Problem 14-1

A woman having a mass M stands in an elevator which has a downward acceleration a starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends a distance s. Explain why the work of these forces is different.

Units Used: $\mathrm{kJ} = 10^3 \mathrm{J}$

Given: $M = 70\ \mathrm{kg}$ $g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $a = 4\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $s = 6\ \mathrm{m}$

Solution:

$$M g - N_p = M a \qquad N_p = M g - M a \qquad N_p = 406.7\ \mathrm{N}$$

$$U_W = M g s \qquad U_W = 4.12\,\mathrm{kJ}$$

$$U_{NP} = -s N_p \qquad U_{NP} = -2.44\,\mathrm{kJ}$$

The difference accounts for a change in kinetic energy.

Problem 14-2

The crate of weight W has a velocity v_A when it is at A. Determine its velocity after it slides down the plane to $s = s'$. The coefficient of kinetic friction between the crate and the plane is μ_k.

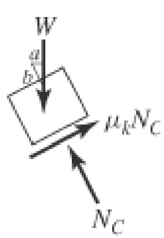
Given:

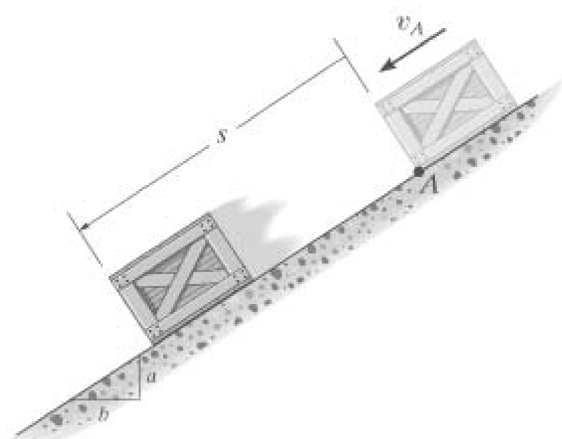
$W = 20\ \mathrm{lb}$ $a = 3$

$v_A = 12\ \dfrac{\mathrm{ft}}{\mathrm{s}}$ $b = 4$

$s' = 6\ \mathrm{ft}$

$\mu_k = 0.2$





Solution:

$$\theta = \operatorname{atan}\left(\frac{a}{b}\right) \qquad N_C = W\cos(\theta) \qquad F = \mu_k N_C$$

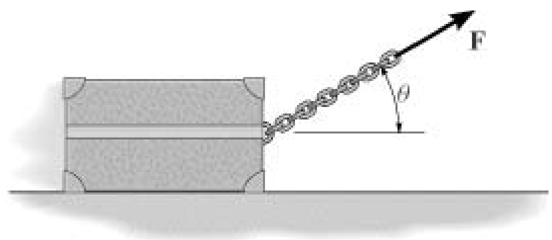
Guess $v' = 1\,\dfrac{\mathrm{m}}{\mathrm{s}}$

Given $$\frac{1}{2}\left(\frac{W}{g}\right)v_A^{\,2} + W\sin(\theta)s' - F s' = \frac{1}{2}\left(\frac{W}{g}\right)v'^{2} \qquad v' = \mathrm{Find}(v') \qquad v' = 17.72\,\frac{\mathrm{ft}}{\mathrm{s}}$$

Problem 14-3

The crate of mass M is subjected to a force having a constant direction and a magnitude F, where s is measured in meters. When $s = s_1$, the crate is moving to the right with a speed v_1. Determine its speed when $s = s_2$. The coefficient of kinetic friction between the crate and the ground is μ_k.

Given:



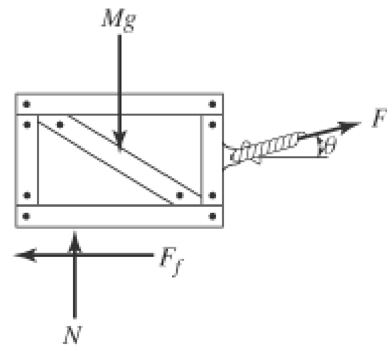
$M = 20\ \text{kg}$ $\quad F = 100\ \text{N}$

$s_1 = 4\ \text{m}$ $\quad \theta = 30\ \text{deg}$

$v_1 = 8\ \dfrac{\text{m}}{\text{s}}$ $\quad a = 1$

$s_2 = 25\ \text{m}$ $\quad b = 1\ \text{m}^{-1}$

$\mu_k = 0.25$



Solution:

Equation of motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N$

$$N + F\sin(\theta) - Mg = 0 \qquad N = Mg - F\sin(\theta)$$

Principle of work and Energy: The horizontal component of force **F** which acts in the direction of displacement does positive work, whereas the friction force $F_f = \mu_k(Mg - F\sin(\theta))$ does negative work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force **F** and the weight of the crate do not displace hence do no work.

$$F\cos(\theta) - \mu_k N = Ma$$

$$F\cos(\theta) - \mu_k(Mg - F\sin(\theta)) = Ma$$

$$a = \frac{F\cos(\theta) - \mu_k(Mg - F\sin(\theta))}{M} \qquad a = 2.503\ \frac{\text{m}}{\text{s}^2}$$

$$v\frac{dv}{ds} = a \qquad \frac{v^2}{2} = \frac{v_1^2}{2} + a(s_2 - s_1)$$

$$v = \sqrt{2\left[\frac{v_1^2}{2} + a(s_2 - s_1)\right]} \qquad v = 13.004\ \frac{\text{m}}{\text{s}}$$

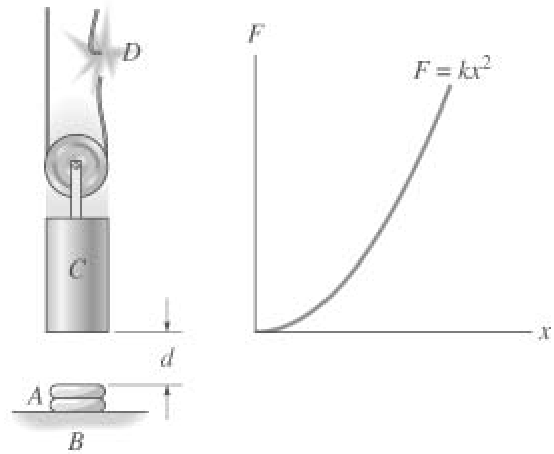
***Problem 14-4**

The “air spring” *A* is used to protect the support structure *B* and prevent damage to the conveyor-belt tensioning weight *C* in the event of a belt failure *D*. The force developed by the spring as a function of its deflection is shown by the graph. If the weight is *W* and it is suspended a height *d* above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.

Given:

$W = 50 \text{ lb} \quad k = 8000 \dfrac{\text{lb}}{\text{ft}^2}$

$d = 1.5 \text{ ft}$



Solution:

$$T_1 + U = T_2$$

$$0 + W(d + \delta) - \int_0^{\delta} k x^2 \, dx = 0$$

Guess $\delta = 1 \text{ in}$

Given $W(d+\delta) - k\left(\dfrac{\delta^3}{3}\right) = 0 \qquad \delta = \text{Find}(\delta) \qquad \delta = 3.896 \text{ in}$

Problem 14-5

A car is equipped with a bumper *B* designed to absorb collisions. The bumper is mounted to the car using pieces of flexible tubing *T*. Upon collision with a rigid barrier at *A*, a constant horizontal force **F** is developed which causes a car deceleration *kg* (the highest safe deceleration for a passenger without a seatbelt). If the car and passenger have a total mass *M* and the car is initially coasting with a speed *v*, determine the magnitude of **F** needed to stop the car and the deformation *x* of the bumper tubing.

Units Used:

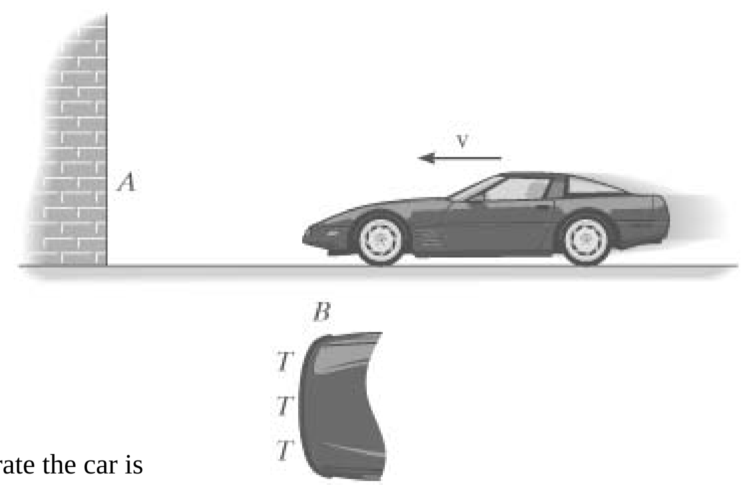
$\text{Mm} = 10^3 \text{ kg}$

$\text{kN} = 10^3 \text{ N}$

Given:

$M = 1.5\,10^3 \text{ kg}$

$v = 1.5 \dfrac{\text{m}}{\text{s}} \qquad k = 3$



Solution:

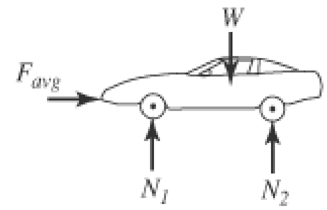
The average force needed to decelerate the car is

$F_{avg} = M k g \qquad F_{avg} = 44.1 \text{ kN}$

The deformation is

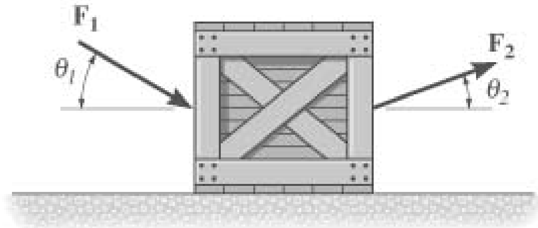
$$T_1 + U_{12} = T_2 \qquad \frac{1}{2}M v^2 - F_{avg}x = 0$$

$$x = \frac{1}{2}M\left(\frac{v^2}{F_{avg}}\right) \qquad x = 38.2\ \text{mm}$$



Problem 14-6

The crate of mass M is subjected to forces $\mathbf{F_1}$ and $\mathbf{F_2}$, as shown. If it is originally at rest, determine the distance it slides in order to attain a speed v. The coefficient of kinetic friction between the crate and the surface is μ_k.



Units Used:

$$\text{kN} = 10^3\ \text{N}$$

Given:

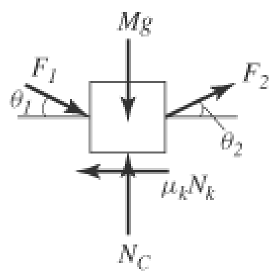
$$M = 100\ \text{kg} \qquad v = 6\ \frac{\text{m}}{\text{s}}$$

$$F_1 = 800\ \text{N} \qquad \mu_k = 0.2$$

$$F_2 = 1.5\ \text{kN} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$\theta_1 = 30\ \text{deg}$$

$$\theta_2 = 20\ \text{deg}$$



Solution:

$$N_C - F_1\sin(\theta_1) - M g + F_2\sin(\theta_2) = 0$$

$$N_C = F_1\sin(\theta_1) + M g - F_2\sin(\theta_2) \qquad N_C = 867.97\ \text{N}$$

$$T_1 + U_{12} = T_2$$

$$F_1\cos(\theta_1)s - \mu_k N_c s + F_2\cos(\theta_2)s = \frac{1}{2}M v^2$$

$$s = \frac{M v^2}{2\left(F_1\cos(\theta_1) - \mu_k N_C + F_2\cos(\theta_2)\right)} \qquad s = 0.933\ \text{m}$$

Problem 14-7

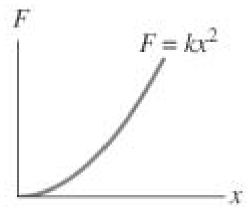
Design considerations for the bumper B on the train car of mass M require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to a distance d when the car, traveling at speed v, strikes the rigid stop. Neglect the mass of the car wheels.

Units Used:

$Mg = 10^3 \text{ kg}$

$kN = 10^3 \text{ N}$

$MN = 10^3 \text{ kN}$

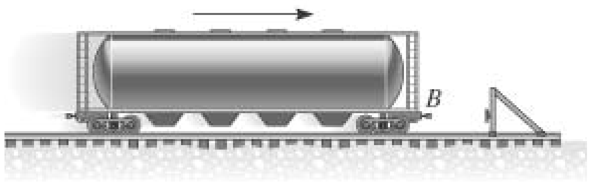


Given:

$M = 5 \text{ Mg}$

$d = 0.2 \text{ m}$

$v = 4 \frac{\text{m}}{\text{s}}$



Solution:

$$\frac{1}{2}Mv^2 - \int_0^d kx^2\, dx = 0 \qquad \frac{1}{2}Mv^2 - \frac{kd^3}{3} = 0 \qquad k = \frac{3Mv^2}{2d^3} \qquad k = 15\frac{\text{MN}}{\text{m}^2}$$

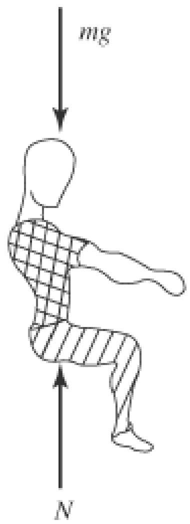
*Problem 14-8

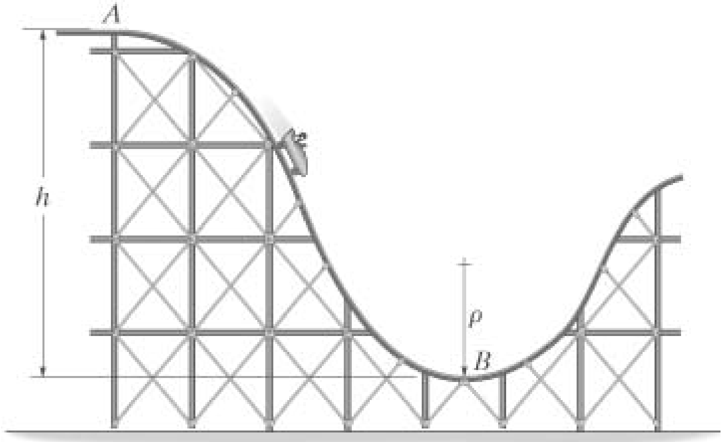
Determine the required height *h* of the roller coaster so that when it is essentially at rest at the crest of the hill it will reach a speed *v* when it comes to the bottom. Also, what should be the minimum radius of curvature ρ for the track at *B* so that the passengers do not experience a normal force greater than *kmg*? Neglect the size of the car and passengers.

Given:

$v = 100 \frac{\text{km}}{\text{hr}}$

$k = 4$





Solution:

$$T_1 + U_{12} = T_2$$

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{1}{2}\frac{v^2}{g}$$

$$h = 39.3 \text{ m}$$

$$kmg - mg = \frac{mv^2}{\rho} \qquad \rho = \frac{v^2}{g(k-1)} \qquad \rho = 26.2 \text{ m}$$

Problem 14-9

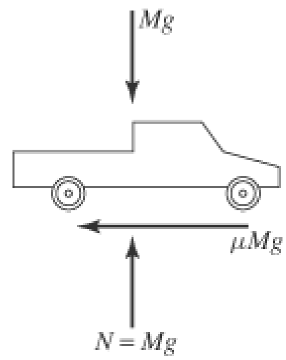
When the driver applies the brakes of a light truck traveling at speed v_1 it skids a distance d_1 before stopping. How far will the truck skid if it is traveling at speed v_2 when the brakes are applied?

Given:

$v_1 = 40\ \frac{\text{km}}{\text{hr}}$

$d_1 = 3\ \text{m}$

$v_2 = 80\ \frac{\text{km}}{\text{hr}}$



Solution:

$\frac{1}{2}M v_1^{\,2} - \mu_k M g d_1 = 0 \qquad \mu_k = \frac{v_1^{\,2}}{2 g d_1} \qquad \mu_k = 2.097$

$\frac{1}{2}M v_2^{\,2} - \mu_k M g d_2 = 0 \qquad d_2 = \frac{v_2^{\,2}}{2\mu_k g} \qquad d_2 = 12\ \text{m}$

Problem 14-10

The ball of mass M of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed a distance δ when $x = 0$. Determine how far x it must be pulled back and released so that the ball will begin to leave the track when $\theta = \theta_1$.

Given:

$M = 0.5\ \text{kg}$

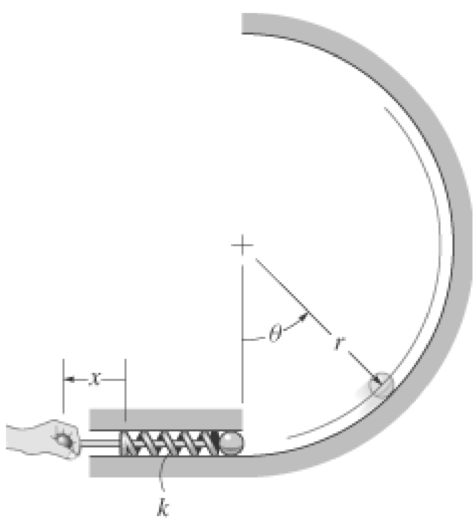
$\delta = 0.08\ \text{m}$

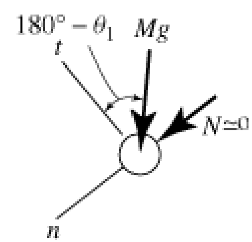
$\theta_1 = 135\ \text{deg}$

$r = 1.5\ \text{m}$

$k = 500\ \frac{\text{N}}{\text{m}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution:

$N = 0 \qquad \theta = \theta_1$

$\Sigma F_n = m a_n \qquad N - M g \cos(\theta) = M\left(\frac{v^2}{r}\right) \qquad v = \sqrt{-g r \cos(\theta)} \qquad v = 3.226\ \frac{\text{m}}{\text{s}}$

Guess $x = 10\ \text{mm}$

Given $\displaystyle\int_{x+\delta}^{\delta} -kx\,dx - Mgr(1-\cos(\theta)) = \frac{1}{2}Mv^2$ $x = \text{Find}(x)$ $x = 178.9\,\text{mm}$

Problem 14-11

The force **F**, acting in a constant direction on the block of mass M, has a magnitude which varies with the position x of the block. Determine how far the block slides before its velocity becomes v_1. When $x = 0$, the block is moving to the right at speed v_0 . The coefficient of kinetic friction between the block and surface is μ_k.

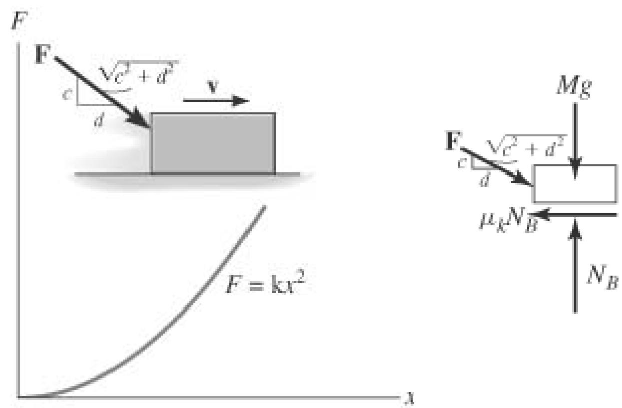
Given:

$M = 20\ \text{kg}$ $c = 3$

$v_1 = 5\ \frac{\text{m}}{\text{s}}$ $d = 4$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$ $k = 50\ \frac{\text{N}}{\text{m}^2}$

$\mu_k = 0.3$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

$$N_B - Mg - \left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2 = 0 \qquad N_B = Mg + \left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2$$

Guess $\delta = 2\ \text{m}$

Given

$$\frac{1}{2}Mv_0^2 + \left(\frac{d}{\sqrt{c^2+d^2}}\right)\int_0^{\delta} kx^2\,dx - \mu_k Mg\delta - \mu_k\int_0^{\delta}\left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2\,dx = \frac{1}{2}Mv_1^2$$

$\delta = \text{Find}(\delta)$ $\delta = 3.413\ \text{m}$

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*__Problem 14-12__

The force **F**, acting in a constant direction on the block of mass M, has a magnitude which varies with position x of the block. Determine the speed of the block after it slides a distance d_1 . When $x = 0$, the block is moving to the right at v_0 . The coefficient of kinetic friction between the block and surface is μ_k.

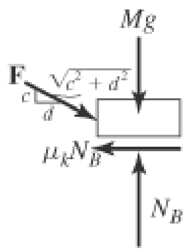
Given:

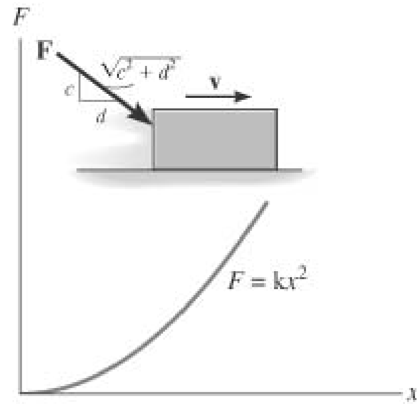
$M = 20\ \text{kg}$ $c = 3$

$d_1 = 3\ \text{m}$ $d = 4$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$ $k = 50\ \frac{\text{N}}{\text{m}^2}$

$\mu_k = 0.3$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution:

$$N_B - Mg - \left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2 = 0 \quad N_B = Mg + \left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2$$

Guess $v_1 = 2\frac{\text{m}}{\text{s}}$

Given

$$\frac{1}{2}Mv_0^2 + \left(\frac{d}{\sqrt{c^2+d^2}}\right)\int_0^{d_1} kx^2\, dx - \mu_k Mg d_1 - \mu_k \int_0^{d_1} \left(\frac{c}{\sqrt{c^2+d^2}}\right)kx^2\, dx = \frac{1}{2}Mv_1^2$$

$v_1 = \text{Find}(v_1)$ $v_1 = 3.774\ \frac{\text{m}}{\text{s}}$

Problem 14-13

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so by considering the block of mass M which rests on the smooth surface and is subjected to horizontal force **F**. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of v_0 and travels a distance d, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of v_B relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.

Given:

$M = 10\ \text{kg}$

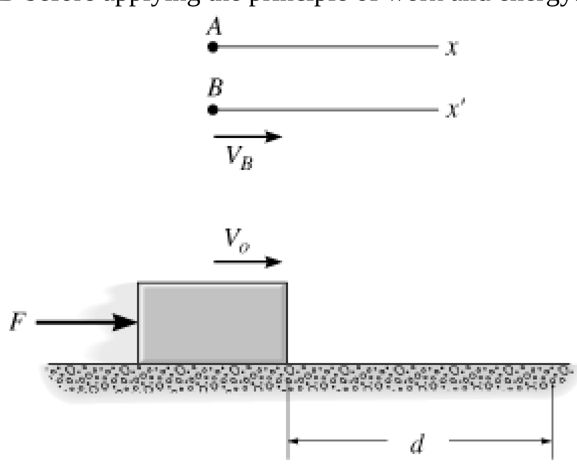
$F = 6\ \text{N}$

$v_0 = 5\ \frac{\text{m}}{\text{s}}$

$d = 10\ \text{m}$

$v_B = 2\ \frac{\text{m}}{\text{s}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Observer *A:*

$$\frac{1}{2}M v_0^2 + F d = \frac{1}{2}M v_2^2 \qquad v_2 = \sqrt{v_0^2 + \frac{2F d}{M}} \qquad v_2 = 6.083\ \frac{\text{m}}{\text{s}}$$

$$F = M a \qquad a = \frac{F}{M} \qquad a = 0.6\ \frac{\text{m}}{\text{s}^2} \qquad \text{Guess} \quad t = 1\text{s}$$

Given $\quad d = 0 + v_0 t + \frac{1}{2}a t^2 \qquad t = \text{Find}(t) \qquad t = 1.805\ \text{s}$

Observer *B:*

$$d' = (v_0 - v_B)t + \frac{1}{2}a t^2 \qquad d' = 6.391\ \text{m}$$

The distance that the block moves as seen by observer *B*.

$$\frac{1}{2}M(v_0 - v_B)^2 + F d' = \frac{1}{2}M v'_2{}^2 \qquad v'_2 = \sqrt{(v_0 - v_B)^2 + \frac{2F d'}{M}} \qquad v'_2 = 4.083\ \frac{\text{m}}{\text{s}}$$

Notice that $\quad v_2 = v'_2 + v_B$

Problem 14-14

Determine the velocity of the block *A* of weight W_A if the two blocks are released from rest and the block *B* of weight W_B moves a distance *d* up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is μ_k.

Given:

$W_A = 60\ \text{lb}$

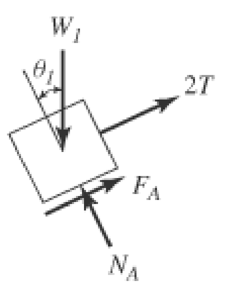
$W_B = 40\ \text{lb}$

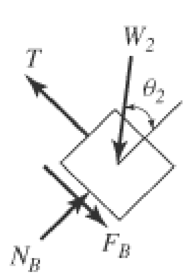
$\theta_1 = 60\ \text{deg}$

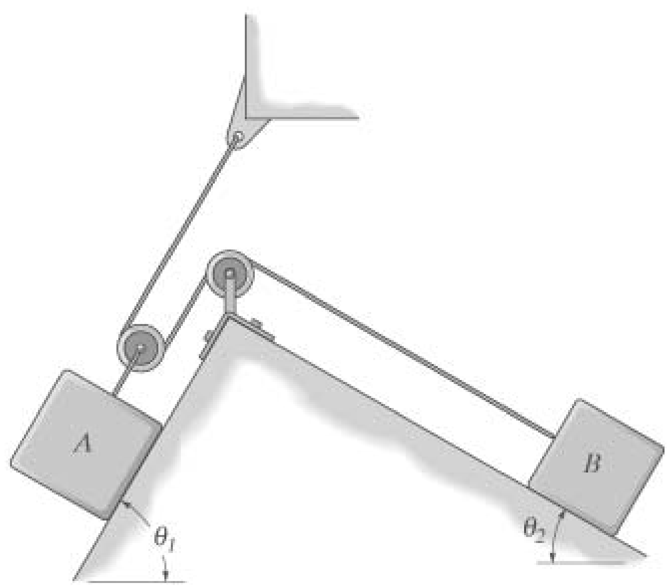
$\theta_2 = 30\ \text{deg}$

$d = 2\ \text{ft}$

$\mu_k = 0.10$







Solution:

$L = 2s_A + s_B$

$0 = 2v_A + v_B$

Guesses

$v_A = 1\frac{\text{ft}}{\text{s}} \qquad v_B = -1\frac{\text{ft}}{\text{s}}$

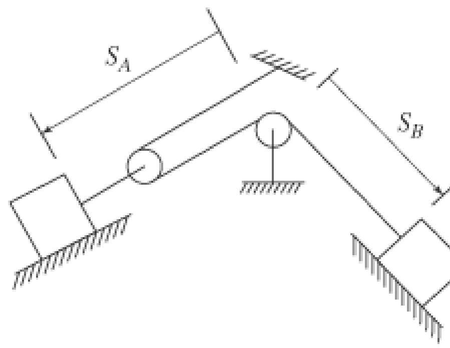
Given

$$0 = 2v_A + v_B$$

$$W_A\left(\frac{d}{2}\right)\sin(\theta_1) - W_B\, d\sin(\theta_2) - \mu_k W_A\cos(\theta_1)\frac{d}{2}\ \ldots = \frac{1}{2g}\left(W_A v_A^2 + W_B v_B^2\right)$$
$$+ -\mu_k W_B\cos(\theta_2) d$$

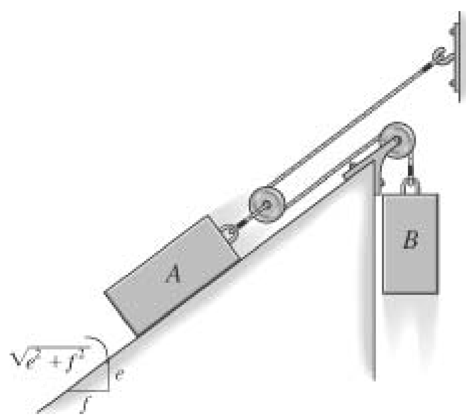
$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \text{Find}(v_A, v_B)$$

$v_B = -1.543\ \dfrac{\text{ft}}{\text{s}}$ $\qquad$ $v_A = 0.771\ \dfrac{\text{ft}}{\text{s}}$



Problem 14-15

Block A has weight W_A and block B has weight W_B. Determine the speed of block A after it moves a distance d down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



Given:

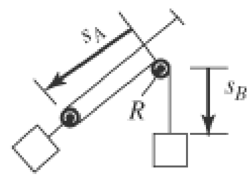
$W_A = 60\ \text{lb}$ $\qquad$ $e = 3$

$W_B = 10\ \text{lb}$ $\qquad$ $f = 4$

$d = 5\ \text{ft}$ $\qquad$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

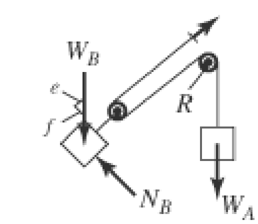
Solution:

$L = 2s_A + s_B$ $\qquad$ $0 = 2\Delta s_A + \Delta s_B$ $\qquad$ $0 = 2v_A + v_B$



$$0 + W_A\left(\frac{e}{\sqrt{e^2 + f^2}}\right)d - W_B 2d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(2v_A)^2$$

$$v_A = \sqrt{\frac{2g\,d}{W_A + 4W_B}\left(W_A\frac{e}{\sqrt{e^2 + f^2}} - 2W_B\right)} \qquad v_A = 7.178\ \frac{\text{ft}}{\text{s}}$$



***Problem 14-16**

The block A of weight W_A rests on a surface for which the coefficient of kinetic friction is μ_k. Determine the distance the cylinder B of weight W_B must descend so that A has a speed v_A starting from rest.

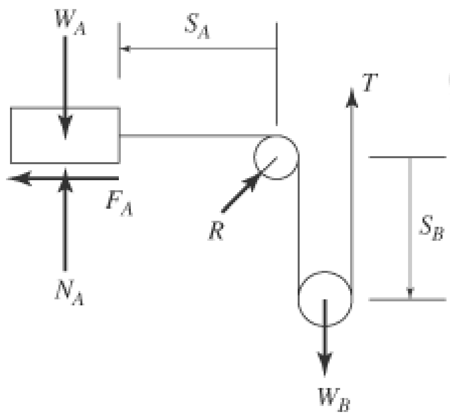
Given:

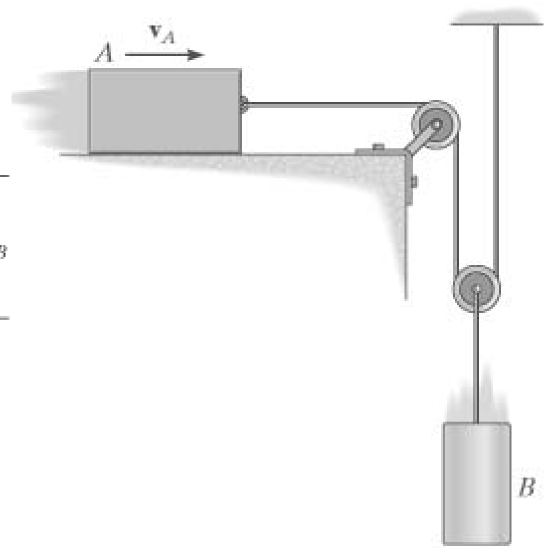
$W_A = 3\text{ lb}$

$W_B = 8\text{ lb}$

$\mu_k = 0.3$

$v_A = 5\ \frac{\text{ft}}{\text{s}}$





Solution:

$L = s_A + 2s_B$

Guesses $d = 1\text{ ft}$

Given

$$W_B d - \mu_k W_A 2d = \frac{1}{2g}\left[W_A v_A^2 + W_B\left(\frac{v_A}{2}\right)^2\right]$$

$d = \text{Find}(d)$ $d = 0.313\text{ ft}$

Problem 14-17

The block of weight W slides down the inclined plane for which the coefficient of kinetic friction is μ_k. If it is moving at speed v when it reaches point A, determine the maximum deformation of the spring needed to momentarily arrest the motion.

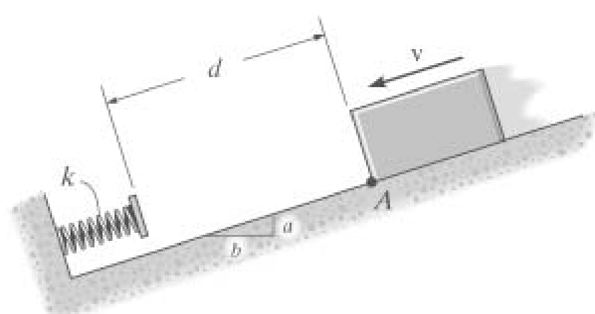
Given:

$W = 100\text{ lb}$ $a = 3\text{ m}$

$v = 10\ \frac{\text{ft}}{\text{s}}$ $b = 4\text{ m}$

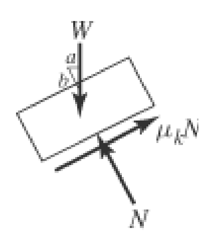
$d = 10\text{ ft}$

$k = 200\ \frac{\text{lb}}{\text{ft}}$ $\mu_k = 0.25$



Solution:

$$N = \left(\frac{b}{\sqrt{a^2 + b^2}}\right)W \qquad N = 80\text{ lb}$$



Initial Guess

$d_{max} = 5\text{ m}$

Given $\frac{1}{2}\left(\frac{W}{g}\right)v^2 - \mu_k N(d + d_{max}) - \frac{1}{2}k\,d_{max}^2 + W(d + d_{max})\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = 0$

$d_{max} = \text{Find}(d_{max}) \qquad d_{max} = 2.56\,\text{ft}$

Problem 14-18

The collar has mass *M* and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length *l*. If the collar is displaced a distance $s = s'$ and released from rest, determine its velocity at the instant it returns to the point $s = 0$.

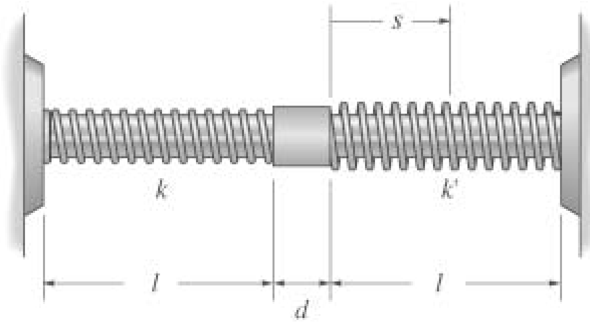
Given:

$M = 20\text{ kg}$ $\quad k = 50\,\frac{\text{N}}{\text{m}}$

$s' = 0.5\text{ m}$

$l = 1\text{ m}$ $\quad k' = 100\,\frac{\text{N}}{\text{m}}$

$d = 0.25\text{ m}$

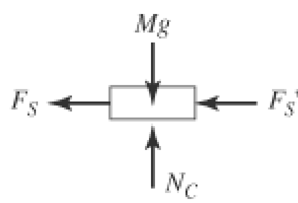


Solution:

$\frac{1}{2}k s'^2 + \frac{1}{2}k' s'^2 = \frac{1}{2}M v_C^2$

$v_C = \sqrt{\frac{k + k'}{M}}\cdot s'$

$v_C = 1.37\,\frac{\text{m}}{\text{s}}$



Problem 14-19

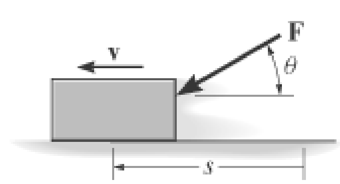
The block of mass *M* is subjected to a force having a constant direction and a magnitude $F = k/(a+bx)$. When $x = x_1$, the block is moving to the left with a speed v_1. Determine its speed when $x = x_2$. The coefficient of kinetic friction between the block and the ground is μ_k.

Given:

$M = 2\text{ kg}$ $\quad b = 1\text{ m}^{-1}$ $\quad x_2 = 12\text{ m}$ $\quad g = 9.81\,\frac{\text{m}}{\text{s}^2}$

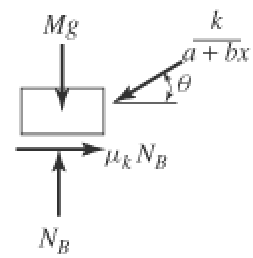
$k = 300\text{ N}$ $\quad x_1 = 4\text{ m}$ $\quad \theta = 30\text{ deg}$

$a = 1$ $\quad v_1 = 8\,\frac{\text{m}}{\text{s}}$ $\quad \mu_k = 0.25$



Solution:

$$N_B - Mg - \left(\frac{k}{a+bx}\right)\sin(\theta) = 0 \qquad N_B = Mg + \frac{k\sin(\theta)}{a+bx}$$



$$U = \int_{x_1}^{x_2} \frac{k\cos(\theta)}{a+bx}\,dx - \mu_k \int_{x_1}^{x_2} Mg + \frac{k\sin(\theta)}{a+bx}\,dx \qquad U = 173.177\ \text{N}\cdot\text{m}$$

$$\frac{1}{2}Mv_1^2 + U = \frac{1}{2}Mv_2^2 \qquad v_2 = \sqrt{v_1^2 + \frac{2U}{M}} \qquad v_2 = 15.401\ \frac{\text{m}}{\text{s}}$$

***Problem 14-20**

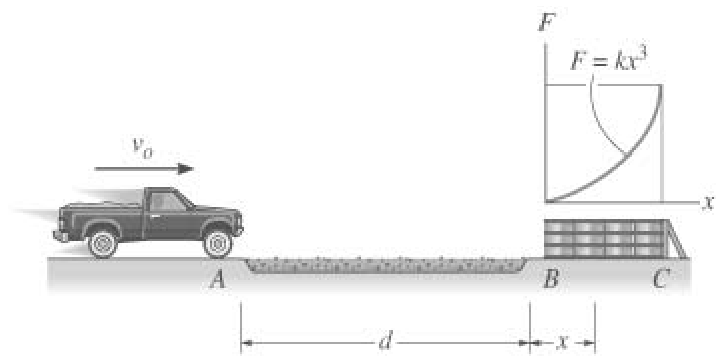
The motion of a truck is arrested using a bed of loose stones *AB* and a set of crash barrels *BC*. If experiments show that the stones provide a rolling resistance F_t per wheel and the crash barrels provide a resistance as shown in the graph, determine the distance x the truck of weight W penetrates the barrels if the truck is coasting at speed v_0 when it approaches A. Neglect the size of the truck.

Given:

$$F_t = 160\ \text{lb} \qquad d = 50\ \text{ft}$$

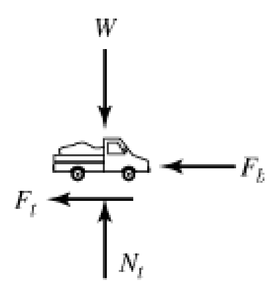
$$W = 4500\ \text{lb} \qquad k = 1000\ \frac{\text{lb}}{\text{ft}^3}$$

$$v_0 = 60\ \frac{\text{ft}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$



Solution:

$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 - 4F_t d - k\frac{x^4}{4} = 0$$



$$x = \left(\frac{2Wv_0^2}{kg} - \frac{16F_t d}{k}\right)^{\frac{1}{4}} \qquad x = 5.444\ \text{ft}$$

Problem 14-21

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having weight W will penetrate the barrier if it is originally traveling at speed v_0 when it strikes the first barrel.

Units Used:

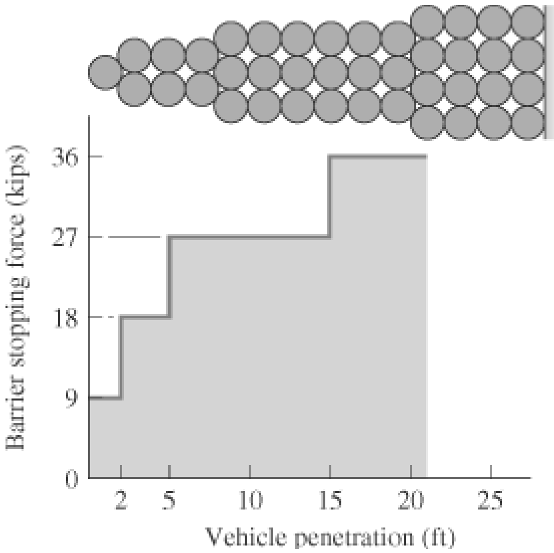
$\text{kip} = 10^3 \text{ lb}$

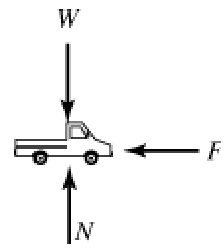
Given:

$W = 4000 \text{ lb}$

$v_0 = 55 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

$$\frac{1}{2}\left(\frac{W}{g}\right)v_0^2 - Area = 0$$

$$Area = \frac{1}{2}\left(\frac{W}{g}\right)v_0^2 \qquad Area = 187.888 \text{ kip}\cdot\text{ft}$$

We must produce this much work with the barrels.

Assume that $5 \text{ ft} < x < 15 \text{ ft}$

$$Area = (2 \text{ ft})(9 \text{ kip}) + (3 \text{ ft})(18 \text{ kip}) + (x - 5 \text{ ft})(27 \text{ kip})$$

$$x = \frac{Area - 72 \text{ kip}\cdot\text{ft}}{27 \text{ kip}} + 5 \text{ ft} \qquad x = 9.292 \text{ ft}$$

Check that the assumption is corrrect!

Problem 14-22

The collar has a mass M and is supported on the rod having a coefficient of kinetic friction μ_k. The attached spring has an unstretched length l and a stiffness k. Determine the speed of the collar after the applied force F causes it to be displaced a distance $s = s_1$ from point A. When s = 0 the collar is held at rest.

Given:

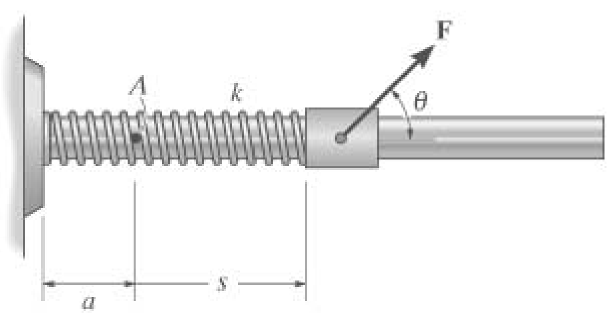
$M = 30 \text{ kg}$ $\quad \mu_k = 0.4$

$a = 0.5 \text{ m}$ $\quad \theta = 45 \text{ deg}$

$F = 200 \text{ N}$ $\quad s_1 = 1.5 \text{ m}$

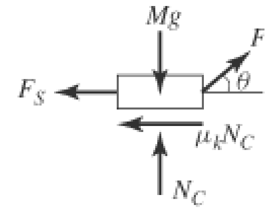
$l = 0.2 \text{ m}$ $\quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$k = 50 \frac{\text{N}}{\text{m}}$



Solution:

Guesses $N_C = 1\ \text{N}$ $v = 1\ \dfrac{\text{m}}{\text{s}}$



Given

$$N_C - Mg + F\sin(\theta) = 0$$

$$F\cos(\theta)s_1 - \mu_k N_C s_1 + \frac{1}{2}k(a-l)^2 - \frac{1}{2}k(s_1+a-l)^2 = \frac{1}{2}Mv^2$$

$$\begin{pmatrix} N_C \\ v \end{pmatrix} = \text{Find}(N_C, v) \qquad N_C = 152.9\ \text{N} \qquad v = 1.666\ \frac{\text{m}}{\text{s}}$$

Problem 14-23

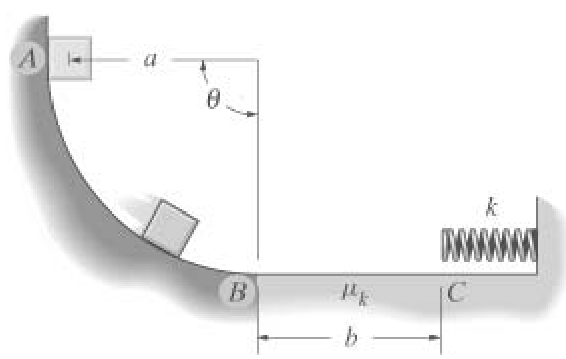
The block of weight W is released from rest at A and slides down the smooth circular surface AB. It then continues to slide along the horizontal rough surface until it strikes the spring. Determine how far it compresses the spring before stopping.

Given:

$W = 5\ \text{lb}$ $\mu_k = 0.2$

$a = 3\ \text{ft}$ $\theta = 90\ \text{deg}$

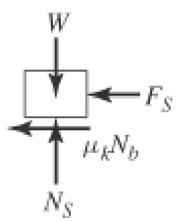
$b = 2\ \text{ft}$ $k = 40\ \dfrac{\text{lb}}{\text{ft}}$



Solution:

Guess $d = 1\ \text{ft}$

Given



$$Wa - \mu_k W(b+d) - \frac{1}{2}kd^2 = 0$$

$$d = \text{Find}(d) \qquad d = 0.782\ \text{ft}$$

---

*__Problem 14-24__

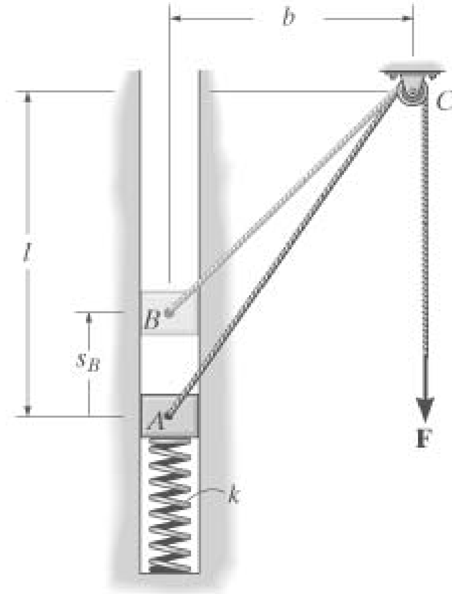
The block has a mass M and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force **F** which must be applied to the cord so that the block attains a speed v_B when it reaches s_B. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\ \Delta l$.

Given:

$M = 0.8\ \text{kg}$ $l = 0.4\ \text{m}$

$v_B = 2.5\ \dfrac{\text{m}}{\text{s}}$ $b = 0.3\ \text{m}$

$s_B = 0.15\ \text{m}$ $\quad k = 100\ \frac{\text{N}}{\text{m}}$



Solution:

$$\Delta l = \sqrt{l^2 + b^2} - \sqrt{(l - s_B)^2 + b^2}$$

Guess $\quad F = 1\ \text{N}$

Given

$$F\Delta l - M g s_B - \frac{1}{2}k s_B^2 = \frac{1}{2}M v_B^2$$

$F = \text{Find}(F)$ $\quad F = 43.9\ \text{N}$

Problem 14-25

The collar has a mass M and is moving at speed v_1 when $x = 0$ and a force of **F** is applied to it. The direction θ of this force varies such that $\theta = ax$, where θ is clockwise, measured in degrees. Determine the speed of the collar when $x = x_1$. The coefficient of kinetic friction between the collar and the rod is μ_k.

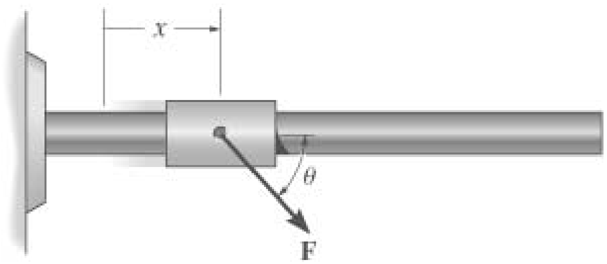
Given:

$M = 5\ \text{kg}$ $\quad v_1 = 8\ \frac{\text{m}}{\text{s}}$

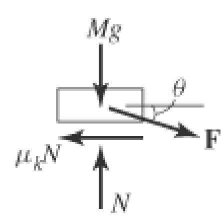
$F = 60\ \text{N}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\mu_k = 0.3$

$x_1 = 3\ \text{m}$ $\quad a = 10\ \frac{\text{deg}}{\text{m}}$



Solution:



$$N = F\sin(\theta) + M g$$

Guess $\quad v = 5\ \frac{\text{m}}{\text{s}}$

Given $\quad \frac{1}{2}M v_1^2 + \int_0^{x_1} F\cos(a x)\ dx - \mu_k \int_0^{x_1} F\sin(a x) + M g\ dx = \frac{1}{2}M v^2$

$v = \text{Find}(v)$ $\quad v = 10.47\ \frac{\text{m}}{\text{s}}$

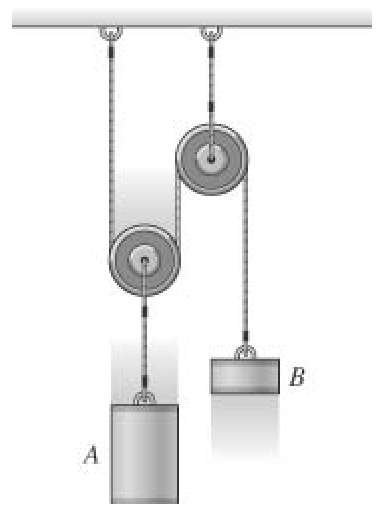
Problem 14-26

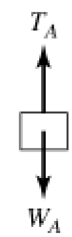
Cylinder *A* has weight W_A and block *B* has weight W_B. Determine the distance *A* must descend from rest before it obtains speed v_A. Also, what is the tension in the cord supporting block *A*? Neglect the mass of the cord and pulleys.

Given:

$$W_A = 60\ \text{lb} \qquad v_A = 8\ \frac{\text{ft}}{\text{s}}$$

$$W_B = 10\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$





Solution:

$$L = 2s_A + s_B \qquad 0 = 2v_A + v_B$$

System

$$0 + W_A d - W_B 2d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(2v_A)^2$$

$$d = \frac{\left(\frac{W_A + 4W_B}{2g}\right)v_A^2}{W_A - 2W_B} \qquad d = 2.484\ \text{ft}$$

Block *A* alone

$$0 + W_A d - T d = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 \qquad T = W_A - \frac{W_A v_A^2}{2g\,d} \qquad T = 36\ \text{lb}$$

Problem 14-27

The conveyor belt delivers crate each of mass *M* to the ramp at *A* such that the crate's velocity is v_A, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is μ_k, determine the speed at which each crate slides off the ramp at *B*. Assume that no tipping occurs.

Given:

$$M = 12\ \text{kg}$$

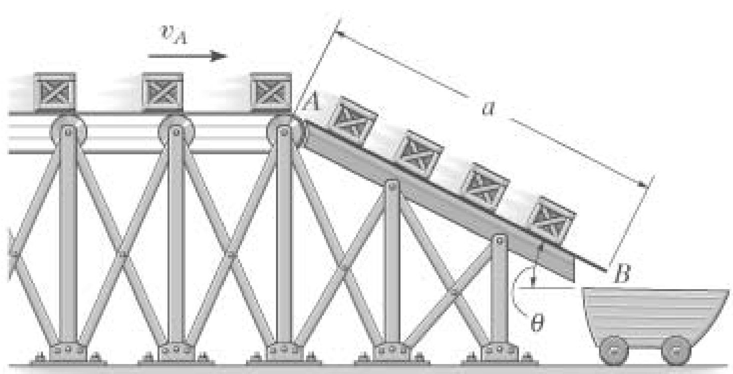
$$v_A = 2.5\ \frac{\text{m}}{\text{s}}$$

$$\mu_k = 0.3$$

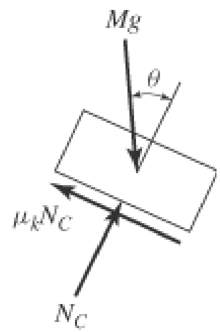
$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

$$\theta = 30\ \text{deg}$$

$$a = 3\ \text{m}$$



Solution:



$$N_C = Mg\cos(\theta)$$

$$\frac{1}{2}Mv_A^2 + (Mga)\sin(\theta) - \mu_k N_C a = \frac{1}{2}Mv_B^2$$

$$v_B = \sqrt{v_A^2 + (2ga)\sin(\theta) - (2\mu_k g)\cos(\theta)a}$$

$$v_B = 4.52\ \frac{\text{m}}{\text{s}}$$

***Problem 14-28**

When the skier of weight W is at point A he has a speed v_A. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.

Given:

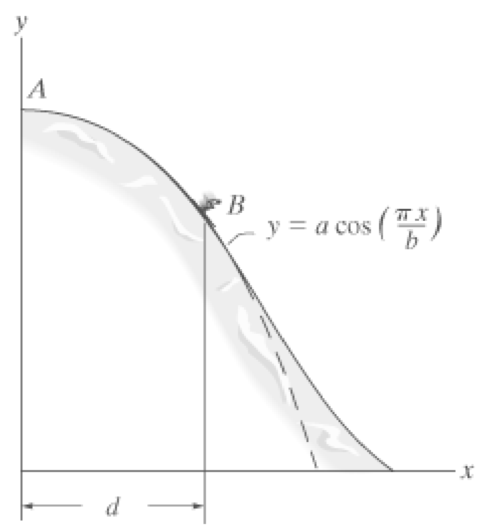
$W = 150\ \text{lb}$

$v_A = 5\ \frac{\text{ft}}{\text{s}}$

$a = 50\ \text{ft}$

$b = 100\ \text{ft}$

$d = 35\ \text{ft}$



Solution:

$$y(x) = (a)\cos\left(\pi\frac{x}{b}\right) \qquad y'(x) = \frac{d}{dx}y(x)$$

$$y''(x) = \frac{d}{dx}y'(x) \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta_B = \operatorname{atan}(y'(d)) \qquad \rho_B = \rho(d)$$

Guesses $F_N = 1\ \text{lb}$ $\quad v' = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad v_B = 1\ \frac{\text{ft}}{\text{s}}$

Given $$\frac{1}{2}\left(\frac{W}{g}\right)v_A^2 + W(y(0\ \text{ft}) - y(d)) = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2$$

$$F_N - W\cos(\theta_B) = \left(\frac{W}{g}\right)\frac{v_B^2}{\rho_B} \qquad -W\sin(\theta_B) = \left(\frac{W}{g}\right)v'$$

$$\begin{pmatrix} v_B \\ F_N \\ v' \end{pmatrix} = \text{Find}(v_B, F_N, v') \qquad v_B = 42.2\,\frac{\text{ft}}{\text{s}} \qquad F_N = 50.6\,\text{lb} \qquad v' = 26.2\,\frac{\text{ft}}{\text{s}^2}$$

Problem 14-29

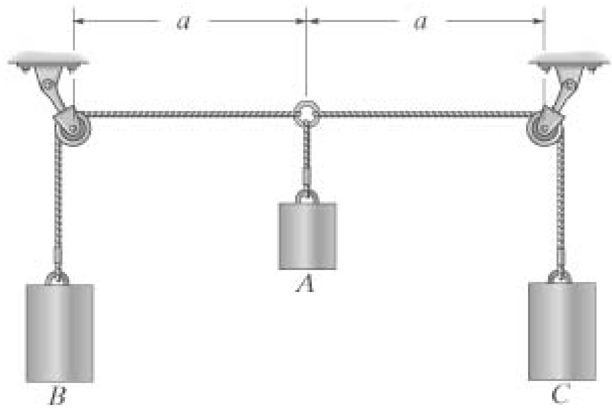
When the block A of weight W_1 is released from rest it lifts the two weights B and C each of weight W_2. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.

Given:

$W_1 = 12\ \text{lb}$

$W_2 = 15\ \text{lb}$

$a = 4\ \text{ft}$



Solution:

Guess $\quad y = 10\ \text{ft}$

Given $\quad W_1 y - 2W_2\left(\sqrt{a^2 + y^2} - a\right) = 0 \qquad y = \text{Find}(y) \qquad y = 3.81\,\text{ft}$

Problem 14-30

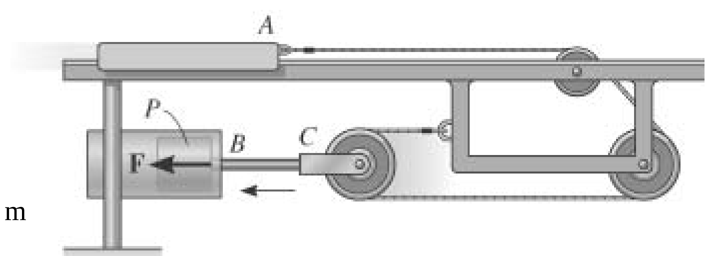
The catapulting mechanism is used to propel slider A of mass M to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies constant force $\mathbf{F}$ to rod BC such that it moves it a distance d, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.

Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 10\ \text{kg} \quad F = 20\ \text{kN} \quad d = 0.2\ \text{m}$



Solution:

$$0 + F\,d = \frac{1}{2}M v^2 \qquad v = \sqrt{\frac{2F\,d}{M}} \qquad v = 28.284\ \frac{\text{m}}{\text{s}}$$

Problem 14-31

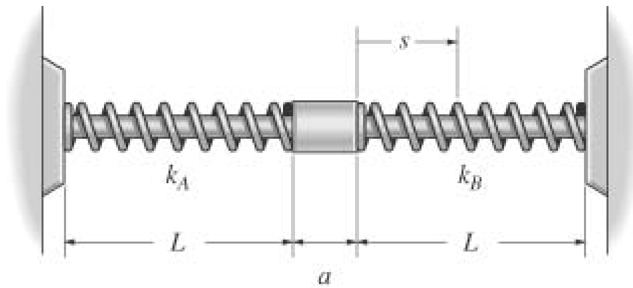
The collar has mass M and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length L and the collar has speed v_0 when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.

Given:

$M = 20\ \text{kg}$ $\quad a = 0.25\ \text{m}$

$L = 1\ \text{m}$ $\quad k_A = 50\ \frac{\text{N}}{\text{m}}$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$ $\quad k_B = 100\ \frac{\text{N}}{\text{m}}$



Solution:

$$\frac{1}{2}M v_0^2 - \frac{1}{2}(k_A + k_B) d^2 = 0 \qquad d = \sqrt{\frac{M}{k_A + k_B}}\, v_0 \qquad d = 0.73\ \text{m}$$

*****Problem 14-32**

The cyclist travels to point A, pedaling until he reaches speed v_A. He then coasts freely up the curved surface. Determine the normal force he exerts on the surface when he reaches point B. The total mass of the bike and man is M. Neglect friction, the mass of the wheels, and the size of the bicycle.

Units Used:

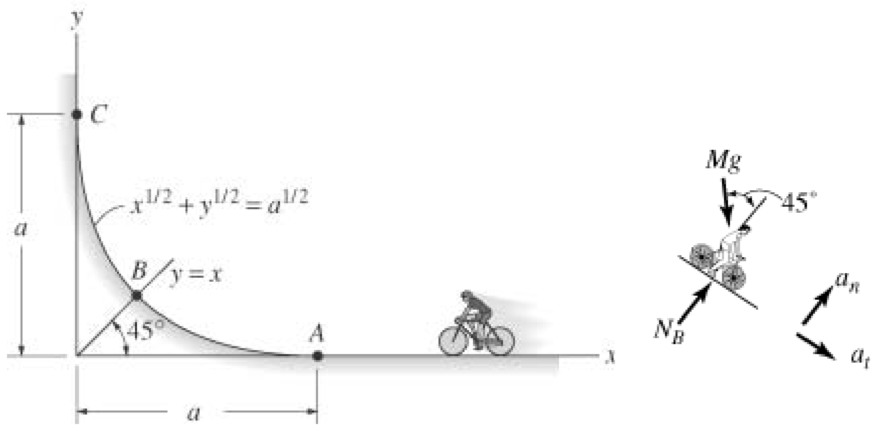
$\text{kN} = 10^3\ \text{N}$

Given:

$v_A = 8\ \frac{\text{m}}{\text{s}}$

$M = 75\ \text{kg}$

$a = 4\ \text{m}$



Solution:

When $\quad y = x \qquad 2\sqrt{y} = \sqrt{a} \qquad y = \frac{a}{4} \qquad y = 1\ \text{m}$

$$\frac{1}{2}M v_A^2 - M g y = \frac{1}{2}M v_B^2 \qquad v_B = \sqrt{v_A^2 - 2 g y} \qquad v_B = 6.662\ \frac{\text{m}}{\text{s}}$$

Now find the radius of curvature $\quad \sqrt{x} + \sqrt{y} = \sqrt{a} \qquad \frac{1}{2\sqrt{x}}dx + \frac{1}{2\sqrt{y}}dy = 0$

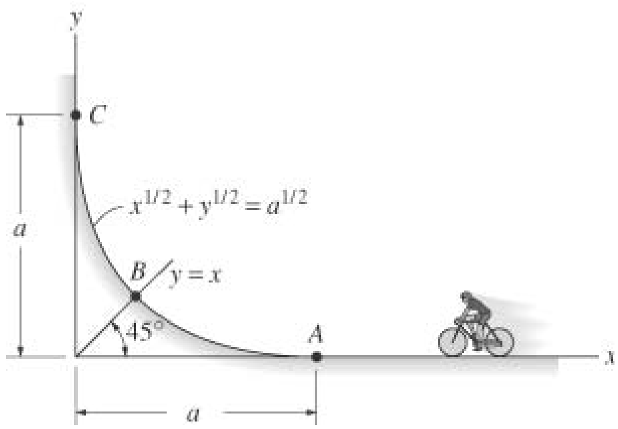
$$y' = -\sqrt{\frac{y}{x}} \qquad y'' = \frac{y - x\frac{d}{dx}y}{2x^2}\sqrt{\frac{x}{y}} \qquad \text{When} \quad y = x \quad y' = -1 \quad y'' = \frac{1}{y}$$

Thus $\rho = \dfrac{\sqrt{\left(1 + y'^2\right)^3}}{y''}$ $\qquad \rho = \sqrt{8}\,y \qquad \rho = 2.828\ \text{m}$

$$N_B - M g \cos(45\ \text{deg}) = M\left(\frac{v_B^2}{\rho}\right) \qquad N_B = M g \cos(45\ \text{deg}) + M\left(\frac{v_B^2}{\rho}\right) \qquad N_B = 1.697\ \text{kN}$$

Problem 14-33

The cyclist travels to point A, pedaling until he reaches speed v_A. He then coasts freely up the curved surface. Determine how high he reaches up the surface before he comes to a stop. Also, what are the resultant normal force on the surface at this point and his acceleration? The total mass of the bike and man is M. Neglect friction, the mass of the wheels, and the size of the bicycle.

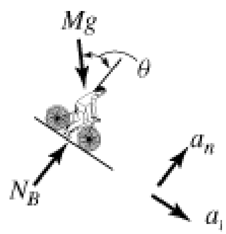


Given:

$v_A = 4\ \frac{\text{m}}{\text{s}} \qquad M = 75\ \text{kg} \qquad a = 4\ \text{m}$

Solution:

$\frac{1}{2}M v_A^2 - M g y = 0 \qquad y = \frac{v_A^2}{2g} \qquad y = 0.815\ \text{m}$



$x = \left(\sqrt{a} - \sqrt{y}\right)^2 \qquad x = 1.203\ \text{m}$

$y' = -\sqrt{\frac{y}{x}} \qquad \theta = \text{atan}\left(|y'|\right) \qquad \theta = 39.462\ \text{deg}$

$N_B - M g \cos(\theta) = 0 \qquad N_B = M g \cos(\theta) \qquad N_B = 568.03\ \text{N}$

$M g \sin(\theta) = M a_t \qquad a_t = g \sin(\theta) \qquad a_t = 6.235\ \frac{\text{m}}{\text{s}^2}$

Problem 14-34

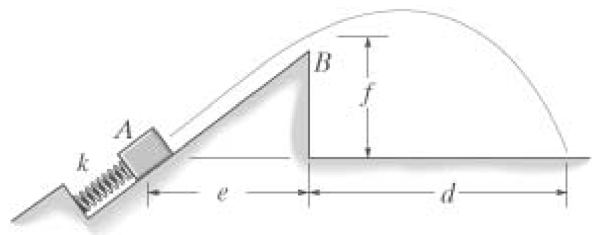
The block of weight W is pressed against the spring so as to compress it a distance δ when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.

Given:

$W = 10$ lb $\quad e = 4$ ft

$\delta = 2$ ft $\quad f = 3$ ft

$k = 100 \frac{\text{lb}}{\text{ft}} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$



Solution: $\theta = \operatorname{atan}\left(\frac{f}{e}\right)$

Guesses $\quad v_B = 1 \frac{\text{ft}}{\text{s}} \quad t = 1$ s $\quad d = 1$ ft

Given

$$\frac{1}{2}k\delta^2 - W f = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2 \qquad d = v_B\cos(\theta)t \qquad 0 = f + v_B\sin(\theta)t - \left(\frac{g}{2}\right)t^2$$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \qquad v_B = 33.08 \frac{\text{ft}}{\text{s}} \qquad t = 1.369 \text{ s} \qquad d = 36.2 \text{ ft}$$

Problem 14-35

The man at the window A wishes to throw a sack of mass M onto the ground. To do this he allows it to swing from rest at B to point C, when he releases the cord at $\theta = \theta_1$. Determine the speed at which it strikes the ground and the distance R.

Given:

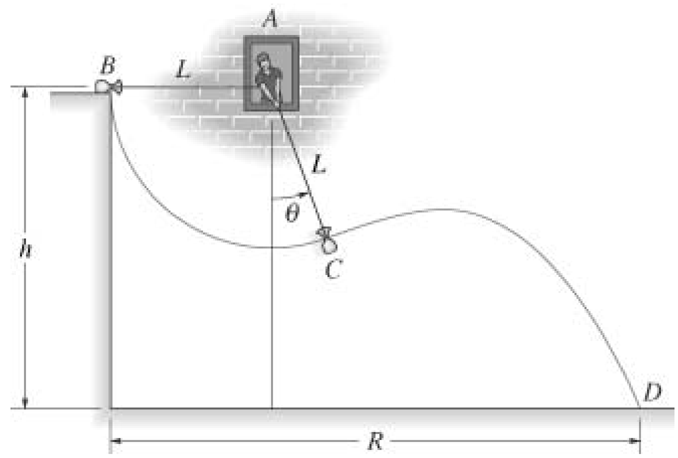
$\theta_1 = 30$ deg

$h = 16$ m

$L = 8$ m

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

$M = 30$ kg



Solution:

$$0 + M g L\cos(\theta_1) = \frac{1}{2}M v_C^2 \qquad v_C = \sqrt{2 g L\cos(\theta_1)} \qquad v_C = 11.659 \frac{\text{m}}{\text{s}}$$

$0 + Mgh = \frac{1}{2}Mv_D^2$ $\quad v_D = \sqrt{2gh}$ $\quad v_D = 17.718\ \frac{m}{s}$

Free Flight Guess $t = 2\ s$ $\quad R = 1\ m$

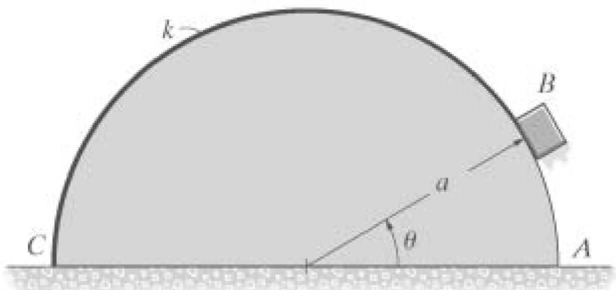
Given $0 = \left(\frac{-g}{2}\right)t^2 + v_C \sin(\theta_1)t + h - L\cos(\theta_1)$ $\quad R = v_C \cos(\theta_1)t + L(1 + \sin(\theta_1))$

$\begin{pmatrix} t \\ R \end{pmatrix} = \text{Find}(t, R)$ $\quad t = 2.078\ s$ $\quad R = 33.0\ m$

---

*Problem 14-36**

A block of weight W rests on the smooth semicylindrical surface. An elastic cord having a stiffness k is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = \theta_1$. Neglect the size of the block.



Given:

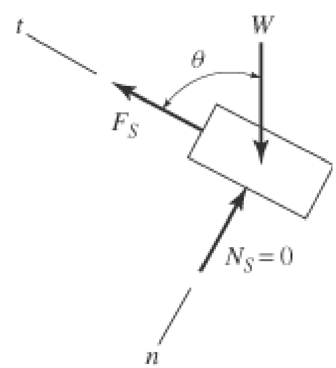
$W = 2\ lb$

$k = 2\ \frac{lb}{ft}$

$\theta_1 = 45\ deg$

$a = 1.5\ ft$

$g = 9.81\ \frac{m}{s^2}$



Solution:

Guess $\delta = 1\ ft$ $\quad v_1 = 1\ \frac{ft}{s}$

Given

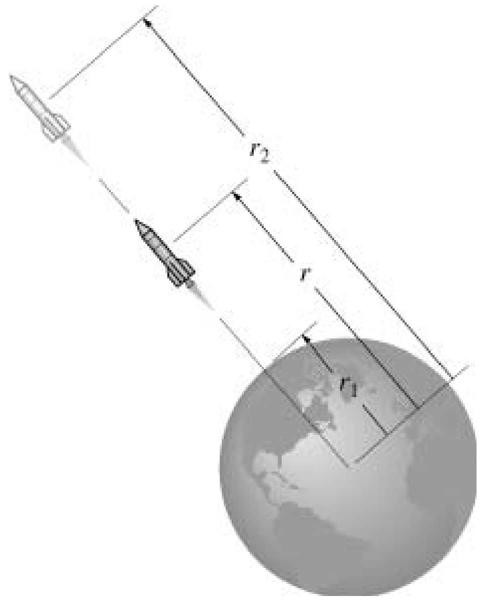
$$W\sin(\theta_1) = \left(\frac{W}{g}\right)\frac{v_1^2}{a}$$

$$\frac{1}{2}k(\pi a - \delta)^2 - \frac{1}{2}k\left[(\pi - \theta_1)a - \delta\right]^2 - Wa\sin(\theta_1) = \left(\frac{W}{g}\right)\frac{v_1^2}{2}$$

$\begin{pmatrix} v_1 \\ \delta \end{pmatrix} = \text{Find}(v_1, \delta)$ $\quad v_1 = 5.843\ \frac{ft}{s}$ $\quad \delta = 2.77\ ft$

Problem 14-37

A rocket of mass *m* is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming that no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13-1), where M_e is the mass of the earth and *r* the distance between the rocket and the center of the earth.



Solution:

$$F = G\left(\frac{M_e m}{r^2}\right)$$

$$U_{12} = \int F\, dr = -G M_e m \int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

$$U_{12} = G M_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Problem 14-38

The spring has a stiffness *k* and an *unstretched length* l_0. As shown, it is confined by the plate and wall using cables so that its length is *l*. A block of weight *W* is given a speed v_A when it is at *A*, and it slides down the incline having a coefficient of kinetic friction μ_k. If it strikes the plate and pushes it forward a distance l_1 before stopping, determine its speed at *A*. Neglect the mass of the plate and spring.

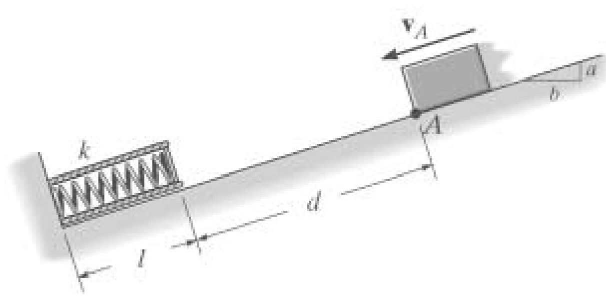
Given:

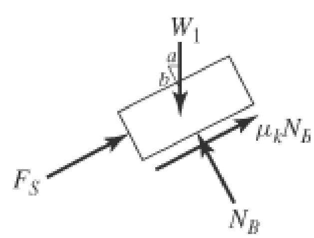
$W = 4\text{ lb}$ $\quad d = 3\text{ ft}$

$l_0 = 2\text{ ft}$ $\quad k = 50\ \frac{\text{lb}}{\text{ft}}$

$l = 1.5\text{ ft}$ $\quad \mu_k = 0.2$

$l_1 = 0.25\text{ ft}$ $\quad a = 3$ $\quad b = 4$





Solution: $\quad \theta = \text{atan}\left(\frac{a}{b}\right)$

Guess $\quad v_A = 1\ \frac{\text{ft}}{\text{s}}$

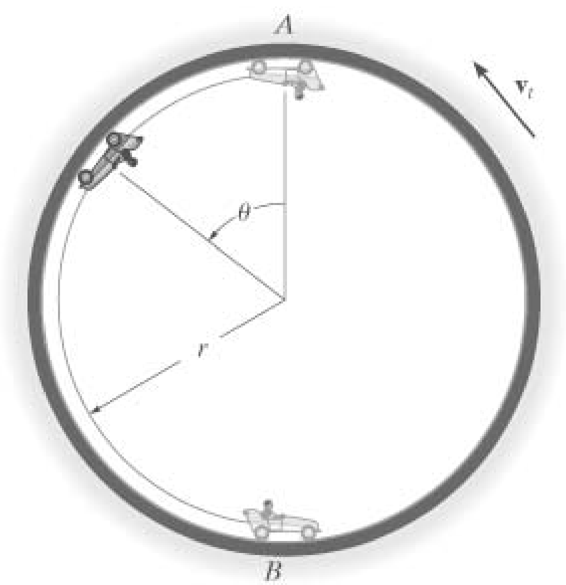
Given

$$\frac{1}{2}\left(\frac{W}{g}\right)v_A^{\,2} + W\left(\sin(\theta) - \mu_k\cos(\theta)\right)\left(d + l_1\right) - \frac{1}{2}k\left[\left(l_0 - l + l_1\right)^2 - \left(l_0 - l\right)^2\right] = 0$$

$v_A = \text{Find}(v_A)$ $\quad v_A = 5.80\ \frac{\text{ft}}{\text{s}}$

Problem 14-39

The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, v_t. If the rider applies the brake when going from *B* to *A* and then releases it at the top of the drum, *A*, so that the car coasts freely down along the track to *B* ($\theta = \pi$ rad), determine the speed of the car at *B* and the normal reaction which the drum exerts on the car at *B*. Neglect friction during the motion from *A* to *B*. The rider and car have a total mass *M* and the center of mass of the car and rider moves along a circular path having a radius *r*.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$M = 250\ \text{kg}$

$r = 8\ \text{m}$

$v_t = 3\ \frac{\text{m}}{\text{s}}$

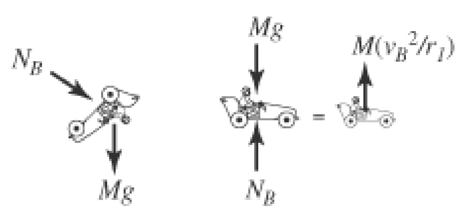
Solution:

$$\frac{1}{2}M v_t^{\,2} + M g 2r = \frac{1}{2}M v_B^{\,2}$$

$v_B = \sqrt{v_t^{\,2} + 4gr}$ $\quad v_B = 18.0\ \frac{\text{m}}{\text{s}}$

$$N_B - Mg = M\left(\frac{v_B^{\,2}}{r}\right)$$

$N_B = M\left(g + \frac{v_B^{\,2}}{r}\right)$ $\quad N_B = 12.5\ \text{kN}$



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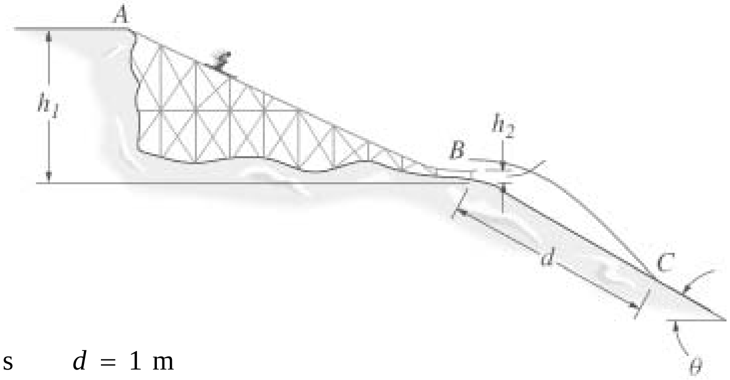
*__Problem 14-40__

The skier starts from rest at *A* and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches *B*. Also, find the distance *d* to where he strikes the ground at *C*, if he makes the jump traveling horizontally at *B*. Neglect the skier's size. He has a mass *M*.

Given:

$M = 70\ \text{kg}$ $\quad h_1 = 50\ \text{m}$

$\theta = 30\ \text{deg}$ $\quad h_2 = 4\ \text{m}$



Solution:

Guesses $\quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad t = 1\ \text{s} \quad d = 1\ \text{m}$

Given $\quad M g(h_1 - h_2) = \frac{1}{2} M v_B^2 \quad v_B t = d\cos(\theta) \quad -h_2 - d\sin(\theta) = \frac{-1}{2} g t^2$

$$\begin{pmatrix} v_B \\ t \\ d \end{pmatrix} = \text{Find}(v_B, t, d) \quad t = 3.753\ \text{s} \quad v_B = 30.0\ \frac{\text{m}}{\text{s}} \quad d = 130.2\ \text{m}$$

Problem 14-41

A spring having a stiffness *k* is compressed a distance *δ*. The stored energy in the spring is used to drive a machine which requires power *P*. Determine how long the spring can supply energy at the required rate.

Units Used: $\quad \text{kN} = 10^3\ \text{N}$

Given: $\quad k = 5\ \frac{\text{kN}}{\text{m}} \quad \delta = 400\ \text{mm} \quad P = 90\ \text{W}$

Solution: $\quad U_{12} = \frac{1}{2} k\delta^2 = P t \quad t = \frac{1}{2} k\left(\frac{\delta^2}{P}\right) \quad t = 4.44\ \text{s}$

Problem 14-42

Determine the power input for a motor necessary to lift a weight *W* at a constant rate *v*. The efficiency of the motor is *ε*.

Given: $\quad W = 300\ \text{lbf} \quad v = 5\ \frac{\text{ft}}{\text{s}} \quad \varepsilon = 0.65$

Solution: $P = \dfrac{W v}{\varepsilon}$ $P = 4.20\,\text{hp}$

Problem 14-43

An electrically powered train car draws a power P. If the car has weight W and starts from rest, determine the maximum speed it attains in time t. The mechanical efficiency is ε.

Given: $P = 30\ \text{kW}$ $W = 40000\ \text{lbf}$ $t = 30\ \text{s}$ $\varepsilon = 0.8$

Solution: $\varepsilon P = F v = \dfrac{W}{g}\left(\dfrac{d}{dt}v\right)v$

$$\int_0^v v\,dv = \int_0^t \frac{\varepsilon P g}{W}\,dt \qquad v = \sqrt{\frac{2\varepsilon P g t}{W}} \qquad v = 29.2\,\frac{\text{ft}}{\text{s}}$$

***Problem 14-44**

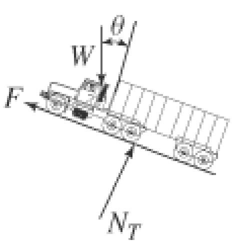
A truck has a weight W and an engine which transmits a power P to *all* the wheels. Assuming that the wheels do not slip on the ground, determine the angle θ of the largest incline the truck can climb at a constant speed v.

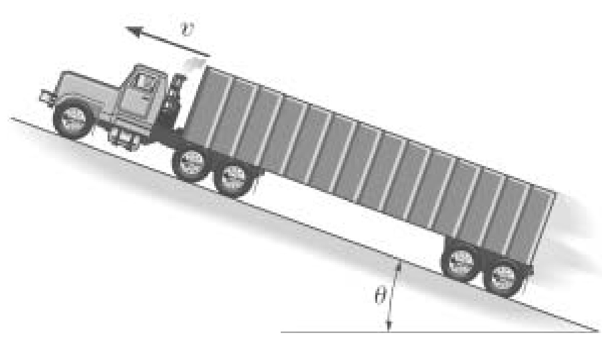
Given:

$W = 25000\ \text{lbf}$

$v = 50\,\dfrac{\text{ft}}{\text{s}}$

$P = 350\ \text{hp}$





Solution:

$F = W\sin(\theta)$ $P = W\sin(\theta)v$

$\theta = \operatorname{asin}\left(\dfrac{P}{W v}\right)$ $\theta = 8.86\,\text{deg}$

Problem 14-45

An automobile having mass M travels up a slope at constant speed v. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has efficiency ε.

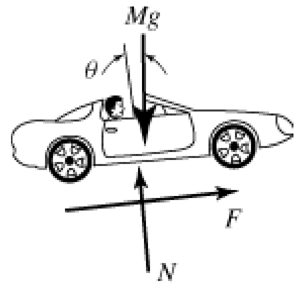
Units Used:

$\text{Mg} = 10^3\ \text{kg}$

Given:

$M = 2\ \text{Mg} \quad v = 100\ \frac{\text{km}}{\text{hr}}$

$\theta = 7\ \text{deg} \qquad \varepsilon = 0.65$



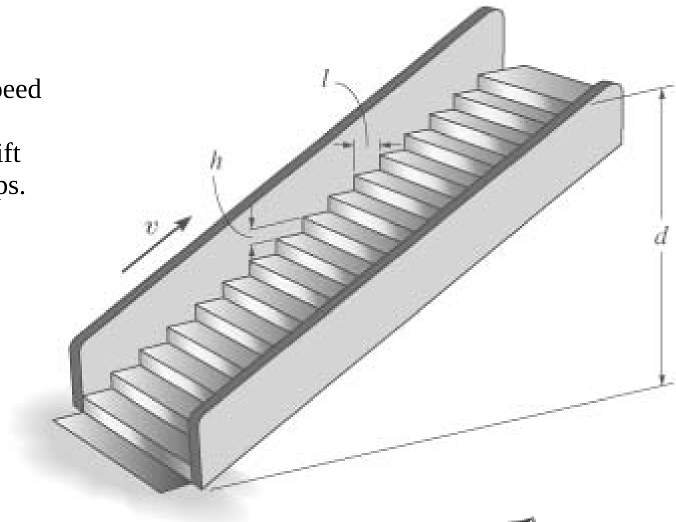
Solution:

$P = M g \sin(\theta) v \qquad P = 66.419\,\text{kW}$

$P_{eng} = \frac{P}{\varepsilon} \qquad P_{eng} = 102.2\,\text{kW}$

Problem 14-46

The escalator steps move with a constant speed *v*. If the steps are of height *h* and length *l*, determine the power of a motor needed to lift an average mass *M* per step.There are *n* steps.



Given:

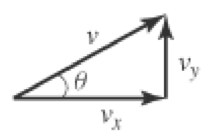
$M = 150\ \text{kg} \qquad h = 125\ \text{mm}$

$n = 32 \qquad l = 250\ \text{mm}$

$v = 0.6\ \frac{\text{m}}{\text{s}} \qquad d = n h$

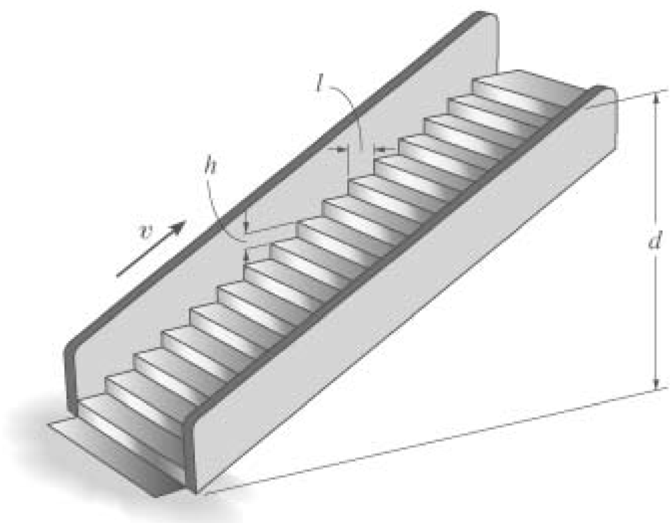
Solution:

$\theta = \text{atan}\left(\frac{h}{l}\right) \qquad P = n M g v \sin(\theta) \qquad P = 12.63\,\text{kW}$



Problem 14-47

If the escalator in Prob. 14−46 is *not moving*, determine the constant speed at which a man having a mass *M* must walk up the steps to generate power *P*—the same amount that is needed to power a standard light bulb.



Given:

$M = 80\ \text{kg} \qquad h = 125\ \text{mm}$

$n = 32 \qquad l = 250\ \text{mm}$

$v = 0.6\ \frac{\text{m}}{\text{s}} \qquad P = 100\ \text{W}$

Solution:

$\theta = \operatorname{atan}\left(\frac{h}{l}\right)$ $\qquad P = F v \sin(\theta)$ $\qquad v = \frac{P}{M g \sin(\theta)}$ $\qquad v = 0.285 \ \frac{\text{m}}{\text{s}}$

***Problem 14-48**

An electric streetcar has a weight W and accelerates along a horizontal straight road from rest such that the power is always P. Determine how far it must travel to reach a speed of v.

Given: $W = 15000 \text{ lbf}$ $\qquad v = 40 \ \frac{\text{ft}}{\text{s}}$ $\qquad P = 100 \text{ hp}$

Solution:

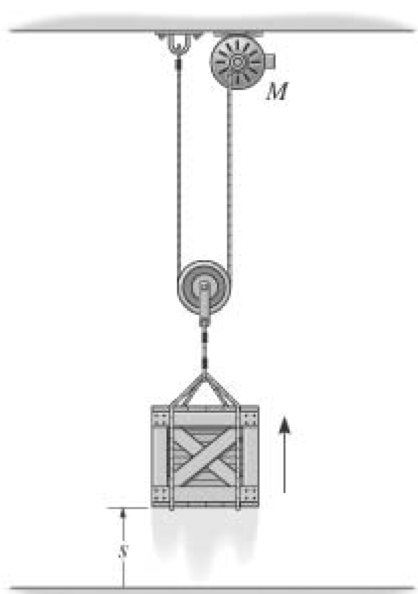
$$P = F v = \left(\frac{W}{g}\right) a v = \left(\frac{W}{g}\right) v^2 \left(\frac{d}{ds_c} v\right)$$

Guess $\quad d = 1 \text{ ft}$

Given $\quad \int_0^d P \, ds_c = \int_0^v \left(\frac{W}{g}\right) v^2 \, dv$ $\qquad d = \text{Find}(d)$ $\qquad d = 180.8 \text{ ft}$

Problem 14-49

The crate of weight W is given speed v in time t_1 starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = t_2$. The motor has an efficiency ε. Neglect the mass of the pulley and cable.



Given:

$W = 50 \text{ lbf}$ $\qquad t_2 = 2 \text{ s}$

$v = 10 \ \frac{\text{ft}}{\text{s}}$ $\qquad \varepsilon = 0.76$

$t_1 = 4 \text{ s}$ $\qquad g = 32.2 \ \frac{\text{ft}}{\text{s}^2}$

Solution:

$a = \frac{v}{t_1}$ $\qquad a = 2.5 \ \frac{\text{ft}}{\text{s}^2}$

$v_2 = a t_2$ $\qquad v_2 = 5 \ \frac{\text{ft}}{\text{s}}$

$$F - W = \left(\frac{W}{g}\right)a \qquad F = W + \left(\frac{W}{g}\right)a \qquad F = 53.882\ \text{lbf}$$

$$P = F v_2 \qquad P = 0.49\ \text{hp} \qquad P_{motor} = \frac{P}{\varepsilon} \qquad P_{motor} = 0.645\ \text{hp}$$

Problem 14-50

A car has a mass M and accelerates along a horizontal straight road from rest such that the power is always a constant amount P. Determine how far it must travel to reach a speed of v.

Solution:

Power: Since the power output is constant, then the traction force F varies with v. Applying Eq. 14-10, we have

$$P = F v \qquad F = \frac{P}{v}$$

Equation of Motion: $\dfrac{P}{v} = M a \qquad a = \dfrac{P}{M v}$

Kinematics: Applying equation $\mathrm{d}s = \dfrac{v\,\mathrm{d}v}{a}$, we have

$$\int_0^s 1\ \mathrm{d}s = \int_0^v \frac{M v^2}{P}\ \mathrm{d}v \qquad s = \frac{M v^3}{3P}$$

Problem 14-51

To dramatize the loss of energy in an automobile, consider a car having a weight W_{car} that is traveling at velocity v. If the car is brought to a stop, determine how long a light bulb with power P_{bulb} must burn to expend the same amount of energy.

Given: $W_{car} = 5000\ \text{lbf} \qquad P_{bulb} = 100\ \text{W}$

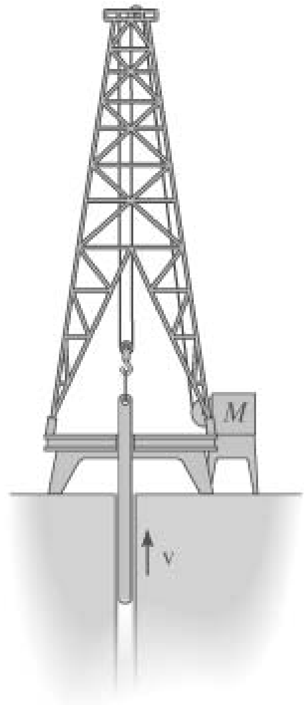
$v = 35\ \dfrac{\text{mi}}{\text{hr}} \qquad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\frac{1}{2}\left(\frac{W_{car}}{g}\right)v^2 = P_{bulb}\,t \qquad t = \frac{W_{car} v^2}{2g P_{bulb}} \qquad t = 46.2\ \text{min}$$

***Problem 14-52**

Determine the power output of the draw-works motor *M* necessary to lift the drill pipe of weight *W* upward with a constant speed *v*. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.



Given:

$W = 600 \text{ lbf}$

$v = 4 \frac{\text{ft}}{\text{s}}$

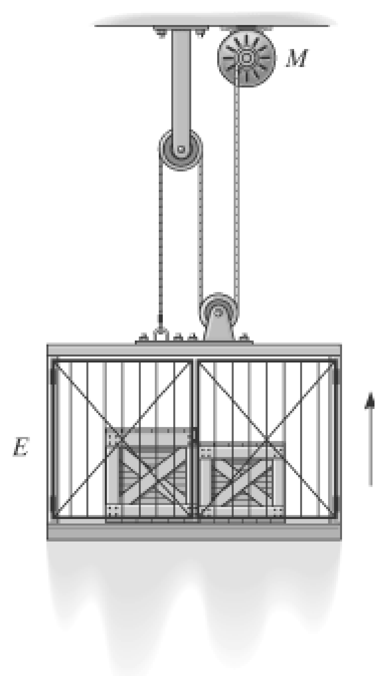
Solution:

$P = W v$

$P = 4.36 \text{ hp}$

Problem 14-53

The elevator of mass m_{el} starts from rest and travels upward with a constant acceleration a_c. Determine the power output of the motor *M* when $t = t_1$. Neglect the mass of the pulleys and cable.



Given:

$m_{el} = 500 \text{ kg}$

$a_c = 2 \frac{\text{m}}{\text{s}^2}$

$t_1 = 3 \text{ s}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

$F - m_{el} g = m_{el} a_c \qquad F = m_{el}(g + a_c)$

$F = 5.905 \times 10^3 \text{ N}$

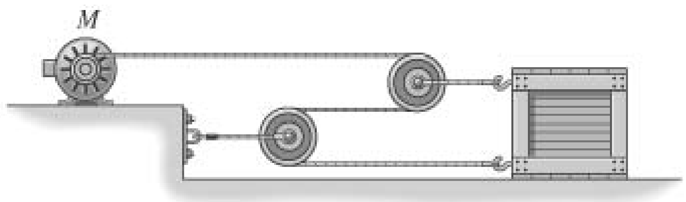
$v_1 = a_c t_1$ $\qquad v_1 = 6 \frac{m}{s}$

$P = F v_1$ $\qquad P = 35.4\,kW$

Problem 14-54

The crate has mass m_c and rests on a surface for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. If the motor M supplies a cable force of $F = at^2 + b$, determine the power output developed by the motor when $t = t_1$.

Given:



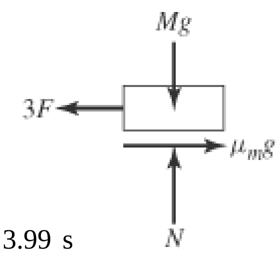
$m_C = 150\ kg$ $\qquad a = 8 \frac{N}{s^2}$

$\mu_S = 0.3$ $\qquad b = 20\ N$

$\mu_k = 0.2$ $\qquad t_1 = 5\ s$

$g = 9.81 \frac{m}{s^2}$

Solution:



Time to start motion

$3\left(a t^2 + b\right) = \mu_S m_C g$ $\qquad t = \sqrt{\frac{1}{a}\left(\frac{\mu_S m_C g}{3} - b\right)}$ $\qquad t = 3.99\ s$

Speed at t_1 $\qquad 3\left(a t^2 + b\right) - \mu_k m_C g = m_C a = m_C \frac{d}{dt} v$

$$v = \int_t^{t_1} \frac{3}{m_C}\left(a t^2 + b\right) - \mu_k g\ dt \qquad v = 1.70 \frac{m}{s}$$

$P = 3\left(a {t_1}^2 + b\right) v$ $\qquad P = 1.12\,kW$

Problem 14-55

The elevator E and its freight have total mass m_E. Hoisting is provided by the motor M and the block C of mass m_C .If the motor has an efficiency ε, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed v_E.

Given:

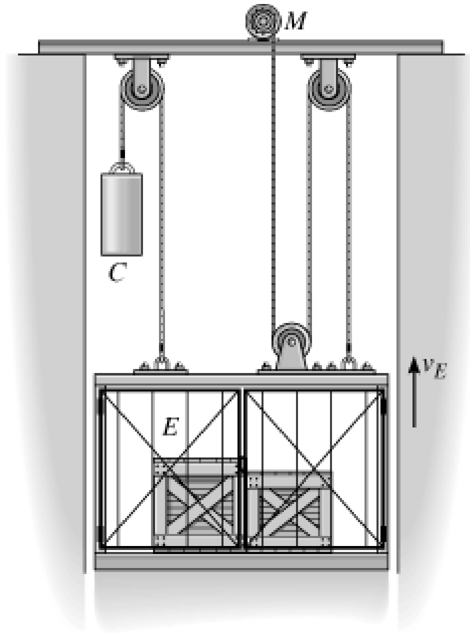
$m_C = 60\ kg$

$m_E = 400\ \text{kg}$

$\varepsilon = 0.6$

$v_E = 4\ \frac{\text{m}}{\text{s}}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



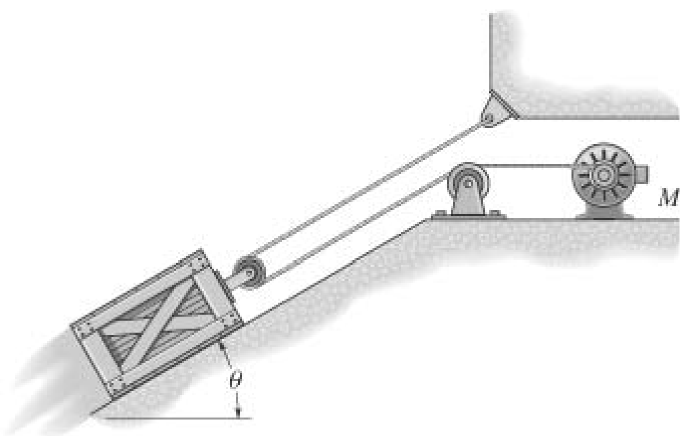
Solution:

$F = (m_E - m_C)g$

$P = \frac{F v_E}{\varepsilon}$ $\qquad P = 22.236\,\text{kW}$

***Problem 14-56**

The crate of mass m_c is hoisted up the incline of angle θ by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains speed v after traveling a distance d along the plane, determine the power that must be supplied to the motor at this instant. Neglect friction along the plane. The motor has efficiency ε.



Given:

$m_C = 50\ \text{kg}$ $\qquad \theta = 30\ \text{deg}$

$d = 8\ \text{m}$ $\qquad \varepsilon = 0.74$

$v = 4\ \frac{\text{m}}{\text{s}}$ $\qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$a_C = \frac{v^2}{2d}$ $\qquad a_C = 1\ \frac{\text{m}}{\text{s}^2}$

$F - (m_C g)\sin(\theta) = m a_C$ $\qquad F = m_C(g\sin(\theta) + a_C)$ $\qquad F = 295.25\ \text{N}$

$P = \frac{F v}{\varepsilon}$ $\qquad P = 1.596\,\text{kW}$

Problem 14-57

The block has mass M and rests on a surface for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. If a force $F = kt^2$ is applied to the cable, determine the power developed by the force at $t = t_2$. *Hint:* First determine the time needed for the force to cause motion.

Given:

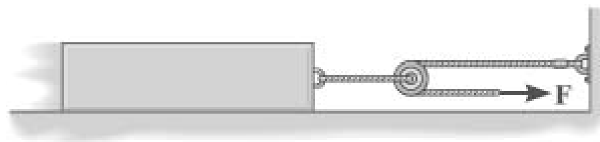
$M = 150 \text{ kg}$ $\quad k = 60 \dfrac{\text{N}}{\text{s}^2}$

$\mu_s = 0.5$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

$\mu_k = 0.4$

$t_2 = 5 \text{ s}$



Solution:

$$2F = 2kt_1^2 = \mu_s M g \qquad t_1 = \sqrt{\frac{\mu_s M g}{2k}} \qquad t_1 = 2.476 \text{ s}$$

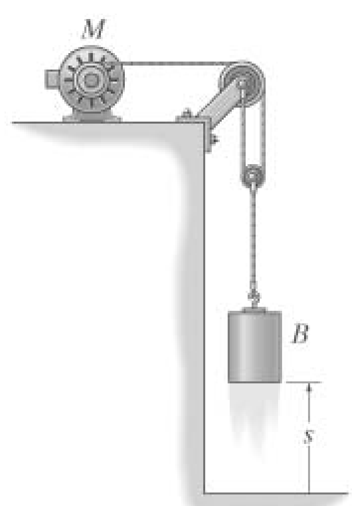
$$2kt^2 - \mu_k M g = M a = M\left(\frac{d}{dt}v\right)$$

$$v_2 = \int_{t_1}^{t_2} \left(\frac{2kt^2}{M} - \mu_k g\right) dt \qquad v_2 = 19.381 \frac{\text{m}}{\text{s}}$$

$$P = 2kt_2^2 v_2 \qquad P = 58.144 \text{ kW}$$

Problem 14-58

The load of weight W is hoisted by the pulley system and motor M. If the crate starts from rest and by constant acceleration attains a speed v after rising a distance $s = s_1$, determine the power that must be supplied to the motor at this instant. The motor has an efficiency ε. Neglect the mass of the pulleys and cable.



Given:

$W = 50 \text{ lbf}$ $\quad \varepsilon = 0.76$

$v = 15 \dfrac{\text{ft}}{\text{s}}$ $\quad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

$s_1 = 6 \text{ ft}$

Solution:

$$a = \frac{v^2}{2s_1} \qquad F = W + \left(\frac{W}{g}\right)a$$

$$P = \frac{F v}{\varepsilon} \qquad P = 2.84\,\text{hp}$$

Problem 14-59

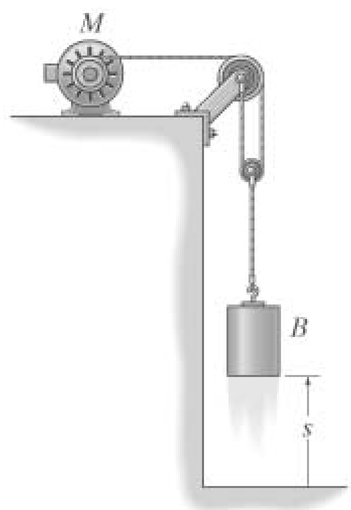
The load of weight W is hoisted by the pulley system and motor M. If the motor exerts a constant force **F** on the cable, determine the power that must be supplied to the motor if the load has been hoisted at $s = s'$ starting from rest.The motor has an efficiency ε.

Given:

$W = 50$ lbf $\quad \varepsilon = 0.76$

$F = 30$ lbf $\quad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$s' = 10$ ft



Solution:

$$2F - W = \frac{W}{g}a$$

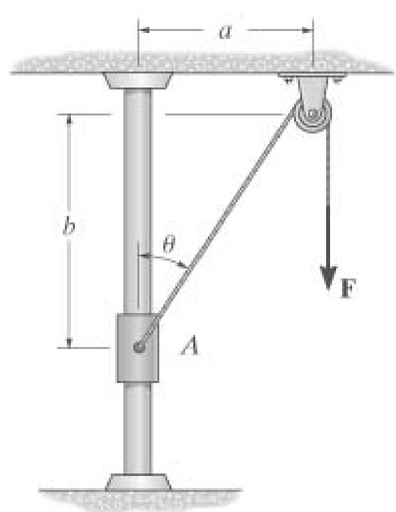
$$a = \left(\frac{2F}{W} - 1\right)g \qquad a = 6.44\,\frac{\text{ft}}{\text{s}^2}$$

$$v = \sqrt{2a s'} \qquad v = 11.349\,\frac{\text{ft}}{\text{s}}$$

$$P = \frac{2F v}{\varepsilon} \qquad P = 1.629\,\text{hp}$$

***Problem 14-60**

The collar of weight W starts from rest at A and is lifted by applying a constant vertical force **F** to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = \theta_2$.



Given:

$W = 10$ lbf $\qquad a = 3$ ft

$F = 25$ lbf $\qquad b = 4$ ft

$\theta_2 = 60$ deg

Solution:

$$h = b - (a)\cot(\theta_2)$$

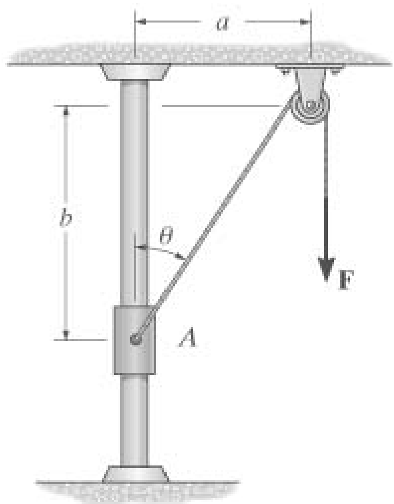
$L_1 = \sqrt{a^2 + b^2}$ $\qquad L_2 = \sqrt{a^2 + (b-h)^2}$

$F(L_1 - L_2) - Wh = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2$ $\qquad v_2 = \sqrt{2\left(\frac{F}{W}\right)(L_1 - L_2)g - 2gh}$

$P = F v_2 \cos(\theta_2)$ $\qquad P = 0.229\,\text{hp}$

Problem 14-61

The collar of weight W starts from rest at A and is lifted with a constant speed v along the smooth rod. Determine the power developed by the force **F** at the instant shown.



Given:

$W = 10\ \text{lbf}$

$v = 2\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$

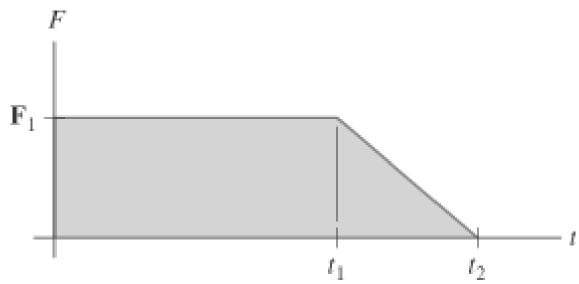
$b = 4\ \text{ft}$

Solution:

$\theta = \text{atan}\left(\frac{a}{b}\right)$ $\qquad F\cos(\theta) - W = 0$ $\qquad F = \frac{W}{\cos(\theta)}$

$P = F v \cos(\theta)$ $\qquad P = 0.0364\,\text{hp}$

Problem 14-62



An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in time $t = t_2$.

Units Used: $\qquad \text{kJ} = 10^3\ \text{J}$

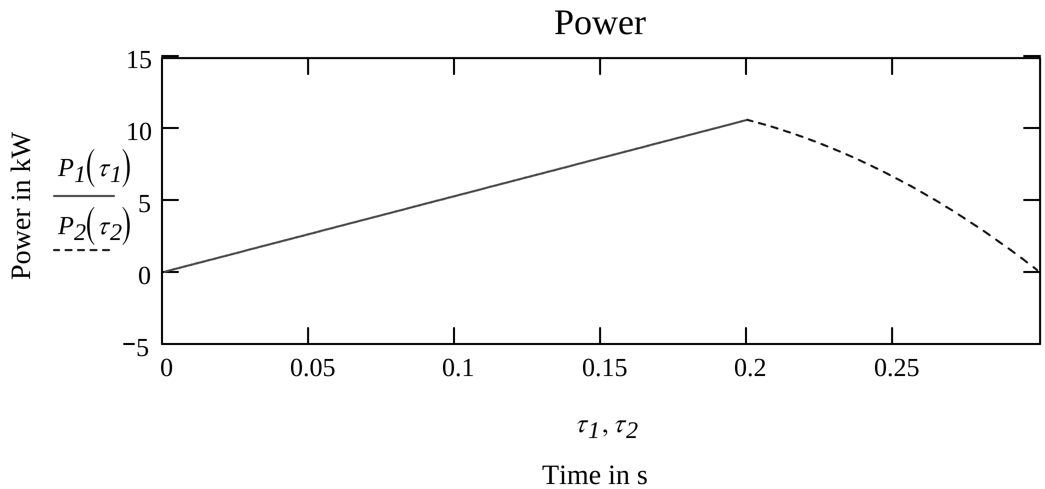
Given:

$F_1 = 800\ \text{N} \qquad t_1 = 0.2\ \text{s}$

$v_2 = 20\ \frac{\text{m}}{\text{s}} \qquad t_2 = 0.3\ \text{s}$

Solution: $\tau_1 = 0, 0.01t_1 .. t_1$ $P_1(\tau_1) = F_1 \frac{v_2}{t_2} \tau_1 \frac{1}{kW}$

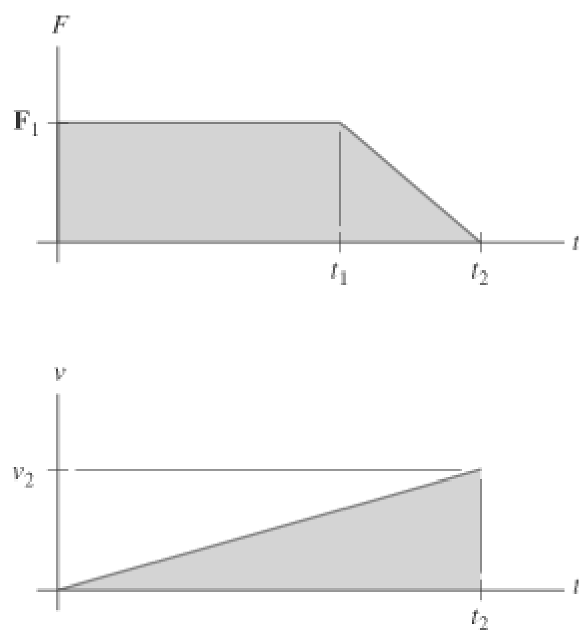
$\tau_2 = t_1, 1.01t_1 .. t_2$ $P_2(\tau_2) = F_1 \left(\frac{\tau_2 - t_2}{t_1 - t_2}\right) \frac{v_2}{t_2} \tau_2 \frac{1}{kW}$



$$U = \left(\int_0^{t_1} P_1(\tau)\, d\tau + \int_{t_1}^{t_2} P_2(\tau)\, d\tau\right) kW \qquad U = 1.689\ kJ$$

Problem 14-63

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the time period $0 < t < t_2$.



Given:

$F_1 = 800\ N$

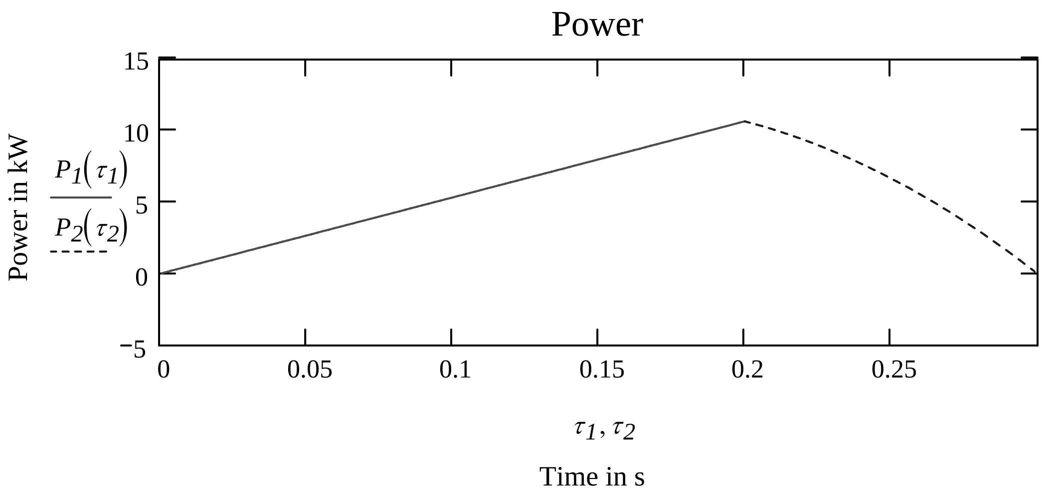
$v_2 = 20\ \frac{m}{s}$

$t_1 = 0.2\ s$

$t_2 = 0.3\ s$

Solution: $\tau_1 = 0, 0.01t_1 .. t_1$ $P_1(\tau_1) = F_1\left(\frac{v_2}{t_2}\right)\tau_1 \frac{1}{\text{kW}}$

$\tau_2 = t_1, 1.01t_1 .. t_2$ $P_2(\tau_2) = F_1\left(\frac{\tau_2 - t_2}{t_1 - t_2}\right)\left(\frac{v_2}{t_2}\right)\tau_2 \frac{1}{\text{kW}}$



$P_{max} = P_1(t_1)\text{kW}$ $P_{max} = 10.667\,\text{kW}$

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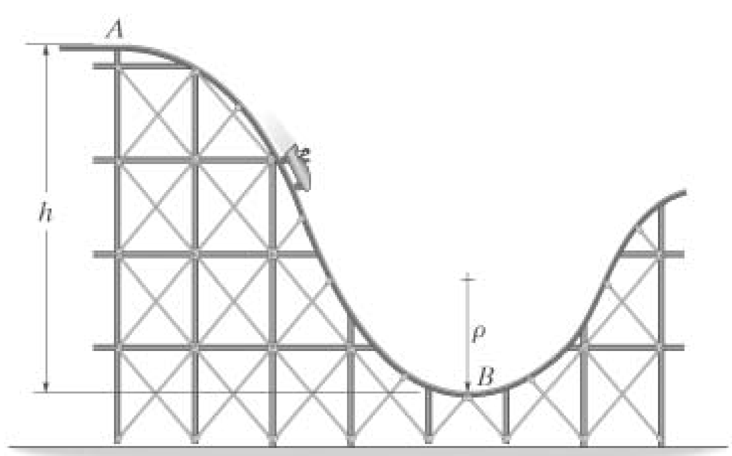
*__Problem 14-64__

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill it will reach a speed v when it comes to the bottom. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than kmg? Neglect the size of the car and passengers.

Given:

$v = 100 \frac{\text{km}}{\text{hr}}$

$k = 4$



Solution:

$T_1 = 0$ $V_1 = m g h$

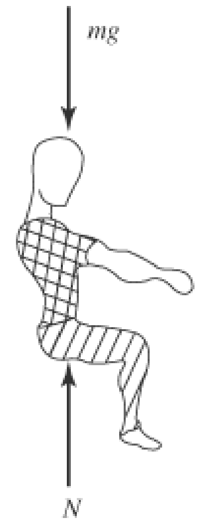
$T_2 = \frac{1}{2}m v^2$ $V_2 = 0$

$0 + m g h = \frac{1}{2}m v^2 + 0$

$$h = \frac{1}{2}\left(\frac{v^2}{g}\right) \qquad h = 39.3\ \text{m}$$

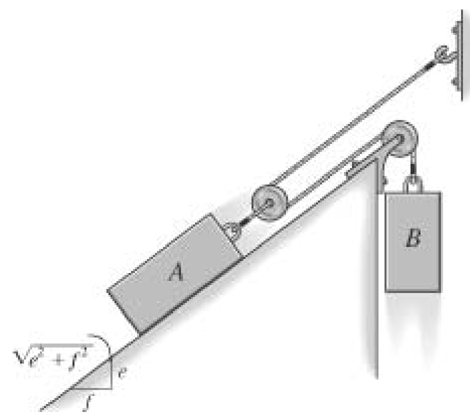
$$kmg - mg = \frac{mv^2}{\rho}$$

$$\rho = \frac{v^2}{g(k-1)} \qquad \rho = 26.2\ \text{m}$$



Problem 14-65

Block A has weight W_A and block B has weight W_B. Determine the speed of block A after it moves a distance d down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



Given:

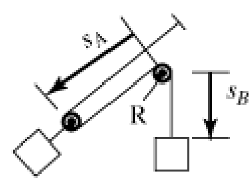
$W_A = 60\ \text{lb}$ $\quad e = 3$

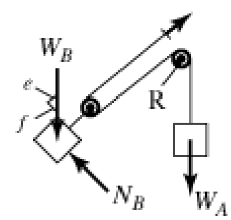
$W_B = 10\ \text{lb}$ $\quad f = 4$

$d = 5\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$L = 2s_A + s_B \qquad 0 = 2\Delta s_A + \Delta s_B \qquad 0 = 2v_A + v_B$$





$$T_1 = 0 \qquad V_1 = 0$$

$$T_2 = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)v_B^2 \qquad V_2 = -W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right)d + W_B 2d$$

$$0 + 0 = \frac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \frac{1}{2}\left(\frac{W_B}{g}\right) - W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right)d + W_B 2d$$

$$v_A = \sqrt{\frac{2gd}{W_A + 4W_B}\left[W_A\left(\frac{e}{\sqrt{e^2+f^2}}\right) - 2W_B\right]} \qquad v_A = 7.178\ \frac{\text{ft}}{\text{s}}$$

Problem 14-66

The collar has mass M and rests on the smooth rod. Two springs are attached to it and the ends of the rod as shown. Each spring has an uncompressed length l. If the collar is displaced a distance $s = s'$ and released from rest, determine its velocity at the instant it returns to the point $s = 0$.

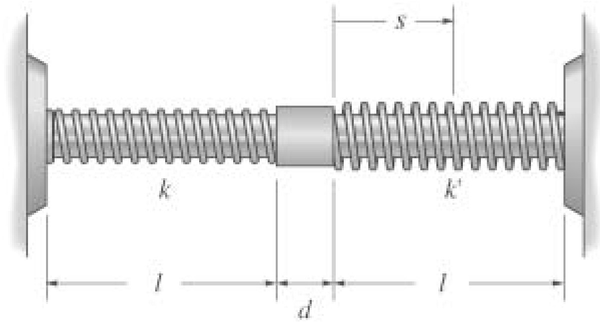
Given:

$M = 20\ \text{kg}$ $\quad k = 50\ \frac{\text{N}}{\text{m}}$

$s' = 0.5\ \text{m}$

$l = 1\ \text{m}$ $\quad k' = 100\ \frac{\text{N}}{\text{m}}$

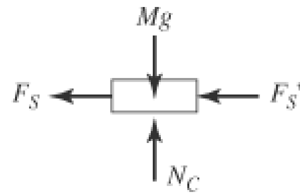
$d = 0.25\ \text{m}$



Solution:

$T_1 = 0 \qquad V_1 = \frac{1}{2}(k + k')\, s'^2$

$T_2 = \frac{1}{2}M v^2 \qquad V_2 = 0$



$$0 + \frac{1}{2}(k + k')\, s'^2 = \frac{1}{2}M v_C^2 + 0$$

$$v_C = \sqrt{\frac{k + k'}{M}}\, s' \qquad v_C = 1.37\ \frac{\text{m}}{\text{s}}$$

Problem 14-67

The collar has mass M and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length L and the collar has speed v_0 when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.

Given:

$M = 20\ \text{kg}$

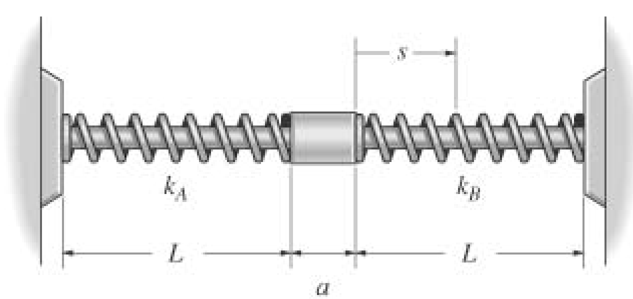
$L = 1\ \text{m}$

$a = 0.25\ \text{m}$

$v_0 = 2\ \frac{\text{m}}{\text{s}}$

$k_A = 50\ \frac{\text{N}}{\text{m}}$

$k_B = 100\ \frac{\text{N}}{\text{m}}$



Solution:

$T_1 = \frac{1}{2}M v_0^2 \qquad V_1 = 0 \qquad T_2 = 0 \qquad V_2 = \frac{1}{2}(k_A + k_B)\, d^2$

$\frac{1}{2}M v_0^2 + 0 = 0 + \frac{1}{2}(k_A + k_B)\, d^2 \qquad d = \sqrt{\frac{M}{k_A + k_B}}\, v_0 \qquad d = 0.73\ \text{m}$

---

*__Problem 14-68__

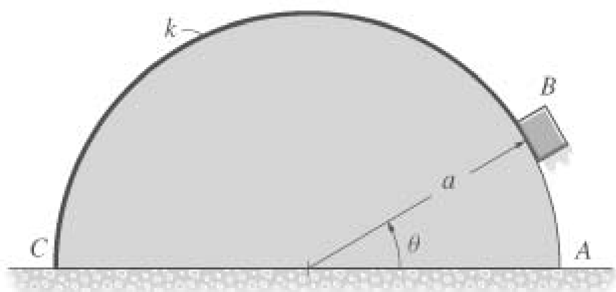
A block of weight W rests on the smooth semicylindrical surface. An elastic cord having a stiffness k is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at $A(\theta = 0°)$, determine the unstretched length of the cord so the block begins to leave the semicylinder at the instant $\theta = \theta_2$. Neglect the size of the block.

Given:

$W = 2\ \text{lb} \qquad a = 1.5\ \text{ft}$

$k = 2\ \frac{\text{lb}}{\text{ft}} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

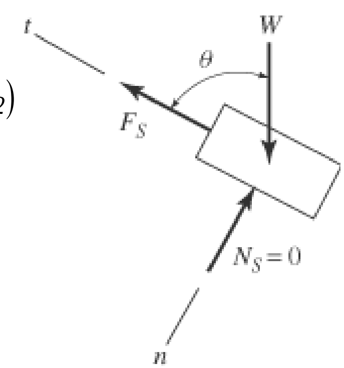
$\theta_2 = 45\ \text{deg}$



Solution:

$T_1 = 0 \qquad V_1 = \frac{1}{2}k(\pi a - \delta)^2$

$T_2 = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 \qquad V_2 = \frac{1}{2}k\left[(\pi - \theta_2)a - \delta\right]^2 + (W a)\sin(\theta_2)$



Guess $\qquad \delta = 1\ \text{ft} \qquad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$W\sin(\theta_2) = \frac{W}{g}\left(\frac{v_2^2}{a}\right)$$

$$0 + \frac{1}{2}k(\pi a - \delta)^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 + \left[\frac{1}{2}k\left[(\pi - \theta_2)a - \delta\right]^2 + (W a)\sin(\theta_2)\right]$$

$\begin{pmatrix} v_2 \\ \delta \end{pmatrix} = \text{Find}(v_2, \delta) \qquad v_2 = 5.843\ \frac{\text{ft}}{\text{s}} \qquad \delta = 2.77\ \text{ft}$

Problem 14-69

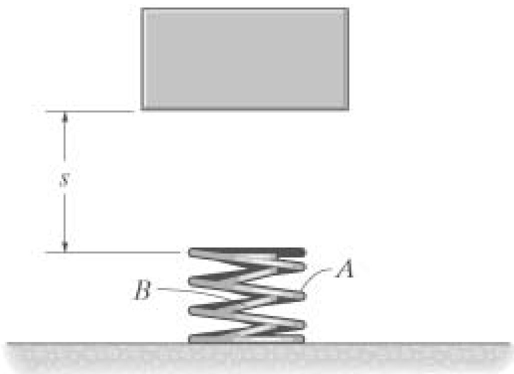
Two equal-length springs are "nested" together in order to form a shock absorber. If it is designed to arrest the motion of mass M that is dropped from a height s_1 above the top of the springs from an at-rest position, and the maximum compression of the springs is to be δ, determine the required stiffness of the inner spring, k_B, if the outer spring has a stiffness k_A.

Given:

$M = 2\ \text{kg}$ $\delta = 0.2\ \text{m}$

$k_A = 400\ \frac{\text{N}}{\text{m}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$s_1 = 0.5\ \text{m}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + Mg(s_1 + \delta) = 0 + \frac{1}{2}(k_A + k_B)\,\delta^2$$

$$k_B = \frac{2Mg(s_1 + \delta)}{\delta^2} - k_A \qquad k_B = 287\ \frac{\text{N}}{\text{m}}$$

Problem 14-70

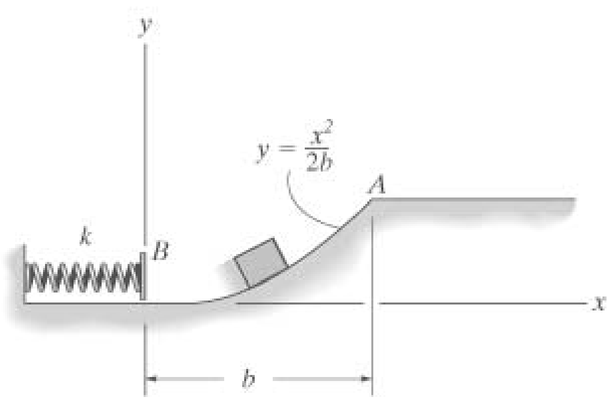
Determine the smallest amount the spring at B must be compressed against the block of weight W so that when it is released from B it slides along the smooth surface and reaches point A.

Given:

$W = 0.5\ \text{lb}$

$b = 1\ \text{ft}$

$k = 5\ \frac{\text{lb}}{\text{in}}$



Solution:

$$y(x) = \frac{x^2}{2b}$$

$T_B = 0$ $V_B = \frac{1}{2}k\delta^2$ $T_A = 0$ $V_A = W\,y(b)$

$$0 + \frac{1}{2}k\delta^2 = 0 + W\,y(b) \qquad \delta = \sqrt{\frac{2W\,y(b)}{k}} \qquad \delta = 1.095\ \text{in}$$

Problem 14-71

If the spring is compressed a distance δ against the block of weight W and it is released from rest, determine the normal force of the smooth surface on the block when it reaches the point x_1.

Given:

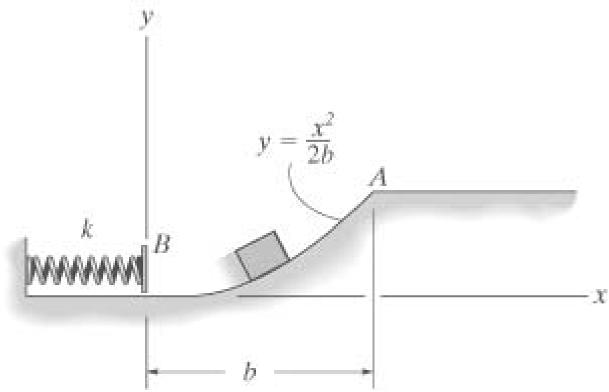
$W = 0.5\ \text{lb}$

$b = 1\ \text{ft}$

$k = 5\ \dfrac{\text{lb}}{\text{in}}$

$\delta = 3\ \text{in}$

$x_1 = 0.5\ \text{ft}$



Solution:

$$y(x) = \frac{x^2}{2b} \qquad y'(x) = \frac{x}{b} \qquad y''(x) = \frac{1}{b} \qquad \rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$$\theta(x) = \operatorname{atan}(y'(x))$$

$$T_1 = 0 \qquad V_1 = \frac{1}{2}k\delta^2 \qquad T_1 = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2$$

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_1^2 + W\,y(x_1) \qquad v_1 = \sqrt{\left(k\delta^2 - 2V_2 = W\,y(x_1)\right)}$$

$$F_N - W\cos(\theta(x_1)) = \frac{W}{g}\left(\frac{v_1^2}{\rho(x_1)}\right)$$

$$F_N = W\cos(\theta(x_1)) + \frac{W}{g}\left(\frac{v_1^2}{\rho(x_1)}\right) \qquad F_N = 3.041\ \text{lb}$$

---

*__Problem 14-72__

The girl has mass M and center of mass at G. If she is swinging to a maximum height defined by $\theta = \theta_1$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.

Given:

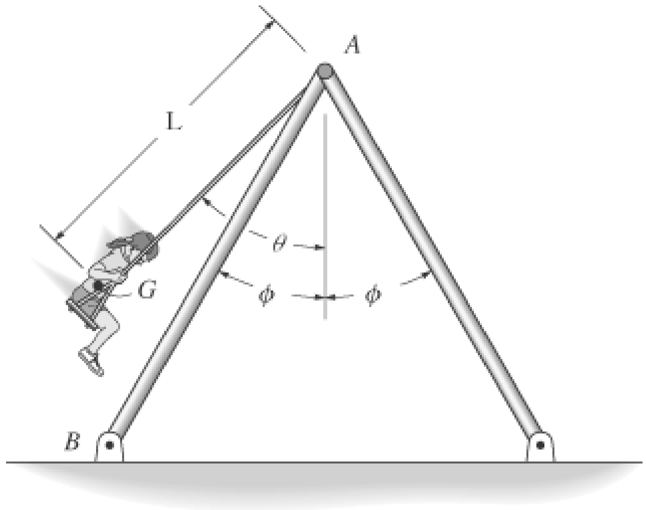
$M = 40\ \text{kg}$

$\theta_1 = 60\ \text{deg}$

$\phi = 30 \text{ deg}$

$L = 2 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 - MgL\cos(\theta_1) = \frac{1}{2}Mv^2 - MgL$$

$v = \sqrt{2gL(1-\cos(\theta_1))}$ $\quad v = 4.429 \dfrac{\text{m}}{\text{s}}$

$T - Mg = M\left(\dfrac{v^2}{L}\right)$ $\quad T = Mg + M\left(\dfrac{v^2}{L}\right)$ $\quad T = 784.8 \text{ N}$

$4F_{AB}\cos(\phi) - T = 0$ $\quad F_{AB} = \dfrac{T}{4\cos(\phi)}$ $\quad F_{AB} = 226.552 \text{ N}$

Problem 14-73

Each of the two elastic rubber bands of the slingshot has an unstretched length l. If they are pulled back to the position shown and released from rest, determine the speed of the pellet of mass M just after the rubber bands become unstretched. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness k.

Given:

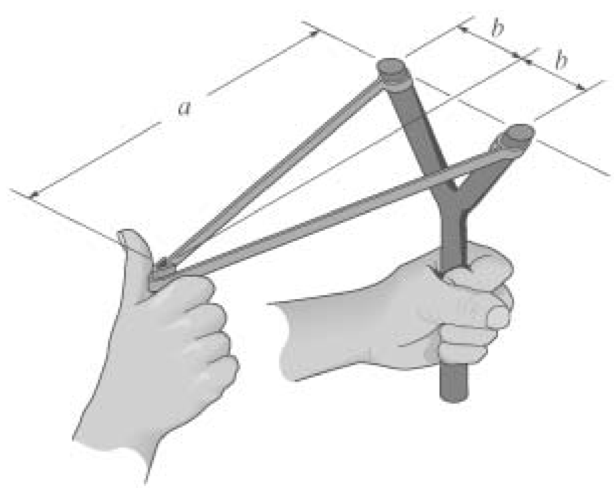
$l = 200 \text{ mm}$

$M = 25 \text{ gm}$

$a = 240 \text{ mm}$

$b = 50 \text{ mm}$

$k = 50 \dfrac{\text{N}}{\text{m}}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k\left(\sqrt{b^2 + a^2} - l\right)^2\right] = \frac{1}{2}Mv^2$$

$$v = \sqrt{\frac{2k}{M}}\left(\sqrt{b^2 + a^2} - l\right) \qquad v = 2.86\ \frac{\text{m}}{\text{s}}$$

Problem 14-74

Each of the two elastic rubber bands of the slingshot has an unstretched length l. If they are pulled back to the position shown and released from rest, determine the maximum height the pellet of mass M will reach if it is fired vertically upward. Neglect the mass of the rubber bands and the change in elevation of the pellet while it is constrained by the rubber bands. Each rubber band has a stiffness k.

Given:

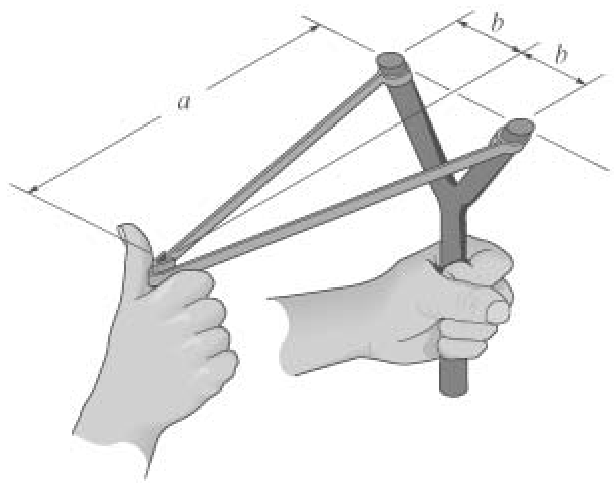
$l = 200\ \text{mm}$

$M = 25\ \text{gm}$

$a = 240\ \text{mm}$

$b = 50\ \text{mm}$

$k = 50\ \frac{\text{N}}{\text{m}}$



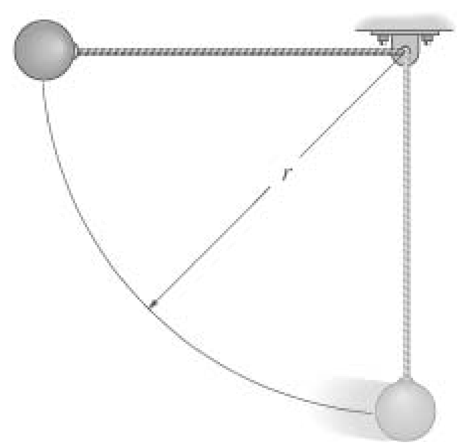
Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k\left(\sqrt{b^2 + a^2} - l\right)^2\right] = Mgh$$

$$h = \frac{k}{Mg}\left(\sqrt{b^2 + a^2} - l\right)^2 \qquad h = 416\,\text{mm}$$

Problem 14-75



The bob of the pendulum has a mass M and is released from rest when it is in the horizontal position shown. Determine its speed and the tension in the cord at the instant the bob passes through its lowest position.

Given:

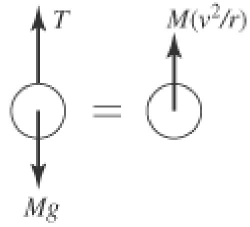
$M = 0.2\ \text{kg}$

$r = 0.75\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$



$$0 + 0 = \frac{1}{2}M v_2^2 - M g r$$

$$v_2 = \sqrt{2gr} \qquad v_2 = 3.84\ \frac{\text{m}}{\text{s}}$$

$$\Sigma F_n = Ma_n$$

$$T - Mg = M\left(\frac{v_2^2}{r}\right)$$

$$T = M\left(g + \frac{v_2^2}{r}\right) \qquad T = 5.89\ \text{N}$$

---

*__Problem 14-76__

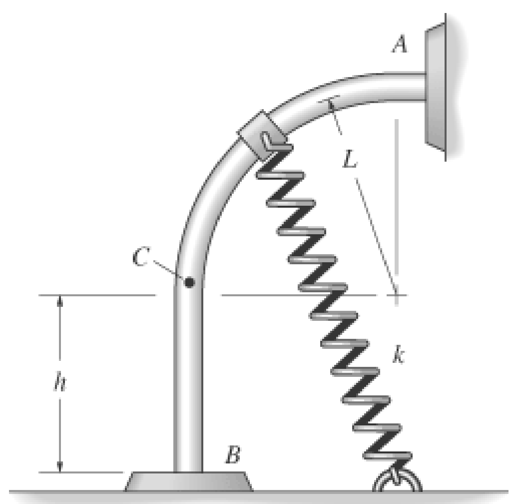
The collar of weight W is released from rest at A and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at B. The spring has an unstretched length L.

Given:

$W = 5\ \text{lb}$ $\quad k = 2\ \frac{\text{lb}}{\text{in}}$

$L = 12\ \text{in}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$h = 10\ \text{in}$



Solution:

$$T_A + V_A = T_B + V_B$$

$$0 + W(L + h) + \frac{1}{2}kh^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2$$

$$v_B = \sqrt{\left(\frac{kg}{W}\right)h^2 + 2g(L + h)} \qquad v_B = 15.013\ \frac{\text{ft}}{\text{s}}$$

Problem 14-77

The collar of weight *W* is released from rest at *A* and travels along the smooth guide. Determine its speed when its center reaches point *C* and the normal force it exerts on the rod at this point. The spring has an unstretched length *L*, and point *C* is located just before the end of the curved portion of the rod.

Given:

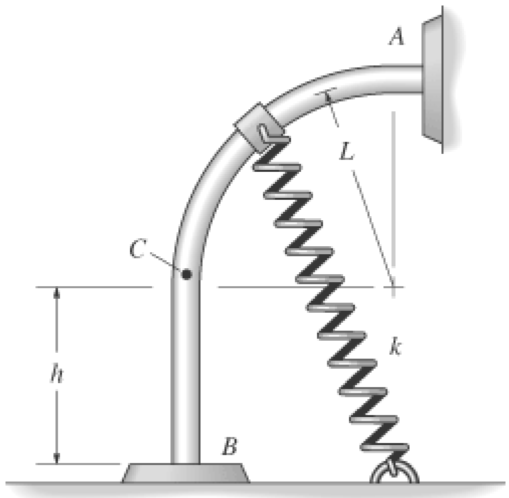
$W = 5 \text{ lb}$

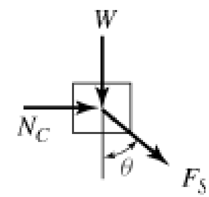
$L = 12 \text{ in}$

$h = 10 \text{ in}$

$k = 2 \frac{\text{lb}}{\text{in}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

$$T_A + V_A = T_C + V_C \qquad 0 + WL + \frac{1}{2}kh^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_C^2 + \frac{1}{2}k\left(\sqrt{L^2+h^2} - L\right)^2$$

$$v_C = \sqrt{2gL + \left(\frac{kg}{W}\right)h^2 - \left(\frac{kg}{W}\right)\left(\sqrt{L^2+h^2} - L\right)^2} \qquad v_C = 12.556 \frac{\text{ft}}{\text{s}}$$

$$N_C + k\left(\sqrt{L^2+h^2} - L\right)\left(\frac{L}{\sqrt{L^2+h^2}}\right) = \frac{W}{g}\left(\frac{v_C^2}{L}\right)$$

$$N_C = \frac{W}{g}\left(\frac{v_C^2}{L}\right) - \left(\frac{kL}{\sqrt{L^2+h^2}}\right)\left(\sqrt{L^2+h^2} - L\right) \qquad N_C = 18.919 \text{ lb}$$

Problem 14-78

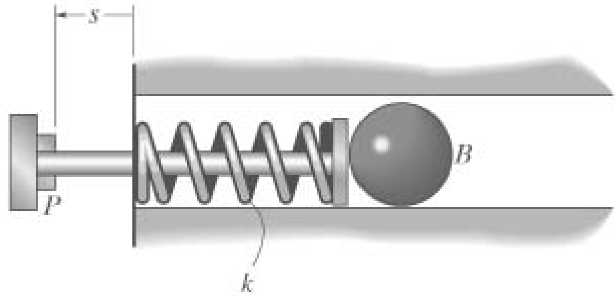
The firing mechanism of a pinball machine consists of a plunger *P* having a mass M_p and a spring stiffness *k*. When *s* = 0, the spring is compressed a distance δ. If the arm is pulled back such that $s = s_1$ and released, determine the speed of the pinball *B* of mass M_b *just before* the plunger strikes the stop, i.e., assume all surfaces of contact to be smooth. The ball moves in the horizontal plane. Neglect friction, the mass of the spring, and the rolling motion of the ball.

Given:

$M_p = 0.25 \text{ kg} \qquad s_1 = 100 \text{ mm}$

$M_b = 0.3 \text{ kg}$ $k = 300 \frac{\text{N}}{\text{m}}$

$\delta = 50 \text{ mm}$



Solution:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}k(s_1 + \delta)^2 = \frac{1}{2}(M_p + M_b)v_2^2 + \frac{1}{2}k\delta^2$$

$$v_2 = \sqrt{\frac{k}{M_p + M_b}\left[(s_1 + \delta)^2 - \delta^2\right]} \qquad v_2 = 3.30 \frac{\text{m}}{\text{s}}$$

Problem 14-79

The roller-coaster car has mass M, including its passenger, and starts from the top of the hill A with a speed v_A. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

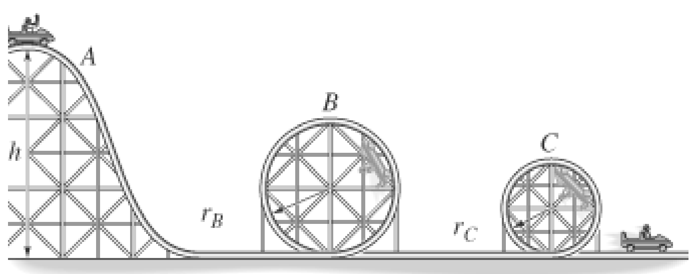
Units Used: $\text{kN} = 10^3 \text{ N}$

Given:

$M = 800 \text{ kg}$ $v_A = 3 \frac{\text{m}}{\text{s}}$

$r_B = 10 \text{ m}$

$r_C = 7 \text{ m}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

Check the loop at B first We require that $N_B = 0$

$$-N_B - Mg = -M\left(\frac{v_B^2}{r_B}\right) \qquad v_B = \sqrt{g r_B} \qquad v_B = 9.907 \frac{\text{m}}{\text{s}}$$

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}Mv_A^2 + Mgh = \frac{1}{2}Mv_B^2 + Mg2r_B$$

$$h = \frac{v_B^2 - v_A^2}{2g} + 2r_B \qquad h = 24.541 \text{ m}$$

Now check the loop at C

$$T_A + V_A = T_C + V_C \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_C^2 + M g 2r_C$$

$$v_C = \sqrt{v_A^2 + 2g(h - 2r_C)} \qquad v_C = 14.694\ \frac{\text{m}}{\text{s}}$$

$$-N_C - M g = -M\left(\frac{v_C^2}{r_C}\right) \qquad N_C = M\left(\frac{v_C^2}{r_C}\right) - M g \qquad N_C = 16.825\ \text{kN}$$

Since $N_C > 0$ then the coaster successfully passes through loop C.

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*__Problem 14-80__

The roller-coaster car has mass M, including its passenger, and starts from the top of the hill A with a speed v_A. Determine the minimum height h of the hill crest so that the car travels around both inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C?

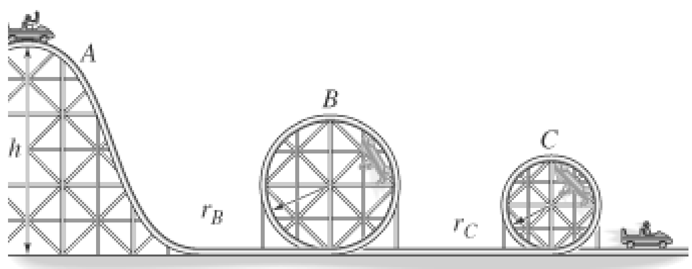
Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$M = 800\ \text{kg}$ $\quad v_A = 0\ \frac{\text{m}}{\text{s}}$

$r_B = 10\ \text{m}$

$r_C = 7\ \text{m}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: Check the loop at B first. We require that $N_B = 0$

$$-N_B - M g = -M\left(\frac{v_B^2}{r_B}\right) \qquad v_B = \sqrt{g r_B} \qquad v_B = 9.907\ \frac{\text{m}}{\text{s}}$$

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_B^2 + M g 2r_B$$

$$h = \frac{v_B^2 - v_A^2}{2g} + 2r_B \qquad h = 25\ \text{m}$$

Now check the loop at C

$$T_A + V_A = T_C + V_C \qquad \frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_C^2 + M g 2r_C$$

$$v_C = \sqrt{v_A^2 + 2g(h - 2r_C)} \qquad v_C = 14.694\ \frac{\text{m}}{\text{s}}$$

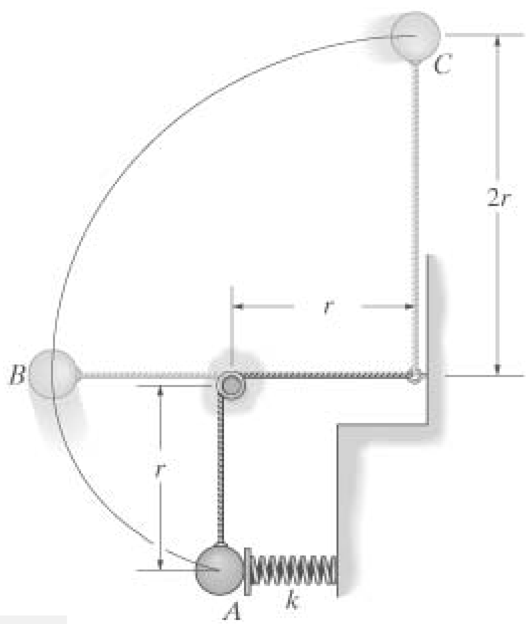
$$-N_C - Mg = -M\left(\frac{v_C^2}{r_C}\right) \qquad N_C = M\left(\frac{v_C^2}{r_C}\right) - Mg \qquad N_C = 16.825\,\text{kN}$$

Since $N_C > 0$ then the coaster successfully passes through loop C.

Problem 14-81

The bob of mass M of a pendulum is fired from rest at position A by a spring which has a stiffness k and is compressed a distance δ. Determine the speed of the bob and the tension in the cord when the bob is at positions B and C. Point B is located on the path where the radius of curvature is still r, i.e., just before the cord becomes horizontal.

Units Used: $\text{kN} = 10^3\ \text{N}$



Given:

$M = 0.75\ \text{kg}$

$k = 6\ \frac{\text{kN}}{\text{m}}$

$\delta = 125\ \text{mm}$

$r = 0.6\ \text{m}$

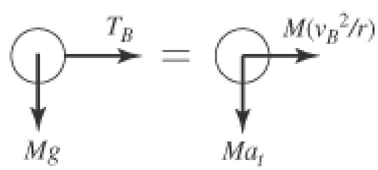
Solution:

At B:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_B^2 + Mgr$$

$$v_B = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 2gr} \qquad v_B = 10.6\ \frac{\text{m}}{\text{s}}$$

$$T_B = M\left(\frac{v_B^2}{r}\right) \qquad T_B = 142\ \text{N}$$

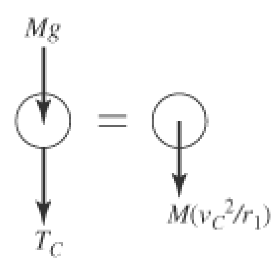


At C:

$$0 + \frac{1}{2}k\delta^2 = \frac{1}{2}Mv_C^2 + Mg3r$$

$$v_C = \sqrt{\left(\frac{k}{M}\right)\delta^2 - 6gr} \qquad v_C = 9.47\ \frac{\text{m}}{\text{s}}$$

$$T_C + Mg = M\left(\frac{v_C^2}{2r}\right)$$



$$T_C = M\left(\frac{v_C^2}{2r} - g\right) \qquad T_C = 48.7\ \text{N}$$

Problem 14-82

The spring has stiffness k and unstretched length L. If it is attached to the smooth collar of weight W and the collar is released from rest at A, determine the speed of the collar just before it strikes the end of the rod at B. Neglect the size of the collar.

Given:

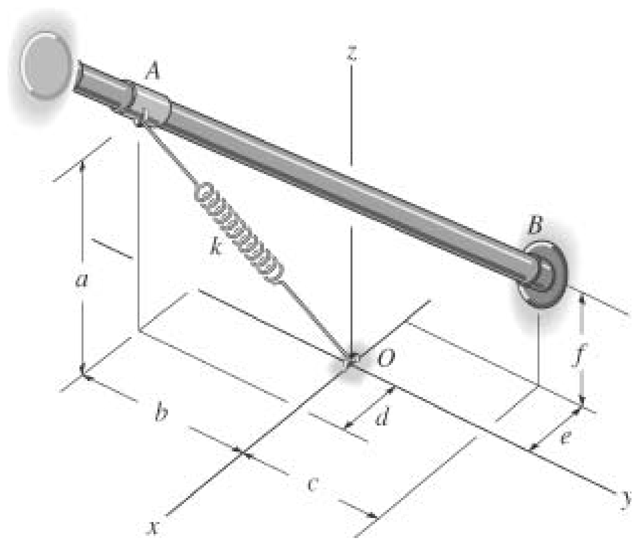
$k = 3\ \frac{\text{lb}}{\text{ft}}$ $\quad c = 3\ \text{ft}$

$L = 2\ \text{ft}$ $\quad d = 1\ \text{ft}$

$W = 5\ \text{lb}$ $\quad e = 1\ \text{ft}$

$a = 6\ \text{ft}$ $\quad f = 2\ \text{ft}$

$b = 4\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

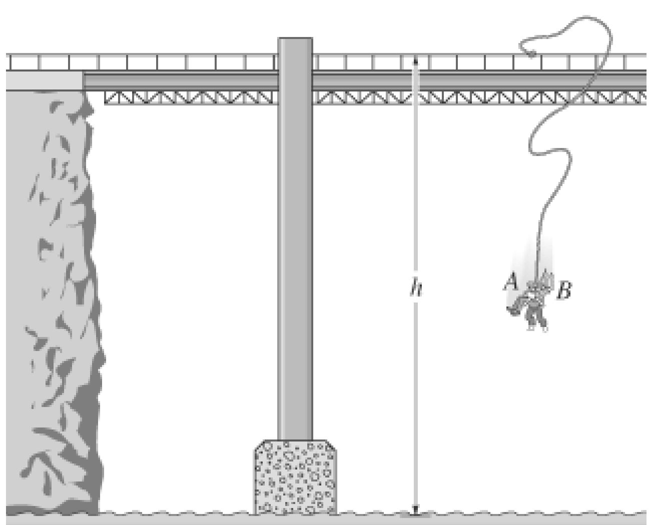
$$T_A + V_A = T_B + V_B$$

$$0 + W(a - f) + \frac{1}{2}k\left(\sqrt{a^2 + b^2 + d^2} - L\right)^2 = \frac{1}{2}\left(\frac{W}{g}\right)v_B^2 + \frac{1}{2}k\left(\sqrt{c^2 + e^2 + f^2} - L\right)^2$$

$$v_B = \sqrt{2g(a - f) + \frac{kg}{W}\left[\left(\sqrt{a^2 + b^2 + d^2} - L\right)^2 - \left(\sqrt{c^2 + e^2 + f^2} - L\right)^2\right]} \qquad v_B = 27.2\ \frac{\text{ft}}{\text{s}}$$

Problem 14-83

Just for fun, two engineering students each of weight W, A and B, intend to jump off the bridge from rest using an elastic cord (bungee cord) having stiffness k. They wish to just reach the surface of the river, when A, attached to the cord, lets go of B at the instant they touch the water. Determine the proper unstretched length of the cord to do the stunt, and calculate the maximum acceleration of student A and the maximum height he reaches above the water after the rebound. From your results, comment on the feasibility of doing this stunt.



Given:

$W = 150\ \text{lb}$ $\quad k = 80\ \frac{\text{lb}}{\text{ft}}$ $\quad h = 120\ \text{ft}$

Solution:

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = 0 - 2Wh + \frac{1}{2}k(h - L)^2$

$L = h - \sqrt{\frac{4Wh}{k}}$ $\quad L = 90\,\text{ft}$

At the bottom, after A lets go of B

$k(h - L) - W = \left(\frac{W}{g}\right)a$ $\quad a = \frac{kg}{W}(h - L) - g$ $\quad a = 483\,\frac{\text{ft}}{\text{s}^2}$ $\quad \frac{a}{g} = 15$

Maximum height

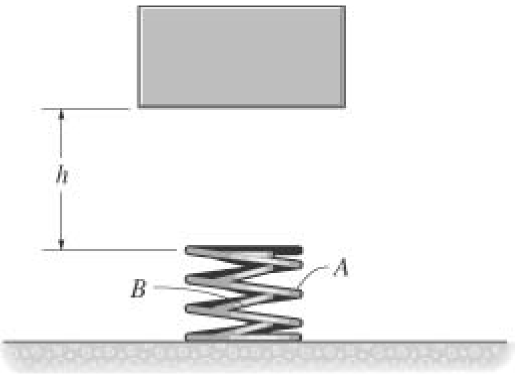
$T_2 + V_2 = T_3 + V_3$ Guess $H = 2h$ Given

$0 + \frac{1}{2}k(h - L)^2 = WH + \frac{1}{2}k(H - h - L)^2$ $\quad H = \text{Find}(H)$ $\quad H = 218.896\,\text{ft}$

This stunt should not be attempted since $\frac{a}{g} = 15$ (excessive) and the rebound height is above the bridge!!

Problem 14-84

Two equal-length springs having stiffnesses k_A and k_B are "nested" together in order to form a shock absorber. If a block of mass M is dropped from an at-rest position a distance h above the top of the springs, determine their deformation when the block momentarily stops.



Given:

$k_A = 300\ \frac{\text{N}}{\text{m}}$ $\quad M = 2\ \text{kg}$

$h = 0.6\ \text{m}$

$k_B = 200\ \frac{\text{N}}{\text{m}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

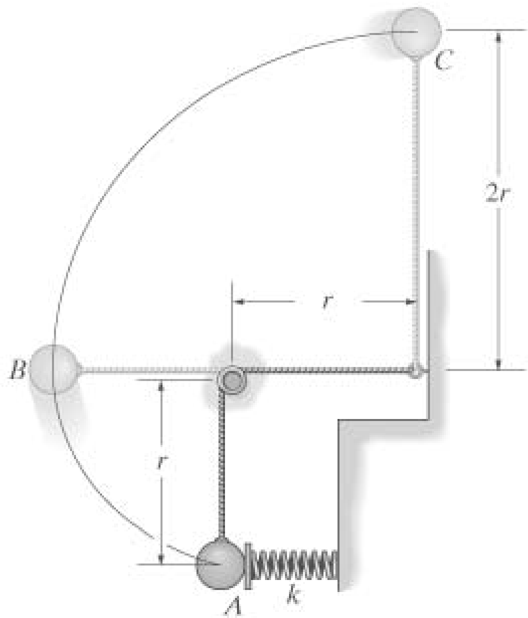
$T_1 + V_1 = T_2 + V_2$

Guess $\delta = 0.1\ \text{m}$

Given $0 + Mgh = \frac{1}{2}(k_A + k_B)\,\delta^2 - Mg\delta$ $\quad \delta = \text{Find}(\delta)$ $\quad \delta = 0.260\ \text{m}$

Problem 14-85

The bob of mass M of a pendulum is fired from rest at position A. If the spring is compressed to a distance δ and released, determine (a) its stiffness k so that the speed of the bob is zero when it reaches point B, where the radius of curvature is still r, and (b) the stiffness k so that when the bob reaches point C the tension in the cord is zero.



Units Used: $\text{kN} = 10^3\ \text{N}$

Given:

$M = 0.75\ \text{kg}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\delta = 50\ \text{mm}$ $\quad r = 0.6\ \text{m}$

Solution:

At B:

$$\frac{1}{2}k\delta^2 = M g r$$

$$k = \frac{2M g r}{\delta^2} \qquad k = 3.53\ \frac{\text{kN}}{\text{m}}$$

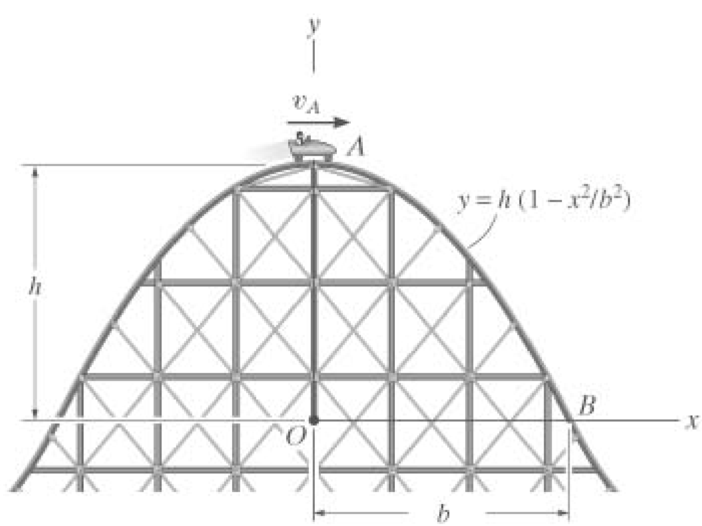
At C:

$$-M g = -M\left(\frac{v_C^{\,2}}{2r}\right) \qquad v_C = \sqrt{2g r}$$

$$\frac{1}{2}k\delta^2 = M g 3r + \frac{1}{2}M v_C^{\,2} \qquad k = \frac{M}{\delta^2}\left(6g r + v_C^{\,2}\right) \qquad k = 14.13\ \frac{\text{kN}}{\text{m}}$$

Problem 14-86

The roller-coaster car has a speed v_A when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is W.



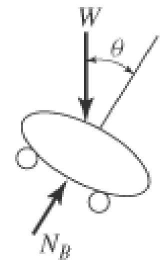
Given:

$W = 350\ \text{lb}$ $\quad b = 200\ \text{ft}$

$v_A = 15\ \dfrac{\text{ft}}{\text{s}}$ $\quad h = 200\ \text{ft}$

Solution:

$$y(x) = h\left(1 - \frac{x^2}{b^2}\right) \qquad y'(x) = -2\left(\frac{h x}{b^2}\right) \qquad y''(x) = -2\left(\frac{h}{b^2}\right)$$



$$\theta_B = \operatorname{atan}(y'(b)) \qquad \rho_B = \frac{\sqrt{\left(1 + y'(b)^2\right)^3}}{y''(b)}$$

$$\frac{1}{2}\left(\frac{W}{g}\right) v_A^2 + W h = \frac{1}{2}\left(\frac{W}{g}\right) v_B^2$$

$$v_B = \sqrt{v_A^2 + 2 g h} \qquad v_B = 114.5\,\frac{\text{ft}}{\text{s}}$$

$$N_B - W\cos(\theta_B) = \frac{W}{g}\left(\frac{v_B^2}{\rho_B}\right) \qquad N_B = W\cos(\theta_B) + \frac{W}{g}\left(\frac{v_B^2}{\rho_B}\right) \qquad N_B = 29.1\ \text{lb}$$

Problem 14-87

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. Determine the minimum speed v_0 at which the cars should coast down from the top of the hill, so that passengers can just make the loop without leaving contact with their seats. Neglect friction, the size of the car and passenger, and assume each passenger and car has a mass m.

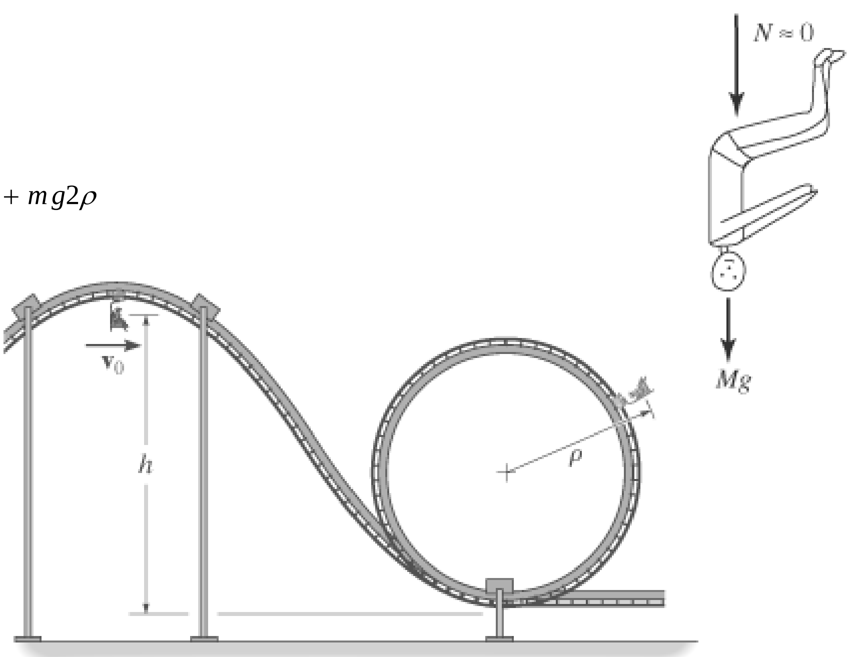
Solution:

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m v_0^2 + m g h = \frac{1}{2} m v_1^2 + m g 2\rho$$

$$v_1 = \sqrt{v_0^2 + 2g(h - 2\rho)}$$



$$m g = m\left(\frac{v_1^2}{\rho}\right)$$

$$v_1 = \sqrt{g\rho}$$

Thus,

$$g\rho = v_0^2 + 2 g h - 4 g\rho$$

$$v_0 = \sqrt{g(5\rho - 2h)}$$

***Problem 14-88**

The Raptor is an outside loop roller coaster in which riders are belted into seats resembling ski-lift chairs. If the cars travel at v_0 when they are at the top of the hill, determine their speed when they are at the top of the loop and the reaction of the passenger of mass M_p on his seat at this instant.The car has a mass M_c. Neglect friction and the size of the car and passenger.

Given:

$M_p = 70 \text{ kg}$

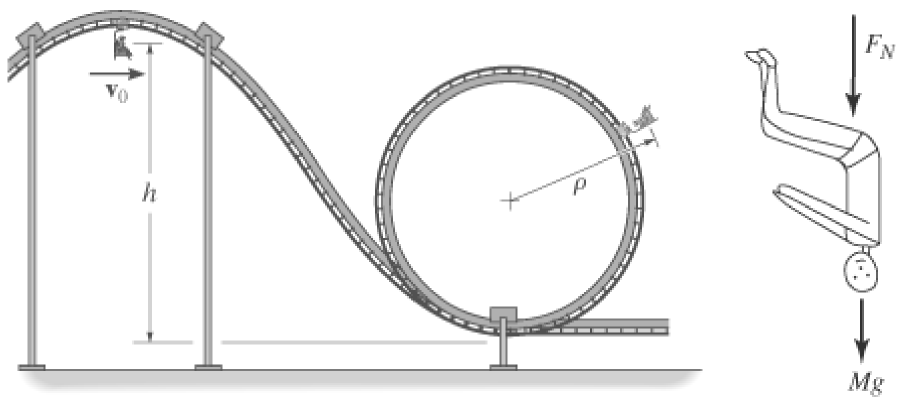
$M_c = 50 \text{ kg}$

$v_0 = 4 \frac{\text{m}}{\text{s}}$

$h = 12 \text{ m}$

$\rho = 5 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$



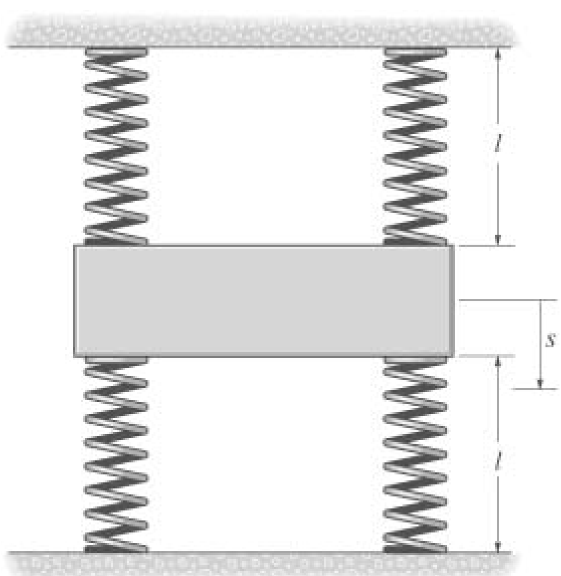
Solution:

$$\frac{1}{2} M v_0^2 + M g h = \frac{1}{2} M v_1^2 + M g 2\rho \qquad v_1 = \sqrt{v_0^2 + 2 g h - 4 g \rho} \qquad v_1 = 7.432 \frac{\text{m}}{\text{s}}$$

$$M_p g + N = M_p \left(\frac{v_1^2}{\rho}\right) \qquad F_N = M_p \left(\frac{v_1^2}{\rho} - g\right) \qquad F_N = 86.7 \text{ N}$$

Problem 14-89

A block having a mass M is attached to four springs. If each spring has a stiffness k and an unstretched length δ, determine the *maximum* downward vertical displacement s_{max} of the block if it is released from rest at $s = 0$.



Units Used: $\text{kN} = 10^3 \text{ N}$

Given:

$M = 20 \text{ kg}$

$k = 2 \frac{\text{kN}}{\text{m}}$

$l = 100 \text{ mm}$

$\delta = 150 \text{ mm}$

Solution:

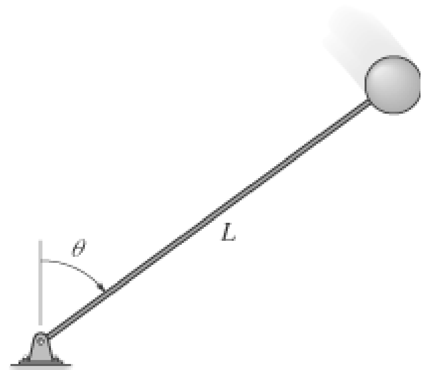
Guess $s_{max} = 100 \text{ mm}$

Given $4\frac{1}{2}k(l-\delta)^2 = -M g s_{max} + 2\left[\frac{1}{2}k(l-\delta+s_{max})^2\right] + 2\left[\frac{1}{2}k(l-\delta-s_{max})^2\right]$

$s_{max} = \text{Find}(s_{max})$ $\qquad s_{max} = 49.0 \text{ mm}$

Problem 14-90

The ball has weight W and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



Given:

$W = 15 \text{ lb}$

$L = 3 \text{ ft}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v = 1 \frac{\text{m}}{\text{s}}$ $\qquad \theta = 10 \text{ deg}$

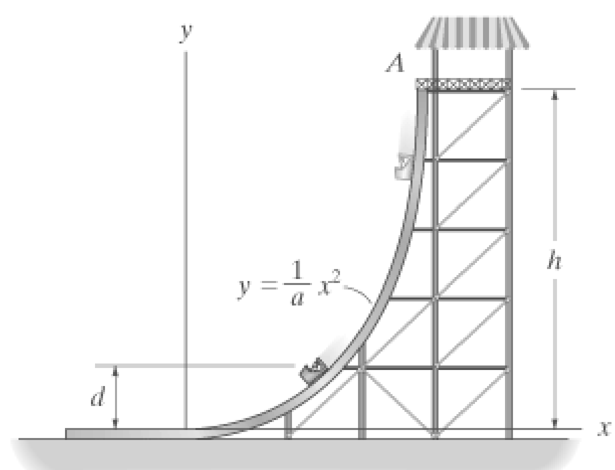
Given

$$W L = \frac{1}{2}\left(\frac{W}{g}\right)v^2 + W L\cos(\theta) \qquad -W\cos(\theta) = \frac{-W}{g}\left(\frac{v^2}{L}\right)$$

$\begin{pmatrix} v \\ \theta \end{pmatrix} = \text{Find}(v, \theta)$ $\qquad v = 8.025 \frac{\text{ft}}{\text{s}}$ $\qquad \theta = 48.2 \text{ deg}$

Problem 14-91

The ride at an amusement park consists of a gondola which is lifted to a height h at A. If it is released from rest and falls along the parabolic track, determine the speed at the instant $y = d$. Also determine the normal reaction of the tracks on the gondola at this instant. The gondola and passenger have a total weight W. Neglect the effects of friction.



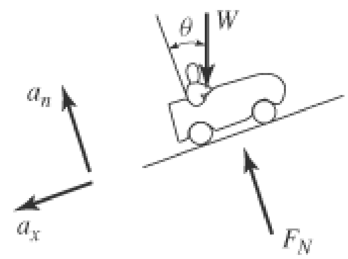
Given:

$W = 500 \text{ lb}$ $\qquad d = 20 \text{ ft}$

$h = 120 \text{ ft}$ $\qquad a = 260 \text{ ft}$

Solution:

$y(x) = \frac{x^2}{a}$ $\quad y'(x) = 2\frac{x}{a}$ $\quad y''(x) = \frac{2}{a}$



$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$\theta(x) = \text{atan}(y'(x))$

Guesses

$x_2 = 1\text{ ft}$ $\quad v_2 = 10\frac{\text{ft}}{\text{s}}$ $\quad F_N = 1\text{ lb}$

Given

$$Wh = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 + Wd \qquad d = y(x_2) \qquad F_N - W\cos(\theta(x_2)) = \frac{W}{g}\left(\frac{v_2^2}{\rho(x_2)}\right)$$

$$\begin{pmatrix} x_2 \\ v_2 \\ F_N \end{pmatrix} = \text{Find}(x_2, v_2, F_N) \qquad x_2 = 72.1\text{ ft} \qquad v_2 = -80.2\frac{\text{ft}}{\text{s}} \qquad F_N = 952\text{ lb}$$

***Problem 14-92**

The collar of weight *W* has a speed *v* at *A*. The attached spring has an unstretched length δ and a stiffness *k*. If the collar moves over the smooth rod, determine its speed when it reaches point *B*, the normal force of the rod on the collar, and the rate of decrease in its speed.

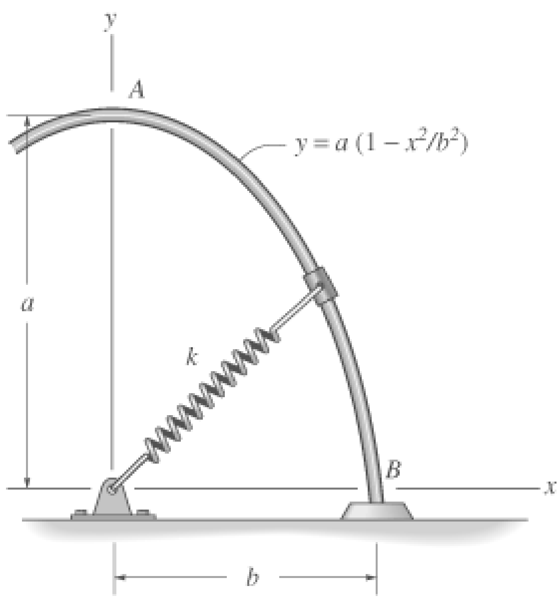
Given:

$W = 2\text{ lb}$ $\quad \delta = 2\text{ ft}$

$a = 4.5\text{ ft}$ $\quad k = 10\frac{\text{lb}}{\text{ft}}$

$b = 3\text{ ft}$ $\quad g = 32.2\frac{\text{ft}}{\text{s}^2}$

$v = 5\frac{\text{ft}}{\text{s}}$



Solution:

$$y(x) = a\left(1 - \frac{x^2}{b^2}\right)$$

$$y'(x) = -2\left(\frac{a\,x}{b^2}\right) \qquad y''(x) = -2\left(\frac{a}{b^2}\right) \qquad \rho(x) = \frac{\sqrt{\left(1+y'(x)^2\right)^3}}{y''(x)}$$

$$\theta = \operatorname{atan}(y'(b)) \qquad \rho_B = \rho(b)$$

Guesses $v_B = 1\ \frac{\text{ft}}{\text{s}}$ $\quad F_N = 1\ \text{lb}$ $\quad v'_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\frac{1}{2}\left(\frac{W}{g}\right)v^2 + \frac{1}{2}k(a-\delta)^2 + W\,a = \frac{1}{2}\left(\frac{W}{g}\right)v_B{}^2 + \frac{1}{2}k(b-\delta)^2$

$$F_N + k(b-\delta)\sin(\theta) - W\cos(\theta) = \frac{W}{g}\left(\frac{v_B{}^2}{\rho_B}\right)$$

$$-k(b-\delta)\cos(\theta) - W\sin(\theta) = \left(\frac{W}{g}\right)v'_B$$

$$\begin{pmatrix} v_B \\ F_N \\ v'_B \end{pmatrix} = \operatorname{Find}(v_B, F_N, v'_B) \qquad v_B = 34.1\ \frac{\text{ft}}{\text{s}} \qquad F_N = 7.84\ \text{lb} \qquad v'_B = -20.4\ \frac{\text{ft}}{\text{s}^2}$$

Problem 14-93

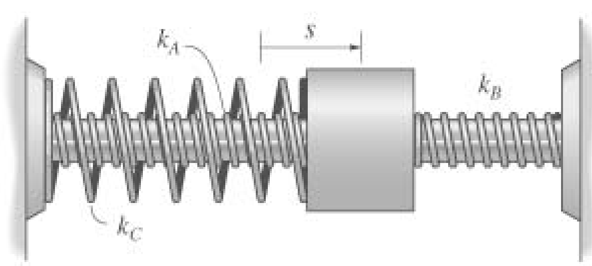
The collar of weight W is constrained to move on the smooth rod. It is attached to the three springs which are unstretched at $s = 0$. If the collar is displaced a distance $s = s_1$ and released from rest, determine its speed when $s = 0$.

Given:

$W = 20\ \text{lb}$ $\quad k_A = 10\ \frac{\text{lb}}{\text{ft}}$

$s_1 = 0.5\ \text{ft}$ $\quad k_B = 10\ \frac{\text{lb}}{\text{ft}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad k_C = 30\ \frac{\text{lb}}{\text{ft}}$



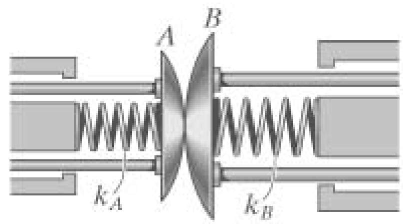
Solution:

$$\frac{1}{2}(k_A + k_B + k_C)s_1{}^2 = \frac{1}{2}\left(\frac{W}{g}\right)v^2$$

$$v = \sqrt{\frac{g}{W}(k_A + k_B + k_C)}\,s_1 \qquad v = 4.49\ \frac{\text{ft}}{\text{s}}$$

Problem 14-94

A tank car is stopped by two spring bumpers A and B, having stiffness k_A and k_B respectively. Bumper A is attached to the car, whereas bumper B is attached to the wall. If the car has a weight W and is freely coasting at speed v_c determine the maximum deflection of each spring at the instant the bumpers stop the car.

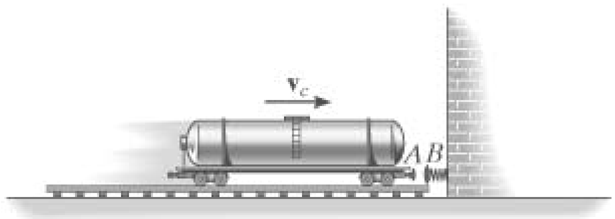


Given:

$k_A = 15 \times 10^3 \ \frac{\text{lb}}{\text{ft}}$ $\quad k_B = 20 \times 10^3 \ \frac{\text{lb}}{\text{ft}}$

$W = 25 \times 10^3 \ \text{lb}$ $\quad v_c = 3 \ \frac{\text{ft}}{\text{s}}$

Solution:



Guesses $\quad s_A = 1 \ \text{ft} \qquad s_B = 1 \ \text{ft}$

Given $\quad \frac{1}{2}\left(\frac{W}{g}\right)v_c^2 = \frac{1}{2}k_A s_A^2 + \frac{1}{2}k_B s_B^2$

$k_A s_A = k_B s_B$

$\begin{pmatrix} s_A \\ s_B \end{pmatrix} = \text{Find}(s_A, s_B) \qquad \begin{pmatrix} s_A \\ s_B \end{pmatrix} = \begin{pmatrix} 0.516 \\ 0.387 \end{pmatrix} \text{ft}$

Problem 14-95

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.

Solution:

$$V = -\int_{\infty}^{r} \frac{-G M_e m}{r^2} \, dr = \frac{-G M_e m}{r} \qquad \text{QED}$$

$$F = -Grad\ V = -\frac{d}{dr}V = -\frac{d}{dr}\frac{-G M_e m}{r} = \frac{-G M_e m}{r^2} \qquad \text{QED}$$

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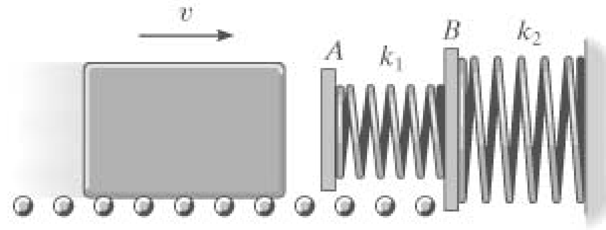
*__Problem 14-96__

The double-spring bumper is used to stop the steel billet of weight W in the rolling mill. Determine the maximum deflection of the plate A caused by the billet if it strikes the plate with a speed v. Neglect the mass of the springs, rollers and the plates A and B.

Given:

$W = 1500\ \text{lb}$ $\quad k_1 = 3000\ \dfrac{\text{lb}}{\text{ft}}$

$v = 8\ \dfrac{\text{ft}}{\text{s}}$ $\quad k_2 = 4500\ \dfrac{\text{lb}}{\text{ft}}$



Solution:

$$k_1 x_1 = k_2 x_2$$

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$

$$\frac{1}{2}\left(\frac{W}{g}\right)v^2 = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2\left(\frac{k_1 x_1}{k_2}\right)^2$$

$$\left(\frac{W}{g}\right)v^2 = \left(k_1 + \frac{k_1^2 x_1^2}{k_2}\right)x_1^2 \qquad x_1 = \sqrt{\frac{W v^2}{g\left(k_1 + \dfrac{k_1^2}{k_2}\right)}} \qquad x_1 = 0.235\ \text{m}$$