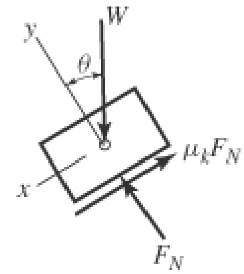
Problem 15-1

A block of weight W slides down an inclined plane of angle θ with initial velocity v_0. Determine the velocity of the block at time t_1 if the coefficient of kinetic friction between the block and the plane is μ_k.

Given:



$W = 20\ \text{lb}$ $\quad t_1 = 3\ \text{s}$

$\theta = 30\ \text{deg}$ $\quad \mu_k = 0.25$

$v_0 = 2\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$(\nwarrow +)\ m v_{y1} + \Sigma \int_{t_1}^{t_2} F_{y'}\, dt = m v_{y2}$$

$$0 + F_N t_1 - W\cos(\theta) t_1 = 0 \qquad F_N = W\cos(\theta) \qquad F_N = 17.32\ \text{lb}$$

$$(\swarrow +)\ m(v_{x'1}) + \Sigma \int_{t_1}^{t_2} F_{x'}\, dt = m(v_{x'2})$$

$$\left(\frac{W}{g}\right) v_0 + W\sin(\theta) t_1 - \mu_k F_N t_1 = \left(\frac{W}{g}\right) v$$

$$v = \frac{W v_0 + W\sin(\theta) t_1 g - \mu_k F_N t_1 g}{W} \qquad v = 29.4\ \frac{\text{ft}}{\text{s}}$$

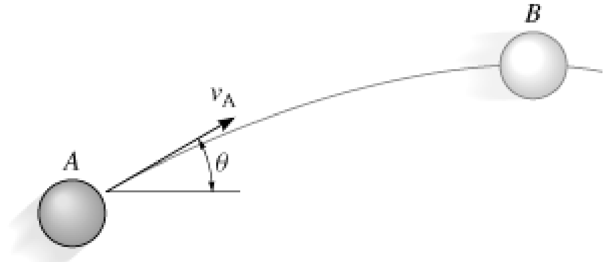
Problem 15-2

A ball of weight W is thrown in the direction shown with an initial speed v_A. Determine the time needed for it to reach its highest point B and the speed at which it is traveling at B. Use the principle of impulse and momentum for the solution.

Given:

$W = 2\ \text{lb}$ $\quad \theta = 30\ \text{deg}$

$v_A = 18\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W}{g}\right)v_A\sin(\theta) - W\,t = \left(\frac{W}{g}\right)0 \qquad t = \frac{v_A\sin(\theta)}{g} \qquad t = 0.280\ \text{s}$$

$$\left(\frac{W}{g}\right)v_A\cos(\theta) + 0 = \left(\frac{W}{g}\right)v_x \qquad v_x = v_A\cos(\theta) \qquad v_x = 15.59\ \frac{\text{ft}}{\text{s}}$$

Problem 15-3

A block of weight W is given an initial velocity v_0 up a smooth slope of angle θ. Determine the time it will take to travel up the slope before it stops.

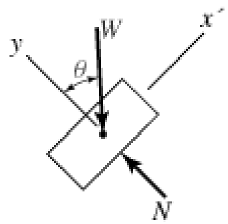
Given:

$W = 5$ lb

$v_0 = 10\ \frac{\text{ft}}{\text{s}}$

$\theta = 45$ deg

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W}{g}\right)v_0 - W\sin(\theta)t = 0 \qquad t = \frac{v_0}{g\sin(\theta)} \qquad t = 0.439\ \text{s}$$

***Problem 15-4**

The baseball has a horizontal speed v_1 when it is struck by the bat B. If it then travels away at an angle θ from the horizontal and reaches a maximum height h, measured from the height of the bat, determine the magnitude of the net impulse of the bat on the ball.The ball has a mass M. Neglect the weight of the ball during the time the bat strikes the ball.

Given:

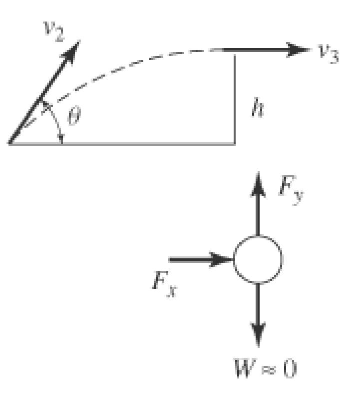
$M = 0.4$ kg

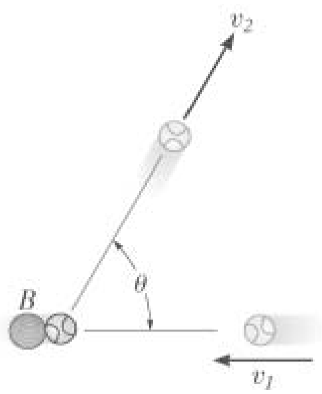
$v_1 = 35\ \frac{\text{m}}{\text{s}}$

$h = 50$ m

$\theta = 60$ deg

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$





Solution:

Guesses

$$v_2 = 20\ \frac{m}{s} \quad Imp_x = 1\ N\cdot s \quad Imp_y = 10\ N\cdot s$$

Given

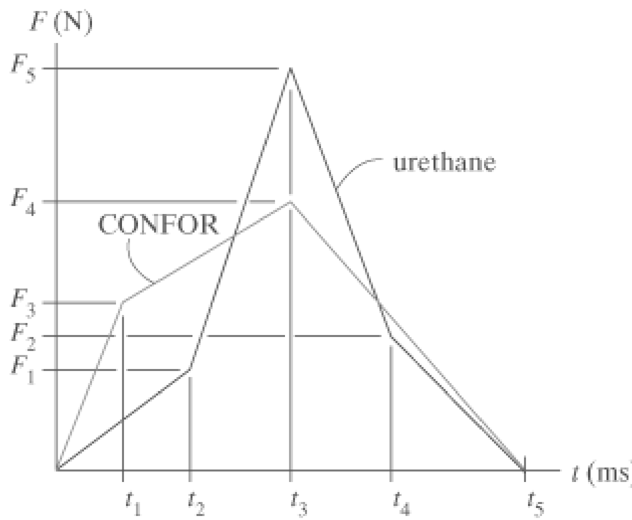
$$\frac{1}{2}M\left(v_2\sin(\theta)\right)^2 = M\,g\,h \qquad -M\,v_1 + Imp_x = M\,v_2\cos(\theta) \qquad 0 + Imp_y = M\,v_2\sin(\theta)$$

$$\begin{pmatrix} v_2 \\ Imp_x \\ Imp_y \end{pmatrix} = \text{Find}\left(v_2, Imp_x, Imp_y\right) \qquad v_2 = 36.2\ \frac{m}{s} \qquad \begin{pmatrix} Imp_x \\ Imp_y \end{pmatrix} = \begin{pmatrix} 21.2 \\ 12.5 \end{pmatrix} N\cdot s$$

$$\left|\begin{pmatrix} Imp_x \\ Imp_y \end{pmatrix}\right| = 24.7\ N\cdot s$$

Problem 15-5

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



Units Used:

$$ms = 10^{-3}\ s$$

Given:

$$F_1 = 0.3\ N \quad t_1 = 2\ ms$$

$$F_2 = 0.4\ N \quad t_2 = 4\ ms$$

$$F_3 = 0.5\ N \quad t_3 = 7\ ms$$

$$F_4 = 0.8\ N \quad t_4 = 10\ ms$$

$$F_5 = 1.2\ N \quad t_5 = 14\ ms$$

Solution:

CONFOR foam:

$$I_c = \frac{1}{2}t_1F_3 + \frac{1}{2}\left(F_3 + F_4\right)\left(t_3 - t_1\right) + \frac{1}{2}F_4\left(t_5 - t_3\right)$$

$$I_c = 6.55\ N\cdot ms$$

Urethane foam:

$$I_U = \frac{1}{2}t_2 F_1 + \frac{1}{2}(F_5 + F_1)(t_3 - t_2) + \frac{1}{2}(F_5 + F_2)(t_4 - t_3) + \frac{1}{2}(t_5 - t_4)F_2$$

$$I_U = 6.05\ \text{N}\cdot\text{ms}$$

Problem 15-6

A man hits the golf ball of mass M such that it leaves the tee at angle θ with the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse of the club C on the ball. Neglect the impulse caused by the ball's weight while the club is striking the ball.

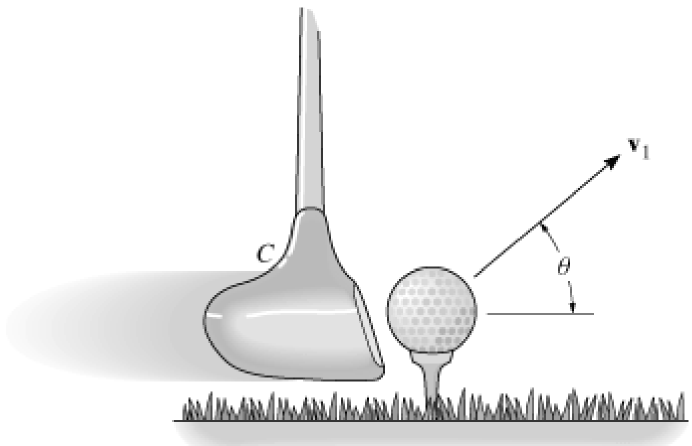
Given:

$M = 50\ \text{gm}$

$\theta = 40\ \text{deg}$

$d = 20\ \text{m}$

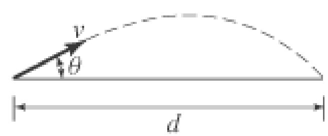
$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: First find the velocity v_1

Guesses $v_1 = 1\ \frac{\text{m}}{\text{s}}$ $t = 1\ \text{s}$

Given $0 = \left(\frac{-g}{2}\right)t^2 + v_1 \sin(\theta)t$ $d = v_1 \cos(\theta)t$



$\begin{pmatrix} t \\ v_1 \end{pmatrix} = \text{Find}(t, v_1)$ $t = 1.85\ \text{s}$ $v_1 = 14.11\ \frac{\text{m}}{\text{s}}$

Impulse - Momentum

$0 + Imp = M v_1$ $Imp = M v_1$ $Imp = 0.706\ \text{N}\cdot\text{s}$

Problem 15-7

A solid-fueled rocket can be made using a fuel grain with either a hole (a), or starred cavity (b), in the cross section. From experiment the engine thrust-time curves (T vs. t) for the same amount of propellant using these geometries are shown. Determine the total impulse in both cases.

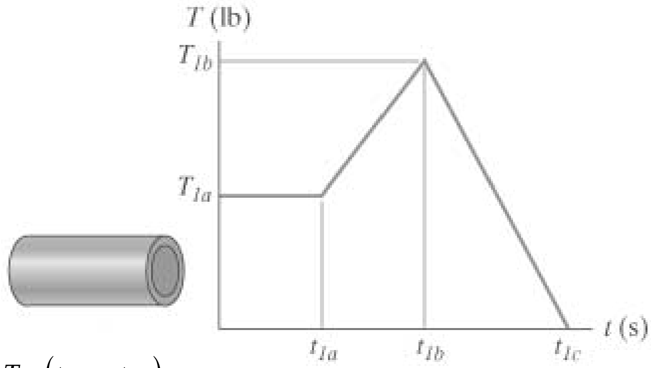
Given:

$T_{1a} = 4\ \text{lb}$ $t_{1a} = 3\ \text{s}$

$T_{1b} = 8\ \text{lb}$ $t_{1b} = 6\ \text{s}$

$T_{2a} = 6 \text{ lb} \quad t_{1c} = 10 \text{ s}$

$t_{2a} = 8 \text{ s} \quad t_{2b} = 10 \text{ s}$

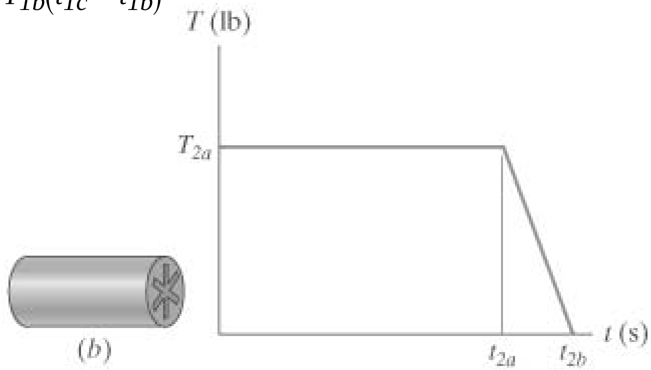


Solution:

Impulse is area under curve for hole cavity.

$$I_a = T_{1a}t_{1a} + \frac{1}{2}(T_{1a} + T_{1b})(t_{1b} - t_{1a}) + \frac{1}{2}T_{1b}(t_{1c} - t_{1b})$$

$$I_a = 46.00 \text{ lb}\cdot\text{s}$$



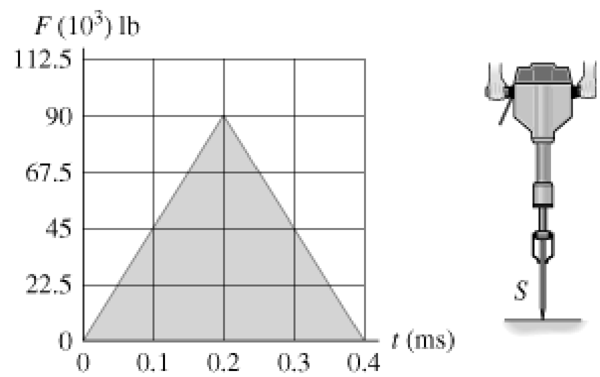
For starred cavity:

$$I_b = T_{2a}t_{2a} + \frac{1}{2}T_{2a}(t_{2b} - t_{2a})$$

$$I_b = 54.00 \text{ lb}\cdot\text{s}$$

***Problem 15-8**

During operation the breaker hammer develops on the concrete surface a force which is indicated in the graph. To achieve this the spike S of weight W is fired from rest into the surface at speed v. Determine the speed of the spike just after rebounding.



Given:

$W = 2 \text{ lb}$

$v = 200 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

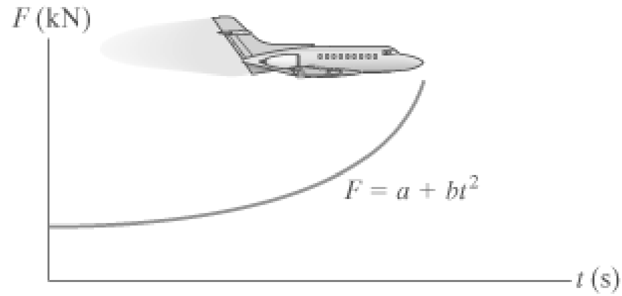
Solution:

$$I = \left(\frac{1}{2} \times 90 \times 10^3 \text{ lb}\right)(0.4\ 10^{-3}\text{s}) \quad I = 18.00 \text{ lb}\cdot\text{s} \quad \Delta t = 0.4 \times 10^{-3} \text{ s}$$

$$\left(\frac{-W}{g}\right)v + I - W\Delta t = \left(\frac{W}{g}\right)v' \qquad v' = -v + \left(\frac{Ig}{W}\right) - g\Delta t \qquad v' = 89.8 \frac{\text{ft}}{\text{s}}$$

Problem 15-9

The jet plane has a mass M and a horizontal velocity v_0 when $t = 0$. If *both* engines provide a horizontal thrust which varies as shown in the graph, determine the plane's velocity at time t_1. Neglect air resistance and the loss of fuel during the motion.



Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$M = 250 \text{ Mg}$$

$$v_0 = 100 \frac{\text{m}}{\text{s}}$$

$$t_1 = 15 \text{ s}$$

$$a = 200 \text{ kN}$$

$$b = 2 \frac{\text{kN}}{\text{s}^2}$$

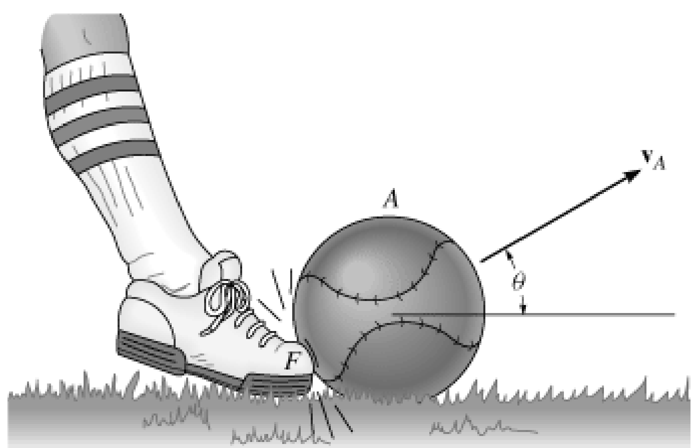
Solution:

$$M v_0 + \int_0^{t_1} a + b t^2 \, dt = M v_1$$

$$v_1 = v_0 + \frac{1}{M} \int_0^{t_1} a + b t^2 \, dt \qquad v_1 = 121.00 \frac{\text{m}}{\text{s}}$$

Problem 15-10

A man kicks the ball of mass M such that it leaves the ground at angle θ with the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse of his foot F on the ball. Neglect the impulse caused by the ball's weight while it's being kicked.

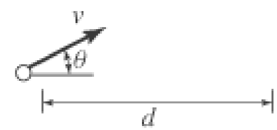


Given:

$$M = 200 \text{ gm} \quad d = 15 \text{ m}$$

$$\theta = 30 \text{ deg} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution: First find the velocity v_A



Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $t = 1\ \text{s}$

Given $0 = \left(\frac{-g}{2}\right)t^2 + v_A \sin(\theta)t$ $d = v_A \cos(\theta)t$

$\begin{pmatrix} t \\ v_A \end{pmatrix} = \text{Find}(t, v_A)$ $t = 1.33\ \text{s}$ $v_A = 13.04\ \frac{\text{m}}{\text{s}}$

Impulse - Momentum

$0 + I = M v_A$ $I = M v_A$ $I = 2.61\ \text{N}\cdot\text{s}$

Problem 15-11

The particle P is acted upon by its weight W and forces $\mathbf{F}_1 = (a\mathbf{i} + bt\mathbf{j} + ct\mathbf{k})$ and $\mathbf{F}_2 = dt^2\mathbf{i}$. If the particle originally has a velocity of $\mathbf{v_1} = (v_{1x}\mathbf{i}+v_{1y}\mathbf{j}+v_{1z}\mathbf{k})$, determine its speed after time t_1.

Given:

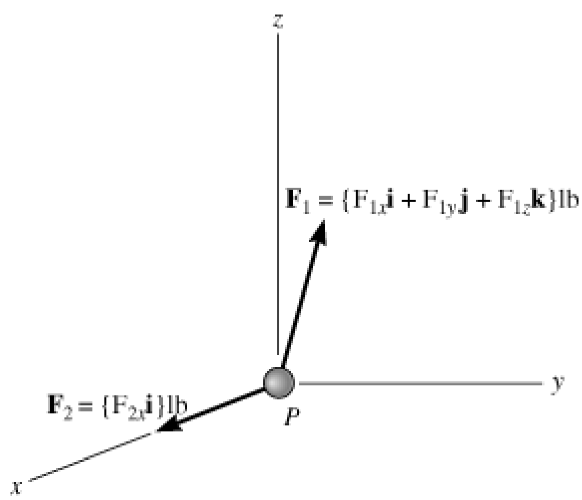
$W = 3\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$v_{1x} = 3\ \frac{\text{ft}}{\text{s}}$ $a = 5\ \text{lb}$

$v_{1y} = 1\ \frac{\text{ft}}{\text{s}}$ $b = 2\ \frac{\text{lb}}{\text{s}}$

$v_{1z} = 6\ \frac{\text{ft}}{\text{s}}$ $c = 1\ \frac{\text{lb}}{\text{s}}$

$t_1 = 2\ \text{s}$ $d = 1\ \frac{\text{lb}}{\text{s}^2}$



Solution:

$$m v_1 + \int_0^{t_1} (F_1 + F_2 - W k)\, dt = m v_2 \qquad v_2 = v_1 + \frac{1}{m}\int_0^{t_1} (F_1 + F_2 - W k)\, dt$$

$$v_{2x} = v_{1x} + \frac{g}{W}\int_0^{t_1} a + d t^2\, dt \qquad v_{2x} = 138.96\ \frac{\text{ft}}{\text{s}}$$

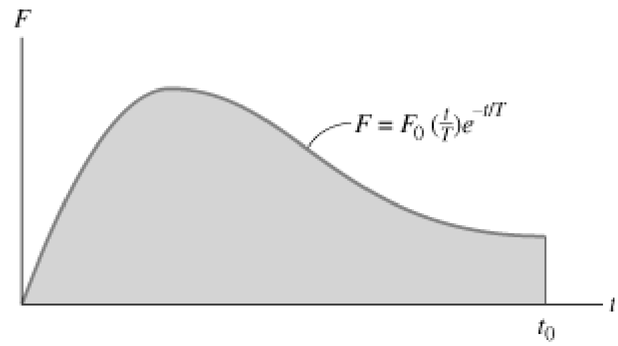
$$v_{2y} = v_{1y} + \frac{g}{W}\int_0^{t_1} b\, t\, dt \qquad v_{2y} = 43.93\ \frac{\text{ft}}{\text{s}}$$

$$v_{2z} = v_{1z} + \frac{g}{W}\int_0^{t_1} c\,t - W\,dt \qquad v_{2z} = -36.93\,\frac{\text{ft}}{\text{s}}$$

$$v_2 = \sqrt{v_{2x}^{\,2} + v_{2y}^{\,2} + v_{2z}^{\,2}} \qquad v_2 = 150.34\,\frac{\text{ft}}{\text{s}}$$

***Problem 15-12**

The twitch in a muscle of the arm develops a force which can be measured as a function of time as shown in the graph. If the effective contraction of the muscle lasts for a time t_0, determine the impulse developed by the muscle.



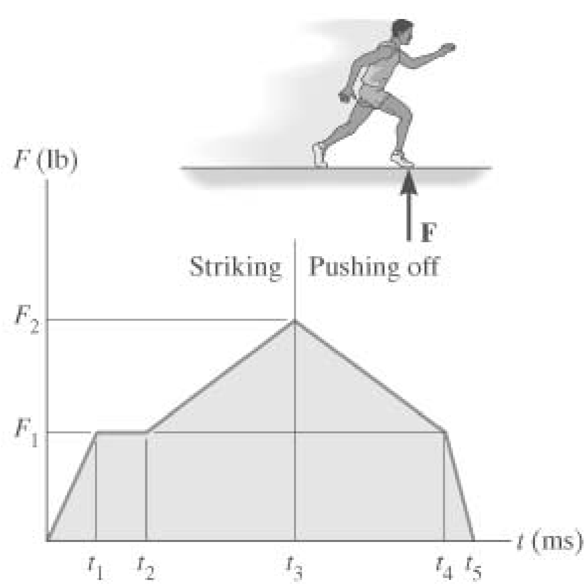
Solution:

$$I = \int_0^{t_0} F_0\left(\frac{t}{T}\right) e^{\frac{-t}{T}}\,dt = F_0\left(-t_0 - T\right) e^{\frac{-t_0}{T}} + T\,F_0$$

$$I = F_0\,T\left[1 - \left(1 + \frac{t_0}{T}\right)e^{\frac{-t_0}{T}}\right]$$

Problem 15-13

From experiments, the time variation of the vertical force on a runner's foot as he strikes and pushes off the ground is shown in the graph.These results are reported for a 1-lb *static* load, i.e., in terms of unit weight. If a runner has weight W, determine the approximate vertical impulse he exerts on the ground if the impulse occurs in time t_5.



Units Used:

$$\text{ms} = 10^{-3}\ \text{s}$$

Given:

$$W = 175\ \text{lb}$$

$$t_1 = 25\ \text{ms} \qquad t = 210\ \text{ms}$$

$t_2 = 50\ \text{ms} \quad t_3 = 125\ \text{ms}$

$t_4 = 200\ \text{ms} \quad t_5 = 210\ \text{ms}$

$F_2 = 3.0\ \text{lb} \quad F_1 = 1.5\ \text{lb}$

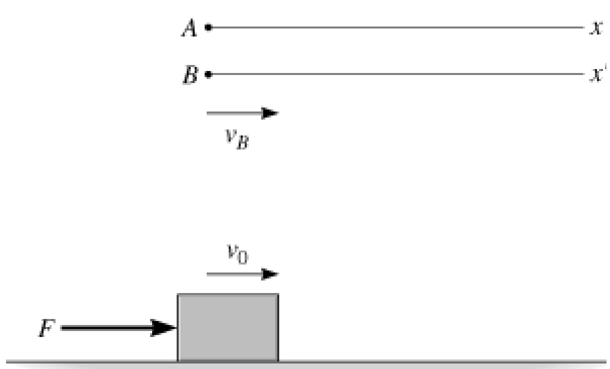
Solution:

$$Area = \frac{1}{2}t_1 F_1 + F_1(t_2 - t_1) + F_1(t_4 - t_2) + \frac{1}{2}(t_5 - t_4)F_1 + \frac{1}{2}(F_2 - F_1)(t_4 - t_2)$$

$$Imp = Area\frac{W}{\text{lb}} \qquad Imp = 70.2\ \text{lb}\cdot\text{s}$$

Problem 15-14

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the block of mass M which rests on the smooth surface and is subjected to horizontal force **F**. If observer A is in a *fixed* frame x, determine the final speed of the block at time t_1 if it has an initial speed v_0 measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at constant velocity v_B relative to A.



Given:

$M = 10\ \text{kg} \quad v_0 = 5\ \frac{\text{m}}{\text{s}}$

$F = 6\ \text{N}$

$t_1 = 4\ \text{s} \quad v_B = 2\ \frac{\text{m}}{\text{s}}$

Solution:

Observer A:

$$M v_0 + F t_1 = M v_{1A} \qquad v_{1A} = v_0 + \left(\frac{F}{M}\right)t_1 \qquad v_{1A} = 7.40\ \frac{\text{m}}{\text{s}}$$

Observer B:

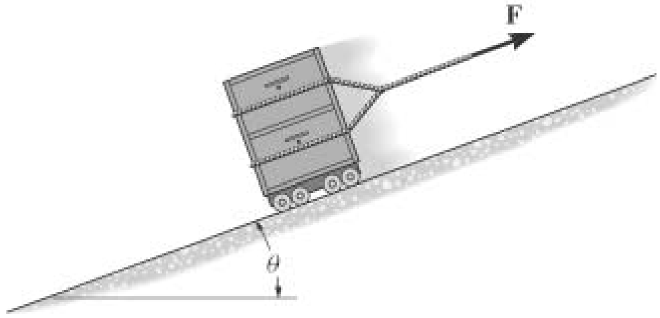
$$M(v_0 - v_B) + F t_1 = M v_{1B} \qquad v_{1B} = v_0 - v_B + \left(\frac{F}{M}\right)t_1 \qquad v_{1B} = 5.40\ \frac{\text{m}}{\text{s}}$$

Note that $v_{1A} = v_{1B} + v_B$

Problem 15-15

The cabinet of weight W is subjected to the force **F** = $a(bt+c)$. If the cabinet is initially moving up the plane with velocity v_0, determine how long it will take before the cabinet comes to a stop. **F** always acts parallel to the plane. Neglect the size of the rollers.

Given:



$W = 4\ \text{lb}$ $\quad v_0 = 10\ \frac{\text{ft}}{\text{s}}$

$a = 20\ \text{lb}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$b = \frac{1}{\text{s}}$ $\quad \theta = 20\ \text{deg}$

$c = 1$

Solution: Guess $t = 10\ \text{s}$ Given

$$\left(\frac{W}{g}\right)v_0 + \int_0^t a(b\tau + c)\,d\tau - W\sin(\theta)t = 0 \qquad t = \text{Find}(t) \qquad t = -0.069256619\ \text{s}$$

*****Problem 15-16**

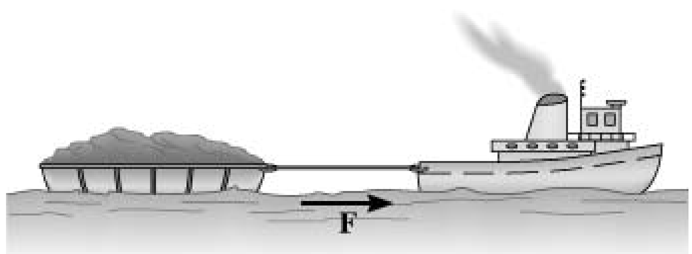
If it takes time t_1 for the tugboat of mass m_t to increase its speed uniformly to v_1 starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine the force F acting on the tugboat. The barge has mass of m_b.

Units Used:

$\text{Mg} = 1000\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

Given:



$t_1 = 35\ \text{s}$

$m_t = 50\ \text{Mg}$

$v_1 = 25\ \frac{\text{km}}{\text{hr}}$

$m_b = 75\ \text{Mg}$

Solution:

The barge alone

$$0 + T t_1 = m_b v_1 \qquad T = \frac{m_b v_1}{t_1} \qquad T = 14.88\,\text{kN}$$

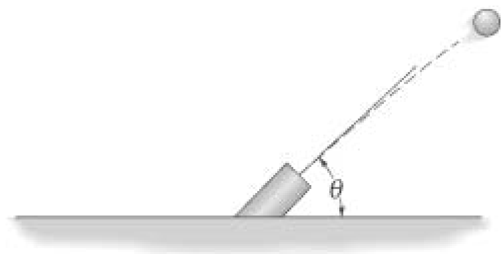
The barge and the tug

$$0 + F t_1 = (m_t + m_b)v_1 \qquad F = \frac{(m_t + m_b)v_1}{t_1} \qquad F = 24.80\,\text{kN}$$

Problem 15-17

When the ball of weight W is fired, it leaves the ground at an angle θ from the horizontal and strikes the ground at the same elevation a distance d away. Determine the impulse given to the ball.

Given:

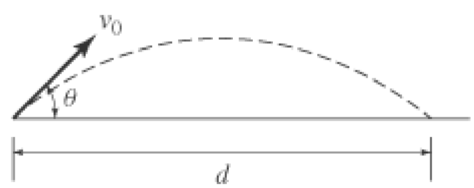


$W = 0.4\ \text{lb}$

$d = 130\ \text{ft}$

$\theta = 40\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad v_0 = 1\ \frac{\text{ft}}{\text{s}} \qquad t = 1\ \text{s} \qquad Imp = 1\ \text{lb}\cdot\text{s}$

Given $\quad v_0 \cos(\theta) t = d \qquad \frac{-1}{2} g t^2 + v_0 \sin(\theta) t = 0 \qquad Imp = \left(\frac{W}{g}\right) v_0$

$$\begin{pmatrix} v_0 \\ t \\ Imp \end{pmatrix} = \text{Find}(v_0, t, Imp) \qquad v_0 = 65.2\ \frac{\text{ft}}{\text{s}} \qquad t = 2.6\ \text{s} \qquad Imp = 0.810\ \text{lb}\cdot\text{s}$$

Problem 15-18

The uniform beam has weight W. Determine the average tension in each of the two cables AB and AC if the beam is given an upward speed v in time t starting from rest. Neglect the mass of the cables.

Units Used:

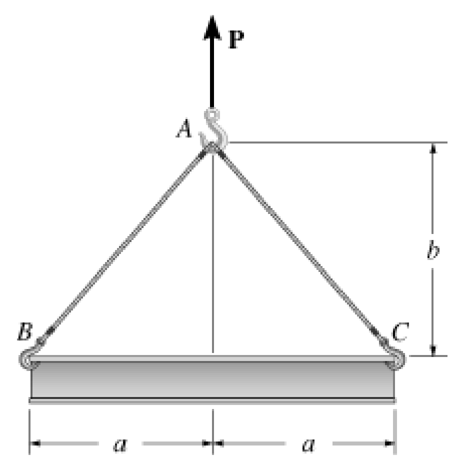
$\text{kip} = 10^3\ \text{lb}$

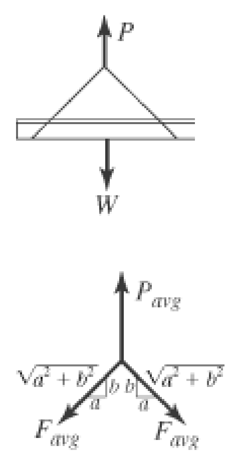
Given:

$W = 5000\ \text{lb} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$v = 8\ \frac{\text{ft}}{\text{s}} \qquad a = 3\ \text{ft}$

$t = 1.5\ \text{s} \qquad b = 4\ \text{ft}$





Solution:

$$0 - W t + 2\left(\frac{b}{\sqrt{a^2 + b^2}}\right) F_{AB} t = \left(\frac{W}{g}\right) v$$

$$F_{AB} = \left(\frac{W}{g} v + W t\right)\left(\frac{\sqrt{a^2 + b^2}}{2 b t}\right) \qquad F_{AB} = 3.64 \text{ kip}$$

Problem 15-19

The block of mass M is moving downward at speed v_1 when it is a distance h from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.

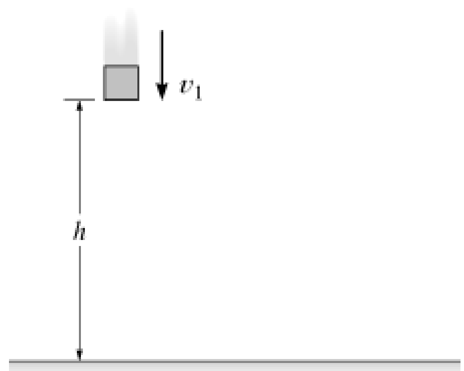
Given:

$M = 5 \text{ kg}$

$v_1 = 2 \frac{\text{m}}{\text{s}}$

$h = 8 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$



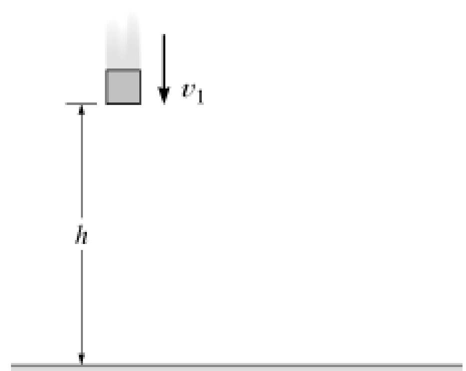
Solution:

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2 g h} \quad v_2 = 12.69 \frac{\text{m}}{\text{s}}$

Collision $\quad M v_2 - I = 0 \quad I = M v_2 \quad I = 63.4 \text{ N}\cdot\text{s}$

***Problem 15-20**

The block of mass M is falling downward at speed v_1 when it is a distance h from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in time Δt once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



Given:

$M = 5\ \text{kg}$

$v_1 = 2\ \frac{\text{m}}{\text{s}}$ $\quad h = 8\ \text{m}$

$\Delta t = 0.9\ \text{s}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh} \quad v_2 = 12.69\ \frac{\text{m}}{\text{s}}$

Collision $\quad M v_2 - F\Delta t = 0 \quad F = \frac{M v_2}{\Delta t} \quad F = 70.5\ \text{N}$

Problem 15-21

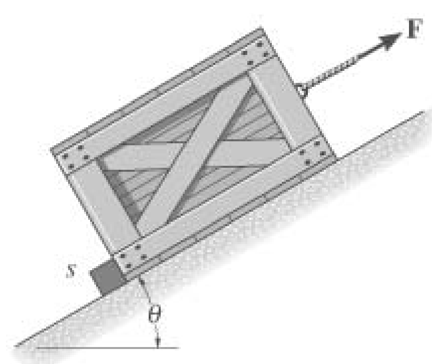
A crate of mass M rests against a stop block s, which prevents the crate from moving down the plane. If the coefficients of static and kinetic friction between the plane and the crate are μ_s and μ_k respectively, determine the time needed for the force **F** to give the crate a speed v up the plane. The force always acts parallel to the plane and has a magnitude of $F = at$. *Hint:* First determine the time needed to overcome static friction and start the crate moving.

Given:

$M = 50\ \text{kg} \quad \theta = 30\ \text{deg} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$v = 2\ \frac{\text{m}}{\text{s}} \quad \mu_s = 0.3$

$a = 300\ \frac{\text{N}}{\text{s}} \quad \mu_k = 0.2$



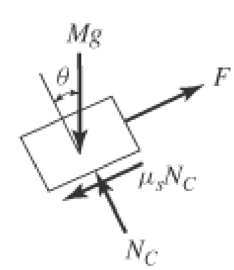
Solution:

Guesses $\quad t_1 = 1\ \text{s} \quad N_C = 1\ \text{N} \quad t_2 = 1\ \text{s}$

Given $\quad N_C - M g \cos(\theta) = 0$

$a t_1 - \mu_s N_C - M g \sin(\theta) = 0$

$$\int_{t_1}^{t_2} \left(a t - M g \sin(\theta) - \mu_k N_C\right) dt = M v$$



$$\begin{pmatrix} t_1 \\ t_2 \\ N_C \end{pmatrix} = \text{Find}(t_1, t_2, N_C) \quad t_1 = 1.24\ \text{s} \quad t_2 = 1.93\ \text{s}$$

Problem 15-22

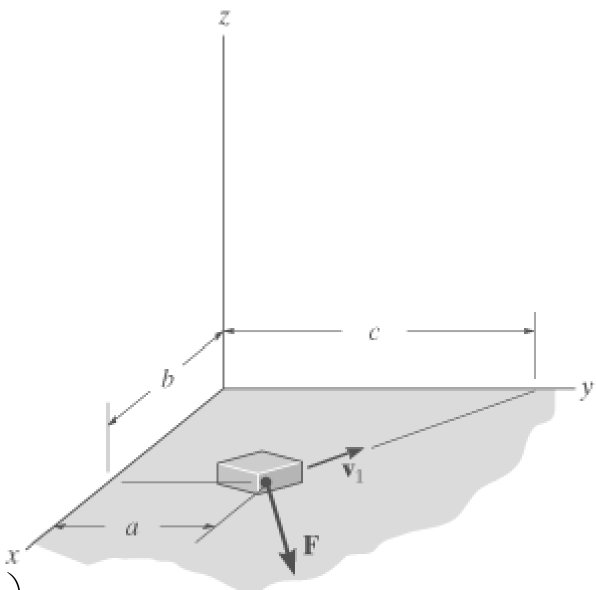
The block of weight W has an initial velocity v_1 in the direction shown. If a force $\mathbf{F} = \{f_1\mathbf{i} + f_2\mathbf{j}\}$ acts on the block for time t, determine the final speed of the block. Neglect friction.

Given:

$W = 2\ \text{lb}$ $\quad a = 2\ \text{ft}$ $\quad f_1 = 0.5\ \text{lb}$

$v_1 = 10\ \frac{\text{ft}}{\text{s}}$ $\quad b = 2\ \text{ft}$ $\quad f_2 = 0.2\ \text{lb}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad c = 5\ \text{ft}$ $\quad t = 5\ \text{s}$



Solution:

$$\theta = \text{atan}\left(\frac{b}{c-a}\right)$$

Guesses $\quad v_{2x} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{2y} = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_1\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix} + \begin{pmatrix}f_1\\ f_2\end{pmatrix}t = \left(\frac{W}{g}\right)\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix}$$

$$\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix} = \text{Find}(v_{2x}, v_{2y}) \qquad \begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix} = \begin{pmatrix}34.7\\ 24.4\end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \left|\begin{pmatrix}v_{2x}\\ v_{2y}\end{pmatrix}\right| = 42.4\ \frac{\text{ft}}{\text{s}}$$

Problem 15-23

The tennis ball has a horizontal speed v_1 when it is struck by the racket. If it then travels away at angle θ from the horizontal and reaches maximum altitude h, measured from the height of the racket, determine the magnitude of the net impulse of the racket on the ball. The ball has mass M. Neglect the weight of the ball during the time the racket strikes the ball.

Given:

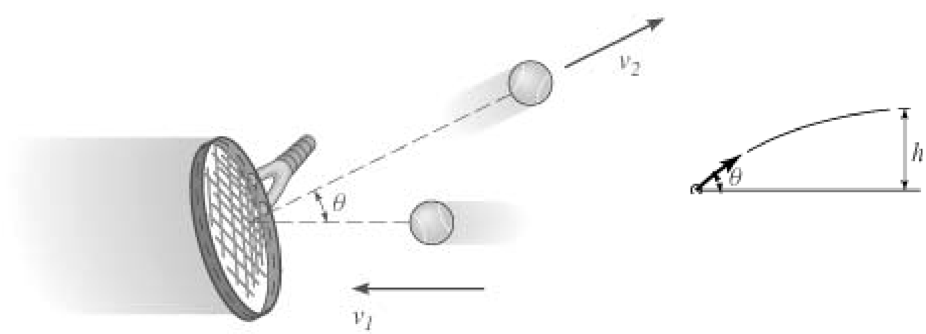
$v_1 = 15\ \frac{\text{m}}{\text{s}}$

$\theta = 25\ \text{deg}$

$h = 10\ \text{m}$

$M = 180\ \text{gm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Free flight $v_2 \sin(\theta) = \sqrt{2 g h}$ $v_2 = \dfrac{\sqrt{2 g h}}{\sin(\theta)}$ $v_2 = 33.14 \dfrac{\text{m}}{\text{s}}$

Impulse - momentum

$-M v_1 + I_x = M v_2 \cos(\theta)$ $I_x = M(v_2 \cos(\theta) + v_1)$ $I_x = 8.11\ \text{N}\cdot\text{s}$

$0 + I_y = M v_2 \sin(\theta)$ $I_y = M v_2 \sin(\theta)$ $I_y = 2.52\ \text{N}\cdot\text{s}$

$I = \sqrt{I_x^{\,2} + I_y^{\,2}}$ $I = 8.49\ \text{N}\cdot\text{s}$

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*__Problem 15-24__

The slider block of mass M is moving to the right with speed v when it is acted upon by the forces $\mathbf{F_1}$ and $\mathbf{F_2}$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = t_3$. Neglect friction and the mass of the pulleys and cords.

Given:

$M = 40\ \text{kg}$

$v = 1.5 \dfrac{\text{m}}{\text{s}}$

$t_3 = 6\ \text{s}$

$t_2 = 4\ \text{s}$

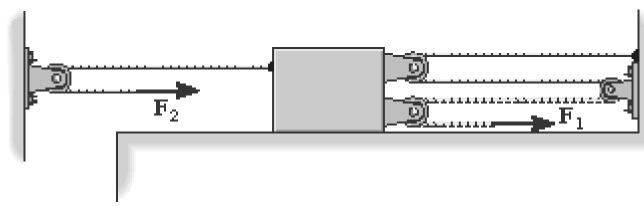
$t_1 = 2\ \text{s}$

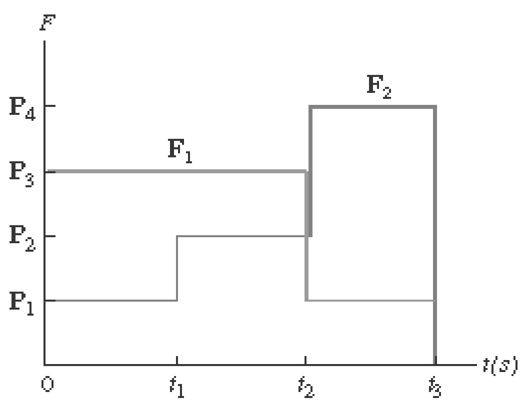
$P_1 = 10\ \text{N}$

$P_2 = 20\ \text{N}$

$P_3 = 30\ \text{N}$

$P_4 = 40\ \text{N}$





Solution: The impulses acting on the block are found from the areas under the graph.

$$I = 4\left[P_3 t_2 + P_1 (t_3 - t_2)\right] - \left[P_1 t_1 + P_2 (t_2 - t_1) + P_4 (t_3 - t_2)\right]$$

$M v + I = M v_3$ $v_3 = v + \dfrac{I}{M}$ $v_3 = 12.00 \dfrac{\text{m}}{\text{s}}$

Problem 15-25

Determine the velocities of blocks A and B at time t after they are released from rest. Neglect the mass of the pulleys and cables.

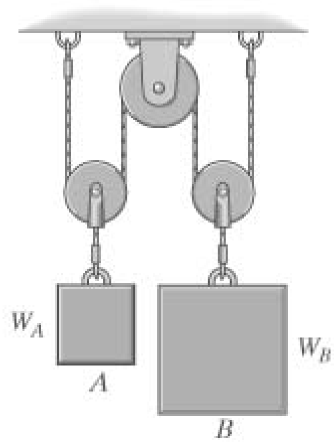
Given:

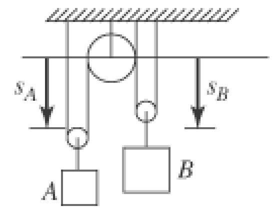
$W_A = 2\ \text{lb}$

$W_B = 4\ \text{lb}$

$t = 2\ \text{s}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

$2s_A + 2s_B = L$

$v_A = -v_B$

Block A $\quad 0 + (2T - W_A)t = \dfrac{W_A}{g} v_A$

Block B $\quad 0 + (2T - W_B)t = \dfrac{W_B}{g}(-v_A)$

Combining $\quad (W_B - W_A)t = \left(\dfrac{W_B + W_A}{g}\right) v_A$

$$v_A = \left(\frac{W_B - W_A}{W_B + W_A}\right) g\, t \qquad v_B = -v_A \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 21.47 \\ -21.47 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 15-26

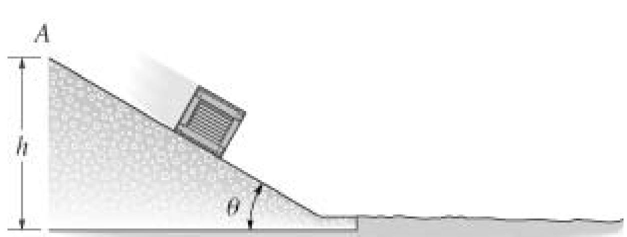
The package of mass M is released from rest at A. It slides down the smooth plane which is inclined at angle θ onto the rough surface having a coefficient of kinetic friction of μ_k. Determine the total time of travel before the package stops sliding. Neglect the size of the package.

Given:

$M = 5\ \text{kg} \qquad h = 3\ \text{m}$

$\theta = 30\ \text{deg} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\mu_k = 0.2$



Solution:

On the slope $v_1 = \sqrt{2gh}$ $v_1 = 7.67\ \frac{m}{s}$ $t_1 = \frac{v_1}{g\sin(\theta)}$ $t_1 = 1.56\ s$

On the flat $M v_1 - \mu_k M g t_2 = 0$ $t_2 = \frac{v_1}{\mu_k g}$ $t_2 = 3.91\ s$

$t = t_1 + t_2$ $t = 5.47\ s$

Problem 15-27

Block A has weight W_A and block B has weight W_B. If B is moving downward with a velocity v_{B0} at $t = 0$, determine the velocity of A when $t = t_1$. Assume that block A slides smoothly.

Given:

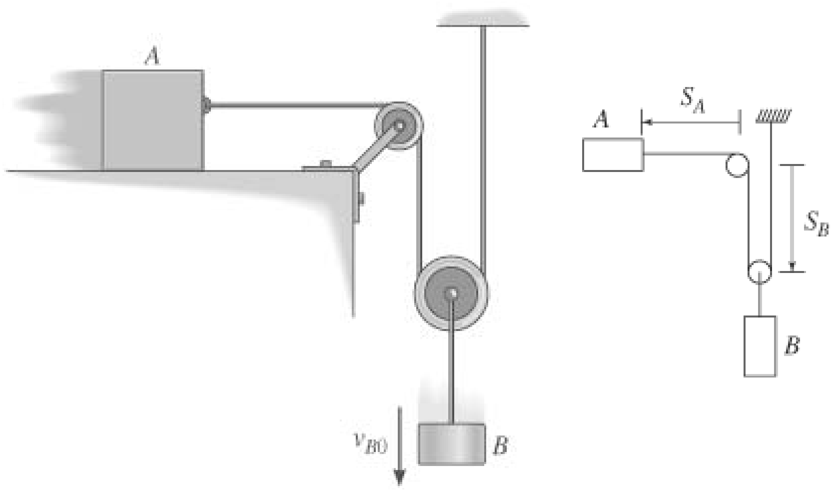
$W_A = 10\ lb$

$W_B = 3\ lb$

$v_{B0} = 3\ \frac{ft}{s}$

$t_1 = 1\ s$

$g = 32.2\ \frac{ft}{s^2}$



Solution: $s_A + 2s_B = L$ $v_A = -2v_B$ Guess $v_{A1} = 1\ \frac{ft}{s}$ $T = 1\ lb$

Given

Block A $\left(\frac{W_A}{g}\right)2v_{B0} + T t_1 = \left(\frac{W_A}{g}\right)v_{A1}$

Block B $\left(\frac{-W_B}{g}\right)v_{B0} + 2T t_1 - W_B t_1 = \left(\frac{-W_B}{g}\right)\left(\frac{v_{A1}}{2}\right)$

$\begin{pmatrix} v_{A1} \\ T \end{pmatrix} = \text{Find}(v_{A1}, T)$ $T = 1.40\ lb$ $v_{A1} = 10.49\ \frac{ft}{s}$

***Problem 15-28**

Block A has weight W_A and block B has weight W_B. If B is moving downward with a velocity v_{B1} at t = 0, determine the velocity of A when $t = t_1$. The coefficient of kinetic friction between the horizontal plane and block A is μ_k.

Given:

$W_A = 10\ \text{lb}$

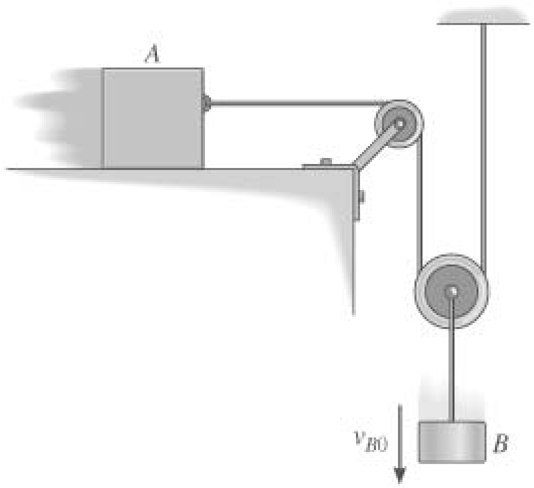
$W_B = 3\ \text{lb}$

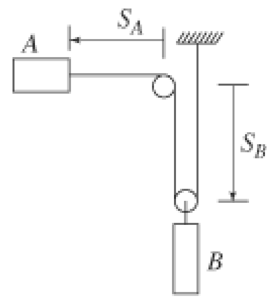
$v_{B1} = 3\ \frac{\text{ft}}{\text{s}}$

$\mu_k = 0.15$

$t_1 = 1\ \text{s}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution: $s_A + 2s_B = L$ $\quad v_A = -2v_B$ $\quad$ Guess $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad T = 1\ \text{lb}$

Given

Block A $\quad \left(\frac{W_A}{g}\right)2v_{B1} + T\,t_1 - \mu_k W_A t_1 = \left(\frac{W_A}{g}\right)v_{A2}$

Block B $\quad \left(\frac{-W_B}{g}\right)v_{B1} + 2T\,t_1 - W_B t_1 = \frac{-W_B}{g}\left(\frac{v_{A2}}{2}\right)$

$\begin{pmatrix} v_{A2} \\ T \end{pmatrix} = \text{Find}(v_{A2}, T) \qquad T = 1.50\,\text{lb} \qquad v_{A2} = 6.00\,\frac{\text{ft}}{\text{s}}$

Problem 15-29

A jet plane having a mass M takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed v, determine the plane's airspeed after time t.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

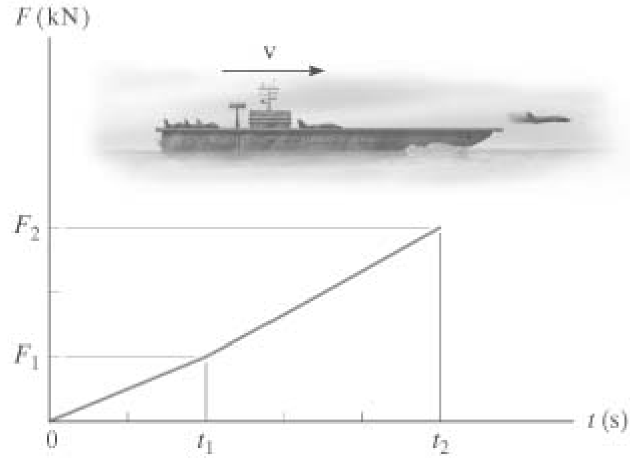
Given:

$M = 7\ \text{Mg}$ $\quad t_1 = 2\ \text{s}$

$v = 40\ \dfrac{\text{km}}{\text{hr}}$ $\quad t_2 = 5\ \text{s}$

$F_1 = 5\ \text{kN}$ $\quad t = 5\ \text{s}$

$F_2 = 15\ \text{kN}$



Solution:

The impulse exerted on the plane is equal to the area under the graph.

$$M v + \frac{1}{2} F_1 t_1 + \frac{1}{2}(F_1 + F_2)(t_2 - t_1) = M v_1$$

$$v_1 = v + \frac{1}{2M}\left[F_1 t_1 + (F_1 + F_2)(t_2 - t_1)\right] \qquad v_1 = 16.11\ \frac{\text{m}}{\text{s}}$$

Problem 15-30

The motor pulls on the cable at *A* with a force **F** = *a* + *b*t^2. If the crate of weight *W* is originally at rest at $t = 0$, determine its speed at time $t = t_2$. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.

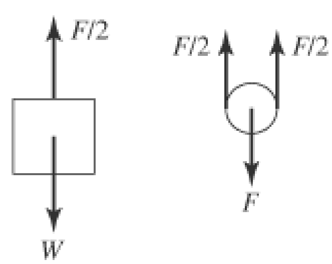
Given:

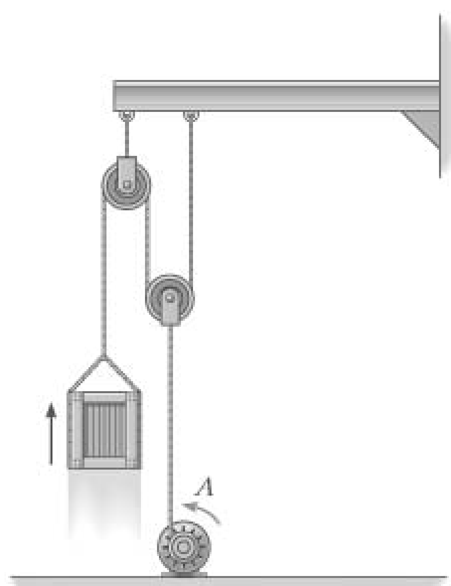
$W = 17\ \text{lb}$

$a = 30\ \text{lb}$

$b = 1\ \dfrac{\text{lb}}{\text{s}^2}$

$t_2 = 4\ \text{s}$





Solution:

$$\frac{1}{2}\left(a + b t_1^2\right) - W = 0$$

$$t_1 = \sqrt{\frac{2W - a}{b}} \qquad t_1 = 2.00\ \text{s}$$

$$\frac{1}{2}\int_{t_1}^{t_2} a + b t^2\,dt - W(t_2 - t_1) = \left(\frac{W}{g}\right) v_2 \qquad v_2 = \frac{g}{2W}\int_{t_1}^{t_2} a + b t^2\,dt - g(t_2 - t_1) \qquad v_2 = 10.10\ \frac{\text{ft}}{\text{s}}$$

Problem 15-31

The log has mass M and rests on the ground for which the coefficients of static and kinetic friction are μ_s and μ_k respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = t_2$. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.

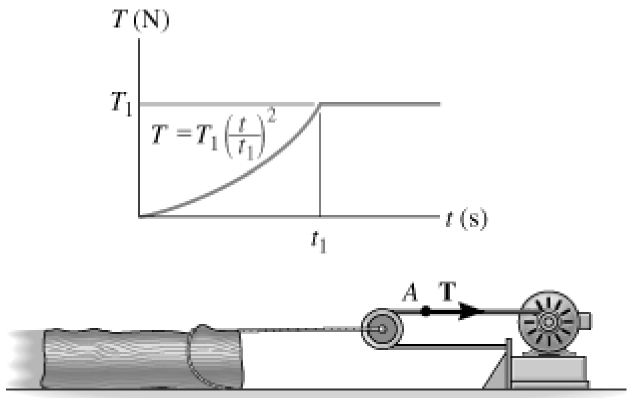
Given:

$M = 500 \text{ kg}$ $\quad t_1 = 3 \text{ s}$

$\mu_s = 0.5$ $\quad T_1 = 1800 \text{ N}$

$\mu_k = 0.4$ $\quad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$t_2 = 5 \text{ s}$



Solution:

To begin motion we need $\quad 2T_1\left(\frac{t_0^{\,2}}{t_1^{\,2}}\right) = \mu_s M g \qquad t_0 = \sqrt{\frac{\mu_s M g}{2T_1}}\, t_1 \qquad t_0 = 2.48 \text{ s}$

Impulse - Momentum

$$0 + \int_{t_0}^{t_1} 2T_1\left(\frac{t}{t_1}\right)^2 dt + 2T_1(t_2 - t_1) - \mu_k M g(t_2 - t_0) = M v_2$$

$$v_2 = \frac{1}{M}\left[\int_{t_0}^{t_1} 2T_1\left(\frac{t}{t_1}\right)^2 dt + 2T_1(t_2 - t_1) - \mu_k M g(t_2 - t_0)\right] \qquad v_2 = 7.65 \frac{\text{m}}{\text{s}}$$

***Problem 15-32**

A railroad car having mass m_1 is coasting with speed v_1 on a horizontal track. At the same time another car having mass m_2 is coasting with speed v_2 in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

Units used: $\quad \text{Mg} = 10^3 \text{ kg} \qquad \text{kJ} = 10^3 \text{ J}$

Given: $\quad m_1 = 15 \text{ Mg} \qquad m_2 = 12 \text{ Mg}$

$v_1 = 1.5\ \frac{m}{s}$ $\quad v_2 = 0.75\ \frac{m}{s}$

Solution:

$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v$ $\quad v = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$ $\quad v = 0.50\ \frac{m}{s}$

$T_1 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$ $\quad T_1 = 20.25\ \text{kJ}$

$T_2 = \frac{1}{2}(m_1 + m_2)v^2$ $\quad T_2 = 3.38\ \text{kJ}$

$\Delta T = T_2 - T_1$ $\quad \Delta T = -16.88\ \text{kJ}$

$\frac{-\Delta T}{T_1}100 = 83.33$ % loss

The energy is dissipated as noise, shock, and heat during the coupling.

Problem 15-33

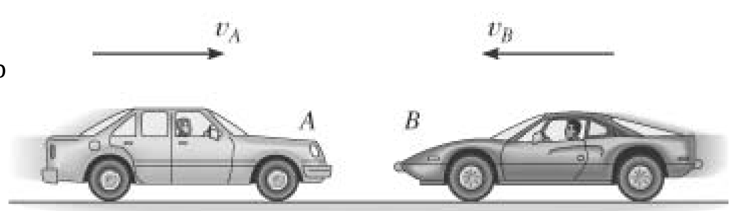
Car A has weight W_A and is traveling to the right at speed v_A Meanwhile car B of weight W_B is traveling at speed v_B to the left. If the cars crash head-on and become entangled, determine their common velocity just after the collision. Assume that the brakes are not applied during collision.

Given:

$W_A = 4500\ \text{lb}$ $\quad W_B = 3000\ \text{lb}$

$v_A = 3\ \frac{ft}{s}$ $\quad v_B = 6\ \frac{ft}{s}$

$g = 32.2\ \frac{ft}{s^2}$



Solution: $\left(\frac{W_A}{g}\right)v_A - \left(\frac{W_B}{g}\right)v_B = \left(\frac{W_A + W_B}{g}\right)v$ $\quad v = \frac{W_A v_A - W_B v_B}{W_A + W_B}$ $\quad v = -0.60\ \frac{ft}{s}$

Problem 15-34

The bus B has weight W_B and is traveling to the right at speed v_B. Meanwhile car A of weight W_A is traveling at speed v_A to the left. If the vehicles crash head-on and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.

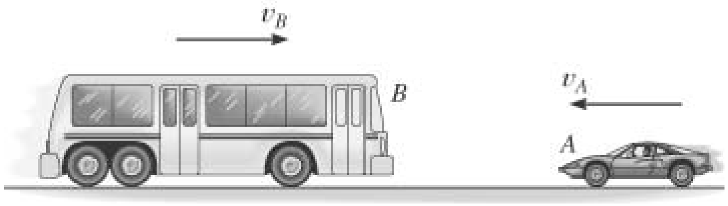
Given:

$W_B = 15000\ \text{lb} \qquad v_B = 5\ \frac{\text{ft}}{\text{s}}$

$W_A = 3000\ \text{lb}$

$v_A = 4\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\left(\frac{W_B}{g}\right)v_B - \left(\frac{W_A}{g}\right)v_A = \left(\frac{W_B + W_A}{g}\right)v \qquad v = \frac{W_B v_B - W_A v_A}{W_B + W_A} \qquad v = 3.50\ \frac{\text{ft}}{\text{s}}$$

Positive means to the right, negative means to the left.

Problem 15-35

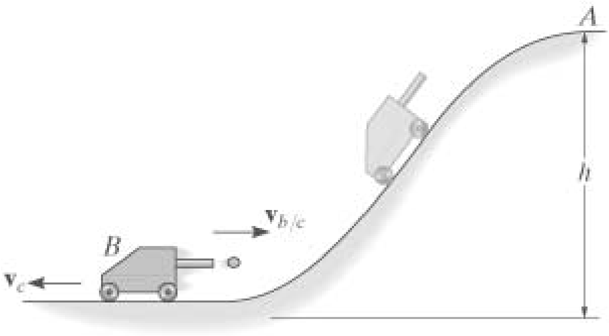
The cart has mass M and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a ball of mass M_1 out the back with a horizontal velocity v_{bc} measured relative to the cart. Determine the final velocity of the cart.

Given:

$M = 3\ \text{kg} \qquad h = 1.25\ \text{m}$

$M_1 = 0.5\ \text{kg} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$v_{bc} = 0.6\ \frac{\text{m}}{\text{s}}$



Solution:

$$v_1 = \sqrt{2gh}$$

$$(M + M_1)v_1 = M v_c + M_1(v_c - v_{bc})$$

$$v_c = v_1 + \left(\frac{M_1}{M + M_1}\right)v_{bc}$$

$$v_c = 5.04\ \frac{\text{m}}{\text{s}}$$

---

*__Problem 15-36__

Two men A and B, each having weight W_m, stand on the cart of weight W_c. Each runs with speed v measured relative to the cart. Determine the final speed of the cart if (a) A runs and jumps off, then B runs and jumps off the same end, and (b) both run at the same time and jump off at the same time. Neglect the mass of the wheels and assume the jumps are made horizontally.

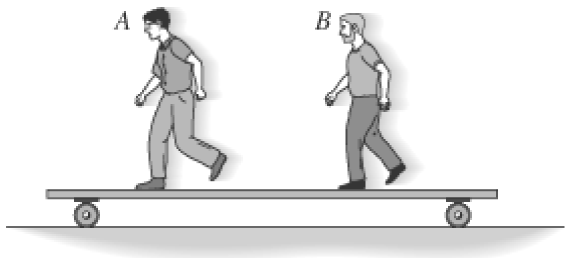
Given:

$W_m = 160\ \text{lb}$

$W_c = 200\ \text{lb}$

$v = 3\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $m_m = \frac{W_m}{g}$ $m_c = \frac{W_c}{g}$

(a) A jumps first

$$0 = -m_m(v - v_c) + (m_m + m_c)v_{c1} \qquad v_{c1} = \frac{m_m v}{m_c + 2m_m} \qquad v_{c1} = 0.923\ \frac{\text{ft}}{\text{s}}$$

And then B jumps

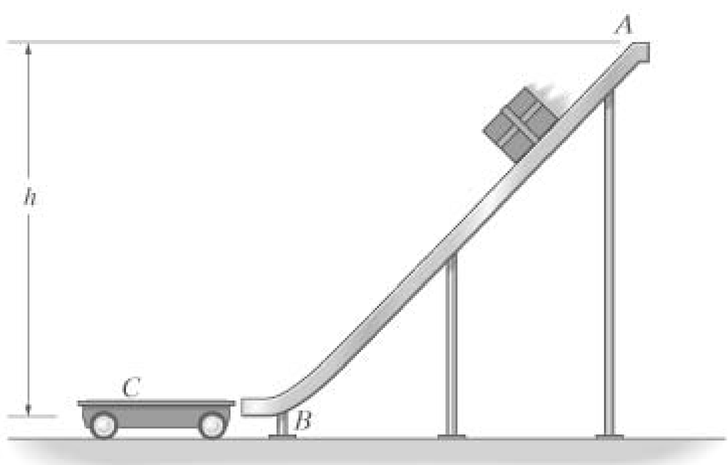
$$(m_m + m_c)v_{c1} = -m_m(v - v_{c2}) + m_c v_{c2} \qquad v_{c2} = \frac{m_m v + (m_m + m_c)v_{c1}}{m_m + m_c} \qquad v_{c2} = 2.26\ \frac{\text{ft}}{\text{s}}$$

(b) Both men jump at the same time

$$0 = -2m_m(v - v_{c3}) + m_c v_{c3} \qquad v_{c3} = \frac{2m_m v}{2m_m + m_c} \qquad v_{c3} = 1.85\ \frac{\text{ft}}{\text{s}}$$

Problem 15-37

A box of weight W_1 slides from rest down the smooth ramp onto the surface of a cart of weight W_2. Determine the speed of the box at the instant it stops sliding on the cart. If someone ties the cart to the ramp at B, determine the horizontal impulse the box will exert at C in order to stop its motion. Neglect friction on the ramp and neglect the size of the box.



Given:

$$W_1 = 40 \text{ lb} \qquad W_2 = 20 \text{ lb} \qquad h = 15 \text{ ft} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

Solution:

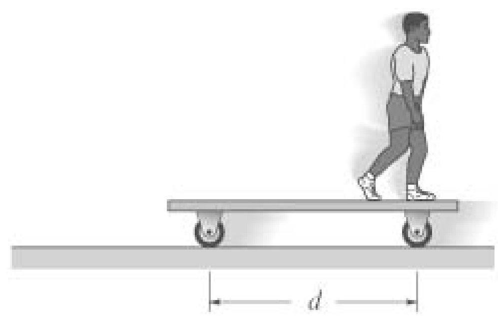
$$v_1 = \sqrt{2gh}$$

$$\frac{W_1}{g}v_1 = \left(\frac{W_1 + W_2}{g}\right)v_2 \qquad v_2 = \left(\frac{W_1}{W_1 + W_2}\right)v_1 \qquad v_2 = 20.7\frac{\text{ft}}{\text{s}}$$

$$\left(\frac{W_1}{g}\right)v_1 - Imp = 0 \qquad Imp = \left(\frac{W_1}{g}\right)v_1 \qquad Imp = 38.6\,\text{lb}\cdot\text{s}$$

Problem 15-38

A boy of weight W_1 walks forward over the surface of the cart of weight W_2 with a constant speed v relative to the cart. Determine the cart's speed and its displacement at the moment he is about to step off. Neglect the mass of the wheels and assume the cart and boy are originally at rest.



Given:

$$W_1 = 100 \text{ lb} \qquad W_2 = 60 \text{ lb} \qquad v = 3\frac{\text{ft}}{\text{s}} \qquad d = 6 \text{ ft}$$

Solution:

$$0 = \left(\frac{W_1}{g}\right)(v_C + v) + \left(\frac{W_2}{g}\right)v_C \qquad v_C = -\frac{W_1}{W_1 + W_2}v \qquad v_C = -1.88\frac{\text{ft}}{\text{s}}$$

Assuming that the boy walks the distance d

$$t = \frac{d}{v} \qquad s_C = v_C t \qquad s_C = -3.75\,\text{ft}$$

Problem 15-39

The barge B has weight W_B and supports an automobile weighing W_a. If the barge is not tied to the pier P and someone drives the automobile to the other side of the barge for unloading, determine how far the barge moves away from the pier. Neglect the resistance of the water.

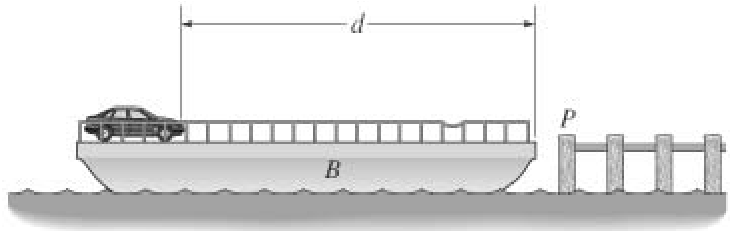
Given:

$W_B = 30000\ \text{lb}$

$W_a = 3000\ \text{lb}$

$d = 200\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$m_B = \frac{W_B}{g} \qquad m_a = \frac{W_a}{g}$$

v is the velocity of the car relative to the barge. The answer is independent of the acceleration so we will do the problem for a constant speed.

$$m_B v_B + m_a(v + v_B) = 0 \qquad v_B = \frac{-m_a v}{m_B + m_a}$$

$$t = \frac{d}{v} \qquad s_B = -v_B t \qquad s_B = \frac{m_a d}{m_a + m_B} \qquad s_B = 18.18\ \text{ft}$$

***Problem 15-40**

A bullet of weight W_1 traveling at speed v_1 strikes the wooden block of weight W_2 and exits the other side at speed v_2 as shown. Determine the speed of the block just after the bullet exits the block, and also determine how far the block slides before it stops. The coefficient of kinetic friction between the block and the surface is μ_k.

Given:

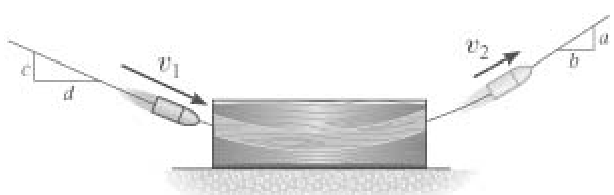
$W_1 = 0.03\ \text{lb}$ $\quad a = 3\ \text{ft}$

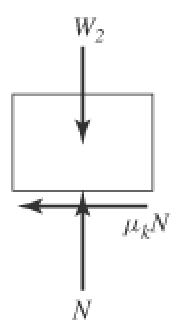
$W_2 = 10\ \text{lb}$ $\quad b = 4\ \text{ft}$

$v_1 = 1300\ \dfrac{\text{ft}}{\text{s}}$ $\quad c = 5\ \text{ft}$

$\quad d = 12\ \text{ft}$

$v_2 = 50\ \dfrac{\text{ft}}{\text{s}}$ $\quad \mu_k = 0.5$





Solution:

$$\left(\frac{W_1}{g}\right)v_1\left(\frac{d}{\sqrt{c^2 + d^2}}\right) = \left(\frac{W_2}{g}\right)v_B + \left(\frac{W_1}{g}\right)v_2\left(\frac{b}{\sqrt{a^2 + b^2}}\right)$$

$$v_B = \frac{W_1}{W_2}\left(\frac{v_1 d}{\sqrt{c^2+d^2}} - \frac{v_2 b}{\sqrt{a^2+b^2}}\right) \qquad v_B = 3.48\ \frac{\text{ft}}{\text{s}}$$

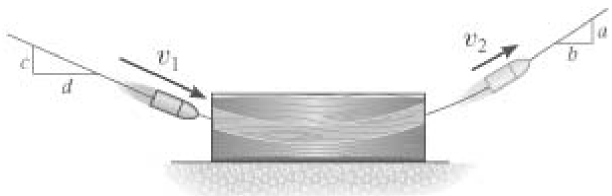
$$\frac{1}{2}\left(\frac{W_2}{g}\right)v_B^{\ 2} - \mu_k W_2 d = 0 \qquad d = \frac{v_B^{\ 2}}{2g\mu_k} \qquad d = 0.38\ \text{ft}$$

Problem 15-41

A bullet of weight W_1 traveling at v_1 strikes the wooden block of weight W_2 and exits the other side at v_2 as shown. Determine the speed of the block just after the bullet exits the block. Also, determine the average normal force on the block if the bullet passes through it in time Δt, and the time the block slides before it stops. The coefficient of kinetic friction between the block and the surface is μ_k.

Units Used: $\text{ms} = 10^{-3}\ \text{s}$

Given:



$W_1 = 0.03\ \text{lb}$ $\quad a = 3\ \text{ft}$

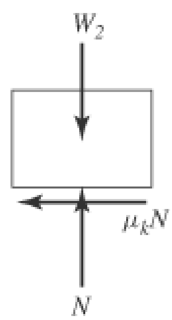
$W_2 = 10\ \text{lb}$ $\quad b = 4\ \text{ft}$

$\mu_k = 0.5$ $\quad c = 5\ \text{ft}$

$\Delta t = 1\ \text{ms}$ $\quad d = 12\ \text{ft}$

$v_1 = 1300\ \frac{\text{ft}}{\text{s}}$ $\quad v_2 = 50\ \frac{\text{ft}}{\text{s}}$

Solution:



$$\frac{W_1}{g}v_1\left(\frac{d}{\sqrt{c^2+d^2}}\right) = \frac{W_2}{g}v_B + \frac{W_1}{g}v_2\left(\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$v_B = \frac{W_1}{W_2}\left(\frac{v_1 d}{\sqrt{c^2+d^2}} - \frac{v_2 b}{\sqrt{a^2+b^2}}\right) \qquad v_B = 3.48\ \frac{\text{ft}}{\text{s}}$$

$$\frac{-W_1}{g}v_1\left(\frac{c}{\sqrt{c^2+d^2}}\right) + (N - W_2)\Delta t = \frac{W_1}{g}v_2\left(\frac{a}{\sqrt{a^2+b^2}}\right)$$

$$N = \frac{W_1}{g\Delta t}\left(\frac{v_2 a}{\sqrt{a^2+b^2}} + \frac{v_1 c}{\sqrt{c^2+d^2}}\right) + W_2 \qquad N = 503.79\ \text{lb}$$

$$\left(\frac{W_2}{g}\right)v_B - \mu_k W_2 t = 0 \qquad t = \frac{v_B}{g\mu_k} \qquad t = 0.22\ \text{s}$$

Problem 15-42

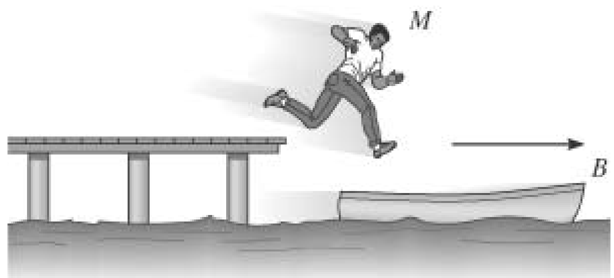
The man M has weight W_M and jumps onto the boat B which has weight W_B. If he has a horizontal component of velocity v *relative to the boat*, just before he enters the boat, and the boat is traveling at speed v_B away from the pier when he makes the jump, determine the resulting velocity of the man and boat.

Given:

$W_M = 150\ \text{lb}$ $\quad v_B = 2\ \frac{\text{ft}}{\text{s}}$

$W_B = 200\ \text{lb}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$v = 3\ \frac{\text{ft}}{\text{s}}$



Solution:

$$\frac{W_M}{g}(v + v_B) + \frac{W_B}{g}v_B = \left(\frac{W_M + W_B}{g}\right)v'$$

$$v' = \frac{W_M v + (W_M + W_B)v_B}{W_M + W_B} \qquad v' = 3.29\ \frac{\text{ft}}{\text{s}}$$

Problem 15-43

The man M has weight W_M and jumps onto the boat B which is originally at rest. If he has a horizontal component of velocity v just before he enters the boat, determine the weight of the boat if it has velocity v' once the man enters it.

Given:

$W_M = 150\ \text{lb}$

$v = 3\ \frac{\text{ft}}{\text{s}}$

$v' = 2\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$\left(\frac{W_M}{g}\right)v = \left(\frac{W_M + W_B}{g}\right)v' \qquad W_B = \left(\frac{v - v'}{v'}\right)W_M \qquad W_B = 75.00\ \text{lb}$$

***Problem 15-44**

A boy A having weight W_A and a girl B having weight W_B stand motionless at the ends of the toboggan, which has weight W_t. If A walks to B and stops, and both walk back together to the original position of A (both positions measured on the toboggan), determine the final position of the toboggan just after the motion stops. Neglect friction.

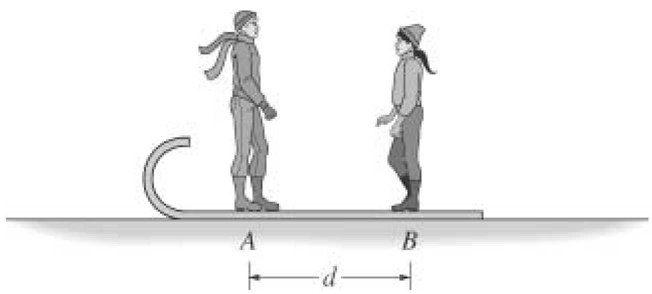
Given:

$W_A = 80 \text{ lb}$

$W_B = 65 \text{ lb}$

$W_t = 20 \text{ lb}$

$d = 4 \text{ ft}$



Solution: The center of mass doesn't move during the motion since there is no friction and therefore no net horizontal force

$$W_B d = (W_A + W_B + W_t)d' \qquad d' = \frac{W_B d}{W_A + W_B + W_t} \qquad d' = 1.58 \text{ ft}$$

Problem 15-45

The projectile of weight W is fired from ground level with initial velocity v_A in the direction shown. When it reaches its highest point B it explodes into two fragments of weight $W/2$. If one fragment travels vertically upward at speed v_1, determine the distance between the fragments after they strike the ground. Neglect the size of the gun.

Given:

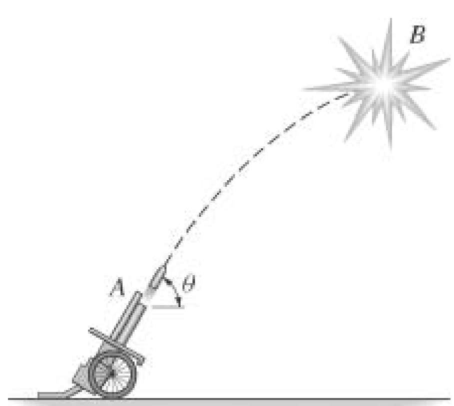
$W = 10 \text{ lb}$

$v_A = 80 \frac{\text{ft}}{\text{s}}$

$v_1 = 12 \frac{\text{ft}}{\text{s}}$

$\theta = 60 \text{ deg}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution: At the top $v = v_A \cos(\theta)$

Explosion $\left(\frac{W}{g}\right)v = 0 + \left(\frac{W}{2g}\right)v_{2x}$ $\quad v_{2x} = 2v$ $\quad v_{2x} = 80.00\ \frac{\text{ft}}{\text{s}}$

$0 = \left(\frac{W}{2g}\right)v_1 - \left(\frac{W}{2g}\right)v_{2y}$ $\quad v_{2y} = v_1$ $\quad v_{2y} = 12.00\ \frac{\text{ft}}{\text{s}}$

Kinematics $h = \frac{(v_A \sin(\theta))^2}{2g}$ $\quad h = 74.53\ \text{ft}$ $\quad$ Guess $t = 1\ \text{s}$

Given $0 = \left(\frac{-g}{2}\right)t^2 - v_{2y}t + h$ $\quad t = \text{Find}(t)$ $\quad t = 1.81\ \text{s}$

$d = v_{2x}t$ $\quad d = 144.9\ \text{ft}$

Problem 15-46

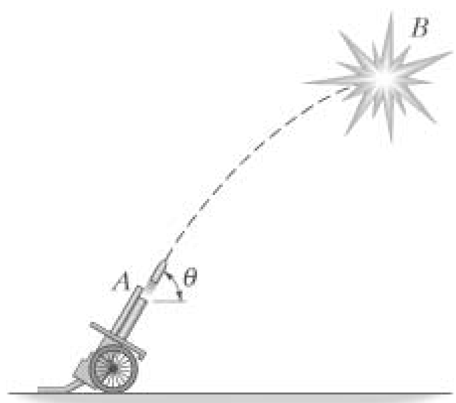
The projectile of weight W is fired from ground level with an initial velocity v_A in the direction shown. When it reaches its highest point B it explodes into two fragments of weight $W/2$. If one fragment is seen to travel vertically upward, and after they fall they are a distance d apart, determine the speed of each fragment just after the explosion. Neglect the size of the gun.

Given:

$W = 10\ \text{lb}$ $\quad \theta = 60\ \text{deg}$

$v_A = 80\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$d = 150\ \text{ft}$



Solution:

$$h = \frac{(v_A \sin(\theta))^2}{2g}$$

Guesses

$v_1 = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{2x} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{2y} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad t = 1\ \text{s}$

Given $\left(\frac{W}{g}\right)v_A\cos(\theta) = \left(\frac{W}{2g}\right)v_{2x}$ $\quad 0 = \left(\frac{W}{2g}\right)v_1 + \left(\frac{W}{2g}\right)v_{2y}$

$d = v_{2x}t$ $\quad 0 = h - \frac{1}{2}gt^2 + v_{2y}t$

$$\begin{pmatrix} v_1 \\ v_{2x} \\ v_{2y} \\ t \end{pmatrix} = \text{Find}(v_1, v_{2x}, v_{2y}, t) \qquad t = 1.87\ \text{s} \qquad \begin{pmatrix} v_1 \\ v_{2x} \\ v_{2y} \end{pmatrix} = \begin{pmatrix} 9.56 \\ 80.00 \\ -9.56 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$v_1 = 9.56\ \frac{\text{ft}}{\text{s}} \qquad \left| \begin{pmatrix} v_{2x} \\ v_{2y} \end{pmatrix} \right| = 80.57\ \frac{\text{ft}}{\text{s}}$$

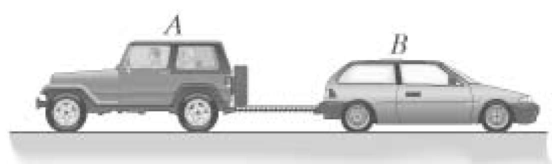
Problem 15-47

The winch on the back of the jeep A is turned on and pulls in the tow rope at speed v_{rel}. If both the car B of mass M_B and the jeep A of mass M_A are free to roll, determine their velocities at the instant they meet. If the rope is of length L, how long will this take?

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$

Given:



$$M_A = 2.5\ \text{Mg} \qquad v_{rel} = 2\ \frac{\text{m}}{\text{s}}$$

$$M_B = 1.25\ \text{Mg} \qquad L = 5\ \text{m}$$

Solution:

Guess $\quad v_A = 1\ \frac{\text{m}}{\text{s}} \qquad v_B = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad 0 = M_A v_A + M_B v_B \qquad v_A - v_B = v_{rel} \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \text{Find}(v_A, v_B)$

$$t = \frac{L}{v_{rel}} \qquad t = 2.50\ \text{s} \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 0.67 \\ -1.33 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

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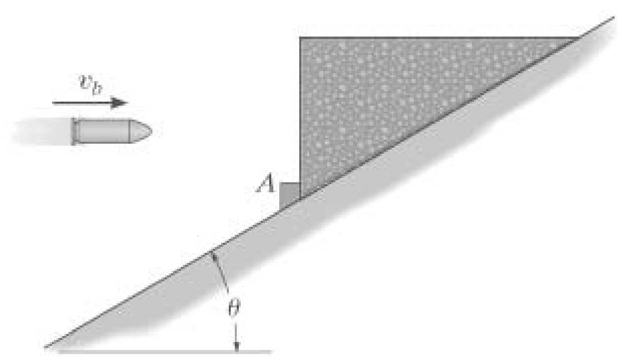
*__Problem 15-48__

The block of mass M_a is held at rest on the smooth inclined plane by the stop block at A. If the bullet of mass M_b is traveling at speed v when it becomes embedded in the block of mass M_c, determine the distance the block will slide up along the plane before momentarily stopping.



Given:

$M_a = 10\ \text{kg}$ $\quad v = 300\ \dfrac{\text{m}}{\text{s}}$

$M_b = 10\ \text{gm}$ $\quad \theta = 30\ \text{deg}$

$M_c = 10\ \text{kg}$

Solution:

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the *FBD*, the *impulsive* force **F** caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the *x* axis

$$M_b v_{bx} = (M_b + M_a) v_x$$

$$M_b v \cos(\theta) = (M_b + M_a) v_x$$

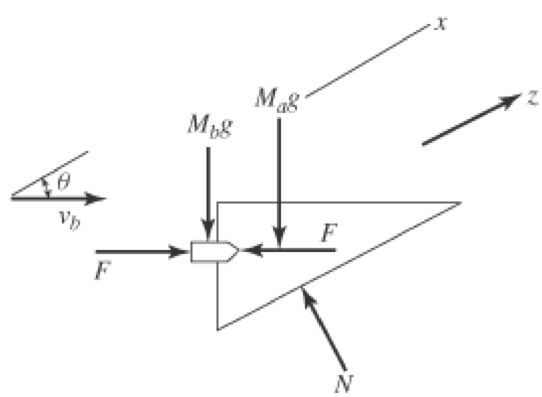
$$v_x = M_b\, v \left(\frac{\cos(\theta)}{M_b + M_a}\right) \qquad v_x = 0.2595\ \frac{\text{m}}{\text{s}}$$

Conservation of Energy: The datum is set at the block's initial position. When the block and the embedded bullet are at their highest point they are a distance *h above* the datum. Their gravitational potential energy is $(M_a + M_b)gh$. Applying Eq. 14-21, we have

$$0 + \frac{1}{2}(M_a + M_b) v_x^2 = 0 + (M_a + M_b) g h$$

$$h = \frac{1}{2}\left(\frac{v_x^2}{g}\right) \qquad h = 3.43\,\text{mm}$$

$$d = \frac{h}{\sin(\theta)} \qquad d = 6.86\,\text{mm}$$



Problem 15-49

A tugboat *T* having mass m_T is tied to a barge *B* having mass m_B. If the rope is "elastic" such that it has stiffness *k*, determine the maximum stretch in the rope during the initial towing. Originally both the tugboat and barge are moving in the same direction with speeds v_{T1} and v_{B1} respectively. Neglect the resistance of the water.

Units Used:

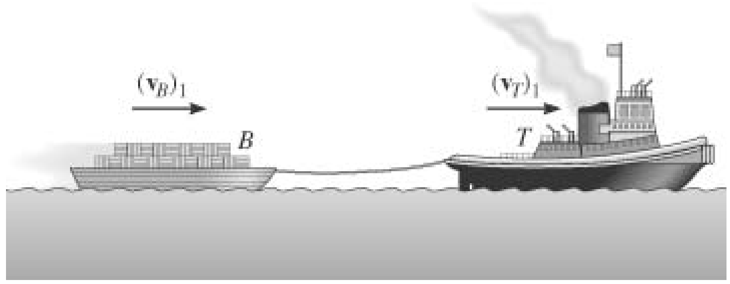
$\text{Mg} = 10^3\ \text{kg} \qquad \text{kN} = 10^3\ \text{N}$

Given:

$m_T = 19\ \text{Mg}$ $\quad v_{B1} = 10\ \dfrac{\text{km}}{\text{hr}}$

$m_B = 75\ \text{Mg}$ $\quad v_{T1} = 15\ \dfrac{\text{km}}{\text{hr}}$

$k = 600\ \dfrac{\text{kN}}{\text{m}}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

At maximum stretch the velocities are the same.

Guesses $\quad v_2 = 1\ \dfrac{\text{km}}{\text{hr}} \quad \delta = 1\ \text{m}$

Given

momentum $\quad m_T v_{T1} + m_B v_{B1} = (m_T + m_B) v_2$

energy $\quad \dfrac{1}{2} m_T v_{T1}^2 + \dfrac{1}{2} m_B v_{B1}^2 = \dfrac{1}{2}(m_T + m_B) v_2^2 + \dfrac{1}{2} k \delta^2$

$\begin{pmatrix} v_2 \\ \delta \end{pmatrix} = \text{Find}(v_2, \delta) \quad v_2 = 11.01\ \dfrac{\text{km}}{\text{hr}} \quad \delta = 0.221\ \text{m}$

Problem 15-50

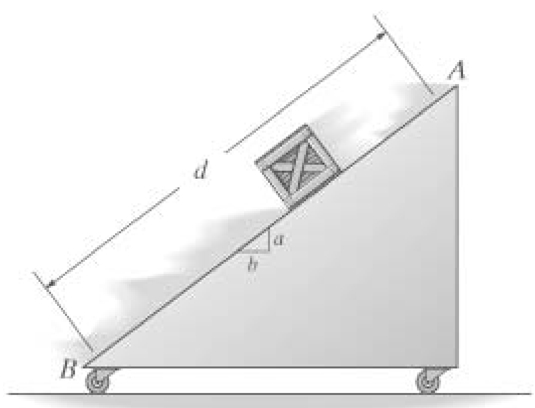
The free-rolling ramp has a weight W_r. The crate, whose weight is W_c, slides a distance d from rest at A, down the ramp to B. Determine the ramp's speed when the crate reaches B. Assume that the ramp is smooth, and neglect the mass of the wheels.

Given:

$W_r = 120\ \text{lb}$ $\quad a = 3$

$W_c = 80\ \text{lb}$ $\quad b = 4$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad d = 15\ \text{ft}$



Solution:

$\theta = \text{atan}\left(\dfrac{a}{b}\right)$

Guesses $\quad v_r = 1\ \dfrac{\text{ft}}{\text{s}} \quad v_{cr} = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

$$W_c\, d \sin(\theta) = \frac{1}{2}\left(\frac{W_r}{g}\right) v_r^2 + \frac{1}{2}\left(\frac{W_c}{g}\right)\left[\left(v_r - v_{cr}\cos(\theta)\right)^2 + \left(v_{cr}\sin(\theta)\right)^2\right]$$

$$0 = \left(\frac{W_r}{g}\right) v_r + \left(\frac{W_c}{g}\right)\left(v_r - v_{cr}\cos(\theta)\right)$$

$$\begin{pmatrix} v_r \\ v_{cr} \end{pmatrix} = \text{Find}(v_r, v_{cr}) \qquad v_{cr} = 27.9\,\frac{\text{ft}}{\text{s}} \qquad v_r = 8.93\,\frac{\text{ft}}{\text{s}}$$

Problem 15-51

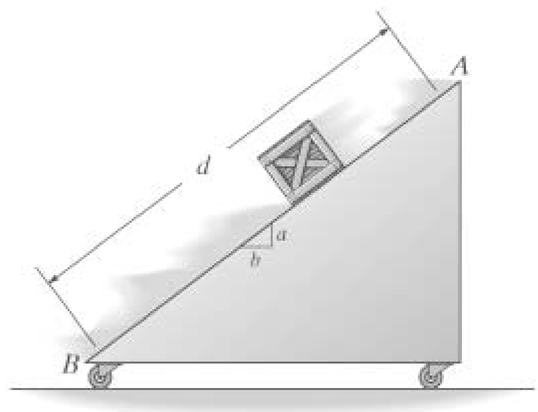
The free-rolling ramp has a weight W_r. If the crate, whose weight is W_c, is released from rest at A, determine the distance the ramp moves when the crate slides a distance d down the ramp and reaches the bottom B.

Given:

$W_r = 120$ lb $a = 3$

$W_c = 80$ lb $b = 4$

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$ $d = 15$ ft



Solution:

$$\theta = \text{atan}\left(\frac{a}{b}\right)$$

Momentum

$$0 = \left(\frac{W_r}{g}\right) v_r + \left(\frac{W_c}{g}\right)\left(v_r - v_{cr}\cos(\theta)\right) \qquad v_r = \left(\frac{W_c}{W_c + W_r}\right)\cos(\theta) v_{cr}$$

Integrate

$$s_r = \left(\frac{W_c}{W_c + W_r}\right)\cos(\theta)\, d \qquad s_r = 4.80\text{ ft}$$

***Problem 15-52**

The boy B jumps off the canoe at A with a velocity v_{BA} relative to the canoe as shown. If he lands in the second canoe C, determine the final speed of both canoes after the motion. Each canoe has a mass M_c. The boy's mass is M_B, and the girl D has a mass M_D. Both canoes are originally at rest.

Given:

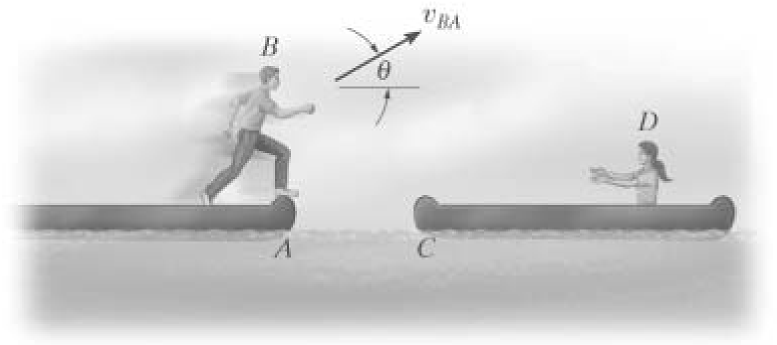
$M_c = 40\ \text{kg}$

$M_B = 30\ \text{kg}$

$M_D = 25\ \text{kg}$

$v_{BA} = 5\ \frac{\text{m}}{\text{s}}$

$\theta = 30\ \text{deg}$



Solution:

Guesses $\quad v_A = 1\ \frac{\text{m}}{\text{s}} \qquad v_C = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad 0 = M_c v_A + M_B\left(v_A + v_{BA}\cos(\theta)\right)$

$$M_B\left(v_A + v_{BA}\cos(\theta)\right) = \left(M_c + M_B + M_D\right)v_C$$

$$\begin{pmatrix} v_A \\ v_C \end{pmatrix} = \text{Find}(v_A, v_C) \qquad \begin{pmatrix} v_A \\ v_C \end{pmatrix} = \begin{pmatrix} -1.86 \\ 0.78 \end{pmatrix}\frac{\text{m}}{\text{s}}$$

Problem 15-53

The free-rolling ramp has a mass M_r. A crate of mass M_c is released from rest at A and slides down d to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?

Given:

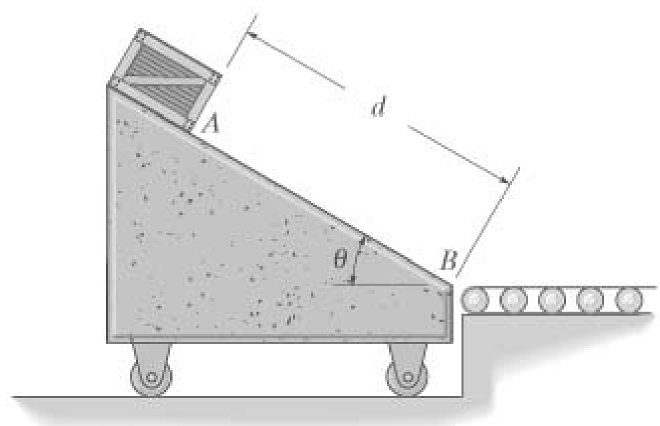
$M_r = 40\ \text{kg}$

$M_c = 10\ \text{kg}$

$d = 3.5\ \text{m}$

$\theta = 30\ \text{deg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guesses $\quad v_c = 1\ \frac{\text{m}}{\text{s}} \qquad v_r = 1\ \frac{\text{m}}{\text{s}} \qquad v_{cr} = 1\ \frac{\text{m}}{\text{s}}$

Given $\quad 0 + M_c g d \sin(\theta) = \frac{1}{2}M_c v_c^2 + \frac{1}{2}M_r v_r^2$

$$\left(v_r + v_{cr}\cos(\theta)\right)^2 + \left(v_{cr}\sin(\theta)\right)^2 = v_c^{\ 2}$$

$$0 = M_r v_r + M_c\left(v_r + v_{cr}\cos(\theta)\right)$$

$$\begin{pmatrix} v_c \\ v_r \\ v_{cr} \end{pmatrix} = \mathrm{Find}(v_c, v_r, v_{cr}) \qquad v_{cr} = 6.36\ \frac{m}{s} \qquad \begin{pmatrix} v_r \\ v_c \end{pmatrix} = \begin{pmatrix} -1.101 \\ 5.430 \end{pmatrix} \frac{m}{s}$$

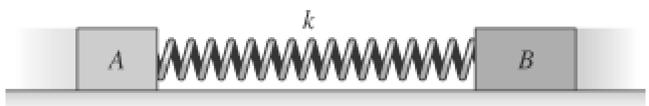
Problem 15-54

Blocks A and B have masses m_A and m_B respectively. They are placed on a smooth surface and the spring connected between them is stretched a distance d. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.

Given:

$m_A = 40\ \text{kg} \quad d = 2\ \text{m}$

$m_B = 60\ \text{kg} \quad k = 180\ \frac{N}{m}$



Solution: Guesses $v_A = 1\ \frac{m}{s}$ $\quad v_B = -1\ \frac{m}{s}$ Given

momentum $\quad 0 = m_A v_A + m_B v_B$

energy $\quad \frac{1}{2}kd^2 = \frac{1}{2}m_A v_A^{\ 2} + \frac{1}{2}m_B v_B^{\ 2}$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \mathrm{Find}(v_A, v_B) \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 3.29 \\ -2.19 \end{pmatrix} \frac{m}{s}$$

Problem 15-55

Block A has a mass M_A and is sliding on a rough horizontal surface with a velocity v_{A1} when it makes a direct collision with block B, which has a mass M_B and is originally at rest. If the collision is perfectly elastic, determine the velocity of each block just after collision and the distance between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is μ_k.

Given:

$M_A = 3\ \text{kg} \quad g = 9.81\ \frac{m}{s^2}$

$M_B = 2\ \text{kg} \quad e = 1$

$v_{A1} = 2\ \frac{m}{s} \quad \mu_k = 0.3$

Solution:

Guesess

$v_{A2} = 3\ \frac{m}{s}$ $v_{B2} = 5\ \frac{m}{s}$ $d_2 = 1\ m$

Given

$M_A v_{A1} = M_A v_{A2} + M_B v_{B2}$ $e v_{A1} = v_{B2} - v_{A2}$ $d_2 = \frac{v_{B2}^2 - v_{A2}^2}{2g\mu_k}$

$\begin{pmatrix} v_{A2} \\ v_{B2} \\ d_2 \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, d_2)$ $\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.40 \\ 2.40 \end{pmatrix} \frac{m}{s}$ $d_2 = 0.951\ m$

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*Problem 15-56

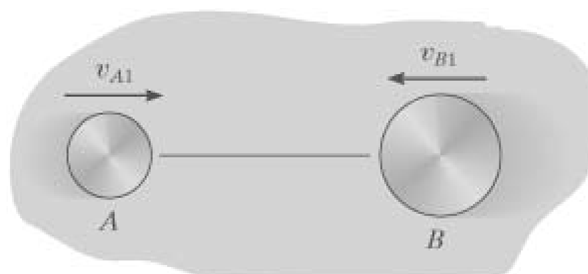
Disks A and B have masses M_A and M_B respectively. If they have the velocities shown, determine their velocities just after direct central impact.

Given:

$M_A = 2\ kg$ $v_{A1} = 2\ \frac{m}{s}$

$M_B = 4\ kg$ $v_{B1} = 5\ \frac{m}{s}$

$e = 0.4$



Solution: Guesses $v_{A2} = 1\ \frac{m}{s}$ $v_{B2} = 1\ \frac{m}{s}$

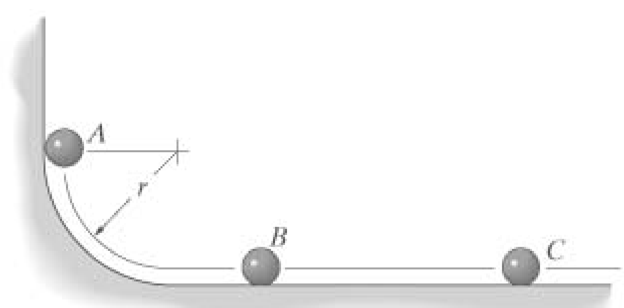
Given $M_A v_{A1} - M_B v_{B1} = M_A v_{A2} + M_B v_{B2}$

$e(v_{A1} + v_{B1}) = v_{B2} - v_{A2}$

$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2})$ $\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -4.53 \\ -1.73 \end{pmatrix} \frac{m}{s}$

Problem 15-57

The three balls each have weight W and have a coefficient of restitution e. If ball A is released from rest and strikes ball B and then ball B strikes ball C, determine the velocity of each ball after the second collision has occurred. The balls slide without friction.



Given:

$W = 0.5\ lb$ $r = 3\ ft$

$e = 0.85 \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$v_A = \sqrt{2gr}$

Guesses

$v_{A'} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B'} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B''} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{C''} = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_A = \left(\frac{W}{g}\right)v_{A'} + \left(\frac{W}{g}\right)v_{B'} \qquad e\,v_A = v_{B'} - v_{A'}$$

$$\left(\frac{W}{g}\right)v_{B'} = \left(\frac{W}{g}\right)v_{B''} + \left(\frac{W}{g}\right)v_{C''} \qquad e\,v_{B'} = v_{C''} - v_{B''}$$

$$\begin{pmatrix} v_{A'} \\ v_{B'} \\ v_{B''} \\ v_{C''} \end{pmatrix} = \text{Find}(v_{A'}, v_{B'}, v_{B''}, v_{C''}) \qquad \begin{pmatrix} v_{A'} \\ v_{B''} \\ v_{C''} \end{pmatrix} = \begin{pmatrix} 1.04 \\ 0.96 \\ 11.89 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 15-58

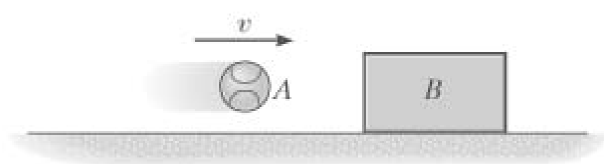
The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. If the coefficient of restitution between A and B is e, and the coefficient of kinetic friction between the plane and the block is μ_k, determine the time before block B stops sliding.

Given:

$W_A = 1\ \text{lb} \qquad \mu_k = 0.4$

$W_B = 10\ \text{lb} \qquad v = 20\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2} \qquad e = 0.6$



Solution:

Guesses $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{ft}}{\text{s}} \qquad t = 1\ \text{s}$

Given $\quad \left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v = v_{B2} - v_{A2}$

$$\left(\frac{W_B}{g}\right)v_{B2} - \mu_k W_B t = 0$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ t \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, t) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad t = 0.23\ \text{s}$$

Problem 15-59

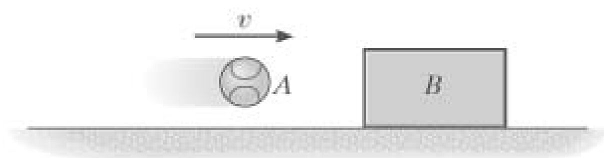
The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. If the coefficient of restitution between A and B is e, and the coefficient of kinetic friction between the plane and the block is μ_k, determine the distance block B slides before stopping.

Given:

$W_A = 1\ \text{lb}$ $\quad \mu_k = 0.4$

$W_B = 10\ \text{lb}$ $\quad v = 20\ \frac{\text{ft}}{\text{s}}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad e = 0.6$



Solution:

Guesses $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{ft}}{\text{s}} \qquad d = 1\ \text{ft}$

Given $\quad \left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e v = v_{B2} - v_{A2}$

$$\frac{1}{2}\left(\frac{W_B}{g}\right)v_{B2}^2 - \mu_k W_B d = 0$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ d \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, d) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad d = 0.33\ \text{ft}$$

Problem 15-60

The ball A of weight W_A is thrown so that when it strikes the block B of weight W_B it is traveling horizontally at speed v. Determine the average normal force exerted between A and B if the impact occurs in time Δt. The coefficient of restitution between A and B is e.

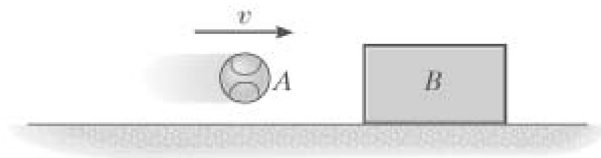
Given:

$W_A = 1\ \text{lb}$ $\quad \mu_k = 0.4$

$W_B = 10 \text{ lb} \quad v = 20 \frac{\text{ft}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2} \quad e = 0.6$

$\Delta t = 0.02 \text{ s}$



Solution:

Guesses $v_{A2} = 1 \frac{\text{ft}}{\text{s}} \qquad v_{B2} = 1 \frac{\text{ft}}{\text{s}} \qquad F_N = 1 \text{ lb}$

Given $\left(\frac{W_A}{g}\right)v = \left(\frac{W_A}{g}\right)v_{A2} + \left(\frac{W_B}{g}\right)v_{B2} \qquad e\,v = v_{B2} - v_{A2}$

$$\left(\frac{W_A}{g}\right)v - F_N\Delta t = \left(\frac{W_A}{g}\right)v_{A2}$$

$\begin{pmatrix} v_{A2} \\ v_{B2} \\ F_N \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, F_N) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -9.09 \\ 2.91 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad F_N = 45.2 \text{ lb}$

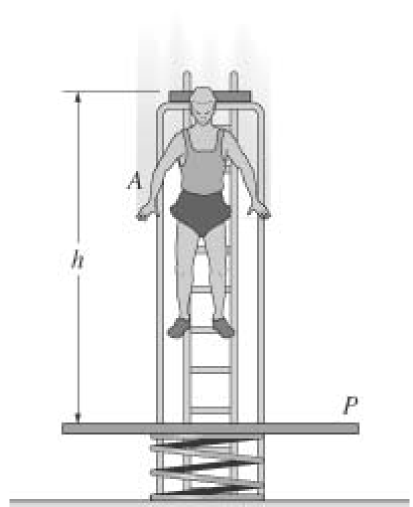
Problem 15-61

The man A has weight W_A and jumps from rest from a height h onto a platform P that has weight W_P. The platform is mounted on a spring, which has stiffness k. Determine (a) the velocities of A and P just after impact and (b) the maximum compression imparted to the spring by the impact. Assume the coefficient of restitution between the man and the platform is e, and the man holds himself rigid during the motion.

Given:

$W_A = 175 \text{ lb} \quad W_P = 60 \text{ lb} \quad k = 200 \frac{\text{lb}}{\text{ft}}$

$h = 8 \text{ ft} \quad e = 0.6 \quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$m_A = \frac{W_A}{g} \qquad m_P = \frac{W_P}{g} \qquad \delta_{st} = \frac{W_P}{k}$

Guesses $v_{A1} = 1 \frac{\text{ft}}{\text{s}} \qquad v_{A2} = 1 \frac{\text{ft}}{\text{s}}$

$v_{P2} = -1 \frac{\text{ft}}{\text{s}} \qquad \delta = 21 \text{ ft}$

Given

energy $W_A h = \frac{1}{2} m_A v_{A1}^2$

momentum $-m_A v_{A1} = m_A v_{A2} + m_P v_{P2}$

restitution $e v_{A1} = v_{A2} - v_{P2}$

energy $\frac{1}{2} m_P v_{P2}^2 + \frac{1}{2} k \delta_{st}^2 = \frac{1}{2} k (\delta + \delta_{st})^2 - W_P \delta$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}(v_{A1}, v_{A2}, v_{P2}, \delta) \qquad \begin{pmatrix} v_{A2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -13.43 \\ -27.04 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \delta = 2.61\ \text{ft}$$

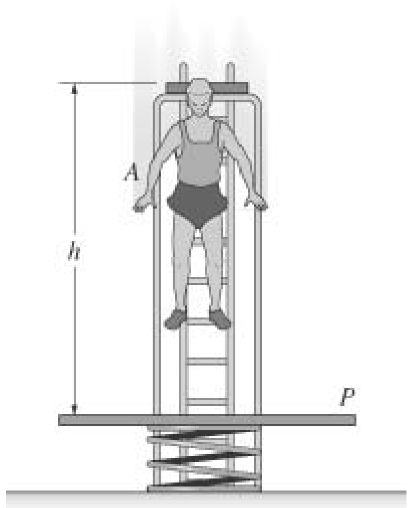
Problem 15-62

The man A has weight W_A and jumps from rest onto a platform P that has weight W_P. The platform is mounted on a spring, which has stiffness k. If the coefficient of restitution between the man and the platform is e, and the man holds himself rigid during the motion, determine the required height h of the jump if the maximum compression of the spring becomes δ.

Given:

$W_A = 100\ \text{lb}$ $\quad W_P = 60\ \text{lb}$ $\quad \delta = 2\ \text{ft}$

$k = 200\ \frac{\text{lb}}{\text{ft}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad e = 0.6$



Solution:

$m_A = \frac{W_A}{g}$ $\quad m_P = \frac{W_P}{g}$ $\quad \delta_{st} = \frac{W_P}{k}$

Guesses $v_{A1} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$

$v_{P2} = -1\ \frac{\text{ft}}{\text{s}}$ $\quad h = 21\ \text{ft}$

Given

energy $W_A h = \frac{1}{2} m_A v_{A1}^2$

momentum $-m_A v_{A1} = m_A v_{A2} + m_P v_{P2}$

restitution $e v_{A1} = v_{A2} - v_{P2}$

energy $\quad \frac{1}{2} m_P v_{P2}^2 + \frac{1}{2} k \delta_{st}^2 = \frac{1}{2} k \delta^2 - W_P(\delta - \delta_{st})$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{P2} \\ h \end{pmatrix} = \text{Find}(v_{A1}, v_{A2}, v_{P2}, h) \qquad \begin{pmatrix} v_{A2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -7.04 \\ -17.61 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad h = 4.82\ \text{ft}$$

Problem 15-63

The collar B of weight W_B is at rest, and when it is in the position shown the spring is unstretched. If another collar A of weight W_A strikes it so that B slides a distance b on the smooth rod before momentarily stopping, determine the velocity of A just after impact, and the average force exerted between A and B during the impact if the impact occurs in time Δt. The coefficient of restitution between A and B is e.

Units Used: $\quad \text{kip} = 10^3\ \text{lb}$

Given:

$W_B = 10\ \text{lb}$

$W_A = 1\ \text{lb}$

$k = 20\ \frac{\text{lb}}{\text{ft}}$

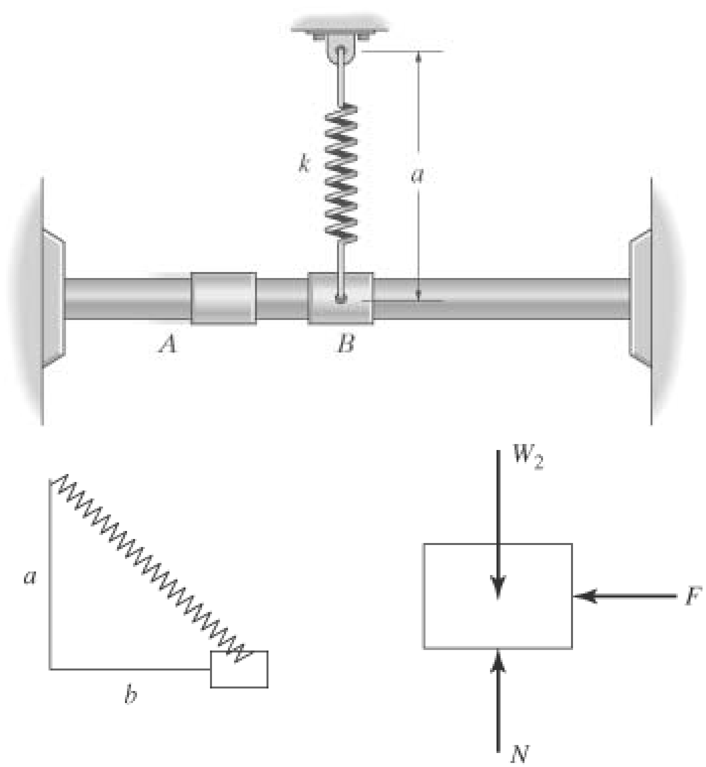
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$a = 3\ \text{ft}$

$b = 4\ \text{ft}$

$\Delta t = 0.002\ \text{s}$

$e = 0.5$



Solution:

Guesses $\quad v_{A1} = 1\ \frac{\text{ft}}{\text{s}} \quad v_{A2} = 1\ \frac{\text{ft}}{\text{s}} \quad v_{B2} = 1\ \frac{\text{ft}}{\text{s}} \quad F = 1\ \text{lb}$

Given $\quad \left(\frac{W_A}{g}\right) v_{A1} = \left(\frac{W_A}{g}\right) v_{A2} + \left(\frac{W_B}{g}\right) v_{B2} \qquad e\, v_{A1} = v_{B2} - v_{A2}$

$$\left(\frac{W_A}{g}\right)v_{A1} - F\Delta t = \left(\frac{W_A}{g}\right)v_{A2} \qquad \frac{1}{2}\left(\frac{W_B}{g}\right)v_{B2}^2 = \frac{1}{2}k\left(\sqrt{a^2+b^2}-a\right)^2$$

$$\begin{pmatrix} v_{A1} \\ v_{A2} \\ v_{B2} \\ F \end{pmatrix} = \text{Find}(v_{A1}, v_{A2}, v_{B2}, F) \qquad v_{A2} = -42.80\ \frac{\text{ft}}{\text{s}} \qquad F = 2.49\ \text{kip}$$

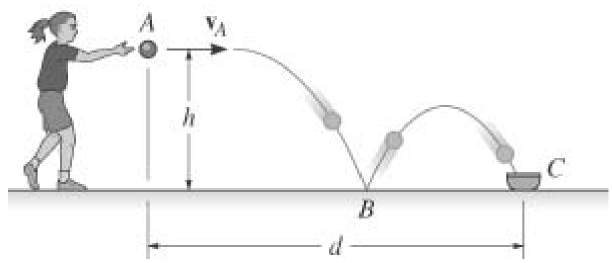
***Problem 15-64**

If the girl throws the ball with horizontal velocity v_A, determine the distance d so that the ball bounces once on the smooth surface and then lands in the cup at C.

Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$e = 0.8 \qquad h = 3\ \text{ft}$



Solution:

$t_B = \sqrt{2\frac{h}{g}} \qquad t_B = 0.43\ \text{s}$

$v_{By1} = g\,t_B \qquad v_{By1} = 13.90\ \frac{\text{ft}}{\text{s}}$

$v_{By2} = e\,v_{By1} \qquad v_{By2} = 11.12\ \frac{\text{ft}}{\text{s}}$

$t_C = \frac{2v_{By2}}{g} \qquad t_C = 0.69\ \text{s}$

$d = v_A(t_B + t_C) \qquad d = 8.98\ \text{ft}$

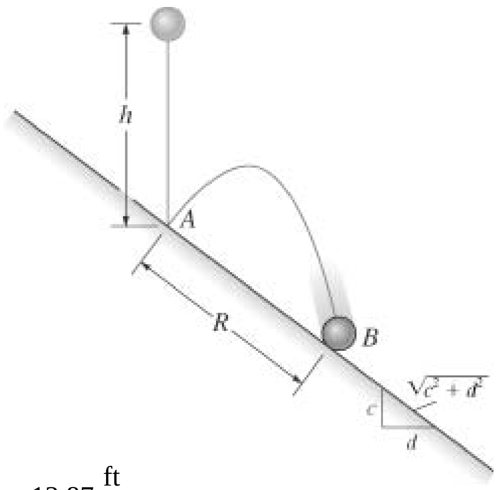
Problem 15-65

The ball is dropped from rest and falls a distance h before striking the smooth plane at A. If the coefficient of restitution is e, determine the distance R to where it again strikes the plane at B.

Given:

$h = 4\ \text{ft} \qquad c = 3$

$e = 0.8 \qquad d = 4 \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$\theta = \text{atan}\left(\frac{c}{d}\right) \qquad \theta = 36.87\ \text{deg}$

$v_{A1} = \sqrt{2gh} \qquad v_{A1} = 16.05\ \frac{\text{ft}}{\text{s}}$

$v_{A1n} = v_{A1}\cos(\theta) \qquad v_{A1t} = v_{A1}\sin(\theta)$

$v_{A2n} = e\,v_{A1n} \qquad v_{A2t} = v_{A1t}$

$v_{A2x} = v_{A2n}\sin(\theta) + v_{A2t}\cos(\theta) \qquad v_{A2x} = 13.87\ \frac{\text{ft}}{\text{s}}$

$v_{A2y} = v_{A2n}\cos(\theta) - v_{A2t}\sin(\theta) \qquad v_{A2y} = 2.44\ \frac{\text{ft}}{\text{s}}$

Guesses $\quad t = 1\ \text{s} \qquad R = 10\ \text{ft}$

Given $\quad R\cos(\theta) = v_{A2x}t \qquad -R\sin(\theta) = \left(\frac{-g}{2}\right)t^2 + v_{A2y}t$

$\begin{pmatrix} R \\ t \end{pmatrix} = \text{Find}(R, t) \qquad t = 0.80\ \text{s} \qquad R = 13.82\ \text{ft}$

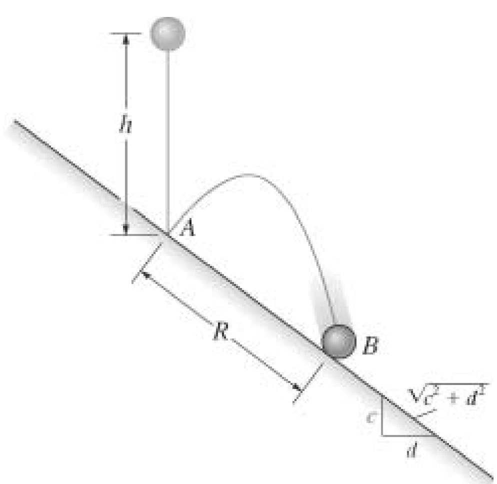
Problem 15-66

The ball is dropped from rest and falls a distance h before striking the smooth plane at A. If it rebounds and in time t again strikes the plane at B, determine the coefficient of restitution e between the ball and the plane. Also, what is the distance R?

Given:

$h = 4\ \text{ft} \qquad c = 3 \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$t = 0.5\ \text{s} \qquad d = 4$



Solution:

$\theta = \text{atan}\left(\frac{c}{d}\right) \qquad \theta = 36.87\ \text{deg}$

$v_{A1} = \sqrt{2gh} \qquad v_{A1} = 16.05\ \frac{\text{ft}}{\text{s}}$

$v_{A1n} = v_{A1}\cos(\theta) \qquad v_{A1t} = v_{A1}\sin(\theta)$

$v_{A2t} = v_{A1t}$

Guesses $e = 0.8$ $R = 10\ \text{ft}$ $v_{A2n} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{A2x} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{A2y} = 1\ \frac{\text{ft}}{\text{s}}$

Given $v_{A2n} = e\, v_{A1n}$

$v_{A2x} = v_{A2n}\sin(\theta) + v_{A2t}\cos(\theta)$ $v_{A2y} = v_{A2n}\cos(\theta) - v_{A2t}\sin(\theta)$

$R\cos(\theta) = v_{A2x}\, t$ $-R\sin(\theta) = \frac{-g}{2}t^2 + v_{A2y}\, t$

$$\begin{pmatrix} e \\ R \\ v_{A2n} \\ v_{A2x} \\ v_{A2y} \end{pmatrix} = \text{Find}(e, R, v_{A2n}, v_{A2x}, v_{A2y})$$ $R = 7.23\,\text{ft}$ $e = 0.502$

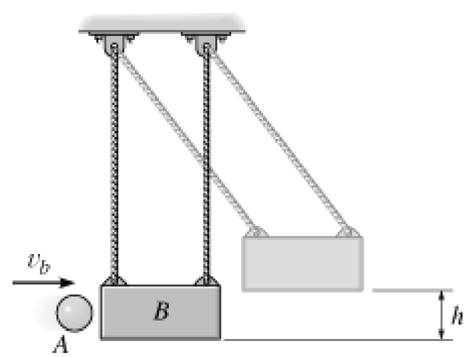
Problem 15-67

The ball of mass m_b is thrown at the suspended block of mass m_B with velocity v_b. If the coefficient of restitution between the ball and the block is e, determine the maximum height h to which the block will swing before it momentarily stops.

Given:

$m_b = 2\ \text{kg}$ $m_B = 20\ \text{kg}$ $e = 0.8$ $v_b = 4\ \frac{\text{m}}{\text{s}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:



Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$ $h = 1\ \text{m}$

Given

momentum $m_b v_b = m_b v_A + m_B v_B$

restitution $e\, v_b = v_B - v_A$

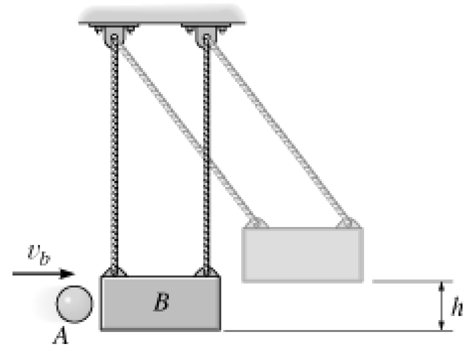
energy $\frac{1}{2}m_B v_B^2 = m_B g\, h$

$$\begin{pmatrix} v_A \\ v_B \\ h \end{pmatrix} = \text{Find}(v_A, v_B, h) \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -2.55 \\ 0.65 \end{pmatrix}\frac{\text{m}}{\text{s}}$$ $h = 21.84\,\text{mm}$

---

*__Problem 15-68__

The ball of mass m_b is thrown at the suspended block of mass m_B with a velocity of v_b. If the time of impact between the ball and the block is Δt, determine the average normal force exerted on the block during this time.



Given: $\text{kN} = 10^3\ \text{N}$

$m_b = 2\ \text{kg}$ $\quad v_b = 4\ \frac{\text{m}}{\text{s}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_B = 20\ \text{kg}$ $\quad e = 0.8$ $\quad \Delta t = 0.005\ \text{s}$

Solution:

Guesses $\quad v_A = 1\ \frac{\text{m}}{\text{s}}$ $\quad v_B = 1\ \frac{\text{m}}{\text{s}}$ $\quad F = 1\ \text{N}$

Given

momentum $\quad m_b v_b = m_b v_A + m_B v_B$

restitution $\quad e v_b = v_B - v_A$

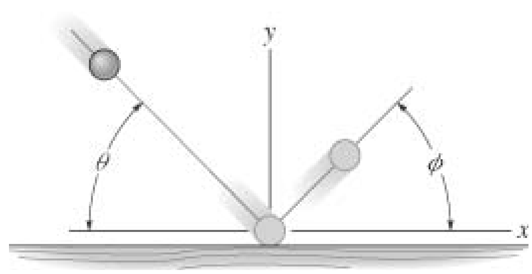
momentum B $\quad 0 + F\Delta t = m_B v_B$

$$\begin{pmatrix} v_A \\ v_B \\ F \end{pmatrix} = \text{Find}(v_A, v_B, F) \qquad \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -2.55 \\ 0.65 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad F = 2.62\ \text{kN}$$

Problem 15-69

A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x\Delta t = \mu F_y\Delta t$ or $F_x = \mu F_y$.

Solution:



$$e v_1 \sin(\theta) = v_2 \sin(\phi)$$

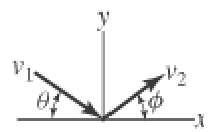
$$\frac{v_2}{v_1} = e\left(\frac{\sin(\theta)}{\sin(\phi)}\right) \qquad [1]$$

$(\overset{+}{\rightarrow})$ $\quad m v_1 \cos(\theta) - F_x\Delta t = m v_2 \cos(\phi)$

$$F_x = \frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} \qquad [2]$$

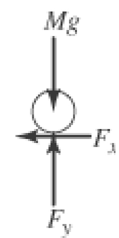
$(+\downarrow)$ $\quad m v_1 \sin(\theta) - F_y\Delta t = -m v_2 \sin(\phi)$

$$F_y = \frac{m v_1 \sin(\theta) + m v_2 \sin(\phi)}{\Delta t} \qquad [\,3\,]$$



Since $F_x = \mu F_y$, from Eqs [2] and [3]

$$\frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} = \frac{\mu\left(m v_1 \sin(\theta) + m v_2 \sin(\phi)\right)}{\Delta t}$$



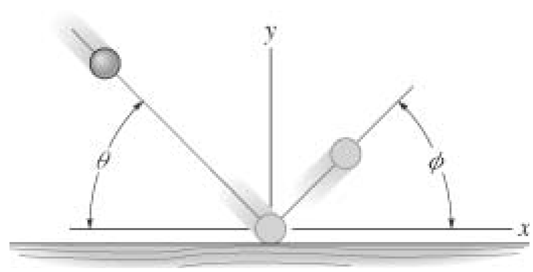
$$\frac{v_2}{v_1} = \frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)} \qquad [\,4\,]$$

Substituting Eq. [4] into [1] yields:

$$e = \frac{\sin(\phi)}{\sin(\theta)}\left(\frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$$

Problem 15-70

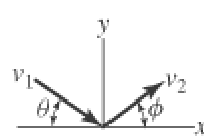
A ball is thrown onto a rough floor at an angle of θ. If it rebounds at the same angle ϕ, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is e. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



Solution:

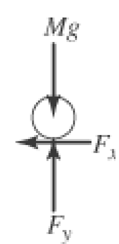
$$e v_1 \sin(\theta) = v_2 \sin(\phi)$$

$$\frac{v_2}{v_1} = e\left(\frac{\sin(\theta)}{\sin(\phi)}\right) \qquad [\,1\,]$$



$(\overset{+}{\rightarrow})$ $\quad m v_1 \cos(\theta) - F_x \Delta t = m v_2 \cos(\phi)$

$$F_x = \frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} \qquad [\,2\,]$$



$(+\downarrow)$ $\quad m v_1 \sin(\theta) - F_y \Delta t = -m v_2 \sin(\phi)$

$$F_y = \frac{m v_1 \sin(\theta) + m v_2 \sin(\phi)}{\Delta t} \qquad [\,3\,]$$

Since $F_x = \mu F_y$, from Eqs [2] and [3]

$$\frac{m v_1 \cos(\theta) - m v_2 \cos(\phi)}{\Delta t} = \frac{\mu(m v_1 \sin(\theta) + m v_2 \sin(\phi))}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)} \qquad [\,4\,]$$

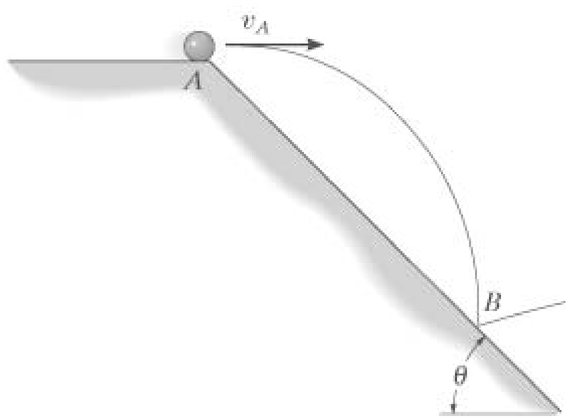
Substituting Eq. [4] into [1] yields: $e = \dfrac{\sin(\phi)}{\sin(\theta)}\left(\dfrac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$

Given $\theta = 45\ \text{deg}$ $\phi = 45\ \text{deg}$ $e = 0.6$ Guess $\mu = 0.2$

Given $e = \dfrac{\sin(\phi)}{\sin(\theta)}\left(\dfrac{\cos(\theta) - \mu \sin(\theta)}{\mu \sin(\phi) + \cos(\phi)}\right)$ $\mu = \text{Find}(\mu)$ $\mu = 0.25$

Problem 15-71

The ball bearing of weight W travels over the edge A with velocity v_A. Determine the speed at which it rebounds from the smooth inclined plane at B. Take $e = 0.8$.



Given:

$W = 0.2\ \text{lb}$ $\theta = 45\ \text{deg}$

$v_A = 3\ \dfrac{\text{ft}}{\text{s}}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$ $e = 0.8$

Solution:

Guesses $v_{B1x} = 1\ \dfrac{\text{ft}}{\text{s}}$ $v_{B1y} = 1\ \dfrac{\text{ft}}{\text{s}}$ $v_{B2n} = 1\ \dfrac{\text{ft}}{\text{s}}$ $v_{B2t} = 1\ \dfrac{\text{ft}}{\text{s}}$

$t = 1\ \text{s}$ $R = 1\ \text{ft}$

Given $v_{B1x} = v_A$ $v_A t = R\cos(\theta)$

$\dfrac{-1}{2} g t^2 = -R\sin(\theta)$ $v_{B1y} = -g t$

$v_{B1x}\cos(\theta) - v_{B1y}\sin(\theta) = v_{B2t}$

$$e\left(-v_{B1y}\cos(\theta) - v_{B1x}\sin(\theta)\right) = v_{B2n}$$

$$\begin{pmatrix} v_{B1x} \\ v_{B1y} \\ v_{B2n} \\ v_{B2t} \\ t \\ R \end{pmatrix} = \text{Find}\left(v_{B1x}, v_{B1y}, v_{B2n}, v_{B2t}, t, R\right)$$

$$\begin{pmatrix} v_{B1x} \\ v_{B1y} \end{pmatrix} = \begin{pmatrix} 3.00 \\ -6.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$t = 0.19 \text{ s}$$

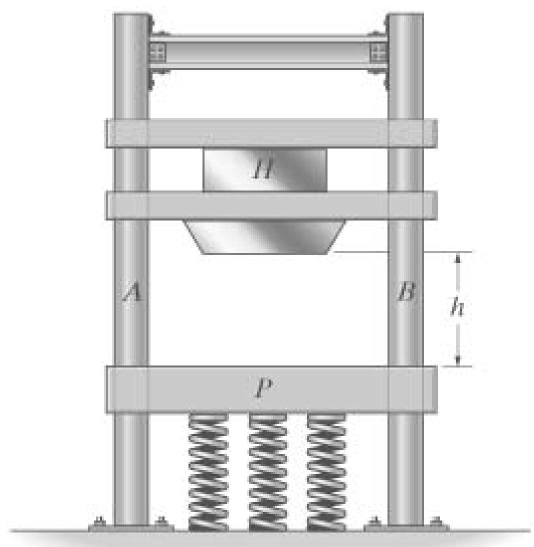
$$R = 0.79 \text{ ft}$$

$$\begin{pmatrix} v_{B2n} \\ v_{B2t} \end{pmatrix} = \begin{pmatrix} 1.70 \\ 6.36 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\begin{pmatrix} v_{B2n} \\ v_{B2t} \end{pmatrix}\right| = 6.59 \frac{\text{ft}}{\text{s}}$$

---

*__Problem 15-72__

The drop hammer H has a weight W_H and falls from rest h onto a forged anvil plate P that has a weight W_P. The plate is mounted on a set of springs that have a combined stiffness k_T. Determine (a) the velocity of P and H just after collision and (b) the maximum compression in the springs caused by the impact. The coefficient of restitution between the hammer and the plate is e. Neglect friction along the vertical guideposts A and B.



Given:

$W_H = 900 \text{ lb}$ $\quad k_T = 500 \frac{\text{lb}}{\text{ft}}$

$W_P = 500 \text{ lb}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$h = 3 \text{ ft}$ $\quad e = 0.6$

Solution:

$$\delta_{st} = \frac{W_P}{k_T} \qquad v_{H1} = \sqrt{2gh}$$

Guesses

$$v_{H2} = 1 \frac{\text{ft}}{\text{s}} \qquad v_{P2} = 1 \frac{\text{ft}}{\text{s}} \qquad \delta = 2 \text{ ft}$$

Given $\quad \left(\frac{W_H}{g}\right)v_{H1} = \left(\frac{W_H}{g}\right)v_{H2} + \left(\frac{W_P}{g}\right)v_{P2}$

$$e\,v_{H1} = v_{P2} - v_{H2}$$

$$\frac{1}{2}k_T\delta_{st}^{\ 2} + \frac{1}{2}\left(\frac{W_P}{g}\right)v_{P2}^{\ 2} = \frac{1}{2}k_T\delta^2 - W_P(\delta - \delta_{st})$$

$$\begin{pmatrix} v_{H2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}(v_{H2}, v_{P2}, \delta) \qquad \begin{pmatrix} v_{H2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} 5.96 \\ 14.30 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \delta = 3.52\,\text{ft}$$

Problem 15-73

It was observed that a tennis ball when served horizontally a distance h above the ground strikes the smooth ground at B a distance d away. Determine the initial velocity v_A of the ball and the velocity v_B (and θ) of the ball just after it strikes the court at B. The coefficient of restitution is e.

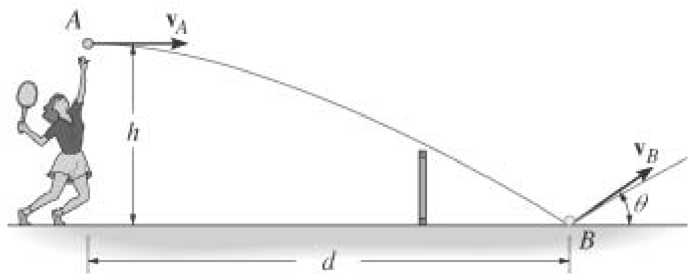
Given:

$h = 7.5$ ft

$d = 20$ ft

$e = 0.7$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $\quad v_A = 1\ \frac{\text{ft}}{\text{s}} \quad v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$

$v_{By1} = 1\ \frac{\text{ft}}{\text{s}} \quad \theta = 10\ \text{deg} \quad t = 1\ \text{s}$

Given $\quad h = \frac{1}{2}g\,t^2 \qquad d = v_A\,t$

$e\,v_{By1} = v_{B2}\sin(\theta) \qquad v_{By1} = g\,t$

$v_A = v_{B2}\cos(\theta)$

$$\begin{pmatrix} v_A \\ t \\ v_{By1} \\ v_{B2} \\ \theta \end{pmatrix} = \text{Find}(v_A, t, v_{By1}, v_{B2}, \theta) \qquad v_A = 29.30\,\frac{\text{ft}}{\text{s}} \qquad v_{B2} = 33.10\,\frac{\text{ft}}{\text{s}} \qquad \theta = 27.70\,\text{deg}$$

Problem 15-74

The tennis ball is struck with a horizontal velocity v_A, strikes the smooth ground at B, and bounces upward at $\theta = \theta_1$. Determine the initial velocity v_A, the final velocity v_B, and the coefficient of restitution between the ball and the ground.

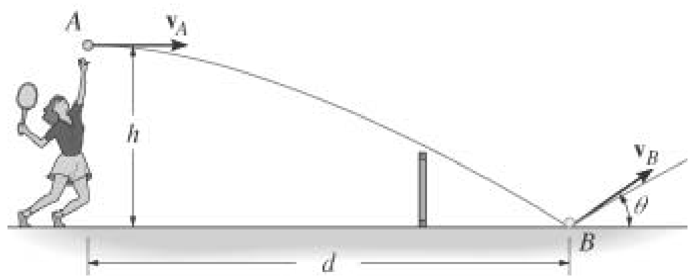
Given:

$h = 7.5\ \text{ft}$

$d = 20\ \text{ft}$

$\theta_1 = 30\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $\theta = \theta_1$

Guesses $v_A = 1\ \frac{\text{ft}}{\text{s}}$ $t = 1\ \text{s}$ $v_{By1} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$ $e = 0.5$

Given $h = \frac{1}{2} g t^2$ $d = v_A t$ $v_{By1} = g t$

$e\, v_{By1} = v_{B2} \sin(\theta)$ $v_A = v_{B2} \cos(\theta)$

$$\begin{pmatrix} v_A \\ t \\ v_{By1} \\ v_{B2} \\ e \end{pmatrix} = \text{Find}(v_A, t, v_{By1}, v_{B2}, e) \qquad v_A = 29.30\,\frac{\text{ft}}{\text{s}} \qquad v_{B2} = 33.84\,\frac{\text{ft}}{\text{s}} \qquad e = 0.77$$

Problem 15-75

The ping-pong ball has mass M. If it is struck with the velocity shown, determine how high h it rises above the end of the smooth table after the rebound. The coefficient of restitution is e.

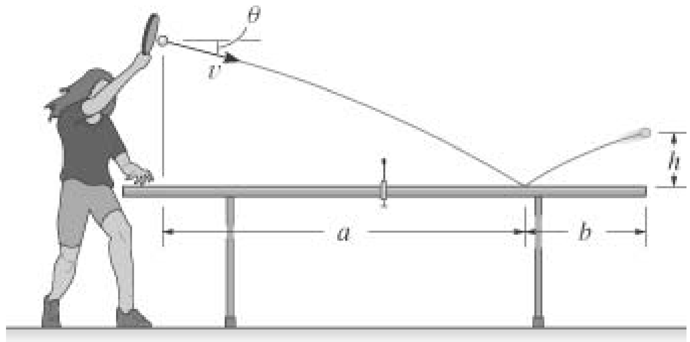
Given:

$M = 2\ \text{gm}$ $a = 2.25\ \text{m}$

$e = 0.8$ $b = 0.75\ \text{m}$

$\theta = 30\ \text{deg}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$v = 18\ \dfrac{\text{m}}{\text{s}}$



Solution: Guesses $v_{1x} = 1\ \dfrac{\text{m}}{\text{s}}$ $v_{1y} = 1\ \dfrac{\text{m}}{\text{s}}$ $v_{2x} = 1\ \dfrac{\text{m}}{\text{s}}$ $v_{2y} = 1\ \dfrac{\text{m}}{\text{s}}$

$t_1 = 1\ \text{s}$ $t_2 = 2\ \text{s}$ $h = 1\ \text{m}$

Given $v_{1x} = v\cos(\theta)$ $a = v\cos(\theta)t_1$ $v_{1y} = g\,t_1 + v\sin(\theta)$

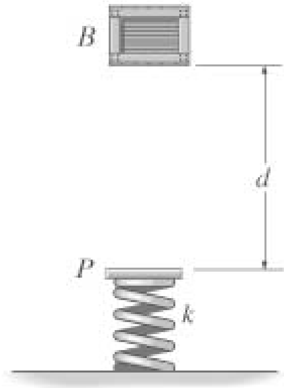
$v_{2x} = v_{1x}$ $e\,v_{1y} = v_{2y}$ $b = v_{2x}t_2$ $h = v_{2y}t_2 - \left(\dfrac{g}{2}\right)t_2^{\,2}$

$$\begin{pmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \\ t_1 \\ t_2 \\ h \end{pmatrix} = \text{Find}(v_{1x}, v_{1y}, v_{2x}, v_{2y}, t_1, t_2, h) \qquad \begin{pmatrix} v_{1x} \\ v_{1y} \\ v_{2x} \\ v_{2y} \end{pmatrix} = \begin{pmatrix} 15.59 \\ 10.42 \\ 15.59 \\ 8.33 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.05 \end{pmatrix} \text{s}$$

$h = 390\,\text{mm}$

***Problem 15-76**

The box B of weight W_B is dropped from rest a distance d from the top of the plate P of weight W_P, which is supported by the spring having a stiffness k. Determine the maximum compression imparted to the spring. Neglect the mass of the spring.



Given: $W_B = 5\ \text{lb}$ $W_P = 10\ \text{lb}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$k = 30\ \dfrac{\text{lb}}{\text{ft}}$ $d = 5\ \text{ft}$ $e = 0.6$

Solution:

$$\delta_{st} = \frac{W_P}{k} \qquad v_{B1} = \sqrt{2 g d}$$

Guesses $v_{B2} = 1\ \dfrac{\text{ft}}{\text{s}}$ $v_{P2} = 1\ \dfrac{\text{ft}}{\text{s}}$ $\delta = 2\ \text{ft}$

Given $$\left(\frac{W_B}{g}\right) v_{B1} = \left(\frac{W_B}{g}\right) v_{B2} + \left(\frac{W_P}{g}\right) v_{P2} \qquad e\, v_{B1} = v_{P2} - v_{B2}$$

$$\frac{1}{2} k \delta_{st}^{\,2} + \frac{1}{2}\left(\frac{W_P}{g}\right) v_{P2}^{\,2} = \frac{1}{2} k \delta^2 - W_P\left(\delta - \delta_{st}\right)$$

$$\begin{pmatrix} v_{B2} \\ v_{P2} \\ \delta \end{pmatrix} = \text{Find}\left(v_{B2}, v_{P2}, \delta\right) \qquad \begin{pmatrix} v_{B2} \\ v_{P2} \end{pmatrix} = \begin{pmatrix} -1.20 \\ 9.57 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \delta = 1.31\ \text{ft}$$

Problem 15-77

A pitching machine throws the ball of weight M towards the wall with an initial velocity v_A as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall and (c) the distance d from the wall to where it strikes the ground at C.

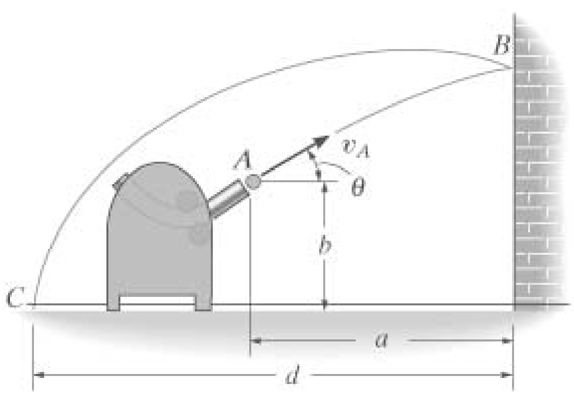
Given:

$M = 0.5\ \text{kg}$ $a = 3\ \text{m}$

$v_A = 10\ \dfrac{\text{m}}{\text{s}}$ $b = 1.5\ \text{m}$

$\theta = 30\ \text{deg}$ $e = 0.5$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution: Guesses

$v_{Bx1} = 1\ \frac{m}{s}$ $v_{Bx2} = 1\ \frac{m}{s}$

$v_{By1} = 1\ \frac{m}{s}$ $v_{By2} = 1\ \frac{m}{s}$

$h = 1\ m$ $d = 1\ m$

$t_1 = 1\ s$ $t_2 = 1\ s$

Given

$v_A \cos(\theta) t_1 = a$ $b + v_A \sin(\theta) t_1 - \frac{1}{2} g t_1^2 = h$

$v_{By2} = v_{By1}$ $v_A \sin(\theta) - g t_1 = v_{By1}$

$d = v_{Bx2} t_2$ $h + v_{By2} t_2 - \frac{1}{2} g t_2^2 = 0$

$v_A \cos(\theta) = v_{Bx1}$ $e v_{Bx1} = v_{Bx2}$

$$\begin{pmatrix} v_{Bx1} \\ v_{By1} \\ v_{Bx2} \\ v_{By2} \\ h \\ t_1 \\ t_2 \\ d \end{pmatrix} = \text{Find}(v_{Bx1}, v_{By1}, v_{Bx2}, v_{By2}, h, t_1, t_2, d)$$

$\left| \begin{pmatrix} v_{Bx1} \\ v_{By1} \end{pmatrix} \right| = 8.81\ \frac{m}{s}$

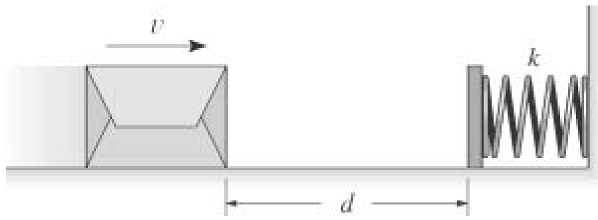
$\left| \begin{pmatrix} v_{Bx2} \\ v_{By2} \end{pmatrix} \right| = 4.62\ \frac{m}{s}$

$d = 3.96\ m$

Problem 15-78

The box of weight W_b slides on the surface for which the coefficient of friction is μ_k. The box has velocity v when it is a distance d from the plate. If it strikes the plate, which has weight W_p and is held in position by an unstretched spring of stiffness k, determine the maximum compression imparted to the spring. The coefficient of restitution between the box and the plate is e. Assume that the plate slides smoothly.

Given:



$W_b = 20\ \text{lb}$ $\quad W_p = 10\ \text{lb}$

$\mu_k = 0.3$ $\quad k = 400\ \frac{\text{lb}}{\text{ft}}$

$v = 15\ \frac{\text{ft}}{\text{s}}$ $\quad e = 0.8$

$d = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $\quad v_{b1} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{b2} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad v_{p2} = 1\ \frac{\text{ft}}{\text{s}}$ $\quad \delta = 1\ \text{ft}$

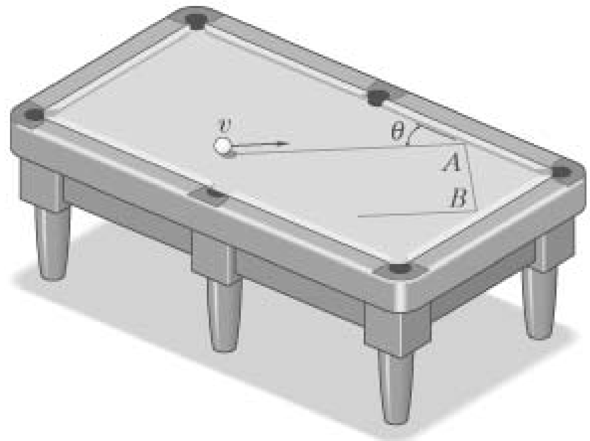
Given $\quad \frac{1}{2}\left(\frac{W_b}{g}\right)v^2 - \mu_k W_b d = \frac{1}{2}\left(\frac{W_b}{g}\right)v_{b1}^2$ $\quad \left(\frac{W_b}{g}\right)v_{b1} = \left(\frac{W_b}{g}\right)v_{b2} + \left(\frac{W_p}{g}\right)v_{p2}$

$e v_{b1} = v_{p2} - v_{b2}$ $\quad \frac{1}{2}\left(\frac{W_p}{g}\right)v_{p2}^2 = \frac{1}{2}k\delta^2$

$$\begin{pmatrix} v_{b1} \\ v_{b2} \\ v_{p2} \\ \delta \end{pmatrix} = \text{Find}(v_{b1}, v_{b2}, v_{p2}, \delta) \qquad \begin{pmatrix} v_{b1} \\ v_{b2} \\ v_{p2} \end{pmatrix} = \begin{pmatrix} 13.65 \\ 5.46 \\ 16.38 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \delta = 0.456\ \text{ft}$$

Problem 15-79

The billiard ball of mass M is moving with a speed v when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is e, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.



Given:

$M = 200\ \text{gm}$

$v = 2.5\ \frac{\text{m}}{\text{s}}$

$\theta = 45\ \text{deg}$

$e = 0.6$

Solution:

Guesses

$v_2 = 1\ \frac{\text{m}}{\text{s}}$ $\quad \theta_2 = 1\ \text{deg}$ $\quad v_3 = 1\ \frac{\text{m}}{\text{s}}$ $\quad \theta_3 = 1\ \text{deg}$

Given $\quad e\,v\sin(\theta) = v_2\sin(\theta_2) \qquad v\cos(\theta) = v_2\cos(\theta_2)$

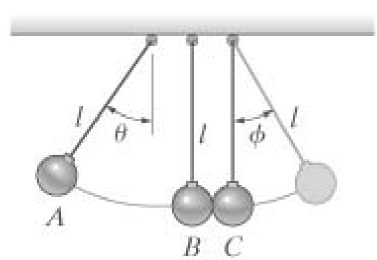
$e\,v_2\cos(\theta_2) = v_3\sin(\theta_3) \qquad v_2\sin(\theta_2) = v_3\cos(\theta_3)$

$$\begin{pmatrix} v_2 \\ v_3 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \text{Find}(v_2, v_3, \theta_2, \theta_3) \qquad \begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 2.06 \\ 1.50 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 31.0 \\ 45.0 \end{pmatrix} \text{deg}$$

$$v_3 = 1.500\ \frac{\text{m}}{\text{s}}$$

***Problem 15-80**

The three balls each have the same mass m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



Solution:

Energy

$$0 + l(1 - \cos(\theta))mg = \frac{1}{2}m{v_A}^2$$

$$v_A = \sqrt{2(1 - \cos(\theta))g\,l}$$

Collision of ball A with B:

$$m v_A + 0 = m v'_A + m v'_B \qquad e v_A = v'_B - v'_A \qquad v'_B = \frac{1}{2}(1 + e)v'_B$$

Collision of ball B with C:

$$m\,v'_B + 0 = m\,v''_B + m\,v''_C \qquad e\,v'_B = v''_C - v''_B \qquad v''_C = \frac{1}{4}(1+e)^2 v_A$$

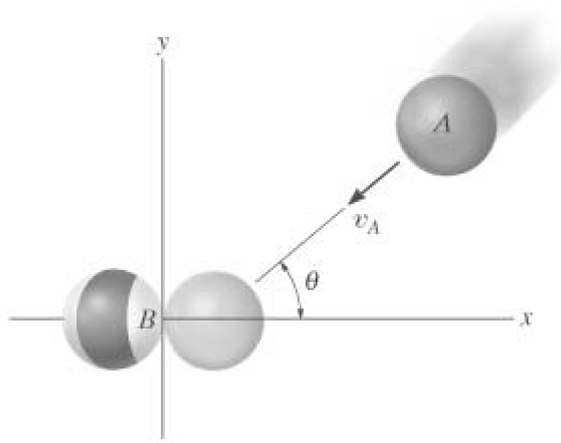
Energy

$$\frac{1}{2}m\,v''_c{}^2 + 0 = 0 + l(1-cos(\phi))m\,g \qquad \frac{1}{2}\left(\frac{1}{16}\right)(1+e)^4(2)(1-\cos(\theta)) = (1-\cos(\phi))$$

$$\left(\frac{1+e}{2}\right)^4(1-\cos(\theta)) = 1-\cos(\phi) \qquad \phi = \operatorname{acos}\left[1-\left(\frac{1+e}{2}\right)^4(1-\cos(\theta))\right]$$

Problem 15-81

Two smooth billiard balls *A* and *B* each have mass *M*. If *A* strikes *B* with a velocity v_A as shown, determine their final velocities just after collision. Ball *B* is originally at rest and the coefficient of restitution is *e*. Neglect the size of each ball.



Given:

$M = 0.2\ \text{kg}$

$\theta = 40\ \text{deg}$

$v_A = 1.5\ \frac{\text{m}}{\text{s}}$

$e = 0.85$

Solution: Guesses $v_{A2} = 1\ \frac{\text{m}}{\text{s}}$ $\quad v_{B2} = 1\ \frac{\text{m}}{\text{s}}$ $\quad \theta_2 = 20\ \text{deg}$

Given $\quad -M\,v_A\cos(\theta) = M\,v_{B2} + M\,v_{A2}\cos(\theta_2)$

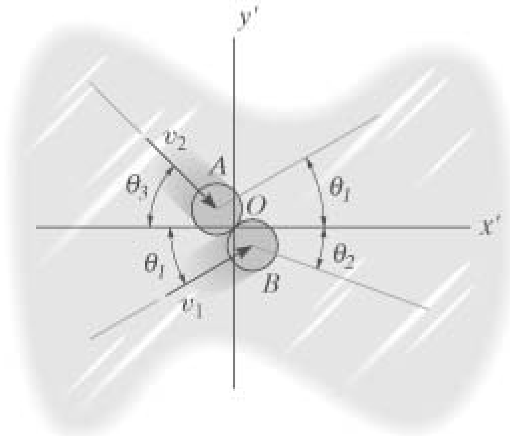
$$e\,v_A\cos(\theta) = v_{A2}\cos(\theta_2) - v_{B2}$$

$$v_A\sin(\theta) = v_{A2}\sin(\theta_2)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_2 \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_2) \qquad \theta_2 = 95.1\,\text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.968 \\ -1.063 \end{pmatrix}\frac{\text{m}}{\text{s}}$$

Problem 15-82

The two hockey pucks *A* and *B* each have a mass *M*. If they collide at *O* and are deflected along the colored paths, determine their speeds just after impact. Assume that the icy surface over which they slide is smooth. *Hint:* Since the *y'* axis is *not* along the line of impact, apply the conservation of momentum along the *x'* and *y'* axes.



Given:

$M = 250\ \text{g}$ $\quad \theta_1 = 30\ \text{deg}$

$v_1 = 40\ \frac{\text{m}}{\text{s}}$ $\quad \theta_2 = 20\ \text{deg}$

$v_2 = 60\ \frac{\text{m}}{\text{s}}$ $\quad \theta_3 = 45\ \text{deg}$

Solution:

Initial Guess:

$v_{A2} = 5\ \frac{\text{m}}{\text{s}}$ $\quad v_{B2} = 4\ \frac{\text{m}}{\text{s}}$

Given

$$M v_2 \cos(\theta_3) + M v_1 \cos(\theta_1) = M v_{A2} \cos(\theta_1) + M v_{B2} \cos(\theta_2)$$

$$-M v_2 \sin(\theta_3) + M v_1 \sin(\theta_1) = M v_{A2} \sin(\theta_1) - M v_{B2} \sin(\theta_2)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 6.90 \\ 75.66 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 15-83

Two smooth coins *A* and *B*, each having the same mass, slide on a smooth surface with the motion shown. Determine the speed of each coin after collision if they move off along the dashed paths. *Hint:* Since the line of impact has not been defined, apply the conservation of momentum along the *x* and *y* axes, respectively.

Given:

$v_{A1} = 0.5\ \frac{ft}{s}$

$v_{B1} = 0.8\ \frac{ft}{s}$

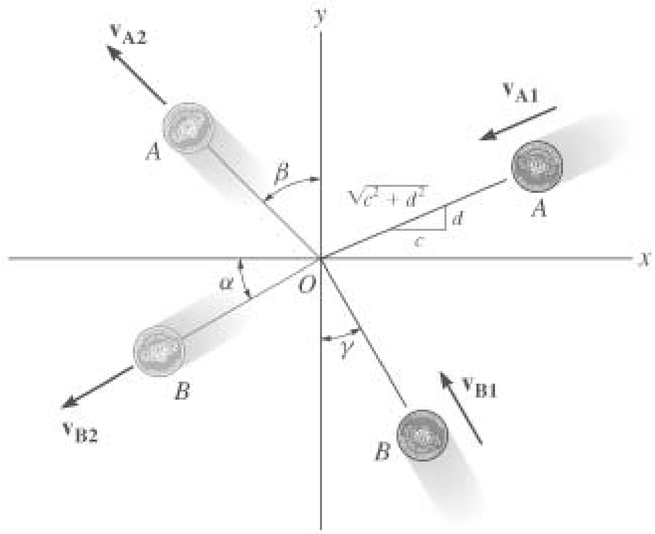
$\alpha = 30\ deg$

$\beta = 45\ deg$

$\gamma = 30\ deg$

$c = 4$

$d = 3$



Solution:

Guesses $v_{B2} = 0.25\ \frac{ft}{s}$ $v_{A2} = 0.5\ \frac{ft}{s}$

Given

$$-v_{A1}\left(\frac{c}{\sqrt{c^2+d^2}}\right) - v_{B1}\sin(\gamma) = -v_{A2}\sin(\beta) - v_{B2}\cos(\alpha)$$

$$-v_{A1}\left(\frac{d}{\sqrt{c^2+d^2}}\right) + v_{B1}\cos(\gamma) = v_{A2}\cos(\beta) - v_{B2}\sin(\alpha)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \mathrm{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 0.766 \\ 0.298 \end{pmatrix} \frac{ft}{s}$$

***Problem 15-84**

The two disks A and B have a mass M_A and M_B, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is e.

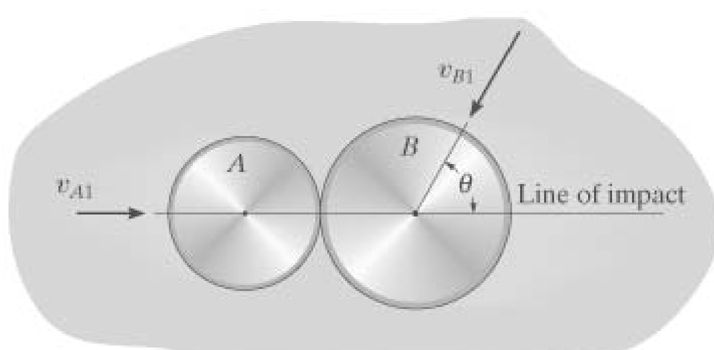
Given:

$M_A = 3\ kg$

$M_B = 5\ kg$

$\theta = 60\ deg$

$v_{B1} = 7\ \frac{m}{s}$



$v_{A1} = 6\ \frac{m}{s}$

$e = 0.65$

Solution: Guesses $v_{A2} = 1\ \frac{m}{s}$ $v_{B2} = 1\ \frac{m}{s}$ $\theta_2 = 20\ \text{deg}$

Given

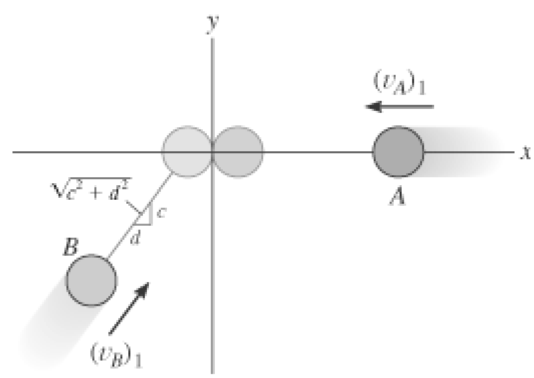
$$M_A v_{A1} - M_B v_{B1}\cos(\theta) = M_A v_{A2} + M_B v_{B2}\cos(\theta_2)$$

$$e\left(v_{A1} + v_{B1}\cos(\theta)\right) = v_{B2}\cos(\theta_2) - v_{A2} \qquad v_{B1}\sin(\theta) = v_{B2}\sin(\theta_2)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_2 \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_2) \qquad \theta_2 = 68.6\,\text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -3.80 \\ 6.51 \end{pmatrix} \frac{m}{s}$$

Problem 15-85

Two smooth disks A and B each have mass M. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is e.



Given:

$M = 0.5\ \text{kg}$ $c = 4$ $v_{A1} = 6\ \frac{m}{s}$

$e = 0.75$ $d = 3$ $v_{B1} = 4\ \frac{m}{s}$

Solution:

Guesses

$v_{A2} = 1\ \frac{m}{s}$ $v_{B2} = 1\ \frac{m}{s}$ $\theta_A = 10\ \text{deg}$ $\theta_B = 10\ \text{deg}$

Given $v_{A1}(0) = v_{A2}\sin(\theta_A)$ $\quad v_{B1}\left(\frac{c}{\sqrt{c^2+d^2}}\right) = v_{B2}\sin(\theta_B)$

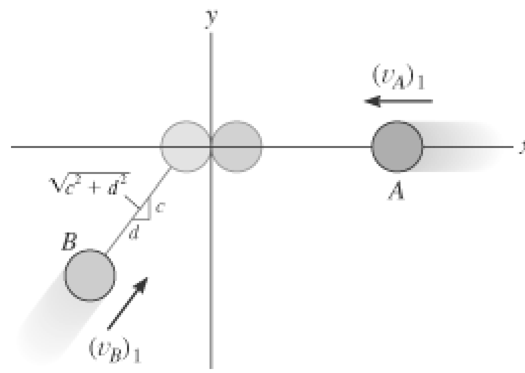
$$M v_{B1}\left(\frac{d}{\sqrt{c^2+d^2}}\right) - M v_{A1} = M v_{A2}\cos(\theta_A) - M v_{B2}\cos(\theta_B)$$

$$e\left[v_{A1} + v_{B1}\left(\frac{d}{\sqrt{c^2+d^2}}\right)\right] = v_{A2}\cos(\theta_A) + v_{B2}\cos(\theta_B)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_A \\ \theta_B \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_A, \theta_B) \qquad \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix} = \begin{pmatrix} 0.00 \\ 32.88 \end{pmatrix} \text{deg} \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} 1.35 \\ 5.89 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

Problem 15-86

Two smooth disks A and B each have mass M. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line angle θ counterclockwise from the y axis.



Given:

$M = 0.5\ \text{kg} \qquad c = 4 \qquad v_{A1} = 6\ \frac{\text{m}}{\text{s}}$

$\theta_B = 30\ \text{deg} \qquad d = 3 \qquad v_{B1} = 4\ \frac{\text{m}}{\text{s}}$

Solution:

Guesses

$v_{A2} = 2\ \frac{\text{m}}{\text{s}} \qquad v_{B2} = 1\ \frac{\text{m}}{\text{s}} \qquad \theta_A = 10\ \text{deg} \qquad e = 0.5$

Given $\quad v_{A1}0 = v_{A2}\sin(\theta_A) \qquad v_{B1}\left(\frac{c}{\sqrt{c^2+d^2}}\right) = v_{B2}\cos(\theta_B)$

$$M v_{B1}\left(\frac{d}{\sqrt{c^2+d^2}}\right) - M v_{A1} = M v_{A2}\cos(\theta_A) - M v_{B2}\sin(\theta_B)$$

$$e\left[v_{A1} + v_{B1}\left(\frac{d}{\sqrt{c^2+d^2}}\right)\right] = v_{A2}\cos(\theta_A) + v_{B2}\sin(\theta_B)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \theta_A \\ e \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}, \theta_A, e) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -1.75 \\ 3.70 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad e = 0.0113$$

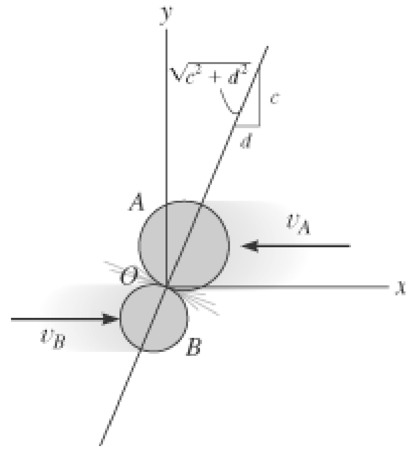
Problem 15-87

Two smooth disks A and B have the initial velocities shown just before they collide at O. If they have masses m_A and m_B, determine their speeds just after impact. The coefficient of restitution is e.

Given:

$v_A = 7\ \frac{m}{s}$ $m_A = 8\ kg$ $c = 12$ $e = 0.5$

$v_B = 3\ \frac{m}{s}$ $m_B = 6\ kg$ $d = 5$



Solution: $\theta = \operatorname{atan}\left(\frac{d}{c}\right)$ $\theta = 22.62\ deg$

Guesses $v_{A2t} = 1\ \frac{m}{s}$ $v_{A2n} = 1\ \frac{m}{s}$

$v_{B2t} = 1\ \frac{m}{s}$ $v_{B2n} = 1\ \frac{m}{s}$

Given $v_B \cos(\theta) = v_{B2t}$ $-v_A \cos(\theta) = v_{A2t}$

$$m_B v_B \sin(\theta) - m_A v_A \sin(\theta) = m_B v_{B2n} + m_A v_{A2n}$$

$$e(v_B + v_A)\sin(\theta) = v_{A2n} - v_{B2n}$$

$$\begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \operatorname{Find}(v_{A2t}, v_{A2n}, v_{B2t}, v_{B2n}) \qquad \begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \begin{pmatrix} -6.46 \\ -0.22 \\ 2.77 \\ -2.14 \end{pmatrix} \frac{m}{s}$$

$v_{A2} = \sqrt{v_{A2t}^2 + v_{A2n}^2}$ $v_{A2} = 6.47\ \frac{m}{s}$

$v_{B2} = \sqrt{v_{B2t}^2 + v_{B2n}^2}$ $v_{B2} = 3.50\ \frac{m}{s}$

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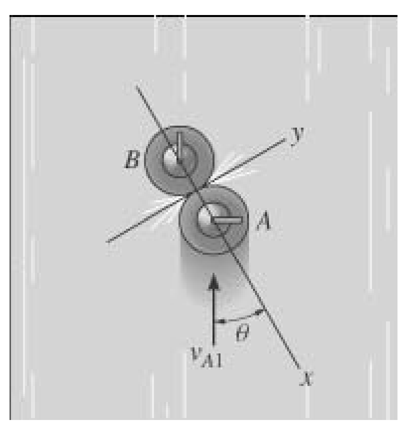
*__Problem 15-88__

The “stone” A used in the sport of curling slides over the ice track and strikes another “stone” B as shown. If each “stone” is smooth and has weight W, and the coefficient of restitution between the “stones” is e, determine their speeds just after collision. Initially A has velocity v_{A1} and B is at rest. Neglect friction.



Given: $W = 47\ lb$ $v_{A1} = 8\ \frac{ft}{s}$

$e = 0.8$ $\theta = 30\ deg$

Solution:

Guesses $v_{A2t} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{A2n} = 1\ \frac{\text{ft}}{\text{s}}$

$v_{B2t} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2n} = 1\ \frac{\text{ft}}{\text{s}}$

Given $v_{A1}\sin(\theta) = v_{A2t}$ $0 = v_{B2t}$

$v_{A1}\cos(\theta) = v_{A2n} + v_{B2n}$

$e\,v_{A1}\cos(\theta) = v_{B2n} - v_{A2n}$

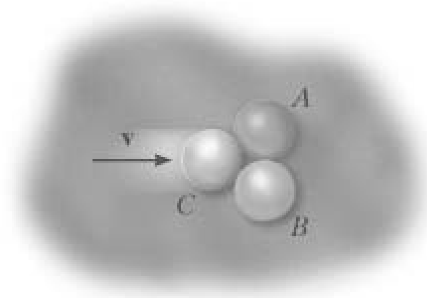
$$\begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \text{Find}(v_{A2t}, v_{A2n}, v_{B2t}, v_{B2n}) \qquad \begin{pmatrix} v_{A2t} \\ v_{A2n} \\ v_{B2t} \\ v_{B2n} \end{pmatrix} = \begin{pmatrix} 4.00 \\ 0.69 \\ 0.00 \\ 6.24 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$v_{A2} = \sqrt{v_{A2t}^2 + v_{A2n}^2}$ $v_{A2} = 4.06\ \frac{\text{ft}}{\text{s}}$

$v_{B2} = \sqrt{v_{B2t}^2 + v_{B2n}^2}$ $v_{B2} = 6.24\ \frac{\text{ft}}{\text{s}}$

Problem 15-89

The two billiard balls A and B are originally in contact with one another when a third ball C strikes each of them at the same time as shown. If ball C remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

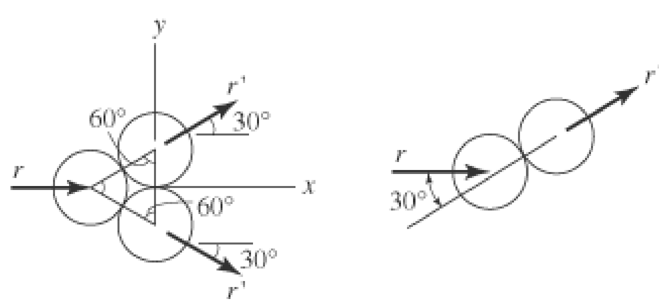


Solution:

Conservation of "x" momentum:

$$m\,v = 2m\,v'\cos(30\ \text{deg})$$

$$v = 2v'\cos(30\ \text{deg}) \qquad (1)$$



Coefficient of restitution:

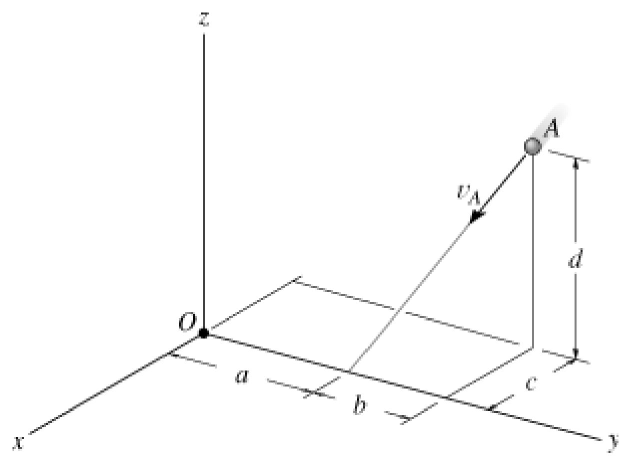
$$e = \frac{v'}{v\cos(30\ \text{deg})} \qquad (2)$$

Substituiting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v' \cos(30\ \text{deg})^2} \qquad e = \frac{2}{3}$$

Problem 15-90

Determine the angular momentum of particle A of weight W about point O. Use a Cartesian vector solution.



Given:

$W = 2\ \text{lb}$ $\quad a = 3\ \text{ft}$

$v_A = 12\ \frac{\text{ft}}{\text{s}}$ $\quad b = 2\ \text{ft}$

$c = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ $\quad d = 4\ \text{ft}$

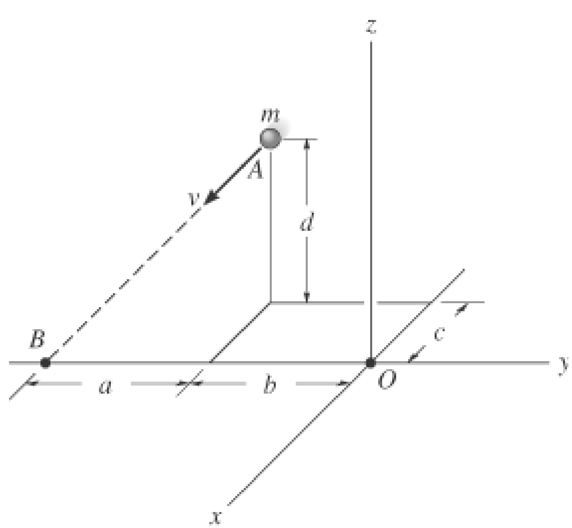
Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -c \\ a+b \\ d \end{pmatrix} \qquad \mathbf{r_v} = \begin{pmatrix} c \\ -b \\ -d \end{pmatrix} \qquad \mathbf{v_{Av}} = v_A \frac{\mathbf{r_v}}{|\mathbf{r_v}|}$$

$$\mathbf{H_O} = \mathbf{r_{OA}} \times (W\mathbf{v_{Av}}) \qquad \mathbf{H_O} = \begin{pmatrix} -1.827 \\ 0.000 \\ -0.914 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

Problem 15-91

Determine the angular momentum $\mathbf{H_O}$ of the particle about point O.



Given:

$M = 1.5\ \text{kg}$

$v = 6\ \frac{\text{m}}{\text{s}}$

$a = 4\ \text{m}$

$b = 3\ \text{m}$

$c = 2\ \text{m}$

$d = 4\ \text{m}$

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -c \\ -b \\ d \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} c \\ -a \\ -d \end{pmatrix} \qquad \mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|}$$

$$\mathbf{H_O} = \mathbf{r_{OA}} \times (M\mathbf{v_A}) \qquad \mathbf{H_O} = \begin{pmatrix} 42.0 \\ 0.0 \\ 21.0 \end{pmatrix} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

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*Problem 15-92

Determine the angular momentum $\mathbf{H_O}$ of each of the particles about point O.

Given: $\theta = 30 \text{ deg}$ $\quad \phi = 60 \text{ deg}$

$m_A = 6 \text{ kg}$ $\quad c = 2 \text{ m}$

$m_B = 4 \text{ kg}$ $\quad d = 5 \text{ m}$

$m_C = 2 \text{ kg}$ $\quad e = 2 \text{ m}$

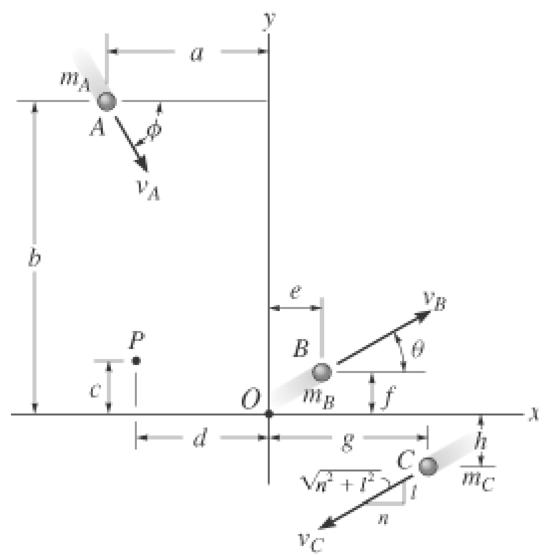
$v_A = 4 \frac{\text{m}}{\text{s}}$ $\quad f = 1.5 \text{ m}$

$v_B = 6 \frac{\text{m}}{\text{s}}$ $\quad g = 6 \text{ m}$

$v_C = 2.6 \frac{\text{m}}{\text{s}}$ $\quad h = 2 \text{ m}$

$a = 8 \text{ m}$ $\quad l = 5$

$b = 12 \text{ m}$ $\quad n = 12$



Solution:

$$\mathbf{H_{AO}} = a\,m_A v_A \sin(\phi) - b\,m_A v_A \cos(\phi) \qquad \mathbf{H_{AO}} = 22.3 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{BO}} = -f\,m_B v_B \cos(\theta) + e\,m_B v_B \sin(\theta) \qquad \mathbf{H_{BO}} = -7.18 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{CO}} = -h\,m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C - g\,m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C \qquad \mathbf{H_{CO}} = -21.60 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-93

Determine the angular momentum $\mathbf{H_P}$ of each of the particles about point P.

Given: $\theta = 30\ \text{deg}$ $\phi = 60\ \text{deg}$ $a = 8\ \text{m}$ $f = 1.5\ \text{m}$

$m_A = 6\ \text{kg}$ $v_A = 4\ \frac{\text{m}}{\text{s}}$ $b = 12\ \text{m}$ $g = 6\ \text{m}$

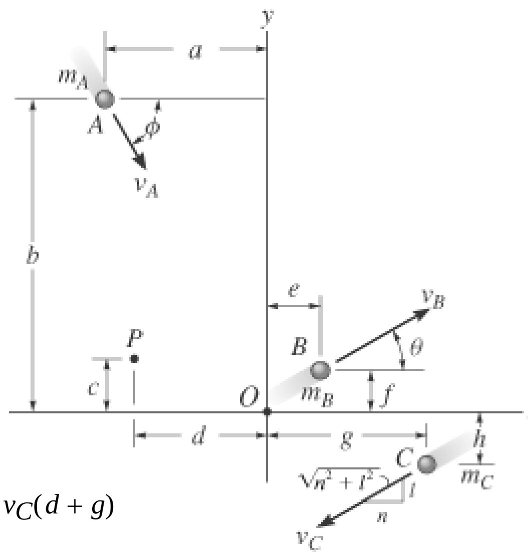
$c = 2\ \text{m}$ $h = 2\ \text{m}$

$m_B = 4\ \text{kg}$ $v_B = 6\ \frac{\text{m}}{\text{s}}$

$d = 5\ \text{m}$ $l = 5$

$m_C = 2\ \text{kg}$ $v_C = 2.6\ \frac{\text{m}}{\text{s}}$ $e = 2\ \text{m}$ $n = 12$

Solution:



$$\mathbf{H_{AP}} = m_A v_A \sin(\phi)(a - d) - m_A v_A \cos(\phi)(b - c)$$

$$\boxed{\mathbf{H_{AP}} = -57.6\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

$$\mathbf{H_{BP}} = m_B v_B \cos(\theta)(c - f) + m_B v_B \sin(\theta)(d + e)$$

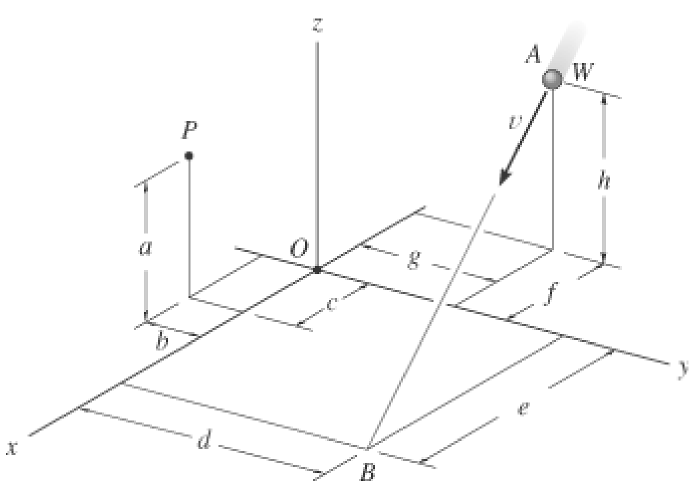
$$\boxed{\mathbf{H_{BP}} = 94.4\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

$$\mathbf{H_{CP}} = -m_C\left(\frac{n}{\sqrt{l^2 + n^2}}\right)v_C(c + h) - m_C\left(\frac{l}{\sqrt{l^2 + n^2}}\right)v_C(d + g)$$

$$\boxed{\mathbf{H_{CP}} = -41.2\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

Problem 15-94

Determine the angular momentum $\mathbf{H_O}$ of the particle about point O.



Given:

$W = 10\ \text{lb}$ $d = 9\ \text{ft}$

$v = 14\ \frac{\text{ft}}{\text{s}}$ $e = 8\ \text{ft}$

$a = 5\ \text{ft}$ $f = 4\ \text{ft}$

$b = 2\ \text{ft}$ $g = 5\ \text{ft}$

$c = 3\ \text{ft}$ $h = 6\ \text{ft}$

Solution:

$$\mathbf{r_{OA}} = \begin{pmatrix} -f \\ g \\ h \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} f+e \\ d-g \\ -h \end{pmatrix}$$

$$\mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{H_O} = \mathbf{r_{OA}} \times (W\mathbf{v_A}) \qquad \mathbf{H_O} = \begin{pmatrix} -16.78 \\ 14.92 \\ -23.62 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

Problem 15-95

Determine the angular momentum $\mathbf{H_P}$ of the particle about point P.

Given:

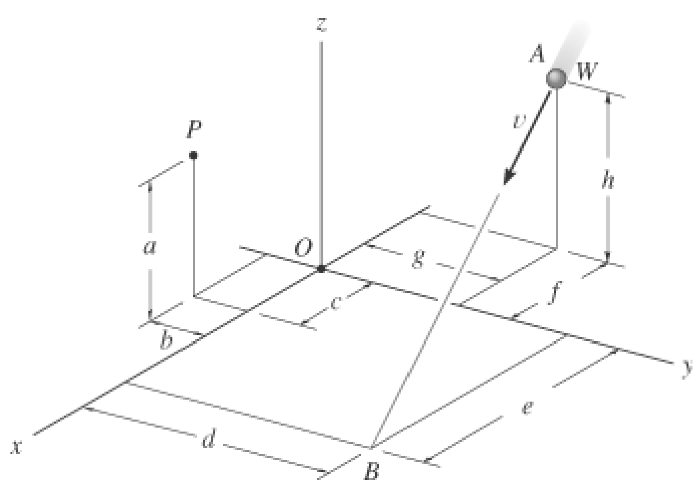
$W = 10\text{ lb}$ $\quad d = 9\text{ ft}$

$v = 14\frac{\text{ft}}{\text{s}}$ $\quad e = 8\text{ ft}$

$a = 5\text{ ft}$ $\quad f = 4\text{ ft}$

$b = 2\text{ ft}$ $\quad g = 5\text{ ft}$

$c = 3\text{ ft}$ $\quad h = 6\text{ ft}$



Solution:

$$\mathbf{r_{PA}} = \begin{pmatrix} -f-c \\ b+g \\ h-a \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} f+e \\ d-g \\ -h \end{pmatrix}$$

$$\mathbf{v_A} = v\frac{\mathbf{r_{AB}}}{|\mathbf{r_{AB}}|} \qquad \mathbf{H_P} = \mathbf{r_{PA}} \times (W\mathbf{v_A}) \qquad \mathbf{H_P} = \begin{pmatrix} -14.30 \\ -9.32 \\ -34.81 \end{pmatrix} \text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

*__Problem 15-96__

Determine the total angular momentum $\mathbf{H_O}$ for the system of three particles about point O. All the particles are moving in the x-y plane.

Given:

$m_A = 1.5\text{ kg}$ $\quad a = 900\text{ mm}$

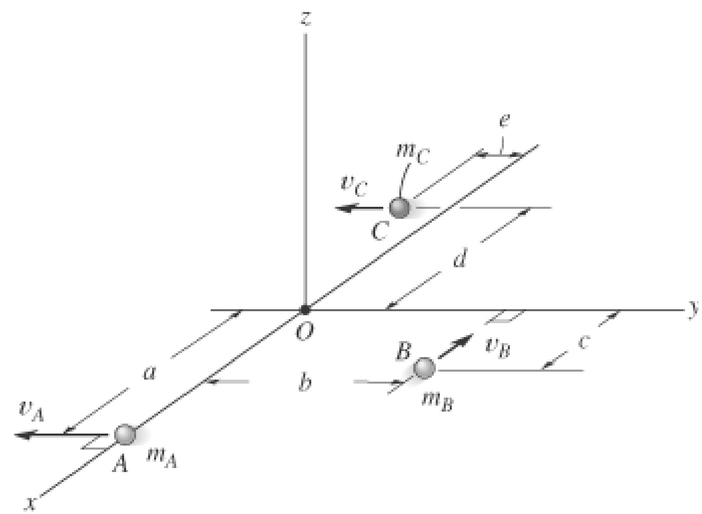
$v_A = 4\ \frac{m}{s}$ $b = 700$ mm

$m_B = 2.5$ kg $c = 600$ mm

$v_B = 2\ \frac{m}{s}$ $d = 800$ mm

$m_C = 3$ kg $e = 200$ mm

$v_C = 6\ \frac{m}{s}$



Solution:

$$\mathbf{H_O} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} \times \left[m_A \begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} \right] + \begin{pmatrix} c \\ b \\ 0 \end{pmatrix} \times \left[m_B \begin{pmatrix} -v_B \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} -d \\ -e \\ 0 \end{pmatrix} \times \left[m_C \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix} \right]$$

$$\mathbf{H_O} = \begin{pmatrix} 0.00 \\ 0.00 \\ 12.50 \end{pmatrix} \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-97

Determine the angular momentum $\mathbf{H_O}$ of each of the two particles about point O. Use a scalar solution.

Given:

$m_A = 2$ kg $c = 1.5$ m

$m_B = 1.5$ kg $d = 2$ m

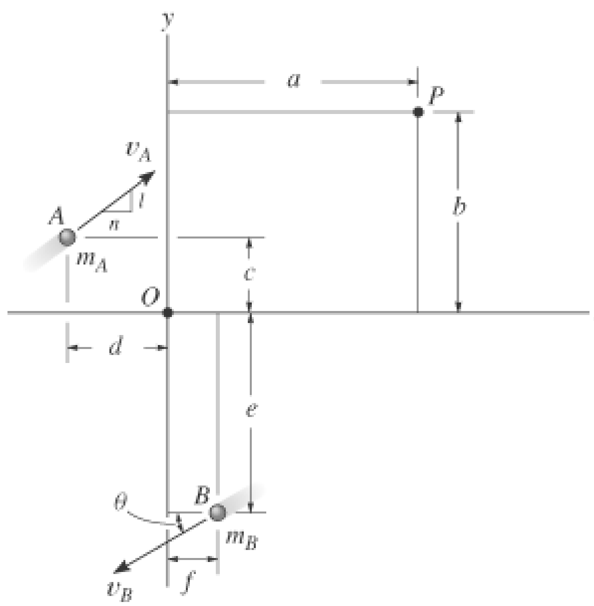
$v_A = 15\ \frac{m}{s}$ $e = 4$ m

$f = 1$ m

$v_B = 10\ \frac{m}{s}$ $\theta = 30$ deg

$a = 5$ m $l = 3$

$b = 4$ m $n = 4$



Solution:

$$\mathbf{H_{OA}} = -m_A\left(\frac{n}{\sqrt{n^2+l^2}}\right)v_A c - m_A\left(\frac{l}{\sqrt{n^2+l^2}}\right)v_A d \qquad \mathbf{H_{OA}} = -72.0\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{OB}} = -m_B v_B \cos(\theta) e - m_B v_B \sin(\theta) f \qquad \mathbf{H_{OB}} = -59.5\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-98

Determine the angular momentum $\mathbf{H_P}$ of each of the two particles about point P. Use a scalar solution.

Given:

$m_A = 2$ kg $\quad c = 1.5$ m

$m_B = 1.5$ kg $\quad d = 2$ m

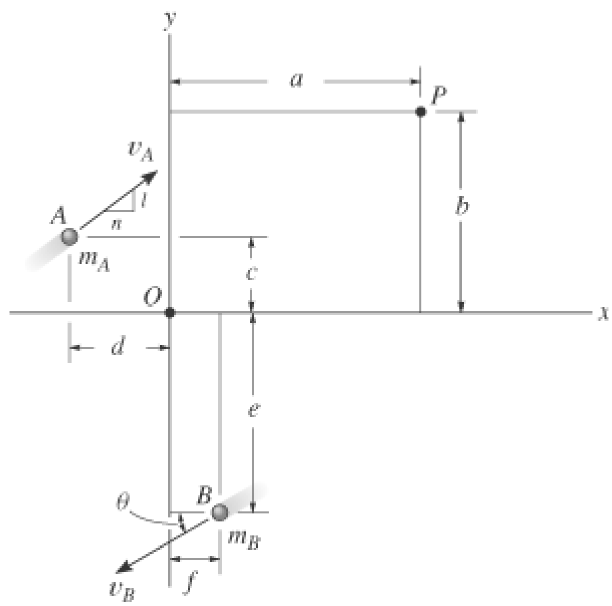
$v_A = 15\,\frac{\text{m}}{\text{s}}$ $\quad e = 4$ m

$v_B = 10\,\frac{\text{m}}{\text{s}}$ $\quad f = 1$ m

$a = 5$ m $\quad \theta = 30$ deg

$b = 4$ m $\quad l = 3$

$n = 4$



Solution:

$$\mathbf{H_{PA}} = m_A\frac{n}{\sqrt{n^2+l^2}}v_A(b-c) - m_A\frac{l}{\sqrt{n^2+l^2}}v_A(a+d) \qquad \mathbf{H_{PA}} = -66.0\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\mathbf{H_{PB}} = -m_B v_B \cos(\theta)(b+e) + m_B v_B \sin(\theta)(a-f) \qquad \mathbf{H_{PB}} = -73.9\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 15-99

The ball B has mass M and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = at^2 + bt + c$, determine the speed of the ball when $t = t_1$. The ball has a speed $v = v_0$ when $t = 0$.

Given:

$M = 10\ \text{kg}$

$a = 3\ \dfrac{\text{N}\cdot\text{m}}{\text{s}^2}$

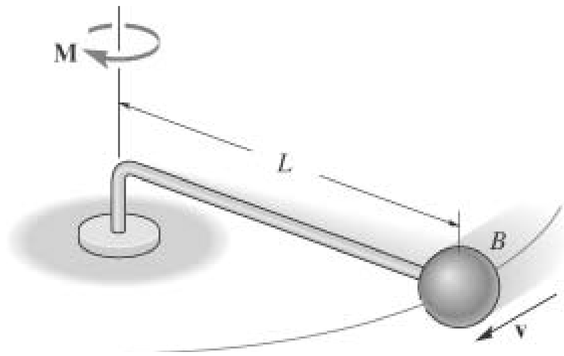
$b = 5\ \dfrac{\text{N}\cdot\text{m}}{\text{s}}$

$c = 2\ \text{N}\cdot\text{m}$

$t_1 = 2\ \text{s}$

$v_0 = 2\ \dfrac{\text{m}}{\text{s}}$

$L = 1.5\ \text{m}$



Solution: Principle of angular impulse momentum

$$M v_0 L + \int_0^{t_1} a t^2 + b t + c\ \mathrm{d}t = M v_1 L$$

$$v_1 = v_0 + \frac{1}{M L}\int_0^{t_1} a t^2 + b t + c\ \mathrm{d}t \qquad v_1 = 3.47\ \frac{\text{m}}{\text{s}}$$

***Problem 15-100**

The two blocks A and B each have a mass M_0. The blocks are fixed to the horizontal rods, and their initial velocity is v' in the direction shown. If a couple moment of M is applied about shaft CD of the frame, determine the speed of the blocks at time t. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.

Given:

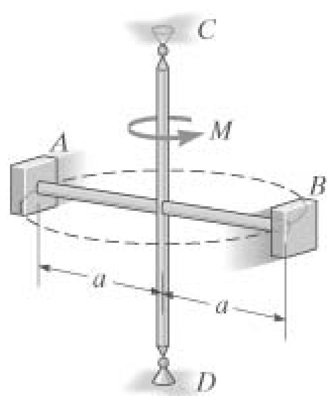
$M_0 = 0.4\ \text{kg}$

$a = 0.3\ \text{m}$

$v' = 2\ \dfrac{\text{m}}{\text{s}}$

$M = 0.6\ \text{N}\cdot\text{m}$

$t = 3\ \text{s}$



Solution:

$$2 a M_0 v' + M t = 2 a M_0 v$$

$$v = v' + \frac{M t}{2a M_0} \qquad v = 9.50 \frac{\text{m}}{\text{s}}$$

Problem 15-101

The small cylinder C has mass m_C and is attached to the end of a rod whose mass may be neglected. If the frame is subjected to a couple $M = at^2 + b$, and the cylinder is subjected to force F, which is always directed as shown, determine the speed of the cylinder when $t = t_1$. The cylinder has a speed v_0 when $t = 0$.

Given:

$m_C = 10$ kg $\qquad t_1 = 2$ s

$a = 8 \text{ N} \frac{\text{m}}{\text{s}^2} \qquad v_0 = 2 \frac{\text{m}}{\text{s}}$

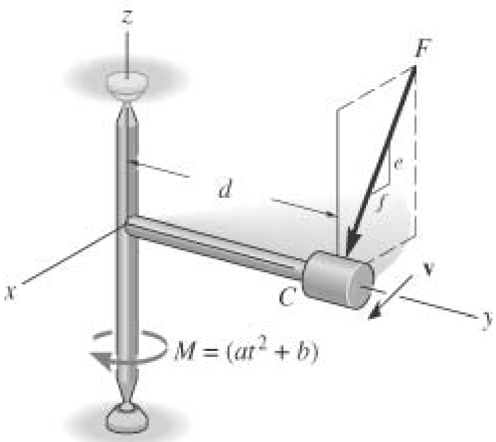
$d = 0.75$ m

$b = 5$ N·m

$e = 4$

$F = 60$ N

$f = 3$



Solution:

$$m_C v_0 d + \int_0^{t_1} a t^2 + b \, dt + \left(\frac{f}{\sqrt{e^2 + f^2}}\right) F d t_1 = m_C v_1 d$$

$$v_1 = v_0 + \frac{1}{m_C d}\left[\int_0^{t_1} a t^2 + b \, dt + \left(\frac{f}{\sqrt{e^2 + f^2}}\right) F d t_1\right] \qquad v_1 = 13.38 \frac{\text{m}}{\text{s}}$$

Problem 15-102

A box having a weight W is moving around in a circle of radius r_A with a speed v_{A1} while connected to the end of a rope. If the rope is pulled inward with a constant speed v_r, determine the speed of the box at the instant $r = r_B$. How much work is done after pulling in the rope from A to B? Neglect friction and the size of the box.

Given:

$W = 8$ lb

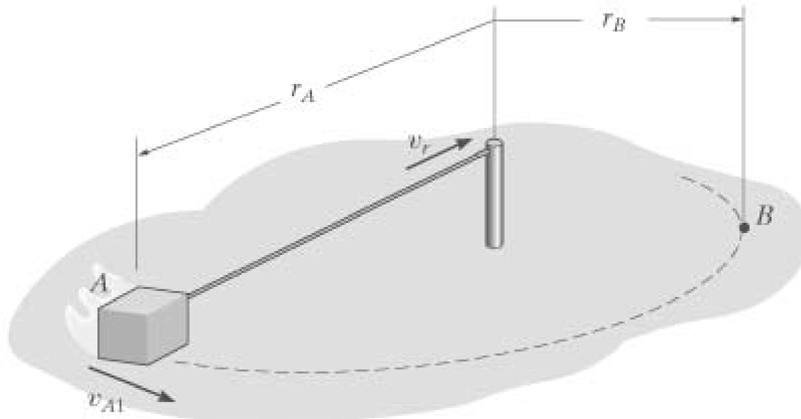
$r_A = 2$ ft

$v_{A1} = 5\ \frac{ft}{s}$

$v_r = 4\ \frac{ft}{s}$

$r_B = 1\ ft$

$g = 32.21\ \frac{ft}{s^2}$



Solution:

$$\left(\frac{W}{g}\right) r_A v_{A1} = \left(\frac{W}{g}\right) r_B v_{Btangent}$$

$$v_{Btangent} = r_A \left(\frac{v_{A1}}{r_B}\right) \qquad v_{Btangent} = 10.00\ \frac{ft}{s}$$

$$v_B = \sqrt{v_{Btangent}^2 + v_r^2} \qquad v_B = 10.8\ \frac{ft}{s}$$

$$U_{AB} = \frac{1}{2}\left(\frac{W}{g}\right) v_B^2 - \frac{1}{2}\left(\frac{W}{g}\right) v_{A1}^2 \qquad U_{AB} = 11.3\ ft \cdot lb$$

Problem 15-103

An earth satellite of mass M is launched into a free-flight trajectory about the earth with initial speed v_A when the distance from the center of the earth is r_A. If the launch angle at this position is ϕ_A determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass M_e. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, **F**, Eq. 13-1. For part of the solution, use the conservation of energy.

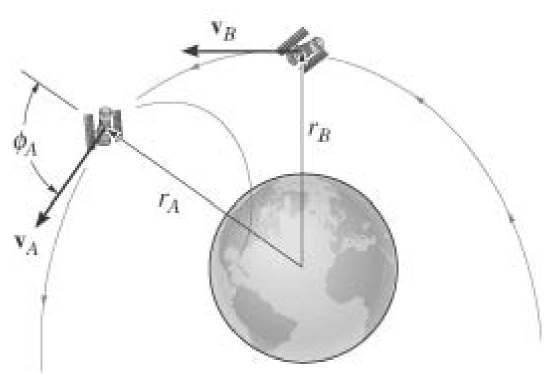
Units used: $\quad Mm = 10^3\ km$

Given:

$M = 700\ kg \qquad \phi_A = 70\ deg$

$v_A = 10\ \frac{km}{s} \qquad G = 6.673 \times 10^{-11}\ \frac{N \cdot m^2}{kg^2}$

$r_A = 15\ Mm \qquad M_e = 5.976 \times 10^{24}\ kg$



Solution: Guesses $v_B = 10\ \frac{\text{km}}{\text{s}}$ $r_B = 10\ \text{Mm}$

Given $M v_A \sin(\phi_A) r_A = M v_B r_B$

$$\frac{1}{2} M v_A^2 - \frac{G M_e M}{r_A} = \frac{1}{2} M v_B^2 - \frac{G M_e M}{r_B}$$

$$\begin{pmatrix} v_B \\ r_B \end{pmatrix} = \text{Find}(v_B, r_B) \qquad v_B = 10.2\ \frac{\text{km}}{\text{s}} \qquad r_B = 13.8\ \text{Mm}$$

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*__Problem 15-104__

The ball B has weight W and is originally rotating in a circle. As shown, the cord AB has a length of L and passes through the hole A, which is a distance h above the plane of motion. If $L/2$ of the cord is pulled through the hole, determine the speed of the ball when it moves in a circular path at C.

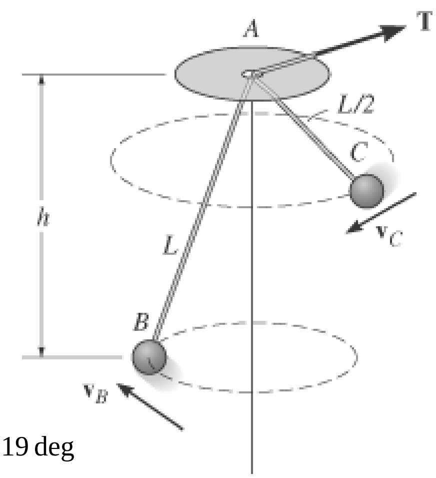
Given:

$W = 5\ \text{lb}$

$L = 3\ \text{ft}$

$h = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution: $\theta_B = \text{acos}\left(\frac{h}{L}\right)$ $\theta_B = 48.19\ \text{deg}$

Guesses $T_B = 1\ \text{lb}$ $T_C = 1\ \text{lb}$ $v_B = 1\ \frac{\text{ft}}{\text{s}}$ $v_C = 1\ \frac{\text{ft}}{\text{s}}$ $\theta_C = 10\ \text{deg}$

Given $T_B \cos(\theta_B) - W = 0$ $\qquad T_B \sin(\theta_B) = \frac{W}{g}\left(\frac{v_B^2}{L \sin(\theta_B)}\right)$

$$T_C \cos(\theta_C) - W = 0 \qquad T_C \sin(\theta_C) = \frac{W}{g}\left(\frac{v_C^2}{\frac{L}{2}\sin(\theta_C)}\right)$$

$$\left(\frac{W}{g}\right) v_B L \sin(\theta_B) = \left(\frac{W}{g}\right) v_C \left(\frac{L}{2}\right) \sin(\theta_C)$$

$$\begin{pmatrix} T_B \\ T_C \\ v_B \\ v_C \\ \theta_C \end{pmatrix} = \text{Find}(T_B, T_C, v_B, v_C, \theta_C) \qquad \begin{pmatrix} T_B \\ T_C \end{pmatrix} = \begin{pmatrix} 7.50 \\ 20.85 \end{pmatrix} \text{lb} \qquad \theta_C = 76.12\,\text{deg}$$

$$v_B = 8.97\,\frac{\text{ft}}{\text{s}} \qquad v_C = 13.78\,\frac{\text{ft}}{\text{s}}$$

Problem 15-105

The block of weight W rests on a surface for which the kinetic coefficient of friction is μ_k. It is acted upon by a radial force $\mathbf{F_R}$ and a horizontal force $\mathbf{F_H}$, always directed at angle θ from the tangent to the path as shown. If the block is initially moving in a circular path with a speed v_1 at the instant the forces are applied, determine the time required before the tension in cord AB becomes T. Neglect the size of the block for the calculation.

Given:

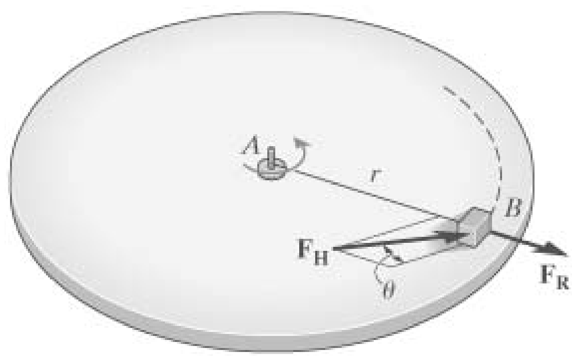
$W = 10\ \text{lb}$ $\mu_k = 0.5$

$F_R = 2\ \text{lb}$ $T = 20\ \text{lb}$

$F_H = 7\ \text{lb}$ $r = 4\ \text{ft}$

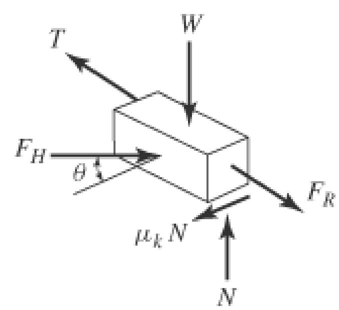
$v_1 = 2\ \frac{\text{ft}}{\text{s}}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta = 30\ \text{deg}$



Solution:

Guesses $t = 1\ \text{s}$ $v_2 = 1\ \frac{\text{ft}}{\text{s}}$



Given

$$\left(\frac{W}{g}\right)v_1 r + F_H\cos(\theta) r t - \mu_k W r t = \left(\frac{W}{g}\right)v_2 r$$

$$F_R + F_H\sin(\theta) - T = -\frac{W}{g}\left(\frac{v_2^{\,2}}{r}\right)$$

$$\begin{pmatrix} t \\ v_2 \end{pmatrix} = \text{Find}(t, v_2) \qquad v_2 = 13.67\,\frac{\text{ft}}{\text{s}} \qquad t = 3.41\ \text{s}$$

Problem 15-106

The block of weight W is originally at rest on the smooth surface. It is acted upon by a radial force $\mathbf{F_R}$ and a horizontal force $\mathbf{F_H}$, always directed at θ from the tangent to the path as shown. Determine the time required to break the cord, which requires a tension T. What is the speed of the block when this occurs? Neglect the size of the block for the calculation.

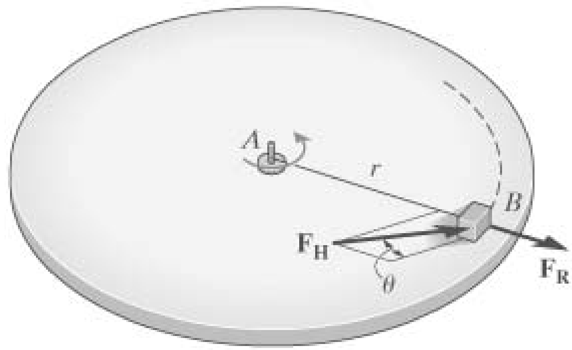
Given:

$W = 10\ \text{lb}$ $\quad \theta = 30\ \text{deg}$

$F_R = 2\ \text{lb}$ $\quad T = 30\ \text{lb}$

$F_H = 7\ \text{lb}$ $\quad r = 4\ \text{ft}$

$v_1 = 0\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

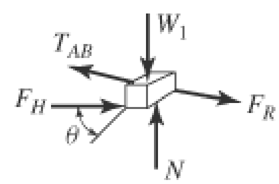


Solution:

Guesses $\quad t = 1\ \text{s} \quad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

Given

$$\left(\frac{W}{g}\right)v_1 r + F_H\cos(\theta) r t = \left(\frac{W}{g}\right)v_2 r$$

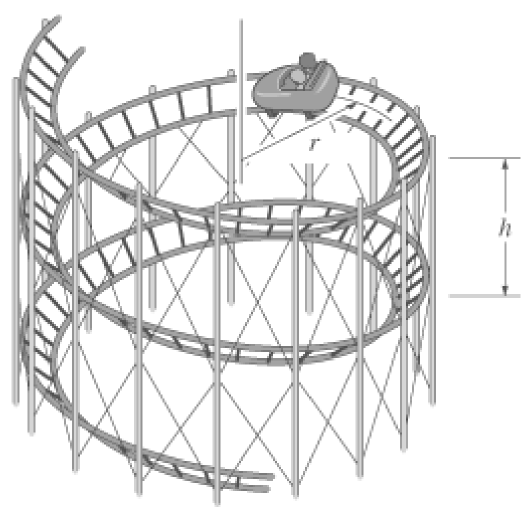


$$F_R + F_H\sin(\theta) - T = -\frac{W}{g}\left(\frac{v_2^{\,2}}{r}\right)$$

$$\begin{pmatrix} t \\ v_2 \end{pmatrix} = \text{Find}(t, v_2) \qquad v_2 = 17.76\ \frac{\text{ft}}{\text{s}} \qquad t = 0.91\ \text{s}$$

Problem 15-107

The roller-coaster car of weight W starts from rest on the track having the shape of a cylindrical helix. If the helix descends a distance h for every one revolution, determine the time required for the car to attain a speed v. Neglect friction and the size of the car.



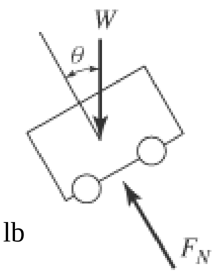
Given:

$W = 800\ \text{lb}$

$h = 8\ \text{ft}$

$v = 60\ \frac{\text{ft}}{\text{s}}$

$r = 8\ \text{ft}$



Solution:

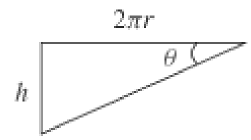
$\theta = \operatorname{atan}\left(\frac{h}{2\pi r}\right) \qquad \theta = 9.04\,\text{deg}$

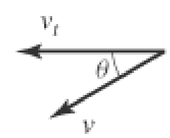
$F_N - W\cos(\theta) = 0 \qquad F_N = W\cos(\theta) \qquad F_N = 790.06\,\text{lb}$

$v_t = v\cos(\theta) \qquad v_t = 59.25\,\frac{\text{ft}}{\text{s}}$

$H_A + \int M\,\text{d}t = H_2 \qquad \int_0^t F_N\sin(\theta) r\,\text{d}t = \left(\frac{W}{g}\right) h\,v_t \qquad F_N\sin(\theta) r\,t = \left(\frac{W}{g}\right) h\,v_t$

$t = W\left(\frac{v_t h}{F_N\sin(\theta) g\,r}\right) \qquad t = 11.9\ \text{s}$





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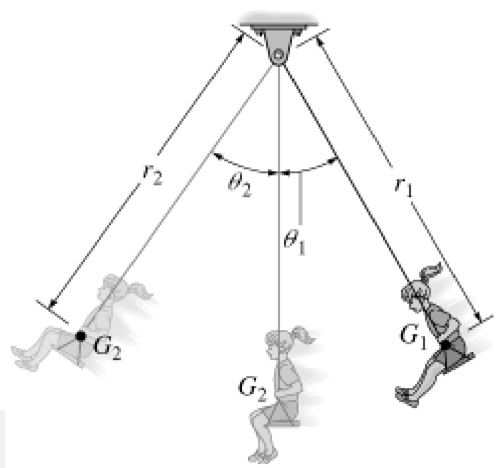
*__Problem 15-108__

A child having mass M holds her legs up as shown as she swings downward from rest at θ_1. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.

Given:

$M = 50\ \text{kg} \qquad r_1 = 2.80\ \text{m} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$\theta_1 = 30\ \text{deg} \qquad r_2 = 3\ \text{m}$



Solution:

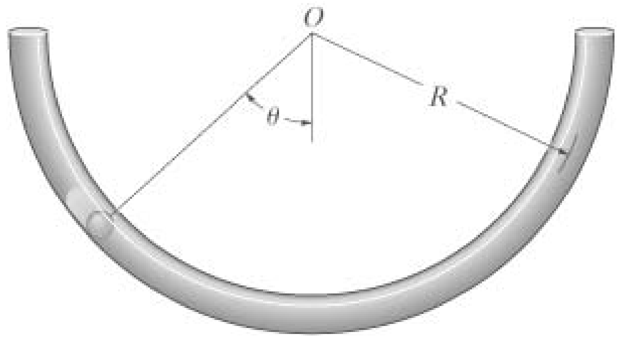
$v_{2b} = \sqrt{2g\,r_1(1 - \cos(\theta_1))} \qquad v_{2b} = 2.71\ \frac{\text{m}}{\text{s}}$

$r_1 v_{2b} = r_2 v_{2a} \qquad v_{2a} = \frac{r_1}{r_2} v_{2b} \qquad v_{2a} = 2.53\ \frac{\text{m}}{\text{s}}$

$\theta_2 = \operatorname{acos}\left(1 - \frac{v_{2a}^2}{2g\,r_2}\right) \qquad \theta_2 = 27.0\,\text{deg}$

Problem 15-109

A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O ($\Sigma M_0 = H_0$), and show that the motion of the particle is governed by the differential equation $\theta'' + (g / R) \sin \theta = 0$.



Solution:

$$\Sigma M_0 = \frac{d}{dt} H_0$$

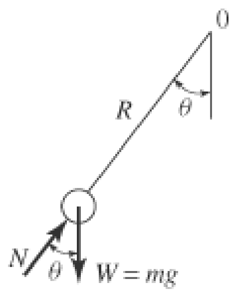
$$-R\, m\, g \sin(\theta) = \frac{d}{dt}(m\, v\, R)$$

$$g \sin(\theta) = -\frac{d}{dt} v = -\frac{d^2}{dt^2} s$$

But, $s = R\theta$

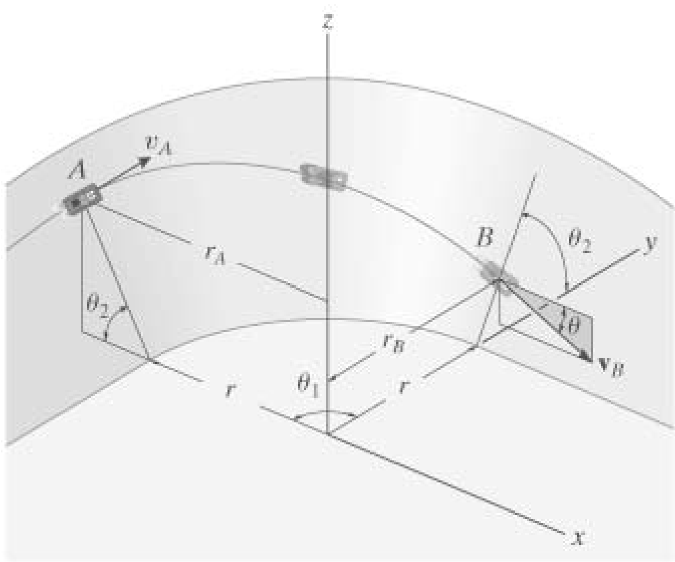
Thus, $g \sin(\theta) = -R\, \theta''$

or, $$\theta'' + \left(\frac{g}{R}\right) \sin(\theta) = 0$$



Problem 15-110

A toboggan and rider, having a total mass M, enter horizontally tangent to a circular curve (θ_1) with a velocity v_A. If the track is flat and banked at angle θ_2, determine the speed v_B and the angle θ of "descent", measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



Given:

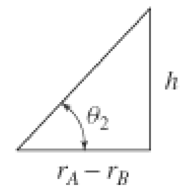
$M = 150$ kg $\quad \theta_1 = 90$ deg $\quad v_A = 70 \dfrac{\text{km}}{\text{hr}} \quad \theta_2 = 60$ deg

$r_A = 60$ m $\quad r_B = 57$ m $\quad r = 55$ m

Solution:

$$h = (r_A - r_B)\tan(\theta_2)$$

Guesses $v_B = 10\ \frac{\text{m}}{\text{s}}$ $\theta = 1\ \text{deg}$



Given $\frac{1}{2}M v_A^2 + M g h = \frac{1}{2}M v_B^2$ $M v_A r_A = M v_B \cos(\theta) r_B$

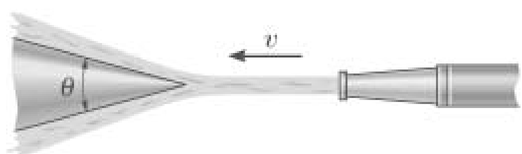
$\begin{pmatrix} v_B \\ \theta \end{pmatrix} = \text{Find}(v_B, \theta)$ $v_B = 21.9\ \frac{\text{m}}{\text{s}}$ $\theta = -1.1 \times 10^3\ \text{deg}$

Problem 15-111

Water is discharged at speed *v* against the fixed cone diffuser. If the opening diameter of the nozzle is *d*, determine the horizontal force exerted by the water on the diffuser.

Units Used:

$$\text{Mg} = 10^3\ \text{kg}$$



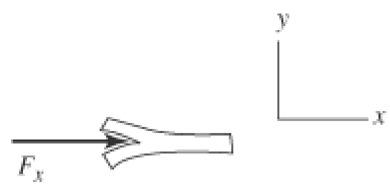
Given:

$v = 16\ \frac{\text{m}}{\text{s}}$ $\theta = 30\ \text{deg}$

$d = 40\ \text{mm}$ $\rho_w = 1\ \frac{\text{Mg}}{\text{m}^3}$

Solution:

$Q = \frac{\pi}{4}d^2 v$ $m' = \rho_w Q$



$$F_x = m'\left(-v\cos\left(\frac{\theta}{2}\right) + v\right)$$

$$F_x = 11.0\ \text{N}$$

***Problem 15-112**

A jet of water having cross-sectional area *A* strikes the fixed blade with speed *v*. Determine the horizontal and vertical components of force which the blade exerts on the water.

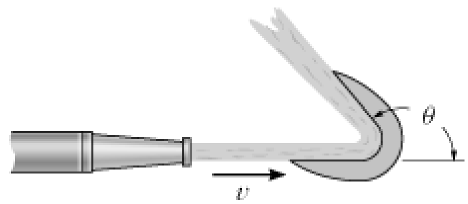
Given:

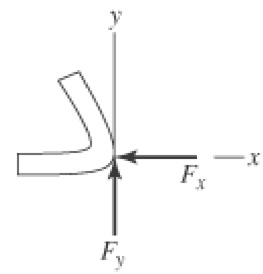
$$A = 4\ \text{in}^2$$

$v = 25\ \frac{\text{ft}}{\text{s}}$

$\theta = 130\ \text{deg}$

$\gamma_w = 62.4\ \frac{\text{lb}}{\text{ft}^3}$





Solution: $Q = A\,v \qquad Q = 0.69\,\frac{\text{ft}^3}{\text{s}}$

$$\frac{d}{dt}m = m' = \rho Q \qquad m' = \gamma_w Q \qquad m' = 1.3468\,\frac{\text{slug}}{\text{s}}$$

$$v_{Ax} = v \qquad v_{Ay} = 0\,\frac{\text{ft}}{\text{s}} \qquad v_{Bx} = v\cos(\theta) \qquad v_{By} = v\sin(\theta)$$

$$F_x = \frac{-m'}{g}\left(v_{Bx} - v_{Ax}\right) \qquad \boxed{F_x = 55.3\,\text{lb}}$$

$$F_y = \frac{m'}{g}\left(v_{By} - v_{Ay}\right) \qquad \boxed{F_y = 25.8\,\text{lb}}$$

Problem 15-113

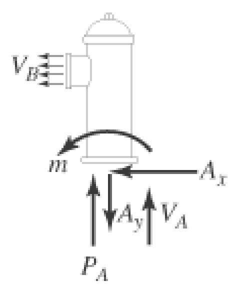
Water is flowing from the fire hydrant opening of diameter d_B with velocity v_B. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is P_A. The diameter of the fire hydrant at A is d_A.

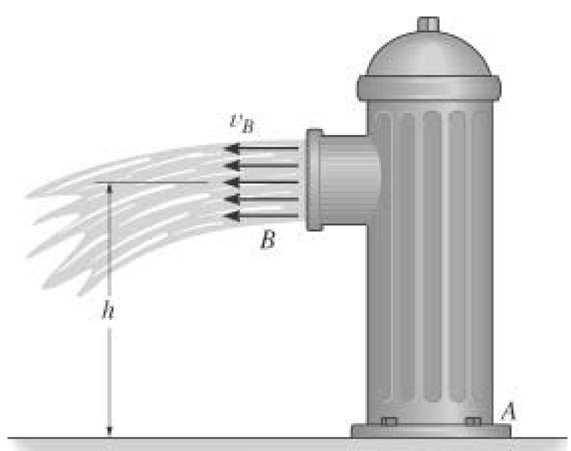
Units Used:

$\text{kPa} = 10^3\ \text{Pa}$

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$





Given:

$d_B = 150\ \text{mm} \qquad h = 500\ \text{mm}$

$v_B = 15\ \frac{\text{m}}{\text{s}} \qquad d_A = 200\ \text{mm}$

$P_A = 50\ \text{kPa} \qquad \rho_w = 1\ \frac{\text{Mg}}{\text{m}^3}$

Solution:

$$A_B = \pi\left(\frac{d_B}{2}\right)^2 \qquad A_A = \pi\left(\frac{d_A}{2}\right)^2 \qquad m' = \rho_w v_B \pi\left(\frac{d_B}{2}\right)^2 \qquad v_A = \frac{m'}{\rho_w A_A}$$

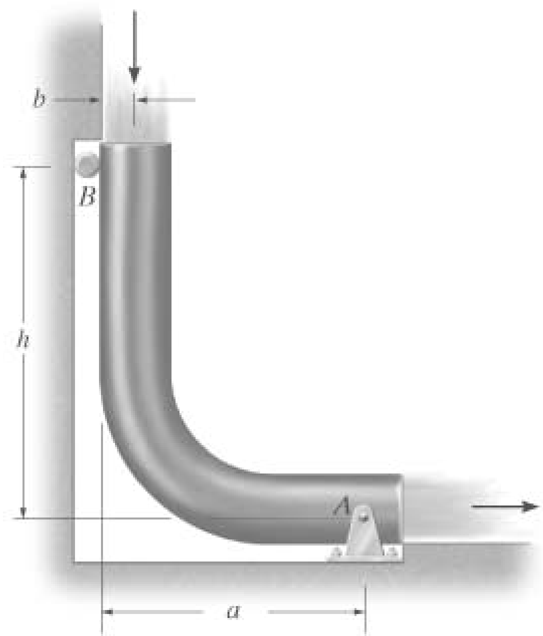
$A_x = m' v_B$ $\quad A_x = 3.98\,\text{kN}$

$-A_y + 50\pi\left(\frac{d_A}{2}\right)^2 = m'(0 - v_A)$ $\quad A_y = m' v_A + P_A\pi\left(\frac{d_A}{2}\right)^2$ $\quad A_y = 3.81\,\text{kN}$

$M = m' h v_B$ $\quad M = 1.99\,\text{kN}\cdot\text{m}$

Problem 15-114

The chute is used to divert the flow of water Q. If the water has a cross-sectional area A, determine the force components at the pin A and roller B necessary for equilibrium. Neglect both the weight of the chute and the weight of the water on the chute.



Units Used:

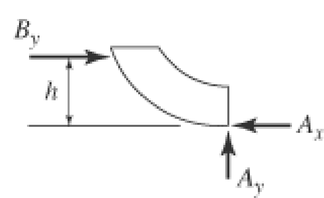
$\text{Mg} = 10^3\ \text{kg} \qquad \text{kN} = 10^3\ \text{N}$

Given:

$Q = 0.6\ \frac{\text{m}^3}{\text{s}} \qquad \rho_w = 1\ \frac{\text{Mg}}{\text{m}^3}$

$A = 0.05\ \text{m}^2 \qquad h = 2\ \text{m}$

$a = 1.5\ \text{m} \qquad b = 0.12\ \text{m}$



Solution:

$\frac{\text{d}}{\text{d}t} m = m' \qquad m' = \rho_w Q$

$v_A = \frac{Q}{A} \qquad v_B = v_A$

$\Sigma F_x = m'(v_{Ax} - v_{Bx}) \qquad B_x - A_x = m'(v_{Ax} - v_{Bx})$

$\Sigma F_y = m'(v_{Ay} - v_{By}) \qquad A_y = m'[0 - (-v_B)]$ $\quad A_y = 7.20\,\text{kN}$

$\Sigma M_A = m'(d_{0A} v_A - d_{0B} v_B) \qquad B_x = \frac{1}{h} m'[b v_A + (a - b) v_A]$ $\quad B_x = 5.40\,\text{kN}$

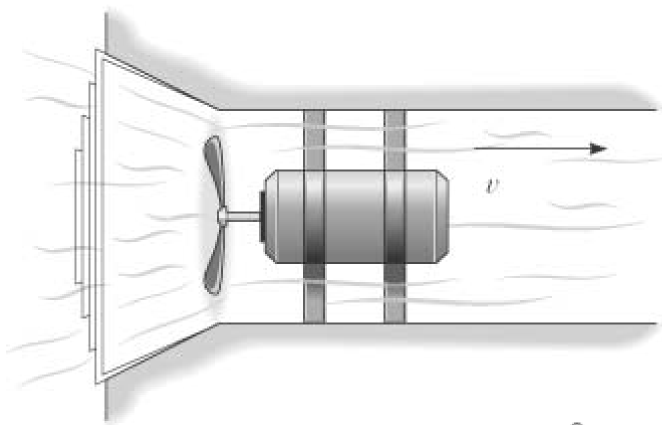
$A_x = B_x - m' v_A$ $\quad A_x = -1.80\,\text{kN}$

Problem 15-115

The fan draws air through a vent with speed v. If the cross-sectional area of the vent is A, determine the horizontal thrust on the blade. The specific weight of the air is γ_a.

Given:

$$v = 12\ \frac{\text{ft}}{\text{s}}$$

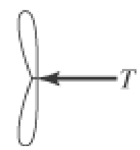


$$A = 2\ \text{ft}^2$$

$$\gamma_a = 0.076\ \frac{\text{lb}}{\text{ft}^3}$$

$$g = 32.20\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

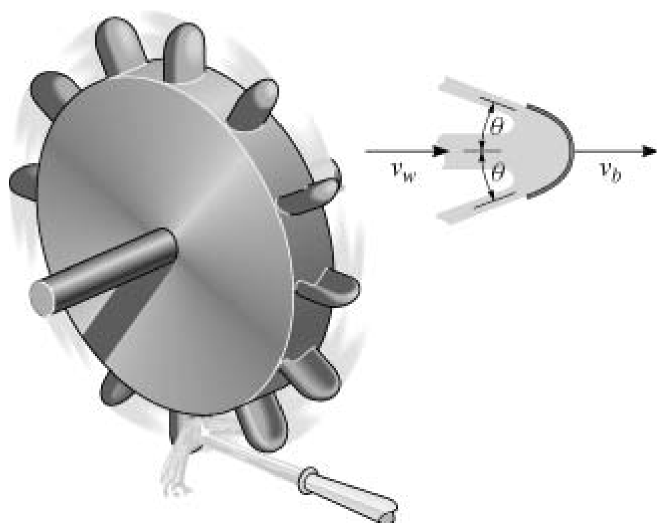


$$m' = \frac{d}{dt}m \qquad m' = \gamma_a v A \qquad m' = 0.05669\ \frac{\text{slug}}{\text{s}}$$

$$T = \frac{m'(v-0)}{g} \qquad \boxed{T = 0.68\ \text{lb}}$$

***Problem 15-116**

The buckets on the *Pelton wheel* are subjected to a jet of water of diameter d, which has velocity v_w. If each bucket is traveling at speed v_b when the water strikes it, determine the power developed by the wheel. The density of water is γ_w.



Given:

$$d = 2\ \text{in} \qquad \theta = 20\ \text{deg}$$

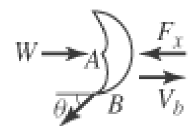
$$v_w = 150\ \frac{\text{ft}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$v_b = 95\ \frac{\text{ft}}{\text{s}}$$

$$\gamma_w = 62.4\ \frac{\text{lbf}}{\text{ft}^3}$$

Solution: $v_A = v_w - v_b \qquad v_A = 55\,\dfrac{\text{ft}}{\text{s}}$

$v_{Bx} = -v_A\cos(\theta) + v_b \qquad v_{Bx} = 43.317\,\dfrac{\text{ft}}{\text{s}}$



$\overset{+}{\longleftarrow}\quad \Sigma F_x = m'(v_{Bx} - v_{Ax})$

$$F_x = \left(\frac{\gamma_w}{g}\right)\pi\left(\frac{d^2}{4}\right)v_A\left[-v_{Bx} - (-v_A)\right] \qquad F_x = 266.41\,\frac{\text{m}}{\text{s}^2}\cdot\text{lb}$$

$P = F_x v_b \qquad P = 4.69\,\text{hp}$

Problem 15-117

The boat of mass M is powered by a fan F which develops a slipstream having a diameter d. If the fan ejects air with a speed v, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density ρ_a and that the entering air is essentially at rest. Neglect the drag resistance of the water.

Given:

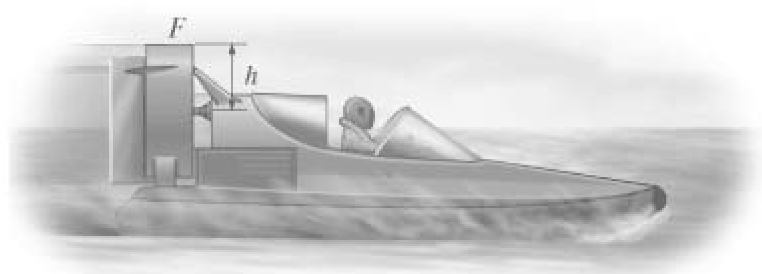
$M = 200\ \text{kg}$

$h = 0.375\ \text{m}$

$d = 0.75\ \text{m}$

$v = 14\ \dfrac{\text{m}}{\text{s}}$

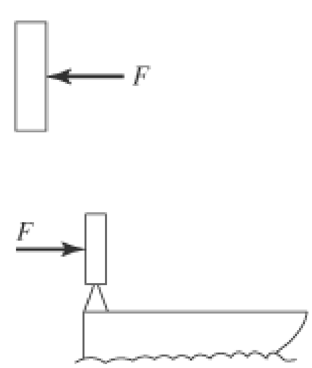
$\rho_a = 1.22\ \dfrac{\text{kg}}{\text{m}^3}$



Solution:

$Q = A v \qquad Q = \dfrac{\pi}{4}d^2 v \qquad Q = 6.1850\ \dfrac{\text{m}^3}{\text{s}}$

$\dfrac{d}{dt}m = m' \qquad m' = \rho_a Q \qquad m' = 7.5457\ \dfrac{\text{kg}}{\text{s}}$



$\Sigma F_x = m'(v_{Bx} - v_{Ax})$

$F = \rho_a Q v \qquad F = 105.64\ \text{N}$

$\Sigma F_x = M a_x \qquad F = M a$

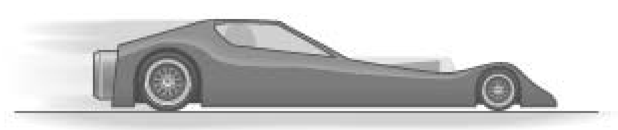
$a = \dfrac{F}{M}$ $\qquad$ $\boxed{a = 0.53\ \dfrac{\text{m}}{\text{s}^2}}$

Problem 15-118

The rocket car has a mass M_C (empty) and carries fuel of mass M_F. If the fuel is consumed at a constant rate c and ejected from the car with a relative velocity v_{DR}, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = kv^2$ and the speed is measured in m/s.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



Given:

$M_C = 3\ \text{Mg}$ $\qquad$ $M_F = 150\ \text{kg}$

$v_{DR} = 250\ \dfrac{\text{m}}{\text{s}}$ $\qquad$ $c = 4\ \dfrac{\text{kg}}{\text{s}}$

$k = 60\ \text{N}\cdot\dfrac{\text{s}^2}{\text{m}^2}$

Solution:

$m_0 = M_C + M_F$ $\qquad$ At time t the mass of the car is $m_0 - c\,t$

Set $F = k\,v^2$, then $\qquad$ $-kv^2 = (m_0 - c\,t)\dfrac{d}{dt}v - v_{DR}c$

Maximum speed occurs at the instant the fuel runs out. $\qquad$ $t = \dfrac{M_F}{c}$ $\qquad$ $t = 37.50\ \text{s}$

Thus, Initial Guess: $\qquad$ $v = 4\ \dfrac{\text{m}}{\text{s}}$

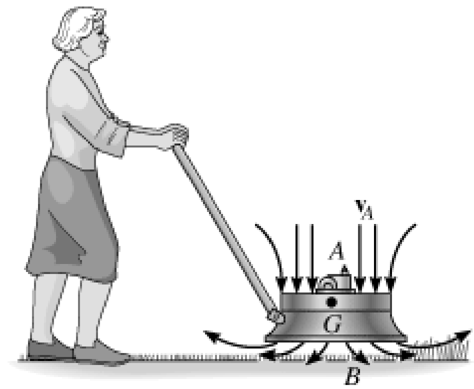
Given $\qquad$ $\displaystyle\int_0^v \frac{1}{c\,v_{DR} - k\,v^2}\,dv = \int_0^t \frac{1}{m_0 - c\,t}\,dt$

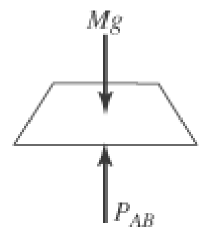
$v = \text{Find}(v)$ $\qquad$ $\boxed{v = 4.06\ \dfrac{\text{m}}{\text{s}}}$

Problem 15-119

A power lawn mower hovers very close over the ground. This is done by drawing air in at speed v_A through an intake unit A, which has cross-sectional area A_A and then discharging it at the ground, B, where the cross-sectional area is A_B. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has mass M with center of mass at G. Assume that air has a constant density of ρ_a.

Given:





$$v_A = 6\ \frac{\text{m}}{\text{s}}$$

$$A_A = 0.25\ \text{m}^2$$

$$A_B = 0.35\ \text{m}^2$$

$$M = 15\ \text{kg}$$

$$\rho_a = 1.22\ \frac{\text{kg}}{\text{m}^3}$$

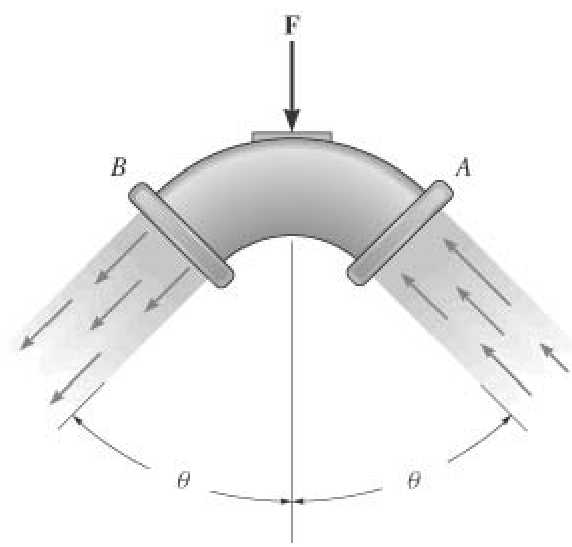
Solution: $m' = \rho_a A_A v_A$ $\quad m' = 1.83\ \frac{\text{kg}}{\text{s}}$

$$+\uparrow\ \Sigma F_y = m'(v_{By} - v_{Ay}) \qquad P A_B - M g = m'[0 - (-v_A)]$$

$$P = \frac{1}{A_B}(m' v_A + M g) \qquad P = 452\ \text{Pa}$$

***Problem 15-120**

The elbow for a buried pipe of diameter d is subjected to static pressure P. The speed of the water passing through it is v. Assuming the pipe connection at A and B do not offer any vertical force resistance on the elbow, determine the resultant vertical force **F** that the soil must then exert on the elbow in order to hold it in equilibrium. Neglect the weight of the elbow and the water within it. The density of water is γ_w.

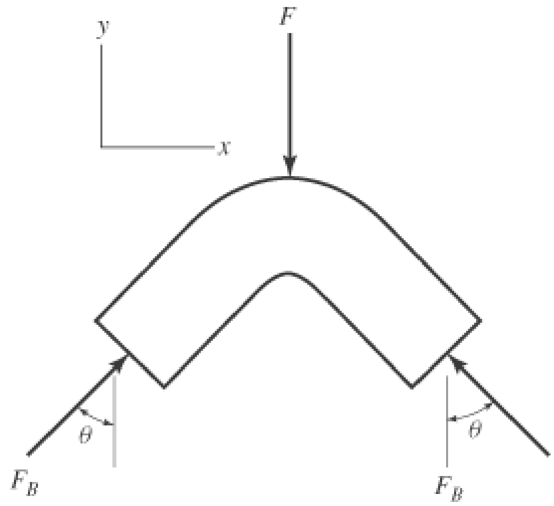


Given:

$$d = 5\ \text{in} \qquad \theta = 45\ \text{deg}$$

$P = 10\ \frac{\text{lb}}{\text{in}^2}$ $\quad \gamma_w = 62.4\ \frac{\text{lb}}{\text{ft}^3}$

$v = 8\ \frac{\text{ft}}{\text{s}}$



Solution:

$$Q = v\left(\frac{\pi}{4}d^2\right)$$

$$m' = \frac{\gamma_w}{g}Q$$

Also, the force induced by the water pressure at A is

$$A = \frac{\pi}{4}d^2$$

$$F = P\,A \qquad F = 196.35\ \text{lb}$$

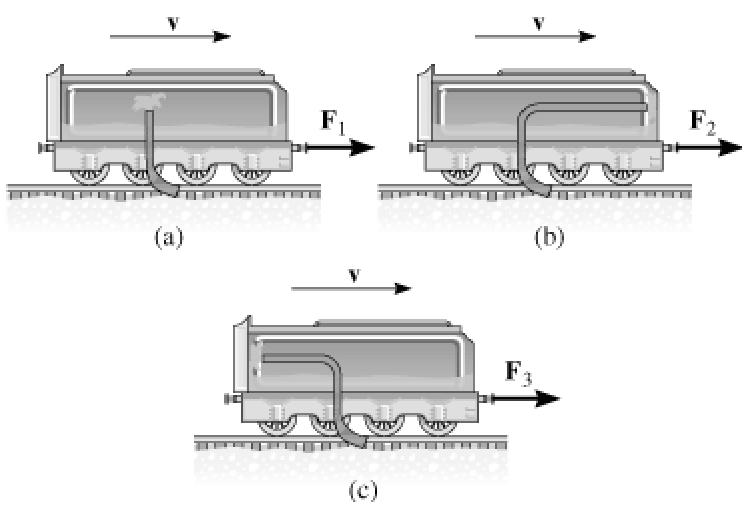
$$2F\cos(\theta) - F_1 = m'(-v\cos(\theta) - v\cos(\theta))$$

$$F_1 = 2(F\cos(\theta) + m'v\cos(\theta))$$

$$F_1 = 302\ \text{lb}$$

Problem 15-121

The car is used to scoop up water that is lying in a trough at the tracks. Determine the force needed to pull the car forward at constant velocity **v** for each of the three cases. The scoop has a cross-sectional area A and the density of water is ρ_w.



Solution:

The system consists of the car and the scoop. In all cases

$$\Sigma F_s = m\frac{d}{dt}v - V_{De}\frac{d}{dt}m_e$$

$$F = 0 - V\rho A\,V \qquad \boxed{F = V^2\rho A}$$

Problem 15-122

A rocket has an empty weight W_1 and carries fuel of weight W_2. If the fuel is burned at the rate c and ejected with a relative velocity v_{DR}, determine the maximum speed attained by the rocket starting from rest. Neglect the effect of gravitation on the rocket.

Given: $W_1 = 500\ \text{lb}$ $\quad W_2 = 300\ \text{lb}$ $\quad c = 15\ \frac{\text{lb}}{\text{s}}$ $\quad v_{DR} = 4400\ \frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: $\quad m_0 = \dfrac{W_1 + W_2}{g}$

The maximum speed occurs when all the fuel is consumed, that is, where $t = \dfrac{W_2}{c}$ $\qquad t = 20.00\ \text{s}$

$$\Sigma F_x = m\frac{d}{dt}v - v_{DR}\frac{d}{dt}m_e$$

At a time t, $M = m_0 - \frac{c}{g}t$, where $\frac{c}{g} = \frac{d}{dt}m_e$. In space the weight of the rocket is zero.

$$0 = (m_0 - c\,t)\frac{d}{dt}v - v_{DR}c$$

Guess $\quad v_{max} = 1\ \frac{\text{ft}}{\text{s}}$

Given $\quad \displaystyle\int_0^{v_{max}} 1\,dv = \int_0^t \frac{\frac{c}{g}v_{DR}}{m_0 - \frac{c}{g}t}\,dt$

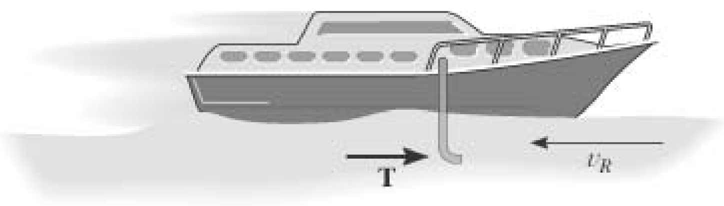
$v_{max} = \text{Find}(v_{max})$ $\qquad \boxed{v_{max} = 2068\ \frac{\text{ft}}{\text{s}}}$

Problem 15-123

The boat has mass M and is traveling forward on a river with constant velocity v_b, measured relative to the river. The river is flowing in the opposite direction at speed v_R. If a tube is placed in the water, as shown, and it collects water of mass M_w in the boat in time t, determine the horizontal thrust T on the tube that is required to overcome the resistance to the water collection.

Units Used:

$\mathrm{Mg} = 10^3\ \mathrm{kg}$



Given:

$M = 180\ \mathrm{kg}$ $\quad M_w = 40\ \mathrm{kg}$

$v_b = 70\ \dfrac{\mathrm{km}}{\mathrm{hr}}$ $\quad t = 80\ \mathrm{s}$

$v_R = 5\ \dfrac{\mathrm{km}}{\mathrm{hr}}$ $\quad \rho w = 1\dfrac{\mathrm{Mg}}{\mathrm{m}^3}$

Solution:

$$m' = \frac{M_w}{t} \qquad m' = 0.50\ \frac{\mathrm{kg}}{\mathrm{s}}$$

$$v_{di} = v_b \qquad v_{di} = 19.44\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\Sigma F_i = m\frac{\mathrm{d}}{\mathrm{d}t}v + v_{di}\,m'$$

$$T = v_{di}\,m' \qquad T = 9.72\ \mathrm{N}$$

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*__Problem 15-124__

The second stage of a two-stage rocket has weight W_2 and is launched from the first stage with velocity v. The fuel in the second stage has weight W_f. If it is consumed at rate r and ejected with relative velocity v_r, determine the acceleration of the second stage just after the engine is fired. What is the rocket's acceleration just before all the fuel is consumed? Neglect the effect of gravitation.

Given:

$W_2 = 2000\ \mathrm{lb}$ $\quad W_f = 1000\ \mathrm{lb}$ $\quad r = 50\ \dfrac{\mathrm{lb}}{\mathrm{s}}$

$v = 3000\ \dfrac{\mathrm{mi}}{\mathrm{hr}}$ $\quad v_r = 8000\ \dfrac{\mathrm{ft}}{\mathrm{s}}$ $\quad g = 32.2\ \dfrac{\mathrm{ft}}{\mathrm{s}^2}$

Solution:

Initially,

$$\Sigma F_s = m\frac{d}{dt}v - v_{di}\left(\frac{d}{dt}m_e\right)$$

$$0 = \left(\frac{W_2 + W_f}{g}\right)a - v_r\frac{r}{g} \qquad a = v_r\left(\frac{r}{W_2 + W_f}\right) \qquad a = 133\frac{\text{ft}}{\text{s}^2}$$

Finally,

$$0 = \left(\frac{W_2}{g}\right)a_1 - v_r\left(\frac{r}{g}\right) \qquad a_1 = v_r\left(\frac{r}{W_2}\right) \qquad a_1 = 200\frac{\text{ft}}{\text{s}^2}$$

Problem 15-125

The earthmover initially carries volume V of sand having a density ρ. The sand is unloaded horizontally through A dumping port P at a rate m' measured relative to the port. If the earthmover maintains a constant resultant tractive force **F** at its front wheels to provide forward motion, determine its acceleration when half the sand is dumped. When empty, the earthmover has a mass M. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.

Units Used:

$$\text{Mg} = 10^3 \text{ kg}$$

$$\text{kN} = 10^3 \text{ N}$$

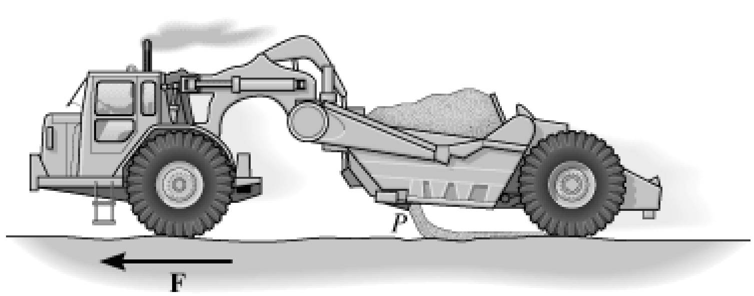
Given:

$$A = 2.5 \text{ m}^2 \qquad \rho = 1520 \frac{\text{kg}}{\text{m}^3}$$

$$m' = 900 \frac{\text{kg}}{\text{s}} \qquad V = 10 \text{ m}^3$$

$$F = 4 \text{ kN}$$

$$M = 30 \text{ Mg}$$



Solution:

When half the sand remains,

$$M_1 = M + \frac{1}{2}V\rho \qquad M_1 = 37600 \text{ kg}$$

$$\frac{d}{dt}m = m' = \rho v A \qquad v = \frac{m'}{\rho A} \quad v = 0.24\ \frac{m}{s}$$

$$\Sigma\ F = m\frac{d}{dt}v - \frac{d}{dt}m\, v_{DR} \qquad F = M_1 a - m' v$$

$$a = \frac{F + m'\ v}{M_1} \qquad a = 0.11\ \frac{m}{s^2}$$

$$a = 112\,\frac{mm}{s^2}$$

Problem 15-126

The earthmover initially carries sand of volume V having density ρ. The sand is unloaded horizontally through a dumping port P of area A at rate of r measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is a when half the sand is dumped. When empty, the earthmover has mass M. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.

Units Used:

$kN = 10^3\ N$

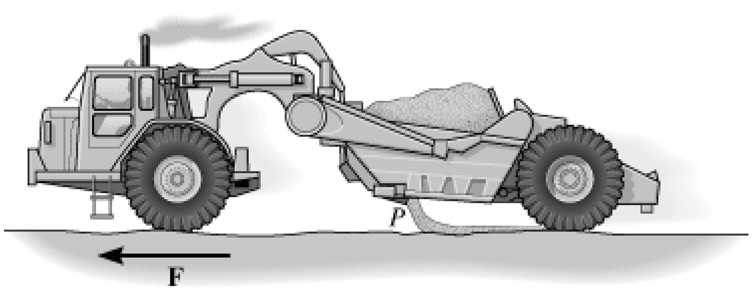
$Mg = 1000\ kg$

Given:

$V = 10\ m^3 \qquad r = 900\ \frac{kg}{s}$

$\rho = 1520\ \frac{kg}{m^3} \qquad a = 0.1\ \frac{m}{s^2}$

$A = 2.5\ m^2 \qquad M = 30\ Mg$



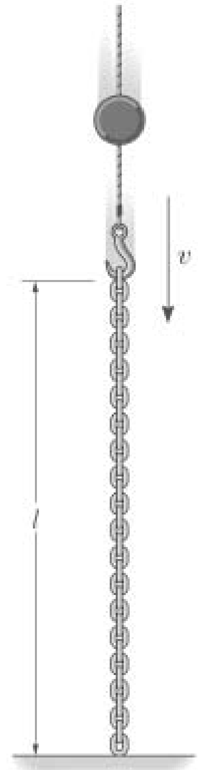
Solution:

When half the sand remains, $\quad M_1 = M + \frac{1}{2}V\rho \qquad M_1 = 37600\ kg$

$$\frac{d}{dt}m = r \qquad r = \rho v A \qquad v = \frac{r}{\rho A} \qquad v = 0.237\ \frac{m}{s}$$

$$F = m\frac{d}{dt}v - \frac{d}{dt}m\, v \qquad F = M_1 a - r v \qquad F = 3.55\,kN$$

Problem 15-127



If the chain is lowered at a constant speed *v*, determine the normal reaction exerted on the floor as a function of time. The chain has a weight W and a total length *l*.

Given:

$$W = 5\ \frac{\text{lb}}{\text{ft}}$$

$$l = 20\ \text{ft}$$

$$v = 4\ \frac{\text{ft}}{\text{s}}$$

Solution:

At time *t*, the weight of the chain on the floor is $W = M\,g(v\,t)$

$$\frac{d}{dt}v = 0 \qquad M_t = M(v\,t)$$

$$\frac{d}{dt}M_t = M\,v$$

$$\Sigma\ F_s = M\frac{d}{dt}v + v_{Dt}\frac{d}{dt}M_t$$

$$R - M\,g(v\,t) = 0 + v(M\,v)$$

$$R = M\left(g\,v\,t + v^2\right) \qquad \boxed{R = \frac{W}{g}\left(g\,v\,t + v^2\right)}$$

***Problem 15-128**

The rocket has mass *M* including the fuel. Determine the constant rate at which the fuel must be burned so that its thrust gives the rocket a speed *v* in time *t* starting from rest. The fuel is expelled from the rocket at relative speed v_r. Neglect the effects of air resistance and assume that *g* is constant.

Given:

$$M = 65000\ \text{lb} \qquad v_r = 3000\ \frac{\text{ft}}{\text{s}}$$

$$v = 200\ \frac{\text{ft}}{\text{s}}$$

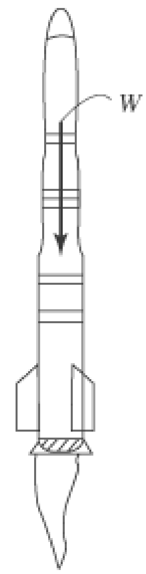
$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$t = 10\ \text{s}$$

Solution:

A System That Losses Mass: Here,

$$W = \left(m_0 - \frac{d}{dt}m_e\, t\right)g$$



Applying Eq. 15-29, we have

$$+\uparrow \Sigma\ F_z = m\frac{d}{dt}v - v_{DE}\frac{d}{dt}m_e \qquad \text{integrating we find}$$

$$v = v_{DE}\ln\left(\frac{m_o}{m_0 - \frac{d}{dt}m_e\, t}\right) - g\,t$$

with $\quad m_0 = M \qquad v_{DE} = v_r$

$$v = v_r\ln\left(\frac{m_0}{m_0 - \frac{d}{dt}m_e\, t}\right) - g(t)$$

$$\frac{d}{dt}m_e = A = \left(\frac{-m_0}{e^{\frac{v+g\,t}{v_r}}} + m_0\right)\frac{1}{t} \qquad A = \left(\frac{-m_0}{e^{\frac{v+g\,t}{v_r}}} + m_0\right)\frac{1}{t} \qquad A = 43.3\,\frac{\text{slug}}{\text{s}}$$

Problem 15-129

The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed v_{er}, determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.

Solution:

$$a_0 = \frac{d}{dt}v$$

$$+\uparrow\ \Sigma F_s = m\frac{d}{dt}v - v_{er}\frac{d}{dt}m_e$$

$$-mg = ma_o - v_{er}\frac{d}{dt}m$$

$$v_{er}\frac{dm}{m} = (a_0 + g)dt$$

Since v_{er} is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{er}\ln\left(\frac{m}{m_0}\right) = \left(a_0 + g\right)t \qquad \frac{m}{m_0} = e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

The time rate fuel consumption is determined from Eq.[1]

$$\frac{d}{dt}m = m\frac{a_0 + g}{v_{er}} \qquad \frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

Note : v_{er} must be considered a negative quantity.

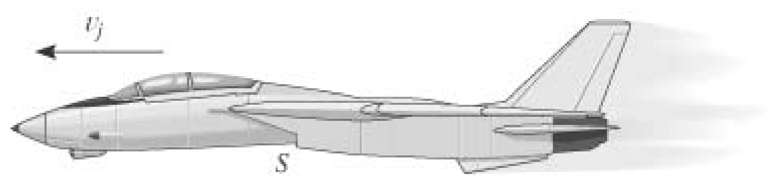
Problem 15-130

The jet airplane of mass M has constant speed v_j when it is flying along a horizontal straight line. Air enters the intake scoops S at rate r_1. If the engine burns fuel at the rate r_2 and the gas (air and fuel) is exhausted relative to the plane with speed v_e, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density ρ. *Hint*: Since mass both enters and exits the plane, Eqs. 15-29 and 15-30 must be combined.

Units Used:

$\mathrm{Mg} = 1000\ \mathrm{kg}$

$\mathrm{kN} = 10^3\ \mathrm{N}$

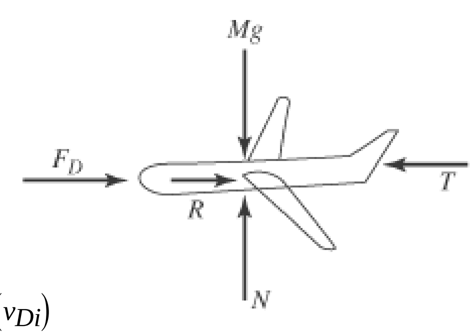


Given:

$M = 12\ \mathrm{Mg}$ $\qquad r_2 = 0.4\ \frac{\mathrm{kg}}{\mathrm{s}}$

$v_j = 950\ \frac{\mathrm{km}}{\mathrm{hr}}$ $\qquad v_e = 450\ \frac{\mathrm{m}}{\mathrm{s}}$

$r_1 = 50\ \frac{\mathrm{m}^3}{\mathrm{s}}$ $\qquad \rho = 1.22\ \frac{\mathrm{kg}}{\mathrm{m}^3}$



Solution:

$$\Sigma F_s = m\frac{d}{dt}v - \frac{d}{dt}m_e\left(v_{DE}\right) + \frac{d}{dt}m_i\left(v_{Di}\right)$$

$$\frac{d}{dt}v = 0 \qquad v_{DE} = V_e \qquad v_{Di} = v_j \qquad \frac{d}{dt}m_i = r_1\rho$$

$$A = r_1 \rho \qquad \frac{d}{dt}m_e = r_2 + A \qquad B = r_2 + A$$

Forces T and F_D are incorporated as the last two terms in the equation,

$$F_D = v_e B - v_j A \qquad F_D = 11.5\,\text{kN}$$

Problem 15-131

The jet is traveling at speed v, angle θ with the horizontal. If the fuel is being spent at rate r_1 and the engine takes in air at r_2 whereas the exhaust gas (air and fuel) has relative speed v_e, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = kv^2$ The jet has weight W. *Hint:* See Prob. 15-130.

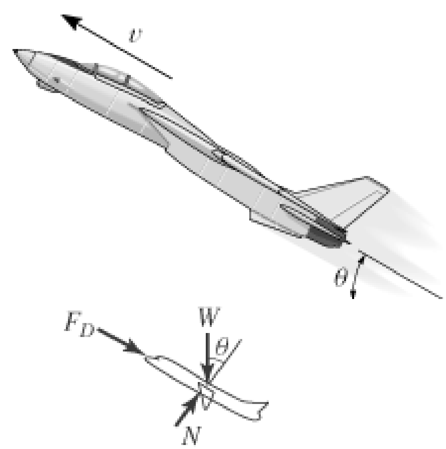
Given:

$$v = 500\ \frac{\text{mi}}{\text{hr}} \qquad v_e = 32800\ \frac{\text{ft}}{\text{s}}$$

$$\theta = 30\ \text{deg} \qquad k_1 = 0.7\ \text{lb}\ \frac{\text{s}^2}{\text{ft}^2}$$

$$r_1 = 3\ \frac{\text{lb}}{\text{s}} \qquad W = 15000\ \text{lb}$$

$$r_2 = 400\ \frac{\text{lb}}{\text{s}} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$



Solution:

$$\frac{d}{dt}m_i = \frac{r_2}{g_1} \qquad A_1 = r_2 \qquad \frac{d}{dt}m_e = \frac{r_1 + r_2}{g_1} \qquad B = r_1 + r_2 \qquad v_1 = v$$

$$+\ \Sigma F_s = m\frac{d}{dt}v - v_{De}\frac{d}{dt}m_e + v_{Di}\frac{d}{dt}m_i$$

$$-W\sin(\theta) - k_1 v_1^2 = W a - v_e B + v_1 A_1$$

$$a = \frac{\left(-W\sin(\theta) - k_1 v_1^2 + v_e \frac{B}{g} - v_1 \frac{A_1}{g}\right)g}{W} \qquad a = 37.5\ \frac{\text{ft}}{\text{s}^2}$$

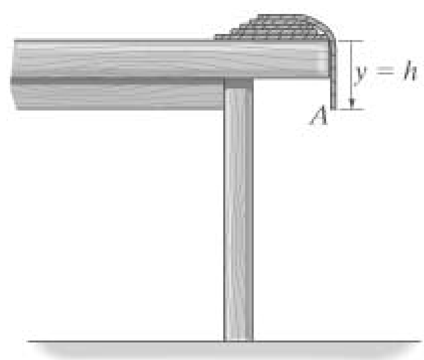
***Problem 15-132**

The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.

Solution:

$$F_s = m\frac{d}{dt}v + v_{Di}\frac{d}{dt}m_i \qquad \text{At a time } t,\ m = m'y \text{ and } \frac{d}{dt}m_i = m'\frac{d}{dt}y = m'v.$$

Here, $v_{Di} = v$, $\frac{d}{dt}v = g$.



$$m'gy = m'y\frac{d}{dt}v + v(m'v)$$

$$gy = y\frac{d}{dt}v + v^2 \qquad \text{Since } v = \frac{d}{dt}y\text{, then } dt = \frac{dy}{v}$$

$$gy = vy\frac{d}{dy}v + v^2$$

Multiply both sides by $2y\,dy$

$$2gy^2\,dy = 2vy^2\,dv + 2yv^2\,dy$$

$$\int 2gy^2\,dy = \int 1\,dv^2y^2 \qquad \frac{2}{3}gy^3 + C = v^2y^2$$

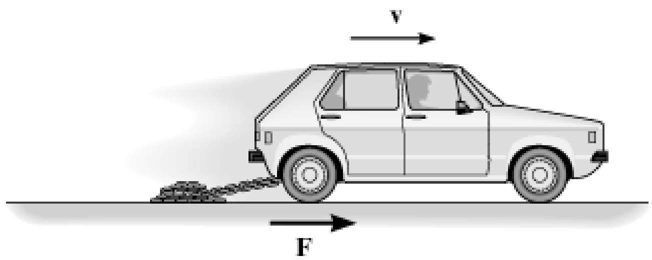
$$v = 0 \quad \text{at} \quad y = h \qquad \frac{2}{3}gh^3 + C = 0 \qquad C = \frac{-2}{3}gh^3$$

$$\frac{2}{3}gy^3 - \frac{2}{3}gh^3 = v^2y^2 \qquad \boxed{v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}}$$

Problem 15-133

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force **F** that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.

Solution:



$\xrightarrow{+}$ $\Sigma F_s = m\frac{d}{dt}v + v_{Di}\frac{d}{dt}m_i$

At a time t, $m = m_0 + c\,t$

Where, $c = \frac{d}{dt}m_i = m'\frac{d}{dt}x = m'v$

Here, $v_{Di} = v$ $\quad \frac{d}{dt}v = 0$

$F = (m_0 - m'vt)(0) + v(m'v) = m'v^2$ $\quad F = m'v^2$

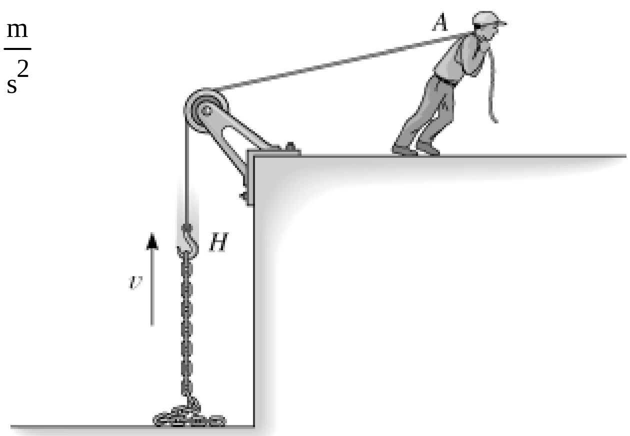
Problem 15-134

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed v. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass density ρ.

Given:

$v = 0.4\,\frac{\text{m}}{\text{s}}$ $\quad \rho = 2\,\frac{\text{kg}}{\text{m}}$ $\quad g = 9.81\,\frac{\text{m}}{\text{s}^2}$

Solution:

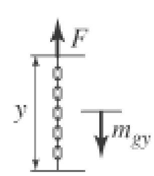


$\frac{d}{dt}v = 0$ $\quad y = v\,t$

$m_i = m\,y = m\,v\,t$

$\frac{d}{dt}m_i = m\,v$

$+\uparrow$ $\Sigma F_s = m\frac{d}{dt}v + v_{Di}\left(\frac{d}{dt}m_i\right)$



$F - m\,g\,v\,t = 0 + v\,m\,v$ $\quad F = m\,g\,v\,t + v\,m\,v$

$F = \rho g\,v\,t + v^2$

$f_1 = \rho g\,v$ $\quad f_1 = 7.85\,\frac{\text{N}}{\text{s}}$ $\quad f_2 = \rho v^2$ $\quad f_2 = 0.320\,\text{N}$

$F = f_1\,t + f_2$
