$v_0 = \left(\frac{1}{r_a\ r_p + r_p^2}\right)\sqrt{2}\,\sqrt{r_p\left(r_a + r_p\right)r_a\,G M_e} \qquad v_0 = 7.82\,\frac{\text{km}}{\text{s}}$$

$$h = r_p v_0 \qquad h = 56.12 \times 10^9\,\frac{\text{m}^2}{\text{s}}$$

$$T = \frac{\pi}{h}\left(r_p + r_a\right)\sqrt{r_p r_a} \qquad T = 1.97\,\text{hr}$$

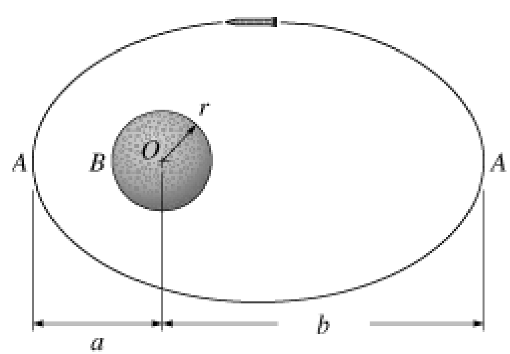
Problem 13-115

The rocket is traveling in free flight along an elliptical trajectory The planet has a mass *k* times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point *A*.

Units Used:

$$\text{Mm} = 10^3\ \text{km}$$

Given:



$$k = 0.60$$

$$a = 6.40\ \text{Mm}$$

$$b = 16\ \text{Mm}$$

$$r = 3.20\ \text{Mm}$$

$$G = 6.673 \times 10^{-11}\ \mathrm{N}\cdot\frac{\mathrm{m}^2}{\mathrm{kg}^2}$$

$$M_e = 5.976 \times 10^{24}\ \mathrm{kg}$$

Solution: *Central - Force Motion*: Substitute Eq 13-27

$$r_a = \frac{r_0}{\dfrac{2GM}{r_0 v_0^{\,2}} - 1}$$ with $r_0 = r_p = a$ and $M = kM_e$

$$b = \frac{a}{\dfrac{2GM}{a v_0^{\,2}} - 1} \qquad \frac{a}{b} = \left(\frac{2GM}{a v_0^{\,2}} - 1\right) \qquad \left(1 + \frac{a}{b}\right) = \frac{2GkM_e}{a v_p^{\,2}}$$

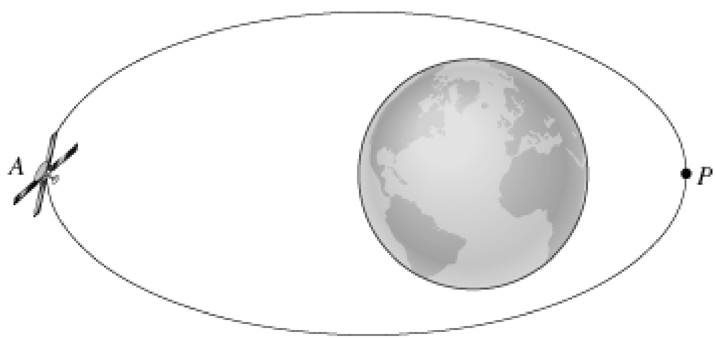
$$v_p = \sqrt{\frac{2GkM_e b}{(a+b)a}} \qquad v_p = 7.308\ \frac{\mathrm{km}}{\mathrm{s}}$$

***Problem 13-116**

An elliptical path of a satellite has an eccentricity e. If it has speed v_p when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?

Units Used:

$$\mathrm{Mm} = 10^3\ \mathrm{km}$$



Given:

$$e = 0.130$$

$$v_p = 15\ \frac{\mathrm{Mm}}{\mathrm{hr}}$$

$$G = 6.673 \times 10^{-11}\ \mathrm{N}\cdot\frac{\mathrm{m}^2}{\mathrm{kg}^2}$$

$$M_e = 5.976 \times 10^{24}\ \mathrm{kg}$$

$$R_e = 6.378 \times 10^{6}\ \mathrm{m}$$

Solution: $v_0 = v_p$ $\qquad e = \left(\dfrac{r_0 v_0^{\,2}}{GM_e} - 1\right)$ $\qquad r_0 = \dfrac{(e+1)GM_e}{v_0^{\,2}}$ $\qquad r_0 = 25.956\ \mathrm{Mm}$

$$r_A = \frac{r_0(e+1)}{1-e} \qquad r_A = 33.7\,\text{Mm} \qquad v_A = \frac{v_0 r_0}{r_A} \qquad v_A = 11.5\,\frac{\text{Mm}}{\text{hr}}$$

$$d = r_A - R_e \qquad d = 27.3\,\text{Mm}$$

Problem 13-117

A communications satellite is to be placed into an equatorial circular orbit around the earth so that it always remains directly over a point on the earth's surface. If this requires the period T (approximately), determine the radius of the orbit and the satellite's velocity.

Units Used: $\text{Mm} = 10^3\ \text{km}$

Given: $T = 24\ \text{hr}$ $\quad G = 66.73 \times 10^{-12}\ \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$ $\quad M_e = 5.976 \times 10^{24}\ \text{kg}$

Solution:

$$\frac{G M_e M_s}{r^2} = \frac{M_s v^2}{r} \qquad \frac{G M_e}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$r = \frac{1}{2\pi}\, 2^{\frac{1}{3}} \left(G\ M_e\ T^2\ \pi\right)^{\frac{1}{3}} \qquad r = 42.2\,\text{Mm}$$

$$v = \frac{2\pi r}{T} \qquad v = 3.07\,\frac{\text{km}}{\text{s}}$$

Problem 13-118

The rocket is traveling in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is c times that of the earth's. If the rocket has the apogee and perigee shown, determine the rocket's velocity when it is at point A.

Given:

$$a = 4000\ \text{mi}$$

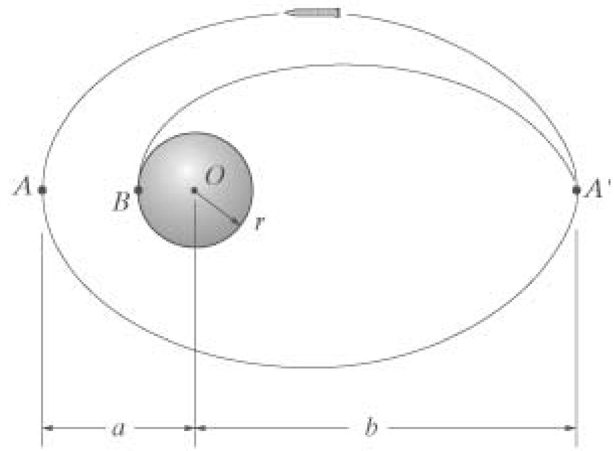
$b = 10000 \text{ mi}$

$c = 0.6$

$r = 2000 \text{ mi}$

$G = 34.4 \times 10^{-9} \dfrac{\text{lbf}\cdot\text{ft}^2}{\text{slug}^2}$

$M_e = 409 \times 10^{21} \text{ slug}$



Solution:

$r_0 = a \qquad OA' = b \qquad M_p = M_e c$

$$OA' = \frac{r_0}{2\left(\dfrac{G M_p}{r_0 v_0^2}\right) - 1} \qquad v_0 = \sqrt{\frac{2 G M_p}{r_0\left(\dfrac{r_0}{OA'} + 1\right)}} \qquad v_0 = 23.9 \times 10^3 \frac{\text{ft}}{\text{s}}$$

Problem 13-119

The rocket is traveling in free flight along an elliptical trajectory $A'A$. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, in going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth's.

Units Used:

$\text{Mm} = 10^3 \text{ km}$

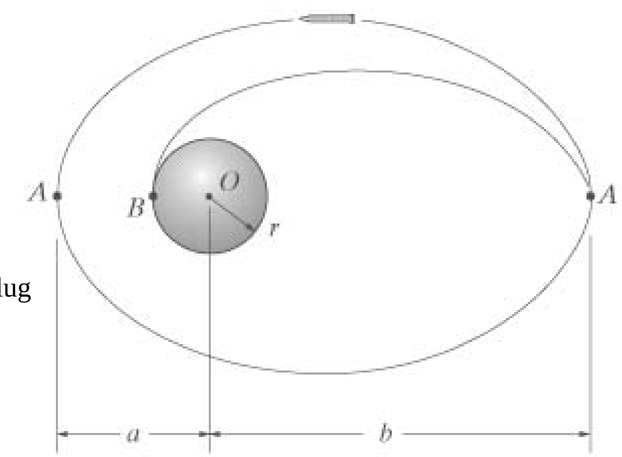
Given:

$a = 4000 \text{ mi} \qquad r = 2000 \text{ mi}$

$b = 10000 \text{ mi} \qquad M_e = 409 \times 10^{21} \text{ slug}$

$c = 0.6$

$G = 34.4 \times 10^{-9} \dfrac{\text{lbf}\cdot\text{ft}^2}{\text{slug}^2}$



Solution:

$$M_p = M_e c \qquad OA' = b \qquad OB = r \qquad OA' = \frac{OB}{2\left(\dfrac{G M_p}{OB\, v_0^2}\right) - 1}$$

$$v_0 = \frac{\sqrt{2}}{OA'\,OB + OB^2}\sqrt{OB\,(OA' + OB)\,OA'\,G\,M_p} \qquad v_0 = 36.5 \times 10^3\ \frac{\text{ft}}{\text{s}} \quad \text{(speed at } B\text{)}$$

$$v_{A'} = \frac{OB\,v_0}{OA'} \qquad v_{A'} = 7.3 \times 10^3\ \frac{\text{ft}}{\text{s}} \qquad h = OB\,v_0 \qquad h = 385.5 \times 10^9\ \frac{\text{ft}^2}{\text{s}}$$

Thus,

$$T = \frac{\pi(OB + OA')}{h}\sqrt{OB\,OA'} \qquad T = 12.19 \times 10^3\ \text{s} \qquad \frac{T}{2} = 1.69\,\text{hr}$$

***Problem 13-120**

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13-25. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit a distance d from the earth's surface.

Given: $d = 800\ \text{km}$ $\quad G = 6.673 \times 10^{-11}\ \text{N}\cdot\frac{\text{m}^2}{\text{kg}^2}$ $\quad M_e = 5.976 \times 10^{24}\ \text{kg}$

$r_e = 6378\ \text{km}$

Solution:

$$v = \sqrt{\frac{G\,M_e}{d + r_e}} \qquad v = 7.454\ \frac{\text{km}}{\text{s}}$$

Problem 13-121

The rocket is traveling in free flight along an elliptical trajectory $A'A$ .The planet has no atmosphere, and its mass is k times that of the earth's. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.

Units used:

$\text{Mm} = 10^3\ \text{km}$

Given:

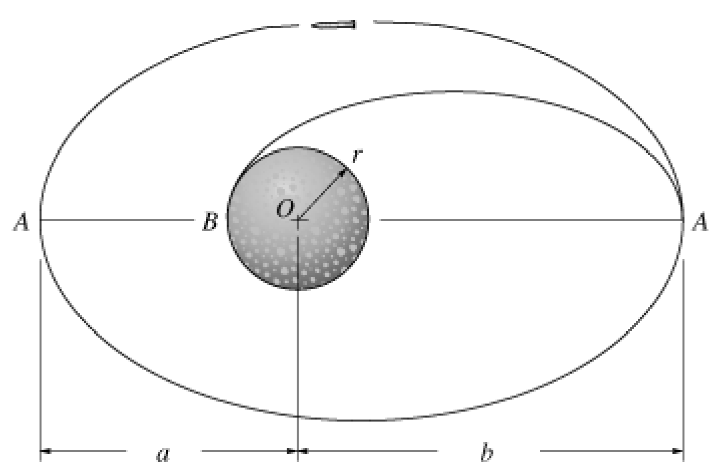
$k = 0.70$

$a = 6\ \text{Mm}$

$b = 9\ \text{Mm}$

$r = 3\ \text{Mm}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$



$$G = 6.673 \times 10^{-11}\ \mathrm{N}\cdot\frac{\mathrm{m}^2}{\mathrm{kg}^2}$$

Solution:

Central - Force motion:

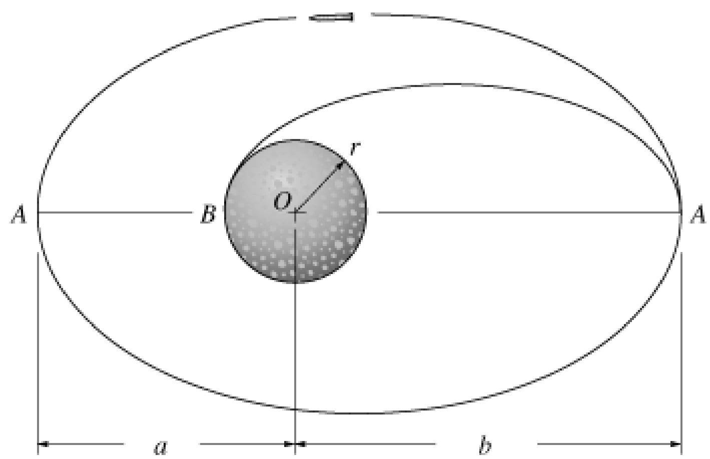
$$r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \qquad b = \frac{a}{\frac{2G(kM_e)}{a v_p^2} - 1} \qquad v_p = \sqrt{\frac{2GkM_e b}{a(a+b)}} \qquad v_p = 7.472\ \frac{\mathrm{km}}{\mathrm{s}}$$

Problem 13-122

The rocket is traveling in free flight along an elliptical trajectory $A'A$ .The planet has no atmosphere, and its mass is k times that of the earth's. The rocket has an apoapsis and periapsis as shown in the figure. If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that it strikes the planet at B. How long does it take for the rocket to land, going from A' to B along an elliptical path?

Units used:

$$\mathrm{Mm} = 10^3\ \mathrm{km}$$



Given:

$k = 0.70$

$a = 6\ \mathrm{Mm}$

$b = 9\ \mathrm{Mm}$

$r = 3\ \mathrm{Mm}$

$M_e = 5.976 \times 10^{24}\ \mathrm{kg}$

$G = 6.673 \times 10^{-11}\ \mathrm{N}\cdot\frac{\mathrm{m}^2}{\mathrm{kg}^2}$

Solution:

Central Force motion:

$$r_a = \frac{r_0}{\frac{2GM}{r_0 v_0^2} - 1} \qquad b = \frac{r}{\frac{2G(kM_e)}{r v_p^2} - 1} \qquad v_p = \sqrt{\frac{2GkM_e b}{r(r+b)}} \qquad v_p = 11.814\ \frac{\mathrm{km}}{\mathrm{s}}$$

$$r_a v_a = r_p v_p \qquad v_a = \left(\frac{r}{b}\right) v_p \qquad v_a = 3.938\ \frac{\mathrm{km}}{\mathrm{s}}$$

Eq.13-20 gives $\quad h = v_p r \qquad h = 35.44 \times 10^{9} \ \frac{\text{m}^2}{\text{s}}$

Thus, applying Eq.13-31 we have $\quad T = \frac{\pi}{h}(r+b)\sqrt{rb} \qquad T = 5.527 \times 10^{3} \text{ s}$

The time required for the rocket to go from A' to B (half the orbit) is given by

$$t = \frac{T}{2} \qquad t = 46.1 \text{ min}$$

Problem 13-123

A satellite S travels in a circular orbit around the earth. A rocket is located at the apogee of its elliptical orbit for which the eccentricity is e. Determine the sudden change in speed that must occur at A so that the rocket can enter the satellite's orbit while in free flight along the blue elliptical trajectory. When it arrives at B, determine the sudden adjustment in speed that must be given to the rocket in order to maintain the circular orbit.

Units used:

$$\text{Mm} = 10^3 \text{ km}$$

Given:

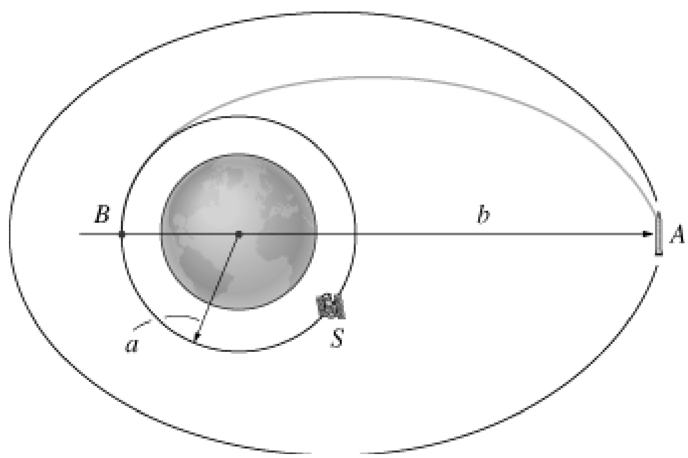
$$e = 0.58$$

$$a = 10 \text{ Mm}$$

$$b = 120 \text{ Mm}$$

$$M_e = 5.976 \times 10^{24} \text{ kg}$$

$$G = 6.673 \times 10^{-11} \text{ N}\cdot\frac{\text{m}^2}{\text{kg}^2}$$



Solution:

Central - Force motion: $\quad C = \frac{1}{r_0}\left(1 - \frac{G M_e}{r_0 v_0^2}\right) \qquad h = r_0 v_0 \qquad e = \frac{C h^2}{G M_e} = \frac{r_0 v_0^2}{G M_e} - 1$

$$v_0 = \sqrt{\frac{(1+e) G M_e}{r_0}} \qquad r_a = \frac{r_0}{\left(\frac{2 G M_e}{r v_0^2}\right) - 1} = \frac{r_0}{2\left(\frac{1}{1+e}\right) - 1}$$

$$r_0 = r_a\left(\frac{1-e}{1+e}\right) \qquad r_0 = b\left(\frac{1-e}{1+e}\right) \qquad r_0 = 31.90 \times 10^6 \text{ m}$$

Substitute $\quad r_{p1} = r_0 \qquad v_{p1} = \sqrt{\frac{(1+e)(G)(M_e)}{r_{p1}}} \qquad v_{p1} = 4.444 \times 10^3 \ \frac{\text{m}}{\text{s}}$

$$v_{a1} = \left(\frac{r_{p1}}{b}\right) v_{p1} \qquad v_{a1} = 1.181 \times 10^3\ \frac{\text{m}}{\text{s}}$$

When the rocket travels along the second elliptical orbit , from Eq.[4] , we have

$$a = \left(\frac{1 - e'}{1 + e'}\right) b \qquad e' = \frac{-a + b}{b + a} \qquad e' = 0.8462$$

Substitute $r_0 = r_{p2} = a \qquad r_{p2} = a \qquad v_{p2} = \sqrt{\frac{(1 + e')(G)(M_e)}{r_{p2}}} \qquad v_{p2} = 8.58 \times 10^3\ \frac{\text{m}}{\text{s}}$

Applying Eq. 13-20, we have $v_{a2} = \frac{r_{p2}}{b} v_{p2} \qquad v_{a2} = 715.021\ \frac{\text{m}}{\text{s}}$

For the rocket to enter into orbit two from orbit one at A, its speed must be decreased by

$$\Delta v = v_{a1} - v_{a2} \qquad \Delta v = 466\ \frac{\text{m}}{\text{s}}$$

If the rocket travels in a circular free - flight trajectory , its speed is given by Eq. 13-25

$$v_c = \sqrt{\frac{G M_e}{a}} \qquad v_c = 6.315 \times 10^3\ \frac{\text{m}}{\text{s}}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

$$\Delta v = v_{p2} - v_c \qquad \Delta v = 2.27\ \frac{\text{km}}{\text{s}}$$

***Problem 13-124**

An asteroid is in an elliptical orbit about the sun such that its perihelion distance is d. If the eccentricity of the orbit is e, determine the aphelion distance of the orbit.

Given: $d = 9.30 \times 10^9\ \text{km} \qquad e = 0.073$

Solution: $r_p = d \qquad r_0 = d$

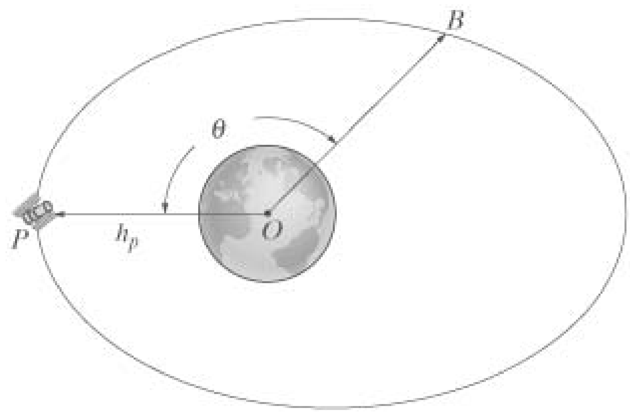
$$e = \frac{Ch^2}{GM_s} = \frac{1}{r_0}\left(1 - \frac{GM_s}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_s}\right) \qquad e = \left(\frac{r_0 v_0^2}{GM_s} - 1\right)$$

$$\frac{GM_s}{r_0 v_0^2} = \frac{1}{e + 1} \qquad r_A = \frac{r_0}{\frac{2}{e + 1} - 1} \qquad r_A = \frac{r_0(e + 1)}{1 - e} \qquad r_A = 10.76 \times 10^9\ \text{km}$$

Problem 13-125

A satellite is in an elliptical orbit around the earth with eccentricity e. If its perigee is h_p, determine its velocity at this point and also the distance OB when it is at point B, located at angle θ from perigee as shown.

Units Used: $\text{Mm} = 10^3\ \text{km}$



Given:

$e = 0.156$

$\theta = 135\ \text{deg}$

$h_p = 5\ \text{Mm}$

$G = 66.73 \times 10^{-12}\ \frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$

Solution:

$$e = \frac{Ch^2}{GM_e} = \frac{1}{h_p}\left(1 - \frac{GM_e}{h_p v_0^2}\right)\left(\frac{h_p^2 v_0^2}{GM_e}\right) \qquad \frac{h_p v^2}{GM_e} = e + 1$$

$$v_0 = \frac{1}{h_p}\sqrt{h_p GM_e(e+1)} \qquad v_0 = 9.6\,\frac{\text{km}}{\text{s}}$$

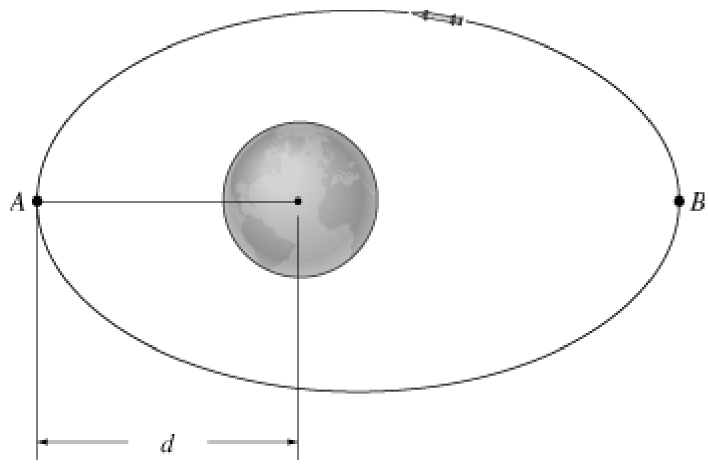
$$\frac{1}{r} = \frac{1}{h_p}\left(1 - \frac{GM_e}{h_p v_0^2}\right)\cos(\theta) + \frac{GM_e}{h_p^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{h_p}\left(1 - \frac{1}{e+1}\right)\cos(\theta) + \frac{1}{h_p}\left(\frac{1}{e+1}\right)$$

$$r = h_p\left(\frac{e+1}{e\cdot\cos(\theta) + 1}\right) \qquad r = 6.5\,\text{Mm}$$

Problem 13-126

The rocket is traveling in a free-flight elliptical orbit about the earth such that the eccentricity is e and its perigee is a distanced d as shown. Determine its speed when it is at point B. Also determine the sudden decrease in speed the rocket must experience at A in order to travel in a circular orbit about the earth.



Given:

$e = 0.76$

$d = 9 \times 10^{6}\ \text{m}$

$G = 6.673 \times 10^{-11}\ \text{N}\cdot\dfrac{\text{m}^2}{\text{kg}^2}$

$M_e = 5.976 \times 10^{24}\ \text{kg}$

Solution:

Central - Force motion:

$$C = \frac{1}{r_0}\left(1 - \frac{G M_e}{r_0 v_0^{\,2}}\right) \qquad h = r_0 v_0$$

$$e = \frac{c h^2}{G M_e} = \frac{r_0 v_0^{\,2}}{G M_e} - 1 \qquad \frac{1}{1+e} = \frac{G M_e}{r_0 v_0^{\,2}} \qquad v_0 = \sqrt{\frac{(1+e) G M_e}{r_0}}$$

$$r_a = \left(\frac{1+e}{1-e}\right) d \qquad r_a = 66 \times 10^{6}\ \text{m} \qquad r_p = d$$

$$v_p = \sqrt{\frac{(1+e) G M_e}{d}} \qquad v_p = 8.831\ \frac{\text{km}}{\text{s}} \qquad v_a = \left(\frac{d}{r_a}\right) v_p \qquad \boxed{v_a = 1.2\ \frac{\text{km}}{\text{s}}}$$

If the rockets in a cicular free - fright trajectory, its speed is given by eq.13-25

$$v_c = \sqrt{\frac{G M_e}{d}} \qquad v_c = 6656.48\ \frac{\text{m}}{\text{s}}$$

The speed for which the rocket must be decreased in order to have a circular orbit is

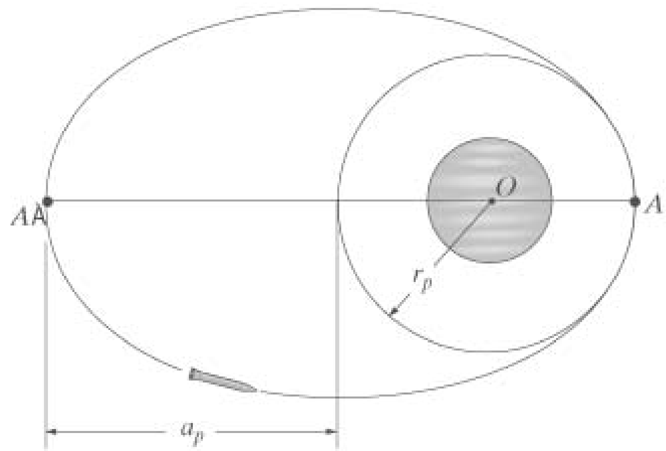
$$\Delta v = v_p - v_c \qquad \boxed{\Delta v = 2.17\ \frac{\text{km}}{\text{s}}}$$

Problem 13-127

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are r_p and a_p, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes a free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is a times the mass of the earth.

Units Used:

$$\mathrm{Mm} = 10^3\ \mathrm{km}$$



Given:

$a = 0.816$ $\quad a_p = 26\ \mathrm{Mm}$

$f = 8\ \mathrm{Mm}$ $\quad r_p = 8\ \mathrm{Mm}$

$$G = 66.73 \times 10^{-12}\ \frac{\mathrm{m}^3}{\mathrm{kg \cdot s^2}}$$

$$M_e = 5.976 \times 10^{24}\ \mathrm{kg}$$

Solution:

$$M_v = a M_e \qquad M_v = 4.876 \times 10^{24}\ \mathrm{kg}$$

$$OA' = \frac{OA}{2\left(\frac{G M_p}{OA\, v_0^{\,2}}\right) - 1} \qquad a_p = \frac{r_p}{\frac{2G M_v}{r_p v_A^{\,2}} - 1}$$

$$v_A = \left(\frac{1}{a_p r_p + r_p^{\,2}}\right)\sqrt{2}\sqrt{r_p\left(a_p + r_p\right) a_p G M_v} \qquad v_A = 7.89\ \frac{\mathrm{km}}{\mathrm{s}}$$

$$v'_A = \frac{r_p v_A}{a_p} \qquad v'_A = 2.43\ \frac{\mathrm{km}}{\mathrm{s}}$$

$$v''_A = \sqrt{\frac{G M_v}{r_p}} \qquad v''_A = 6.38\ \frac{\mathrm{km}}{\mathrm{s}}$$

Circular Orbit: $\quad T_c = \dfrac{2\pi r_p}{v''_A} \qquad T_c = 2.19\ \mathrm{hr}$

Elliptic Orbit: $\quad T_e = \dfrac{\pi}{r_p v_A}\left(r_p + a_p\right)\sqrt{r_p a_p} \qquad T_e = 6.78\ \mathrm{hr}$