Problem 16-1

A wheel has an initial clockwise angular velocity ω and a constant angular acceleration α. Determine the number of revolutions it must undergo to acquire a clockwise angular velocity ω_f. What time is required?

Units Used: $\text{rev} = 2\pi \text{ rad}$

Given: $\omega = 10 \frac{\text{rad}}{\text{s}}$ $\alpha = 3 \frac{\text{rad}}{\text{s}^2}$ $\omega_f = 15 \frac{\text{rad}}{\text{s}}$

Solution: $\omega_f^2 = \omega^2 + 2\alpha\,\theta$ $\theta = \frac{\omega_f^2 - \omega^2}{2\alpha}$ $\theta = 3.32 \text{ rev}$

$\omega_f = \omega + \alpha t$ $t = \frac{\omega_f - \omega}{\alpha}$ $t = 1.67 \text{ s}$

Problem 16-2

A flywheel has its angular speed increased uniformly from ω_1 to ω_2 in time t. If the diameter of the wheel is D, determine the magnitudes of the normal and tangential components of acceleration of a point on the rim of the wheel at time t, and the total distance the point travels during the time period.

Given: $\omega_1 = 15 \frac{\text{rad}}{\text{s}}$ $\omega_2 = 60 \frac{\text{rad}}{\text{s}}$ $t = 80 \text{ s}$ $D = 2 \text{ ft}$

Solution: $r = \frac{D}{2}$

$\omega_2 = \omega_1 + \alpha t$ $\alpha = \frac{\omega_2 - \omega_1}{t}$ $\alpha = 0.56 \frac{\text{rad}}{\text{s}^2}$

$a_t = \alpha r$ $a_t = 0.563 \frac{\text{ft}}{\text{s}^2}$

$a_n = \omega_2^2 r$ $a_n = 3600 \frac{\text{ft}}{\text{s}^2}$

$\theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$ $\theta = 3000 \text{ rad}$

$d = \theta r$ $d = 3000 \text{ ft}$

Problem 16-3

The angular velocity of the disk is defined by $\omega = at^2 + b$. Determine the magnitudes of the velocity and acceleration of point A on the disk when $t = t_1$.

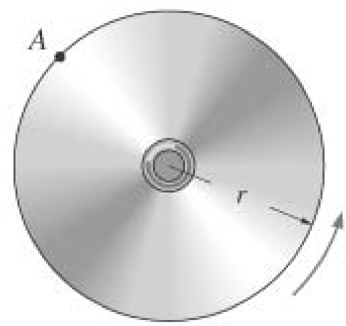
Given:

$a = 5\ \frac{\text{rad}}{\text{s}^3}$

$b = 2\ \frac{\text{rad}}{\text{s}}$

$r = 0.8\ \text{m}$

$t_1 = 0.5\ \text{s}$



Solution: $t = t_1$

$\omega = a t^2 + b$ $\quad\quad \omega = 3.25\ \frac{\text{rad}}{\text{s}}$

$\alpha = 2a t$ $\quad\quad \alpha = 5.00\ \frac{\text{rad}}{\text{s}^2}$

$v = \omega r$ $\quad\quad v = 2.60\ \frac{\text{m}}{\text{s}}$

$a = \sqrt{(\alpha r)^2 + \left(\omega^2 r\right)^2}$ $\quad\quad a = 9.35\ \frac{\text{m}}{\text{s}^2}$

***Problem 16-4**

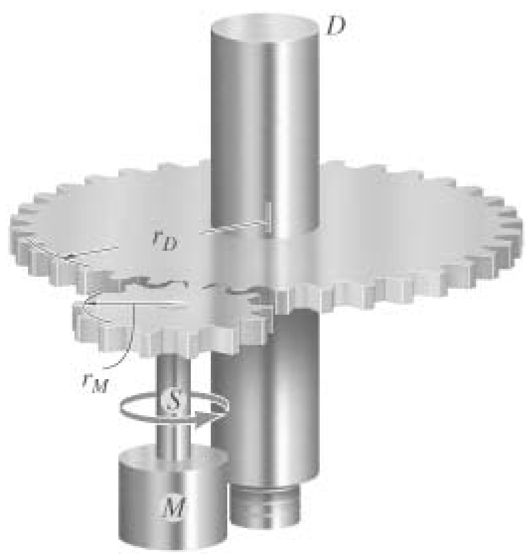
The figure shows the internal gearing of a "spinner" used for drilling wells. With constant angular acceleration, the motor M rotates the shaft S to angular velocity ω_M in time t starting from rest. Determine the angular acceleration of the drill-pipe connection D and the number of revolutions it makes during the start up at t.

Units Used: $\text{rev} = 2\pi$

Given:

$\omega_M = 100\ \frac{\text{rev}}{\text{min}}$ $\quad\quad r_M = 60\ \text{mm}$

$r_D = 150\ \text{mm}$ $\quad\quad t = 2\ \text{s}$



Solution:

$\omega_M = \alpha_M t$

$\alpha_M = \frac{\omega_M}{t}$ $\quad\quad \alpha_M = 5.24\ \frac{\text{rad}}{\text{s}^2}$

$\alpha_M r_M = \alpha_D r_D$

$$\alpha_D = \alpha_M \left(\frac{r_M}{r_D}\right) \qquad \alpha_D = 2.09\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = \frac{1}{2}\alpha_D t^2 \qquad \theta = 0.67\ \text{rev}$$

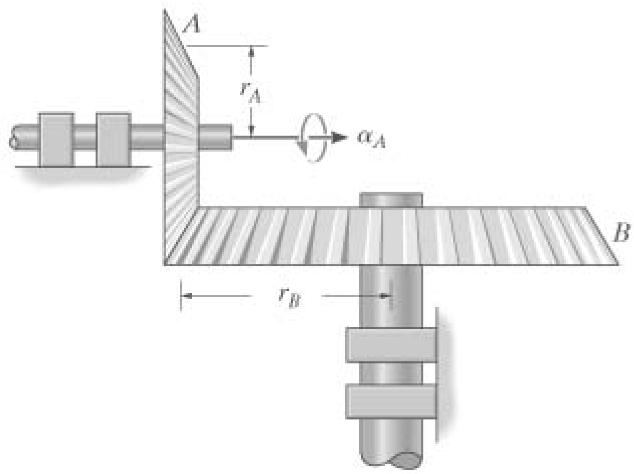
Problem 16-5

If gear *A* starts from rest and has a constant angular acceleration α_A, determine the time needed for gear *B* to attain an angular velocity ω_B.

Given:

$$\alpha_A = 2\ \frac{\text{rad}}{\text{s}^2} \qquad r_B = 0.5\ \text{ft}$$

$$\omega_B = 50\ \frac{\text{rad}}{\text{s}} \qquad r_A = 0.2\ \text{ft}$$



Solution:

The point in contact with both gears has a speed of

$$v_p = \omega_B r_B \qquad v_p = 25.00\ \frac{\text{ft}}{\text{s}}$$

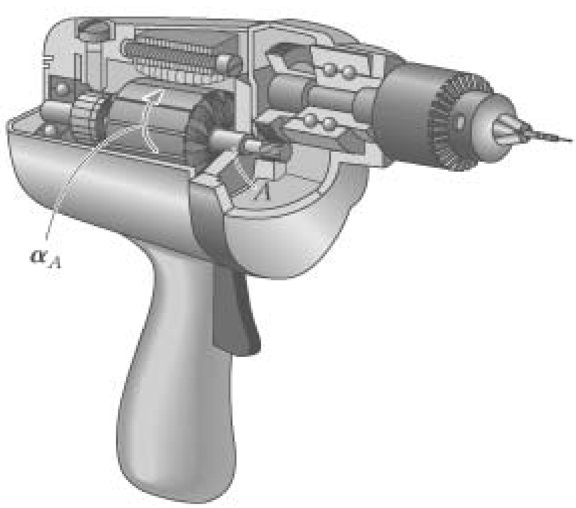
Thus,

$$\omega_A = \frac{v_p}{r_A} \qquad \omega_A = 125.00\ \frac{\text{rad}}{\text{s}}$$

So that $\quad \omega = \alpha_c t \qquad t = \dfrac{\omega_A}{\alpha_A} \qquad t = 62.50\ \text{s}$

Problem 16-6

If the armature *A* of the electric motor in the drill has a constant angular acceleration α_A, determine its angular velocity and angular displacement at time *t*. The motor starts from rest.



Given:

$$\alpha_A = 20\ \frac{\text{rad}}{\text{s}^2} \qquad t = 3\ \text{s}$$

Solution:

$$\omega = \alpha_c t \qquad \omega = \alpha_A t \qquad \omega = 60.00\ \frac{\text{rad}}{\text{s}}$$

$\theta = \frac{1}{2}\alpha_A t^2$ $\qquad$ $\theta = 90.00\ \text{rad}$

Problem 16-7

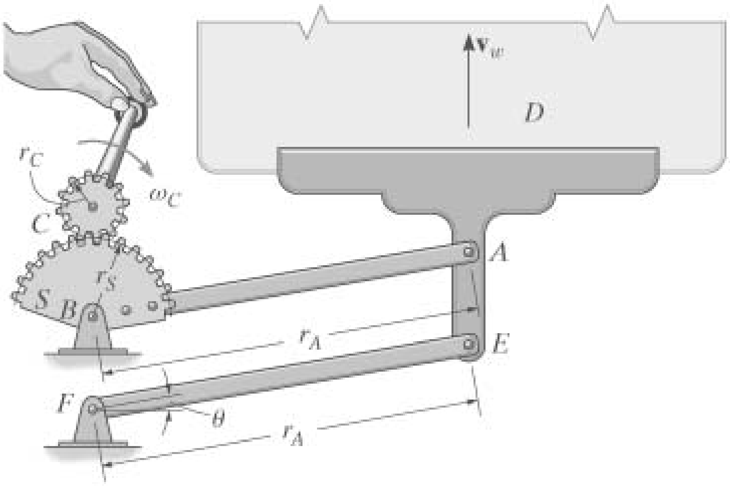
The mechanism for a car window winder is shown in the figure. Here the handle turns the small cog *C*, which rotates the spur gear *S*, thereby rotating the fixed-connected lever *AB* which raises track *D* in which the window rests. The window is free to slide on the track. If the handle is wound with angular velocity ω_c, determine the speed of points *A* and *E* and the speed v_w of the window at the instant θ.

Given:

$\omega_C = 0.5\ \frac{\text{rad}}{\text{s}}$ $\qquad$ $r_C = 20\ \text{mm}$

$\theta = 30\ \text{deg}$ $\qquad$ $r_S = 50\ \text{mm}$

$r_A = 200\ \text{mm}$



Solution:

$v_C = \omega_C r_C$

$v_C = 0.01\ \frac{\text{m}}{\text{s}}$

$\omega_S = \frac{v_C}{r_S}$ $\qquad$ $\omega_S = 0.20\ \frac{\text{rad}}{\text{s}}$

$v_A = v_E = \omega_S r_A$

$v_A = \omega_S r_A$ $\qquad$ $v_A = v_E = 40.00\ \frac{\text{mm}}{\text{s}}$

Points *A* and *E* move along circular paths. The vertical component closes the window.

$v_w = v_A \cos(\theta)$ $\qquad$ $v_w = 34.6\ \frac{\text{mm}}{\text{s}}$

***Problem 16-8**

The pinion gear *A* on the motor shaft is given a constant angular acceleration α. If the gears *A* and *B* have the dimensions shown, determine the angular velocity and angular displacement of the output shaft *C*, when $t = t_1$ starting from rest. The shaft is fixed to *B* and turns with it.

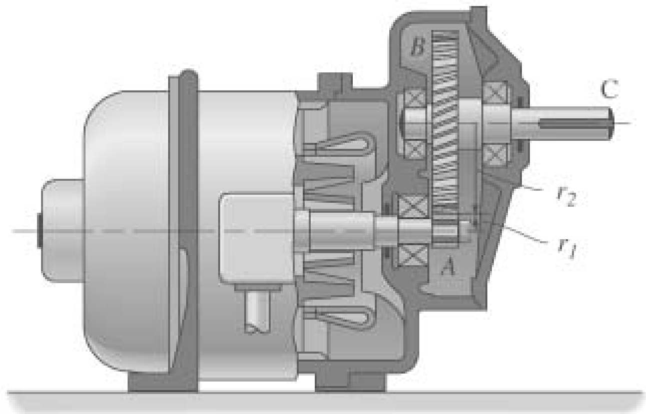
Given:

$\alpha = 3\ \frac{\text{rad}}{\text{s}^2}$

$t_1 = 2\text{ s}$

$r_1 = 35\text{ mm}$

$r_2 = 125\text{ mm}$



Solution:

$\alpha_A = \alpha$

$r_1\alpha_A = r_2\alpha_C \qquad \alpha_C = \left(\frac{r_1}{r_2}\right)\alpha_A$

$\omega_C = \alpha_C t_1 \qquad \omega_C = 1.68\ \frac{\text{rad}}{\text{s}}$

$\theta_C = \frac{1}{2}\alpha_C t_1^2 \qquad \theta_C = 1.68\text{ rad}$

Problem 16-9

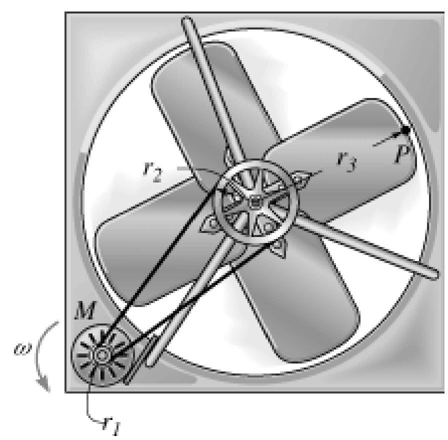
The motor M begins rotating at an angular rate $\omega = a(1 - e^{bt})$. If the pulleys and fan have the radii shown, determine the magnitudes of the velocity and acceleration of point P on the fan blade when $t = t_1$. Also, what is the maximum speed of this point?

Given:

$a = 4\ \frac{\text{rad}}{\text{s}} \qquad r_1 = 1\text{ in}$

$b = -1\ \frac{1}{\text{s}} \qquad r_2 = 4\text{ in}$

$t_1 = 0.5\text{ s} \qquad r_3 = 16\text{ in}$



Solution:

$t = t_1 \qquad r_1\omega_1 = r_2\omega_2$

$\omega_1 = a\left(1 - e^{bt}\right) \qquad \omega_2 = \left(\frac{r_1}{r_2}\right)\omega_1$

$v_P = r_3\omega_2 \qquad v_P = 6.30\ \frac{\text{in}}{\text{s}}$

$$\alpha_1 = -a\,b\,e^{b\,t} \qquad \alpha_2 = \left(\frac{r_1}{r_2}\right)\alpha_1$$

$$a_P = \sqrt{\left(\alpha_2\,r_3\right)^2 + \left(\omega_2^{\,2}\,r_3\right)^2} \qquad a_P = 10.02\,\frac{\text{in}}{\text{s}^2}$$

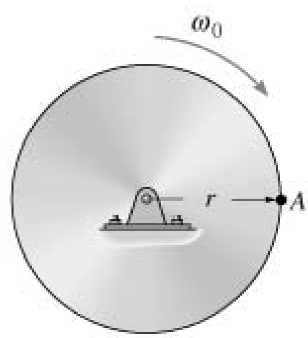
As t approaches ∞

$$\omega_1 = a \qquad \omega_f = \frac{r_1}{r_2}\omega_1 \qquad v_f = r_3\,\omega_f \qquad v_f = 16.00\,\frac{\text{in}}{\text{s}}$$

Problem 16-10

The disk is originally rotating at angular velocity ω_0. If it is subjected to a constant angular acceleration α, determine the magnitudes of the velocity and the n and t components of acceleration of point A at the instant t.

Given:



$$\omega_0 = 8\,\frac{\text{rad}}{\text{s}}$$

$$\alpha = 6\,\frac{\text{rad}}{\text{s}^2}$$

$$t = 0.5\ \text{s}$$

$$r = 2\ \text{ft}$$

Solution:

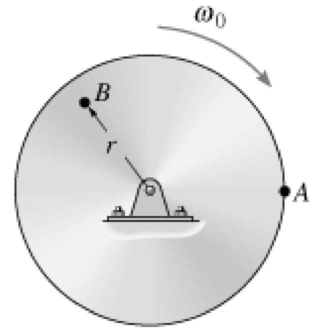
$$\omega = \omega_0 + \alpha t \qquad v_A = r\omega \qquad v_A = 22.00\,\frac{\text{ft}}{\text{s}}$$

$$a_t = r\alpha \qquad a_t = 12.00\,\frac{\text{ft}}{\text{s}^2}$$

$$a_n = r\omega^2 \qquad a_n = 242.00\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-11

The disk is originally rotating at angular velocity ω_0. If it is subjected to a constant angular acceleration α, determine the magnitudes of the velocity and the *n* and *t* components of acceleration of point *B* just after the wheel undergoes a rotation θ.



Given:

$\text{rev} = 2\pi\ \text{rad} \qquad \alpha = 6\ \frac{\text{rad}}{\text{s}^2} \qquad r = 1.5\ \text{ft}$

$\omega_0 = 8\ \frac{\text{rad}}{\text{s}} \qquad \theta = 2\ \text{rev}$

Solution:

$$\omega = \sqrt{\omega_0^{\,2} + 2\,\alpha\,\theta} \qquad \omega = 14.66\ \frac{\text{rad}}{\text{s}}$$

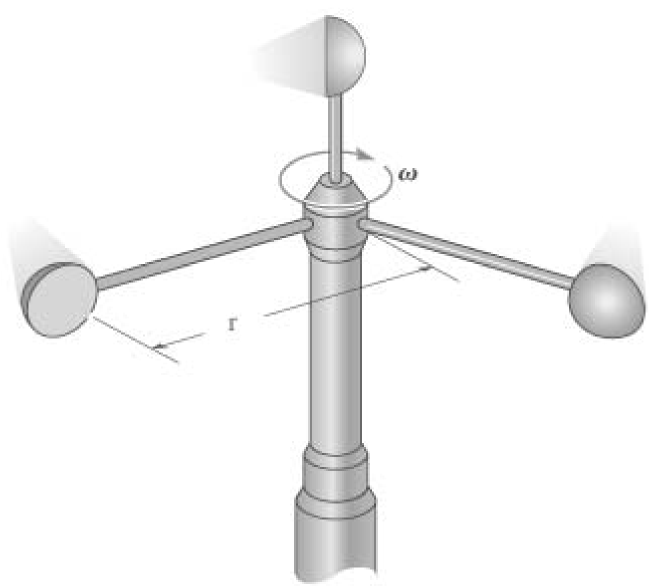
$$v_B = r\omega \qquad v_B = 22\ \frac{\text{ft}}{\text{s}}$$

$$a_{Bt} = r\alpha \qquad a_{Bt} = 9\ \frac{\text{ft}}{\text{s}^2}$$

$$a_{Bn} = r\omega^2 \qquad a_{Bn} = 322\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 16-12**

The anemometer measures the speed of the wind due to the rotation of the three cups. If during a time period t_1 a wind gust causes the cups to have an angular velocity $\omega = (At^2 + B)$, determine (a) the speed of the cups when $t = t_2$, (b) the total distance traveled by each cup during the time period t_1, and (c) the angular acceleration of the cups when $t = t_2$. Neglect the size of the cups for the calculation.



Given:

$t_1 = 3\ \text{s} \qquad t_2 = 2\ \text{s} \qquad r = 1.5\ \text{ft}$

$A = 2\ \frac{1}{\text{s}^3} \qquad B = 3\ \frac{1}{\text{s}}$

Solution:

$$\omega_2 = A t_2^2 + B \qquad v_2 = r\omega_2 \qquad v_2 = 16.50\ \frac{\text{ft}}{\text{s}}$$

$$d = r\int_0^{t_1} A t^2 + B\ dt \qquad d = 40.50\ \text{ft}$$

$$\alpha = \frac{d\omega_2}{dt} \qquad \alpha = 2A t_2 \qquad \alpha = 8.00\ \frac{\text{rad}}{\text{s}^2}$$

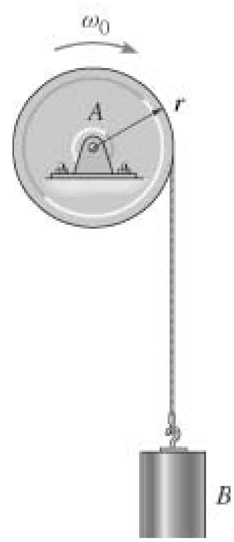
Problem 16-13

A motor gives disk A a clockwise angular acceleration $\alpha_A = at^2 + b$. If the initial angular velocity of the disk is ω_0, determine the magnitudes of the velocity and acceleration of block B when $t = t_1$.

Given:

$$a = 0.6\ \frac{\text{rad}}{\text{s}^4} \qquad \omega_0 = 6\ \frac{\text{rad}}{\text{s}} \qquad r = 0.15\ \text{m}$$

$$b = 0.75\ \frac{\text{rad}}{\text{s}^2} \qquad t_1 = 2\ \text{s}$$



Solution:

$$\alpha_A = a t_1^2 + b$$

$$\omega_A = \frac{a}{3} t_1^3 + b t_1 + \omega_0$$

$$v_B = \omega_A r \qquad v_B = 1.365\ \frac{\text{m}}{\text{s}}$$

$$a_B = \alpha_A r \qquad a_B = 0.472\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-14

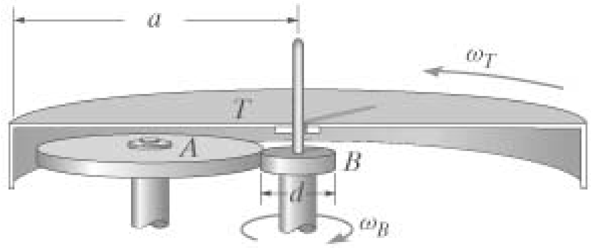
The turntable T is driven by the frictional idler wheel A, which simultaneously bears against the inner rim of the turntable and the motor-shaft spindle B. Determine the required diameter d of the spindle if the motor turns it with angular velocity ω_B and it is required that the turntable rotate with angular velocity ω_T.

Given:

$\omega_B = 25\ \frac{\text{rad}}{\text{s}}$

$\omega_T = 2\ \frac{\text{rad}}{\text{s}}$

$a = 9\ \text{in}$



Solution:

$$\omega_B \frac{d}{2} = \omega_A \left(\frac{a - \frac{d}{2}}{2}\right) \qquad \omega_A = \frac{\omega_B d}{a - \frac{d}{2}}$$

$$\omega_A \left(\frac{a - \frac{d}{2}}{2}\right) = \omega_T a \qquad \frac{\omega_B d}{2} = \omega_T a \qquad d = \frac{2\omega_T a}{\omega_B} \qquad d = 1.44\,\text{in}$$

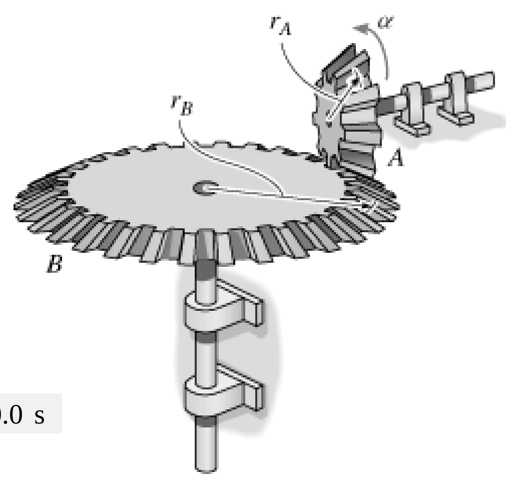
Problem 16-15

Gear A is in mesh with gear B as shown. If A starts from rest and has constant angular acceleration α_A, determine the time needed for B to attain an angular velocity ω_B.

Given:

$\alpha_A = 2\ \frac{\text{rad}}{\text{s}^2}$ $\qquad r_A = 25\ \text{mm}$

$\omega_B = 50\ \frac{\text{rad}}{\text{s}}$ $\qquad r_B = 100\ \text{mm}$



Solution:

$$\alpha_A r_A = \alpha_B r_B \qquad \alpha_B = \left(\frac{r_A}{r_B}\right)\alpha_A$$

$$\omega_B = \alpha_B t \qquad t = \frac{\omega_B}{\alpha_B} \qquad t = 100.0\ \text{s}$$

***Problem 16-16**

The blade on the horizontal-axis windmill is turning with an angular velocity ω_0. Determine the distance point *P* on the tip of the blade has traveled if the blade attains an angular velocity ω in time *t*. The angular acceleration is constant. Also, what is the magnitude of the acceleration of this point at time *t*?

Given:

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\omega = 5\ \frac{\text{rad}}{\text{s}}$

$t = 3\ \text{s}$ $r_p = 15\ \text{ft}$

Solution:

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$d_p = r_p \int_0^t \omega_0 + \alpha t\ dt \qquad d_p = 157.50\ \text{ft}$$

$$a_n = r_p \omega^2 \qquad a_t = r_p \alpha$$

$$a_p = \left| \begin{pmatrix} a_n \\ a_t \end{pmatrix} \right| \qquad a_p = 375.30\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-17

The blade on the horizontal-axis windmill is turning with an angular velocity ω_0. If it is given an angular acceleration α, determine the angular velocity and the magnitude of acceleration of point *P* on the tip of the blade at time *t*.

Given:

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\alpha = 0.6\ \frac{\text{rad}}{\text{s}^2}$ $r = 15\ \text{ft}$ $t = 3\ \text{s}$

Solution:

$\omega = \omega_0 + \alpha t$ $\omega = 3.80\ \frac{\text{rad}}{\text{s}}$

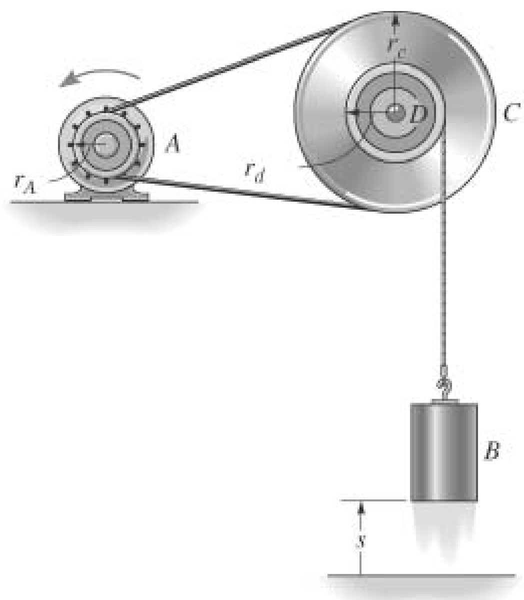
$a_{pt} = \alpha r$ $a_{pt} = 9.00\ \frac{\text{ft}}{\text{s}^2}$

$a_{pn} = \omega^2 r$ $a_{pn} = 216.60\ \frac{\text{ft}}{\text{s}^2}$

$a_p = \sqrt{a_{pt}^2 + a_{pn}^2}$ $a_p = 217\ \frac{\text{ft}}{\text{s}^2}$

Problem 16-18

Starting from rest when $s = 0$, pulley A is given an angular acceleration $\alpha_A = k\theta$. Determine the speed of block B when it has risen to $s = s_1$. The pulley has an inner hub D which is fixed to C and turns with it.



Given:

$k = 6\ \text{s}^{-2}$ $r_C = 150\ \text{mm}$

$s_1 = 6\ \text{m}$ $r_D = 75\ \text{mm}$

$r_A = 50\ \text{mm}$

Solution:

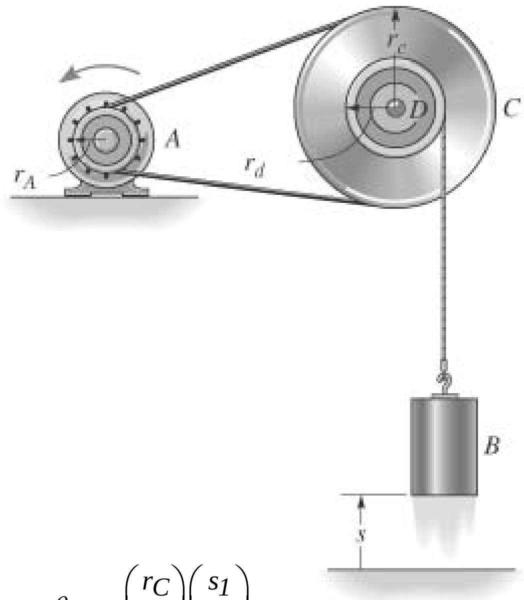
$\theta_A r_A = \theta_C r_C$ $\theta_C r_D = s_1$ $\theta_A = \left(\frac{r_C}{r_A}\right)\frac{s_1}{r_D}$

$\alpha_A = k\theta$ $\frac{\omega_A^2}{2} = k\left(\frac{\theta_A^2}{2}\right)$ $\omega_A = \sqrt{k}\,\theta_A$

$\omega_A r_A = \omega_C r_C$ $\omega_C = \left(\frac{r_A}{r_C}\right)\omega_A$ $v_B = \omega_C r_D$ $v_B = 14.70\ \frac{\text{m}}{\text{s}}$

Problem 16-19

Starting from rest when $s = 0$, pulley A is given a constant angular acceleration α_A. Determine the speed of block B when it has risen to $s = s_1$. The pulley has an inner hub D which is fixed to C and turns with it.



Given:

$\alpha_A = 6\ \frac{\text{rad}}{\text{s}^2}$

$s_1 = 6\ \text{m}$

$r_A = 50\ \text{mm}$

$r_C = 150\ \text{mm}$

$r_D = 75\ \text{mm}$

Solution:

$\theta_A r_A = \theta_C r_C$ $\qquad$ $\theta_C r_D = s_1$ $\qquad$ $\theta_A = \left(\frac{r_C}{r_A}\right)\left(\frac{s_1}{r_D}\right)$

$\frac{\omega_A^2}{2} = \alpha_A \theta_A$ $\qquad$ $\omega_A = \sqrt{2\alpha_A\theta_A}$

$\omega_A r_A = \omega_C r_C$ $\qquad$ $\omega_C = \frac{r_A}{r_C}\omega_A$ $\qquad$ $v_B = \omega_C r_D$ $\qquad$ $v_B = 1.34\ \frac{\text{m}}{\text{s}}$

***Problem 16-20**

Initially the motor on the circular saw turns its drive shaft at $\omega = kt^{2/3}$. If the radii of gears A and B are r_A and r_B respectively, determine the magnitudes of the velocity and acceleration of a tooth C on the saw blade after the drive shaft rotates through angle $\theta = \theta_1$ starting from rest.

Given:

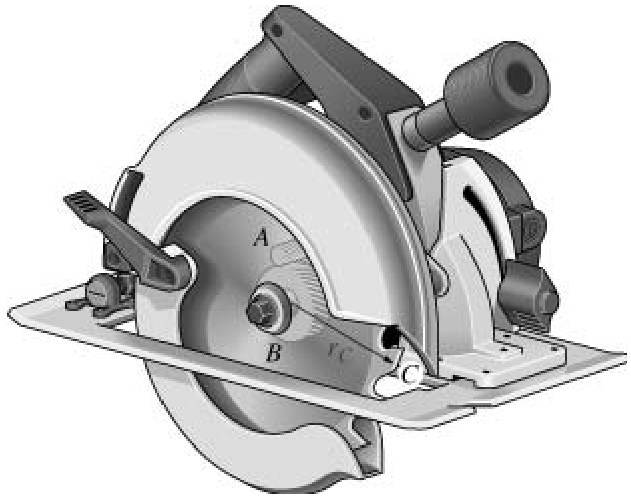
$r_A = 0.25\ \text{in}$

$r_B = 1\ \text{in}$

$r_C = 2.5\ \text{in}$

$\theta_1 = 5\ \text{rad}$

$k = 20\ \dfrac{\text{rad}}{\text{s}^{\frac{5}{3}}}$



Solution:

$\omega_A = k t^{\frac{2}{3}} \qquad \theta_A = \dfrac{3}{5} k t^{\frac{5}{3}}$

$t_1 = \left(\dfrac{5\theta_1}{3k}\right)^{\frac{3}{5}} \qquad t_1 = 0.59\ \text{s}$

$\omega_A = k t_1^{\frac{2}{3}} \qquad \omega_A = 14.09\ \dfrac{\text{rad}}{\text{s}}$

$\omega_B = \dfrac{r_A}{r_B}\omega_A \qquad \omega_B = 3.52\ \dfrac{\text{rad}}{\text{s}}$

$\alpha_A = \dfrac{2}{3} k t_1^{\frac{-1}{3}} \qquad \alpha_A = 15.88\ \dfrac{\text{rad}}{\text{s}^2} \qquad \alpha_B = \dfrac{r_A}{r_B}\alpha_A \qquad \alpha_B = 3.97\ \dfrac{\text{rad}}{\text{s}^2}$

$v_C = r_C\omega_B \qquad v_C = 8.81\ \dfrac{\text{in}}{\text{s}}$

$a_C = \sqrt{\left(r_C\alpha_B\right)^2 + \left(r_C\omega_B^{\,2}\right)^2} \qquad a_C = 32.6\ \dfrac{\text{in}}{\text{s}^2}$

Problem 16-21

Due to the screw at E, the actuator provides linear motion to the arm at F when the motor turns the gear at A. If the gears have the radii listed, and the screw at E has pitch p, determine the speed at F when the motor turns A with angular velocity ω_A. *Hint*: The screw pitch indicates the amount of advance of the screw for each full revolution.

Given:

$rev = 2\pi\ rad$

$p = 2\ \frac{mm}{rev}$

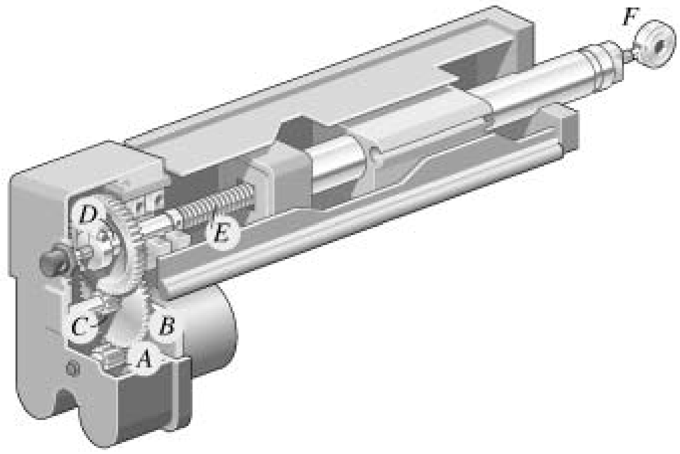
$\omega_A = 20\ \frac{rad}{s}$

$r_A = 10\ mm$

$r_B = 50\ mm$

$r_C = 15\ mm$

$r_D = 60\ mm$



Solution:

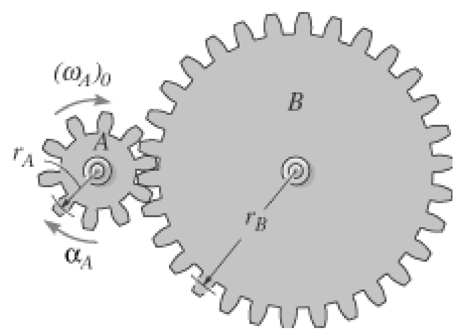
$\omega_A r_A = \omega_B r_B \qquad \omega_B r_C = \omega_D r_D$

$\omega_D = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega_A \qquad \omega_D = 1\,\frac{rad}{s}$

$v_F = \omega_D\,p \qquad v_F = 0.318\,\frac{mm}{s}$

Problem 16-22

A motor gives gear A angular acceleration $\alpha_A = a\theta^3 + b$. If this gear is initially turning with angular velocity ω_{A0}, determine the angular velocity of gear B after A undergoes an angular displacement θ_I.



Given:

$rev = 2\pi\ rad$

$a = 0.25\ \frac{rad}{s^2}$

$b = 0.5\ \frac{rad}{s^2}$

$\omega_{A0} = 20\ \frac{rad}{s}$

$r_A = 0.05\ \text{m}$

$r_B = 0.15\ \text{m}$

$\theta_1 = 10\ \text{rev}$

Solution:

$\alpha_A = a\theta^3 + b$ $\quad\omega_A^2 = \omega_{A0}^2 + 2\int_0^{\theta_1} \left(a\theta^3 + b\right) d\theta$

$\omega_A = \sqrt{\omega_{A0}^2 + 2\int_0^{\theta_1} a\theta^3 + b\ d\theta}$ $\quad\omega_A = 1395.94\ \frac{\text{rad}}{\text{s}}$

$\omega_B = \frac{r_A}{r_B}\omega_A$ $\quad\omega_B = 465\ \frac{\text{rad}}{\text{s}}$

Problem 16-23

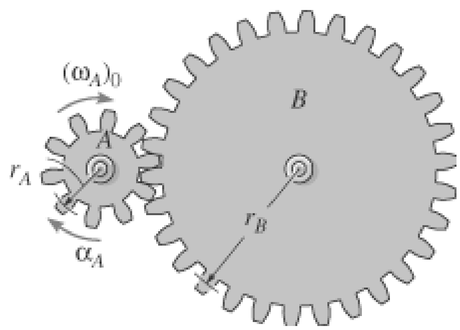
A motor gives gear A angular acceleration $\alpha_A = kt^3$. If this gear is initially turning with angular velocity ω_{A0}, determine the angular velocity of gear B when $t = t_1$.

Given:

$k = 4\ \frac{\text{rad}}{\text{s}^5}$ $\quad t_1 = 2\ \text{s}$

$r_A = 0.05\ \text{m}$

$\omega_{A0} = 20\ \frac{\text{rad}}{\text{s}}$ $\quad r_B = 0.15\ \text{m}$



Solution: $\quad t = t_1$

$\alpha_A = kt^3$ $\quad\omega_A = \left(\frac{k}{4}\right)t^4 + \omega_{A0}$ $\quad\omega_A = 36.00\ \frac{\text{rad}}{\text{s}}$

$\omega_B = \frac{r_A}{r_B}\omega_A$ $\quad\omega_B = 12.00\ \frac{\text{rad}}{\text{s}}$

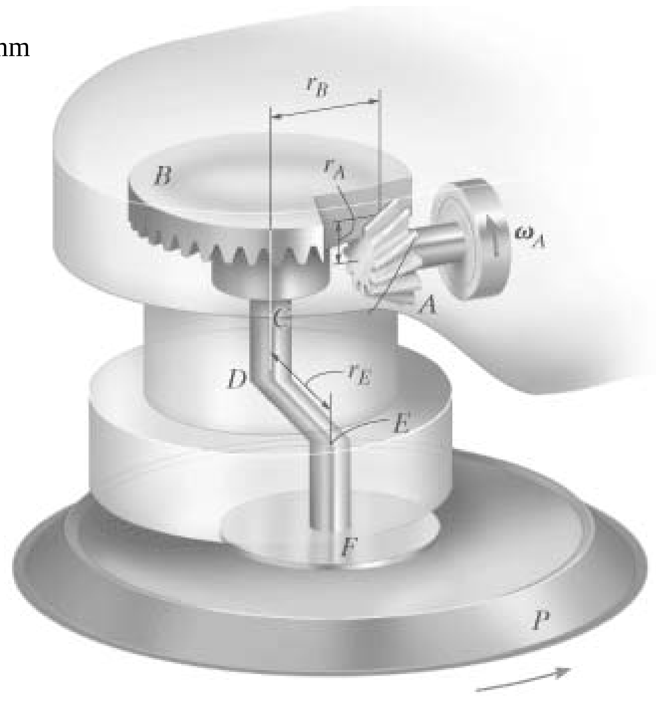
***Problem 16-24**

For a short time a motor of the random-orbit sander drives the gear A with an angular velocity $\omega_A = A(t^3 + Bt)$. This gear is connected to gear B, which is fixed connected to the shaft CD. The end of this shaft is connected to the eccentric spindle EF and pad P, which causes the pad to orbit around shaft CD at a radius r_E. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle *EF* at time *t* after starting from rest.

Given:

$r_A = 10\ \text{mm} \quad r_B = 40\ \text{mm} \quad r_E = 15\ \text{mm}$

$A = 40\ \frac{\text{rad}}{\text{s}^4} \quad B = 6\ \text{s}^2 \qquad t = 2\ \text{s}$



Solution:

$\omega_A = A\left(t^3 + B\,t\right) \qquad \omega_B = \frac{r_A}{r_B}\omega_A$

$\alpha_A = A\left(3t^2 + B\right) \qquad \alpha_B = \frac{r_A}{r_B}\alpha_A$

$v = \omega_B r_E \qquad v = 3.00\ \frac{\text{m}}{\text{s}}$

$a_t = \alpha_B r_E \qquad a_t = 2.70\ \frac{\text{m}}{\text{s}^2}$

$a_n = \omega_B^{\ 2} r_E \qquad a_n = 600.00\ \frac{\text{m}}{\text{s}^2}$

Problem 16-25

For a short time the motor of the random-orbit sander drives the gear *A* with an angular velocity $\omega_A = k\theta^2$. This gear is connected to gear *B*, which is fixed connected to the shaft *CD*. The end of this shaft is connected to the eccentric spindle *EF* and pad *P*, which causes the pad to orbit around shaft *CD* at a radius r_E. Determine the magnitudes of the velocity and the tangential and normal components of acceleration of the spindle *EF* when $\theta = \theta_1$ starting from rest.

Units Used:

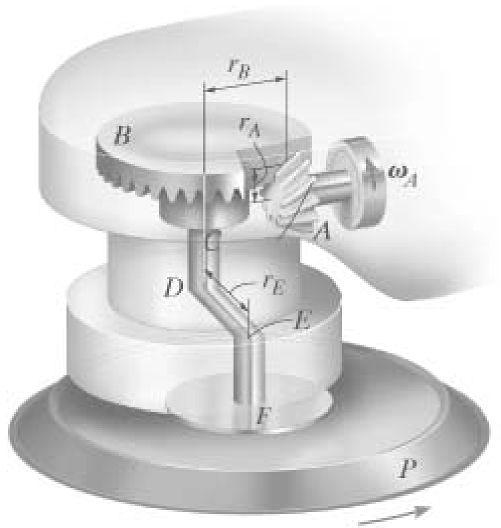
$\text{rev} = 2\pi\ \text{rad}$

Given:

$k = 5 \frac{\text{rad}}{\text{s}}$ $r_A = 10 \text{ mm}$

$r_B = 40 \text{ mm}$

$\theta_1 = 0.5 \text{ rev}$ $r_E = 15 \text{ mm}$



Solution:

$\omega_A = k\theta_1^{\,2}$

$\alpha_A = \left(k\theta_1^{\,2}\right)\left(2k\theta_1\right)$

$\omega_B = \frac{r_A}{r_B}\omega_A$ $\alpha_B = \frac{r_A}{r_B}\alpha_A$

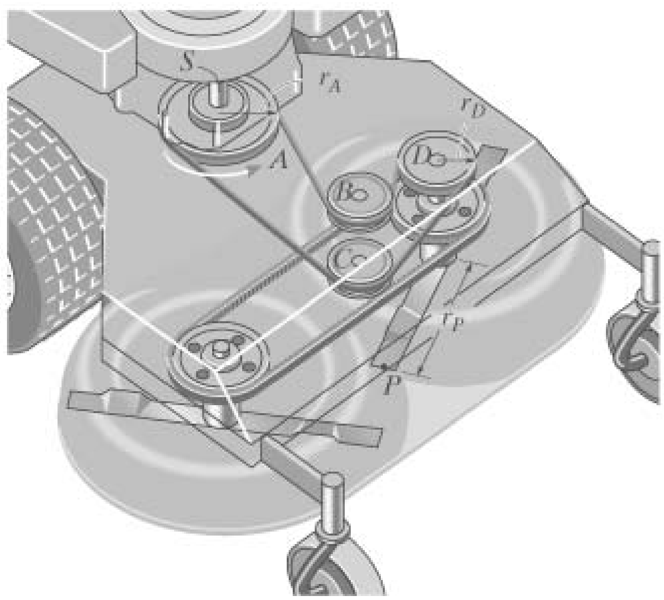
$v = \omega_B r_E$ $v = 0.19 \frac{\text{m}}{\text{s}}$

$a_t = \alpha_B r_E$ $a_t = 5.81 \frac{\text{m}}{\text{s}^2}$

$a_n = \omega_B^{\,2} r_E$ $a_n = 2.28 \frac{\text{m}}{\text{s}^2}$

Problem 16-26

The engine shaft S on the lawnmower rotates at a constant angular rate ω_A. Determine the magnitudes of the velocity and acceleration of point P on the blade and the distance P travels in time t. The shaft S is connected to the driver pulley A, and the motion is transmitted to the belt that passes over the idler pulleys at B and C and to the pulley at D. This pulley is connected to the blade and to another belt that drives the other blade.



Given:

$\omega_A = 40 \frac{\text{rad}}{\text{s}}$ $r_P = 200 \text{ mm}$

$r_A = 75 \text{ mm}$ $\alpha_A = 0$

$r_D = 50 \text{ mm}$ $t = 3 \text{ s}$

Solution:

$$\omega_D = \frac{r_A}{r_D}\omega_A$$

$$v_P = \omega_D r_P \qquad v_P = 12.00\ \frac{\text{m}}{\text{s}}$$

$$a_P = \omega_D^{\,2} r_P \qquad a_P = 720.00\ \frac{\text{m}}{\text{s}^2}$$

$$s_P = r_P\left(\frac{\omega_A t\, r_A}{r_D}\right) \qquad s_P = 36.00\ \text{m}$$

Problem 16-27

The operation of “reverse” for a three-speed automotive transmission is illustrated schematically in the figure. If the crank shaft *G* is turning with angular speed ω_G , determine the angular speed of the drive shaft *H*. Each of the gears rotates about a fixed axis. Note that gears *A* and *B*, *C* and *D*, and *E* and *F* are in mesh. The radii of each of these gears are listed.

Given:

$$\omega_G = 60\ \frac{\text{rad}}{\text{s}}$$

$r_A = 90\ \text{mm}$

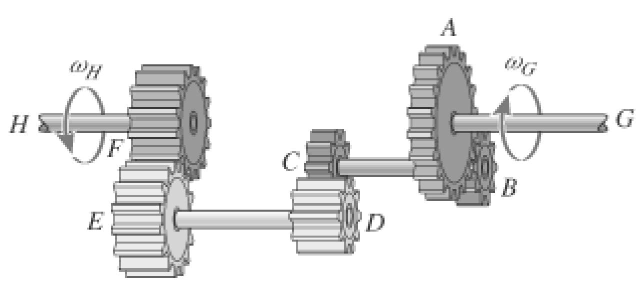
$r_B = 30\ \text{mm}$

$r_C = 30\ \text{mm}$

$r_D = 50\ \text{mm}$

$r_E = 70\ \text{mm}$

$r_F = 60\ \text{mm}$



Solution:

$$\omega_B = \frac{r_A}{r_B}\omega_G \qquad \omega_B = 180.00\ \frac{\text{rad}}{\text{s}}$$

$$\omega_D = \frac{r_C}{r_D}\omega_B \qquad \omega_D = 108.00\ \frac{\text{rad}}{\text{s}}$$

$$\omega_H = \frac{r_E}{r_F}\omega_D \qquad \omega_H = 126.00\ \frac{\text{rad}}{\text{s}}$$

***Problem 16-28**

Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = ke^{bt}$, determine the angular velocity of shaft E at time t after starting from rest. The radius of each gear is listed. Note that gears B and C are fixed connected to the same shaft.

Given:

$r_A = 20\ \text{mm}$

$r_B = 80\ \text{mm}$

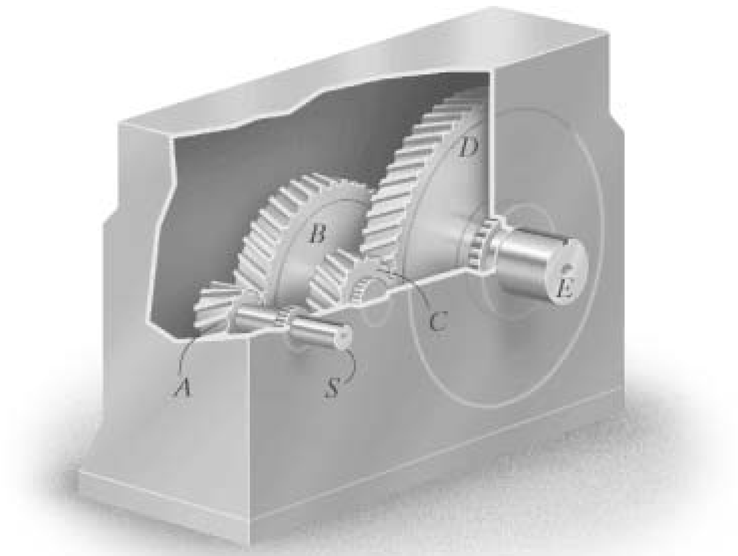
$r_C = 30\ \text{mm}$

$r_D = 120\ \text{mm}$

$k = 0.4\ \dfrac{\text{rad}}{\text{s}^2}$

$b = 1\ \text{s}^{-1}$

$t = 2\ \text{s}$



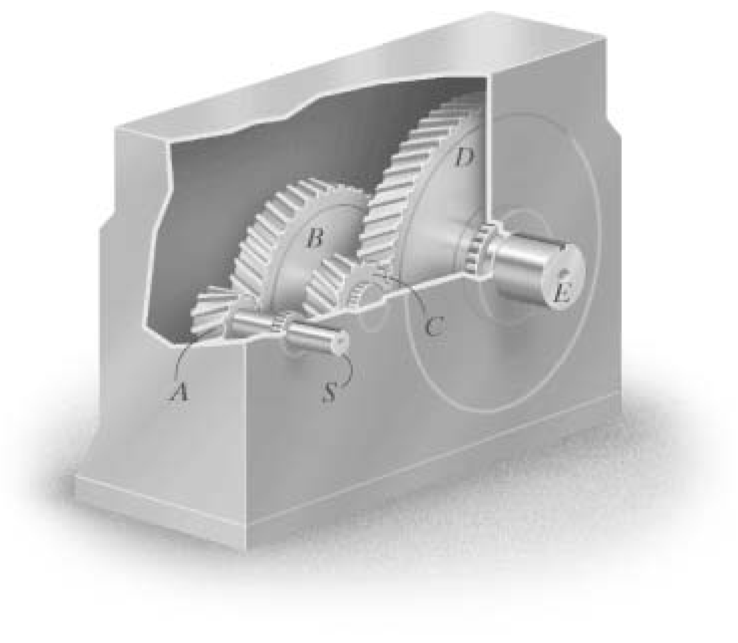
Solution:

$$\omega = \int_0^t k e^{b\,t}\,dt \qquad \omega = 2.56\ \frac{\text{rad}}{\text{s}}$$

$$\omega_E = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega \qquad \omega_E = 0.160\ \frac{\text{rad}}{\text{s}}$$

Problem 16-29

Morse Industrial manufactures the speed reducer shown. If a motor drives the gear shaft S with an angular acceleration $\alpha = k\omega^3$, determine the angular velocity of shaft E at time t_1 after gear S starts from an angular velocity ω_0 when $t = 0$. The radius of each gear is listed. Note that gears B and C are fixed connected to the same shaft.



Given:

$r_A = 20\ \text{mm}$

$r_B = 80\ \text{mm}$

$r_C = 30\ \text{mm}$

$r_D = 120\ \text{mm}$

$\omega_0 = 1\ \frac{\text{rad}}{\text{s}}$

$k = 4\ \frac{\text{rad}}{\text{s}^5}$

$t_1 = 2\ \text{s}$

Solution:

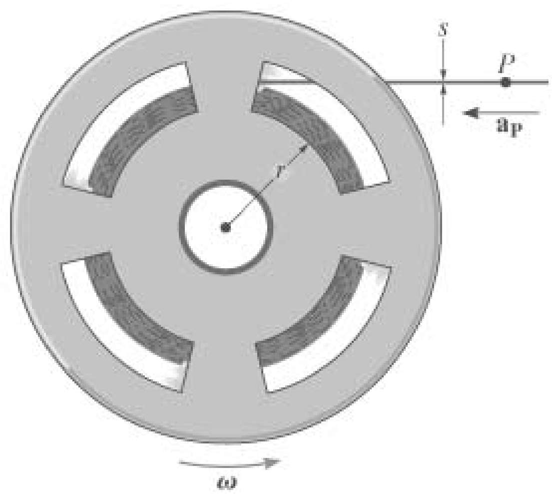
Guess $\omega_1 = 1\ \frac{\text{rad}}{\text{s}}$

Given $\int_0^{t_1} k\, dt = \int_{\omega_0}^{\omega_1} \omega^3\, d\omega \qquad \omega_1 = \text{Find}(\omega_1)$

$\omega_1 = 2.40\ \frac{\text{rad}}{\text{s}} \qquad \omega_E = \left(\frac{r_A}{r_B}\right)\left(\frac{r_C}{r_D}\right)\omega_1 \qquad \omega_E = 0.150\ \frac{\text{rad}}{\text{s}}$

Problem 16-30

A tape having a thickness *s* wraps around the wheel which is turning at a constant rate ω. Assuming the unwrapped portion of tape remains horizontal, determine the acceleration of point *P* of the unwrapped tape when the radius of the wrapped tape is *r*. *Hint:* Since $v_P = \omega r$, take the time derivative and note that $dr/dt = \omega(s/2\pi)$.



Solution:

$v_P = \omega r$

$a_p = \frac{dv_p}{dt} = \frac{d\omega}{dt} r + \omega \frac{dr}{dt}$

since $\frac{d\omega}{dt} = 0, \qquad a_p = \omega\left(\frac{dr}{dt}\right)$

In one revolution *r* is increased by *s*, so that

$\frac{2\pi}{\Delta\theta} = \frac{s}{\Delta r}$

Hence,

$$\Delta r = \frac{s}{2\pi}\Delta\theta \qquad \frac{dr}{dt} = \frac{s}{2\pi}\omega$$

$$a_p = \frac{s}{2\pi}\omega^2$$

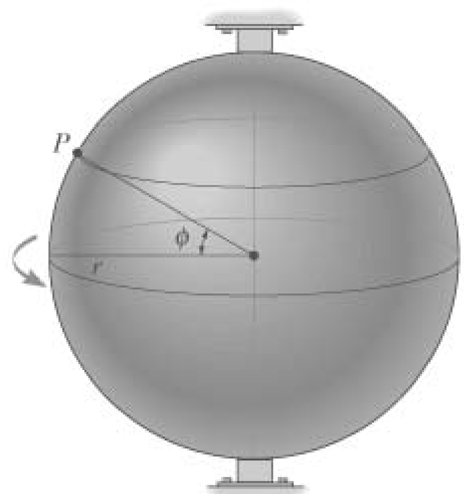
Problem 16-31

The sphere starts from rest at $\theta = 0°$ and rotates with an angular acceleration $\alpha = k\theta$. Determine the magnitudes of the velocity and acceleration of point P on the sphere at the instant $\theta = \theta_1$.

Given:

$\theta_1 = 6 \text{ rad}$ $\qquad r = 8 \text{ in}$

$\phi = 30 \text{ deg}$ $\qquad k = 4 \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\alpha = k\theta_1$$

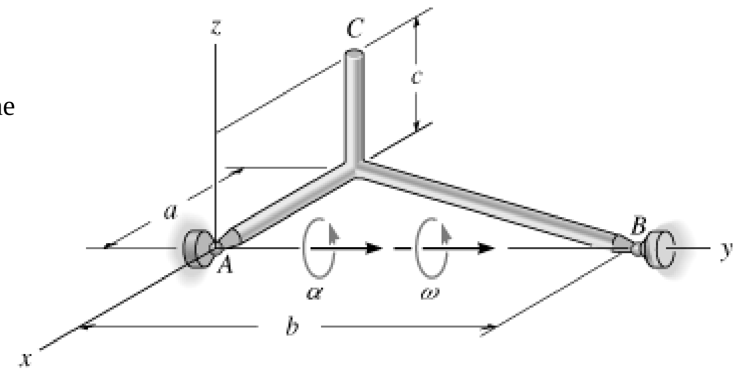
$$\frac{\omega^2}{2} = k\left(\frac{\theta_1^{\,2}}{2}\right) \qquad \omega = \sqrt{k}\,\theta_1$$

$$v_P = \omega r\cos(\phi) \qquad v_P = 6.93 \frac{\text{ft}}{\text{s}}$$

$$a_P = \sqrt{(\alpha r\cos(\phi))^2 + \left(\omega^2 r\cos(\phi)\right)^2} \qquad a_P = 84.3 \frac{\text{ft}}{\text{s}^2}$$

***Problem 16-32**

The rod assembly is supported by ball-and-socket joints at A and B. At the instant shown it is rotating about the y axis with angular velocity ω and has angular acceleration α. Determine the magnitudes of the velocity and acceleration of point C at this instant. Solve the problem using Cartesian vectors and Eqs. 16-9 and 16-13.



Given:

$\omega = 5\ \frac{\text{rad}}{\text{s}}$ $a = 0.4\ \text{m}$

$b = 0.4\ \text{m}$

$\alpha = 8\ \frac{\text{rad}}{\text{s}^2}$ $c = 0.3\ \text{m}$

Solution:

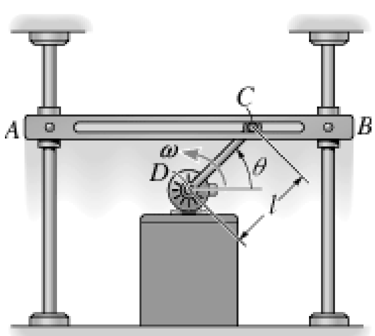
$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{r_{AC}} = \begin{pmatrix} -a \\ 0 \\ c \end{pmatrix}$$

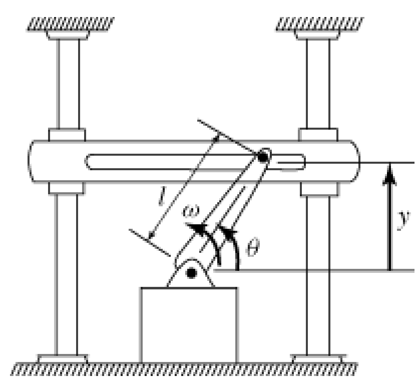
$$\mathbf{v_C} = (\omega\mathbf{j}) \times \mathbf{r_{AC}} \qquad \mathbf{v_C} = \begin{pmatrix} 1.50 \\ 0.00 \\ 2.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad |\mathbf{v_C}| = 2.50\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = (\alpha\mathbf{j}) \times \mathbf{r_{AC}} + (\omega\mathbf{j}) \times \left[(\omega\mathbf{j}) \times \mathbf{r_{AC}}\right] \qquad \mathbf{a_C} = \begin{pmatrix} 12.40 \\ 0.00 \\ -4.30 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_C}| = 13.12\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-33

The bar *DC* rotates uniformly about the shaft at *D* with a constant angular velocity ω. Determine the velocity and acceleration of the bar *AB*, which is confined by the guides to move vertically.





Solution: $\theta' = \omega$ $\theta'' = \alpha = 0$

$$y = l\sin(\theta)$$

$$y' = v_y = l\cos(\theta)\theta' \qquad v_{AB} = \omega l\cos(\theta)$$

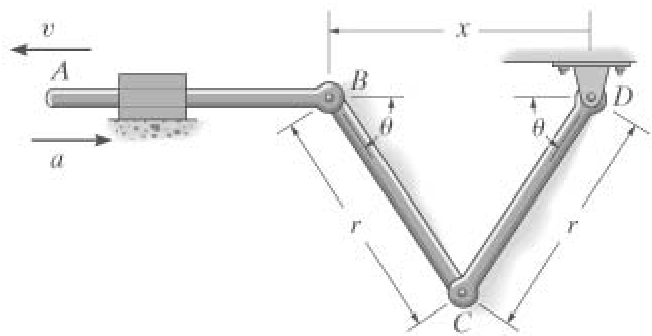
$$y'' = a_y = l\left(\cos(\theta)\,\theta'' - \sin(\theta)\,\theta'^2\right) \qquad a_{AB} = -\omega^2 l\sin(\theta)$$

Problem 16-34

At the instant shown, θ is given, and rod *AB* is subjected to a deceleration *a* when the velocity is *v*. Determine the angular velocity and angular acceleration of link *CD* at this instant.

Given:

$v = 10\ \frac{\text{m}}{\text{s}}$ $a = 16\ \frac{\text{m}}{\text{s}^2}$



$\theta = 60\ \text{deg}$ $r = 300\ \text{mm}$

Solution:

$x = 2r\cos(\theta)$ $x = 0.30\ \text{m}$

$x' = -2r\sin(\theta)\theta'$

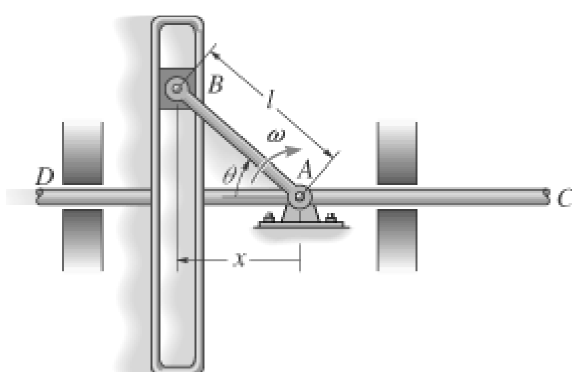
$\omega = \frac{-v}{2r\sin(\theta)}$ $\omega = -19.2\ \frac{\text{rad}}{\text{s}}$

$x'' = -2r\cos(\theta)\,\theta'^2 - 2r\sin(\theta)\theta''$

$\alpha = \frac{a - 2r\cos(\theta)\,\omega^2}{2r\sin(\theta)}$ $\alpha = -183\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-35

The mechanism is used to convert the constant circular motion ω of rod *AB* into translating motion of rod *CD*. Determine the velocity and acceleration of *CD* for any angle θ of *AB*.



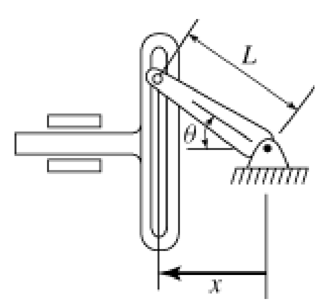
Solution:

$x = l\cos(\theta)$ $x' = v_x = -l\sin(\theta)\theta'$

$x'' = a_x = -l\left(\sin(\theta)\,\theta'' + \cos(\theta)\,\theta'^2\right)$

$v_x = v_{CD}$ $a_x = a_{CD}$ and $\theta' = \omega$ $\theta'' = \alpha = 0$

$v_{CD} = -\omega l\sin(\theta)$ $a_{CD} = -\omega^2 l\cos(\theta)$



***Problem 16-36**

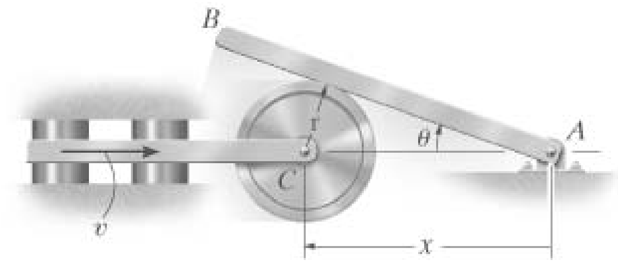
Determine the angular velocity of rod *AB* for the given θ. The shaft and the center of the roller *C* move forward at a constant rate *v*.

Given:

$$v = 5\ \frac{\text{m}}{\text{s}}$$

$$\theta = 30\ \text{deg}$$

$$r = 100\ \text{mm}$$



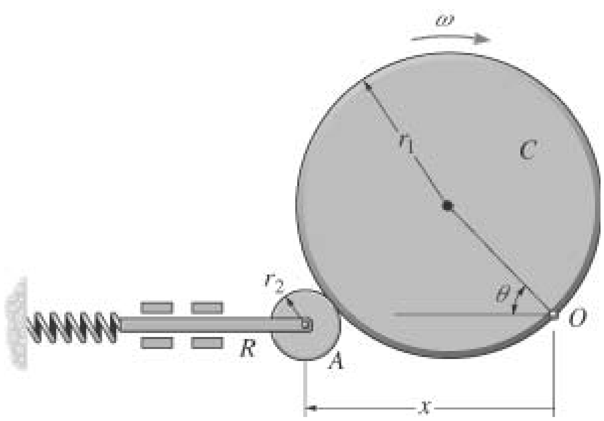
Solution:

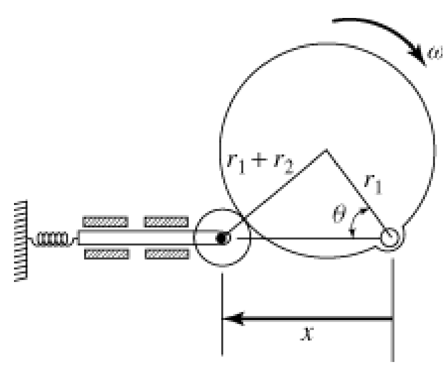
$$r = x\sin(\theta) \qquad 0 = x'\sin(\theta) + x\cos(\theta)\theta' = -v\sin(\theta) + x\cos(\theta)\omega$$

$$x = \frac{r}{\sin(\theta)} \qquad \omega = \left(\frac{v}{x}\right)\tan(\theta) \qquad \omega = 14.43\ \frac{\text{rad}}{\text{s}}$$

Problem 16-37

Determine the velocity of rod *R* for any angle θ of the cam *C* if the cam rotates with a constant angular velocity ω. The pin connection at *O* does not cause an interference with the motion of *A* on *C*.





Solution:

Position Coordinate Equation: Using law of cosines.

$$(r_1 + r_2)^2 = x^2 + r_1^2 - 2r_1 x\cos(\theta)$$

$$x = r_1\cos(\theta) + \sqrt{r_1^2\cos(\theta)^2 + 2r_1 r_2 + r_2^2}$$

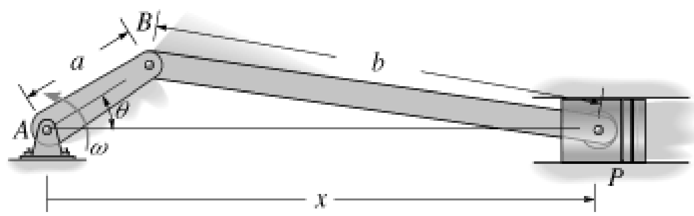
$$0 = 2xx' - 2r_1 x'\cos(\theta) + 2r_1 x\sin(\theta)\theta'$$

$$x' = \frac{-r_1 x \sin(\theta)\theta'}{x - r_1 \cos(\theta)} \qquad v = -r_1 \sin(\theta)\,\omega\left(1 + \frac{r_1 \cos(\theta)}{\sqrt{r_1^{\,2}\cos(\theta)^2 + 2r_1 r_2 + r_2^{\,2}}}\right)$$

Problem 16-38

The crankshaft *AB* is rotating at constant angular velocity ω. Determine the velocity of the piston *P* for the given θ.

Given:



$$\omega = 150\ \frac{\text{rad}}{\text{s}}$$

$$\theta = 30\ \text{deg}$$

$$a = 0.2\ \text{ft}$$

$$b = 0.75\ \text{ft}$$

Solution:

$$x = (a)\cos(\theta) + \sqrt{b^2 - a^2\sin^2(\theta)}$$

$$x' = -(a)\sin(\theta)\theta' - \frac{a^2\cos(\theta)\sin(\theta)\theta'}{\sqrt{b^2 - a^2\sin(\theta)^2}}$$

$$v = -(a)\sin(\theta)\omega - \frac{a^2\cos(\theta)\sin(\theta)\omega}{\sqrt{b^2 - a^2\sin(\theta)^2}} \qquad v = -18.50\ \frac{\text{ft}}{\text{s}}$$

Problem 16-39

At the instant $\theta = \theta_1$ the slotted guide is moving upward with acceleration *a* and velocity *v*. Determine the angular acceleration and angular velocity of link *AB* at this instant. *Note*: The upward motion of the guide is in the negative *y* direction.

Given:

$$\theta_1 = 50\ \text{deg} \quad v = 2\ \frac{\text{m}}{\text{s}}$$

$$a = 3\ \frac{\text{m}}{\text{s}^2} \quad L = 300\ \text{mm}$$

Solution:

$y = L\cos(\theta)$

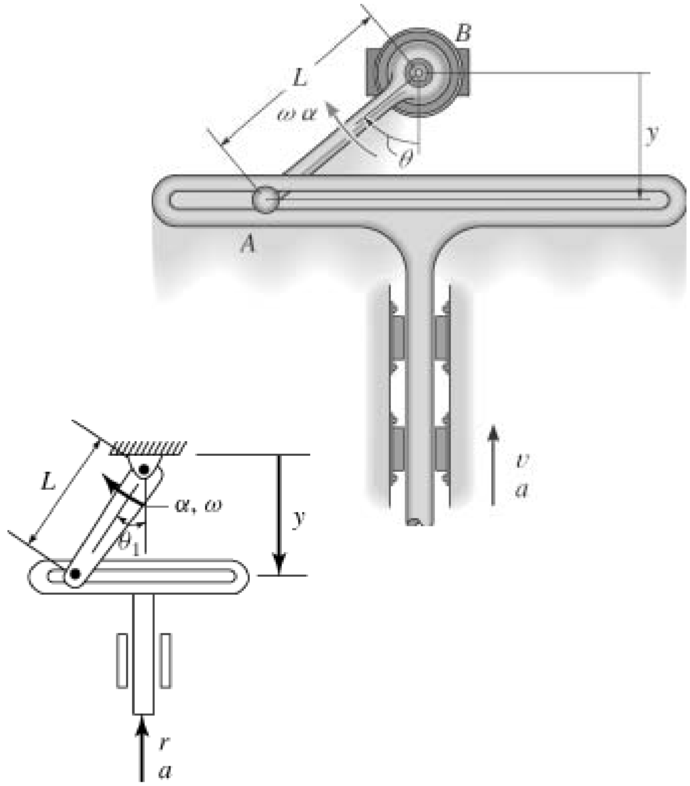
$y' = -L\sin(\theta)\theta'$

$y'' = -L\sin(\theta)\theta'' - L\cos(\theta)\theta'^2$

$\omega = \dfrac{v}{L\sin(\theta_1)}$ $\omega = 8.70 \dfrac{\text{rad}}{\text{s}}$

$\alpha = \dfrac{a - L\cos(\theta_1)\omega^2}{L\sin(\theta_1)}$

$\alpha = -50.50 \dfrac{\text{rad}}{\text{s}^2}$



***Problem 16-40**

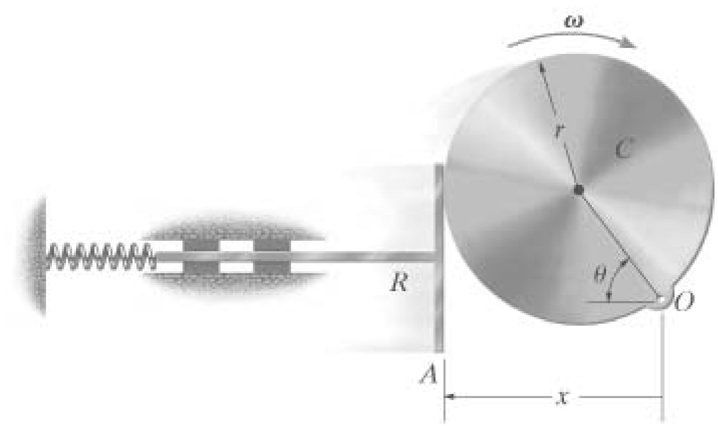
Determine the velocity of the rod R for any angle θ of cam C as the cam rotates with a constant angular velocity ω. The pin connection at O does not cause an interference with the motion of plate A on C.

Solution:

$x = r + r\cos(\theta)$

$x' = -r\sin(\theta)\theta'$

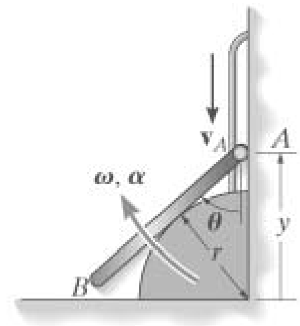
$v = -r\omega\sin(\theta)$



Problem 16-41

The end A of the bar is moving downward along the slotted guide with a constant velocity v_A. Determine the angular velocity ω and angular acceleration a of the bar as a function of its position y.

Solution: $y' = -v_A \qquad y'' = 0$



$$r = y\sin(\theta)$$

$$0 = y'\sin(\theta) + y\cos(\theta)\theta'$$

$$0 = y''\sin(\theta) + 2y'\cos(\theta)\theta' - y\sin(\theta)\,\theta'^2 + y\cos(\theta)\theta''$$

$$\omega = \left(\frac{v_A}{y}\right)\tan(\theta) \qquad \omega = \frac{v_A}{y}\left(\frac{r}{\sqrt{y^2 - r^2}}\right)$$

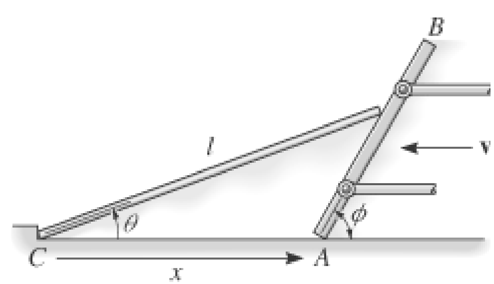
$$\alpha = 2\left(\frac{v_A}{y}\right)\omega + \tan(\theta)\,\omega^2$$

$$\alpha = 2\frac{v_A^2}{y^2}\left(\frac{r}{\sqrt{y^2 - r^2}}\right) + \frac{v_A^2 r^3}{y^2\sqrt{(y^2 - r^2)^3}} \qquad \alpha = \frac{r v_A^2(2y^2 - r^2)}{y^2\sqrt{(y^2 - r^2)^3}}$$

Problem 16-42

The inclined plate moves to the left with a constant velocity **v**. Determine the angular velocity and angular acceleration of the slender rod of length *l*. The rod pivots about the step at *C* as it slides on the plate.

Solution: $x' = -v$



$$\frac{x}{\sin(\phi - \theta)} = \frac{l}{\sin(180\text{ deg} - \phi)} = \frac{l}{\sin(\phi)}$$

$$x\sin(\phi) = l\sin(\phi - \theta)$$

$$x'\sin(\phi) = -l\cos(\phi - \theta)\theta'$$

Thus, $\omega = \dfrac{-v\sin(\phi)}{l\cos(\phi - \theta)}$

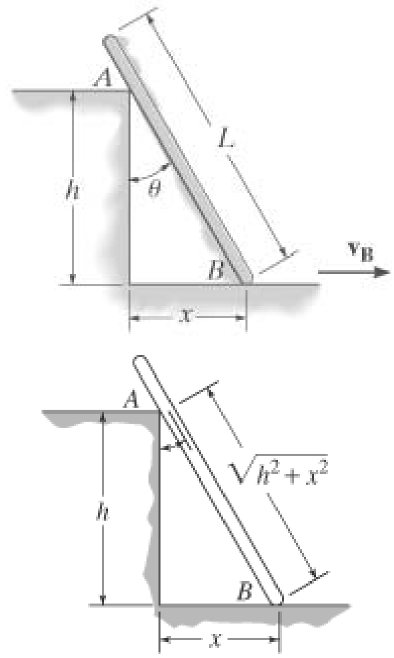
$$x''\sin(\phi) = -l\cos(\phi - \theta)\theta'' - l\sin(\phi - \theta)\,\theta'^2$$

$$0 = -\cos(\phi - \theta)\,\alpha - \sin(\phi - \theta)\,\omega^2$$

$$\alpha = \frac{-\sin(\phi - \theta)}{\cos(\phi - \theta)}\left[\frac{v^2\sin\phi^2}{l^2\cos(\phi - \theta)^2}\right] \qquad \alpha = \frac{-v^2\sin^2(\phi)\sin(\phi - \theta)}{l^2\cos(\phi - \theta)^3}$$

Problem 16-43

The bar remains in contact with the floor and with point A. If point B moves to the right with a constant velocity v_B, determine the angular velocity and angular acceleration of the bar as a function of x.



Solution: $x' = v_B$ $\quad x'' = 0$

$$x = h\tan(\theta)$$

$$x' = h\sec(\theta)^2\theta'$$

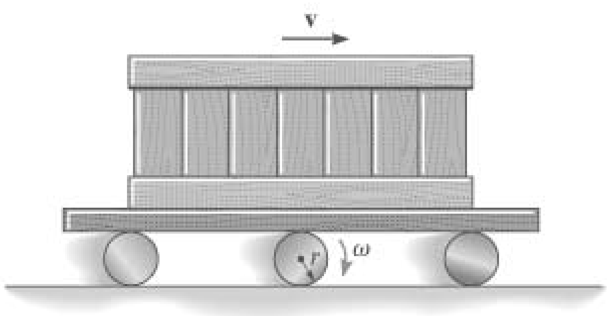
$$x'' = h\sec(\theta)^2\theta'' + 2h\sec(\theta)^2\tan(\theta)\,\theta'^2$$

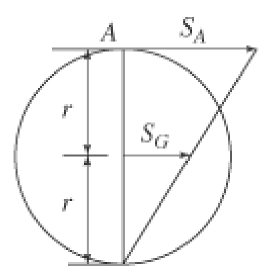
$$\sec(\theta) = \frac{\sqrt{h^2+x^2}}{h} \qquad \tan(\theta) = \frac{x}{h}$$

$$\omega = \frac{h\,v_B}{h^2+x^2} \qquad \alpha = \frac{-2h\,x\,v_B^{\,2}}{\left(h^2+x^2\right)^2}$$

***Problem 16-44**

The crate is transported on a platform which rests on rollers, each having a radius r. If the rollers do not slip, determine their angular velocity if the platform moves forward with a velocity **v**.





Solution:

Position coordinate equation: $s_G = r\theta$. Using similar triangles $s_A = 2s_G = 2r\theta$

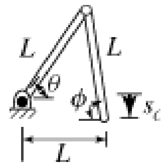
$s'_A = v = 2r\theta'$ $\qquad$ where $\theta' = \omega$

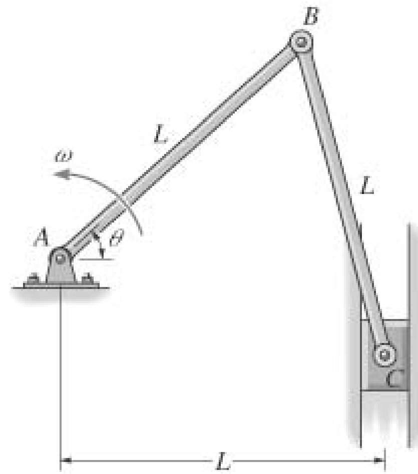
$$\omega = \frac{v}{2r}$$

Problem 16-45

Bar *AB* rotates uniformly about the fixed pin *A* with a constant angular velocity ω. Determine the velocity and acceleration of block *C* when $\theta = \theta_1$.

Given:





$$L = 1\ \text{m}$$

$$\theta_1 = 60\ \text{deg}$$

$$\omega = 2\ \frac{\text{rad}}{\text{s}}$$

$$\alpha = 0\ \frac{\text{rad}}{\text{s}^2}$$

Solution:

$$\theta = \theta_1 \qquad \theta' = \omega \qquad \theta'' = \alpha$$

Guesses $\quad \phi = 60\ \text{deg} \quad \phi' = 1\ \frac{\text{rad}}{\text{s}} \quad \phi'' = 1\ \frac{\text{rad}}{\text{s}^2}$

$$s_C = 1\ \text{m} \qquad v_C = -1\ \frac{\text{m}}{\text{s}} \qquad a_C = -2\ \frac{\text{m}}{\text{s}^2}$$

Given

$$L\cos(\theta) + L\cos(\phi) = L$$

$$\sin(\theta)\,\theta' + \sin(\phi)\,\phi' = 0$$

$$\cos(\theta)\,\theta'^2 + \sin(\theta)\,\theta'' + \sin(\phi)\,\phi'' + \cos(\phi)\,\phi'^2 = 0$$

$$s_C = L\sin(\phi) - L\sin(\theta)$$

$$v_C = L\cos(\phi)\phi' - L\cos(\theta)\theta'$$

$$a_C = -L\sin(\phi)\,\phi'^2 + L\cos(\phi)\phi'' + L\sin(\theta)\,\theta'^2 - L\cos(\theta)\theta''$$

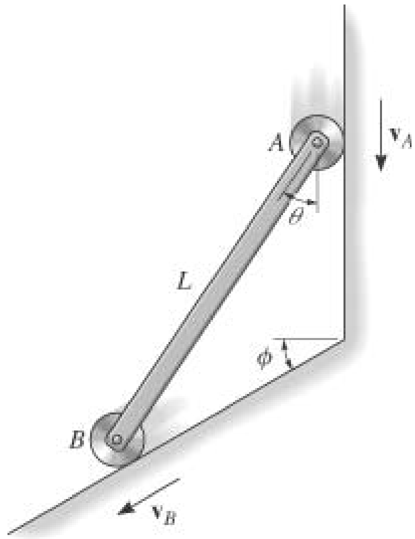
$$\begin{pmatrix} \phi \\ \phi' \\ \phi'' \\ s_C \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\phi, \phi', \phi'', s_C, v_C, a_C)$$

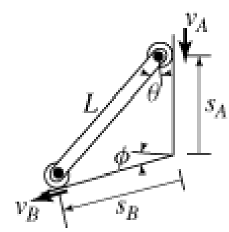
$$\phi = 60.00\ \text{deg} \qquad \phi' = -2.00\ \frac{\text{rad}}{\text{s}} \qquad \phi'' = -4.62\ \frac{\text{rad}}{\text{s}^2}$$

$$s_C = 0.00\ \text{m} \qquad v_C = -2.00\ \frac{\text{m}}{\text{s}} \qquad a_C = -2.31\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-46

The bar is confined to move along the vertical and inclined planes. If the velocity of the roller at A is v_A downward when $\theta = \theta_1$. determine the bar's angular velocity and the velocity of roller B at this instant.





Given:

$$v_A = 6\ \frac{ft}{s}$$

$$\theta_1 = 45\ deg$$

$$\phi = 30\ deg$$

$$L = 5\ ft$$

Solution: $\theta = \theta_1$

Guesses

$$s_A = 1\ ft \qquad s_B = 1\ ft \qquad \omega = 1\ \frac{rad}{s} \qquad v_B = 1\ \frac{ft}{s}$$

Given

$$L\sin(\theta) = s_B\cos(\phi) \qquad L\cos(\theta)\omega = v_B\cos(\phi)$$

$$L\cos(\theta) = s_A + s_B\sin(\phi) \qquad -L\sin(\theta)\omega = -v_A + v_B\sin(\phi)$$

$$\begin{pmatrix} s_A \\ s_B \\ \omega \\ v_B \end{pmatrix} = \text{Find}(s_A, s_B, \omega, v_B) \qquad \begin{pmatrix} s_A \\ s_B \end{pmatrix} = \begin{pmatrix} 1.49 \\ 4.08 \end{pmatrix} ft \qquad \omega = 1.08\ \frac{rad}{s} \qquad v_B = 4.39\ \frac{ft}{s}$$

Problem 16-47

When the bar is at the angle θ the rod is rotating clockwise at ω and has an angular acceleration α. Determine the velocity and acceleration of the weight A at this instant. The cord is of length L.

Given:

$$L = 20\ ft$$

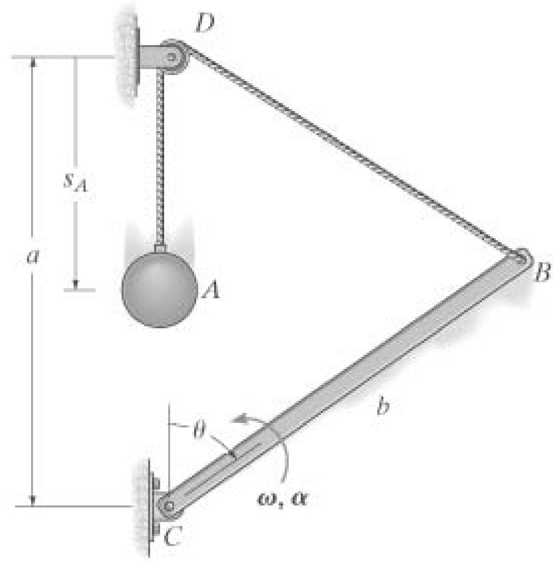
$$a = 10\ ft$$

$b = 10\ \text{ft}$

$\theta = 30\ \text{deg}$

$\omega = 3\ \dfrac{\text{rad}}{\text{s}}$

$\alpha = 5\ \dfrac{\text{rad}}{\text{s}^2}$



Solution: $\theta' = -\omega \qquad \theta'' = -\alpha$

$$s_A = L - \sqrt{a^2 + b^2 - 2ab\cos(\theta)}$$

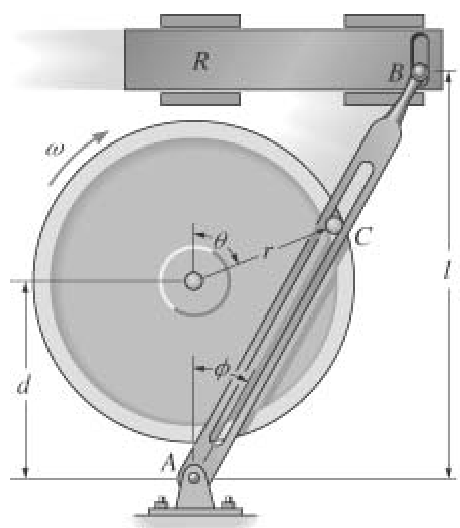
$$v_A = \frac{-ab\sin(\theta)\theta'}{\sqrt{a^2 + b^2 - 2ab\cos(\theta)}}$$

$$a_A = \frac{-ab\sin(\theta)\theta'' - ab\cos(\theta)\theta'^2}{\sqrt{a^2 + b^2 - 2ab\cos(\theta)}} + \frac{(ab\sin(\theta)\theta')^2}{\sqrt{\left(a^2 + b^2 - 2ab\cos(\theta)\right)^3}}$$

$s_A = 14.82\ \text{ft} \qquad v_A = 29.0\ \dfrac{\text{ft}}{\text{s}} \qquad a_A = 59.9\ \dfrac{\text{ft}}{\text{s}^2}$

***Problem 16-48**

The slotted yoke is pinned at A while end B is used to move the ram R horizontally. If the disk rotates with a constant angular velocity ω, determine the velocity and acceleration of the ram. The crank pin C is fixed to the disk and turns with it. The length of AB is L.

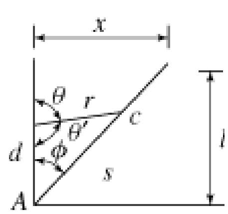


Solution:

$$x = L\sin(\phi)$$

$$s = \sqrt{d^2 + r^2 + 2rd\cos(\theta)}$$

$$s\sin(\phi) = r\sin(\theta)$$



Thus $$x = \frac{Lr\sin(\theta)}{\sqrt{d^2 + r^2 + 2rd\cos(\theta)}}$$

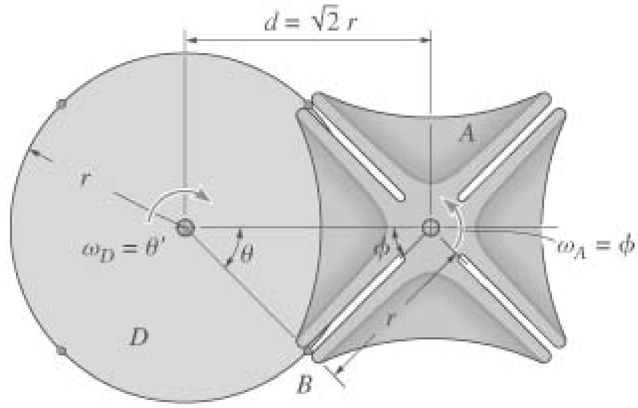
$$v = \frac{L r \cos(\theta)\omega}{\sqrt{d^2 + r^2 + 2rd\cos(\theta)}} + \frac{d L r^2 \sin(\theta)\omega}{\sqrt{\left(d^2 + r^2 + 2rd\cos(\theta)\right)^3}}$$

$$a = \frac{-L r \sin(\theta)\,\omega^2}{\sqrt{d^2 + r^2 + 2rd\cos(\theta)}} + \frac{3d L r^2 \sin(\theta)\cos(\theta)\,\omega^2}{\sqrt{\left(d^2 + r^2 + 2rd\cos(\theta)\right)^3}} + \frac{3d^2 L r^3 \sin(\theta)\,\omega^2}{\sqrt{\left(d^2 + r^2 + 2rd\cos(\theta)\right)^5}}$$

Problem 16-49

The Geneva wheel *A* provides intermittent rotary motion ω_A for continuous motion ω_D of disk *D*. By choosing $d = \sqrt{2}\,r$, the wheel has zero angular velocity at the instant pin *B* enters or leaves one of the four slots. Determine the magnitude of the angular velocity ω_A of the Geneva wheel when $\theta = \theta_1$ so that pin *B* is in contact with the slot.

Given:



$\omega_D = 2\ \frac{\text{rad}}{\text{s}}$

$r = 100\ \text{mm}$

$\theta_1 = 30\ \text{deg}$

Solution:

$$\theta = \theta_1$$

Guesses $\phi = 10\ \text{deg}$ $\omega_A = 1\ \frac{\text{rad}}{\text{s}}$

$s_{BA} = 10\ \text{mm}$ $s'_{BA} = 10\ \frac{\text{mm}}{\text{s}}$

Given $r\cos(\theta) + s_{BA}\cos(\phi) = \sqrt{2}\,r$

$$-r\sin(\theta)\omega_D + s'_{BA}\cos(\phi) - s_{BA}\sin(\phi)\omega_A = 0$$

$$r\sin(\theta) = s_{BA}\sin(\phi)$$

$$r\cos(\theta)\omega_D = s'_{BA}\sin(\phi) + s_{BA}\cos(\phi)\omega_A$$

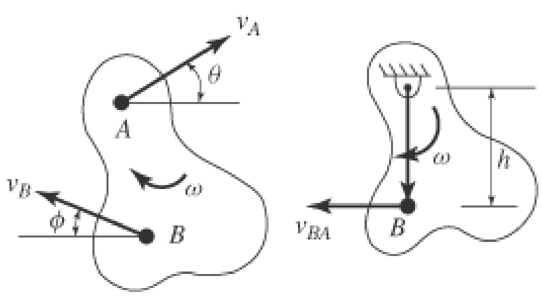
$$\begin{pmatrix} \phi \\ \omega_A \\ s_{BA} \\ s'_{BA} \end{pmatrix} = \text{Find}(\phi, \omega_A, s_{BA}, s'_{BA}) \qquad \phi = 42.37\,\text{deg} \qquad \omega_A = 0.816\,\frac{\text{rad}}{\text{s}}$$

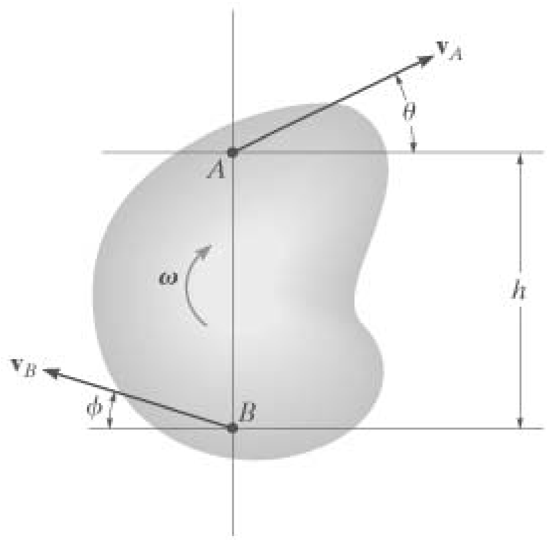
$$s_{BA} = 74.20\,\text{mm} \qquad s'_{BA} = 190.60\,\frac{\text{mm}}{\text{s}}$$

The general solution is $\omega_A = \omega_D\left(\dfrac{\sqrt{2}\cos(\theta) - 1}{3 - 2\sqrt{2}\cos(\theta)}\right)$

Problem 16-50

If h and θ are known, and the speed of A and B is $v_A = v_B = v$, determine the angular velocity ω of the body and the direction ϕ of v_B.





Solution:

$$v_B = v_A + \omega \times r_{BA}$$

$$-v\cos(\phi)\mathbf{i} + v\sin(\phi)\mathbf{j} = v\cos(\theta)\mathbf{i} + v\sin(\theta)\mathbf{j} + (-\omega\mathbf{k}) \times (-h\mathbf{j})$$

$$-v\cos(\phi) = v\cos(\theta) - \omega h \qquad (1)$$

$$v\sin(\phi) = v\sin(\theta) \qquad (2)$$

From Eq. (2), $\phi = \theta$

From Eq. (1), $\omega = \dfrac{2v}{h}\cos(\theta)$

Problem 16-51

The wheel is rotating with an angular velocity ω. Determine the velocity of the collar A for the given values of θ and ϕ.

Given:

$\theta = 30 \text{ deg}$

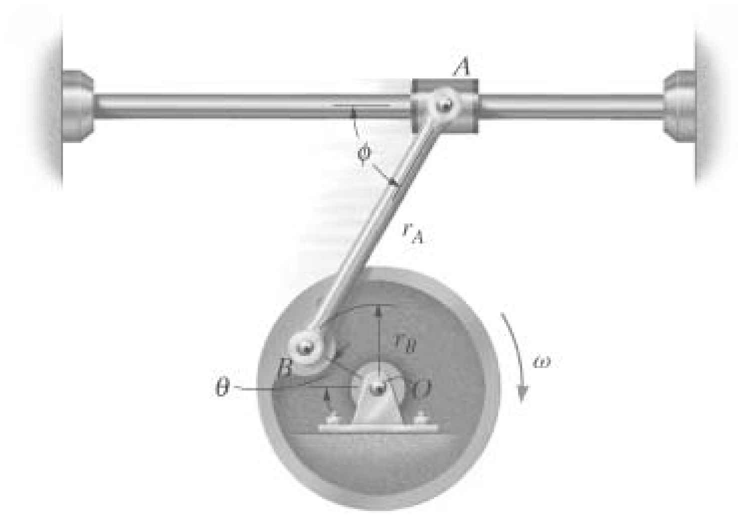
$\phi = 60 \text{ deg}$

$\omega = 8 \frac{\text{rad}}{\text{s}}$

$r_A = 500 \text{ mm}$

$r_B = 150 \text{ mm}$

$v_B = 1.2 \frac{\text{m}}{\text{s}}$



Solution:

Guesses $\omega_{AB} = 1 \frac{\text{rad}}{\text{s}}$ $v_A = 1 \frac{\text{m}}{\text{s}}$

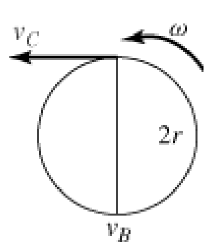
Given $$\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_B\cos(\theta) \\ r_B\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_A\cos(\phi) \\ r_A\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

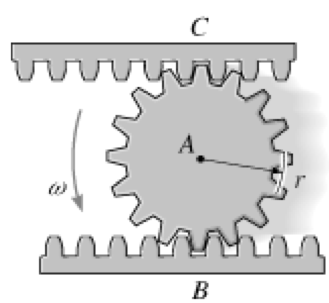
$$\begin{pmatrix} \omega_{AB} \\ v_A \end{pmatrix} = \text{Find}(\omega_{AB}, v_A) \qquad \omega_{AB} = -4.16 \frac{\text{rad}}{\text{s}} \qquad v_A = 2.40 \frac{\text{m}}{\text{s}}$$

***Problem 16-52**

The pinion gear A rolls on the fixed gear rack B with angular velocity ω. Determine the velocity of the gear rack C.

Given: $\omega = 4 \frac{\text{rad}}{\text{s}}$ $r = 0.3 \text{ ft}$





Solution: $v_C = v_B + v_{CB}$

$v_C = \omega(2r)$ $v_C = -6.56 \frac{\text{ft}}{\text{s}}$

Problem 16-53

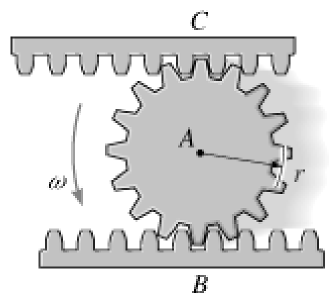
The pinion gear rolls on the gear racks. If B is moving to the right at speed v_B and C is moving to the left at speed v_C determine the angular velocity of the pinion gear and the velocity of its center A.

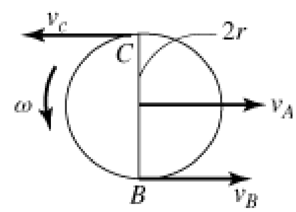
Given:

$$v_B = 8\ \frac{\text{ft}}{\text{s}}$$

$$v_C = 4\ \frac{\text{ft}}{\text{s}}$$

$$r = 0.3\ \text{ft}$$





Solution:

$$v_C = v_B + v_{CB}$$

$$-v_C = v_B - (2r)\,\omega$$

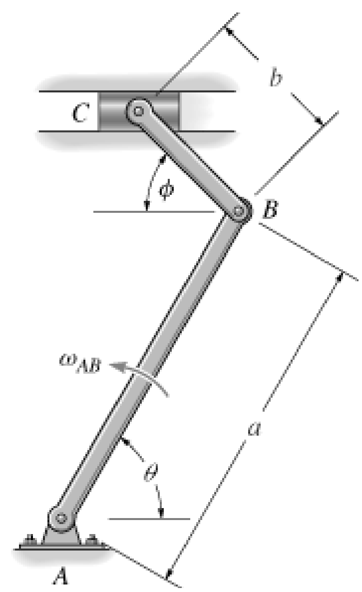
$$\omega = \frac{v_C + v_B}{2r} \qquad \omega = 20.00\ \frac{\text{rad}}{\text{s}}$$

$$v_A = v_B + v_{AB}$$

$$v_A = v_B + (-\omega)r \qquad v_A = 2\ \frac{\text{ft}}{\text{s}}$$

Problem 16-54

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at C. Determine the velocity of the slider block C at the instant shown, if link AB is rotating at angular velocity ω_{AB}.



Given:

$$\theta = 60\ \text{deg}$$

$$\phi = 45\ \text{deg}$$

$$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$$

$$a = 300\ \text{mm}$$

$$b = 125\ \text{mm}$$

Solution:

Guesses $\omega_{BC} = 1\,\frac{\text{rad}}{\text{s}}$ $v_C = 1\,\frac{\text{m}}{\text{s}}$

Given
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 6.79\,\frac{\text{rad}}{\text{s}} \qquad v_C = -1.64\,\frac{\text{m}}{\text{s}}$$

Problem 16-55

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at *C*. Determine the velocity of the slider block *C* at the instant shown, if link *AB* is rotating at angular velocity ω_{AB}.

Given:

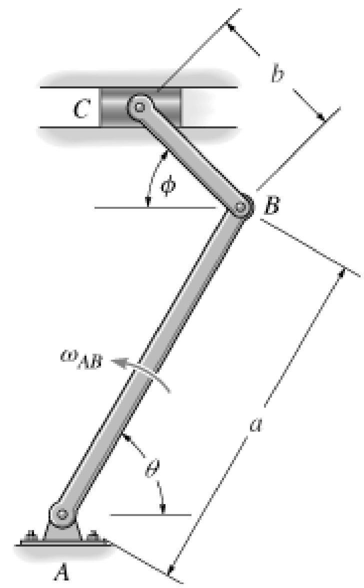
$\theta = 45\text{ deg}$

$\phi = 45\text{ deg}$

$\omega_{AB} = 4\,\frac{\text{rad}}{\text{s}}$

$a = 300\text{ mm}$

$b = 125\text{ mm}$



Solution:

Guesses $\omega_{BC} = 1\,\frac{\text{rad}}{\text{s}}$ $v_C = 1\,\frac{\text{m}}{\text{s}}$

Given
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[a \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[b \begin{pmatrix} -\cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \right] = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

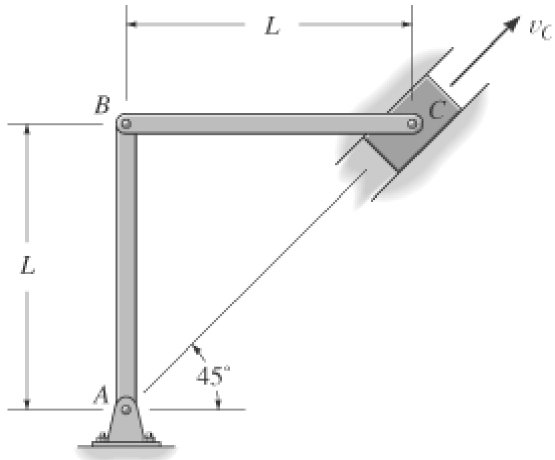
$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 9.60\,\frac{\text{rad}}{\text{s}} \qquad v_C = -1.70\,\frac{\text{m}}{\text{s}}$$

Problem 16-56

The velocity of the slider block C is v_C up the inclined groove. Determine the angular velocity of links AB and BC and the velocity of point B at the instant shown.

Given:

$v_C = 4\ \frac{ft}{s}$ $L = 1\ ft$



Guesses

$v_{Bx} = 1\ \frac{ft}{s}$ $v_{By} = 1\ \frac{ft}{s}$

$\omega_{AB} = 1\ \frac{rad}{s}$ $\omega_{BC} = 1\ \frac{rad}{s}$

Given $\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ L \\ 0 \end{pmatrix} = \begin{pmatrix} v_{Bx} \\ v_{By} \\ 0 \end{pmatrix}$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \cos(45\ deg) \\ v_C \sin(45\ deg) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \\ \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \text{Find}(v_{Bx}, v_{By}, \omega_{AB}, \omega_{BC})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} -2.83 \\ 2.83 \end{pmatrix} \frac{rad}{s}$$

$$\begin{pmatrix} v_{Bx} \\ v_{By} \end{pmatrix} = \begin{pmatrix} 2.83 \\ 0.00 \end{pmatrix} \frac{ft}{s} \qquad \left| \begin{pmatrix} v_{Bx} \\ v_{By} \end{pmatrix} \right| = 2.83\ \frac{ft}{s}$$

Problem 16-57

Rod AB is rotating with an angular velocity ω_{AB}. Determine the velocity of the collar C for the given angles θ and ϕ.

Given:

$\omega_{AB} = 5\ \frac{rad}{s}$

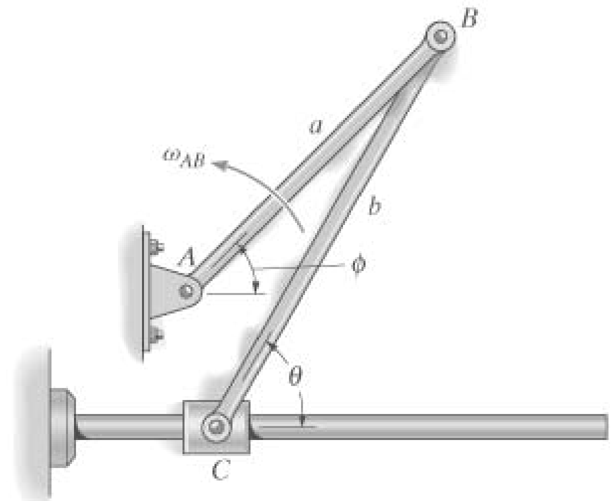
$v_B = 10\ \frac{ft}{s}$

$a = 2\ \text{ft}$

$b = 2.5\ \text{ft}$

$\theta = 60\ \text{deg}$

$\phi = 45\ \text{deg}$



Solution:

Guesses

$v_C = 4\ \frac{\text{ft}}{\text{s}} \qquad \omega_{CB} = 7\ \frac{\text{rad}}{\text{s}}$

Given

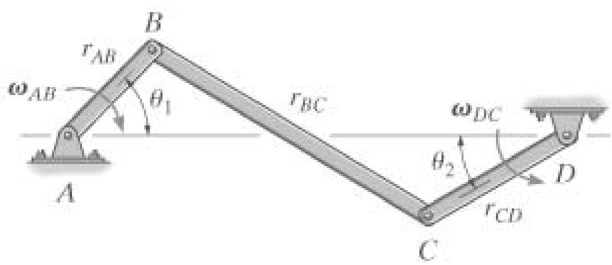
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{bmatrix} (a)\cos(\phi) \\ (a)\sin(\phi) \\ 0 \end{bmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CB} \end{pmatrix} \times \begin{pmatrix} -b\cos(\theta) \\ -b\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_C \\ \omega_{CB} \end{pmatrix} = \text{Find}(v_C, \omega_{CB}) \qquad \omega_{CB} = 5.66\ \frac{\text{rad}}{\text{s}} \qquad v_C = 5.18\ \frac{\text{ft}}{\text{s}}$$

Problem 16-58

If rod *CD* is rotating with an angular velocity ω_{DC}, determine the angular velocities of rods *AB* and *BC* at the instant shown.

Given:



$\omega_{DC} = 8\ \frac{\text{rad}}{\text{s}}$

$\theta_1 = 45\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$r_{AB} = 150\ \text{mm}$

$r_{BC} = 400\ \text{mm}$

$r_{CD} = 200\ \text{mm}$

Solution:

Guesses $\theta_3 = 20\ \text{deg} \quad \omega_{AB} = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$r_{AB}\sin(\theta_1) - r_{BC}\sin(\theta_3) + r_{CD}\sin(\theta_2) = 0$$

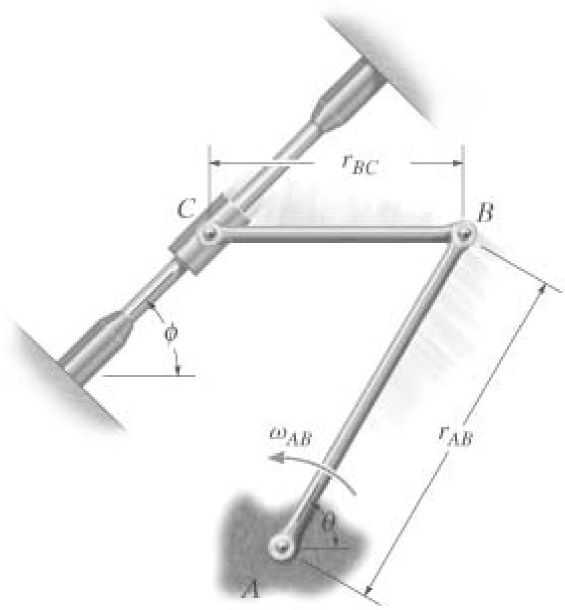
$$\begin{pmatrix} 0 \\ 0 \\ \omega_{DC} \end{pmatrix} \times \begin{pmatrix} -r_{CD}\cos(\theta_2) \\ -r_{CD}\sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -r_{BC}\cos(\theta_3) \\ r_{BC}\sin(\theta_3) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -r_{AB}\cos(\theta_1) \\ -r_{AB}\sin(\theta_1) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} \theta_3 \\ \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \text{Find}(\theta_3, \omega_{AB}, \omega_{BC}) \qquad \theta_3 = 31.01\text{ deg} \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} -9.615 \\ -1.067 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

Positive means *CCW*
Negative means *CW*

Problem 16-59

The angular velocity of link *AB* is ω_{AB}. Determine the velocity of the collar at *C* and the angular velocity of link *CB* for the given angles θ and ϕ. Link *CB* is horizontal at this instant.



Given:

$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$ $\qquad \phi = 45\text{ deg}$

$r_{AB} = 500\text{ mm}$ $\qquad \theta = 60\text{ deg}$

$r_{BC} = 350\text{ mm}$ $\qquad \theta_1 = 30\text{ deg}$

Solution:

Guesses $\quad v_C = 1\ \frac{\text{m}}{\text{s}} \qquad \omega_{CB} = 1\ \frac{\text{rad}}{\text{s}}$

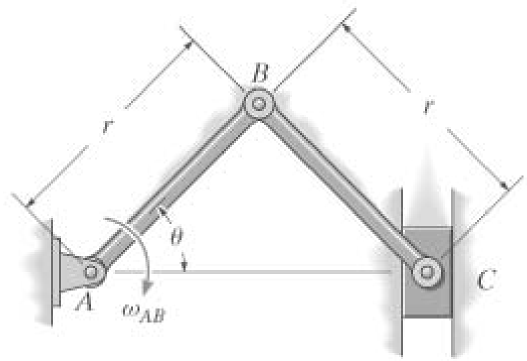
Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_{AB}\cos(\theta) \\ r_{AB}\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CB} \end{pmatrix} \times \begin{pmatrix} -r_{BC} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -v_C\cos(\phi) \\ -v_C\sin(\phi) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_C \\ \omega_{CB} \end{pmatrix} = \text{Find}(v_C, \omega_{CB}) \qquad \omega_{CB} = 7.81\ \frac{\text{rad}}{\text{s}} \qquad v_C = 2.45\ \frac{\text{m}}{\text{s}}$$

***Problem 16-60**

The link *AB* has an angular velocity ω_{AB}. Determine the velocity of block *C* at the instant shown when θ = 45 deg.



Given:

$$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}} \qquad r = 15\ \text{in}$$

$$\theta = 45\ \text{deg} \qquad r = 15\ \text{in}$$

Solution:

Guesses

$$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \qquad v_C = 1\ \frac{\text{in}}{\text{s}}$$

Given

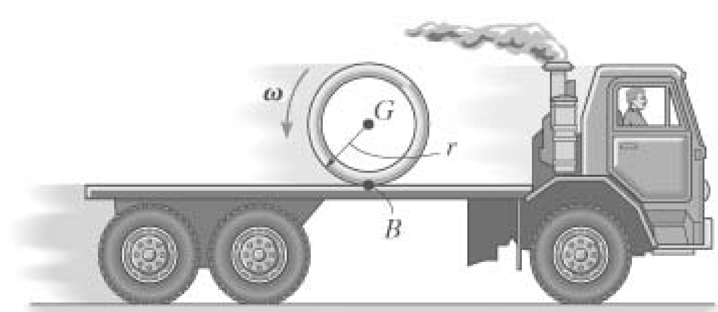
$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{BC} \end{pmatrix} \times \begin{pmatrix} r\cos(\theta) \\ -r\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C) \qquad \omega_{BC} = 2.00\ \frac{\text{rad}}{\text{s}} \qquad v_C = 3.54\ \frac{\text{ft}}{\text{s}}$$

Problem 16-61

At the instant shown, the truck is traveling to the right at speed *v*, while the pipe is rolling counterclockwise at angular velocity ω without slipping at *B*. Determine the velocity of the pipe's center *G*.

Given:



$$v = 3\ \frac{\text{m}}{\text{s}}$$

$$\omega = 8\ \frac{\text{rad}}{\text{s}}$$

$$r = 1.5\ \text{m}$$

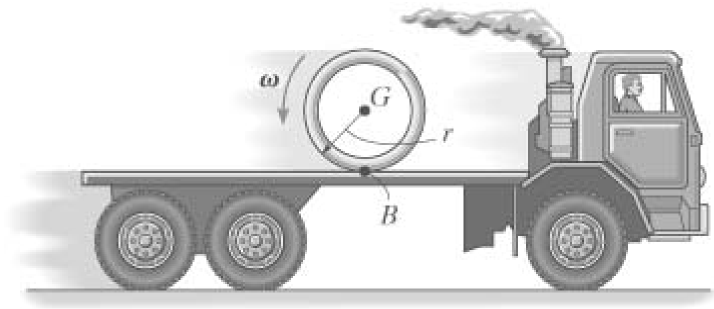
Solution:

$$v_G = v + v_{GB}$$

$$v_G = v - \omega r \qquad v_G = -9.00\ \frac{\text{m}}{\text{s}}$$

Problem 16-62

At the instant shown, the truck is traveling to the right at speed v. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary.



Given:

$$v = 8\ \frac{m}{s}$$

$$r = 1.5\ m$$

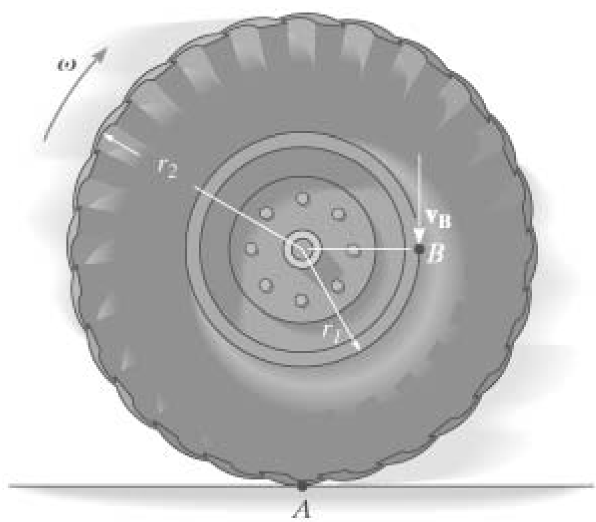
Solution:

$$v_G = v + v_{GB} \qquad 0 = v - \omega r \qquad \omega = \frac{v}{r} \qquad \omega = 5.33\ \frac{rad}{s}$$

Problem 16-63

If, at a given instant, point B has a downward velocity of v_B, determine the velocity of point A at this instant. Notice that for this motion to occur, the wheel must slip at A.

Given:



$$v_B = 3\ \frac{m}{s}$$

$$r_1 = 0.15\ m$$

$$r_2 = 0.4\ m$$

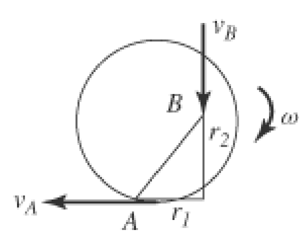
Solution:

Guesses

$$v_A = 1\ \frac{m}{s} \qquad \omega = 1\ \frac{rad}{s}$$

Given

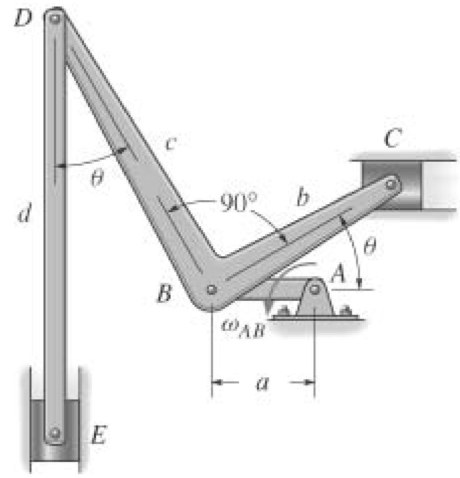
$$\begin{pmatrix} -v_A \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_B \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r_1 \\ -r_2 \\ 0 \end{pmatrix}$$



$$\begin{pmatrix} v_A \\ \omega \end{pmatrix} = \text{Find}(v_A, \omega) \qquad \omega = 20.00\ \frac{rad}{s} \qquad v_A = 8.00\ \frac{m}{s}$$

Problem 16-64

If the link AB is rotating about the pin at A with angular velocity ω_{AB}, determine the velocities of blocks C and E at the instant shown.



Given:

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$a = 1\ \text{ft}$

$b = 2\ \text{ft}$

$\theta = 30\ \text{deg}$

$c = 3\ \text{ft}$

$d = 4\ \text{ft}$

Solution:

Guesses $\omega_{BCD} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{DE} = 1\ \frac{\text{rad}}{\text{s}}$ $v_C = 1\ \frac{\text{ft}}{\text{s}}$ $v_E = 1\ \frac{\text{ft}}{\text{s}}$

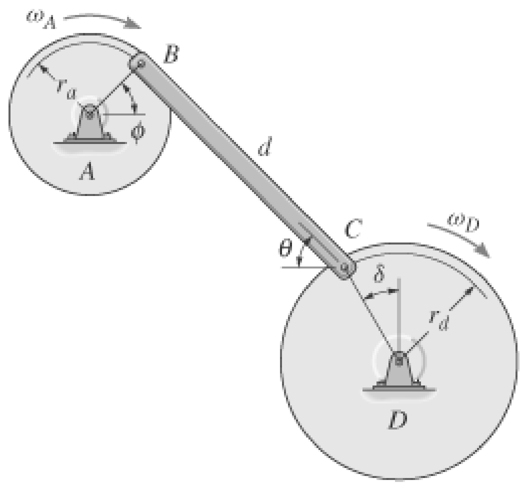
Given $\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BCD} \end{pmatrix} \times \begin{pmatrix} b\cos(\theta) \\ b\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} -v_C \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BCD} \end{pmatrix} \times \begin{pmatrix} -c\sin(\theta) \\ c\cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{DE} \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_E \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{BCD} \\ \omega_{DE} \\ v_C \\ v_E \end{pmatrix} = \text{Find}(\omega_{BCD}, \omega_{DE}, v_C, v_E) \quad \begin{pmatrix} \omega_{BCD} \\ \omega_{DE} \end{pmatrix} = \begin{pmatrix} 2.89 \\ 1.88 \end{pmatrix} \frac{\text{rad}}{\text{s}} \quad \begin{pmatrix} v_E \\ v_C \end{pmatrix} = \begin{pmatrix} 9.33 \\ 2.89 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 16-65

If disk D has constant angular velocity ω_D, determine the angular velocity of disk A at the instant shown.



Given:

$\omega_D = 2\,\frac{\text{rad}}{\text{s}}$ $\quad r_a = 0.5\text{ ft}$

$\theta = 60\text{ deg}$ $\quad r_d = 0.75\text{ ft}$

$\phi = 45\text{ deg}$ $\quad d = 2\text{ ft}$

$\delta = 30\text{ deg}$

Solution:

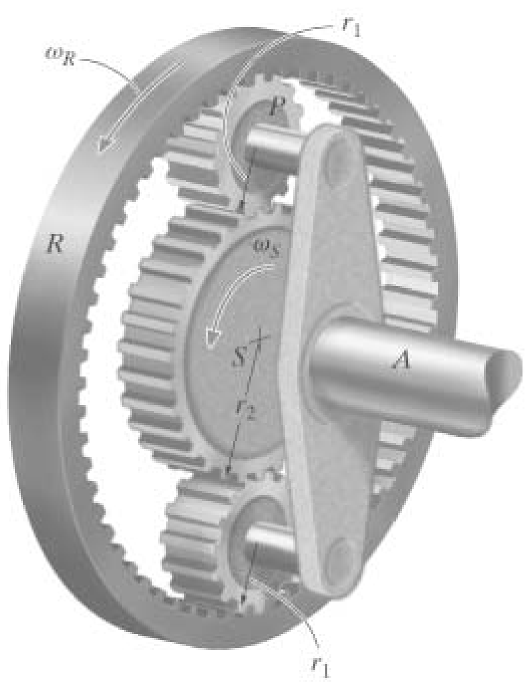
Guesses $\quad \omega_A = 1\,\frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1\,\frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_D \end{pmatrix} \times \begin{pmatrix} -r_d \sin(\delta) \\ r_d \cos(\delta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -d\cos(\theta) \\ d\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_A \end{pmatrix} \times \begin{pmatrix} -r_a \cos(\phi) \\ -r_a \sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_A \\ \omega_{BC} \end{pmatrix} = \text{Find}(\omega_A, \omega_{BC}) \qquad \omega_{BC} = -0.75\,\frac{\text{rad}}{\text{s}} \qquad \omega_A = 0.00\,\frac{\text{rad}}{\text{s}}$$

Problem 16-66



The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear *R* is rotating with angular velocity ω_R, and the sun gear *S* is held fixed, $\omega_S = 0$. Determine the angular velocity of each of the planet gears *P* and shaft *A*.

Given:

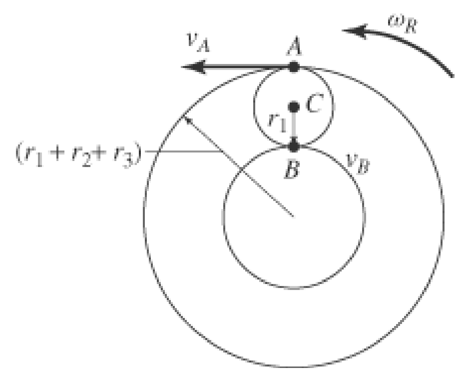
$r_1 = 40\text{ mm}$ $\quad \omega_R = 3\,\frac{\text{rad}}{\text{s}}$

$r_2 = 80\text{ mm}$ $\quad v_B = 0$

Solution:

$$v_A = \omega_R(r_2 + 2r_1)$$

$\omega_P = \dfrac{v_A}{2r_1}$ $\qquad \omega_P = 6.00\,\dfrac{\text{rad}}{\text{s}}$



$v_C = \omega_P r_1$

$\omega_A = \dfrac{v_C}{r_2 + r_1}$ $\qquad \omega_A = 2.00\,\dfrac{\text{rad}}{\text{s}}$

Problem 16-67

If bar *AB* has an angular velocity ω_{AB}, determine the velocity of the slider block *C* at the instant shown.

Given:

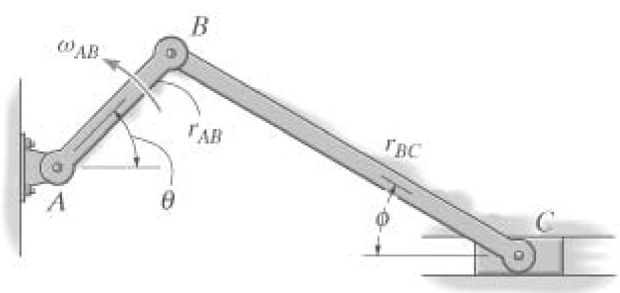
$\omega_{AB} = 6\,\dfrac{\text{rad}}{\text{s}}$

$\theta = 45\text{ deg}$

$\phi = 30\text{ deg}$

$r_{AB} = 200\text{ mm}$

$r_{BC} = 500\text{ mm}$



Solution:

Guesses $\quad \omega_{BC} = 2\,\dfrac{\text{rad}}{\text{s}}$

$v_C = 4\,\dfrac{\text{m}}{\text{s}}$

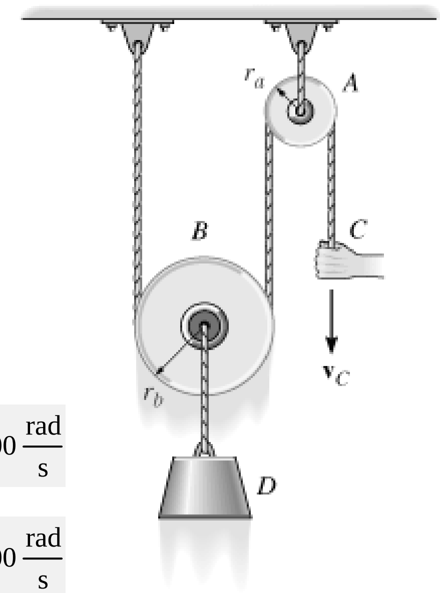
Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} r_{AB}\cos(\theta) \\ r_{AB}\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} r_{BC}\cos(\phi) \\ -r_{BC}\sin(\phi) \\ 0 \end{pmatrix} = \begin{pmatrix} v_C \\ 0 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} \omega_{BC} \\ v_C \end{pmatrix} = \text{Find}(\omega_{BC}, v_C)$ $\qquad \omega_{BC} = -1.96\,\dfrac{\text{rad}}{\text{s}}$ $\qquad v_C = -1.34\,\dfrac{\text{m}}{\text{s}}$

***Problem 16-68**

If the end of the cord is pulled downward with speed v_C, determine the angular velocities of pulleys A and B and the speed of block D. Assume that the cord does not slip on the pulleys.



Given:

$$v_C = 120 \frac{\text{mm}}{\text{s}}$$

$$r_a = 30 \text{ mm}$$

$$r_b = 60 \text{ mm}$$

Solution:

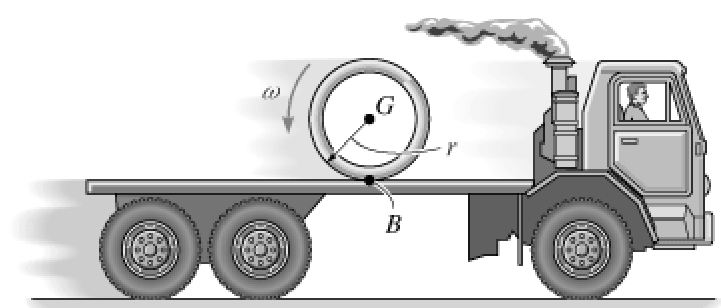
$$v_C = \omega_A r_a \qquad \omega_A = \frac{v_C}{r_a} \qquad \omega_A = 4.00 \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega_B 2 r_b \qquad \omega_B = \frac{v_C}{2 r_b} \qquad \omega_B = 1.00 \frac{\text{rad}}{\text{s}}$$

$$v_D = \omega_B r_b \qquad v_D = 60.00 \frac{\text{mm}}{\text{s}}$$

Problem 16-69

At the instant shown, the truck is traveling to the right at speed $v = at$, while the pipe is rolling counterclockwise at angular velocity $\omega = bt$, without slipping at B. Determine the velocity of the pipe's center G at time t.



Given:

$$a = 8 \frac{\text{m}}{\text{s}^2}$$

$$b = 2 \frac{\text{rad}}{\text{s}^2}$$

$$r = 1.5 \text{ m}$$

$$t = 3 \text{ s}$$

Solution:

$$v = a t \qquad \omega = b t$$

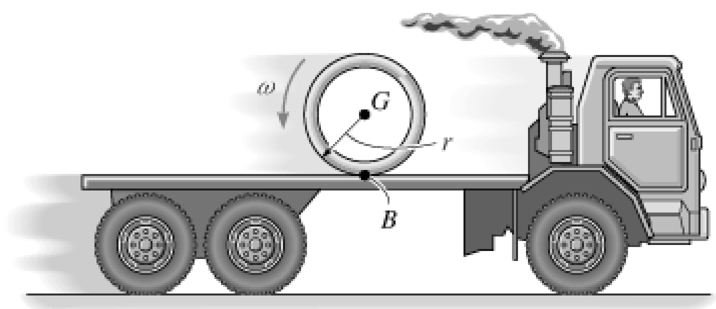
$v_G = v - \omega r$ $\quad v_G = (a - b\,r)t$

where $\quad a - b\,r = 5.00\ \frac{\text{m}}{\text{s}^2}$

Problem 16-70

At the instant shown, the truck is traveling to the right at speed v_t. If the spool does not slip at B, determine its angular velocity if its mass center appears to an observer on the ground to be moving to the right at speed v_G.

Given:



$v_t = 12\ \frac{\text{m}}{\text{s}}$

$v_G = 3\ \frac{\text{m}}{\text{s}}$

$r = 1.5\ \text{m}$

Solution:

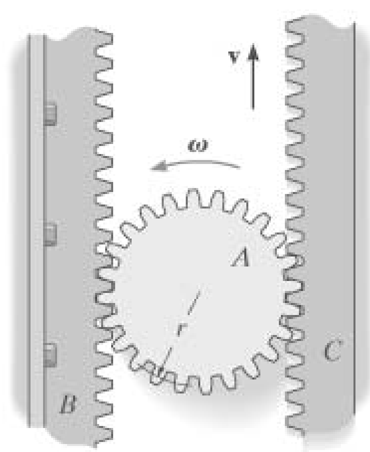
$v_G = v_t - \omega r$

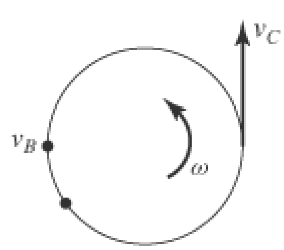
$\omega = \frac{v_t - v_G}{r}$ $\quad \omega = 6.00\ \frac{\text{rad}}{\text{s}}$

Problem 16-71

The pinion gear A rolls on the fixed gear rack B with an angular velocity ω. Determine the velocity of the gear rack C.

Given:





$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$r = 0.3\ \text{ft}$

Solution:

$v_C = v_B + v_{CB}$

$v_C = 2\omega r$

$v_C = 2.40\ \frac{\text{ft}}{\text{s}}$

***Problem 16-72**

Part of an automatic transmission consists of a *fixed* ring gear R, three equal planet gears P, the sun gear S, and the planet carrier C, which is shaded. If the sun gear is rotating with angular velocity ω_s determine the angular velocity ω_c of the *planet carrier*. Note that C is pin-connected to the center of each of the planet gears.

Given:

$$\omega_S = 6\ \frac{\text{rad}}{\text{s}}$$

$$r_S = 4\ \text{in}$$

$$r_P = 2\ \text{in}$$

Solution:

$$\omega_S r_S = \omega_P 2r_P$$

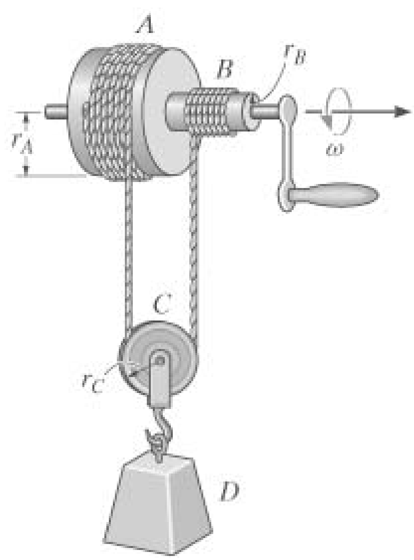
$$\omega_P = \omega_S\left(\frac{r_S}{2r_P}\right) \qquad \omega_P = 6.00\ \frac{\text{rad}}{\text{s}}$$

$$\omega_P r_P = \omega_C\left(r_S + r_P\right)$$

$$\omega_C = \omega_P\left(\frac{r_P}{r_S + r_P}\right) \qquad \omega_C = 2.00\ \frac{\text{rad}}{\text{s}}$$

Problem 16-73

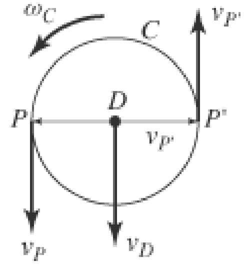
When the crank on the Chinese windlass is turning, the rope on shaft A unwinds while that on shaft B winds up. Determine the speed of block D if the crank is turning with an angular velocity ω. What is the angular velocity of the pulley at C? The rope segments on each side of the pulley are both parallel and vertical, and the rope does not slip on the pulley.



Given:

$$\omega = 4\ \frac{\text{rad}}{\text{s}} \qquad r_A = 75\ \text{mm}$$

$$r_C = 50\ \text{mm} \qquad r_B = 25\ \text{mm}$$



Solution:

$$v_P = \omega r_A \qquad v_{P'} = \omega r_B$$

$$\omega_C = \frac{v_P + v_{P'}}{2r_C} \qquad \omega_C = 4.00\,\frac{\text{rad}}{\text{s}}$$

$$v_D = -v_{P'} + \omega_C r_C \qquad v_D = 100.00\,\frac{\text{mm}}{\text{s}}$$

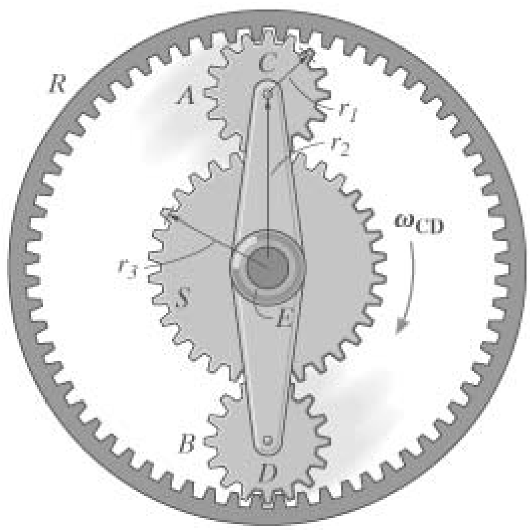
Problem 16-74

In an automobile transmission the planet pinions *A* and *B* rotate on shafts that are mounted on the planet pinion carrier *CD*. As shown, *CD* is attached to a shaft at *E* which is aligned with the center of the *fixed* sun-gear *S*. This shaft is not attached to the sun gear. If *CD* is rotating with angular velocity ω_{CD}, determine the angular velocity of the ring gear *R*.

Given:

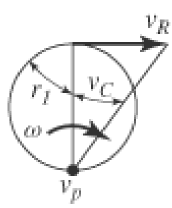
$$\omega_{CD} = 8\,\frac{\text{rad}}{\text{s}} \qquad r_1 = 50\text{ mm}$$

$$r_2 = 125\text{ mm} \qquad r_3 = 75\text{ mm}$$



Solution:

$$v_C = \omega_{CD} r_2 \qquad \omega_A = \frac{v_C}{r_1}$$



$$v_R = \omega_A 2r_1$$

$$\omega_R = \frac{v_R}{r_2 + r_1} \qquad \omega_R = 11.4\,\frac{\text{rad}}{\text{s}}$$

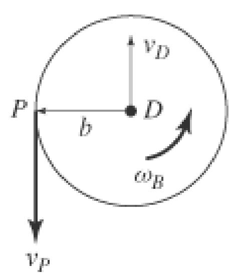
Problem 16-75

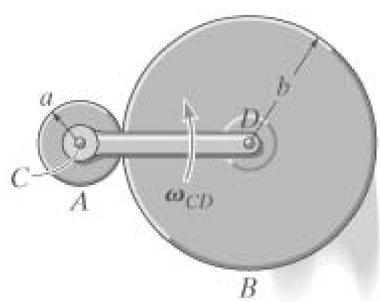
The cylinder *B* rolls on the *fixed cylinder A* without slipping. If the connected bar *CD* is rotating with an angular velocity ω_{CD}. Determine the angular velocity of cylinder *B*.

Given:

$$\omega_{CD} = 5\,\frac{\text{rad}}{\text{s}} \qquad a = 0.1\text{ m}$$

$$b = 0.3\text{ m}$$





Solution:

$$v_D = \omega_{CD}(a + b)$$

$$\omega_B = \frac{v_D}{b} \qquad \omega_B = 6.67\,\frac{\text{rad}}{\text{s}}$$

***Problem 16-76**

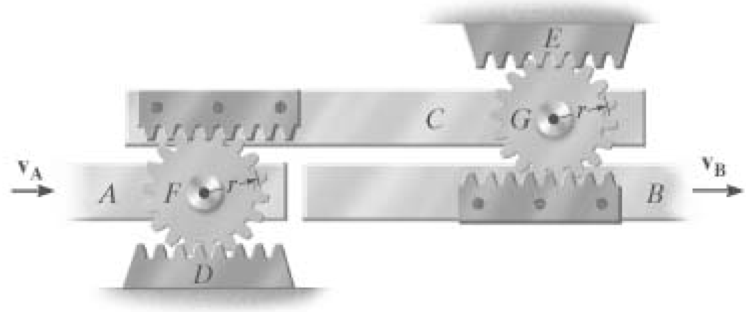
The slider mechanism is used to increase the stroke of travel of one slider with respect to that of another. As shown, when the slider *A* is moving forward, the attached pinion *F* rolls on the fixed rack *D*, forcing slider *C* to move forward. This in turn causes the attached pinion *G* to roll on the fixed rack *E*, thereby moving slider *B*. If *A* has a velocity $\mathbf{v_A}$ at the instant shown, determine the velocity of *B*.

Given:

$$v_A = 4\ \frac{\text{ft}}{\text{s}}$$

$$r = 0.2\ \text{ft}$$

$$v_c = 8\ \frac{\text{ft}}{\text{s}}$$



Solution:

$$\omega_F = \frac{v_A}{r} \qquad v_C = \omega_F(2r)$$

$$\omega_G = \frac{v_C}{r} \qquad v_B = \omega_G(2r) \qquad v_B = 16.00\ \frac{\text{ft}}{\text{s}}$$

Problem 16-77

The gauge is used to indicate the safe load acting at the end of the boom, *B*, when it is in any angular position. It consists of a fixed dial plate *D* and an indicator arm *ACE* which is pinned to the plate at *C* and to a short link *EF*. If the boom is pin-connected to the trunk frame at *G* and is rotating downward with angular velocity ω_B, determine the velocity of the dial pointer *A* at the instant shown, i.e., when *EF* and *AC* are in the vertical position.

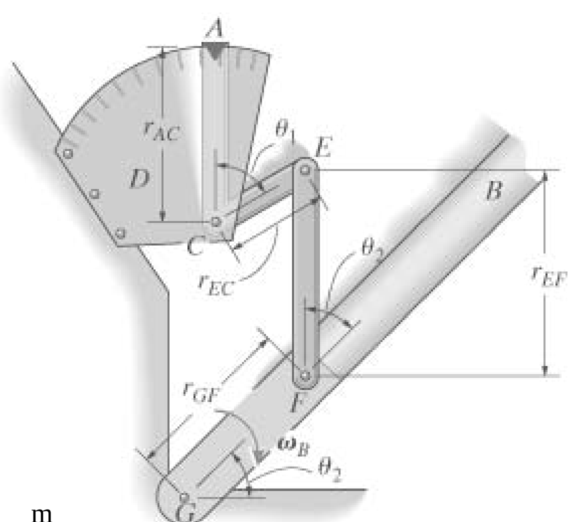
Given:

$$r_{AC} = 250\ \text{mm} \qquad \omega_B = 4\ \frac{\text{rad}}{\text{s}}$$

$$r_{EC} = 150\ \text{mm} \qquad \theta_1 = 60\ \text{deg}$$

$$r_{GF} = 250\ \text{mm} \qquad \theta_2 = 45\ \text{deg}$$

$$r_{EF} = 300\ \text{mm}$$



Solution:

Guesses

$$\omega_{EF} = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_{ACE} = 1\ \frac{\text{rad}}{\text{s}} \qquad v_A = 1\ \frac{\text{m}}{\text{s}}$$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_B \end{pmatrix} \times \begin{pmatrix} r_{GF}\cos(\theta_2) \\ r_{GF}\sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{EF} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{EF} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{ACE} \end{pmatrix} \times \begin{pmatrix} -r_{EC}\sin(\theta_1) \\ -r_{EC}\cos(\theta_1) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{ACE} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{AC} \\ 0 \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{EF} \\ \omega_{ACE} \\ v_A \end{pmatrix} = \text{Find}(\omega_{EF}, \omega_{ACE}, v_A) \qquad \begin{pmatrix} \omega_{EF} \\ \omega_{ACE} \end{pmatrix} = \begin{pmatrix} 1.00 \\ -5.44 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_A = 1.00\ \frac{\text{m}}{\text{s}}$$

Problem 16-78

The wheel is rotating with an angular velocity ω. Determine the velocity of the collar A at the instant θ and ϕ using the method of instantaneous center of zero velocity.

Given:

$r_A = 500\ \text{mm}$

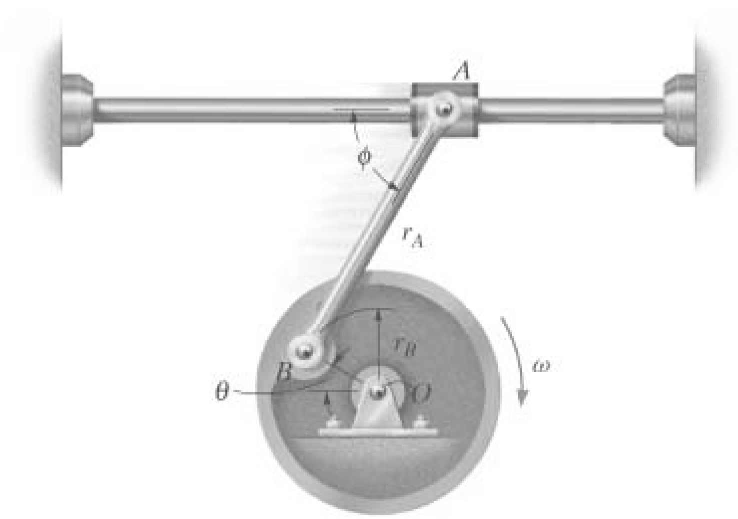
$r_B = 150\ \text{mm}$

$\theta = 30\ \text{deg}$

$\theta_1 = 90\ \text{deg}$

$\phi = 60\ \text{deg}$

$\omega = 8\ \frac{\text{rad}}{\text{s}}$



Solution:

$v_B = \omega r_B$

$v_B = 1.20\ \frac{\text{m}}{\text{s}}$

$r_{BC} = r_A \tan(\theta)$

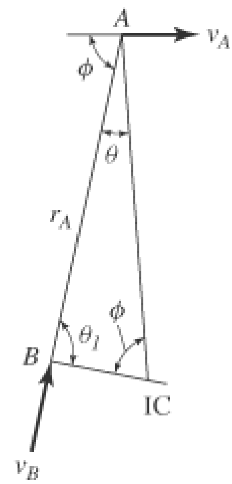
$r_{BC} = 0.289\ \text{m}$

$$\omega_{AB} = \frac{v_B}{r_{BC}}$$

$$\omega_{AB} = 4.16\,\frac{\text{rad}}{\text{s}}$$

$$r_{AC} = \frac{r_A}{\sin(\phi)} \qquad r_{AC} = 0.577\ \text{m}$$

$$v_A = r_{AC}\,\omega_{AB} \qquad v_A = 2.40\,\frac{\text{m}}{\text{s}}$$



Problem 16-79

The shaper mechanism is designed to give a slow cutting stroke and a quick return to a blade attached to the slider at *C*. Determine the velocity of the slider block *C* at the instant shown, if link *AB* is rotating with angular velocity ω_{AB}. Solve using the method of instantaneous center of zero velocity.

Given:

$r_{AB} = 300\ \text{mm}$

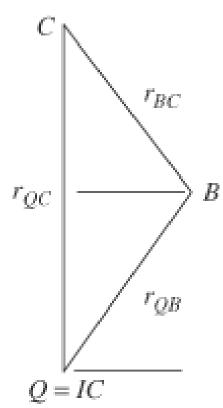
$r_{BC} = 125\ \text{mm}$

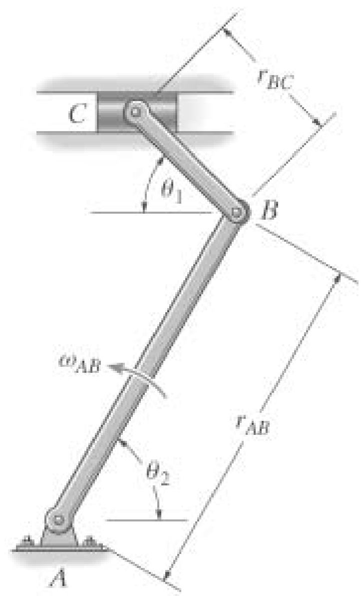
$\omega_{AB} = 4\,\frac{\text{rad}}{\text{s}}$

$\theta = 90\ \text{deg}$

$\theta_1 = 45\ \text{deg}$

$\theta_2 = 60\ \text{deg}$





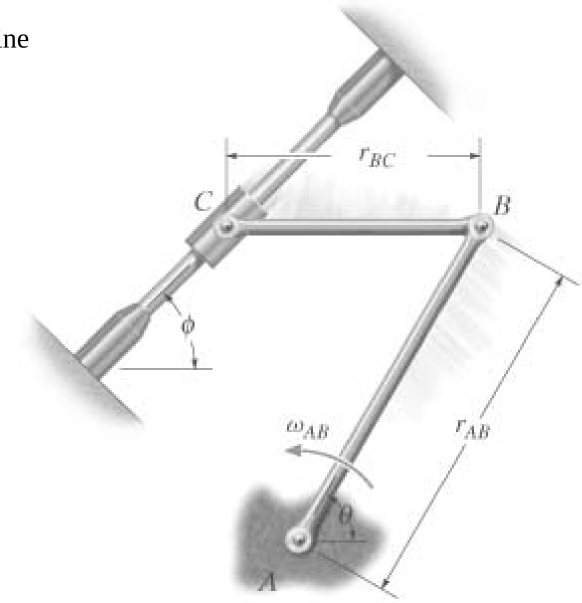
Solution:

$$r_{QB} = r_{BC}\frac{\cos(\theta_1)}{\cos(\theta_2)}$$

$$r_{QC} = r_{BC}\sin(\theta_1) + r_{QB}\sin(\theta_2)$$

$$v_B = \omega_{AB}\,r_{AB} \qquad \omega_{BC} = \frac{v_B}{r_{QB}} \qquad v_C = \omega_{BC}\,r_{QC} \qquad v_C = 1.64\,\frac{\text{m}}{\text{s}}$$

***Problem 16-80**



The angular velocity of link *AB* is ω_{AB}. Determine the velocity of the collar at *C* and the angular velocity of link *CB* in the position shown using the method of instantaneous center of zero velocity. Link *CB* is horizontal at this instant.

Given:

$$\omega_{AB} = 4\ \frac{\text{rad}}{\text{s}}$$

$$r_{AB} = 500\ \text{mm}$$

$$r_{BC} = 350\ \text{mm}$$

$$\phi = 45\ \text{deg}$$

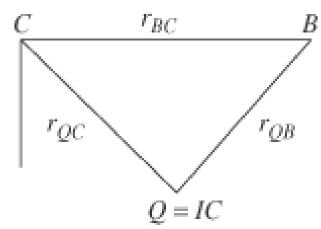
$$\theta = 60\ \text{deg}$$

Solution:

Guesses $r_{QB} = 1\ \text{mm}$ $r_{QC} = 1\ \text{mm}$

Given $r_{BC} = r_{QB}\cos(\theta) + r_{QC}\sin(\phi)$

$$r_{QB}\sin(\theta) = r_{QC}\cos(\phi)$$

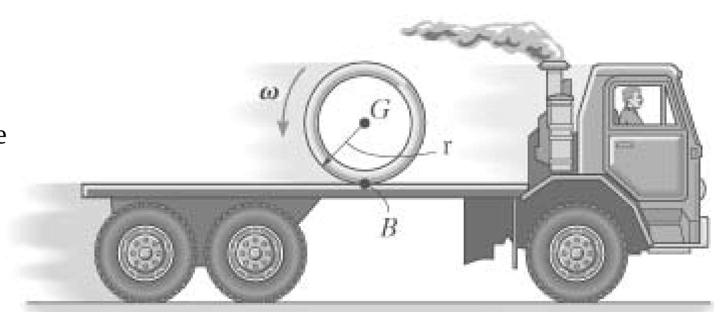


$$\begin{pmatrix} r_{QC} \\ r_{QB} \end{pmatrix} = \text{Find}(r_{QC}, r_{QB}) \qquad \begin{pmatrix} r_{QC} \\ r_{QB} \end{pmatrix} = \begin{pmatrix} 314 \\ 256 \end{pmatrix}\text{mm}$$

$$v_B = \omega_{AB} r_{AB} \qquad \omega_{CB} = \frac{v_B}{r_{QB}} \qquad v_C = \omega_{CB} r_{QC}$$

$$\omega_{CB} = 7.81\ \frac{\text{rad}}{\text{s}} \qquad v_C = 2.45\ \frac{\text{m}}{\text{s}}$$

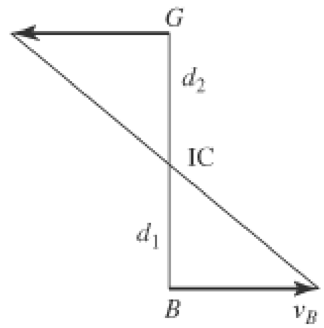
Problem 16-81



At the instant shown, the truck is traveling to the right with speed v_B, while the pipe is rolling counterclockwise with angular velocity ω without slipping at *B*. Determine the velocity of the pipe's center *G* using the method of instantaneous center of zero velocity.

Given:

$\omega = 8\ \dfrac{\text{rad}}{\text{s}}$ $\quad v_B = 3\ \dfrac{\text{m}}{\text{s}}$ $\quad r = 1.5\ \text{m}$



Solution:

$d_1 = \dfrac{v_B}{\omega}$ $\quad d_1 = 0.38\ \text{m}$

$d_2 = r - d_1$ $\quad d_2 = 1.13\ \text{m}$

$v_G = \omega d_2$ $\quad v_G = 9.00\ \dfrac{\text{m}}{\text{s}}$

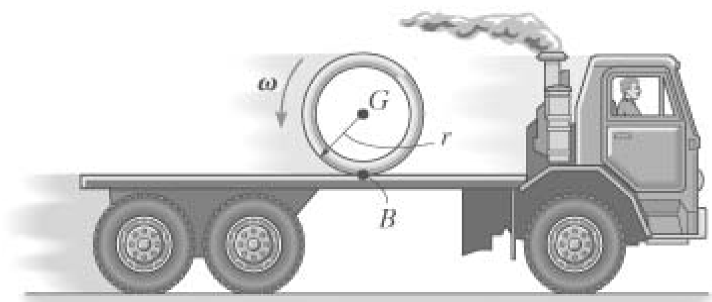
Problem 16-82

At the instant shown, the truck is traveling to the right with speed v_B. If the spool does not slip at B, determine its angular velocity so that its mass center G appears to an observer on the ground to remain stationary. Use the method of instantaneous center of zero velocity.

Given:

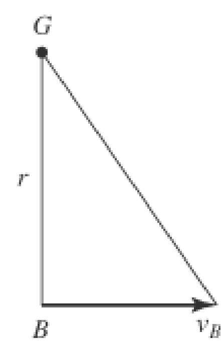
$r = 1.5\ \text{m}$

$v_B = 8\ \dfrac{\text{m}}{\text{s}}$



Solution:

Mass center G is the instantaneous center.



$r\omega = v_B$

$\omega = \dfrac{v_B}{r}$

$\omega = 5.33\ \dfrac{\text{rad}}{\text{s}}$

Problem 16-83

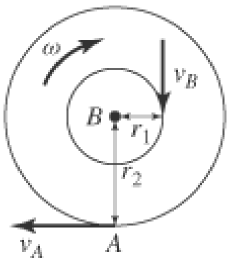
If, at a given instant, point B has a downward velocity v_B determine the velocity of point A at this instant using the method of instantaneous center of zero velocity. Notice that for this motion to occur, the wheel must slip at A.

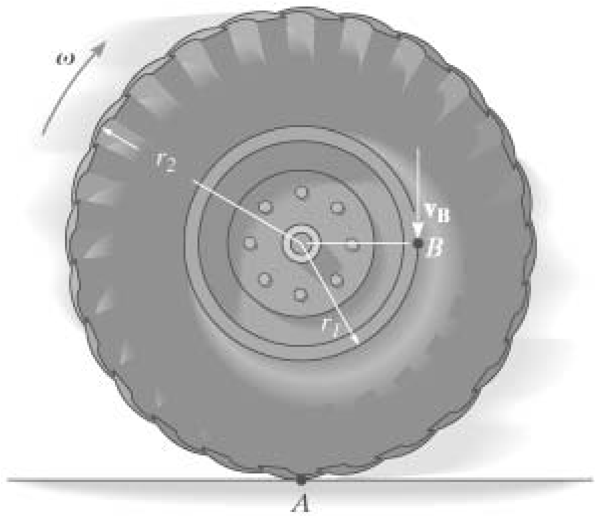
Given:

$v_B = 3\ \frac{\text{m}}{\text{s}}$

$r_1 = 0.15\ \text{m}$

$r_2 = 0.4\ \text{m}$





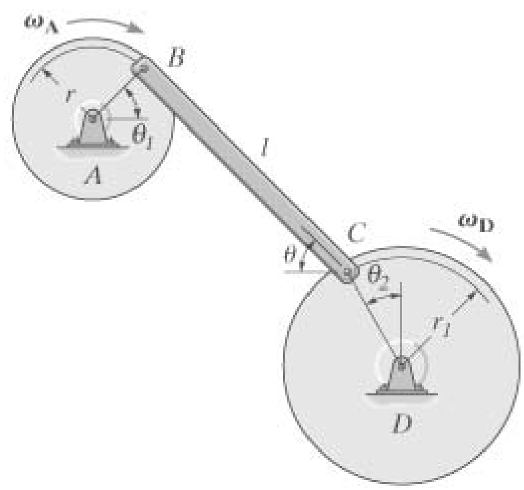
Solution:

$\omega = \frac{v_B}{r_1}$ $\qquad \omega = 20.00\ \frac{\text{rad}}{\text{s}}$

$v_A = r_2 \omega$ $\qquad v_A = 8.00\ \frac{\text{m}}{\text{s}}$

***Problem 16-84**

If disk D has a constant angular velocity ω_D, determine the angular velocity of disk A at the instant θ, using the method of instantaneous center of zero velocity.



Given:

$r = 0.5\ \text{ft}$ $\qquad \theta = 60\ \text{deg}$

$r_1 = 0.75\ \text{ft}$ $\qquad \theta_1 = 45\text{deg}$

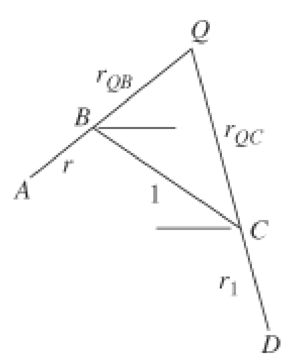
$l = 2\ \text{ft}$ $\qquad \theta_2 = 30\ \text{deg}$ $\qquad \omega_D = 2\ \frac{\text{rad}}{\text{s}}$

Solution:

$\alpha = \theta_1 + \theta$ $\qquad \beta = \frac{\pi}{2} - \theta - \theta_2$ $\qquad \gamma = \pi - \alpha - \beta$

$r_{QB} = l\left(\frac{\sin(\beta)}{\sin(\gamma)}\right)$ $\qquad r_{QB} = 0\ \text{m}$

$r_{QC} = l\left(\frac{\sin(\alpha)}{\sin(\gamma)}\right)$ $\qquad r_{QC} = 0.61\ \text{m}$



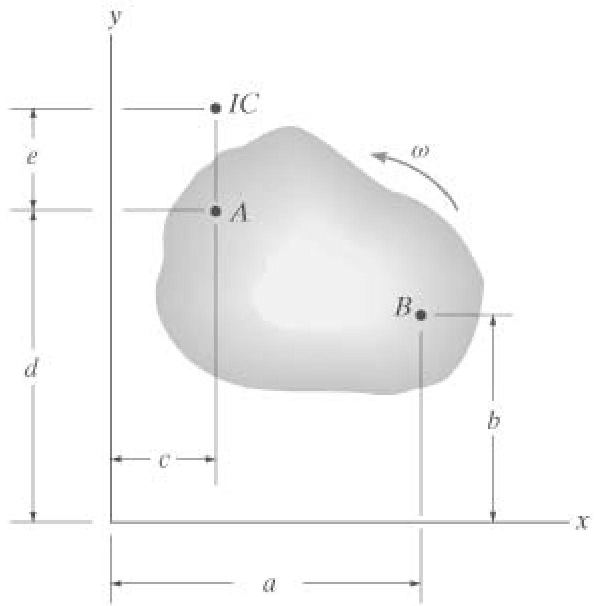
$v_C = \omega_D r_1$ $\qquad \omega_{BC} = \frac{v_C}{r_{QC}}$ $\qquad v_B = \omega_{BC} r_{QB}$

$\omega_A = \frac{v_B}{r}$ $\qquad \omega_A = 0\ \frac{1}{\text{s}}$

Problem 16-85

The instantaneous center of zero velocity for the body is located at point *IC*. If the body has an angular velocity ω, as shown, determine the velocity of *B* with respect to *A*.

Given:



$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$a = 1.5\ \text{m}$

$b = 1\ \text{m}$

$c = 0.5\ \text{m}$

$d = 1.5\ \text{m}$

$e = 0.5\ \text{m}$

Solution:

$$\phi = \operatorname{atan}\left(\frac{c+d-b}{a-c}\right)$$

$$\mathbf{v_A} = \begin{pmatrix} \omega e \\ 0 \\ 0 \end{pmatrix}$$

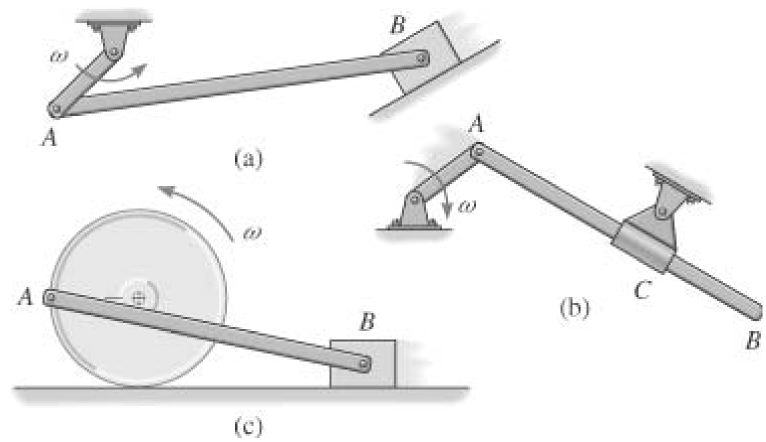
$$\mathbf{v_B} = \omega\sqrt{(a-c)^2 + (e+d-b)^2}\begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix}$$

$$\mathbf{v_{BA}} = \mathbf{v_B} - \mathbf{v_A} \qquad \mathbf{v_{BA}} = \begin{pmatrix} 2.00 \\ 4.00 \\ 0.00 \end{pmatrix}\frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_{BA}}\right| = 4.47\ \frac{\text{m}}{\text{s}}$$

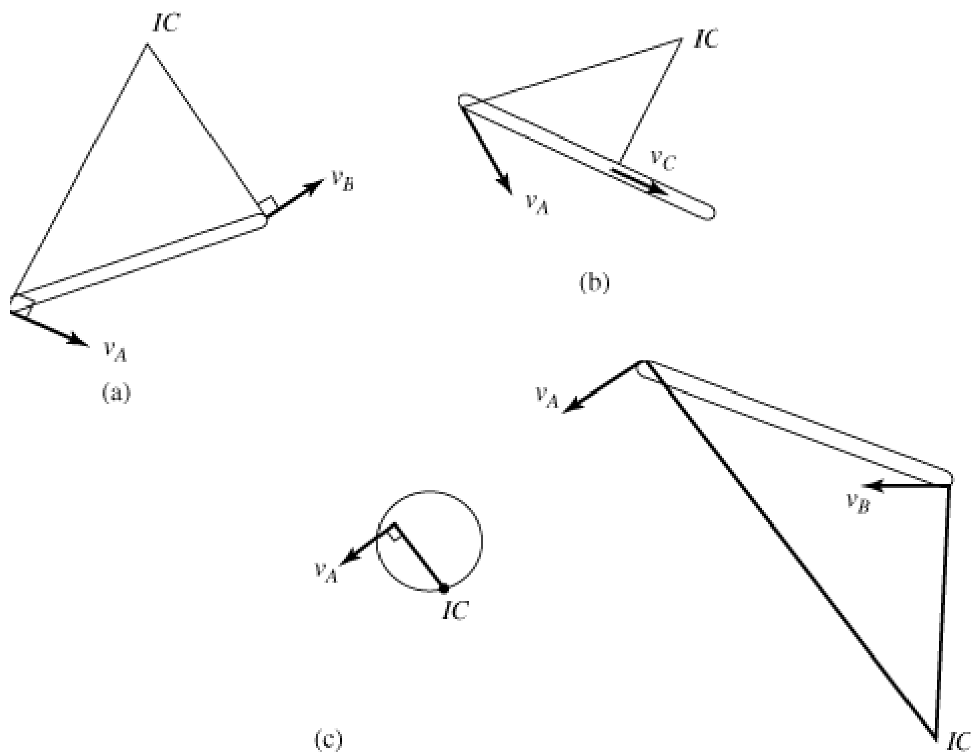
$$\theta = \operatorname{atan}\left(\frac{\mathbf{v_{BA}}_1}{\mathbf{v_{BA}}_0}\right) \qquad \theta = 63.4\ \text{deg}$$

Problem 16-86

In each case show graphically how to locate the instantaneous center of zero velocity of link *AB*. Assume the geometry is known.

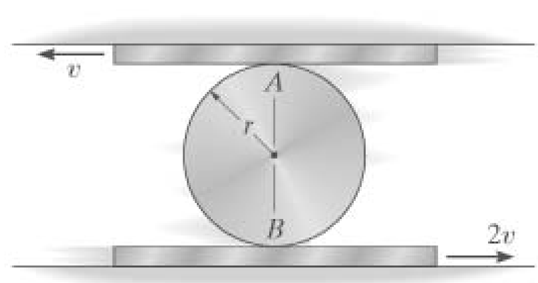


Solution:



Problem 16-87

The disk of radius *r* is confined to roll without slipping at *A* and *B*. If the plates have the velocities shown, determine the angular velocity of the disk.



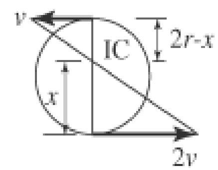
Solution:

$$\frac{v}{2r - x} = \frac{2v}{x}$$

$x = 4r - 2x$

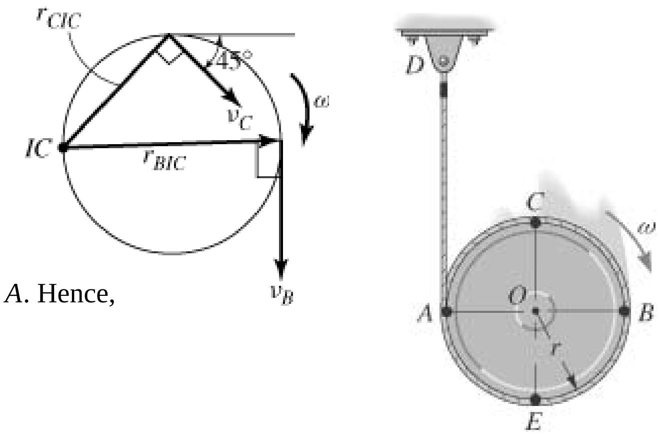
$3x = 4\mathbf{r}$ $\quad x = \frac{4}{3}r$

$\omega = \frac{2v}{\frac{4r}{3}}$ $\quad \omega = \frac{3v}{2r}$



***Problem 16-88**

At the instant shown, the disk is rotating with angular velocity ω. Determine the velocities of points A, B, and C.



Given:

$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$r = 0.15\ \text{m}$

Solution:

The instantaneous center is located at point A. Hence,

$v_A = 0$

$v_C = \sqrt{2}r\omega$ $\quad v_C = 0.849\ \frac{\text{m}}{\text{s}}$

$v_B = 2r\omega$ $\quad v_B = 1.20\ \frac{\text{m}}{\text{s}}$

Problem 16-89

The slider block C is moving with speed v_C up the incline. Determine the angular velocities of links AB and BC and the velocity of point B at the instant shown.

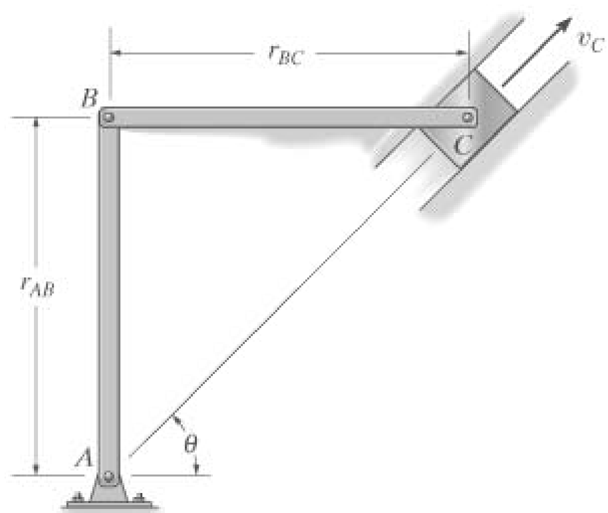
Given:

$v_C = 4\ \frac{\text{ft}}{\text{s}}$

$r_{AB} = 1\ \text{ft}$

$r_{BC} = 1\ \text{ft}$

$\theta = 45\ \text{deg}$



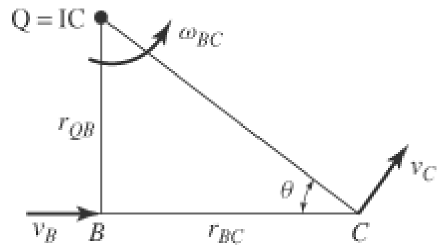
Solution:

$$r_{QB} = r_{BC}\tan(\theta)$$

$$\omega_{BC} = \frac{v_C}{r_{QB}} \qquad \omega_{BC} = 4.00\ \frac{\text{rad}}{\text{s}}$$

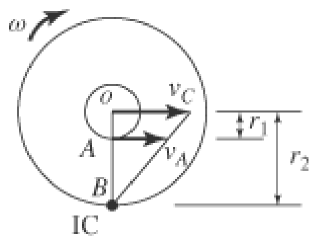
$$v_B = \omega_{BC} r_{QB} \qquad v_B = 4.00\ \frac{\text{ft}}{\text{s}}$$

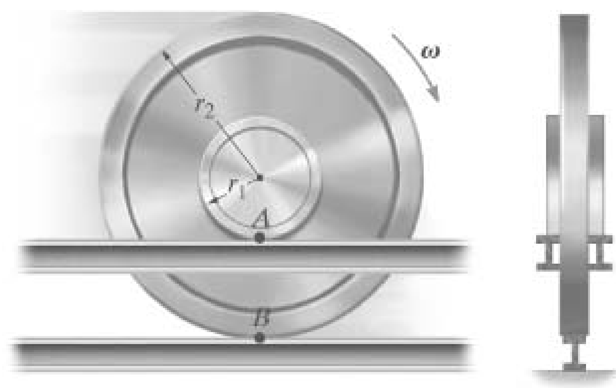
$$\omega_{AB} = \frac{v_B}{r_{AB}} \qquad \omega_{AB} = 4.00\ \frac{\text{rad}}{\text{s}}$$



Problem 16-90

Show that if the rim of the wheel and its hub maintain contact with the three tracks as the wheel rolls, it is necessary that slipping occurs at the hub *A* if no slipping occurs at *B*. Under these conditions, what is the speed at *A* if the wheel has an angular velocity ω?





Solution:

IC is at *B*. $\qquad v_A = \omega(r_2 - r_1)$

Problem 16-91

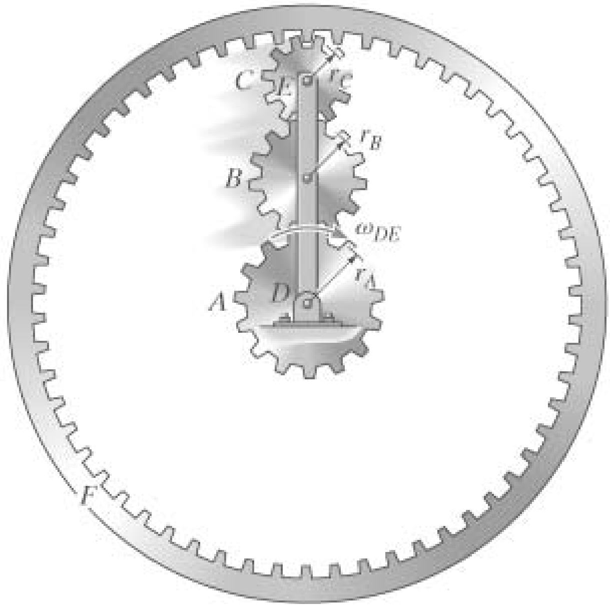
The epicyclic gear train is driven by the rotating link *DE*, which has an angular velocity ω_{DE}. If the ring gear *F* is fixed, determine the angular velocities of gears *A*, *B*, and *C*.

Given:

$$r_A = 50\ \text{mm} \qquad r_C = 30\ \text{mm}$$

$$r_B = 40\ \text{mm} \qquad \omega_{DE} = 5\ \frac{\text{rad}}{\text{s}}$$

Solution:



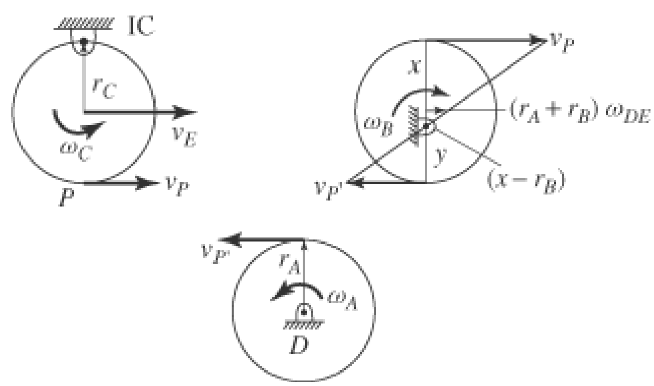
$$v_E = (r_A + 2r_B + r_C)\omega_{DE}$$

$$\omega_C = \frac{v_E}{r_C} \qquad \omega_C = 26.7\ \frac{\text{rad}}{\text{s}}$$

$$v_P = 2r_C\omega_C$$

$$\frac{v_P}{x} = \frac{(r_A + r_B)\omega_{DE}}{x - r_B}$$

$$x = \frac{r_B v_P}{v_P - (r_A + r_B)\omega_{DE}}$$



$$\omega_B = \frac{v_P}{x} \qquad \omega_B = 28.75\ \frac{\text{rad}}{\text{s}}$$

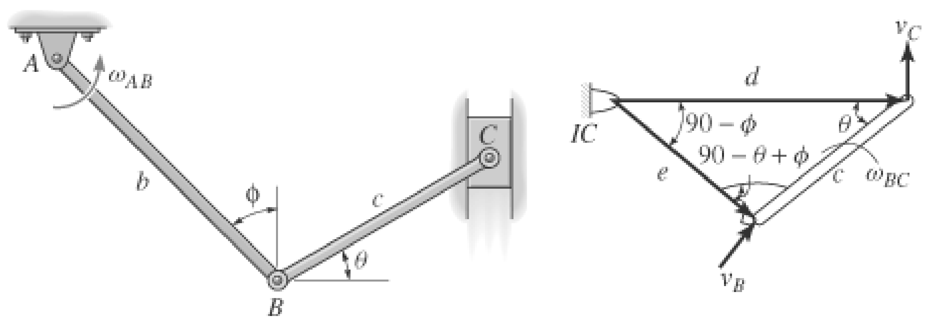
$$v_{P'} = \omega_B(2r_B - x)$$

$$\omega_A = \frac{v_{P'}}{r_A} \qquad \omega_A = 14.0\ \frac{\text{rad}}{\text{s}}$$

***Problem 16-92**

Determine the angular velocity of link *AB* at the instant shown if block *C* is moving upward at speed v_C.

Given:



$$v_C = 12\ \frac{\text{in}}{\text{s}}$$

$$c = 4\ \text{in}$$

$$b = 5\ \text{in}$$

$$\theta = 30\ \text{deg}$$

$$\phi = 45\ \text{deg}$$

Solution:

$$d = c\left(\frac{\sin(90\ \text{deg} - \theta + \phi)}{\sin(90\ \text{deg} - \phi)}\right) \qquad d = 5.46\ \text{in}$$

$$e = c\left(\frac{\sin(\theta)}{\sin(90\ \text{deg} - \phi)}\right) \qquad e = 2.83\ \text{in}$$

$$\omega_{BC} = \frac{v_C}{d} \qquad \omega_{BC} = 2.20\ \frac{\text{rad}}{\text{s}}$$

$$v_B = \omega_{BC}\, e \qquad v_B = 6.21\ \frac{\text{in}}{\text{s}}$$

$$\omega_{AB} = \frac{v_B}{b} \qquad \omega_{AB} = 1.24\ \frac{\text{rad}}{\text{s}}$$

Problem 16-93

In an automobile transmission the planet pinions *A* and *B* rotate on shafts that are mounted on the planet pinion carrier *CD*. As shown, *CD* is attached to a shaft at *E* which is aligned with the center of the *fixed* sun gear *S*. This shaft is not attached to the sun gear. If *CD* is rotating with angular velocity ω_{CD}, determine the angular velocity of the ring gear *R*.

Given:

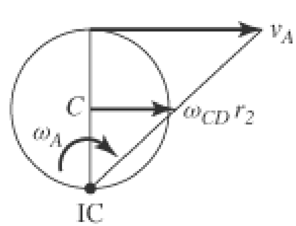
$r_1 = 50\ \text{mm} \qquad r_2 = r_1 + r_3$

$r_3 = 75\ \text{mm} \qquad \omega_{CD} = 8\ \frac{\text{rad}}{\text{s}}$

Solution:

Pinion *A*:

$$\omega_A = \frac{r_2\omega_{CD}}{r_1} \qquad \omega_A = 20.00\ \frac{\text{rad}}{\text{s}}$$



$$v_R = \omega_A(2r_1) \qquad v_R = 2.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_R = \frac{v_R}{r_2 + r_1} \qquad \omega_R = 11.4\ \frac{\text{rad}}{\text{s}}$$

Problem 16-94

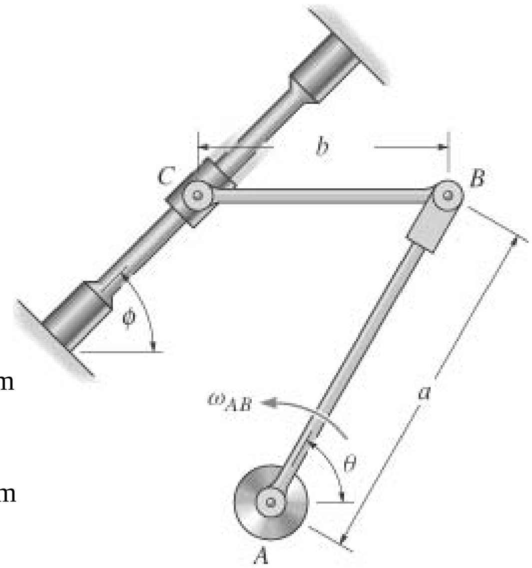
Knowing that the angular velocity of link *AB* is ω_{AB}, determine the velocity of the collar at *C* and the angular velocity of link *CB* at the instant shown. Link *CB* is horizontal at this instant.

Given:

$\omega_{AB} = 4\ \frac{rad}{s}$ $\quad \theta = 60\ deg$

$a = 500\ mm$ $\quad \phi = 45\ deg$

$b = 350\ mm$

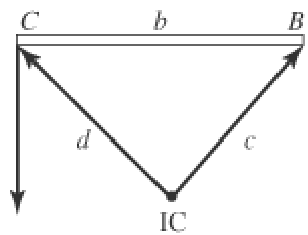


Solution:

$$c = b\left(\frac{\sin(90\ deg - \phi)}{\sin(90\ deg - \theta + \phi)}\right) \qquad c = 256.22\ mm$$

$$d = b\left(\frac{\sin(\theta)}{\sin(90\ deg - \theta + \phi)}\right) \qquad d = 313.80\ mm$$

$$v_B = \omega_{AB}\, a \qquad v_B = 2.00\ \frac{m}{s}$$



$$\omega_{BC} = \frac{v_B}{c} \qquad \omega_{BC} = 7.81\ \frac{rad}{s}$$

$$v_C = \omega_{BC}\, d \qquad v_C = 2.45\ \frac{m}{s}$$

Problem 16-95

If the collar at *C* is moving downward to the left with speed v_C, determine the angular velocity of link *AB* at the instant shown.

Given:

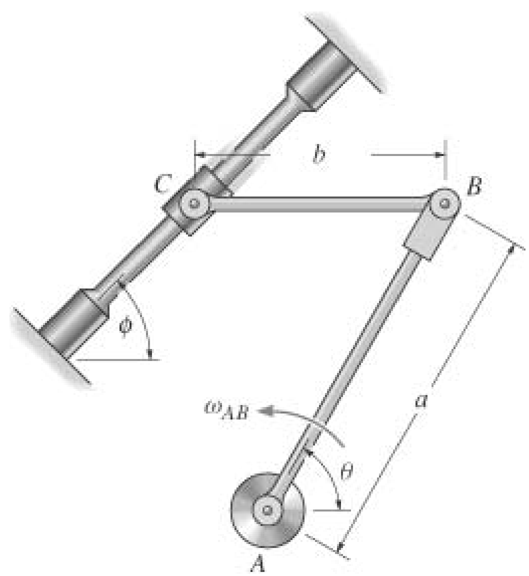
$v_C = 8\ \frac{m}{s}$

$a = 500\ mm$

$b = 350\ mm$

$\theta = 60\ deg$

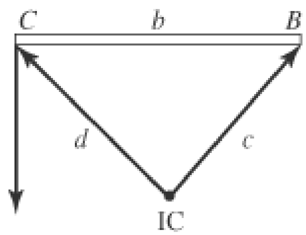
$\phi = 45\ deg$



Solution:

$$c = b\left(\frac{\sin(90\ \text{deg} - \phi)}{\sin(90\ \text{deg} - \theta + \phi)}\right) \qquad c = 256.22\ \text{mm}$$

$$d = b\left(\frac{\sin(\theta)}{\sin(90\ \text{deg} - \theta + \phi)}\right) \qquad d = 313.80\ \text{mm}$$



$$\omega_{BC} = \frac{v_C}{d} \qquad \omega_{BC} = 25.49\ \frac{\text{rad}}{\text{s}}$$

$$v_B = \omega_{BC}\, c \qquad v_B = 6.53\ \frac{\text{m}}{\text{s}}$$

$$\omega_{AB} = \frac{v_B}{a} \qquad \omega_{AB} = 13.1\ \frac{\text{rad}}{\text{s}}$$

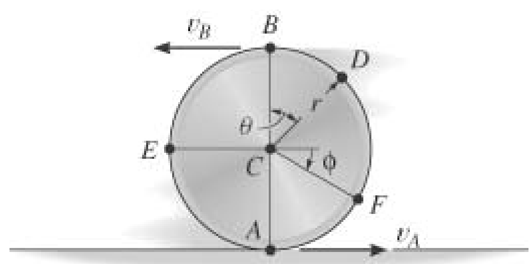
***Problem 16-96**

Due to slipping, points A and B on the rim of the disk have the velocities v_A and v_B. Determine the velocities of the center point C and point D at this instant.

Given:

$$v_A = 5\ \frac{\text{ft}}{\text{s}} \qquad \theta = 45\ \text{deg} \qquad r = 0.8\ \text{ft}$$

$$v_B = 10\ \frac{\text{ft}}{\text{s}} \qquad \phi = 30\ \text{deg}$$

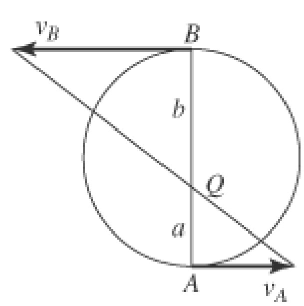


Solution:

Guesses $a = 1\ \text{ft} \qquad b = 1\ \text{ft}$

Given $\dfrac{a}{v_A} = \dfrac{b}{v_B} \qquad a + b = 2r$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b) \qquad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0.53 \\ 1.07 \end{pmatrix} \text{ft}$$



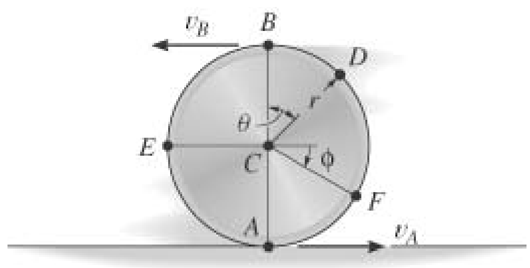
$$\omega = \frac{v_A}{a} \qquad \omega = 9.38\ \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega(r - a) \qquad v_C = 2.50\ \frac{\text{ft}}{\text{s}}$$

$$v_D = \omega\sqrt{(r - a + r\cos(\theta))^2 + (r\sin(\theta))^2} \qquad v_D = 9.43\ \frac{\text{ft}}{\text{s}}$$

Problem 16-97

Due to slipping, points A and B on the rim of the disk have the velocities v_A and v_B. Determine the velocities of the center point C and point E at this instant.



Given:

$v_A = 5\ \frac{\text{ft}}{\text{s}}$ $\theta = 45\ \text{deg}$ $r = 0.8\ \text{ft}$

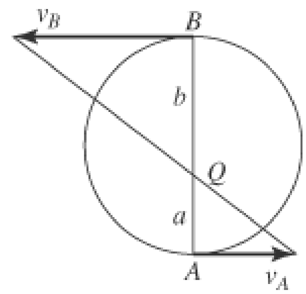
$v_B = 10\ \frac{\text{ft}}{\text{s}}$ $\phi = 30\ \text{deg}$

Solution:

Guesses $a = 1\ \text{ft}$ $b = 1\ \text{ft}$

Given $\frac{a}{v_A} = \frac{b}{v_B}$ $a + b = 2r$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \text{Find}(a, b) \qquad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0.53 \\ 1.07 \end{pmatrix} \text{ft}$$



$$\omega = \frac{v_A}{a} \qquad \omega = 9.38\ \frac{\text{rad}}{\text{s}}$$

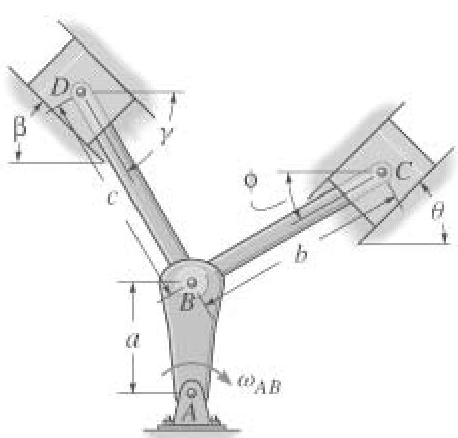
$$v_C = \omega(r - a) \qquad v_C = 2.50\ \frac{\text{ft}}{\text{s}}$$

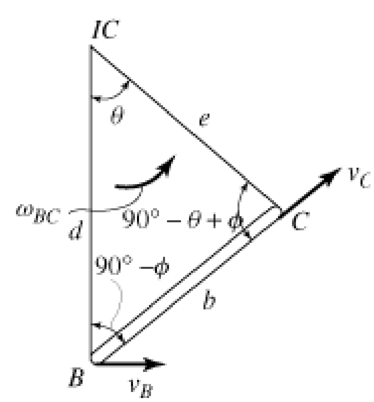
$$v_E = \omega\sqrt{(r - a)^2 + r^2} \qquad v_E = 7.91\ \frac{\text{ft}}{\text{s}}$$

Problem 16-98

The mechanism used in a marine engine consists of a single crank AB and two connecting rods BC and BD. Determine the velocity of the piston at C the instant the crank is in the position shown and has an angular velocity ω_{AB}.

Given:





$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.2\ \text{m}$

$b = 0.4\ \text{m}$

$c = 0.4\ \text{m}$

$\theta = 45\ \text{deg}$

$\phi = 30\ \text{deg}$

$\beta = 45\ \text{deg}$

Solution:

$$d = b\left(\frac{\sin(90\ \text{deg} - \phi)}{\sin(\theta)}\right) \qquad d = 0.49\ \text{m}$$

$$e = b\left(\frac{\sin(90\ \text{deg} + \phi - \theta)}{\sin(\theta)}\right) \qquad e = 0.55\ \text{m}$$

$$v_B = \omega_{AB}\, a \qquad v_B = 1.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83\ \frac{\text{rad}}{\text{s}}$$

$$v_C = \omega_{BC}\, d \qquad v_C = 0.897\ \frac{\text{m}}{\text{s}}$$

Problem 16-99

The mechanism used in a marine engine consists of a single crank *AB* and two connecting rods *BC* and *BD*. Determine the velocity of the piston at *D* the instant the crank is in the position shown and has an angular velocity ω_{AB}.

Given:

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.2\ \text{m}$

$b = 0.4\ \text{m}$

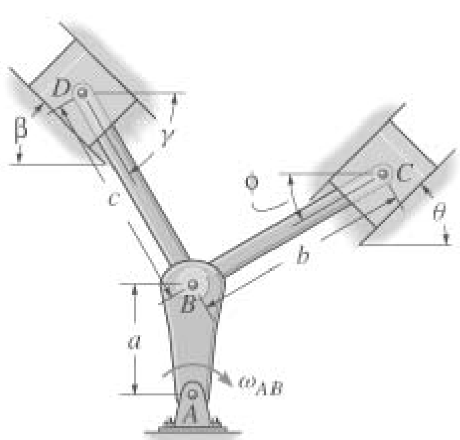
$c = 0.4\ \text{m}$

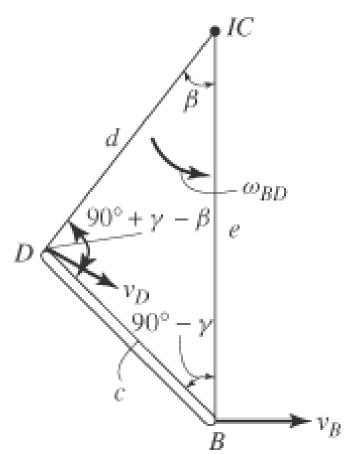
$\theta = 45\ \text{deg}$

$\phi = 30\ \text{deg}$

$\gamma = 60\ \text{deg}$

$\beta = 45\ \text{deg}$





Solution:

$$d = c\left(\frac{\sin(90\ \text{deg} - \gamma)}{\sin(\beta)}\right) \qquad d = 0.28\ \text{m}$$

$$e = c\left(\frac{\sin(90\ \text{deg} + \gamma - \beta)}{\sin(\beta)}\right) \qquad e = 0.55\ \text{m}$$

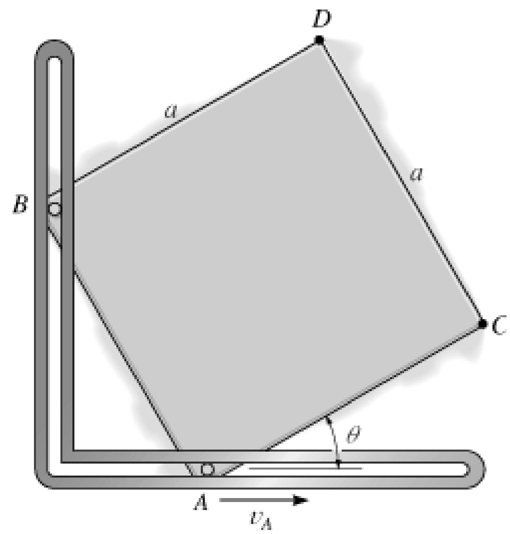
$$v_B = \omega_{AB}\, a \qquad v_B = 1.00\ \frac{\text{m}}{\text{s}}$$

$$\omega_{BC} = \frac{v_B}{e} \qquad \omega_{BC} = 1.83\,\frac{\text{rad}}{\text{s}}$$

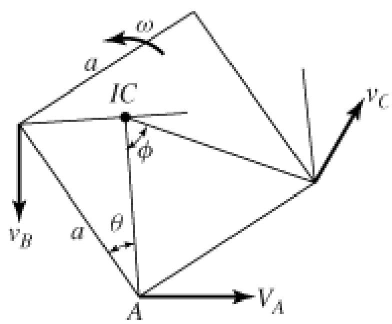
$$v_D = \omega_{BC}\,d \qquad v_D = 0.518\,\frac{\text{m}}{\text{s}}$$

***Problem 16-100**

The square plate is confined within the slots at A and B. In the position shown, point A is moving to the right with speed v_A. Determine the velocity of point C at this instant.



Given:



$\theta = 30$ deg

$v_A = 8\,\frac{\text{m}}{\text{s}}$

$a = 0.3$ m

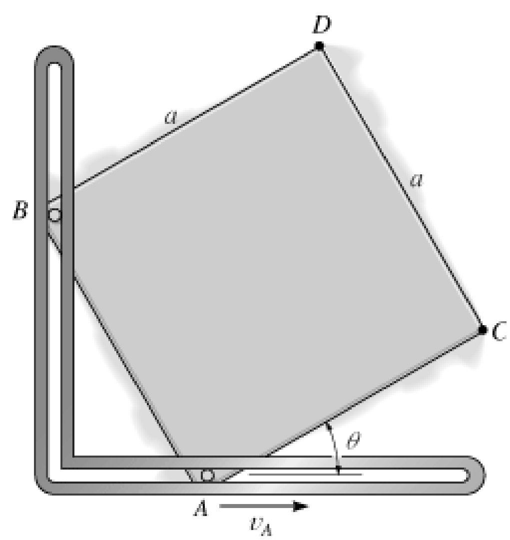
Solution:

$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79\,\frac{\text{rad}}{\text{s}}$$

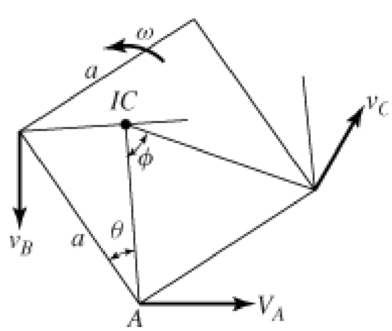
$$v_C = \omega\sqrt{(a\cos(\theta))^2 + (a\cos(\theta) - a\sin(\theta))^2} \qquad v_C = 8.69\,\frac{\text{m}}{\text{s}}$$

Problem 16-101

The square plate is confined within the slots at A and B. In the position shown, point A is moving to the right at speed v_A. Determine the velocity of point D at this instant.



Given:



$\theta = 30$ deg

$v_A = 8\,\frac{\text{m}}{\text{s}}$

$a = 0.3$ m

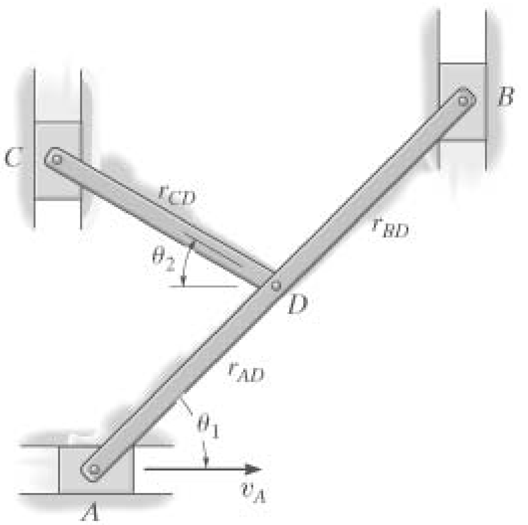
Solution:

$$\omega = \frac{v_A}{a\cos(\theta)} \qquad \omega = 30.79\ \frac{\text{rad}}{\text{s}}$$

$$v_D = \omega\sqrt{(-a\sin(\theta) + a\cos(\theta))^2 + (a\sin(\theta))^2} \qquad v_D = 5.72\ \frac{\text{m}}{\text{s}}$$

Problem 16-102

If the slider block A is moving to the right with speed v_A, determine the velocities of blocks B and C at the instant shown.



Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}}$ $\qquad r_{AD} = 2\ \text{ft}$

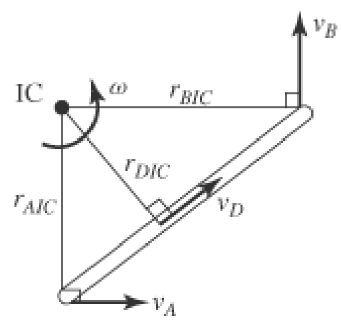
$\theta_1 = 45\ \text{deg}$ $\qquad r_{BD} = 2\ \text{ft}$

$\theta_2 = 30\ \text{deg}$ $\qquad r_{CD} = 2\ \text{ft}$

Solution:

$$r_{AIC} = (r_{AD} + r_{BD})\sin(\theta_1)$$

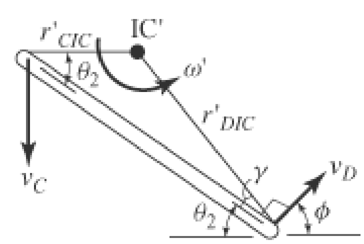
$$r_{BIC} = (r_{AD} + r_{BD})\cos(\theta_1)$$



$$r_{CID} = \sqrt{r_{AIC}^2 + r_{AD}^2 - 2r_{AIC}r_{AD}\sin(\theta_1)}$$

$$\phi = \operatorname{asin}\left(\frac{r_{AD}}{r_{CID}}\cos(\theta_1)\right)$$

$$\gamma = 90\ \text{deg} - \phi - \theta_2$$



$$r'_{CIC} = r_{CD}\left(\frac{\sin(\gamma)}{\sin(90\ \text{deg} + \phi)}\right)$$

$$r'_{DIC} = r_{CD}\left(\frac{\sin(\theta_2)}{\sin(90\ \text{deg} + \phi)}\right)$$

$$\omega_{AB} = \frac{v_A}{r_{AIC}} \qquad v_B = \omega_{AB}r_{BIC} \qquad v_B = 8.00\ \frac{\text{ft}}{\text{s}}$$

$v_D = \omega_{AB} r_{CID}$ $\qquad \omega_{CD} = \frac{v_D}{r'_{DIC}}$ $\qquad v_C = \omega_{CD} r'_{CIC}$ $\qquad v_C = 2.93 \frac{ft}{s}$

Problem 16-103

The crankshaft AB rotates with angular velocity ω_{AB} about the fixed axis through point A, and the disk at C is held fixed in its support at E. Determine the angular velocity of rod CD at the instant shown where CD is perpendicular to BF.

Given:

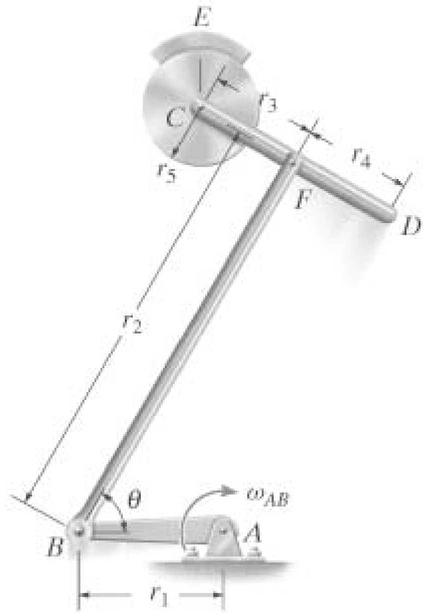
$r_1 = 100\ \text{mm}$ $\qquad \theta = 60\ \text{deg}$

$r_2 = 300\ \text{mm}$ $\qquad \omega_{AB} = 50\ \frac{rad}{s}$

$r_3 = 75\ \text{mm}$

$r_4 = 75\ \text{mm}$

$r_5 = 40\ \text{mm}$



Solution:

$r_{BIC} = \frac{r_2}{\cos(\theta)}$ $\qquad r_{BIC} = 0.60\ \text{m}$

$r_{FIC} = r_2 \tan(\theta)$ $\qquad r_{FIC} = 0.5196\ \text{m}$

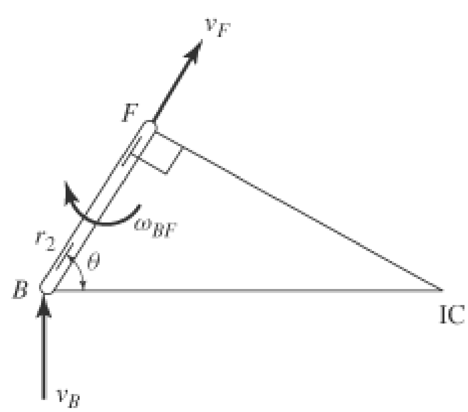
$v_B = \omega_{AB} r_1$ $\qquad v_B = 5.00\ \frac{m}{s}$

$\omega_{BF} = \frac{v_B}{r_{BIC}}$ $\qquad \omega_{BF} = 8.33\ \frac{rad}{s}$

$v_F = \omega_{BF} r_{FIC}$ $\qquad v_F = 4.330\ \frac{m}{s}$

Thus,

$\omega_{CD} = \frac{v_F}{r_3}$ $\qquad \omega_{CD} = 57.7\ \frac{rad}{s}$



***Problem 16-104**

The mechanism shown is used in a riveting machine. It consists of a driving piston A, three members, and a riveter which is attached to the slider block D. Determine the velocity of D at the instant shown, when the piston at A is traveling at v_A.

Given:

$r_{AC} = 300\ \text{mm}$

$r_{BC} = 200\ \text{mm}$

$r_{CD} = 150\ \text{mm}$

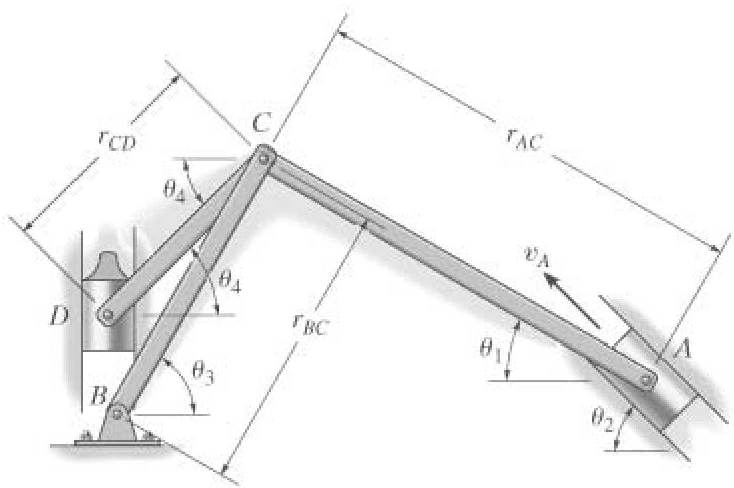
$v_A = 30\ \frac{\text{m}}{\text{s}}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 45\ \text{deg}$

$\theta_3 = 60\ \text{deg}$

$\theta_4 = 45\ \text{deg}$



Solution:

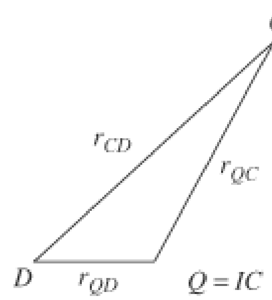
$\alpha = \theta_3 - \theta_2$

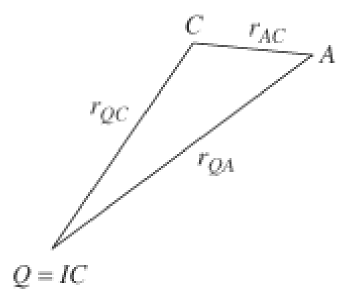
$\beta = 90\ \text{deg} - \theta_2 + \theta_1$

$\gamma = 180\ \text{deg} - \alpha - \beta$

$\delta = \theta_3 - \theta_4$

$\varepsilon = 180\ \text{deg} - \theta_4 - \delta$





$r_{QC} = r_{AC}\left(\frac{\sin(\beta)}{\sin(\alpha)}\right) \qquad r_{QA} = r_{AC}\left(\frac{\sin(\gamma)}{\sin(\alpha)}\right)$

$r_{Q'D} = r_{CD}\left(\frac{\sin(\delta)}{\sin(\varepsilon)}\right) \qquad r_{Q'C} = r_{CD}\left(\frac{\sin(\theta_4)}{\sin(\varepsilon)}\right)$

$\omega_{AC} = \frac{v_A}{r_{QA}} \qquad v_C = \omega_{AC} r_{QC} \qquad \omega_{CD} = \frac{v_C}{r_{Q'C}} \qquad v_D = \omega_{CD} r_{Q'D}$

$v_D = 10.61\ \frac{\text{m}}{\text{s}}$

Problem 16-105

At a given instant the bottom A of the ladder has acceleration a_A and velocity v_A, both acting to the left. Determine the acceleration of the top of the ladder, B, and the ladder's angular acceleration at this same instant.

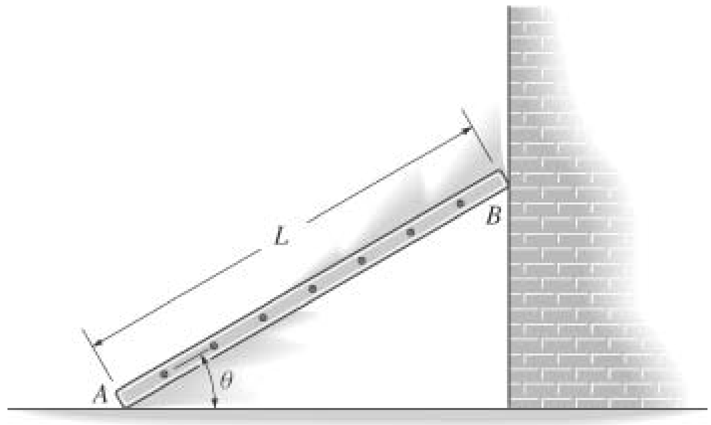
Given:

$a_A = 4\ \frac{ft}{s^2}$

$v_A = 6\ \frac{ft}{s}$

$L = 16\ ft$

$\theta = 30\ deg$



Solution: Guesses $\omega = 1\ \frac{rad}{s}$ $v_B = 1\ \frac{ft}{s}$ $\alpha = 1\ \frac{rad}{s^2}$ $a_B = 1\ \frac{ft}{s^2}$

Given $\begin{pmatrix} -v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix}$

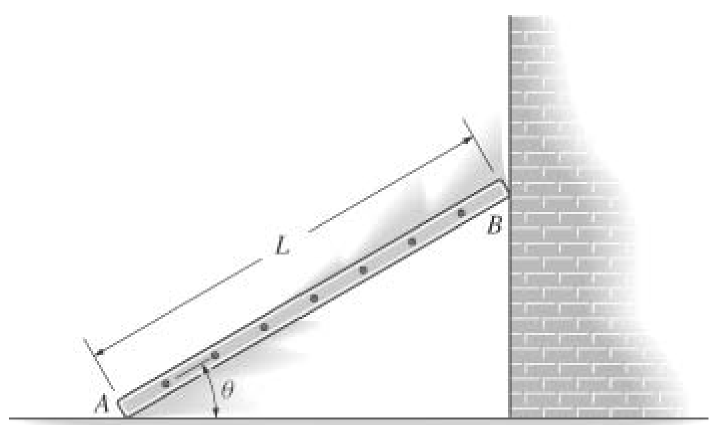
$$\begin{pmatrix} -a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix}$$

$\begin{pmatrix} \omega \\ \alpha \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega, \alpha, v_B, a_B)$ $\omega = -0.75\ \frac{rad}{s}$ $v_B = -10.39\ \frac{ft}{s}$

$\alpha = -1.47\ \frac{rad}{s^2}$ $a_B = -24.93\ \frac{ft}{s^2}$

Problem 16-106

At a given instant the top B of the ladder has acceleration a_B and velocity v_B both acting downward. Determine the acceleration of the bottom A of the ladder, and the ladder's angular acceleration at this instant.



Given:

$a_B = 2\ \frac{ft}{s^2}$ $v_B = 4\ \frac{ft}{s}$

$L = 16\ \text{ft} \qquad \theta = 30\ \text{deg}$

Solution: Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}} \qquad v_A = 1\ \frac{\text{ft}}{\text{s}} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ L\sin(\theta) \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ -a_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega \\ \alpha \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\omega, \alpha, v_A, a_A) \qquad \omega = -0.289\ \frac{\text{rad}}{\text{s}} \qquad v_A = -2.31\ \frac{\text{ft}}{\text{s}}$$

$$\alpha = -0.0962\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 0.385\ \frac{\text{ft}}{\text{s}^2}$$

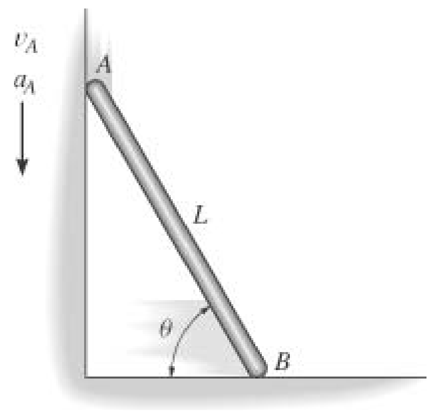
Problem 16-107

At a given instant the top end *A* of the bar has the velocity and acceleration shown. Determine the acceleration of the bottom *B* and the bar’s angular acceleration at this instant.

Given:

$v_A = 5\ \frac{\text{ft}}{\text{s}} \qquad L = 10\ \text{ft}$

$a_A = 7\ \frac{\text{ft}}{\text{s}^2} \qquad \theta = 60\ \text{deg}$



Solution:

Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $$\begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} L\cos(\theta) \\ -L\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$$

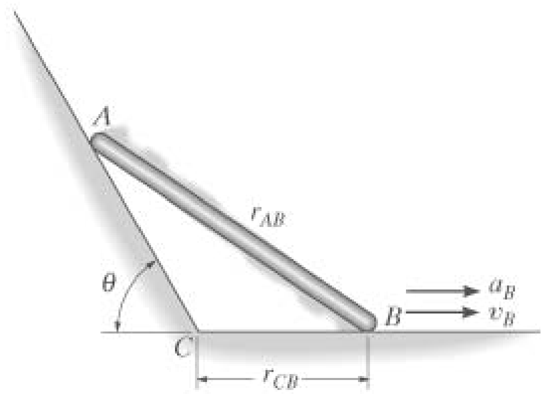
$$\begin{pmatrix}0\\-a_A\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\alpha\end{pmatrix}\times\begin{pmatrix}L\cos(\theta)\\-L\sin(\theta)\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}L\cos(\theta)\\-L\sin(\theta)\\0\end{pmatrix}\right]=\begin{pmatrix}a_B\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}\omega\\v_B\\\alpha\\a_B\end{pmatrix}=\text{Find}(\omega,v_B,\alpha,a_B)\qquad \omega = 1.00\ \frac{\text{rad}}{\text{s}}\qquad v_B = 8.66\ \frac{\text{ft}}{\text{s}}$$

$$\alpha = -0.332\ \frac{\text{rad}}{\text{s}^2}\qquad a_B = -7.88\ \frac{\text{ft}}{\text{s}^2}$$

***Problem 16-108**

The rod of length r_{AB} slides down the inclined plane, such that when it is at B it has the motion shown. Determine the velocity and acceleration of A at this instant.



Given:

$r_{AB} = 10$ ft $\qquad v_B = 2\ \frac{\text{ft}}{\text{s}}$

$r_{CB} = 4$ ft $\qquad \theta = 60$ deg

$a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}\quad v_A = 1\ \frac{\text{ft}}{\text{s}}\quad a_A = 1\ \frac{\text{ft}}{\text{s}^2}\quad \phi = 1$ deg

Given $\quad r_{AB}\sin(\theta-\phi) = r_{CB}\sin(\theta)$

$$\begin{pmatrix}v_B\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}-r_{AB}\cos(\phi)\\r_{AB}\sin(\phi)\\0\end{pmatrix}=\begin{pmatrix}v_A\cos(\theta)\\-v_A\sin(\theta)\\0\end{pmatrix}$$

$$\begin{pmatrix}a_B\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\alpha\end{pmatrix}\times\begin{pmatrix}-r_{AB}\cos(\phi)\\r_{AB}\sin(\phi)\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega\end{pmatrix}\times\begin{pmatrix}-r_{AB}\cos(\phi)\\r_{AB}\sin(\phi)\\0\end{pmatrix}\right]=\begin{pmatrix}a_A\cos(\theta)\\-a_A\sin(\theta)\\0\end{pmatrix}$$

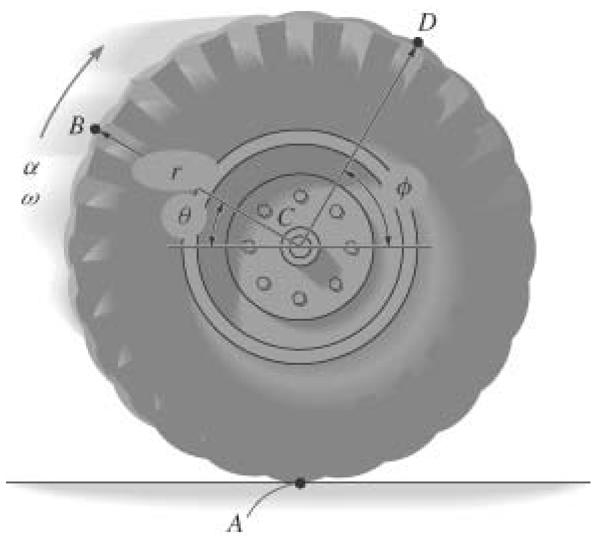
$$\begin{pmatrix} \phi \\ \omega \\ \alpha \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\phi, \omega, \alpha, v_A, a_A) \qquad \phi = 39.73\,\text{deg}$$

$$\omega = 0.18\,\frac{\text{rad}}{\text{s}} \qquad \alpha = 0.1049\,\frac{\text{rad}}{\text{s}^2} \qquad v_A = 1.640\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1.18\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-109

The wheel is moving to the right such that it has angular velocity ω and angular acceleration α at the instant shown. If it does not slip at *A*, determine the acceleration of point *B*.

Given:



$$\omega = 2\,\frac{\text{rad}}{\text{s}}$$

$$\alpha = 4\,\frac{\text{rad}}{\text{s}^2}$$

$$r = 1.45\ \text{ft}$$

$$\theta = 30\ \text{deg}$$

$$\phi = 60\ \text{deg}$$

Solution:

$$\mathbf{a_B} = \begin{pmatrix} \alpha r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha \end{pmatrix} \times \begin{pmatrix} -r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -r\cos(\theta) \\ r\sin(\theta) \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} 13.72 \\ 2.12 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 13.89\,\frac{\text{ft}}{\text{s}^2}$$

Problem 16-110

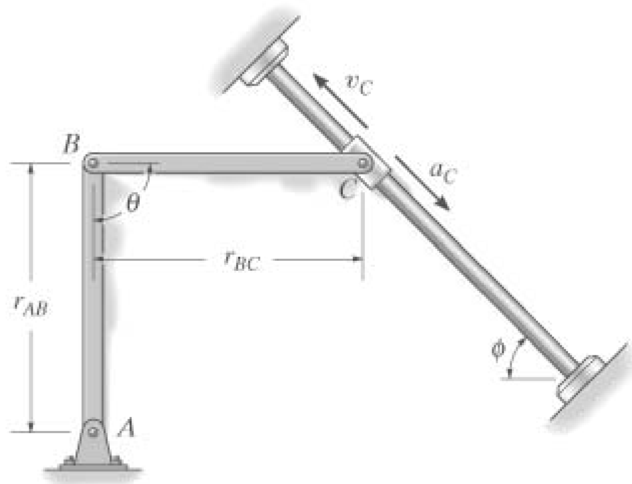
Determine the angular acceleration of link *AB* at the instant shown if the collar *C* has velocity v_c and deceleration a_c as shown.

Given:

$v_C = 4\ \frac{\text{ft}}{\text{s}}$ $r_{AB} = 0.5\ \text{ft}$

$a_C = 3\ \frac{\text{ft}}{\text{s}^2}$ $r_{BC} = 0.5\ \text{ft}$

$\theta = 90\ \text{deg}$ $\phi = 45\ \text{deg}$



Solution:

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{AB} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} r_{BC}\sin(\theta) \\ -r_{BC}\cos(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} -v_C\cos(\phi) \\ v_C\sin(\phi) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{AB} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ r_{AB} \\ 0 \end{pmatrix}\right] \dots = \begin{pmatrix} a_C\cos(\phi) \\ -a_C\sin(\phi) \\ 0 \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} r_{BC}\sin(\theta) \\ -r_{BC}\cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} r_{BC}\sin(\theta) \\ -r_{BC}\cos(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 5.66 \\ 5.66 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$\alpha_{BC} = 27.8\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AB} = -36.2\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-111

The flywheel rotates with angular velocity ω and angular acceleration α. Determine the angular acceleration of links *AB* and *BC* at the instant shown.

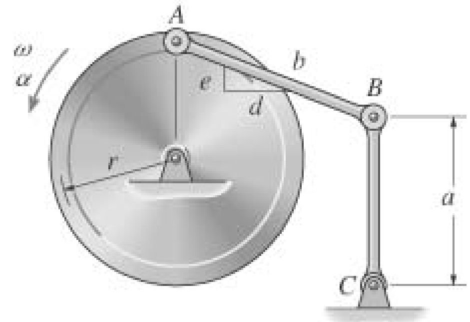
Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad a = 0.4\ \text{m}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 0.5\ \text{m}$

$r = 0.3\ \text{m}$ $\quad e = 3$

$d = 4$



Solution:

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} \quad \mathbf{r_2} = \frac{b}{\sqrt{e^2 + d^2}}\begin{pmatrix} d \\ -e \\ 0 \end{pmatrix} \quad \mathbf{r_3} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\omega\mathbf{k} \times \mathbf{r_1} + \omega_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{BC}\mathbf{k} \times \mathbf{r_3} = 0$$

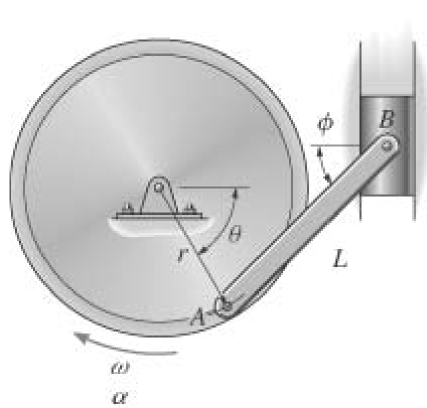
$$\alpha\mathbf{k} \times \mathbf{r_1} + \omega\mathbf{k} \times (\omega\mathbf{k} \times \mathbf{r_1}) + \alpha_{AB}\mathbf{k} \times \mathbf{r_2} + \omega_{AB}\mathbf{k} \times (\omega_{AB}\mathbf{k} \times \mathbf{r_2}) \ ... = 0$$
$$+ \alpha_{BC}\mathbf{k} \times \mathbf{r_3} + \omega_{BC}\mathbf{k} \times (\omega_{BC}\mathbf{k} \times \mathbf{r_3})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.50 \end{pmatrix}\frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 3.94 \end{pmatrix}\frac{\text{rad}}{\text{s}^2}$$

***Problem 16-112**

At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block *B* at this instant.



Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad \theta = 60\ \text{deg}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\quad L = 0.5\ \text{m}$

$r = 0.3\ \text{m}$ $\quad \phi = 45\ \text{deg}$

Solution:

$$\mathbf{r_1} = r\begin{pmatrix}\cos(\theta)\\ -\sin(\theta)\\ 0\end{pmatrix} \qquad \mathbf{r_2} = L\begin{pmatrix}\cos(\phi)\\ \sin(\phi)\\ 0\end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$ $a_B = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$-\omega\mathbf{k}\times\mathbf{r_1} + \omega_{AB}\mathbf{k}\times\mathbf{r_2} = \begin{pmatrix}0\\ v_B\\ 0\end{pmatrix}$$

$$-\alpha\,\mathbf{k}\times\mathbf{r_1} - \omega\mathbf{k}\times\left(-\omega\mathbf{k}\times\mathbf{r_1}\right) + \alpha_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{AB}\mathbf{k}\times\left(\omega_{AB}\mathbf{k}\times\mathbf{r_2}\right) = \begin{pmatrix}0\\ a_B\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\omega_{AB}\\ \alpha_{AB}\\ v_B\\ a_B\end{pmatrix} = \text{Find}\left(\omega_{AB}, \alpha_{AB}, v_B, a_B\right) \qquad \omega_{AB} = -1.47\ \frac{\text{rad}}{\text{s}} \qquad v_B = -0.82\ \frac{\text{m}}{\text{s}}$$

$$\alpha_{AB} = -8.27\ \frac{\text{rad}}{\text{s}^2} \qquad a_B = -3.55\ \frac{\text{m}}{\text{s}^2}$$

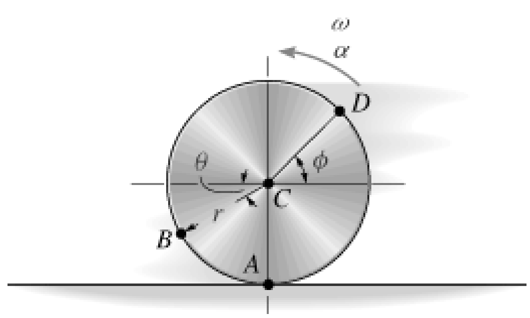
Problem 16-113

The disk is moving to the left such that it has angular acceleration α and angular velocity ω at the instant shown. If it does not slip at A, determine the acceleration of point B.

Given:

$$\alpha = 8\ \frac{\text{rad}}{\text{s}^2} \qquad r = 0.5\ \text{m} \qquad \phi = 45\ \text{deg}$$

$$\omega = 3\ \frac{\text{rad}}{\text{s}} \qquad \theta = 30\ \text{deg}$$



Solution:

$$\mathbf{a_B} = \begin{pmatrix}-\alpha r\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}0\\0\\ \alpha\end{pmatrix}\times\begin{pmatrix}-r\cos(\theta)\\ -r\sin(\theta)\\ 0\end{pmatrix} + \begin{pmatrix}0\\0\\ \omega\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\ \omega\end{pmatrix}\times\begin{pmatrix}-r\cos(\theta)\\ -r\sin(\theta)\\ 0\end{pmatrix}\right]$$

$$\mathbf{a_B} = \begin{pmatrix} 1.90 \\ -1.21 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_B}\right| = 2.25 \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{-\alpha r\cos(\theta) + \omega^2 r\sin(\theta)}{-\alpha r + \alpha r\sin(\theta) + \omega^2 r\cos(\theta)}\right) \qquad \theta = -32.6\ \text{deg} \qquad |\theta| = 32.62\ \text{deg}$$

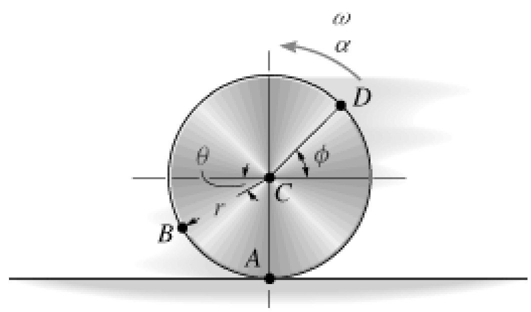
Problem 16-114

The disk is moving to the left such that it has angular acceleration α and angular velocity ω at the instant shown. If it does not slip at *A*, determine the acceleration of point *D*.

Given:

$$\alpha = 8 \frac{\text{rad}}{\text{s}^2} \qquad r = 0.5\ \text{m} \qquad \phi = 45\ \text{deg}$$

$$\omega = 3 \frac{\text{rad}}{\text{s}} \qquad \theta = 30\ \text{deg}$$



Solution:

$$\mathbf{a_D} = \begin{pmatrix} -\alpha r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ r\sin(\phi) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ r\sin(\phi) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_D} = \begin{pmatrix} -10.01 \\ -0.35 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_D}\right| = 10.02 \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\alpha r\cos(\phi) - \omega^2 r\sin(\phi)}{-\alpha r - \alpha r\sin(\phi) - \omega^2 r\cos(\phi)}\right) \qquad \theta = 2.02\ \text{deg}$$

Problem 16-115

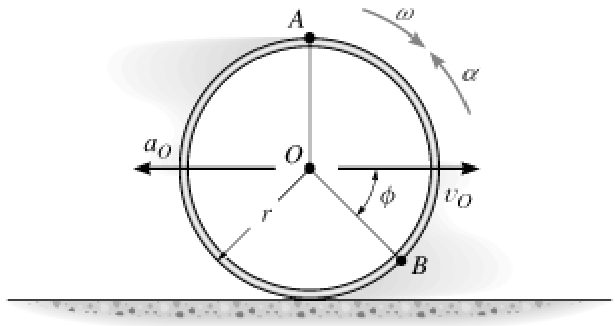
The hoop is cast on the rough surface such that it has angular velocity ω and angular acceleration α.. Also, its center has a velocity v_0 and a deceleration a_0. Determine the acceleration of point *A* at this instant.

Given:

$$\omega = 4 \frac{\text{rad}}{\text{s}} \qquad a_0 = 2 \frac{\text{m}}{\text{s}^2}$$

$\alpha = 5\ \frac{\text{rad}}{\text{s}^2}$ $\quad r = 0.3\ \text{m}$

$v_0 = 5\ \frac{\text{m}}{\text{s}}$ $\quad \phi = 45\ \text{deg}$



Solution:

$$\mathbf{a_A} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_A} = \begin{pmatrix} -3.50 \\ -4.80 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_A}\right| = 5.94\ \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{-\omega^2 r}{-a_0 - \alpha r}\right) \qquad \theta = 53.9\ \text{deg}$$

***Problem 16-116**

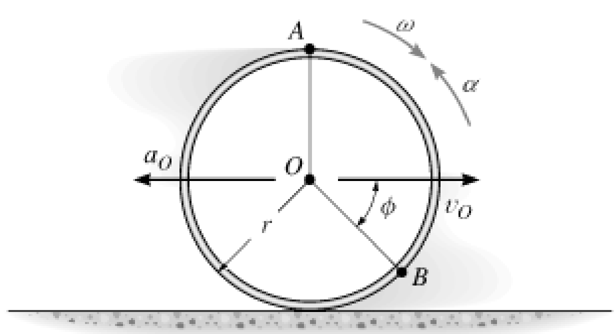
The hoop is cast on the rough surface such that it has angular velocity ω and angular acceleration α. Also, its center has a velocity v_0 and a deceleration a_0. Determine the acceleration of point B at this instant.

Given:

$\omega = 4\ \frac{\text{rad}}{\text{s}}$ $\quad a_0 = 2\ \frac{\text{m}}{\text{s}^2}$

$\alpha = 5\ \frac{\text{rad}}{\text{s}^2}$ $\quad r = 0.3\ \text{m}$

$v_0 = 5\ \frac{\text{m}}{\text{s}}$ $\quad \phi = 45\ \text{deg}$



Solution:

$$\mathbf{a_B} = \begin{pmatrix} -a_0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ -r\sin(\phi) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} r\cos(\phi) \\ -r\sin(\phi) \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_B} = \begin{pmatrix} -4.33 \\ 4.45 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_B}\right| = 6.21\ \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\alpha r\cos(\phi) + \omega^2 r\sin(\phi)}{-a_0 + \alpha r\sin(\phi) - \omega^2 r\cos(\phi)}\right) \qquad \theta = -45.8\ \text{deg} \qquad |\theta| = 45.8\ \text{deg}$$

Problem 16-117

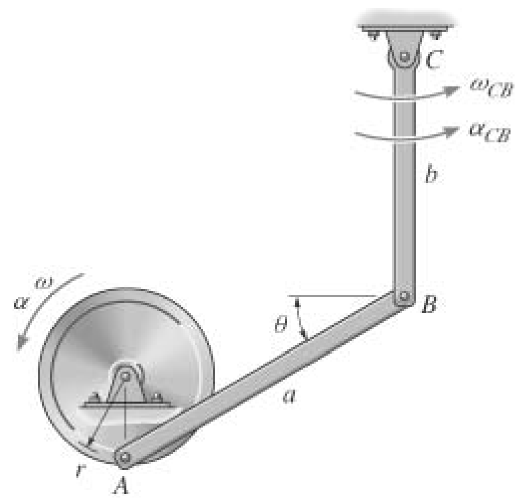
The disk rotates with angular velocity ω and angular acceleration α. Determine the angular acceleration of link *CB* at this instant.

Given:

$\omega = 5\ \frac{\text{rad}}{\text{s}}$ $\quad a = 2\ \text{ft}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 1.5\ \text{ft}$

$r = 0.5\ \text{ft}$ $\quad \theta = 30\ \text{deg}$



Solution:

$$\mathbf{r_1} = \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{r_3} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{AB} = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_{BC} = 1\ \frac{\text{rad}}{\text{s}} \quad \alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

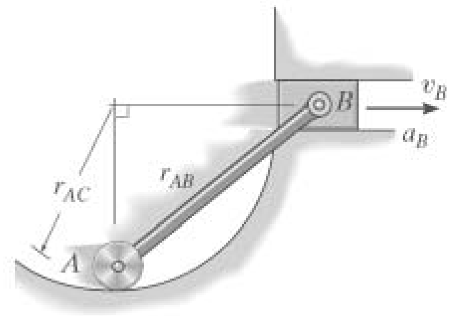
Given $\quad \omega\mathbf{k}\times\mathbf{r_1} + \omega_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{BC}\mathbf{k}\times\mathbf{r_3} = 0$

$$\alpha\mathbf{k}\times\mathbf{r_1} + \omega\mathbf{k}\times(\omega\mathbf{k}\times\mathbf{r_1}) + \alpha_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{AB}\mathbf{k}\times(\omega_{AB}\mathbf{k}\times\mathbf{r_2}) \ldots = 0$$
$$+ \alpha_{BC}\mathbf{k}\times\mathbf{r_3} + \omega_{BC}\mathbf{k}\times(\omega_{BC}\mathbf{k}\times\mathbf{r_3})$$

$$\begin{pmatrix} \omega_{AB} \\ \omega_{BC} \\ \alpha_{AB} \\ \alpha_{BC} \end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC}) \qquad \begin{pmatrix} \omega_{AB} \\ \omega_{BC} \end{pmatrix} = \begin{pmatrix} 0.00 \\ 1.67 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\alpha_{AB} = -4.81\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{BC} = 5.21\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-118



At a given instant the slider block *B* is moving to the right with the motion shown. Determine the angular acceleration of link *AB* and the acceleration of point *A* at this instant.

Given:

$a_B = 2\ \frac{\text{ft}}{\text{s}^2}$ $\quad r_{AB} = 5\ \text{ft}$

$v_B = 6\ \frac{\text{ft}}{\text{s}}$ $\quad r_{AC} = 3\ \text{ft}$

Solution: $\quad d = \sqrt{r_{AB}^2 - r_{AC}^2}$

From an instantaneous center analysis we find that $\quad \omega_{AB} = 0$

Guesses $\quad \alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad a_{Ax} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{Ax} \\ \frac{v_B^2}{r_{AC}} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} d \\ r_{AC} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{AB} \\ a_{Ax} \end{pmatrix} = \text{Find}(\alpha_{AB}, a_{Ax}) \qquad \mathbf{a_A} = \begin{pmatrix} a_{Ax} \\ \frac{v_B^2}{r_{AC}} \end{pmatrix} \qquad \mathbf{a_A} = \begin{pmatrix} -7.00 \\ 12.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$\alpha_{AB} = -3.00\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\alpha_{AB}\, d}{a_{Ax}}\right) \qquad \theta = 59.7\ \text{deg}$$

Problem 16-119

The closure is manufactured by the LCN Company and is used to control the restricted motion of a heavy door. If the door to which is it connected has an angular acceleration α, determine the angular accelerations of links *BC* and *CD*. Originally the door is not rotating but is hinged at *A*.

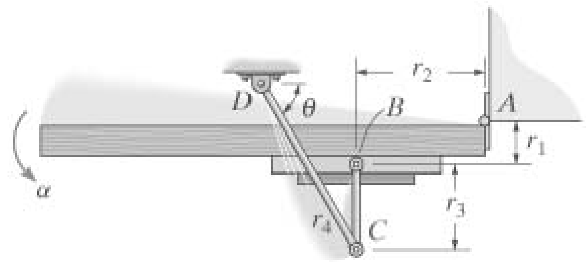
Given:

$r_1 = 2.5\ \text{in}$ $\quad \alpha = 3\ \frac{\text{rad}}{\text{s}^2}$

$r_2 = 6\ \text{in}$

$r_3 = 4\ \text{in}$ $\quad \theta = 60\ \text{deg}$

$r_4 = 12\ \text{in}$



Solution:

$$\mathbf{r_{AB}} = \begin{pmatrix} -r_2 \\ -r_1 \\ 0 \end{pmatrix} \quad \mathbf{r_{BC}} = \begin{pmatrix} 0 \\ -r_3 \\ 0 \end{pmatrix} \quad \mathbf{r_{CD}} = \begin{pmatrix} -r_4\cos(\theta) \\ r_4\sin(\theta) \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2} \quad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\quad \alpha\mathbf{k}\times\mathbf{r_{AB}} + \alpha_{BC}\mathbf{k}\times\mathbf{r_{BC}} + \alpha_{CD}\mathbf{k}\times\mathbf{r_{CD}} = 0$

$$\begin{pmatrix} \alpha_{BC} \\ \alpha_{CD} \end{pmatrix} = \text{Find}(\alpha_{BC}, \alpha_{CD}) \qquad \begin{pmatrix} \alpha_{BC} \\ \alpha_{CD} \end{pmatrix} = \begin{pmatrix} -9.67 \\ -3.00 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

***Problem 16-120**

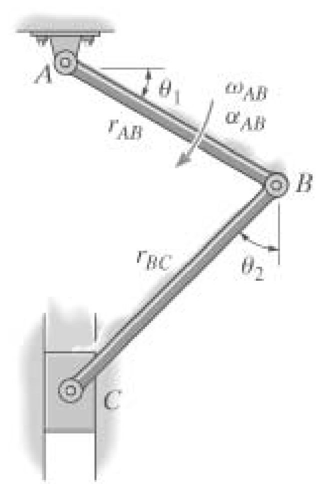
Rod *AB* has the angular motion shown. Determine the acceleration of the collar *C* at this instant.

Given:

$\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{AB} = 5\ \frac{\text{rad}}{\text{s}^2}$

$r_{AB} = 0.5\ \text{m}$ $\quad r_{BC} = 0.6\ \text{m}$

$\theta_1 = 30\ \text{deg}$ $\quad \theta_2 = 45\ \text{deg}$



Solution:

$$\mathbf{r_1} = \begin{pmatrix} r_{AB}\cos(\theta_1) \\ -r_{AB}\sin(\theta_1) \\ 0 \end{pmatrix} \qquad \mathbf{r_2} = \begin{pmatrix} -r_{BC}\sin(\theta_2) \\ -r_{BC}\cos(\theta_2) \\ 0 \end{pmatrix}$$

Guesses $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_C = 1\ \frac{\text{m}}{\text{s}}$ $a_C = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \mathbf{r_1} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \mathbf{r_2} = \begin{pmatrix} 0 \\ -v_C \\ 0 \end{pmatrix}$$

$$\left[\begin{pmatrix} 0 \\ 0 \\ -\alpha_{AB} \end{pmatrix} \times \mathbf{r_1} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \mathbf{r_1}\right] \ldots = \begin{pmatrix} 0 \\ -a_C \\ 0 \end{pmatrix} \\ + \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \mathbf{r_2} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \mathbf{r_2}\right]\right]$$

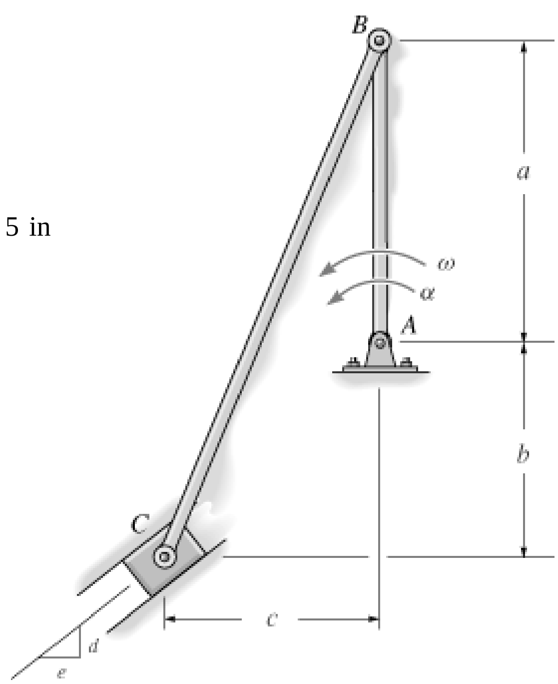
$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\omega_{BC}, \alpha_{BC}, v_C, a_C)$$

$\omega_{BC} = 1.77\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 9.01\ \frac{\text{rad}}{\text{s}^2}$

$v_C = 2.05\ \frac{\text{m}}{\text{s}}$ $a_C = 2.41\ \frac{\text{m}}{\text{s}^2}$

Problem 16-121

At the given instant member *AB* has the angular motions shown. Determine the velocity and acceleration of the slider block *C* at this instant.



Given:

$\omega = 3\ \frac{\text{rad}}{\text{s}}$ $a = 7\ \text{in}$ $d = 3$ $c = 5\ \text{in}$

$\alpha = 2\ \frac{\text{rad}}{\text{s}^2}$ $b = 5\ \text{in}$ $e = 4$

Solution:

Guesses

$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_C = 1\ \frac{\text{in}}{\text{s}}$ $a_C = 1\ \frac{\text{in}}{\text{s}^2}$

Given $$\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix} = \frac{v_C}{\sqrt{e^2+d^2}} \begin{pmatrix} e \\ d \\ 0 \end{pmatrix}$$

$$\left[\begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}\right]\right] \dots = \frac{a_C}{\sqrt{e^2+d^2}} \begin{pmatrix} e \\ d \\ 0 \end{pmatrix}$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -c \\ -a-b \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_C \\ a_C \end{pmatrix} = \text{Find}(\omega_{BC}, \alpha_{BC}, v_C, a_C)$$

$\omega_{BC} = 1.13 \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = -3.00 \frac{\text{rad}}{\text{s}^2}$

$v_C = -9.38 \frac{\text{in}}{\text{s}}$ $a_C = -54.7 \frac{\text{in}}{\text{s}^2}$

Problem 16-122

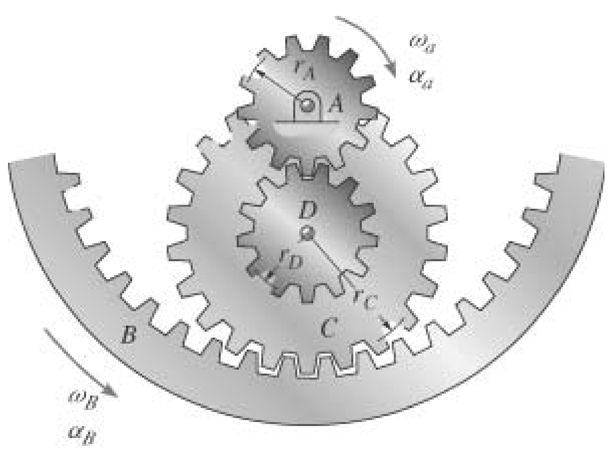
At a given instant gears *A* and *B* have the angular motions shown. Determine the angular acceleration of gear *C* and the acceleration of its center point *D* at this instant. Note that the inner hub of gear *C* is in mesh with gear *A* and its outer rim is in mesh with gear *B*.

Given:

$\omega_B = 1 \frac{\text{rad}}{\text{s}}$ $\omega_A = 4 \frac{\text{rad}}{\text{s}}$

$\alpha_B = 6 \frac{\text{rad}}{\text{s}^2}$ $\alpha_A = 8 \frac{\text{rad}}{\text{s}^2}$

$r_A = 5 \text{ in}$ $r_C = 10 \text{ in}$ $r_D = 5 \text{ in}$



Solution:

Guesses $\omega_C = 1 \frac{\text{rad}}{\text{s}}$ $\alpha_C = 1 \frac{\text{rad}}{\text{s}^2}$

Given $$-\omega_A r_A = \omega_B(r_A + r_D + r_C) - \omega_C(r_C + r_D)$$

$$-\alpha_A r_A = \alpha_B(r_A + r_D + r_C) - \alpha_C(r_C + r_D)$$

$$\begin{pmatrix} \omega_C \\ \alpha_C \end{pmatrix} = \text{Find}(\omega_C, \alpha_C) \qquad \omega_C = 2.67 \frac{\text{rad}}{\text{s}} \qquad \alpha_C = 10.67 \frac{\text{rad}}{\text{s}^2}$$

$$v_D = -\omega_A r_A + \omega_C r_D \qquad v_D = -6.67 \frac{\text{in}}{\text{s}}$$

$$a_{Dt} = -\alpha_A r_A + \alpha_C r_D \qquad a_{Dt} = 13.33 \frac{\text{in}}{\text{s}^2}$$

$$a_{Dn} = \frac{v_D^2}{r_A + r_D} \qquad a_{Dn} = 4.44 \frac{\text{in}}{\text{s}^2}$$

$$\mathbf{a_D} = \begin{pmatrix} a_{Dt} \\ a_{Dn} \end{pmatrix} \qquad \mathbf{a_D} = \begin{pmatrix} 13.33 \\ 4.44 \end{pmatrix} \frac{\text{in}}{\text{s}^2} \qquad |\mathbf{a_D}| = 14.05 \frac{\text{in}}{\text{s}^2}$$

Problem 16-123

The tied crank and gear mechanism gives rocking motion to crank *AC*, necessary for the operation of a printing press. If link *DE* has the angular motion shown, determine the respective angular velocities of gear *F* and crank *AC* at this instant, and the angular acceleration of crank *AC*.

Given:

$\omega_{DE} = 4 \frac{\text{rad}}{\text{s}}$

$\alpha_{DE} = 20 \frac{\text{rad}}{\text{s}^2}$

$a = 100 \text{ mm}$

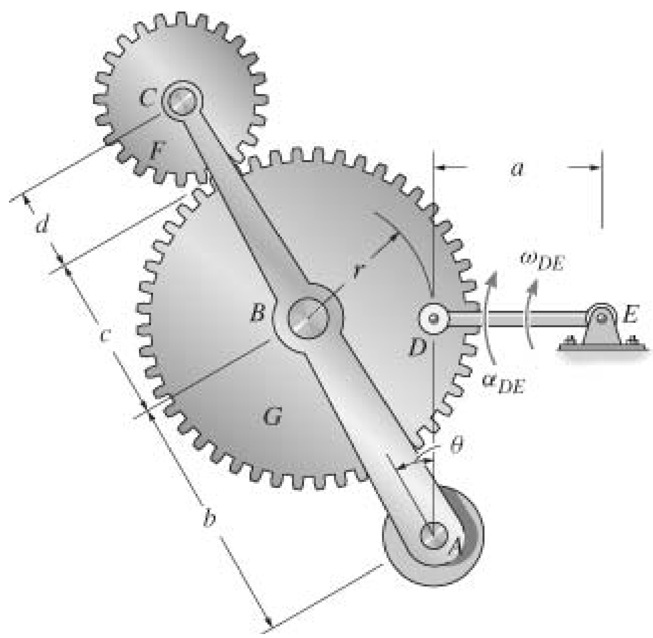
$b = 150 \text{ mm}$

$c = 100 \text{ mm}$

$d = 50 \text{ mm}$

$r = 75 \text{ mm}$

$\theta = 30 \text{ deg}$



Solution: Guesses

$\omega_G = 1 \frac{\text{rad}}{\text{s}} \qquad \omega_{AC} = 1 \frac{\text{rad}}{\text{s}}$

$\alpha_G = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix} = 0$

$$\begin{pmatrix} 0 \\ 0 \\ -\alpha_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega_{DE} \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix} \dots = 0$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_G \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_G \end{pmatrix} \times \begin{pmatrix} -r \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix} \dots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} b\sin(\theta) \\ -b\cos(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_G \\ \omega_{AC} \\ \alpha_G \\ \alpha_{AC} \end{pmatrix} = \text{Find}(\omega_G, \omega_{AC}, \alpha_G, \alpha_{AC})$$

$\omega_G = -5.33\ \frac{\text{rad}}{\text{s}}$ $\omega_{AC} = 0.00\ \frac{\text{rad}}{\text{s}}$

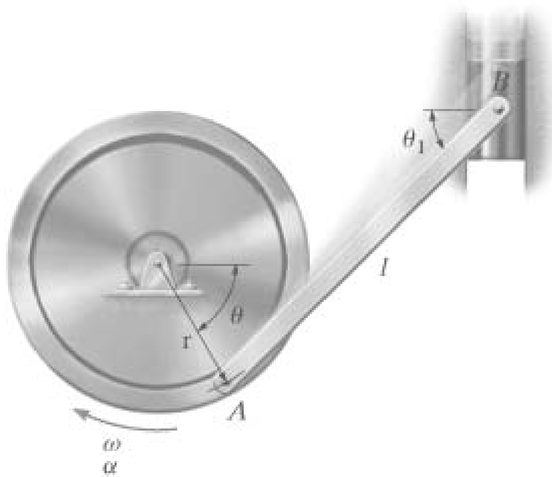
$\alpha_G = -2.07\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AC} = -28.7\ \frac{\text{rad}}{\text{s}^2}$

Now find the motion of gear F.

$\omega_{AC}\, b + \omega_G\, c = \omega_{AC}(b + c + d) - \omega_F\, d$ $\omega_F = \frac{\omega_{AC}(c + d) - \omega_G\, c}{d}$ $\omega_F = 10.67\ \frac{\text{rad}}{\text{s}}$

***Problem 16-124**

At a given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the acceleration of block *B* at this instant.



Given:

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\theta = 60\ \text{deg}$

$\alpha = 6\ \frac{\text{rad}}{\text{s}^2}$ $\phi = 45\ \text{deg}$

$l = 1.5\ \text{m}$ $r = 0.3\ \text{m}$

Solution:

$$\mathbf{r_1} = r\begin{pmatrix}\cos(\theta)\\ -\sin(\theta)\\ 0\end{pmatrix} \qquad \mathbf{r_2} = l\begin{pmatrix}\cos(\phi)\\ \sin(\phi)\\ 0\end{pmatrix} \qquad \mathbf{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$ $a_B = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$-\omega\mathbf{k}\times\mathbf{r_1} + \omega_{AB}\mathbf{k}\times\mathbf{r_2} = \begin{pmatrix}0\\ v_B\\ 0\end{pmatrix}$$

$$-\alpha\,\mathbf{k}\times\mathbf{r_1} - \omega\mathbf{k}\times\left(-\omega\mathbf{k}\times\mathbf{r_1}\right) + \alpha_{AB}\mathbf{k}\times\mathbf{r_2} + \omega_{AB}\mathbf{k}\times\left(\omega_{AB}\mathbf{k}\times\mathbf{r_2}\right) = \begin{pmatrix}0\\ a_B\\ 0\end{pmatrix}$$

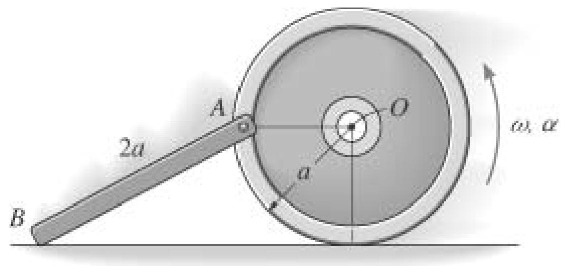
$$\begin{pmatrix}\omega_{AB}\\ \alpha_{AB}\\ v_B\\ a_B\end{pmatrix} = \text{Find}\left(\omega_{AB}, \alpha_{AB}, v_B, a_B\right)$$

$\omega_{AB} = -0.49\ \frac{\text{rad}}{\text{s}}$ $v_B = -0.82\ \frac{\text{m}}{\text{s}}$

$\alpha_{AB} = -2.28\ \frac{\text{rad}}{\text{s}^2}$ $a_B = -2.53\ \frac{\text{m}}{\text{s}^2}$

Problem 16-125

The wheel rolls without slipping such that at the instant shown it has an angular velocity ω and angular acceleration α. Determine the velocity and acceleration of point B on the rod at this instant.



Solution:

Velocity

$$v_B = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -a \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix} = \begin{bmatrix} (\omega_{AB} - \omega)a \\ -(\sqrt{3}\,\omega_{AB} + \omega)\,a \\ 0 \end{bmatrix}$$

Since B stays in contact with the ground we have $\omega_{AB} = \dfrac{-\omega}{\sqrt{3}}$ $\quad v_B = -\dfrac{1+\sqrt{3}}{\sqrt{3}}\,\omega a$

Acceleration

$$a_B = \begin{pmatrix} -\alpha\, a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix} \ldots$$

$$+ \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} -\sqrt{3}\,a \\ -a \\ 0 \end{pmatrix}\right]$$

$$a_B = a\begin{pmatrix} -\alpha + \omega^2 + \alpha_{AB} + \sqrt{3}\,\omega_{AB}^2 \\ -\alpha - \sqrt{3}\,\alpha_{AB} + \omega_{AB}^2 \\ 0 \end{pmatrix}$$

Since B stays in contact with the ground we find

$$\alpha_{AB} = \frac{\omega_{AB}^2 - \alpha}{\sqrt{3}} = \frac{\omega^2}{3\sqrt{3}} - \frac{\alpha}{\sqrt{3}}$$

$$a_B = \left(\frac{4 + 3\sqrt{3}}{3\sqrt{3}}\,\omega^2 - \frac{1+\sqrt{3}}{\sqrt{3}}\,\alpha\right)a$$

Problem 16-126

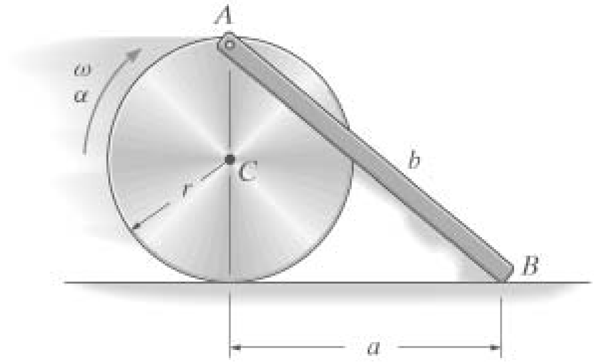
The disk rolls without slipping such that it has angular acceleration α and angular velocity ω at the instant shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant. Assume point A lies on the periphery of the disk, a distance r from C.

Given:

$\alpha = 4\ \frac{\text{rad}}{\text{s}^2}$ $a = 400\ \text{mm}$

$b = 500\ \text{mm}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $r = 150\ \text{mm}$

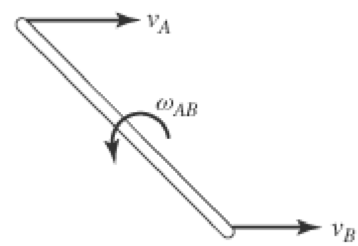


Solution:

The IC is at ∞, so $\omega_{AB} = 0$

$a_C = \alpha r$

$\mathbf{a_A} = a_C + \alpha \times r_{AC} - \omega^2 r_{AC}$



$$\mathbf{a_A} = \begin{pmatrix} a_C \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} - \omega^2 \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix}$$

$$\mathbf{a_A} = \begin{pmatrix} 1.20 \\ -0.60 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_A}| = 1.342\ \frac{\text{m}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{-\omega^2 r}{a_C + \alpha r}\right) \qquad \theta = -26.6\ \text{deg} \qquad |\theta| = 26.6\ \text{deg}$$

Guesses $a_B = 1\ \frac{\text{m}}{\text{s}^2}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\mathbf{a_A} + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a \\ -2r \\ 0 \end{pmatrix} = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} a_B \\ \alpha_{AB} \end{pmatrix} = \text{Find}(a_B, \alpha_{AB})$$

$$\alpha_{AB} = 1.500\ \frac{\text{rad}}{\text{s}^2} \qquad a_B = 1.650\ \frac{\text{m}}{\text{s}^2}$$

Problem 16-127

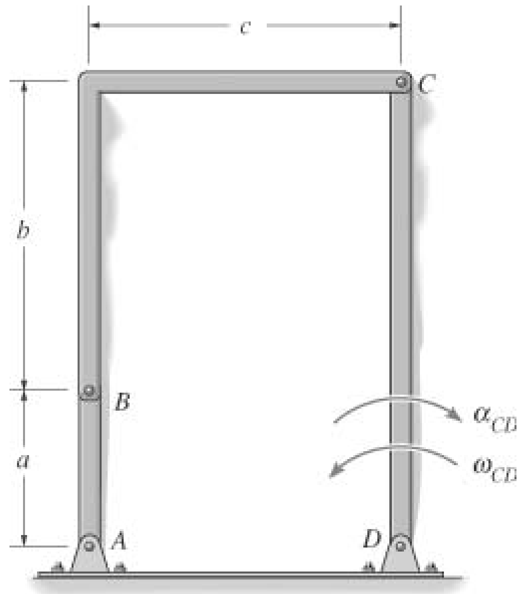
Determine the angular acceleration of link *AB* if link *CD* has the angular velocity and angular deceleration shown.

Given:

$\alpha_{CD} = 4\ \frac{\text{rad}}{\text{s}^2}$ $a = 0.3\ \text{m}$

$b = 0.6\ \text{m}$

$\omega_{CD} = 2\ \frac{\text{rad}}{\text{s}}$ $c = 0.6\ \text{m}$



Solution:

$\omega_{BC} = 0 \quad \omega_{AB} = \omega_{CD}\frac{a+b}{a}$

Guesses

$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2} \quad \alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

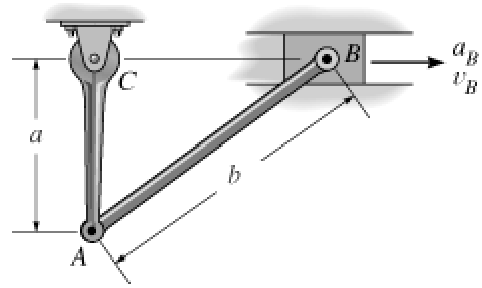
Given

$$\begin{pmatrix}0\\0\\-\alpha_{CD}\end{pmatrix}\times\begin{pmatrix}0\\a+b\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{CD}\end{pmatrix}\times\begin{pmatrix}0\\a+b\\0\end{pmatrix}\right]+\begin{pmatrix}0\\0\\\alpha_{BC}\end{pmatrix}\times\begin{pmatrix}-c\\-b\\0\end{pmatrix}\ \ldots = \begin{pmatrix}0\\0\\0\end{pmatrix}\frac{\text{m}}{\text{s}^2}$$
$$+\begin{pmatrix}0\\0\\\alpha_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}+\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\left[\begin{pmatrix}0\\0\\\omega_{AB}\end{pmatrix}\times\begin{pmatrix}0\\-a\\0\end{pmatrix}\right]$$

$\begin{pmatrix}\alpha_{AB}\\\alpha_{BC}\end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{BC}) \qquad \alpha_{BC} = 12.00\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{AB} = -36.00\ \frac{\text{rad}}{\text{s}^2}$

***Problem 16-128**

The slider block B is moving to the right with acceleration a_B. At the instant shown, its velocity is v_B. Determine the angular acceleration of link AB and the acceleration of point A at this instant.



Given: $a_B = 2\ \frac{\text{ft}}{\text{s}^2}$ $v_B = 6\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$ $b = 5\ \text{ft}$

Solution: $\omega_{AB} = 0$ $v_A = v_B$

$\omega_{AC} = \frac{v_A}{a}$ $\omega_{AC} = 2\ \frac{\text{rad}}{\text{s}}$

Guesses $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$ Given

$$\begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} \sqrt{b^2 - a^2} \\ a \\ 0 \end{pmatrix} = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{AB} \\ \alpha_{AC} \end{pmatrix} = \text{Find}(\alpha_{AB}, \alpha_{AC}) \qquad \alpha_{AC} = -2.33\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_{AB} = -3.00\ \frac{\text{rad}}{\text{s}^2}$$

$$\mathbf{a_A} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix}\right] \qquad \mathbf{a_A} = \begin{pmatrix} -7.00 \\ 12.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$|\mathbf{a_A}| = 13.89\ \frac{\text{ft}}{\text{s}^2}$$

$$\theta = \text{atan}\left(\frac{\omega_{AC}^{\ 2} a}{-\alpha_{AC}\, a}\right) \qquad \theta = 59.74\ \text{deg}$$

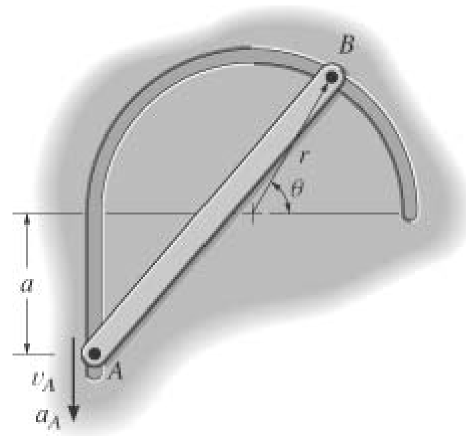
Problem 16-129

The ends of the bar *AB* are confined to move along the paths shown. At a given instant, *A* has velocity v_A and acceleration a_A. Determine the angular velocity and angular acceleration of *AB* at this instant.

Given:

$v_A = 4\ \frac{\text{ft}}{\text{s}}$ $a = 2\ \text{ft}$

$a_A = 7\ \frac{ft}{s^2}$ $r = 2\ ft$ $\theta = 60\ deg$



Solution: Guessses

$\omega = 1\ \frac{rad}{s}$ $\alpha = 1\ \frac{rad}{s^2}$

$v_B = 1\ \frac{ft}{s}$ $a_{Bt} = 1\ \frac{ft}{s^2}$

Given

$$\begin{pmatrix} -v_B\sin(\theta) \\ v_B\cos(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -v_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -a_{Bt}\sin(\theta) \\ a_{Bt}\cos(\theta) \\ 0 \end{pmatrix} + \frac{v_B{}^2}{r}\begin{pmatrix} -\cos(\theta) \\ -\sin(\theta) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -a_A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix} \dots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(\theta) \\ a + r\sin(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega \\ \alpha \\ v_B \\ a_{Bt} \end{pmatrix} = \text{Find}(\omega, \alpha, v_B, a_{Bt})$$

$v_B = 20.39\ \frac{ft}{s}$ $a_{Bt} = -607.01\ \frac{ft}{s^2}$

$\omega = 4.73\ \frac{rad}{s}$ $\alpha = -131.00\ \frac{rad}{s^2}$

Problem 16-130

The mechanism produces intermittent motion of link *AB*. If the sprocket *S* is turning with an angular acceleration α_s and has an angular velocity ω_s at the instant shown, determine the angular velocity and angular acceleration of link *AB* at this instant. The sprocket *S* is mounted on a shaft which is *separate* from a collinear shaft attached to *AB* at *A*. The pin at *C* is attached to one of the chain links such that it moves vertically downward.

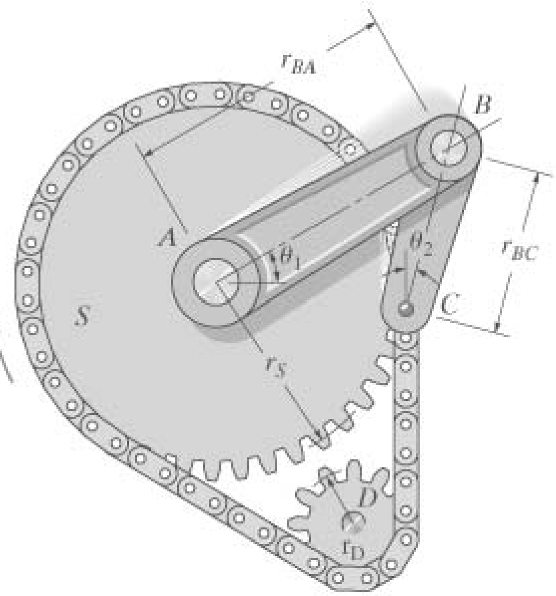
Given:

$\omega_s = 6\ \frac{rad}{s}$ $r_{BA} = 200\ mm$

$\alpha_S = 2\ \frac{\text{rad}}{\text{s}^2}$ $r_{BC} = 150\ \text{mm}$

$\theta_1 = 30\ \text{deg}$ $r_S = 175\ \text{mm}$

$\theta_2 = 15\ \text{deg}$ $r_D = 50\ \text{mm}$



Solution:

$$\mathbf{r_1} = r_{BA}\begin{pmatrix}\cos(\theta_1)\\ \sin(\theta_1)\\ 0\end{pmatrix} \qquad \mathbf{r_2} = r_{BC}\begin{pmatrix}-\sin(\theta_2)\\ -\cos(\theta_2)\\ 0\end{pmatrix}$$

$$\mathbf{k} = \begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \mathbf{j} = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

Guesses $\omega_{AB} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_{BC} = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\omega_{AB}\mathbf{k}\times\mathbf{r_1} + \omega_{BC}\mathbf{k}\times\mathbf{r_2} = -\omega_S r_S\mathbf{j}$

$$\alpha_{AB}\mathbf{k}\times\mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} + \alpha_{BC}\mathbf{k}\times\mathbf{r_2} - \omega_{BC}^2\mathbf{r_2} = -\alpha_S r_S\mathbf{j}$$

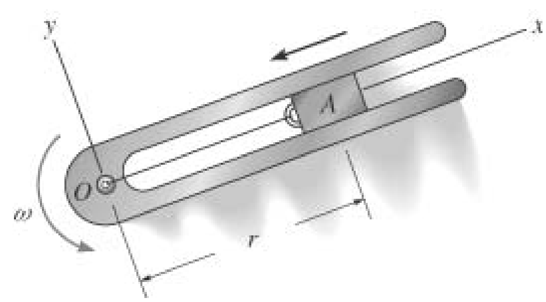
$$\begin{pmatrix}\omega_{AB}\\ \omega_{BC}\\ \alpha_{AB}\\ \alpha_{BC}\end{pmatrix} = \text{Find}(\omega_{AB}, \omega_{BC}, \alpha_{AB}, \alpha_{BC})$$

$\omega_{BC} = -4.95\ \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 70.8\ \frac{\text{rad}}{\text{s}^2}$

$\omega_{AB} = -7.17\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 23.1\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-131

Block *A*, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at *O* with acceleration *a* and velocity *v*. Determine the acceleration of the block at this instant. The rod rotates about *O* with constant angular velocity.



Given:

$a = 4\ \frac{\text{m}}{\text{s}^2}$ $\omega = 4\ \frac{\text{rad}}{\text{s}}$

$$v = 2\ \frac{\text{m}}{\text{s}} \qquad r = 100\ \text{mm}$$

Solution:

$$\mathbf{a_A} = \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_A} = \begin{pmatrix} -5.60 \\ -16.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_A}| = 16.95\ \frac{\text{m}}{\text{s}^2}$$

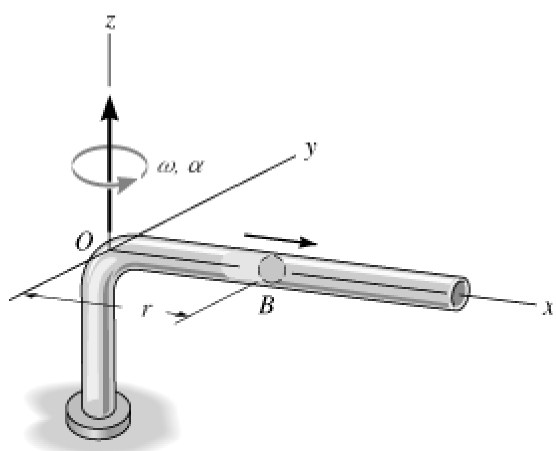
Problem 16-132

The ball B of negligible size rolls through the tube such that at the instant shown it has velocity v and acceleration a, measured relative to the tube. If the tube has angular velocity ω and angular acceleration α at this same instant, determine the velocity and acceleration of the ball.

Given:

$$v = 5\ \frac{\text{ft}}{\text{s}} \qquad \omega = 3\ \frac{\text{rad}}{\text{s}} \qquad r = 2\ \text{ft}$$

$$a = 3\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 5\ \frac{\text{rad}}{\text{s}^2}$$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

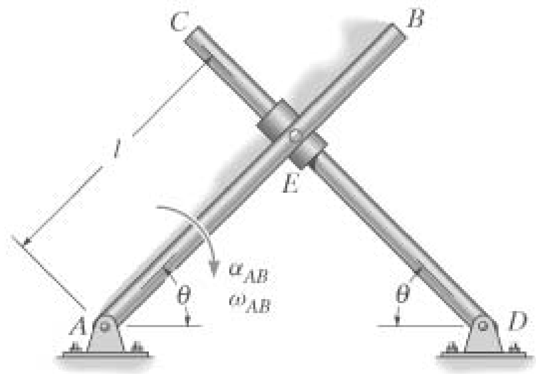
$$\mathbf{v_B} = \begin{pmatrix} 5.00 \\ 6.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_B}| = 7.81\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -15.00 \\ 40.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 42.72\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-133

The collar E is attached to, and pivots about, rod AB while it slides on rod CD. If rod AB has an angular velocity of ω_{AB} and an angular acceleration of α_{AB} both acting clockwise, determine the angular velocity and the angular acceleration of rod CD at the instant shown.



Given:

$\alpha_{AB} = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad \omega_{AB} = 6\ \frac{\text{rad}}{\text{s}}$

$l = 4\ \text{ft}$ $\quad \theta = 45\ \text{deg}$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{r_1} = l\mathbf{u_1} \qquad \mathbf{r_2} = l\mathbf{u_2} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \quad v_{rel} = 1\ \frac{\text{ft}}{\text{s}} \quad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2} \quad a_{rel} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$-\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

$$-\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times (v_{rel}\mathbf{u_2})$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}) \qquad v_{rel} = -24.00\ \frac{\text{ft}}{\text{s}} \qquad a_{rel} = -4.00\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega_{CD} = -0.00\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 36\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-134

Block B moves along the slot in the platform with constant speed v, measured relative to the platform in the direction shown. If the platform is rotating at constant rate ω,determine the velocity and acceleration of the block at the instant shown.

Given:

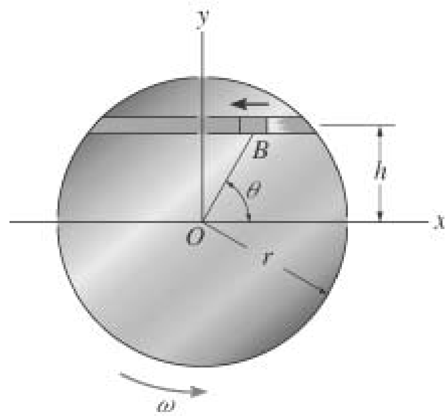
$v = 2\ \frac{\text{ft}}{\text{s}}$

$\theta = 60\ \text{deg}$

$\omega = 5\ \frac{\text{rad}}{\text{s}}$

$r = 3\ \text{ft}$

$h = 2\ \text{ft}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} h\cot(\theta) \\ h \\ 0 \end{pmatrix}$$

$$\mathbf{v_B} = \begin{pmatrix} -12.00 \\ 5.77 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_B}| = 13.32\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} h\cot(\theta) \\ h \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} -v \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -28.9 \\ -70.0 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 75.7\ \frac{\text{ft}}{\text{s}^2}$$

Problem 16-135

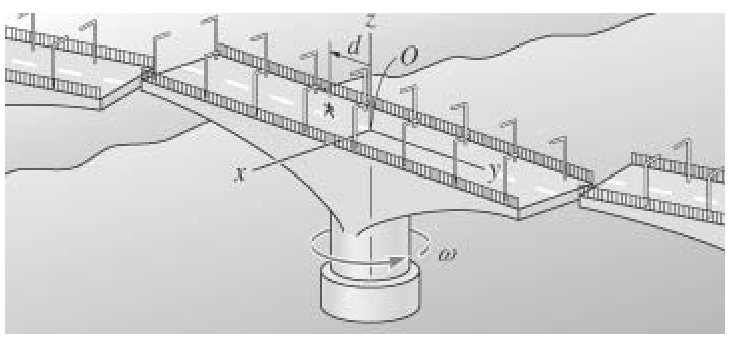
While the swing bridge is closing with constant rotation ω, a man runs along the roadway at constant speed v relative to the roadway. Determine his velocity and acceleration at the instant shown.

Given:

$\omega = 0.5\ \frac{\text{rad}}{\text{s}}$

$v = 5\ \frac{\text{ft}}{\text{s}}$

$d = 15\ \text{ft}$



Solution:

$$\mathbf{v_{man}} = \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \qquad \mathbf{v_{man}} = \begin{pmatrix} 7.50 \\ -5.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_{man}}| = 9.01\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 5.00 \\ 3.75 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{man}}\right| = 6.25 \frac{\text{ft}}{\text{s}^2}$$

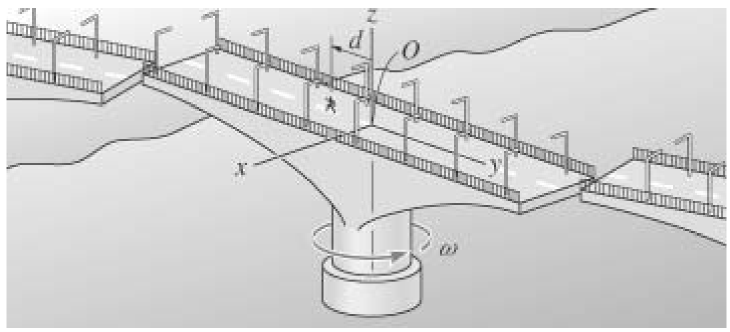
***Problem 16-136**

While the swing bridge is closing with constant rotation ω, a man runs along the roadway such that he is running outward from the center at speed v with acceleration a, both measured relative to the roadway. Determine his velocity and acceleration at this instant.

Given:

$\omega = 0.5 \frac{\text{rad}}{\text{s}}$ $\quad a = 2 \frac{\text{ft}}{\text{s}^2}$

$v = 5 \frac{\text{ft}}{\text{s}}$ $\quad d = 10\ \text{ft}$



Solution:

$$\mathbf{v_{man}} = \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \qquad \mathbf{v_{man}} = \begin{pmatrix} 5.00 \\ -5.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_{man}}\right| = 7.07 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ -v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{man}} = \begin{pmatrix} 5.00 \\ 0.50 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_{man}}\right| = 5.02 \frac{\text{ft}}{\text{s}^2}$$

Problem 16-137

A girl stands at A on a platform which is rotating with constant angular velocity ω. If she walks at constant speed v measured relative to the platform, determine her acceleration (a) when she reaches point D in going along the path ADC, and (b) when she reaches point B if she follows the path ABC.

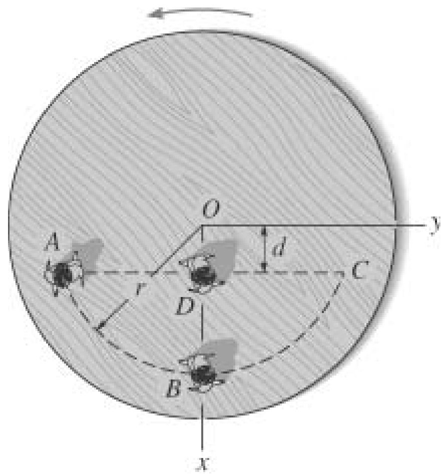
Given:

$\omega = 0.5\ \frac{\text{rad}}{\text{s}}$

$v = 0.75\ \frac{\text{m}}{\text{s}}$

$d = 1\ \text{m}$

$r = 3\ \text{m}$



Solution:

(a)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.00 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

(b)

$$\mathbf{a_{girl}} = \begin{pmatrix} \frac{-v^2}{r} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.69 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-138

A girl stands at *A* on a platform which is rotating with angular acceleration α and at the instant shown has angular velocity ω.If she walks at constant speed *v* measured relative to the platform, determine her acceleration (a) when she reaches point *D* in going along the path *ADC*, and (b) when she reaches point *B* if she follows the path *ABC*.

Given:

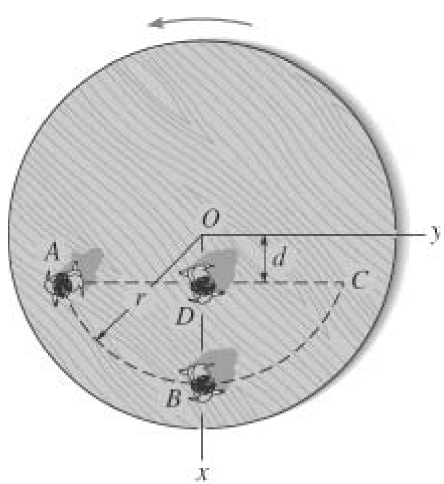
$\alpha = 0.2\ \frac{\text{rad}}{\text{s}^2}$

$\omega = 0.5\ \frac{\text{rad}}{\text{s}}$

$v = 0.75\ \frac{\text{m}}{\text{s}}$

$d = 1\ \text{m}$

$r = 3\ \text{m}$



Solution:

(a)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} d \\ 0 \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \qquad \mathbf{a_{girl}} = \begin{pmatrix} -1.00 \\ 0.20 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

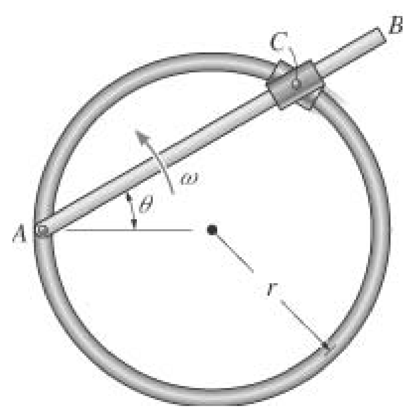
(b)

$$\mathbf{a_{girl}} = \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{-v^2}{r} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \right] + 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}$$

$$\mathbf{a_{girl}} = \begin{pmatrix} -1.69 \\ 0.60 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-139

Rod *AB* rotates counterclockwise with constant angular velocity ω. Determine the velocity and acceleration of point *C* located on the double collar when at the position shown. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod *AB*.



Given: $\omega = 3 \frac{\text{rad}}{\text{s}}$ $\quad \theta = 45 \text{ deg} \quad r = 0.4 \text{ m}$

Solution: Guesses

$v_{rel} = 1 \frac{\text{m}}{\text{s}} \quad v_C = 1 \frac{\text{m}}{\text{s}} \quad a_{rel} = 1 \frac{\text{m}}{\text{s}^2} \quad a_{Ct} = 1 \frac{\text{m}}{\text{s}^2}$

Given

$$v_C \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} = v_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(2\theta) \\ r\sin(2\theta) \\ 0 \end{pmatrix}$$

$$a_{Ct} \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} + \frac{v_C^2}{r} \begin{pmatrix} -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{pmatrix} = a_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r + r\cos(2\theta) \\ r\sin(2\theta) \\ 0 \end{pmatrix} \right] \dots$$
$$+ 2 \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[v_{rel} \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \right]$$

$$\begin{pmatrix} v_{rel} \\ a_{rel} \\ v_C \\ a_{Ct} \end{pmatrix} = \text{Find}(v_{rel}, a_{rel}, v_C, a_{Ct}) \qquad \begin{pmatrix} v_{rel} \\ v_C \end{pmatrix} = \begin{pmatrix} -1.70 \\ 2.40 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} a_{rel} \\ a_{Ct} \end{pmatrix} = \begin{pmatrix} -5.09 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\mathbf{v_{Cv}} = v_C \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Cv}} = a_{Ct} \begin{pmatrix} -\sin(2\theta) \\ \cos(2\theta) \\ 0 \end{pmatrix} + \frac{v_C^2}{r} \begin{pmatrix} -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{pmatrix}$$

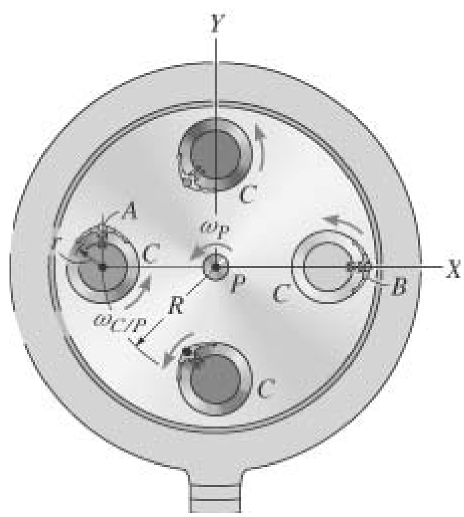
$$\mathbf{v_{Cv}} = \begin{pmatrix} -2.40 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_{Cv}} = \begin{pmatrix} -0.00 \\ -14.40 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 16-140

A ride in an amusement park consists of a rotating platform P, having constant angular velocity ω_P and four cars, C, mounted on the platform, which have constant angular velocities ω_{CP} measured relative to the platform. Determine the velocity and acceleration of the passenger at B at the instant shown.

Given: $\omega_P = 1.5 \frac{\text{rad}}{\text{s}}$ $\quad r = 0.75\ \text{m}$

$\omega_{CP} = 2 \frac{\text{rad}}{\text{s}}$ $\quad R = 3\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

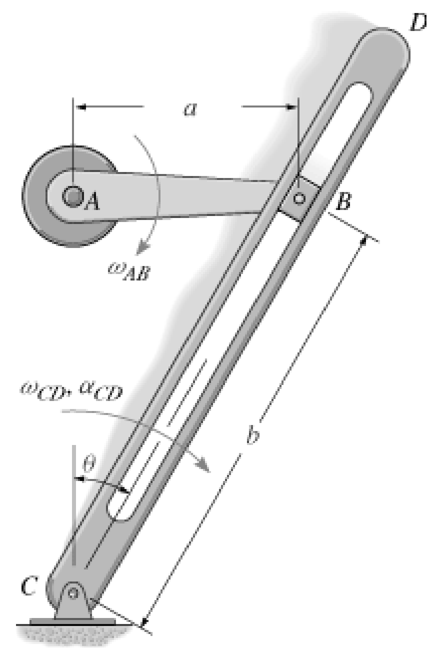
$$\mathbf{v_B} = \begin{pmatrix} 0.00 \\ 7.13 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad |\mathbf{v_B}| = 7.13 \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P \end{pmatrix} \times \begin{pmatrix} R \\ 0 \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_P + \omega_{CP} \end{pmatrix} \times \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} -15.94 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{a_B}| = 15.94 \frac{\text{m}}{\text{s}^2}$$

Problem 16-141

Block B of the mechanism is confined to move within the slot member CD. If AB is rotating at constant rate ω_{AB}, determine the angular velocity and angular acceleration of member CD at the instant shown.



Given: $\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\quad a = 100\ \text{mm}$

$\theta = 30\ \text{deg}$ $\quad b = 200\ \text{mm}$

Solution: Guesses

$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_{rel} = 1\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix} \times \begin{pmatrix}a\\0\\0\end{pmatrix} = v_{rel}\begin{pmatrix}\sin(\theta)\\\cos(\theta)\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-\omega_{CD}\end{pmatrix} \times \begin{pmatrix}b\sin(\theta)\\b\cos(\theta)\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix} \times \left[\begin{pmatrix}0\\0\\-\omega_{AB}\end{pmatrix} \times \begin{pmatrix}a\\0\\0\end{pmatrix}\right] = a_{rel}\begin{pmatrix}\sin(\theta)\\\cos(\theta)\\0\end{pmatrix} + \begin{pmatrix}0\\0\\-\alpha_{CD}\end{pmatrix} \times \begin{pmatrix}b\sin(\theta)\\b\cos(\theta)\\0\end{pmatrix} \ldots$$
$$+ \begin{pmatrix}0\\0\\-\omega_{CD}\end{pmatrix} \times \left[\begin{pmatrix}0\\0\\-\omega_{CD}\end{pmatrix} \times \begin{pmatrix}b\sin(\theta)\\b\cos(\theta)\\0\end{pmatrix}\right] \ldots$$
$$+ 2\begin{pmatrix}0\\0\\-\omega_{CD}\end{pmatrix} \times \left[v_{rel}\begin{pmatrix}\sin(\theta)\\\cos(\theta)\\0\end{pmatrix}\right]$$

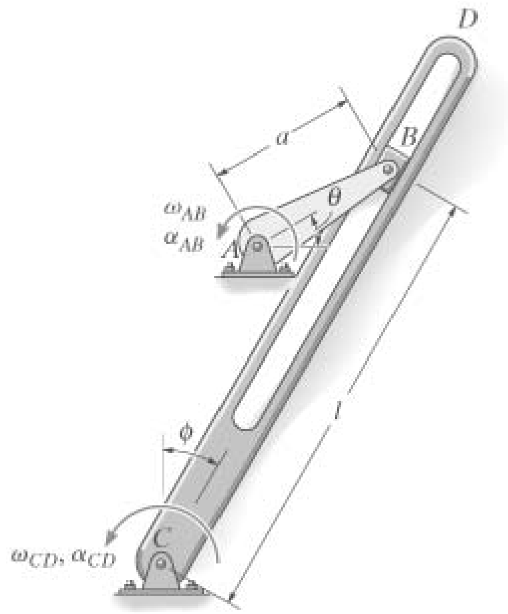
$$\begin{pmatrix}\omega_{CD}\\\alpha_{CD}\\v_{rel}\\a_{rel}\end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel})$$

$v_{rel} = -0.26\ \frac{\text{m}}{\text{s}}$ $\quad a_{rel} = -0.34\ \frac{\text{m}}{\text{s}^2}$

$\omega_{CD} = 0.75\ \frac{\text{rad}}{\text{s}}$ $\quad \alpha_{CD} = -1.95\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-142

The "quick-return" mechanism consists of a crank *AB*, slider block *B*, and slotted link *CD*. If the crank has the angular motion shown, determine the angular motion of the slotted link at this instant.



Given:

$\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\qquad a = 100\ \text{mm}$

$l = 300\ \text{mm}$

$\alpha_{AB} = 9\ \frac{\text{rad}}{\text{s}^2}$ $\qquad \theta = 30\ \text{deg}$

$\phi = 30\ \text{deg}$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix} \qquad \mathbf{r_1} = a\mathbf{u_1} \qquad \mathbf{r_2} = l\mathbf{u_2} \qquad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2} \qquad v_{rel} = 1\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

$$\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}{}^2\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}{}^2\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times \left(v_{rel}\mathbf{u_2}\right)$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}\right) \qquad v_{rel} = 0.15\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = -0.10\ \frac{\text{m}}{\text{s}^2}$$

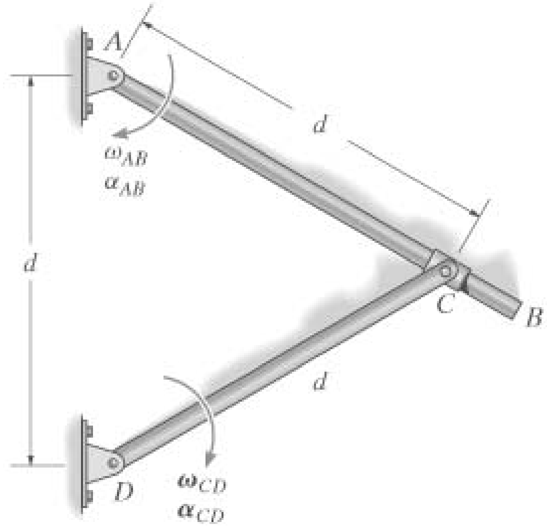
$$\omega_{CD} = 0.87\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 3.23\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-143

At a given instant, rod *AB* has the angular motions shown. Determine the angular velocity and angular acceleration of rod *CD* at this instant.There is a collar at *C*.

Given:

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 12\ \frac{\text{rad}}{\text{s}^2}$ $d = 2\ \text{ft}$



Solution:

$$\mathbf{u_1} = \begin{pmatrix} \sin(60\ \text{deg}) \\ -\cos(60\ \text{deg}) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \sin(60\ \text{deg}) \\ \cos(60\ \text{deg}) \\ 0 \end{pmatrix}$$

$$\mathbf{r_1} = d\mathbf{u_1} \quad \mathbf{r_2} = d\mathbf{u_2} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Guesses $\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$

$v_{rel} = 1\ \frac{\text{ft}}{\text{s}}$ $a_{rel} = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$-\omega_{CD}\mathbf{k} \times \mathbf{r_2} = -\omega_{AB}\mathbf{k} \times \mathbf{r_1} + v_{rel}\mathbf{u_1}$$

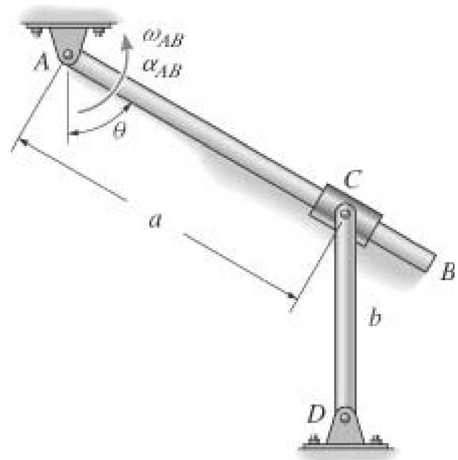
$$-\alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\mathbf{r_2} = -\alpha_{AB}\mathbf{k} \times \mathbf{r_1} - \omega_{AB}^2\mathbf{r_1} + a_{rel}\mathbf{u_1} - 2\omega_{AB}\mathbf{k} \times \left(v_{rel}\mathbf{u_1}\right)$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}\left(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}\right) \qquad v_{rel} = 17.32\ \frac{\text{ft}}{\text{s}} \qquad a_{rel} = -8.43\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega_{CD} = 10.00\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 24.00\ \frac{\text{rad}}{\text{s}^2}$$

Problem 16-144

At the instant shown, rod *AB* has angular velocity ω_{AB} and angular acceleration $\alpha_{AB.}$ Determine the angular velocity and angular acceleration of rod *CD* at this instant. The collar at *C* is pin-connected to CD and slides over AB.



Given: $\theta = 60\ \text{deg}$ $a = 0.75\ \text{m}$

$\omega_{AB} = 3\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AB} = 5\ \frac{\text{rad}}{\text{s}^2}$ $b = 0.5\ \text{m}$

Solution: Guesses

$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_{rel} = 1\ \frac{\text{m}}{\text{s}}$

$a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$ $a_{Cx} = 1\ \frac{\text{m}}{\text{s}^2}$ $a_{Cy} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix} + v_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\sin(\theta) \\ -a\cos(\theta) \\ 0 \end{pmatrix}\right] \ldots$$
$$+ a_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[v_{rel}\begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix}\right]$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \\ a_{Cx} \\ a_{Cy} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}, a_{Cx}, a_{Cy})$$

$$\begin{pmatrix} a_{Cx} \\ a_{Cy} \end{pmatrix} = \begin{pmatrix} 124.41 \\ -40.50 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$v_{rel} = 3.90\ \frac{\text{m}}{\text{s}}$ $\omega_{CD} = -9.00\ \frac{\text{rad}}{\text{s}}$

$a_{rel} = 134.75\ \frac{\text{m}}{\text{s}^2}$ $\alpha_{CD} = -249\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-145

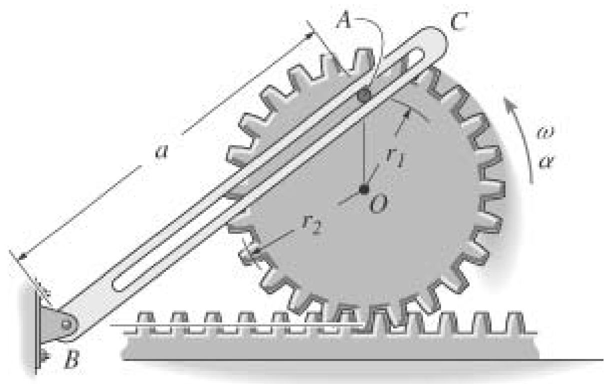
The gear has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link *BC* at this instant. The peg at *A* is fixed to the gear.

Given:

$\omega = 2 \frac{\text{rad}}{\text{s}}$ $r_1 = 0.5 \text{ ft}$

$r_2 = 0.7 \text{ ft}$

$\alpha = 4 \frac{\text{rad}}{\text{s}^2}$ $a = 2 \text{ ft}$



Solution: $b = \sqrt{a^2 - (r_1 + r_2)^2}$

$$\theta = \operatorname{atan}\left(\frac{r_1 + r_2}{b}\right)$$

Guesses $\omega_{BC} = 1 \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 1 \frac{\text{rad}}{\text{s}^2}$ $v_{rel} = 1 \frac{\text{ft}}{\text{s}}$ $a_{rel} = 1 \frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{bmatrix} -\omega(r_1 + r_2) \\ 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} b \\ r_1 + r_2 \\ 0 \end{pmatrix} + v_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -\alpha(r_1 + r_2) \\ -r_1\,\omega^2 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_{BC} \end{pmatrix} \times \begin{pmatrix} b \\ r_1 + r_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{bmatrix} b \\ (r_1 + r_2) \\ 0 \end{bmatrix}\right] \ldots$$
$$+ a_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \left[v_{rel}\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}\right]$$

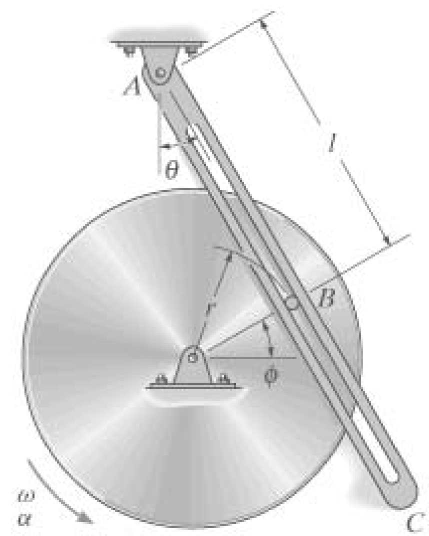
$$\begin{pmatrix} \omega_{BC} \\ \alpha_{BC} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \operatorname{Find}(\omega_{BC}, \alpha_{BC}, v_{rel}, a_{rel})$$

$v_{rel} = -1.92 \frac{\text{ft}}{\text{s}}$ $a_{rel} = -4.00 \frac{\text{ft}}{\text{s}^2}$

$\omega_{BC} = 0.72 \frac{\text{rad}}{\text{s}}$ $\alpha_{BC} = 2.02 \frac{\text{rad}}{\text{s}^2}$

Problem 16-146

The disk rotates with the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link *AC* at this instant. The peg at *B* is fixed to the disk.



Given:

$\omega = 6\ \frac{\text{rad}}{\text{s}}$ $\alpha = 10\ \frac{\text{rad}}{\text{s}^2}$ $l = 0.75\ \text{m}$

$\theta = 30\ \text{deg}$ $\phi = 30\ \text{deg}$ $r = 0.3\ \text{m}$

Solution:

$$\mathbf{u_1} = \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{r_1} = l\mathbf{u_1} \quad \mathbf{r_2} = r\mathbf{u_2}$$

Guesses $\omega_{AC} = 1\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AC} = 1\ \frac{\text{rad}}{\text{s}^2}$ $v_{rel} = 1\ \frac{\text{m}}{\text{s}}$ $a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\omega\mathbf{k} \times \mathbf{r_2} = \omega_{AC}\mathbf{k} \times \mathbf{r_1} + v_{rel}\mathbf{u_1}$$

$$\alpha\mathbf{k} \times \mathbf{r_2} - \omega^2\mathbf{r_2} = \alpha_{AC}\mathbf{k} \times \mathbf{r_1} - \omega_{AC}^2\mathbf{r_1} + a_{rel}\mathbf{u_1} + 2\omega_{AC}\mathbf{k} \times (v_{rel}\mathbf{u_1})$$

$$\begin{pmatrix} \omega_{AC} \\ \alpha_{AC} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}(\omega_{AC}, \alpha_{AC}, v_{rel}, a_{rel})$$

$v_{rel} = -1.80\ \frac{\text{m}}{\text{s}}$ $a_{rel} = -3.00\ \frac{\text{m}}{\text{s}^2}$

$\omega_{AC} = 0.00\ \frac{\text{rad}}{\text{s}}$ $\alpha_{AC} = -14.40\ \frac{\text{rad}}{\text{s}^2}$

Problem 16-147

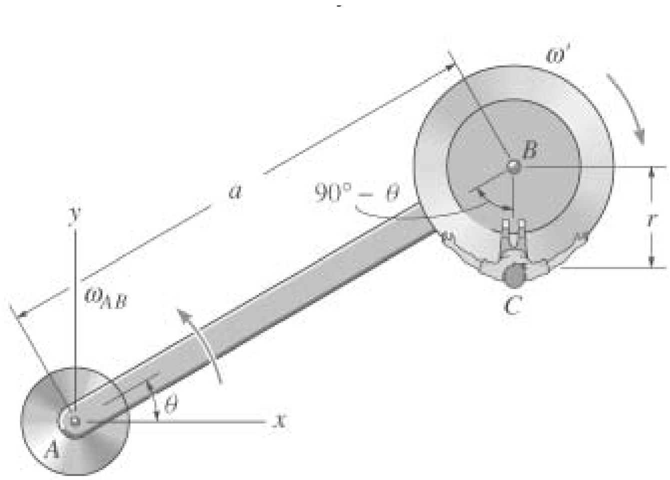
A ride in an amusement park consists of a rotating arm *AB* having constant angular velocity ω_{AB} about point *A* and a car mounted at the end of the arm which has constant angular velocity $-\omega'\ \mathbf{k}$ measured relative to the arm. At the instant shown, determine the velocity and acceleration of the passenger at *C*.

Given:

$\omega_{AB} = 2\ \frac{rad}{s}$ $\quad \omega' = 0.5\ \frac{rad}{s}$

$a = 10\ ft$ $\quad r = 2\ ft$

$\theta = 30\ deg$



Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.00 \\ 17.32 \\ 0.00 \end{pmatrix} \frac{ft}{s}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_C} = \begin{pmatrix} -34.64 \\ -15.50 \\ 0.00 \end{pmatrix} \frac{ft}{s^2}$$

Problem 16-148

A ride in an amusement park consists of a rotating arm *AB* that has angular acceleration α_{AB} when the angular velocity is ω_{AB} at the instant shown. Also at this instant the car mounted at the end of the arm has a relative angular acceleration $-\alpha'\,\mathbf{k}$ of when the angular velocity is $-\omega'\,\mathbf{k}$. Determine the velocity and acceleration of the passenger *C* at this instant.

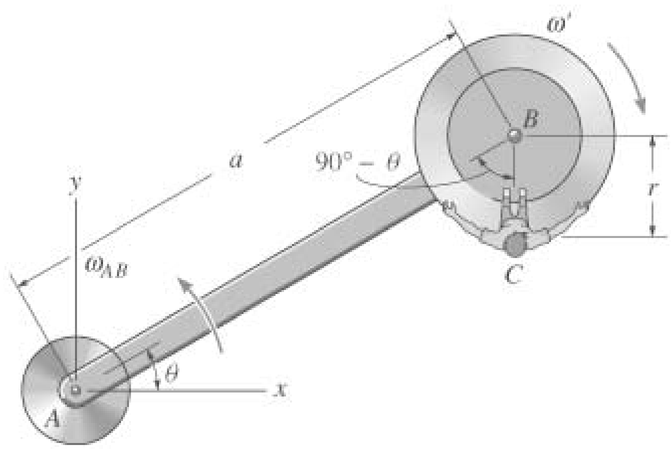
Given:

$\omega_{AB} = 2\ \frac{rad}{s}$ $\quad \alpha_{AB} = 1\ \frac{rad}{s^2}$

$\omega' = 0.5\ \frac{rad}{s}$ $\quad \alpha' = 0.6\ \frac{rad}{s^2}$

$a = 10\ ft$ $\quad r = 2\ ft$

$\theta = 30\ deg$



Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.00 \\ 17.32 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_C} = \left[\begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} \end{pmatrix} \times \begin{pmatrix} a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} \right] \right] \ldots$$
$$+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{AB} - \alpha' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{AB} - \omega' \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_C} = \begin{pmatrix} -38.84 \\ -6.84 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 16-149

The cars on the amusement-park ride rotate around the axle at *A* with constant angular velocity ω_{Af} measured relative to the frame *AB*. At the same time the frame rotates around the main axle support at *B* with constant angular velocity ω_f .Determine the velocity and acceleration of the passenger at *C* at the instant shown.

Given:

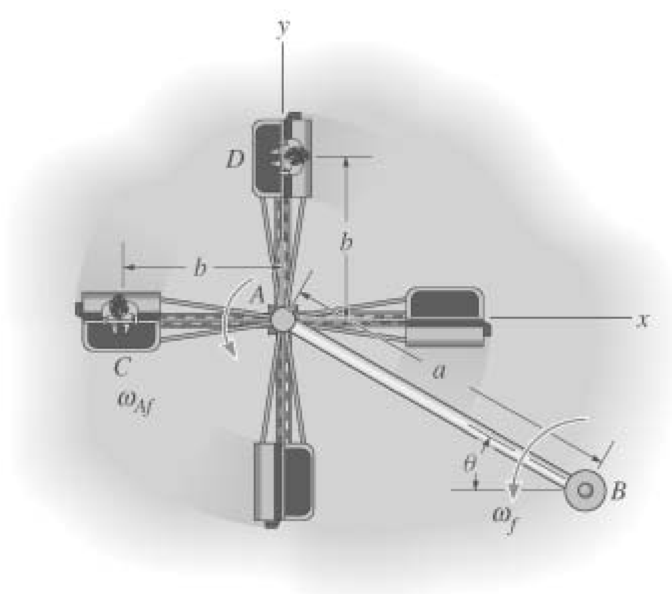
$\omega_{Af} = 2\ \frac{\text{rad}}{\text{s}}$

$\omega_f = 1\ \frac{\text{rad}}{\text{s}}$

$a = 15\ \text{ft}$

$b = 8\ \text{ft}$

$\theta = 30\ \text{deg}$



Solution:

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \begin{pmatrix} -a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -7.50 \\ -36.99 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_f \end{pmatrix} \times \begin{pmatrix} -a\cos(\theta) \\ a\sin(\theta) \\ 0 \end{pmatrix}\right] + \begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_f + \omega_{Af} \end{pmatrix} \times \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix}\right]$$

$$\mathbf{a_C} = \begin{pmatrix} 84.99 \\ -7.50 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 16-150

The block *B* of the “quick-return” mechanism is confined to move within the slot in member *CD*. If *AB* is rotating at a constant rate of ω_{AB}, determine the angular velocity and angular acceleration of member *CD* at the instant shown.

Given:

$$\omega_{AB} = 3\,\frac{\text{rad}}{\text{s}}$$

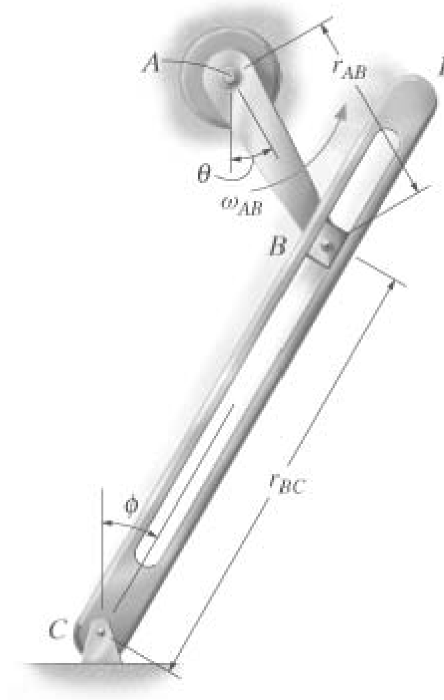
$$r_{AB} = 50\ \text{mm}$$

$$r_{BC} = 200\ \text{mm}$$

$$\theta = 30\ \text{deg}$$

$$\phi = 30\ \text{deg}$$

Solution:



$$\mathbf{u_1} = \begin{pmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{pmatrix} \quad \mathbf{u_2} = \begin{pmatrix} \sin(\phi) \\ \cos(\phi) \\ 0 \end{pmatrix} \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{r_1} = r_{AB}\mathbf{u_1} \qquad \mathbf{r_2} = r_{BC}\mathbf{u_2}$$

Guesses

$$\omega_{CD} = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$v_{rel} = 1\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 1\ \frac{\text{m}}{\text{s}^2}$$

Given

$$\omega_{AB}\mathbf{k} \times \mathbf{r_1} = \omega_{CD}\mathbf{k} \times \mathbf{r_2} + v_{rel}\mathbf{u_2}$$

$$-\omega_{AB}^2\,\mathbf{r_1} = \alpha_{CD}\mathbf{k} \times \mathbf{r_2} - \omega_{CD}^2\,\mathbf{r_2} + a_{rel}\mathbf{u_2} + 2\omega_{CD}\mathbf{k} \times (v_{rel}\mathbf{u_2})$$

$$\begin{pmatrix} \omega_{CD} \\ \alpha_{CD} \\ v_{rel} \\ a_{rel} \end{pmatrix} = \text{Find}(\omega_{CD}, \alpha_{CD}, v_{rel}, a_{rel}) \qquad v_{rel} = 0.13\ \frac{\text{m}}{\text{s}} \qquad a_{rel} = 0.25\ \frac{\text{m}}{\text{s}^2}$$

$$\omega_{CD} = -0.38\ \frac{\text{rad}}{\text{s}} \qquad \alpha_{CD} = 2.44\ \frac{\text{rad}}{\text{s}^2}$$