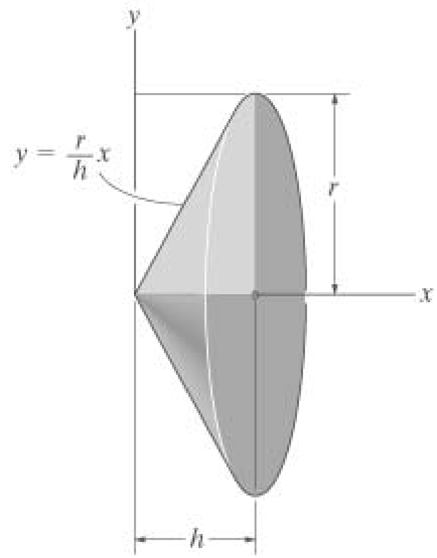
**Problem 17-1**

The right circular cone is formed by revolving the shaded area around the *x* axis. Determine the moment of inertia $I_x$ and express the result in terms of the total mass *m* of the cone. The cone has a constant density $\rho$.




Solution:

$$m = \int_0^h \rho\,\pi\left(\frac{r x}{h}\right)^2 \mathrm{d}x = \frac{1}{3}\,h\,\rho\,\pi r^2$$

$$\rho = \frac{3m}{h\pi r^2}$$

$$I_x = \frac{3m}{h\pi r^2}\int_0^h \frac{1}{2}\pi\left(\frac{r x}{h}\right)^2\left(\frac{r x}{h}\right)^2 \mathrm{d}x \qquad \boxed{I_x = \frac{3}{10} m\,r^2}$$

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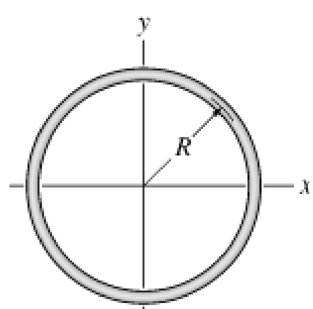
**Problem 17-2**

Determine the moment of inertia of the thin ring about the *z* axis. The ring has a mass *m*.

Solution:




$$m = \int_0^{2\pi} \rho R\,\mathrm{d}\theta = 2\,\pi\,\rho R \qquad \rho = \frac{m}{2\pi R}$$

$$I_z = \frac{m}{2\pi R}\int_0^{2\pi} R\,R^2\,\mathrm{d}\theta = m\,R^2 \qquad \boxed{I_z = m\,R^2}$$

---

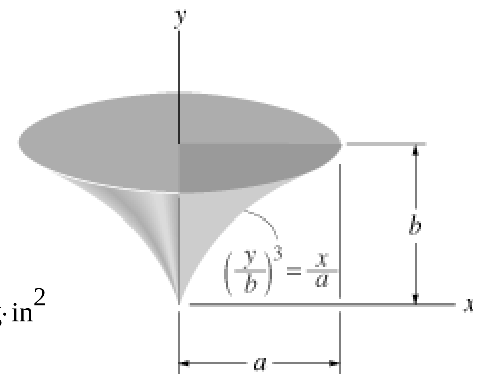
**Problem 17-3**

The solid is formed by revolving the shaded area around the *y* axis. Determine the radius of gyration $k_y$. The specific weight of the material is $\gamma$.

Given:

$$\gamma = 380\,\frac{\text{lb}}{\text{ft}^3} \qquad a = 3\text{ in} \qquad b = 3\text{ in}$$

Solution:




$$m = \int_0^b \gamma \pi \left(a \frac{y^3}{b^3}\right)^2 \mathrm{d}y \qquad m = 0.083\,\mathrm{slug}$$

$$I_y = \int_0^b \gamma \frac{1}{2} \pi \left(a \frac{y^3}{b^3}\right)^2 \left(a \frac{y^3}{b^3}\right)^2 \mathrm{d}y \qquad I_y = 0.201\,\mathrm{slug \cdot in^2}$$

$$k = \sqrt{\frac{I_y}{m}} \qquad k = 1.56\,\mathrm{in}$$

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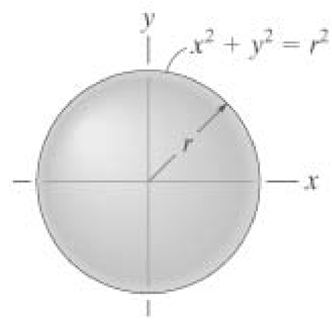
***Problem 17-4**

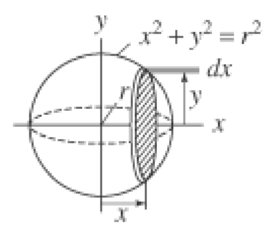
Determine the moment of inertia $I_x$ of the sphere and express the result in terms of the total mass $m$ of the sphere.The sphere has a constant density $\rho$.




Solution:

$$m = \int_{-r}^{r} \rho \pi \left(r^2 - x^2\right) \mathrm{d}x = \frac{4}{3} r^3 \rho \pi \qquad \rho = \frac{3m}{4\pi r^3}$$

$$I_x = \frac{3m}{4\pi r^3} \int_{-r}^{r} \frac{\pi}{2} \left(r^2 - x^2\right)^2 \mathrm{d}x \qquad I_x = \frac{2}{5} m r^2$$

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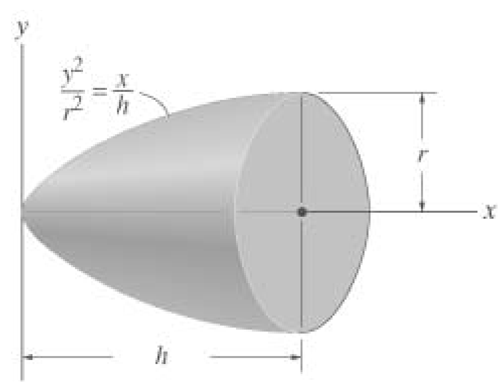
**Problem 17-5**

Determine the radius of gyration $k_x$ of the paraboloid. The density of the material is $\rho$.




Units Used: $\mathrm{Mg} = 10^6\ \mathrm{gm}$

Given:

$h = 200\ \mathrm{mm}$

$r = 100\ \mathrm{mm}$

$\rho = 5\ \frac{\mathrm{Mg}}{\mathrm{m}^3}$

Solution:

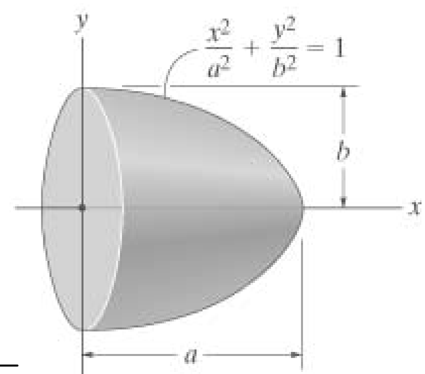
$$M = \int_0^h \rho\,\pi\left(\frac{x r^2}{h}\right) dx \qquad M = 15.708\ \text{kg}$$

$$I_x = \int_0^h \frac{1}{2}\rho\,\pi\left(\frac{x r^2}{h}\right)^2 dx \qquad I_x = 0.052\ \text{kg}\cdot\text{m}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 57.7\,\text{mm}$$

**Problem 17-6**

Determine the moment of inertia of the semiellipsoid with respect to the $x$ axis and express the result in terms of the mass $m$ of the semiellipsoid. The material has a constant density $\rho$.



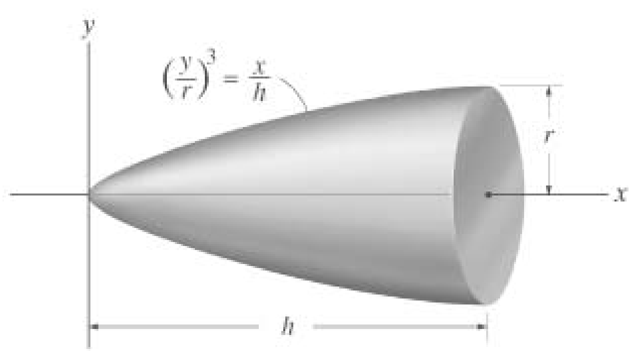

Solution:

$$m = \int_0^a \rho\,\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{2}{3}a\,\rho\,\pi b^2 \qquad \rho = \frac{3m}{2a\pi b^2}$$

$$I_x = \frac{3m}{2a\pi b^2}\int_0^a \frac{1}{2}\pi\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right]^2 dx \qquad I_x = \frac{2}{5}m\,b^2$$

**Problem 17-7**

Determine the radius of gyration $k_x$ of the body. The specific weight of the material is $\gamma$.




Given:

$$\gamma = 380\ \frac{\text{lb}}{\text{ft}^3}$$

$$h = 8\ \text{in}$$

$$r = 2\ \text{in}$$

Solution:

$$M = \int_0^h \gamma \pi \left[ r \left( \frac{x}{h} \right)^{\frac{1}{3}} \right]^2 dx \qquad M = 0.412 \text{ slug}$$

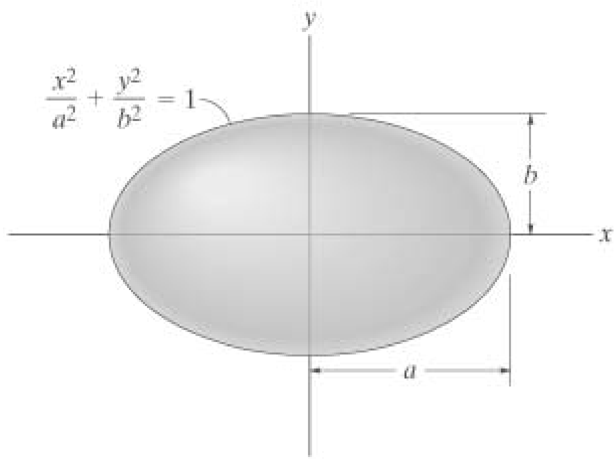
$$I_x = \int_0^h \frac{1}{2} \gamma \pi \left[ r \left( \frac{x}{h} \right)^{\frac{1}{3}} \right]^4 dx \qquad I_x = 0.589 \text{ slug} \cdot \text{in}^2$$

$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 1.20 \text{ in}$$

---

***Problem 17-8**

Determine the moment of inertia of the ellipsoid with respect to the $x$ axis and express the result in terms of the mass $m$ of the ellipsoid. The material has a constant density $\rho$.




Solution:

$$m = \int_{-a}^{a} \rho \pi b^2 \left( 1 - \frac{x^2}{a^2} \right) dx = \frac{4}{3} a \rho \pi b^2$$

$$\rho = \frac{3m}{4a\pi b^2}$$

$$I_x = \frac{3m}{4a\pi b^2} \int_{-a}^{a} \frac{1}{2} \pi \left[ b^2 \left( 1 - \frac{x^2}{a^2} \right) \right]^2 dx \qquad I_x = \frac{2}{5} m b^2$$

---

**Problem 17-9**

Determine the moment of inertia of the homogeneous pyramid of mass $m$ with respect to the $z$ axis.The density of the material is $\rho$. *Suggestion*: Use a rectangular plate element having a volume of $dV = (2x)(2y)\, dz$.

Solution:

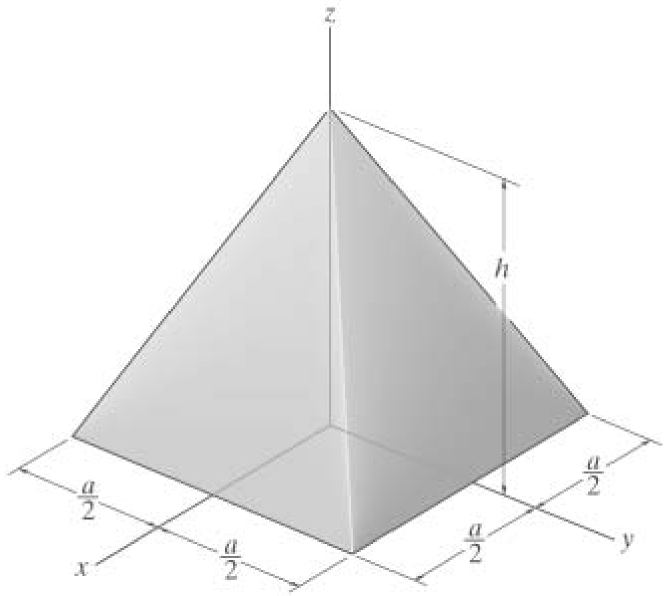
$$x = \frac{a(h-z)}{2h}$$

$$m = \int_0^h \rho \left[\frac{a(h-z)}{h}\right]^2 dz = \frac{1}{3} h \rho a^2$$

$$\rho = \frac{3m}{h a^2}$$

$$I_z = \frac{3m}{h a^2} \int_0^h \frac{1}{6} \left[\frac{a(h-z)}{h}\right]^4 dz$$

$$I_z = \frac{1}{10} m a^2$$




**Problem 17-10**

The concrete shape is formed by rotating the shaded area about the $y$ axis. Determine the moment of inertia $I_y$. The specific weight of concrete is $\gamma$.

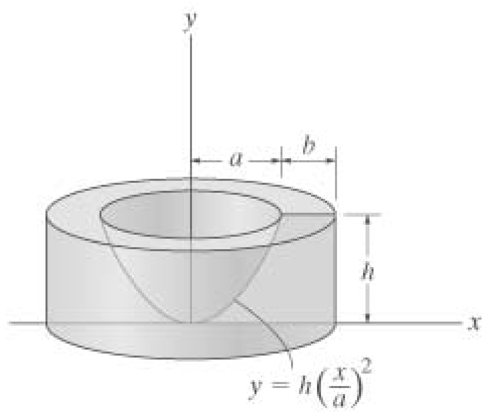
Given:

$a = 6$ in

$b = 4$ in

$h = 8$ in

$\gamma = 150 \frac{\text{lb}}{\text{ft}^3}$




Solution:

$$I_y = \int_0^h \gamma \left[\frac{1}{2}\pi (a+b)^4 - \frac{1}{2}\pi \left(a^2 \frac{y}{h}\right)^2\right] dy \qquad I_y = 2.25\ \text{slug}\cdot\text{ft}^2$$

**Problem 17-11**

Determine the moment of inertia of the thin plate about an axis perpendicular to the page and passing through the pin at $O$. The plate has a hole in its center. Its thickness is $t$, and the material has a density of $\rho$.

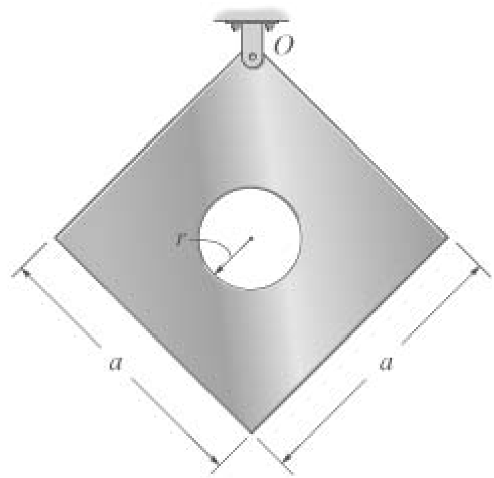
Given:

$a = 1.40\ \text{m}$

$r = 150\ \text{mm}$

$t = 50\ \text{mm}$

$\rho = 50\ \dfrac{\text{kg}}{\text{m}^3}$




Solution:

$$I_O = \rho a^2 t\left(\frac{2a^2}{3}\right) - \left[\rho\,\pi r^2 t\left(\frac{r^2}{2}\right) + \rho\,\pi r^2 t\left(\frac{\sqrt{2}\,a}{2}\right)^2\right] \qquad I_O = 6.227\ \text{kg}\cdot\text{m}^2$$

***Problem 17-12**

Determine the moment of inertia $I_z$ of the frustum of the cone which has a conical depression. The material has a density $\rho$.

Given:

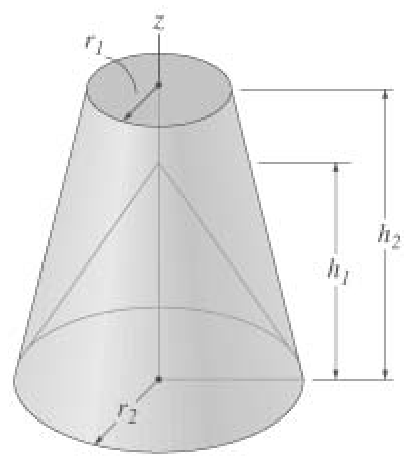
$r_1 = 0.2\ \text{m}$

$r_2 = 0.4\ \text{m}$

$h_1 = 0.6\ \text{m}$

$h_2 = 0.8\ \text{m}$

$\rho = 200\ \dfrac{\text{kg}}{\text{m}^3}$




Solution:

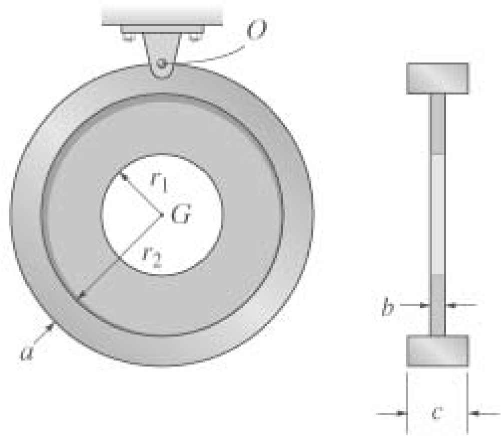
$$h_3 = \frac{r_2 h_2}{r_2 - r_1} \qquad h_4 = h_3 - h_2$$

$$I_z = \rho\left(\frac{\pi r_2^2 h_3}{3}\right)\left(\frac{3}{10}\right)r_2^2 - \left(\frac{\rho\,\pi r_1^2 h_4}{3}\right)\left(\frac{3}{10}\right)r_1^2 - \left(\frac{\rho\,\pi r_2^2 h_1}{3}\right)\left(\frac{3}{10}\right)r_2^2$$

$$I_z = 1.53\ \text{kg}\cdot\text{m}^2$$

**Problem 17-13**

Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center of mass *G*. The material has a specific weight $\gamma$.




Given:

$a = 0.5\ \text{ft}$ $\quad r_1 = 1\ \text{ft}$

$b = 0.25\ \text{ft}$ $\quad r_2 = 2\ \text{ft}$

$c = 1\ \text{ft}$ $\quad \gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

Solution:

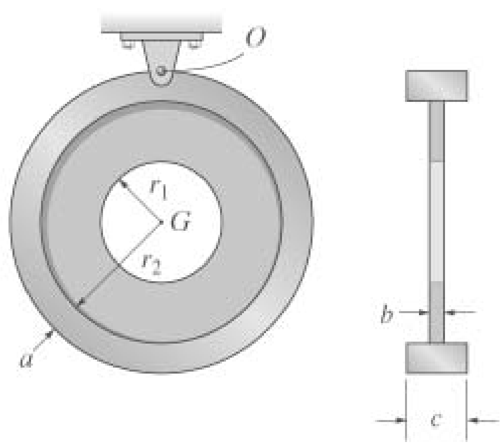
$$I_G = \frac{1}{2}\gamma\pi c\left(r_2 + a\right)^4 - \frac{1}{2}\gamma\pi(c-b)\,r_2^{\,4} - \frac{1}{2}\gamma\pi b\,r_1^{\,4}$$

$$I_G = 118\,\text{slug}\cdot\text{ft}^2$$

**Problem 17-14**

Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through point *O*. The material has a specific weight $\gamma$.

Given:




$a = 0.5\ \text{ft}$

$b = 0.25\ \text{ft}$

$c = 1\ \text{ft}$

$r_1 = 1\ \text{ft}$

$r_2 = 2\ \text{ft}$

$\gamma = 90\ \dfrac{\text{lb}}{\text{ft}^3}$

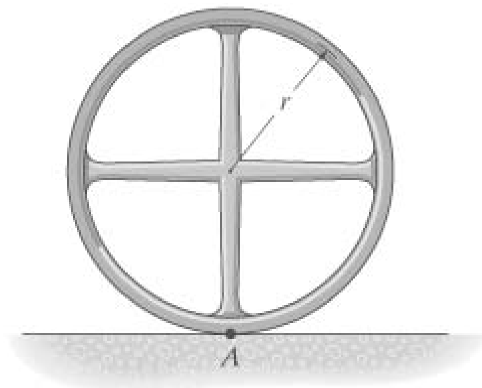
Solution:

$$I_O = \frac{3}{2}\gamma\pi c\left(r_2+a\right)^4 - \left[\frac{1}{2}\gamma\pi(c-b)\,r_2^{\,4} + \gamma\pi(c-b)\,r_2^{\,2}\left(r_2+a\right)^2\right]\ \dots$$
$$+ -\left[\frac{1}{2}\gamma\pi b\,r_1^{\,4} + \gamma\pi b\,r_1^{\,2}\left(r_2+a\right)^2\right]$$

$$I_O = 283\,\text{slug}\cdot\text{ft}^2$$

**Problem 17-15**

The wheel consists of a thin ring having a mass $M_1$ and four spokes made from slender rods, each having a mass $M_2$. Determine the wheel's moment of inertia about an axis perpendicular to the page and passing through point $A$.




Given:

$M_1 = 10\ \text{kg}$

$M_2 = 2\ \text{kg}$

$r = 500\ \text{mm}$

Solution:

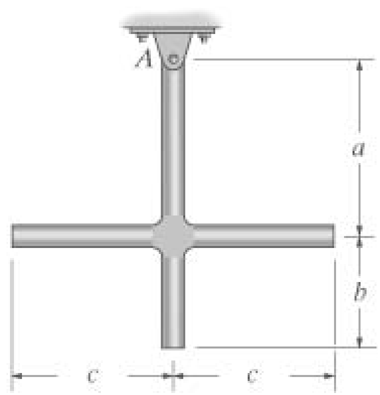
$$I_G = M_1 r^2 + 4M_2\left(\frac{r^2}{3}\right)$$

$$I_A = I_G + \left(M_1 + 4M_2\right)r^2$$

$$I_A = 7.67\ \text{kg}\cdot\text{m}^2$$

**Problem 17-16**

The slender rods have a weight density $\gamma$. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point $A$.




Given:

$\gamma = 3\ \frac{\text{lb}}{\text{ft}}$

$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

$c = 1.5\ \text{ft}$

Solution:

$$I_A = \gamma(a+b)\left[\frac{(a+b)^2}{3}\right] + \gamma(2c)\,\frac{(2c)^2}{12} + \gamma(2c)\,a^2$$

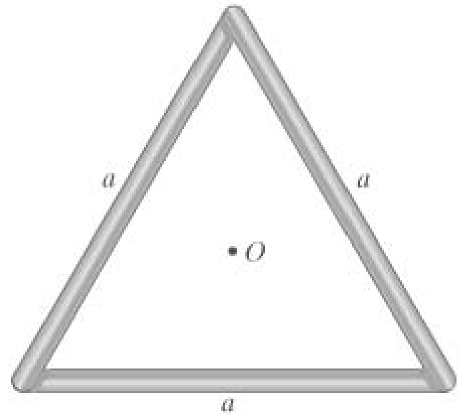
$$I_A = 2.17\ \text{slug}\cdot\text{ft}^2$$

**Problem 17-17**

Each of the three rods has a mass $m$. Determine the moment of inertia of the assembly about an axis which is perpendicular to the page and passes through the center point $O$.

Solution:

$$I_O = 3\left[\frac{1}{12}m\,a^2 + m\left[\frac{(a)\sin(60\ \mathrm{deg})}{3}\right]^2\right]$$




$$I_O = \frac{1}{2}m\,a^2$$

## Problem 17-18

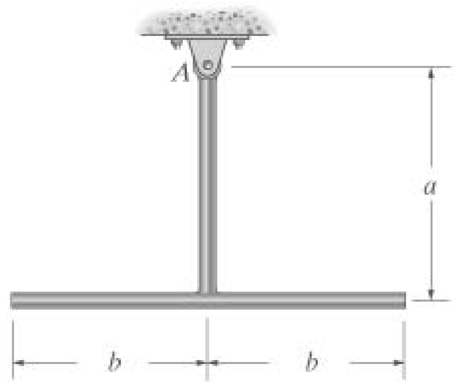
The slender rods have weight density $\gamma$. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through the pin at $A$.

Given:

$\gamma = 3\ \frac{\mathrm{lb}}{\mathrm{ft}}$

$a = 2\ \mathrm{ft}$

$b = 1.5\ \mathrm{ft}$



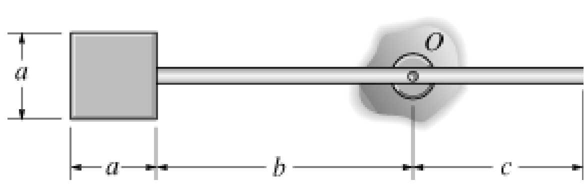

Solution:

$$I_A = \frac{1}{3}\gamma a^3 + \frac{1}{12}\gamma(2b)^3 + \gamma(2b)\,a^2 \qquad I_A = 1.58\,\mathrm{slug\cdot ft^2}$$

## Problem 17-19

The pendulum consists of a plate having weight $W_p$ and a slender rod having weight $W_r$. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point $O$.

Given:




$W_p = 12\ \mathrm{lb}$

$W_r = 4\ \mathrm{lb}$

$a = 1\ \mathrm{ft}$

$b = 3\ \mathrm{ft}$

$c = 2\ \mathrm{ft}$

Solution:

$$I_O = \frac{1}{3}\left(\frac{b}{b+c}\right)W_r b^2 + \frac{1}{3}\left(\frac{c}{b+c}\right)W_r c^2 + \frac{1}{6}W_p a^2 + W_p\left(b+\frac{a}{2}\right)^2$$

$$I_O = 4.921\,\mathrm{slug}\cdot\mathrm{ft}^2$$

$$k_O = \sqrt{\frac{I_O}{W_r + W_p}} \qquad k_O = 3.146\,\mathrm{ft}$$

***Problem 17-20**

Determine the moment of inertia of the overhung crank about the $x$ axis. The material is steel having a density $\rho$.

Units Used:

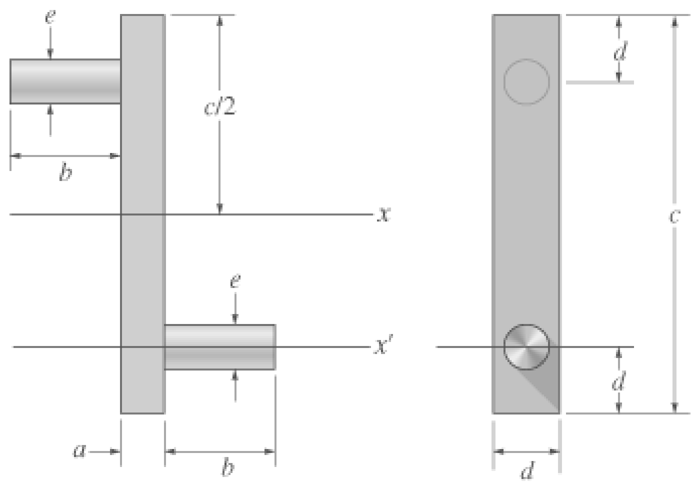
$$\mathrm{Mg} = 10^3\ \mathrm{kg}$$

Given:

$$\rho = 7.85\ \frac{\mathrm{Mg}}{\mathrm{m}^3} \qquad c = 180\ \mathrm{mm}$$

$$a = 20\ \mathrm{mm} \qquad d = 30\ \mathrm{mm}$$

$$b = 50\ \mathrm{mm} \qquad e = 20\ \mathrm{mm}$$




Solution:

$$I_x = 2\left[\frac{\rho\pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho\pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a d c}{12}\left(d^2+c^2\right)$$

$$I_x = 3.25\times 10^{-3}\,\mathrm{kg}\cdot\mathrm{m}^2$$

**Problem 17-21**

Determine the moment of inertia of the overhung crank about the $x'$ axis. The material is steel having a density $\rho$.

Units Used:

$$\mathrm{Mg} = 10^3\ \mathrm{kg}$$

Given:

$\rho = 7.85 \frac{\text{Mg}}{\text{m}^3}$

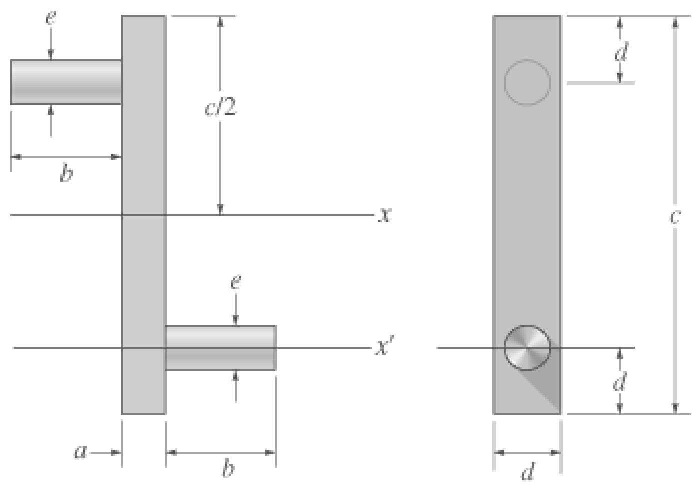
$a = 20 \text{ mm}$

$b = 50 \text{ mm}$

$c = 180 \text{ mm}$

$d = 30 \text{ mm}$

$e = 20 \text{ mm}$




Solution:

$$I_x = 2\left[\frac{\rho\,\pi}{2}\left(\frac{e}{2}\right)^2 b\left(\frac{e}{2}\right)^2 + \rho\,\pi\left(\frac{e}{2}\right)^2 b\left(\frac{c-2d}{2}\right)^2\right] + \frac{\rho a\,d\,c}{12}\left(d^2+c^2\right)$$

$$I_{x'} = I_x + \left[2\rho\,\pi\left(\frac{e}{2}\right)^2 b + \rho a\,d\,c\right]\left(\frac{c-2d}{2}\right)^2 \qquad I_{x'} = 7.19 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

---

**Problem 17-22**

Determine the moment of inertia of the solid steel assembly about the $x$ axis. Steel has specific weight $\gamma_{st}$.

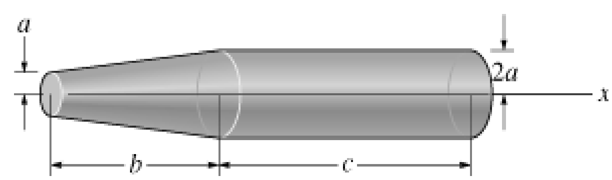
Given:

$a = 0.25 \text{ ft}$

$b = 2 \text{ ft}$

$c = 3 \text{ ft}$

$\gamma_{st} = 490 \frac{\text{lb}}{\text{ft}^3}$



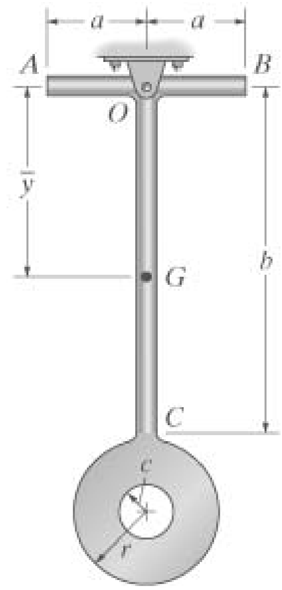

Solution:

$$I_x = \left[\frac{1}{2}\pi(2a)^2 c(2a)^2 + \frac{3}{10}\frac{1}{3}\pi(2a)^2(2b)(2a)^2 - \frac{3}{10}\frac{1}{3}\pi a^2 b\,a^2\right]\gamma_{st}$$

$$I_x = 5.644\,\text{slug}\cdot\text{ft}^2$$

---

**Problem 17-23**




The pendulum consists of two slender rods *AB* and *OC* which have a mass density $\rho_1$. The thin plate has a mass density $\rho_2$. Determine the location *y'* of the center of mass *G* of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through *G*.

Given:

$\rho_1 = 3\ \frac{\text{kg}}{\text{m}}$  $a = 0.4\ \text{m}$  $c = 0.1\ \text{m}$

$\rho_2 = 12\ \frac{\text{kg}}{\text{m}^2}$  $b = 1.5\ \text{m}$  $r = 0.3\ \text{m}$

Solution:

$$y' = \frac{\rho_1 b\left(\frac{b}{2}\right) + \rho_2\pi\left(r^2 - c^2\right)(b + r)}{\rho_1(b + 2a) + \rho_2\pi\left(r^2 - c^2\right)} \qquad y' = 0.888\ \text{m}$$

$$I_O = \frac{1}{12}\rho_1(2a)^3 + \frac{1}{3}\rho_1 b^3 + \left(\frac{\rho_2}{2}\right)\pi r^4 + \rho_2\pi r^2(r + b)^2 - \left[\left(\frac{\rho_2}{2}\right)\pi c^4 + \rho_2\pi c^2(r + b)^2\right]$$

$$I_G = I_O - \left[\rho_1(2a + b) + \rho_2\pi\left(r^2 - c^2\right)\right]y'^2$$

$$I_G = 5.61\ \text{kg}\cdot\text{m}^2$$

---

***Problem 17-24**

Determine the greatest possible acceleration of the race car of mass *M* so that its front tires do not leave the ground or the tires slip on the track. The coefficients of static and kinetic friction are $\mu_s$ and $\mu_k$ respectively. Neglect the mass of the tires. The car has rear-wheel drive and the front tires are free to roll.

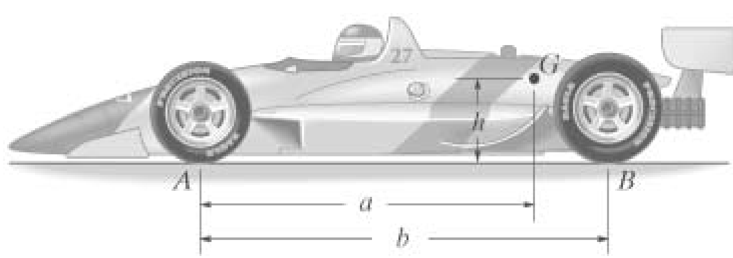
Given:

$M = 975\ \text{kg}$  $\mu_s = 0.8$

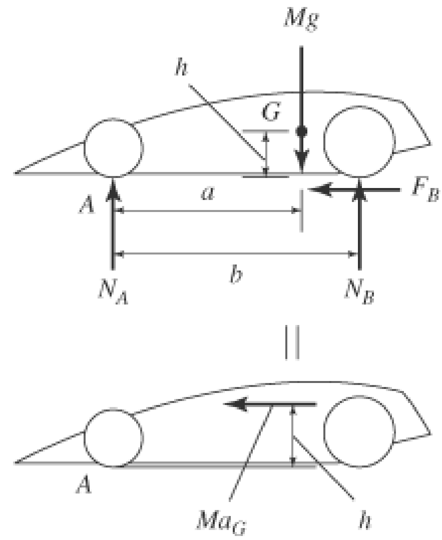
$a = 1.82\ \text{m}$  $\mu_k = 0.6$

$b = 2.20\ \text{m}$  $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$h = 0.55\ \text{m}$



Solution:




First assume that the rear wheels are on the verge of slipping $\quad F_B = \mu_s N_B$

Guesses

$N_A = 1\ \text{N} \qquad N_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$\mu_s N_B = M a_G \qquad N_A + N_B - M g = 0$

$-N_A a + N_B(b - a) - \mu_s N_B h = 0$

$$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -326 \\ 9891 \end{pmatrix} \text{N} \qquad a_{G1} = 8.12\ \frac{\text{m}}{\text{s}^2}$$

Next assume that the front wheels lose contact with the ground $\quad N_A = 0$

Guesses $\quad N_B = 1\ \text{N} \qquad F_B = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $\quad F_B = M a_G \qquad N_B - M g = 0 \qquad N_B(b - a) - F_B h = 0$

$$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G) \qquad \begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N} \qquad a_{G2} = 6.78\ \frac{\text{m}}{\text{s}^2}$$

Choose the critical case $\quad a_G = \min(a_{G1}, a_{G2}) \qquad a_G = 6.78\ \frac{\text{m}}{\text{s}^2}$

---

**Problem 17-25**

Determine the greatest possible acceleration of the race car of mass $M$ so that its front tires do not leave the ground nor the tires slip on the track. The coefficients of static and kinetic friction are $\mu_s$ and $\mu_k$ respectively. Neglect the mass of the tires. The car has four-wheel drive.

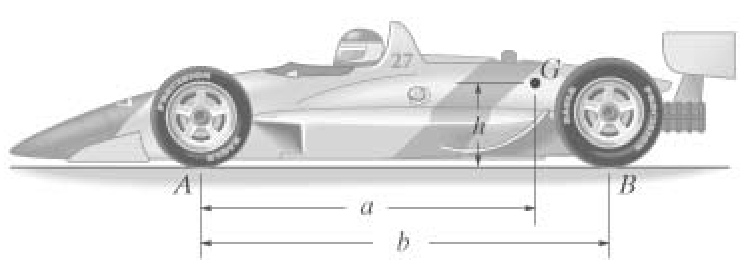
Given:

$M = 975\ \text{kg} \quad \mu_s = 0.8$

$a = 1.82\ \text{m} \quad \mu_k = 0.6$

$b = 2.20\ \text{m} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

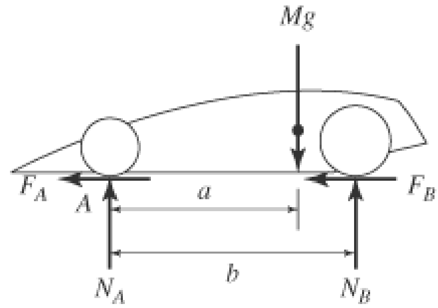
$h = 0.55\ \text{m}$



Solution:

First assume that all wheels are on the verge of slipping $F_A = \mu_s N_A$

$F_B = \mu_s N_B$




Guesses

$N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$\mu_s N_B + \mu_s N_A = M a_G$ $N_A + N_B - M g = 0$

$-N_A a + N_B(b-a) - \mu_s N_B h - \mu_s N_A h = 0$

$\begin{pmatrix} N_A \\ N_B \\ a_{G1} \end{pmatrix} = \text{Find}(N_A, N_B, a_G)$ $\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} -261 \\ 9826 \end{pmatrix} \text{N}$ $a_{G1} = 7.85\ \frac{\text{m}}{\text{s}^2}$

Next assume that the front wheels lose contact with the ground $N_A = 0$

Guesses $N_B = 1\ \text{N}$ $F_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $F_B = M a_G$ $N_B - M g = 0$ $N_B(b-a) - F_B h = 0$

$\begin{pmatrix} N_B \\ F_B \\ a_{G2} \end{pmatrix} = \text{Find}(N_B, F_B, a_G)$ $\begin{pmatrix} N_B \\ F_B \end{pmatrix} = \begin{pmatrix} 9565 \\ 6608 \end{pmatrix} \text{N}$ $a_{G2} = 6.78\ \frac{\text{m}}{\text{s}^2}$

Choose the critical case $a_G = \min(a_{G1}, a_{G2})$ $a_G = 6.78\ \frac{\text{m}}{\text{s}^2}$

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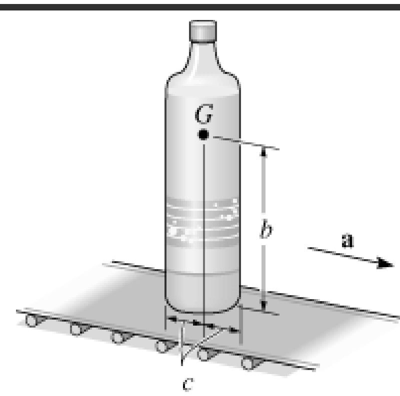
**Problem 17-26**

The bottle of weight $W$ rests on the check-out conveyor at a grocery store. If the coefficient of static friction is $\mu_s$, determine the largest acceleration the conveyor can have without causing the bottle to slip or tip. The center of gravity is at $G$.




Given:

$W = 2\ \text{lb}$

$\mu_s = 0.2$

$b = 8 \text{ in}$

$c = 1.5 \text{ in}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

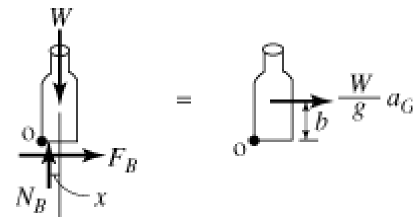
Solution: Assume that bottle tips before slipping

$x = c$

Guesses $a_G = 1 \dfrac{\text{ft}}{\text{s}^2}$ $F_B = 1 \text{ lb}$ $N_B = 1 \text{ lb}$ $F_{max} = 1 \text{ lb}$

Given $F_B = \left(\dfrac{W}{g}\right) a_G$ $N_B - W = 0$

$F_B b - N_B x = 0$ $F_{max} = \mu_s N_B$




$$\begin{pmatrix} a_{Gt} \\ F_B \\ N_B \\ F_{max} \end{pmatrix} = \text{Find}(a_G, F_B, N_B, F_{max}) \qquad \begin{pmatrix} F_B \\ N_B \\ F_{max} \end{pmatrix} = \begin{pmatrix} 0.375 \\ 2 \\ 0.4 \end{pmatrix} \text{lb} \qquad a_{Gt} = 6.037 \frac{\text{ft}}{\text{s}^2}$$

If $F_B = 0.375 \text{ lb} < F_{max} = 0.4 \text{ lb}$ then we have the correct answer.

If $F_B = 0.375 \text{ lb} > F_{max} = 0.4 \text{ lb}$ then we know that slipping occurs first. If this is the case,

$F_B = \mu_s N_B$

Given $F_B = \left(\dfrac{W}{g}\right) a_G$ $N_B - W = 0$ $F_B b - N_B x = 0$

$$\begin{pmatrix} a_{Gs} \\ N_B \\ x \end{pmatrix} = \text{Find}(a_G, N_B, x) \qquad N_B = 2 \text{ lb} \qquad x = 1.6 \text{ in} \qquad a_{Gs} = 6.44 \frac{\text{ft}}{\text{s}^2}$$

As a check, we should have $x = 1.6 \text{ in} < c = 1.5 \text{ in}$ if slipping occurs first

In either case, the answer is $a_G = \min(a_{Gs}, a_{Gt})$ $\boxed{a_G = 6.037 \dfrac{\text{ft}}{\text{s}^2}}$

**Problem 17-27**

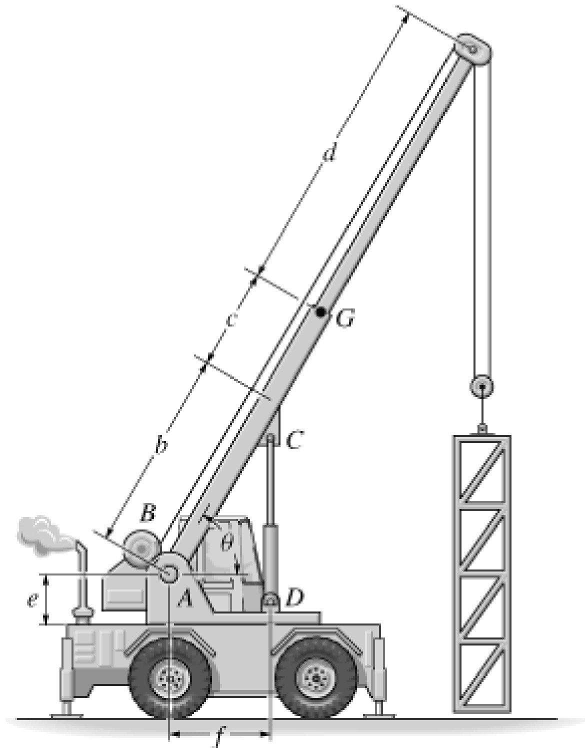
The assembly has mass $m_a$ and is hoisted using the boom and pulley system. If the winch at $B$

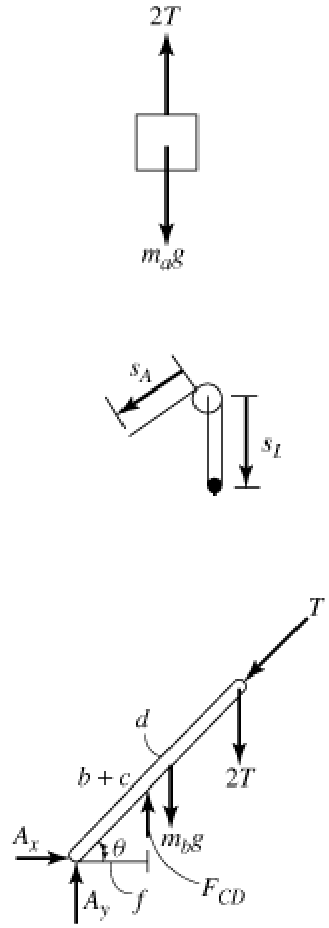
draws in the cable with acceleration $a$, determine the compressive force in the hydraulic cylinder needed to support the boom. The boom has mass $m_b$ and mass center at $G$.

Units Used:

$$\mathrm{Mg} = 10^3\ \mathrm{kg}$$

$$\mathrm{kN} = 10^3\ \mathrm{N}$$




Given:

$$m_a = 8\ \mathrm{Mg}$$

$$m_b = 2\ \mathrm{Mg}$$

$$a = 2\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$b = 4\ \mathrm{m}$$

$$c = 2\ \mathrm{m}$$

$$d = 6\ \mathrm{m}$$

$$e = 1\ \mathrm{m}$$

$$f = 2\ \mathrm{m}$$

$$\theta = 60\ \mathrm{deg}$$

$$g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Solution:

$$2T - m_a g = m_a \frac{a}{2} \qquad T = \frac{1}{2}\left(m_a g + m_a \frac{a}{2}\right) \qquad T = 43.24\,\mathrm{kN}$$

$$\left[-2T(b + c + d) - m_b g(b + c) + F_{CD} b\right]\cos(\theta) = 0$$

$$F_{CD} = \frac{2T(b + c + d) + m_b g(b + c)}{b} \qquad F_{CD} = 289\,\mathrm{kN}$$

***Problem 17-28**

The jet aircraft has total mass $M$ and a center of mass at $G$. Initially at take-off the engines provide thrusts $2T$ and $T'$. Determine the acceleration of the plane and the normal reactions on the nose wheel and each of the *two* wing wheels located at $B$. Neglect the mass of the wheels and, due to low velocity, neglect any lift caused by the wings.

Units Used:

$$\mathrm{Mg} = 10^3\ \mathrm{kg}$$

$kN = 10^3 \ N$

Given:

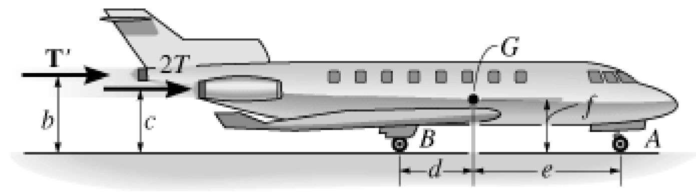
$b = 2.5 \ m$ $M = 22 \ Mg$

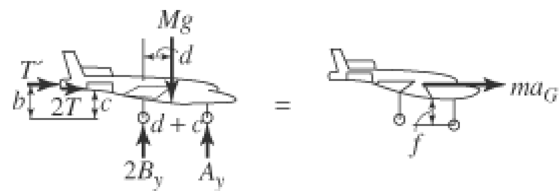
$c = 2.3 \ m$ $T = 2 \ kN$

$d = 3 \ m$ $T' = 1.5 \ kN$

$e = 6 \ m$ $g = 9.81 \ \frac{m}{s^2}$

$f = 1.2 \ m$




Solution:

Guesses $a_G = 1 \ \frac{m}{s^2}$ $B_y = 1 \ kN$ $A_y = 1 \ kN$

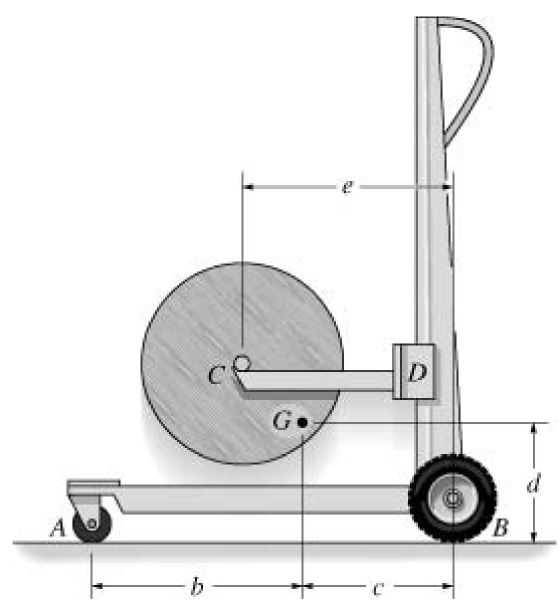
Given $T' + 2T = M a_G$ $2B_y + A_y - M g = 0$

$$-T' b - 2T c - M g d + A_y(d + e) = -M a_G f$$

$$\begin{pmatrix} a_G \\ B_y \\ A_y \end{pmatrix} = \mathrm{Find}(a_G, B_y, A_y) \qquad a_G = 0.250 \ \frac{m}{s^2} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 72.6 \\ 71.6 \end{pmatrix} kN$$

**Problem 17-29**

The lift truck has mass $m_t$ and mass center at $G$. If it lifts the spool of mass $m_s$ with acceleration $a$, determine the reactions of each of the four wheels on the ground. The loading is symmetric. Neglect the mass of the movable arm $CD$.




Given: $a = 3 \ \frac{m}{s^2}$ $d = 0.4 \ m$

$b = 0.75 \ m$ $e = 0.7 \ m$

$c = 0.5 \ m$ $g = 9.81 \ \frac{m}{s^2}$

$m_t = 70 \ kg$ $m_s = 120 \ kg$

Solution:

Guesses $N_A = 1\ \text{N}$ $\quad N_B = 1\ \text{N}$

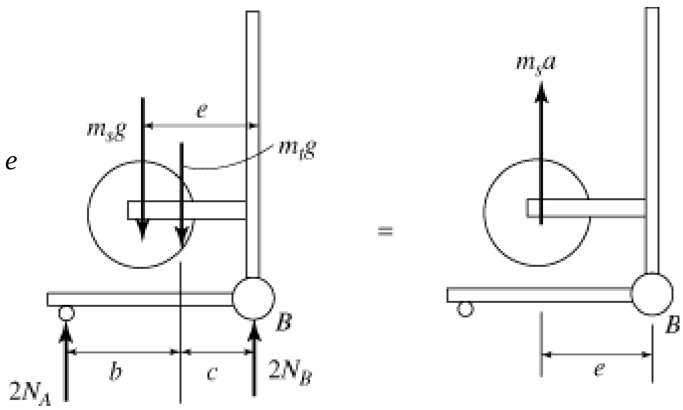
Given

$$2(N_A + N_B) - (m_t + m_s)g = m_s a$$

$$-2N_A(b+c) + m_s g e + m_t g c = -m_s a e$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 568 \\ 544 \end{pmatrix} \text{N}$$

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**Problem 17-30**

The lift truck has mass $m_t$ and mass center at $G$. Determine the largest upward acceleration of the spool of mass $m_s$ so that no reaction of the wheels on the ground exceeds $F_{max}$.

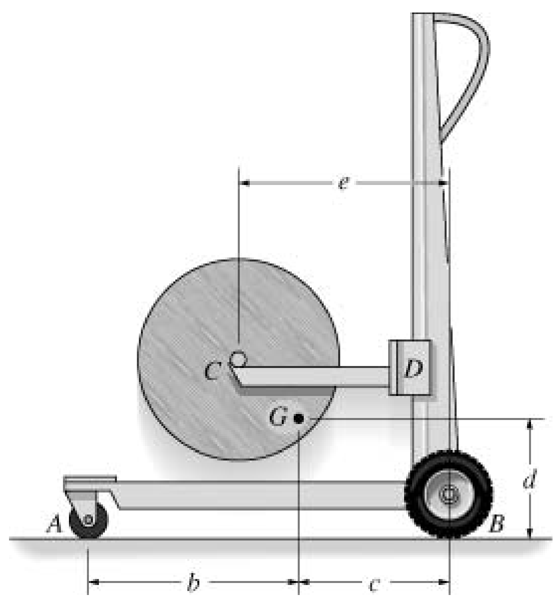
Given:

$m_t = 70\ \text{kg}$ $\quad b = 0.75\ \text{m}$

$m_s = 120\ \text{kg}$ $\quad c = 0.5\ \text{m}$

$F_{max} = 600\ \text{N}$ $\quad d = 0.4\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $\quad e = 0.7\ \text{m}$




Solution: Assume $N_A = F_{max}$

Guesses $a = 1\ \dfrac{\text{m}}{\text{s}^2}$ $\quad N_B = 1\ \text{N}$

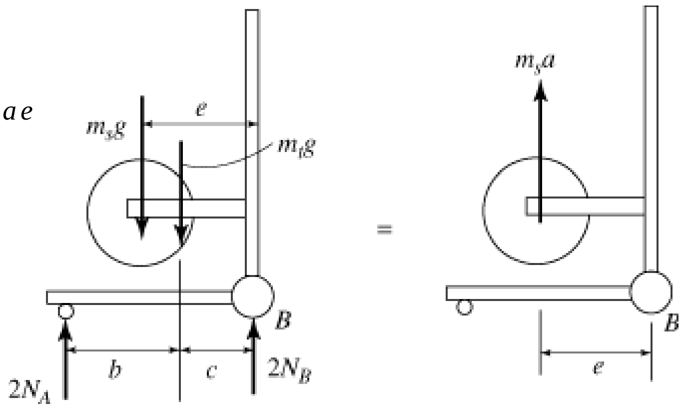
Given

$2(N_A + N_B) - (m_t + m_s)g = m_s a$

$-2N_A(b + c) + m_s g e + m_t g c = -m_s a e$

$\begin{pmatrix} a \\ N_B \end{pmatrix} = \text{Find}(a, N_B)$

$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 600 \\ 569.529 \end{pmatrix} \text{N}$




$a = 3.96 \ \frac{\text{m}}{\text{s}^2}$

Check: Since $N_B = 570\ \text{N}$ < $F_{max} = 600\ \text{N}$ then our assumption is good.

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**Problem 17-31**

The door has weight *W* and center of gravity at *G*. Determine how far the door moves in time *t* starting from rest, if a man pushes on it at *C* with a horizontal force *F*. Also, find the vertical reactions at the rollers *A* and *B*.

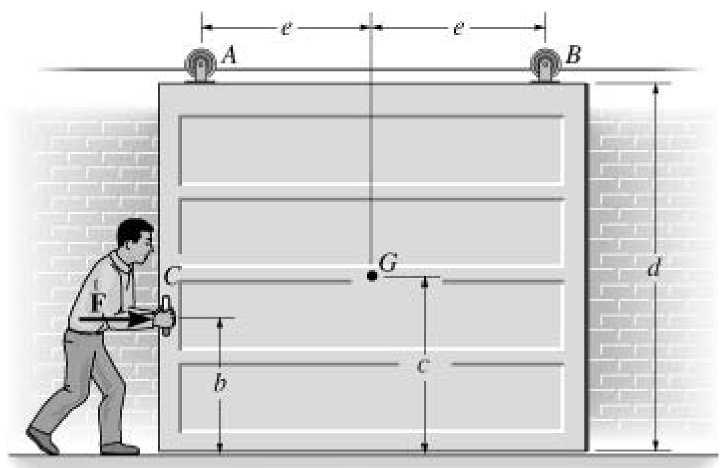
Given:

$W = 200\ \text{lb}$ $\quad c = 5\ \text{ft}$

$t = 2\ \text{s}$ $\quad d = 12\ \text{ft}$

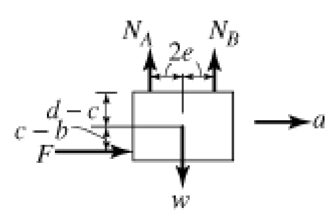
$F = 30\ \text{lb}$ $\quad e = 6\ \text{ft}$

$b = 3\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution:

Guesses $\quad a = 1\ \frac{\text{ft}}{\text{s}^2} \quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb}$




Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$F(c - b) + N_B e - N_A e = 0$

$$\begin{pmatrix} a \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a, N_A, N_B) \qquad a = 4.83\,\frac{\text{ft}}{\text{s}^2} \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 105.0 \\ 95.0 \end{pmatrix}\text{lb}$$

$$d = \frac{1}{2}a t^2 \qquad d = 9.66\,\text{ft}$$

***Problem 17-32**

The door has weight $W$ and center of gravity at $G$. Determine the constant force $F$ that must be applied to the door to push it open a distance $d$ to the right in time $t$, starting from rest. Also, find the vertical reactions at the rollers $A$ and $B$.

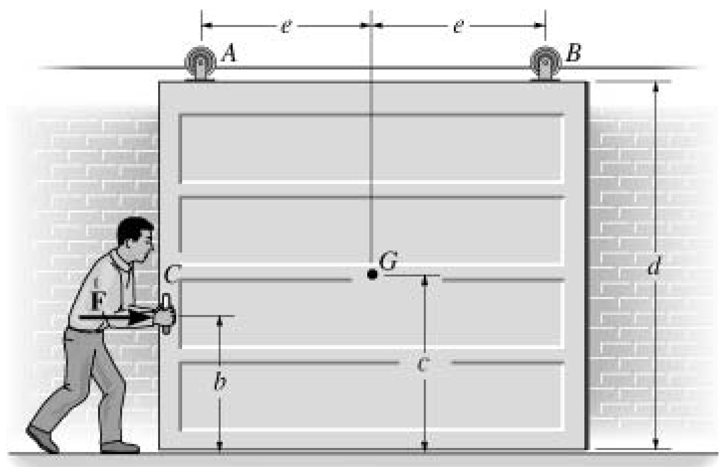
Given:

$W = 200\ \text{lb} \qquad c = 5\ \text{ft}$

$t = 5\ \text{s} \qquad d = 12\ \text{ft}$

$d = 12\ \text{ft} \qquad e = 6\ \text{ft}$

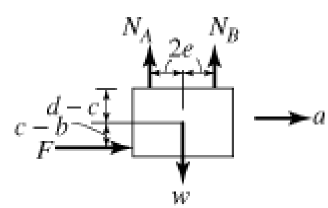
$b = 3\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution:

$$a = 2\left(\frac{d}{t^2}\right) \qquad a = 0.96\,\frac{\text{ft}}{\text{s}^2}$$

Guesses $\quad F = 1\ \text{lb} \quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb}$




Given $\quad F = \left(\frac{W}{g}\right)a \qquad N_A + N_B - W = 0$

$$F(c-b) + N_B e - N_A e = 0$$

$$\begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \text{Find}(F, N_A, N_B) \qquad \begin{pmatrix} F \\ N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.96 \\ 100.99 \\ 99.01 \end{pmatrix}\text{lb}$$

**Problem 17-33**

The fork lift has a boom with mass $M_1$ and a mass center at $G$. If the vertical acceleration of the boom is $a_G$, determine the horizontal and vertical reactions at the pin $A$ and on the short link $BC$ when the load $M_2$ is lifted.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$ $\qquad$ $\text{kN} = 10^3\ \text{N}$

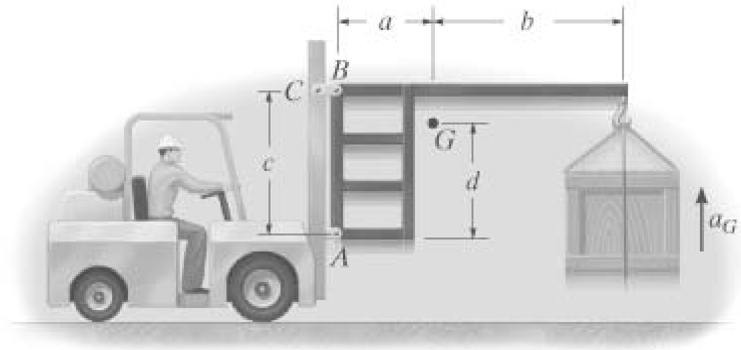
Given:

$M_1 = 800\ \text{kg}$ $\qquad$ $a = 1\ \text{m}$

$M_2 = 1.25\ \text{Mg}$ $\qquad$ $b = 2\ \text{m}$

$c = 1.5\ \text{m}$

$a_G = 4\ \dfrac{\text{m}}{\text{s}^2}$ $\qquad$ $d = 1.25\ \text{m}$




Solution:

Guesses

$A_x = 1\ \text{N}$ $\qquad$ $F_{CB} = 1\ \text{N}$

$A_y = 1\ \text{N}$

Given

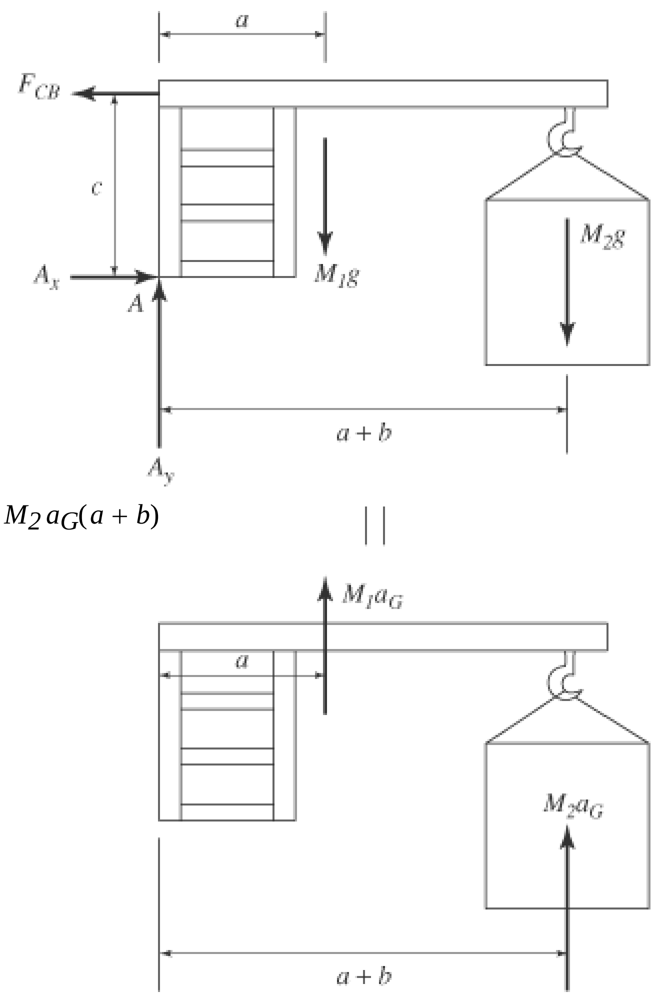
$-F_{CB} + A_x = 0$

$A_y - (M_1 + M_2)g = (M_1 + M_2)a_G$

$F_{CB}\,c - M_1\,g\,a - M_2\,g(a+b) = M_1\,a_G\,a + M_2\,a_G(a+b)$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \text{Find}(A_x, A_y, F_{CB})$$

$$\begin{pmatrix} A_x \\ A_y \\ F_{CB} \end{pmatrix} = \begin{pmatrix} 41.9 \\ 28.3 \\ 41.9 \end{pmatrix} \text{kN}$$



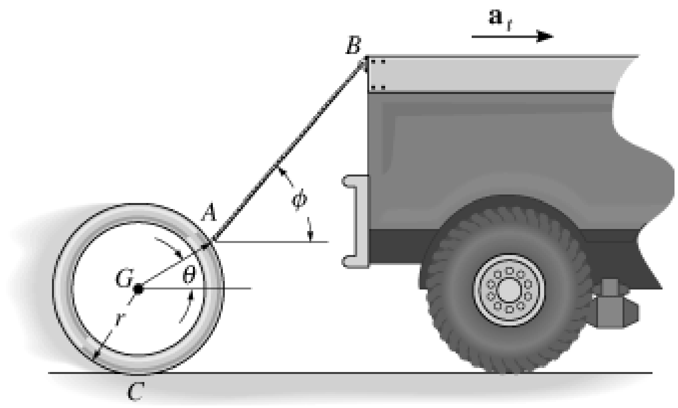

**Problem 17-34**

The pipe has mass $M$ and is being towed behind the truck. If the acceleration of the truck is $a_t$, determine the angle $\theta$ and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k$.

Units Used:

$$kN = 10^3\ N$$

Given:



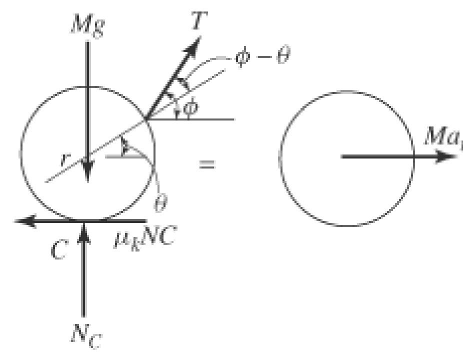

$M = 800\ kg \qquad r = 0.4\ m$

$a_t = 0.5\ \frac{m}{s^2} \qquad \phi = 45\ deg$

$\mu_k = 0.1 \qquad g = 9.81\ \frac{m}{s^2}$

Solution:

Guesses $\quad \theta = 10\ deg \quad N_C = 1\ N \quad T = 1\ N$




Given

$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

$$T\sin(\phi - \theta)r - \mu_k N_C r = 0$$

$$\begin{pmatrix} \theta \\ N_C \\ T \end{pmatrix} = \text{Find}(\theta, N_C, T) \qquad N_C = 6.771\,kN \qquad T = 1.523\,kN \qquad \theta = 18.608\,deg$$

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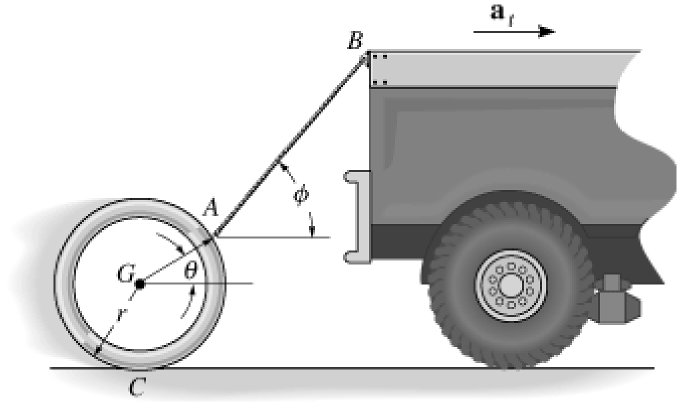
**Problem 17-35**

The pipe has mass $M$ and is being towed behind a truck. Determine the acceleration of the truck and the tension in the cable. The coefficient of kinetic friction between the pipe and the ground is $\mu_k$.




Units Used: $kN = 10^3\ N$

Given:

$M = 800\ kg$ $r = 0.4\ m$

$\theta = 30\ deg$ $\phi = 45\ deg$

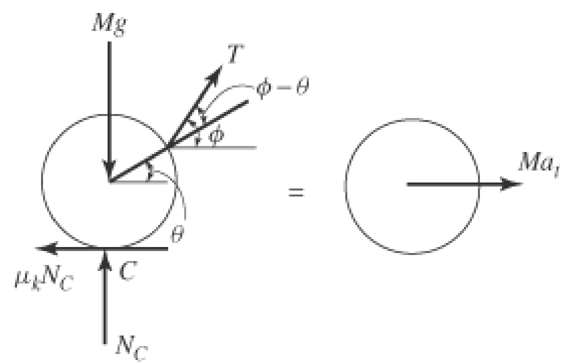
$\mu_k = 0.1$ $g = 9.81\ \frac{m}{s^2}$

Solution:

Guesses $a_t = 1\ \frac{m}{s^2}$

$N_C = 1\ N$

$T = 1\ N$




Given

$$T\cos(\phi) - \mu_k N_C = M a_t$$

$$T\sin(\phi) - M g + N_C = 0$$

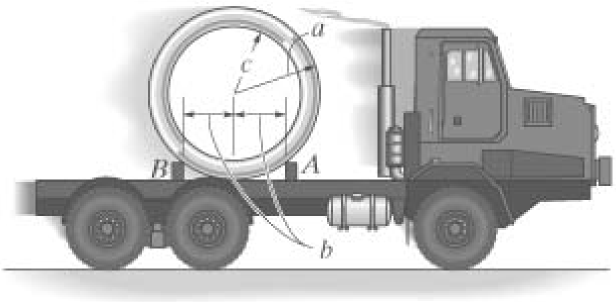
$$T\sin(\phi - \theta) r - \mu_k N_C r = 0$$

$$\begin{pmatrix} a_t \\ N_C \\ T \end{pmatrix} = \text{Find}(a_t, N_C, T) \qquad N_C = 6.164\,\text{kN} \qquad T = 2.382\,\text{kN} \qquad a_t = 1.335\,\frac{m}{s^2}$$

***Problem 17-36**

The pipe has a mass $M$ and is held in place on the truck bed using the two boards $A$ and $B$. Determine the acceleration of the truck so that the pipe begins to lose contact at $A$ and the bed of the truck and starts to pivot about $B$. Assume board $B$ will not slip on the bed of the truck, and the pipe is smooth. Also, what force does board $B$ exert on the pipe during the acceleration?

Units Used: $\text{kN} = 10^3 \text{ N}$




Given:

$M = 460 \text{ kg}$

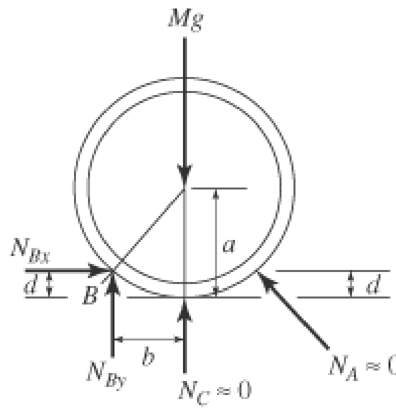
$a = 0.5 \text{ m}$

$b = 0.3 \text{ m}$

$c = 0.4 \text{ m}$

$d = 0.1 \text{ m}$

$g = 9.81 \dfrac{\text{m}}{\text{s}^2}$




Solution: $\theta = \operatorname{asin}\left(\dfrac{b}{a}\right)$

Guesses

$N_{Bx} = 1 \text{ N} \qquad N_{By} = 1 \text{ N} \qquad a_t = 1 \dfrac{\text{m}}{\text{s}^2}$

Given

$N_{Bx} = M a_t \qquad N_{By} - M g = 0 \qquad N_{Bx}(a)\cos(\theta) - N_{By} b = 0$

$$\begin{pmatrix} N_{Bx} \\ N_{By} \\ a_t \end{pmatrix} = \text{Find}(N_{Bx}, N_{By}, a_t) \qquad \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} = \begin{pmatrix} 3.384 \\ 4.513 \end{pmatrix} \text{kN} \qquad a_t = 7.36 \frac{\text{m}}{\text{s}^2}$$

$$\left| \begin{pmatrix} N_{Bx} \\ N_{By} \end{pmatrix} \right| = 5.64 \text{ kN}$$

---

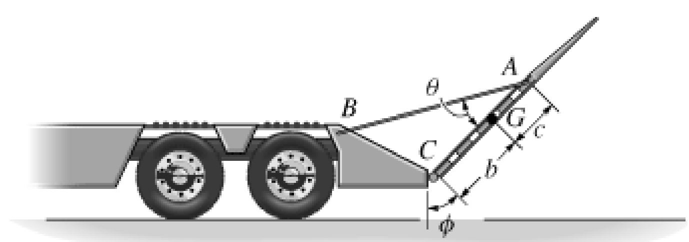
**Problem 17-37**

The drop gate at the end of the trailer has mass $M$ and mass center at $G$. If it is supported by the cable $AB$ and hinge at $C$, determine the tension in the cable when the truck begins to accelerate at rate $a$. Also, what are the horizontal and vertical components of reaction at the hinge $C$?

Given: $\text{kN} = 10^3 \text{ N}$

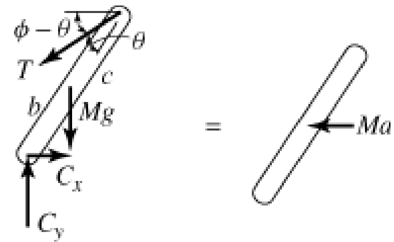
$M = 1.25 \times 10^3 \text{ kg}$

$a = 5 \dfrac{\text{m}}{\text{s}^2} \qquad \theta = 30 \text{ deg}$



$b = 1.5\ \text{m}$ $\phi = 45\ \text{deg}$

$c = 1\ \text{m}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$




Solution:

Guesses $T = 1\ \text{N}$ $C_x = 1\ \text{N}$ $C_y = 1\ \text{N}$

Given $-T\cos(\phi - \theta) + C_x = -M a$

$-T\sin(\phi - \theta) - M g + C_y = 0$

$T\sin(\theta)(b + c) - M g b\cos(\phi) = M a b\sin(\phi)$

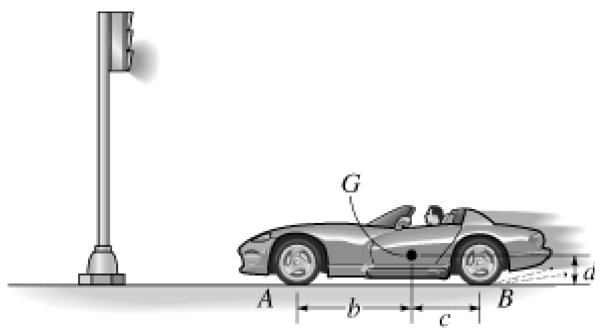
$$\begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \text{Find}(T, C_x, C_y) \qquad \begin{pmatrix} T \\ C_x \\ C_y \end{pmatrix} = \begin{pmatrix} 15.708 \\ 8.923 \\ 16.328 \end{pmatrix} \text{kN}$$

---

**Problem 17-38**

The sports car has mass *M* and a center of mass at *G*. Determine the shortest time it takes for it to reach speed *v*, starting from rest, if the engine only drives the rear wheels, whereas the front wheels are free rolling. The coefficient of static friction between the wheels and the road is $\mu_s$. Neglect the mass of the wheels for the calculation. If driving power could be supplied to all four wheels, what would be the shortest time for the car to reach a speed of *v*?

Given:




$M = 1.5 \times 10^3\ \text{kg}$

$v = 80\ \dfrac{\text{km}}{\text{hr}}$

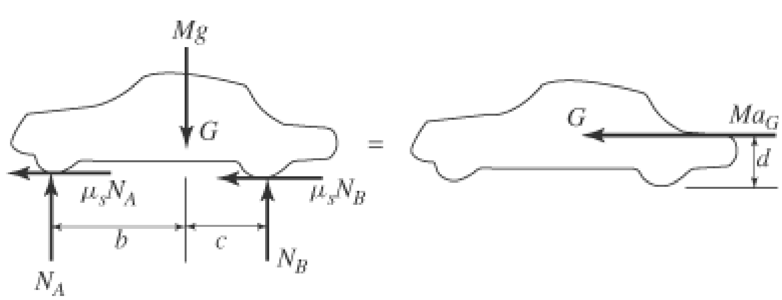
$\mu_s = 0.2$

$b = 1.25\ \text{m}$

$c = 0.75\ \text{m}$

$d = 0.35\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

(a) Rear wheel drive only Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B = M a_G$

$Mgc - N_A(b+c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.185 \times 10^3 \\ 9.53 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.271\ \frac{\text{m}}{\text{s}^2}$$

$$t_{rw} = \frac{v}{a_G} \qquad t_{rw} = 17.488\ \text{s}$$

(b) Four wheel drive Guesses $N_A = 1\ \text{N}$ $N_B = 1\ \text{N}$ $a_G = 1\ \frac{\text{m}}{\text{s}^2}$

Given $N_A + N_B - Mg = 0$ $\mu_s N_B + \mu_s N_A = M a_G$

$Mgc - N_A(b+c) = M a_G d$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 5.003 \times 10^3 \\ 9.712 \times 10^3 \end{pmatrix} \text{N} \qquad a_G = 1.962\ \frac{\text{m}}{\text{s}^2}$$

$$t_{rw} = \frac{v}{a_G} \qquad t_{rw} = 11.326\ \text{s}$$

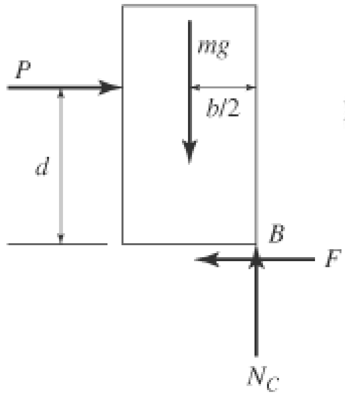
---

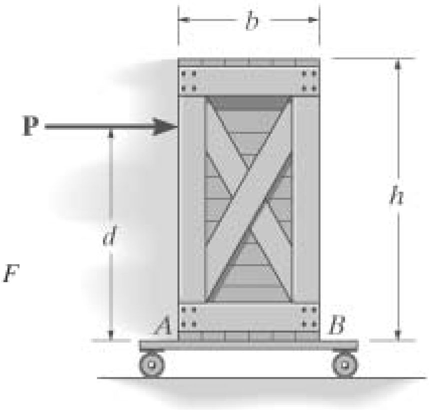
**Problem 17-39**

The crate of mass *m* is supported on a cart of negligible mass. Determine the maximum force *P* that can be applied a distance *d* from the cart bottom without causing the crate to tip on the cart.

Solution:

Require $N_c$ to act at corner $B$ for tipping.




$$P\,d - m\,g\left(\frac{b}{2}\right) = m\,a_G\left(\frac{h}{2}\right)$$

$$P = m\,a_G$$

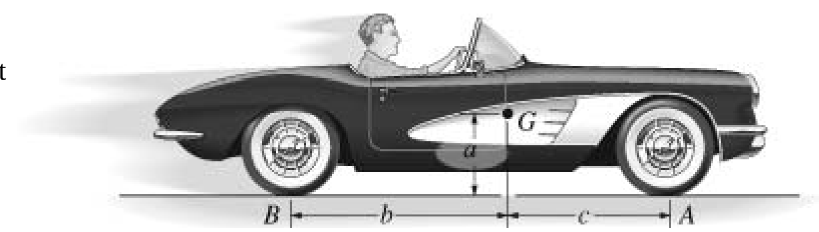
$$P\,d - m\mathbf{g}\left(\frac{b}{2}\right) = P\left(\frac{h}{2}\right)$$

$$P_{max} = \frac{m\,g\,b}{2\left(d - \frac{h}{2}\right)}$$

---

***Problem 17-40**

The car accelerates uniformly from rest to speed $v$ in time $t$. If it has weight $W$ and a center of gravity at $G$, determine the normal reaction of *each wheel* on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of the wheels and take the coefficients of static and kinetic friction to be $\mu_s$ and $\mu_k$ respectively.



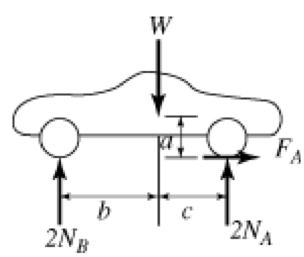

Given: $a = 2.5\ \text{ft}$

$v = 88\ \frac{\text{ft}}{\text{s}}$ $\mu_k = 0.2$

$t = 15\ \text{s}$ $b = 4\ \text{ft}$

$W = 3800\ \text{lb}$ $c = 3\ \text{ft}$

$\mu_s = 0.4$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution: Assume no slipping $a_G = \frac{v}{t}$ $a_G = 5.867\,\frac{\text{ft}}{\text{s}^2}$

Guesses $N_B = 1\ \text{lb}$ $N_A = 1\ \text{lb}$ $F_A = 1\ \text{lb}$

Given $2N_B + 2N_A - W = 0$

$$2F_A = \left(\frac{W}{g}\right)a_G$$

$$-2N_B(b + c) + W\,c = \left(\frac{-W}{g}\right)a_G\,a$$

$$\begin{pmatrix} N_A \\ N_B \\ F_A \end{pmatrix} = \text{Find}(N_A, N_B, F_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 962 \\ 938 \end{pmatrix} \text{lb} \qquad F_A = 346\,\text{lb}$$

$$F_{max} = \mu_s N_A \qquad F_{max} = 385\,\text{lb}$$

Check: Our no-slip assumption is true if $F_A = 346\,\text{lb} < F_{max} = 385\,\text{lb}$

**Problem 17-41**

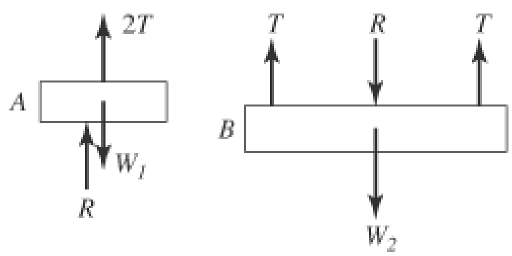
Block $A$ has weight $W_1$ and the platform has weight $W_2$. Determine the normal force exerted by block $A$ on $B$. Neglect the weight of the pulleys and bars of the triangular frame.

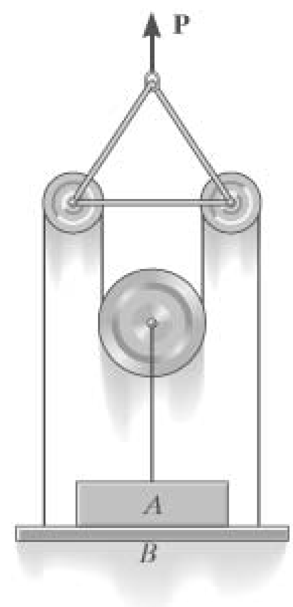
Given:

$W_1 = 50$ lb

$W_2 = 10$ lb

$P = 100$ lb

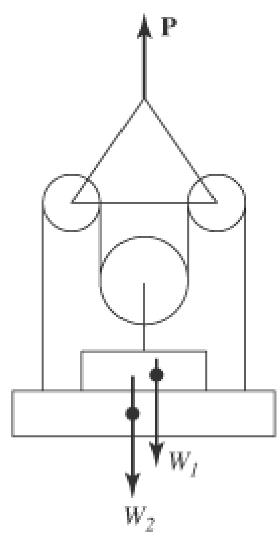



Solution:

$$P - (W_1 + W_2) = \left(\frac{W_1 + W_2}{g}\right) a_G$$

$$a_G = g\left(\frac{P - 1\,W_1 - 1\,W_2}{W_1 + W_2}\right) \qquad a_G = 21.47\,\frac{\text{ft}}{\text{s}^2}$$




Guesses $R = 25$ lb $T = 22$ lb

Given

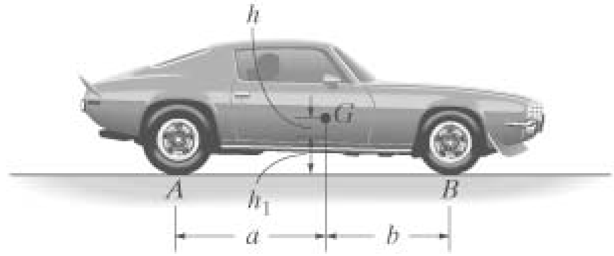
$$2T + R - W_1 = \left(\frac{W_1}{g}\right) a_G$$

$$2T - R - W_2 = \left(\frac{W_2}{g}\right) a_G$$

$$\begin{pmatrix} R \\ T \end{pmatrix} = \text{Find}(R, T) \qquad R = 33.3\,\text{lb} \qquad T = 25\,\text{lb}$$

**Problem 17-42**

The car of mass $M$ shown has been "raked" by increasing the height of its center of mass to $h$ This was done by raising the springs on the rear axle. If the coefficient of kinetic friction between the rear wheels and the ground is $\mu_k$, show that the car can accelerate slightly faster than its counterpart for which $h = 0$. Neglect the mass of the wheels and driver and assume the front wheels at $B$ are free to roll while the rear wheels slip.




Units Used:

$\text{Mg} = 10^3 \text{ kg} \quad \text{kN} = 10^3 \text{ N}$

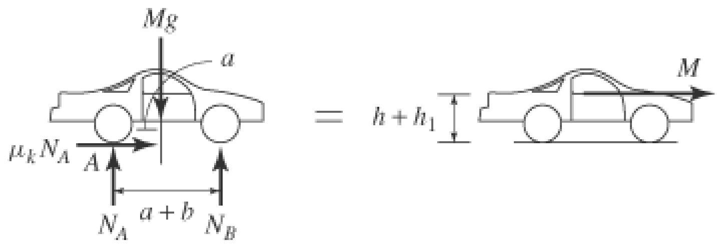
Given:

$M = 1.6 \text{ Mg} \quad a = 1.6 \text{ m}$

$\mu_k = 0.3 \quad b = 1.3 \text{ m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2} \quad h = 0.2 \text{ m}$

$h_1 = 0.4 \text{ m}$




Solution:

In the raised position

Guesses $\quad a_G = 1 \frac{\text{m}}{\text{s}^2} \quad N_A = 4 \text{ N} \quad N_B = 5 \text{ N}$

Given $\quad \mu_k N_A = M a_G$

$N_A + N_B - M g = 0$

$-M g a + N_B(a + b) = -M a_G(h + h_1)$

$$\begin{pmatrix} a_{Gr} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gr} = 1.41 \frac{\text{m}}{\text{s}^2}$$

In the lower (regular) position

Given

$\mu_k N_A = M a_G \qquad N_A + N_B - M g = 0 \qquad -M g a + N_B(a + b) = -M a_G h_1$

$$\begin{pmatrix} a_{Gl} \\ N_A \\ N_B \end{pmatrix} = \text{Find}(a_G, N_A, N_B) \qquad a_{Gl} = 1.38 \frac{\text{m}}{\text{s}^2}$$

Thus the advantage in the raised position is $\quad a_{Gr} - a_{Gl} = 0.03 \frac{\text{m}}{\text{s}^2}$

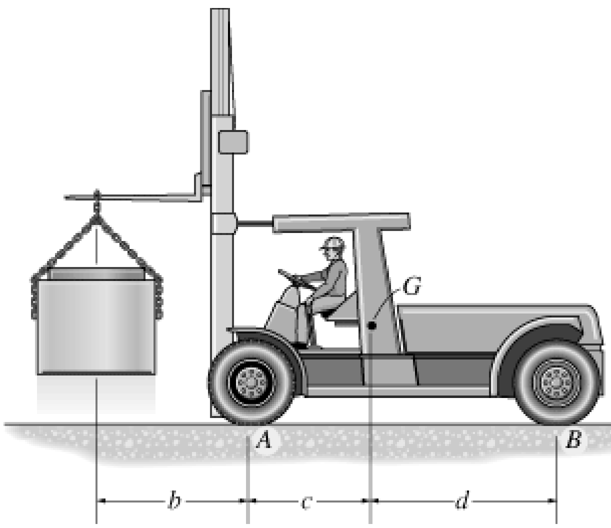
**Problem 17-43**

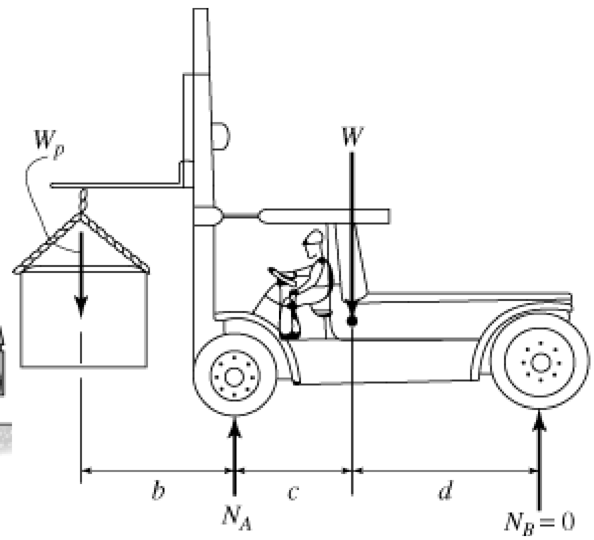
The forklift and operator have combined weight $W$ and center of mass at $G$. If the forklift is used to lift the concrete pipe of weight $W_p$ determine the maximum vertical acceleration it can give to the pipe so that it does not tip forward on its front wheels.

Given:

$W = 10000 \text{ lb}$ $\quad b = 5 \text{ ft}$ $\quad d = 6 \text{ ft}$

$W_p = 2000 \text{ lb}$ $\quad c = 4 \text{ ft}$ $\quad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



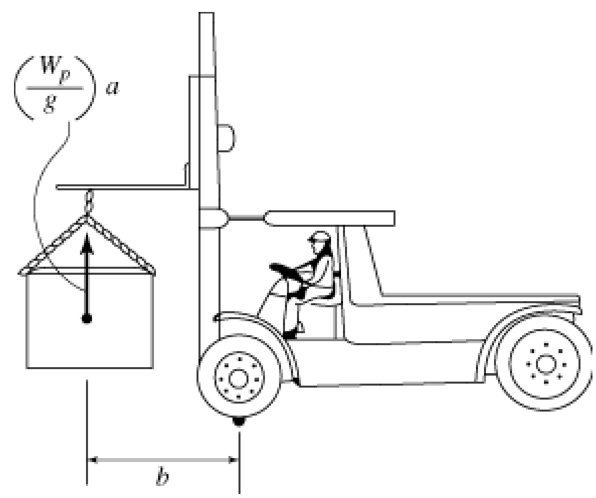

Solution:

It is required that $N_B = 0$

$$W_p b - W c = -\frac{W_p}{g} a b$$

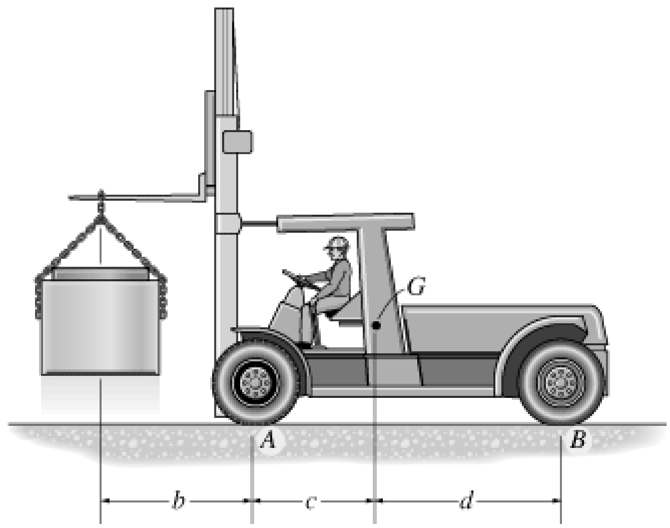
$$a = \left(\frac{W\, c - W_p b}{W_p}\right)\frac{g}{b}$$

$$a = 96.6 \frac{\text{ft}}{\text{s}^2}$$




***Problem 17-44**

The forklift and operator have combined weight $W$ and center of mass at $G$. If the forklift is used to lift the concrete pipe of weight $W_p$ determine the normal reactions on each of its four wheels if the pipe is given upward acceleration $a$.




Units Used:

$$\text{kip} = 10^3 \text{ lb}$$

Given:

$$W_p = 2000 \text{ lb}$$

$$W = 10000 \text{ lb}$$

$$a = 4 \frac{\text{ft}}{\text{s}^2}$$

$$b = 5 \text{ ft}$$

$$c = 4 \text{ ft}$$

$$d = 6 \text{ ft}$$

$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

Guesses

$$N_A = 1 \text{ lb}$$

$$N_B = 1 \text{ lb}$$

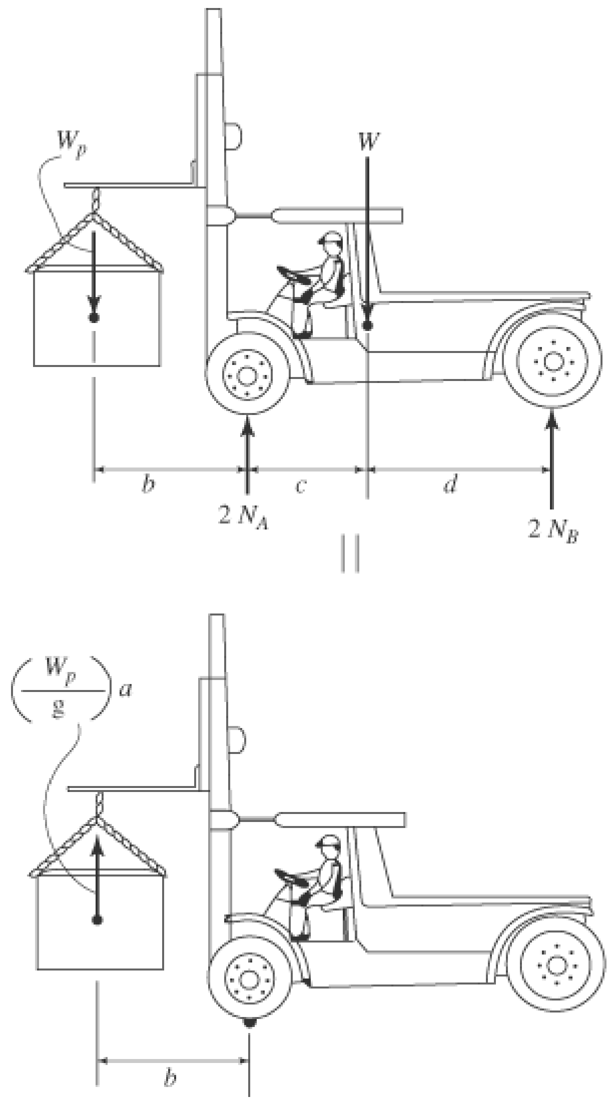
Given

$$2N_A + 2N_B - W - W_p = \left(\frac{W_p}{g}\right)a$$

$$2N_A b - W(b + c) + 2N_B(b + c + d) = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 4.686 \\ 1.438 \end{pmatrix} \text{kip}$$



**Problem 17-45**

The van has weight $W_v$ and center of gravity at $G_v$. It carries fixed load $W_l$ which has center of gravity at $G_l$. If the van is traveling at speed $v$, determine the distance it skids before stopping. The brakes cause *all* the wheels to lock or skid. The coefficient of kinetic friction between the wheels and the pavement is $\mu_k$. Compare this distance with that of the van being empty. Neglect the mass of the wheels.

Given:

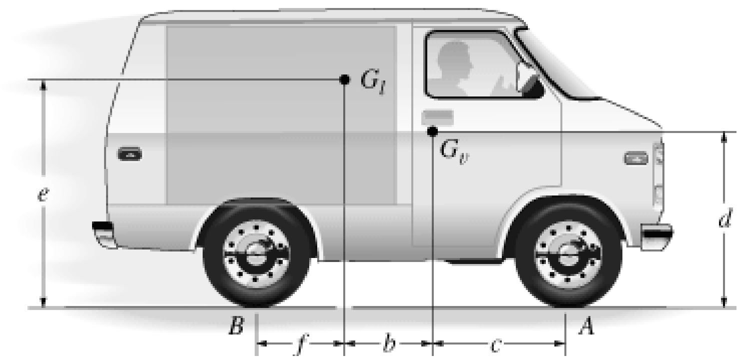
$W_v = 4500\ \text{lb}$ $\quad b = 2\ \text{ft}$

$W_l = 800\ \text{lb}$ $\quad c = 3\ \text{ft}$

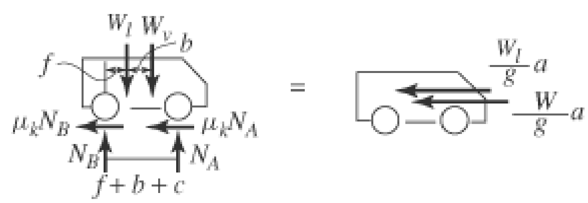
$v = 40\ \frac{\text{ft}}{\text{s}}$ $\quad d = 4\ \text{ft}$

$\mu_k = 0.3$ $\quad e = 6\ \text{ft}$

$f = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution: Loaded




Guesses $\quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb} \quad a = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$N_A + N_B - W_v - W_l = 0$$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3777 \\ 1523 \end{pmatrix}\text{lb}$$

$$a = 9.66\ \frac{\text{ft}}{\text{s}^2} \qquad d_l = \frac{v^2}{2a} \qquad d_l = 82.816\ \text{ft}$$

Unloaded $\quad W_l = 0\ \text{lb}$

Guesses $\quad N_A = 1\ \text{lb} \quad N_B = 1\ \text{lb} \quad a = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $\quad N_A + N_B - W_v - W_l = 0$

$$\mu_k(N_A + N_B) = \left(\frac{W_v + W_l}{g}\right)a$$

$$-N_B(f + b + c) + W_l(b + c) + W_v c = \left(\frac{W_l}{g}\right)a\,e + \left(\frac{W_v}{g}\right)a\,d$$

$$\begin{pmatrix} N_A \\ N_B \\ a \end{pmatrix} = \text{Find}(N_A, N_B, a) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 3343 \\ 1157 \end{pmatrix} \text{lb}$$

$$a = 9.66\ \frac{\text{ft}}{\text{s}^2} \qquad d_{ul} = \frac{v^2}{2a}$$

$$d_{ul} = 82.816\ \text{ft}$$

The distance is the same in both cases although the forces on the tires are different.

---

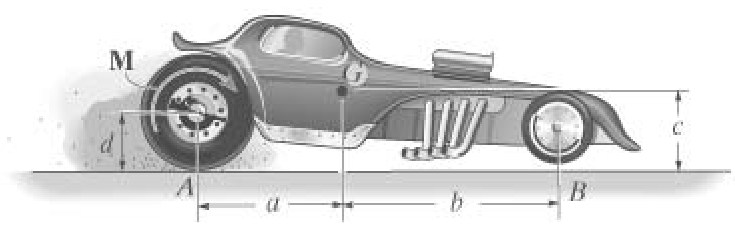
**Problem 17-46**

The “muscle car” is designed to do a “wheeley”, i.e., to be able to lift its front wheels off the ground in the manner shown when it accelerates. If the car of mass $M_1$ has a center of mass at $G$, determine the minimum torque that must be developed at both rear wheels in order to do this. Also, what is the smallest necessary coefficient of static friction assuming the thick-walled rear wheels do not slip on the pavement? Neglect the mass of the wheels.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$




Given:

$M_1 = 1.35\ \text{Mg}$

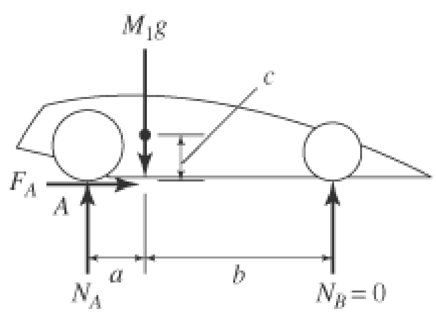
$a = 1.10\ \text{m}$

$b = 1.76\ \text{m}$

$c = 0.67\ \text{m}$

$d = 0.31\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$




Solution:

Guesses $\quad a_G = 1\ \frac{\text{m}}{\text{s}^2} \quad F_A = 1\ \text{N} \quad N_A = 1\ \text{N} \quad M = 1\ \text{N m} \quad \mu_s = 0.1$

Given

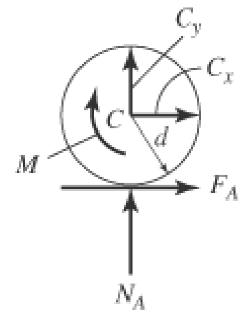
$F_A = M_1 a_G$

$N_A - M_1 g = 0$

$M_1 g a = M_1 a_G c$

$-M + F_A d = 0$

$F_A = \mu_s N_A$




$$\begin{pmatrix} a_G \\ F_A \\ N_A \\ M \\ \mu_s \end{pmatrix} = \mathrm{Find}(a_G, F_A, N_A, M, \mu_s)$$

$$\begin{pmatrix} F_A \\ N_A \end{pmatrix} = \begin{pmatrix} 21.7 \\ 13.2 \end{pmatrix} \mathrm{kN} \qquad M = 6.74\,\mathrm{kN\cdot m}$$

$$a_G = 16.11\,\frac{\mathrm{m}}{\mathrm{s}^2} \qquad \mu_s = 1.642$$

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**Problem 17-47**

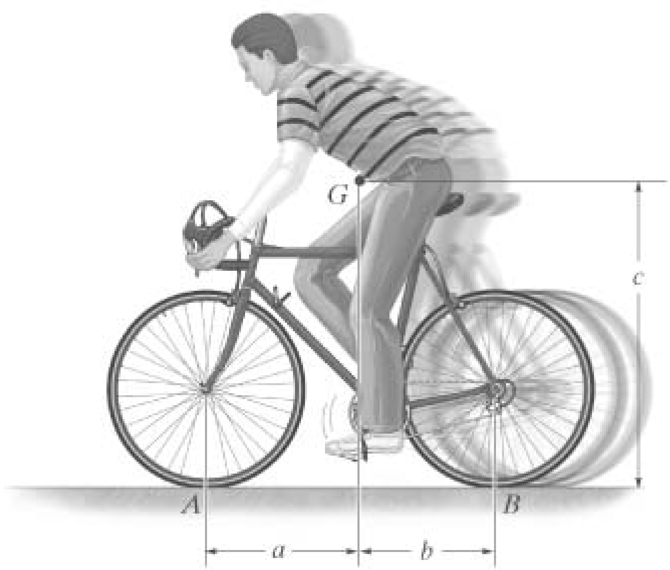
The bicycle and rider have a mass $M$ with center of mass located at $G$. If the coefficient of kinetic friction at the rear tire is $\mu_B$, determine the normal reactions at the tires $A$ and $B$, and the deceleration of the rider, when the rear wheel locks for braking. What is the normal reaction at the rear wheel when the bicycle is traveling at constant velocity and the brakes are not applied? Neglect the mass of the wheels.

Given:

$M = 80\ \mathrm{kg}$ $\quad \mu_B = 0.8$

$a = 0.55\ \mathrm{m}$ $\quad b = 0.4\ \mathrm{m}$

$c = 1.2\ \mathrm{m}$




Solution: Deceleration:

Guesses $\quad a_G = 1\,\frac{\mathrm{m}}{\mathrm{s}^2}$

$N_B = 1\ \mathrm{N}$ $\quad N_A = 1\ \mathrm{N}$

Given $\quad \mu_B N_B = M a_G \qquad N_A + N_B - M g = 0$

$-N_B(a + b) + M g a = M a_G c$

$$\begin{pmatrix} a_G \\ N_B \\ N_A \end{pmatrix} = \text{Find}(a_G, N_B, N_A) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 559 \\ 226 \end{pmatrix} \text{N} \qquad a_G = 2.26 \ \frac{\text{m}}{\text{s}^2}$$

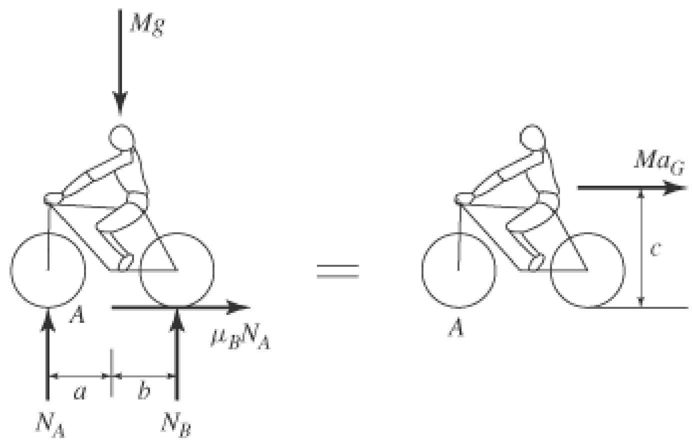
Equilibrium

Given $\quad N_A + N_B - M g = 0$

$$-N_B(a+b) + M g a = 0$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \text{Find}(N_A, N_B)$$

$$\begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 330 \\ 454 \end{pmatrix} \text{N}$$




***Problem 17-48**

The bicycle and rider have a mass $M$ with center of mass located at $G$. Determine the minimum coefficient of kinetic friction between the road and the wheels so that the rear wheel $B$ starts to lift off the ground when the rider applies the brakes to the front wheel. Neglect the mass of the wheels.

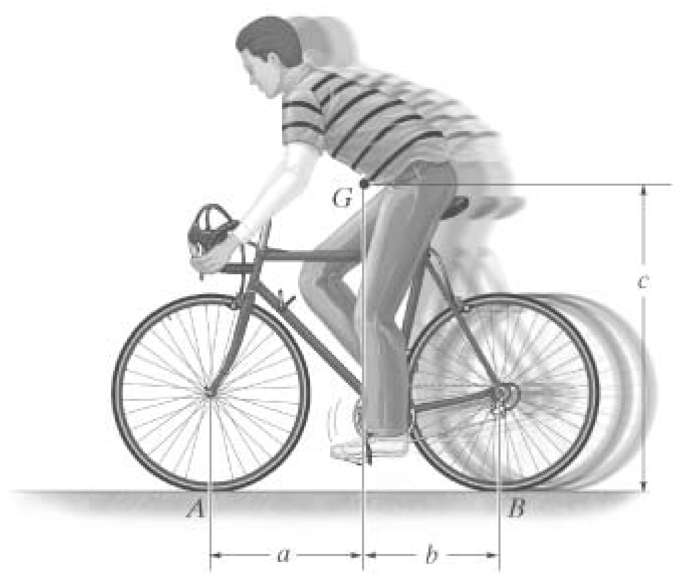
Given:

$M = 80 \text{ kg}$

$a = 0.55 \text{ m}$

$b = 0.4 \text{ m}$

$c = 1.2 \text{ m}$




Solution: $\quad N_B = 0$

Guesses $\quad a_G = 1 \ \frac{\text{m}}{\text{s}^2}$

$\mu_k = 0.1$

$N_A = 1 \text{ N}$

Given $\quad \mu_k N_A = M a_G$

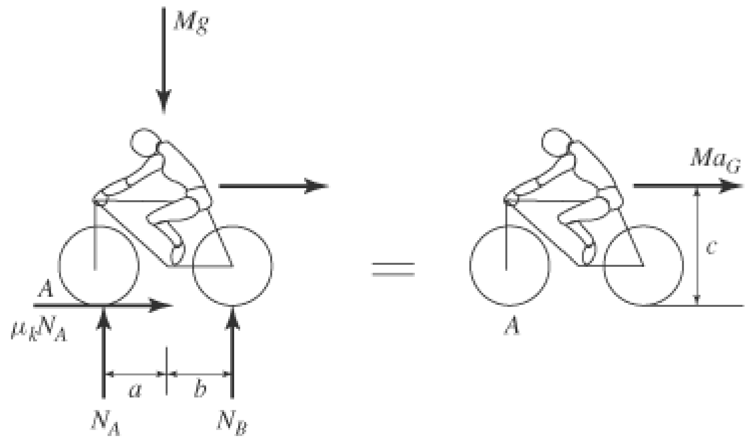
$$N_A - M g = 0$$

$$M g a = M a_G c$$

$$\begin{pmatrix} a_G \\ \mu_k \\ N_A \end{pmatrix} = \text{Find}(a_G, \mu_k, N_A)$$

$N_A = 785\ \text{N}$

$a_G = 4.50\ \dfrac{\text{m}}{\text{s}^2}$ $\quad \mu_k = 0.458$

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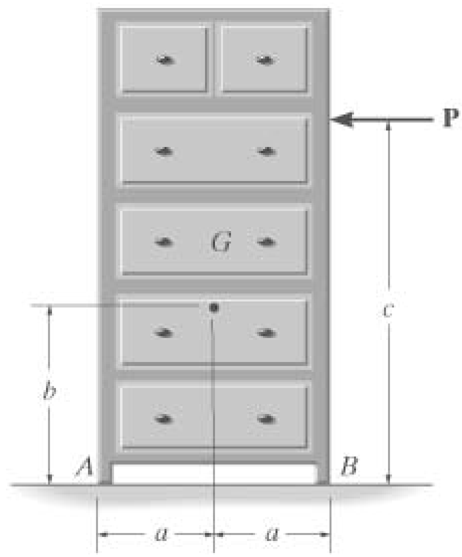
**Problem 17-49**




The dresser has a weight $W$ and is pushed along the floor. If the coefficient of static friction at $A$ and $B$ is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$, determine the smallest horizontal force $P$ needed to cause motion. If this force is increased slightly, determine the acceleration of the dresser. Also, what are the normal reactions at $A$ and $B$ when it begins to move?

Given:

$W = 80\ \text{lb}$

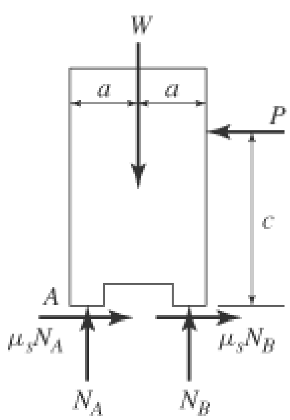
$\mu_s = 0.3$

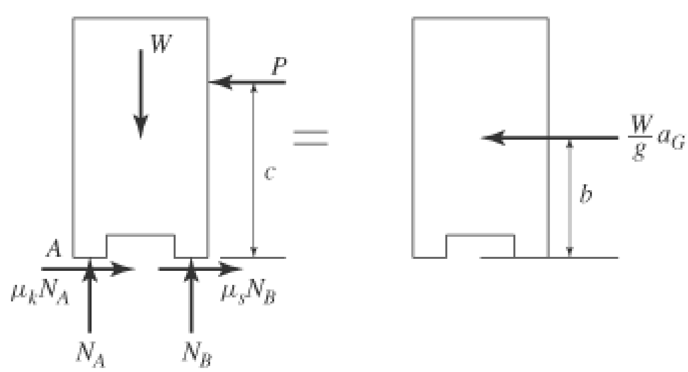
$\mu_k = 0.2$

$a = 1.5\ \text{ft}$

$b = 2.5\ \text{ft}$

$c = 4\text{ft}$




Solution: Impending Motion

Guesses $P = 1\ \text{lb}$

$N_A = 1\ \text{lb}$ $\quad N_B = 1\ \text{lb}$

Given $N_A + N_B - W = 0$

$\mu_s N_A + \mu_s N_B - P = 0$

$P\,c + W\,a - N_A(2a) = 0$

$$\begin{pmatrix} P \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, N_A, N_B) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 72 \\ 8 \end{pmatrix} \text{lb} \qquad P = 24\,\text{lb}$$

Motion Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

Given $N_A + N_B - W = 0 \qquad \mu_k N_A + \mu_k N_B - P = \left(\frac{-W}{g}\right) a_G$

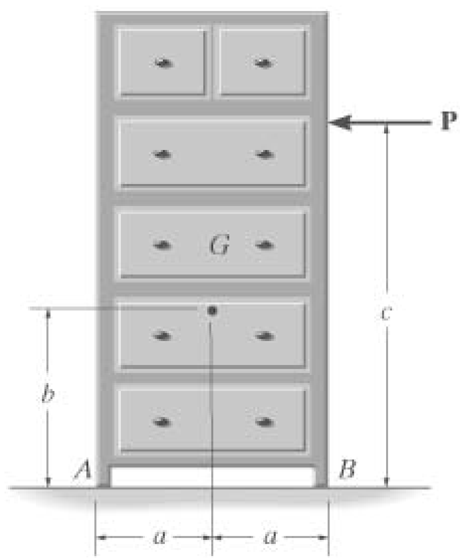
$$P(c - b) + \mu_k(N_A + N_B)b + N_B a - N_A a = 0$$

$$\begin{pmatrix} N_A \\ N_B \\ a_G \end{pmatrix} = \text{Find}(N_A, N_B, a_G) \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 65.3 \\ 14.7 \end{pmatrix} \text{lb} \qquad a_G = 3.22\ \frac{\text{ft}}{\text{s}^2}$$

---

**Problem 17-50**

The dresser has a weight $W$ and is pushed along the floor. If the coefficient of static friction at $A$ and $B$ is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$, determine the maximum horizontal force $P$ that can be applied without causing the dresser to tip over.




Given:

$W = 80\ \text{lb}$

$\mu_s = 0.3$

$\mu_k = 0.2$

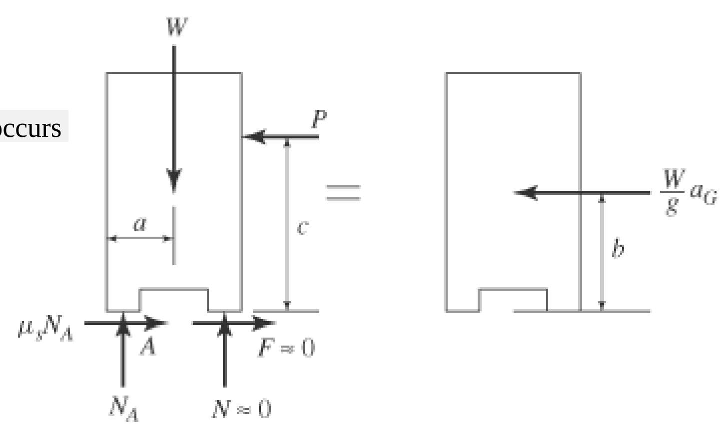
$a = 1.5\ \text{ft}$

$b = 2.5\ \text{ft}$

$c = 4\ \text{ft}$

Solution:

Dresser slides before tipping occurs




Guesses

$N_A = 1\ \text{lb}$

$P = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$N_A - W = 0 \qquad \mu_k N_A - P = \left(\frac{-W}{g}\right) a_G \qquad P(c-b) - N_A a + \mu_k N_A b = 0$$

$$\begin{pmatrix} N_A \\ P \\ a_G \end{pmatrix} = \text{Find}(N_A, P, a_G) \qquad a_G = 15.02\,\frac{\text{ft}}{\text{s}^2} \qquad N_A = 80\,\text{lb} \qquad P = 53.3\,\text{lb}$$

---

**Problem 17-51**

The crate *C* has weight *W* and rests on the truck elevator for which the coefficient of static friction is $\mu_s$. Determine the largest initial angular acceleration $\alpha$ starting from rest, which the parallel links *AB* and *DE* can have without causing the crate to slip. No tipping occurs.

Given:

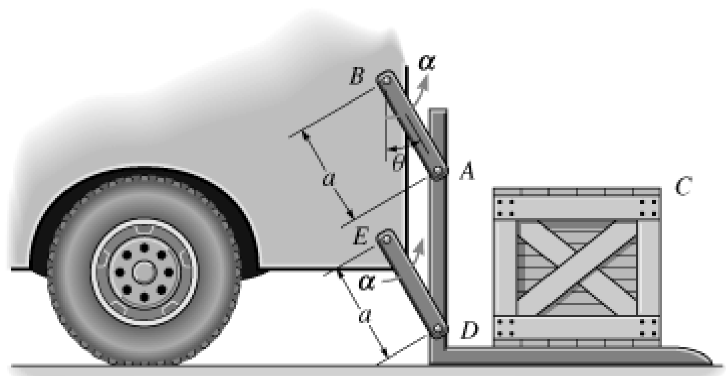
$W = 150\ \text{lb}$

$\mu_s = 0.4$

$a = 2\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



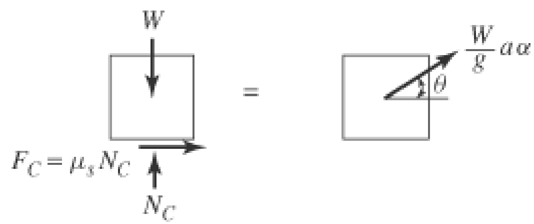

Solution:

Initial Guesses: $N_C = 1\ \text{lb}$ $\qquad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$\mu_s N_C = \left(\frac{W}{g}\right) \alpha a \cos(\theta)$$

$$N_C - W = \left(\frac{W}{g}\right) \alpha (a) \sin(\theta)$$




$$\begin{pmatrix} N_C \\ \alpha \end{pmatrix} = \text{Find}(N_C, \alpha) \qquad N_C = 195.043\,\text{lb} \qquad \alpha = 9.669\,\frac{\text{rad}}{\text{s}^2}$$

---

***Problem 17-52**

The two rods *EF* and *HI* each of weight *W* are fixed (welded) to the link *AC* at *E*. Determine the normal force $N_E$, shear force $V_E$, and moment $M_E$, which the bar *AC* exerts on *FE* at *E* if at the instant $\theta$ link *AB* has an angular velocity $\omega$ and an angular acceleration $\alpha$ as shown.

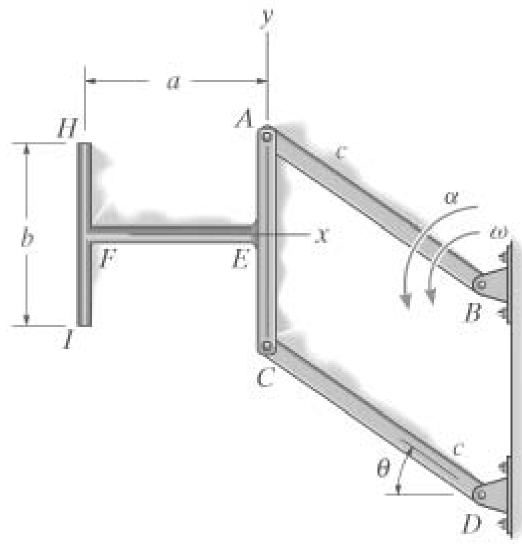
Given:

$W = 3\ \text{lb} \quad a = 2\ \text{ft} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$\theta = 30\ \text{deg} \quad b = 2\ \text{ft}$

$\omega = 5\ \frac{\text{rad}}{\text{s}} \quad c = 3\ \text{ft}$

$\alpha = 8\ \frac{\text{rad}}{\text{s}^2} \quad d = 3\ \text{ft}$



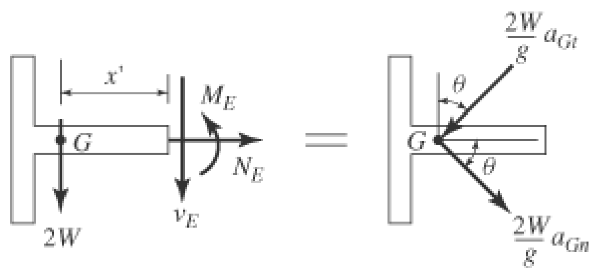

Solution:

$$x' = \frac{W a + W\left(\frac{a}{2}\right)}{2W}$$

$$a_{Gn} = c\omega^2$$

$$a_{Gt} = c\alpha$$

Guesses




$V_E = 1\ \text{lb}$

$N_E = 1\ \text{lb}$

$M_E = 1\ \text{lb}\cdot\text{ft}$

Given $\quad -2W - V_E = -2\left(\frac{W}{g}\right)a_{Gt}\cos(\theta) - 2\left(\frac{W}{g}\right)a_{Gn}\sin(\theta)$

$$N_E = 2\left(\frac{W}{g}\right)a_{Gn}\cos(\theta) - 2\left(\frac{W}{g}\right)a_{Gt}\sin(\theta) \qquad M_E - V_E x' = 0$$

$$\begin{pmatrix} V_E \\ N_E \\ M_E \end{pmatrix} = \text{Find}(V_E, N_E, M_E) \qquad \begin{pmatrix} N_E \\ V_E \end{pmatrix} = \begin{pmatrix} 9.87 \\ 4.86 \end{pmatrix}\text{lb} \qquad M_E = 7.29\ \text{lb}\cdot\text{ft}$$

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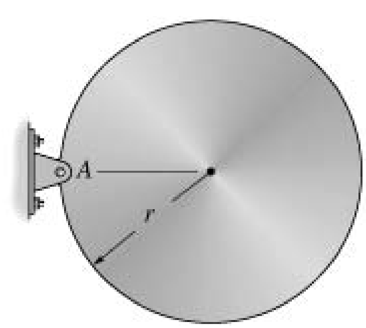
**Problem 17-53**

The disk of mass $M$ is supported by a pin at $A$. If it is released from rest from the position shown, determine the initial horizontal and vertical components of reaction at the pin.

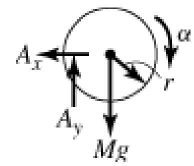



Given:

$M = 80\ \text{kg} \quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$r = 1.5\ \text{m}$

Solution: Guesses $A_x = 1\ \text{N}$ $A_y = 1\ \text{N}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$




Given $M g r = \dfrac{3}{2} M r^2 \alpha$ $-A_x = 0$ $A_y - M g = -M r \alpha$

$$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha) \qquad \alpha = 4.36\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 262 \end{pmatrix} \text{N}$$

---

**Problem 17-54**

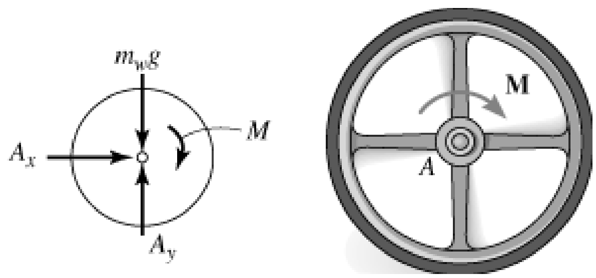
The wheel of mass $m_w$ has a radius of gyration $k_A$. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time $t$ starting from rest. Also, compute the reactions which the fixed pin $A$ exerts on the wheel during the motion.

Given:

$m_w = 10\ \text{kg}$ $t = 3\ \text{s}$

$k_A = 200\ \text{mm}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$b = 5\ \text{N}\ \dfrac{\text{m}}{\text{s}}$




Solution:

$b t = m_w k_A^2 \alpha$ $\alpha = \dfrac{b t}{m_w k_A^2}$

$\omega = \dfrac{b t^2}{2 m_w k_A^2}$ $\omega = 56.2\ \dfrac{\text{rad}}{\text{s}}$

$A_x = 0\ \text{N}$ $A_y - m_w g = 0$

$A_y = m_w g$ $\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 98.1 \end{pmatrix} \text{N}$

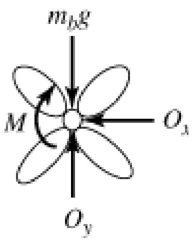
---

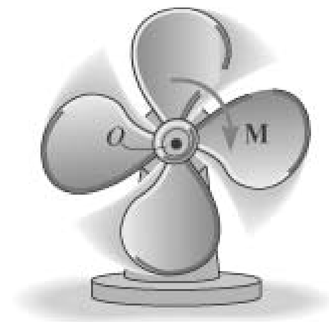
**Problem 17-55**

The fan blade has mass $m_b$ and a moment of inertia $I_0$ about an axis passing through its center $O$. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.

Given:

$m_b = 2\ \text{kg}$




$I_O = 0.18\ \text{kg}\cdot\text{m}^2$

$A = 3\ \text{N}\cdot\text{m}$

$b = -0.2\ \text{s}^{-1}$

$t_1 = 4\ \text{s}$

Solution:

$$A\left(1 - e^{b\,t}\right) = I_O\alpha \qquad \alpha = \frac{A}{I_O}\left(1 - e^{b\,t_1}\right)$$

$$\omega = \frac{A}{I_O}\left(t_1 + \frac{1}{b} - \frac{1}{b}e^{b\,t_1}\right) \qquad \boxed{\omega = 20.8\ \frac{\text{rad}}{\text{s}}}$$

***Problem 17-56**

The rod of weight $W$ is pin-connected to its support at $A$ and has an angular velocity $\omega$ when it is in the horizontal position shown. Determine its angular acceleration and the horizontal and vertical components of reaction which the pin exerts on the rod at this instant.

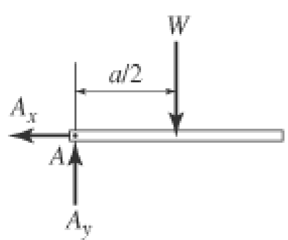
Given:

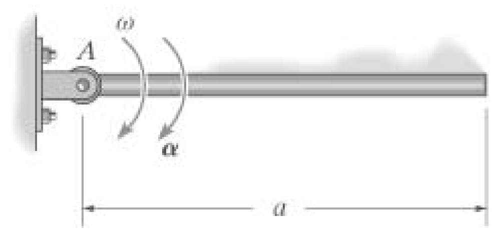
$\omega = 4\ \frac{\text{rad}}{\text{s}}$

$W = 10\ \text{lb}$

$a = 6\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution:

$$A_x = \left(\frac{W}{g}\right)\omega^2\left(\frac{a}{2}\right) \qquad \boxed{A_x = 14.9\,\text{lb}}$$

$$W\frac{a}{2} = \frac{1}{3}\left(\frac{W}{g}\right)a^2\alpha \qquad \alpha = \frac{3}{2a}g \qquad \boxed{\alpha = 8.05\,\frac{\text{rad}}{\text{s}^2}}$$

$$W - A_y = \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right)$$

$$A_y = W - \left(\frac{W}{g}\right)\alpha\left(\frac{a}{2}\right) \qquad \boxed{A_y = 2.50\,\text{lb}}$$

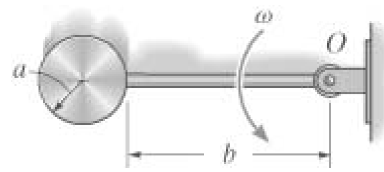
**Problem 17-57**

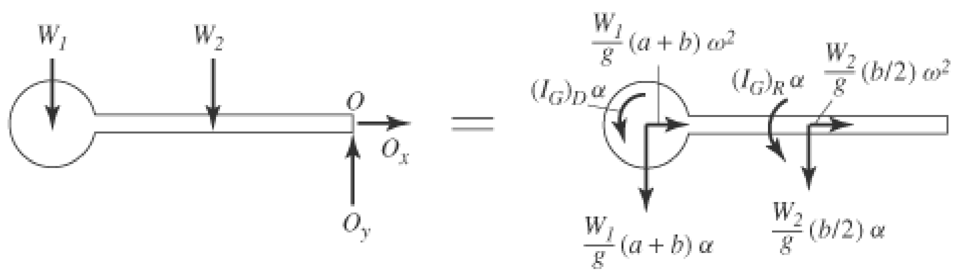
The pendulum consists of a disk of weight $W_1$ and a slender rod of weight $W_2$. Determine the horizontal and vertical components of reaction that the pin $O$ exerts on the rod just as it passes the horizontal position, at which time its angular velocity is $\omega$.

Given:

$W_1 = 15 \text{ lb} \quad a = 0.75 \text{ ft} \quad \omega = 8 \frac{\text{rad}}{\text{s}}$

$W_2 = 10 \text{ lb} \quad b = 3 \text{ ft}$




Solution: $I_O = \frac{1}{2}\left(\frac{W_1}{g}\right)a^2 + \left(\frac{W_1}{g}\right)(a+b)^2 + \frac{1}{3}\left(\frac{W_2}{g}\right)b^2$

Guesses $\quad \alpha = 10 \frac{\text{rad}}{\text{s}^2} \quad O_x = 100 \text{ lb} \quad O_y = 5 \text{ lb}$

Given

$$O_y - W_1 - W_2 = -\frac{W_1}{g}(a+b)\alpha - \left(\frac{W_2}{g}\right)\frac{b}{2}\alpha$$

$$O_x = \left(\frac{W_1}{g}\right)(a+b)\,\omega^2 + \left(\frac{W_2}{g}\right)\left(\frac{b}{2}\right)\omega^2$$

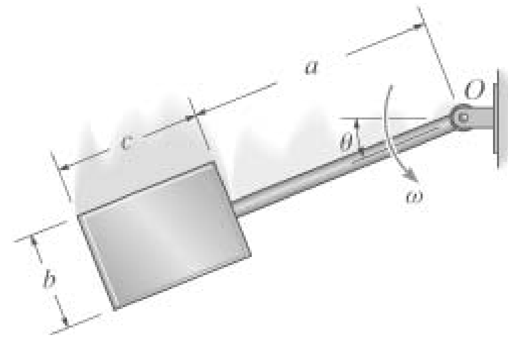
$$W_1(a+b) + W_2\left(\frac{b}{2}\right) = I_O\alpha$$

$\begin{pmatrix} \alpha \\ O_x \\ O_y \end{pmatrix} = \text{Find}(\alpha, O_x, O_y) \qquad \alpha = 9.36 \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 141.61 \\ 4.29 \end{pmatrix} \text{lb}$

**Problem 17-58**

The pendulum consists of a uniform plate of mass $M_1$ and a slender rod of mass $M_2$. Determine the horizontal and vertical components of reaction that the pin $O$ exerts on the rod at the instant shown at which time its angular velocity is $\omega$.

Given:

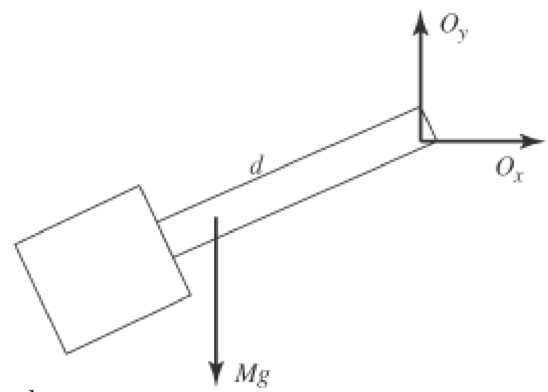



$M_1 = 5\ \text{kg}$ $\quad a = 0.5\ \text{m}$

$M_2 = 2\ \text{kg}$ $\quad b = 0.2\ \text{m}$

$\omega = 3\ \dfrac{\text{rad}}{\text{s}}$ $\quad c = 0.3\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$ $\quad \theta = 30\ \text{deg}$




Solution: $\quad M = M_1 + M_2$

$$I_O = \frac{1}{12}M_1\left(b^2 + c^2\right) + M_1\left(a + \frac{c}{2}\right)^2 + \frac{1}{3}M_2 a^2$$

$$d = \frac{M_1\left(a + \frac{c}{2}\right) + M_2\left(\frac{a}{2}\right)}{M}$$

Guesses $\quad O_x = 1\ \text{N} \quad O_y = 1\ \text{N} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $\quad O_x = M d\alpha \sin(\theta) + M d\omega^2 \cos(\theta) \qquad M g d \cos(\theta) = I_O\alpha$

$$O_y - M g = -M d\alpha \cos(\theta) + M d\omega^2 \sin(\theta)$$

$$\begin{pmatrix} O_x \\ O_y \\ \alpha \end{pmatrix} = \text{Find}\left(O_x, O_y, \alpha\right) \qquad \alpha = 13.65\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 54.8 \\ 41.2 \end{pmatrix}\ \text{N}$$

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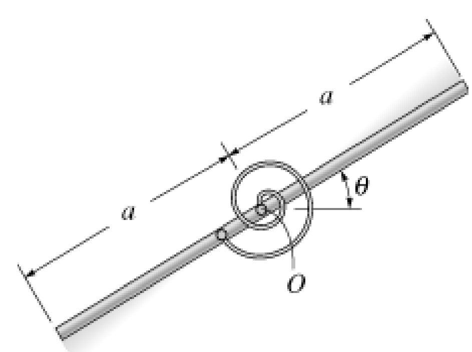
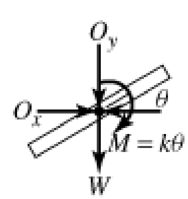
**Problem 17-59**




The bar of weight $W$ is pinned at its center $O$ and connected to a torsional spring. The spring has a stiffness $k$, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$.

Given:




$W = 10\ \text{lb}$

$k = 5\ \dfrac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$-k\theta = \dfrac{1}{12}\left(\dfrac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \dfrac{-3kg}{Wa^2}\theta \qquad \dfrac{\omega^2}{2} - \dfrac{\omega_0{}^2}{2} = \dfrac{-3kg}{Wa^2}\left(\dfrac{\theta^2}{2} - \dfrac{\theta_0{}^2}{2}\right)$

$\omega = \sqrt{\dfrac{3kg}{Wa^2}\left(\dfrac{\pi}{2}\right)^2} \qquad \omega = 10.917 \dfrac{\text{rad}}{\text{s}}$

---

***Problem 17-60**

The bar of weight $w$ is pinned at its center $O$ and connected to a torsional spring. The spring has a stiffness $k$, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = \theta_1$.

Given:

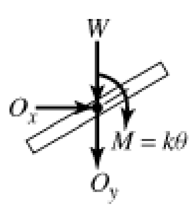
$W = 10 \text{ lb}$

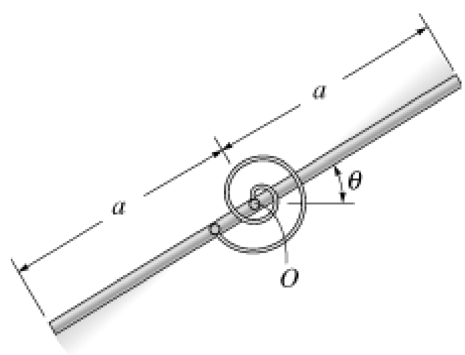
$k = 5 \dfrac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

$\theta_1 = 45 \text{ deg}$




Solution:

$-k\theta = \dfrac{1}{12}\left(\dfrac{W}{g}\right)(2a)^2\alpha \qquad \alpha = \dfrac{-3kg}{Wa^2}\theta$

$\dfrac{\omega^2}{2} - \dfrac{\omega_0{}^2}{2} = \dfrac{-3kg}{Wa^2}\left(\dfrac{\theta^2}{2} - \dfrac{\theta_0{}^2}{2}\right)$

$\omega = \sqrt{\dfrac{3kg}{Wa^2}\left[(90 \text{ deg})^2 - \theta_1{}^2\right]} \qquad \omega = 9.454 \dfrac{\text{rad}}{\text{s}}$

---

**Problem 17-61**

The roll of paper of mass $M$ has radius of gyration $k_A$ about an axis passing through point $A$. It is pin-supported at both ends by two brackets $AB$. If the roll rests against a wall for which the coefficient of kinetic friction is $\mu_k$ and a vertical force $F$ is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls.

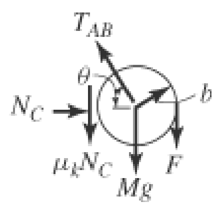
Given:

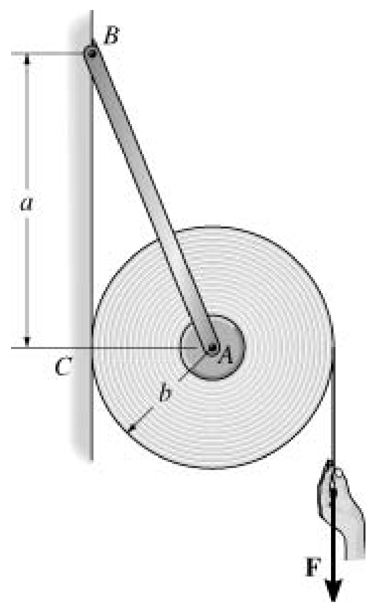
$M = 20\ \text{kg}$ $\quad a = 300\ \text{mm}$

$k_A = 90\ \text{mm}$ $\quad b = 125\ \text{mm}$

$\mu_k = 0.2$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$F = 30\ \text{N}$




Solution: $\quad \theta = \text{atan}\left(\dfrac{a}{b}\right)$

Guesses $\quad N_C = 1\ \text{N} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2} \quad T_{AB} = 1\ \text{N}$

Given $\quad N_C - T_{AB}\cos(\theta) = 0$

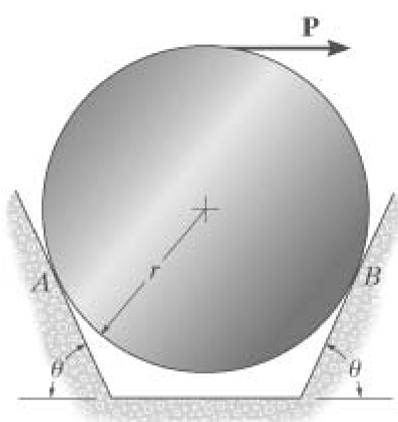
$$T_{AB}\sin(\theta) - \mu_k N_C - M g - F = 0$$

$$F b - \mu_k N_C b = M k_A^{\,2} \alpha$$

$$\begin{pmatrix} N_C \\ \alpha \\ T_{AB} \end{pmatrix} = \text{Find}(N_C, \alpha, T_{AB}) \qquad \begin{pmatrix} N_C \\ T_{AB} \end{pmatrix} = \begin{pmatrix} 102.818 \\ 267.327 \end{pmatrix} \text{N} \qquad \alpha = 7.281\ \frac{\text{rad}}{\text{s}^2}$$

---

**Problem 17-62**



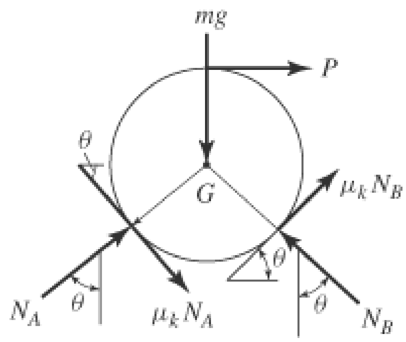

The cylinder has a radius $r$ and mass $m$ and rests in the trough for which the coefficient of kinetic friction at $A$ and $B$ is $\mu_k$. If a horizontal force **P** is applied to the cylinder, determine the cylinder's angular acceleration when it begins to spin.

Solution:

$$P - (N_B - N_A)\sin(\theta) + \mu_k(N_A + N_B)\cos(\theta) = 0$$

$$(N_A + N_B)\cos(\theta) + \mu_k(N_B - N_A)\sin(\theta) - m g = 0$$

$$\left[\mu_k(N_A + N_B) - P\right]r = \frac{-1}{2}m r^2 \alpha$$




Solving

$$N_A + N_B = \frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k^{\,2}\right)}$$

$$N_B - N_A = \frac{\mu_k m g + P}{\sin(\theta)\left(1 + \mu_k^{\,2}\right)}$$

$$\alpha = \frac{-2\mu_k}{m r}\left[\frac{m g - \mu_k P}{\cos(\theta)\left(1 + \mu_k^{\,2}\right)}\right] + \frac{2P}{m r}$$

**Problem 17-63**

The uniform slender rod has a mass $M$. If the cord at $A$ is cut, determine the reaction at the pin $O$, (a) when the rod is still in the horizontal position, and (b) when the rod swings to the vertical position.

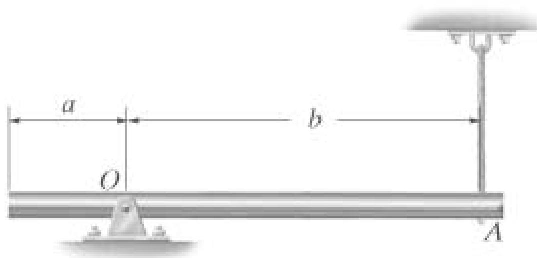
Given:

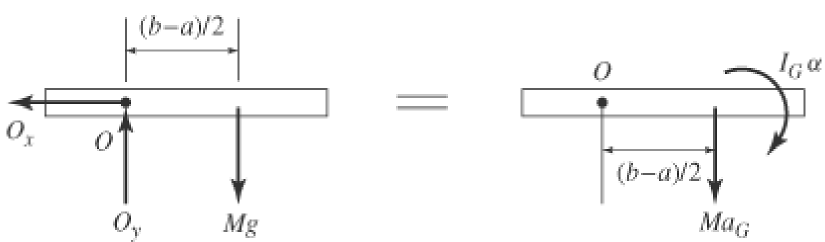
$M = 5$ kg

$a = 200$ mm

$b = 600$ mm

$g = 9.81 \frac{\text{m}}{\text{s}^2}$




Solution: $L = a + b$ $\quad d = \frac{b - a}{2}$ $\quad I_O = \frac{1}{12}M L^2 + M d^2$

(a) In the horizontal position

Guesses $a_G = 1 \frac{\text{m}}{\text{s}^2}$ $\quad \alpha = 1 \frac{\text{rad}}{\text{s}^2}$ $\quad O_x = 1$ N $\quad O_y = 1$ N

Given $a_G = \alpha d$ $\quad M g d = I_O\alpha$ $\quad -O_x = 0$ $\quad O_y - M g = -M a_G$

$$\begin{pmatrix} O_x \\ O_y \\ a_G \\ \alpha \end{pmatrix} = \text{Find}\left(O_x, O_y, a_G, \alpha\right) \qquad a_G = 4.20\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 21.0\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 28.0 \end{pmatrix}\ \text{N}$$

$$\left|\begin{pmatrix} O_x \\ O_y \end{pmatrix}\right| = 28.0\ \text{N}$$

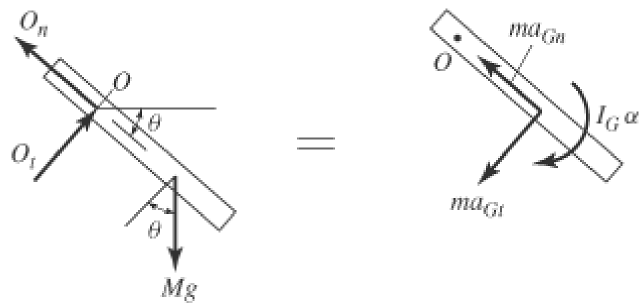
Next examine a general position

$$M g d \cos(\theta) = I_O \alpha$$

$$\alpha = \frac{M g d}{I_O}\cos(\theta)$$

$$\frac{\omega^2}{2} = \frac{M g d}{I_O}\sin(\theta)$$

$$\omega = \sqrt{\frac{2 M g d}{I_O}\sin(\theta)}$$




(b) In the vertical position ($\theta$ = 90 deg) $\qquad \omega = \sqrt{\frac{2 M g d}{I_O}\sin(90\ \text{deg})}$

Guesses $\qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2} \qquad O_x = 1\ \text{N} \qquad O_y = 1\ \text{N}$

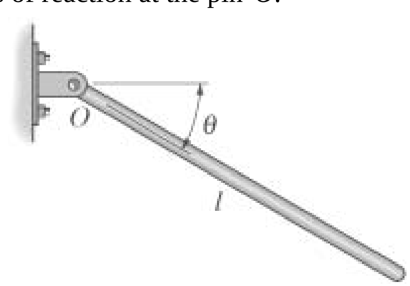
Given $\qquad 0 = I_O \alpha \qquad -O_x = -M \alpha d \qquad O_y - M g = M d \omega^2$

$$\begin{pmatrix} \alpha \\ O_x \\ O_y \end{pmatrix} = \text{Find}\left(\alpha, O_x, O_y\right) \qquad \alpha = 0.0\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} O_x \\ O_y \end{pmatrix} = \begin{pmatrix} 0.0 \\ 91.1 \end{pmatrix}\ \text{N} \qquad \left|\begin{pmatrix} O_x \\ O_y \end{pmatrix}\right| = 91.1\ \text{N}$$

---

***Problem 17-64**

The bar has a mass *m* and length *l*. If it is released from rest from the position shown, determine its angular acceleration and the horizontal and vertical components of reaction at the pin *O*.




Given:

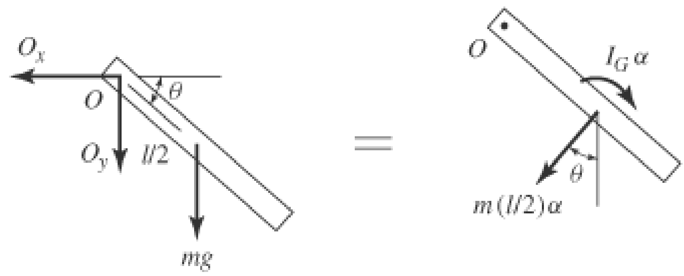
$\theta = 30\ \text{deg}$

Solution:

$$m g \frac{l}{2}\cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$O_x = m\frac{l}{2}\alpha\sin(\theta)$$

$$O_y - mg = -m\frac{l}{2}\alpha\cos(\theta)$$




Solving

$$\alpha = \frac{3g}{2l}\cos(\theta) \qquad O_x = \frac{3mg}{8}\sin(2\theta) \qquad O_y = mg\left(1 - \frac{3}{4}\cos(\theta)^2\right)$$

$$k_1 = \frac{3}{2}\cos(\theta) \qquad k_2 = \frac{3}{8}\sin(2\theta) \qquad k_3 = 1 - \frac{3}{4}\cos(\theta)^2$$

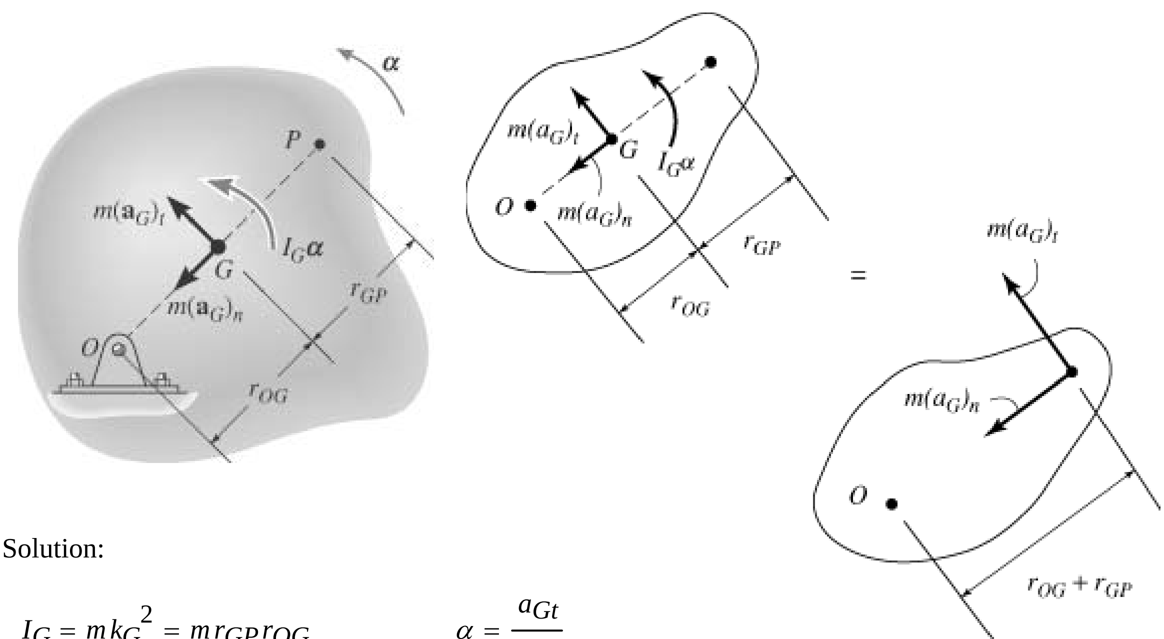
$$\alpha = k_1\frac{g}{l} \qquad O_x = k_2 mg \qquad O_y = k_3 mg$$

$$k_1 = 1.30 \qquad k_2 = 0.325 \qquad k_3 = 0.437$$

---

**Problem 17-65**

The kinetic diagram representing the general rotational motion of a rigid body about a fixed axis at $O$ is shown in the figure. Show that $I_G\alpha$ may be eliminated by moving the vectors $m(a_{Gt})$ and $m(a_{Gn})$ to point $P$, located a distance $r_{GP} = k_G^2/r_{OG}$ from the center of mass $G$ of the body. Here $k_G$ represents the radius of gyration of the body about $G$. The point $P$ is called the *center of percussion* of the body.




Solution:

$$I_G = mk_G^2 = mr_{GP}r_{OG} \qquad \alpha = \frac{a_{Gt}}{r_{OG}}$$

$$m a_{Gt} r_{OG} + I_G \alpha = m a_{Gt} r_{OG} + \left(m r_{OG} r_{GP}\right)\left(\frac{a_{Gt}}{r_{OG}}\right)$$

$$m a_{Gt} r_{OG} + I_G \alpha = m a_{Gt}\left(r_{OG} + r_{GP}\right)$$ Q.E.D.

---

**Problem 17-66**

Determine the position of the center of percussion *P* of the slender bar of weight *W*. (See Prob. 17-65.) What is the horizontal force at the pin when the bar is struck at *P* with force *F?*

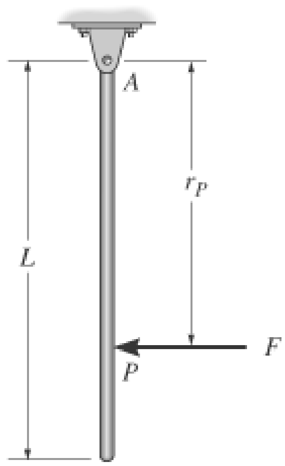
Given:

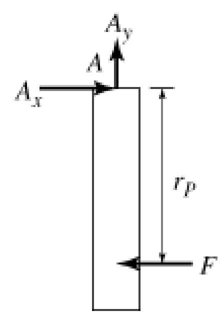
$W = 10 \text{ lb}$

$F = 20 \text{ lb}$

$L = 4 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$




Solution: $k_G = \dfrac{L}{\sqrt{12}}$

From Prob 17-65

$$r_P = \frac{L}{2} + \frac{k_G^2}{\frac{L}{2}} \qquad r_P = 2.667 \text{ ft}$$

Guesses $A_x = 1 \text{ lb}$ $\alpha = 1 \dfrac{\text{rad}}{\text{s}^2}$

Given $A_x - F = \left(\dfrac{-W}{g}\right)\alpha\left(\dfrac{L}{2}\right)$ $-F r_P = \dfrac{-1}{3}\left(\dfrac{W}{g}\right)L^2\alpha$

$$\begin{pmatrix} A_x \\ \alpha \end{pmatrix} = \text{Find}(A_x, \alpha) \qquad \alpha = 32.2 \frac{\text{rad}}{\text{s}^2} \qquad A_x = 2.35 \times 10^{-14} \text{ lb}$$

A zero horizontal force is the condition used to define the center of percussion.

---

**Problem 17-67**

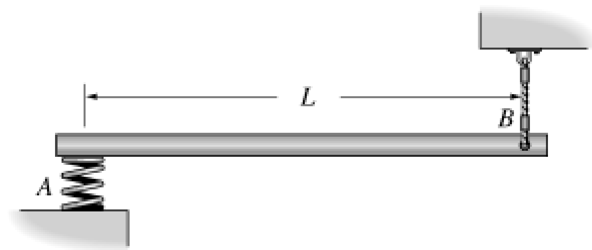
The slender rod of mass *M* is supported horizontally by a spring at *A* and a cord at *B*. Determine the angular acceleration of the rod and the acceleration of the rod's mass center at the instant the cord at *B* is cut. *Hint*: The stiffness of the spring is not needed for the calculation.

Given:

$M = 4 \text{ kg}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

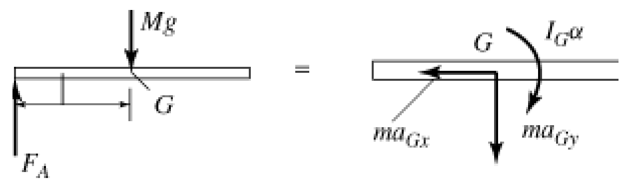
$L = 2\ \text{m}$




Solution:

Since the deflection of the spring is unchanged, we have

$F_A = \frac{Mg}{2}$




$F_A \frac{L}{2} = \frac{1}{12} M L^2 \alpha$

$\alpha = \frac{6F_A}{ML}$ $\qquad \alpha = 14.7\ \frac{\text{rad}}{\text{s}^2}$

$F_A - Mg = -M a_{Gy}$ $\qquad a_{Gy} = g - \frac{F_A}{M}$ $\qquad a_{Gy} = 4.91\ \frac{\text{m}}{\text{s}^2}$ $\qquad a_{Gx} = 0\ \frac{\text{m}}{\text{s}^2}$

---

***Problem 17-68**

In order to experimentally determine the moment of inertia $I_G$ of a connecting rod of mass $M$, the rod is suspended horizontally at $A$ by a cord and at $B$ by a bearing and piezoelectric sensor, an instrument used for measuring force. Under these equilibrium conditions, the force at $B$ is measured as $F_1$. If, at the instant the cord is released, the reaction at $B$ is measured as $F_2$, determine the value of $I_G$. The support at $B$ does not move when the measurement is taken. For the calculation, the horizontal location of $G$ must be determined.

Given:

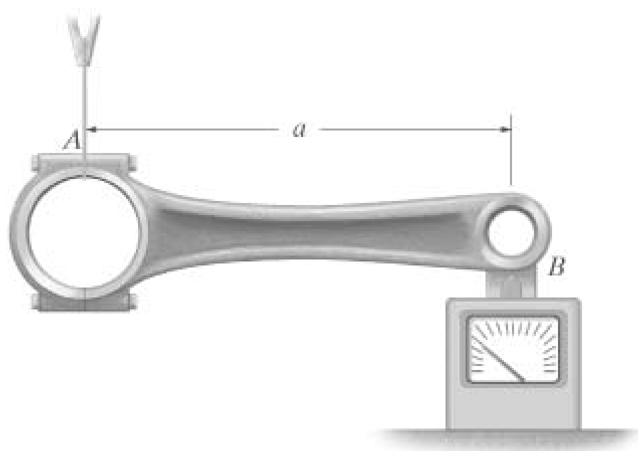
$M = 4\ \text{kg}$

$F_1 = 14.6\ \text{N}$

$F_2 = 9.3\ \text{N}$

$a = 350\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$




Solution:

Guesses

$x = 1\ \text{mm}$ $\qquad I_G = 1\ \text{kg}\cdot\text{m}^2$ $\qquad A_y = 1\ \text{N}$ $\qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

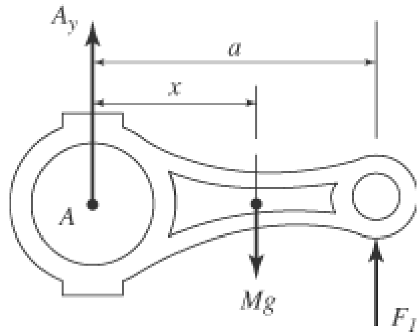
Given

$$A_y - Mg + F_1 = 0$$

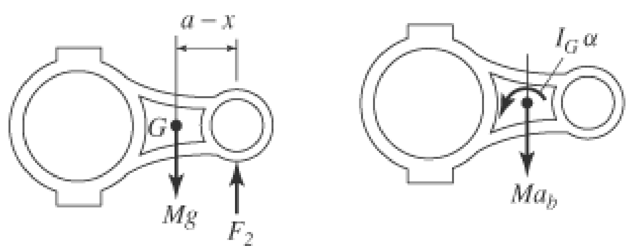
$$-Mgx + F_1 a = 0$$

$$F_2 - Mg = -M\alpha(a - x)$$

$$Mg(a - x) = \left[I_G + M(a - x)^2\right]\alpha$$




$$\begin{pmatrix} x \\ I_G \\ A_y \\ \alpha \end{pmatrix} = \mathrm{Find}(x, I_G, A_y, \alpha)$$




$x = 130\,\mathrm{mm}$ $\quad A_y = 24.6\ \mathrm{N}$ $\quad \alpha = 34.1\ \frac{\mathrm{rad}}{\mathrm{s}^2}$ $\quad I_G = 0.0600\ \mathrm{kg \cdot m^2}$

**Problem 17-69**

Disk $D$ of weight $W$ is subjected to counterclockwise moment $M = bt$. Determine the angular velocity of the disk at time $t$ after the moment is applied. Due to the spring the plate $P$ exerts constant force $P$ on the disk. The coefficients of static and kinetic friction between the disk and the plate are $\mu_s$ and $\mu_k$ respectively. *Hint:* First find the time needed to start the disk rotating.

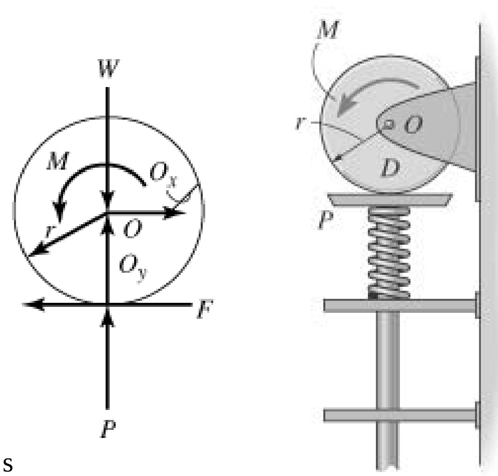
Given:

$W = 10\ \mathrm{lb}$ $\quad \mu_s = 0.3$

$b = 10\ \frac{\mathrm{lb \cdot ft}}{\mathrm{s}}$ $\quad \mu_k = 0.2$

$t = 2\ \mathrm{s}$ $\quad r = 0.5\ \mathrm{ft}$

$P = 100\ \mathrm{lb}$ $\quad g = 32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}$




Solution: When motion begins

$b t_1 = \mu_s P r \qquad t_1 = \frac{\mu_s P r}{b} \qquad t_1 = 1.5\ \mathrm{s}$

At a later time we have

$$b t - \mu_k P r = \frac{1}{2}\left(\frac{W}{g}\right) r^2 \alpha \qquad \alpha = \frac{2g}{W r^2}\left(b t - \mu_k P r\right)$$

$$\omega = \frac{2g}{W r^2}\left[\frac{b}{2}\left(t^2 - t_1^{\,2}\right) - \mu_k P\, r\left(t - t_1\right)\right]$$ $$\omega = 96.6\,\frac{\text{rad}}{\text{s}}$$

**Problem 17-70**

The furnace cover has a mass $M$ and a radius of gyration $k_G$ about its mass center $G$. If an operator applies a force $F$ to the handle in order to open the cover, determine the cover's initial angular acceleration and the horizontal and vertical components of reaction which the pin at $A$ exerts on the cover at the instant the cover begins to open. Neglect the mass of the handle $BAC$ in the calculation.

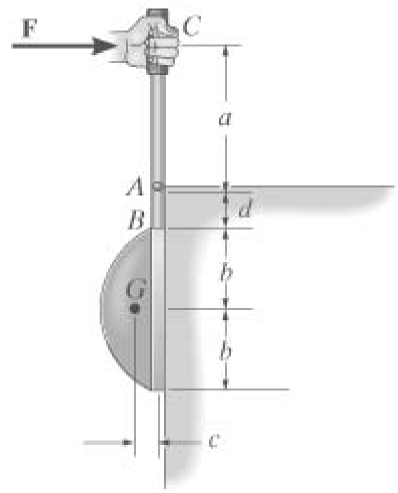
Given:

$M = 20\ \text{kg}$ $\quad a = 0.7\ \text{m}$

$k_G = 0.25\ \text{m}$ $\quad b = 0.4\ \text{m}$

$F = 120\ \text{N}$ $\quad c = 0.25\ \text{m}$

$d = 0.2\ \text{m}$




Solution: $\quad \theta = \text{atan}\left(\frac{c}{b+d}\right)$

Guesses $\quad \alpha = 5\,\frac{\text{rad}}{\text{s}^2}$

$A_x = 50\ \text{N}$

$A_y = 20\ \text{N}$

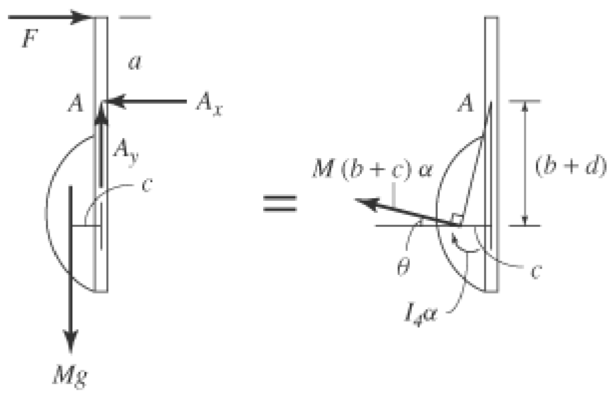
Given

$$A_x - F = M(c+b)\,\alpha\cos(\theta)$$

$$A_y - M g = M(c+b)\,\alpha\sin(\theta)$$

$$F a - M g c = M c^2 \alpha + M(c+b)^2 \alpha$$

$$\begin{pmatrix}\alpha \\ A_x \\ A_y\end{pmatrix} = \text{Find}\left(\alpha, A_x, A_y\right)$$




$$\alpha = 3.60\,\frac{\text{rad}}{\text{s}^2}$$ $$\begin{pmatrix}A_x \\ A_y\end{pmatrix} = \begin{pmatrix}163 \\ 214\end{pmatrix}\ \text{N}$$

**Problem 17-71**

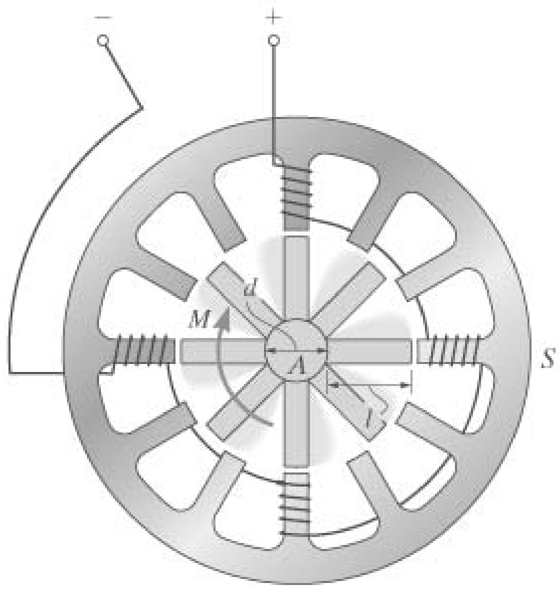
The variable-resistance motor is often used for appliances, pumps, and blowers. By applying a current through the stator $S$, an electromagnetic field is created that "pulls in" the nearest rotor poles. The result of this is to create a torque $M$ about the bearing at $A$. If the rotor is made from iron and has a cylindrical core of mass $M_1$, diameter $d$ and eight extended slender rods, each having a mass $M_2$ and length $l$, determine its angular velocity at time $t$ starting from rest.

Given:

$M_1 = 3\ \text{kg}$ $\quad l = 100\text{mm}$

$M_2 = 1\ \text{kg}$ $\quad d = 50\ \text{mm}$

$M = 4\ \text{N}\cdot\text{m}$ $\quad t = 5\ \text{s}$




Solution:

$$I_A = \frac{1}{2}M_1\left(\frac{d}{2}\right)^2 + 8\left[\frac{1}{12}M_2 l^2 + M_2\left(\frac{d}{2}+\frac{l}{2}\right)^2\right]$$

$$M = I_A\alpha \qquad \alpha = \frac{M}{I_A}$$

$$\omega = \alpha t \qquad \omega = 380\ \frac{\text{rad}}{\text{s}}$$

---

***Problem 17-72**

Determine the angular acceleration of the diving board of mass $M$ and the horizontal and vertical components of reaction at the pin $A$ the instant the man jumps off. Assume that the board is uniform and rigid, and that at the instant he jumps off the spring is compressed a maximum amount $\delta$ and the board is horizontal.

Units Used:

$\text{kN} = 10^3\ \text{N}$

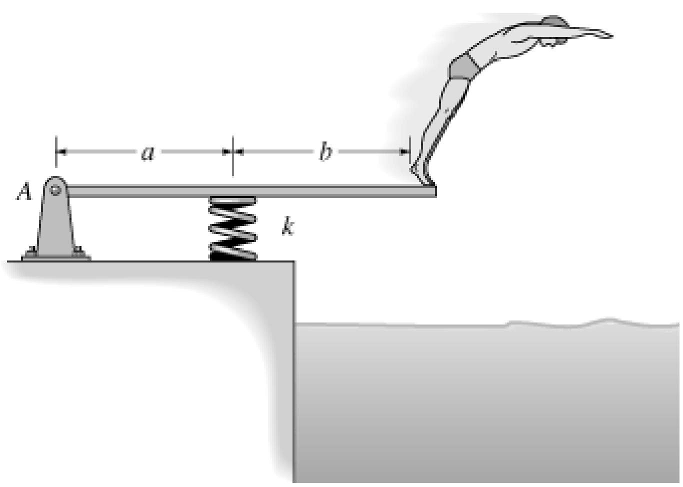
Given:

$M = 25\ \text{kg}$

$\delta = 200\ \text{mm}$

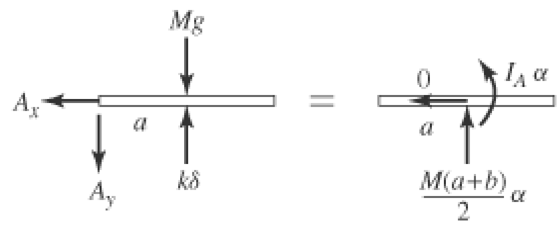
$k = 7\ \frac{\text{kN}}{\text{m}}$

$a = 1.5\ \text{m}$



$b = 1.5\ \text{m}$

$g = 9.815\ \dfrac{\text{m}}{\text{s}^2}$




Solution:

Guesses $\quad A_x = 1\ \text{N} \quad A_y = 1\ \text{N} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $\quad -M g a + k\delta a = \dfrac{1}{3} M (a+b)^2 \alpha$

$$A_x = 0 \qquad -A_y - M g + k\delta = M\alpha\left(\frac{a+b}{2}\right)$$

$$\begin{pmatrix} A_x \\ A_y \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \alpha) \qquad \alpha = 23.1\ \frac{\text{rad}}{\text{s}^2} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0 \\ 289 \end{pmatrix}\ \text{N}$$

---

**Problem 17-73**

The disk has mass $M$ and is originally spinning at the end of the strut with angular velocity $\omega$. If it is then placed against the wall, for which the coefficient of kinetic friction is $\mu_k$, determine the time required for the motion to stop. What is the force in strut $BC$ during this time?

Given:

$M = 20\ \text{kg}$

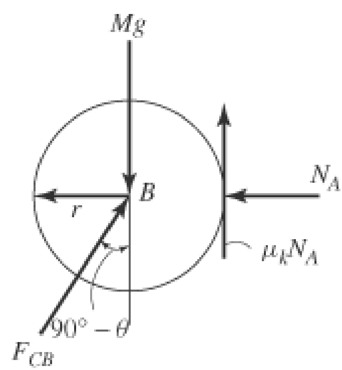
$\omega = 60\ \dfrac{\text{rad}}{\text{s}}$

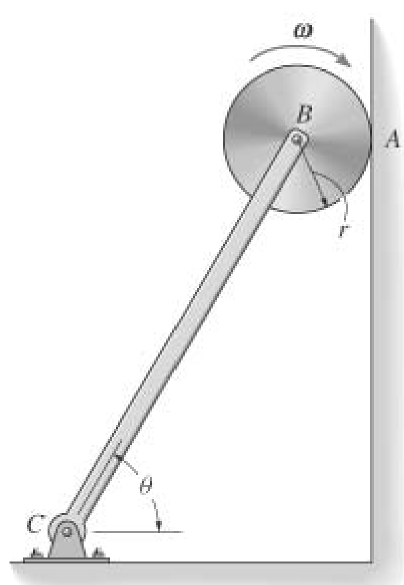
$\mu_k = 0.3$

$\theta = 60\ \text{deg}$

$r = 150\ \text{mm}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$




Solution:

Initial Guess:

$F_{CB} = 1\ \text{N} \qquad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2} \qquad N_A = 1\ \text{N}$

Given $F_{CB}\cos(\theta) - N_A = 0$

$F_{CB}\sin(\theta) - M g + \mu_k N_A = 0$

$\mu_k N_A r = \frac{1}{2} M r^2 \alpha$

$\begin{pmatrix} F_{CB} \\ N_A \\ \alpha \end{pmatrix} = \text{Find}(F_{CB}, N_A, \alpha)$ $\alpha = 19.311 \frac{\text{rad}}{\text{s}^2}$ $N_A = 96.6 \text{ N}$ $F_{CB} = 193 \text{ N}$

$t = \frac{\omega}{\alpha}$ $t = 3.107 \text{ s}$

**Problem 17-74**

The relay switch consists of an electromagnet *E* and an armature *AB* (slender bar) of mass *M* which is pinned at *A* and lies in the vertical plane.When the current is turned off, the armature is held open against the smooth stop at *B* by the spring *CD*, which exerts an upward vertical force $F_s$ on the armature at *C*. When the current is turned on, the electromagnet attracts the armature at *E* with a vertical force *F*. Determine the initial angular acceleration of the armature when the contact *BF* begins to close.

Given:

$M = 20 \text{ gm}$

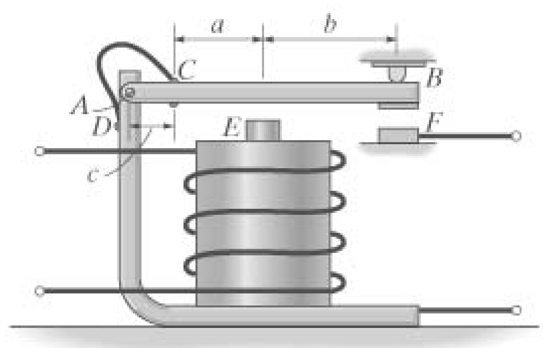
$F = 0.8 \text{ N}$

$F_s = 0.85 \text{ N}$

$a = 20 \text{ mm}$

$b = 30 \text{ mm}$

$c = 10 \text{ mm}$

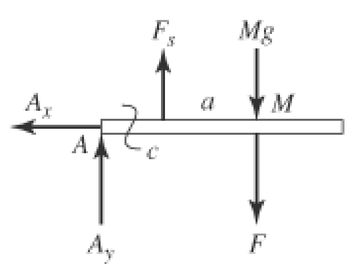



Solution:

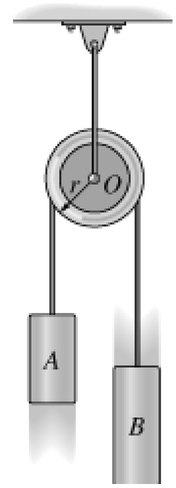
$$F_s c - M g\left(\frac{a+b+c}{2}\right) - F(a+c) = -\frac{1}{3} M (a+b+c)^2 \alpha$$

$$\alpha = 3\left[\frac{M g\left(\frac{a+b+c}{2}\right) + F(a+c) - F_s c}{M (a+b+c)^2}\right]$$

$$\alpha = 891 \frac{\text{rad}}{\text{s}^2}$$



**Problem 17-75**




The two blocks $A$ and $B$ have a mass $m_A$ and $m_B$, respectively, where $m_B > m_A$. If the pulley can be treated as a disk of mass $M$, determine the acceleration of block $A$. Neglect the mass of the cord and any slipping on the pulley.

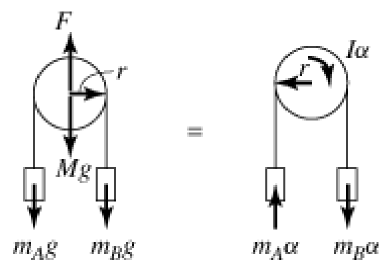
Solution:

$$a = \alpha r$$

$$m_B g r - m_A g r = \frac{1}{2} M r^2 \alpha + m_B r^2 \alpha + m_A r^2 \alpha$$

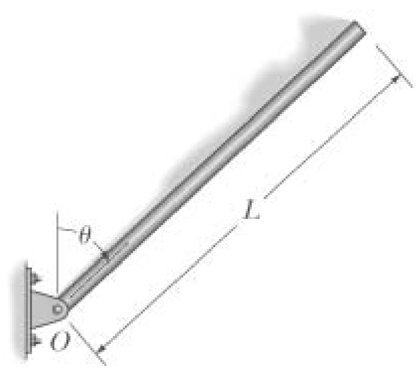
$$\alpha = \frac{g(m_B - m_A)}{r\left(\frac{1}{2} M + m_B + m_A\right)}$$

$$a = \frac{g(m_B - m_A)}{\frac{1}{2} M + m_B + m_A}$$




***Problem 17-76**

The rod has a length $L$ and mass $m$. If it is released from rest when $\theta = 0°$, determine its angular velocity as a function of $\theta$. Also, express the horizontal and vertical components of reaction at the pin $O$ as a function of $\theta$.




Solution:

$$m g \frac{L}{2} \sin(\theta) = \frac{1}{3} m L^2 \alpha$$

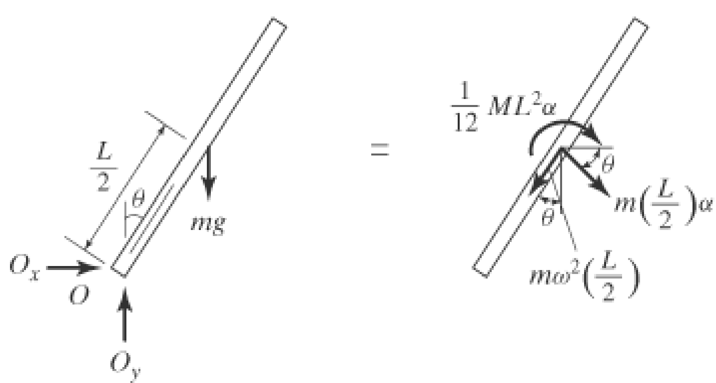
$$\alpha = \frac{3g}{2L} \sin(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2L}(1 - \cos(\theta))$$

$$\omega = \sqrt{\frac{3g}{L}(1 - \cos(\theta))}$$

$$O_x = m \frac{L}{2} \alpha \cos(\theta) - m \frac{L}{2} \omega^2 \sin(\theta)$$

$$O_x = m g \sin(\theta)\left(\frac{9}{4} \cos(\theta) - \frac{3}{2}\right)$$

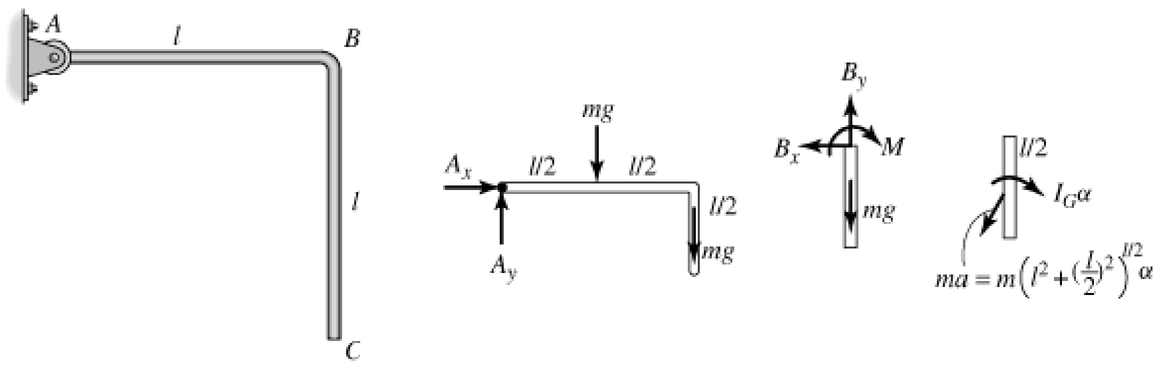


$$O_y - mg = -m\left(\frac{L}{2}\right)\alpha \sin(\theta) - m\left(\frac{L}{2}\right)\omega^2 \cos(\theta)$$

$$O_y = mg\left(1 - \frac{3}{2}\cos(\theta) + \frac{3}{2}\cos(\theta)^2 - \frac{3}{4}\sin(\theta)^2\right)$$

**Problem 17-77**

The two-bar assembly is released from rest in the position shown. Determine the initial bending moment at the fixed joint *B*. Each bar has mass *m* and length *l*.




Solution:

$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}ml^2 + m\left[l^2 + \left(\frac{l}{2}\right)^2\right] = \frac{5}{3}ml^2$$

$$mg\frac{l}{2} + mgl = I_A\alpha \qquad \alpha = \frac{9}{10}\frac{g}{l}$$

$$M = \frac{1}{12}ml^2\alpha + m\left(\frac{l}{2}\right)\alpha\left(\frac{l}{2}\right) = \frac{1}{3}ml^2\alpha \qquad M_A = \frac{3}{10}mgl$$

**Problem 17-78**

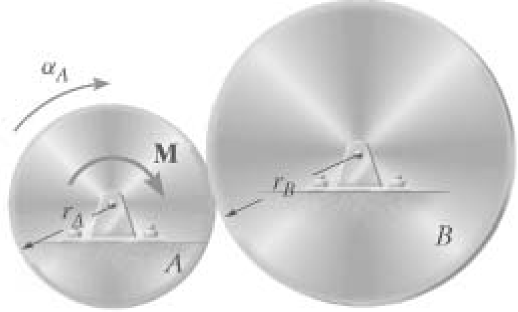
Disk *A* has weight $W_A$ and disk *B* has weight $W_B$. If no slipping occurs between them, determine the couple moment *M* which must be applied to disk *A* to give it an angular acceleration $\alpha_A$.

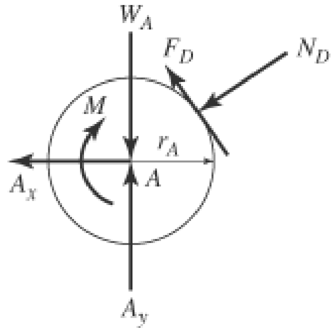
Given:

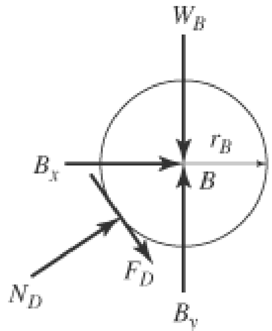
$W_A = 5$ lb $\quad r_A = 0.5$ ft

$W_B = 10$ lb $\quad r_B = 0.75$ ft

$\alpha_A = 4 \frac{\text{rad}}{\text{s}^2}$






Solution:

Guesses $\alpha_B = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad M = 1\ \text{lb}\cdot\text{ft}$ $\quad F_D = 1\ \text{lb}$

Given

$$M - F_D r_A = \frac{1}{2}\left(\frac{W_A}{g}\right) r_A^{\,2}\alpha_A \qquad F_D r_B = \frac{1}{2}\left(\frac{W_B}{g}\right) r_B^{\,2}\alpha_B \qquad r_A\alpha_A = r_B\alpha_B$$

$$\begin{pmatrix} M \\ \alpha_B \\ F_D \end{pmatrix} = \text{Find}(M, \alpha_B, F_D) \qquad \alpha_B = 2.67\ \frac{\text{rad}}{\text{s}^2} \qquad F_D = 0.311\ \text{lb} \qquad M = 0.233\ \text{lb}\cdot\text{ft}$$

---

**Problem 17-79**

The wheel has mass $M$ and radius of gyration $k_B$. It is originally spinning with angular velocity $\omega_1$. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_c$, determine the time required for the motion to stop. What are the horizontal and vertical components of reaction which the pin at $A$ exerts on $AB$ during this time? Neglect the mass of $AB$.

Given:

$M = 25\ \text{kg}$

$k_B = 0.15\ \text{m}$

$\omega_1 = 40\ \frac{\text{rad}}{\text{s}}$

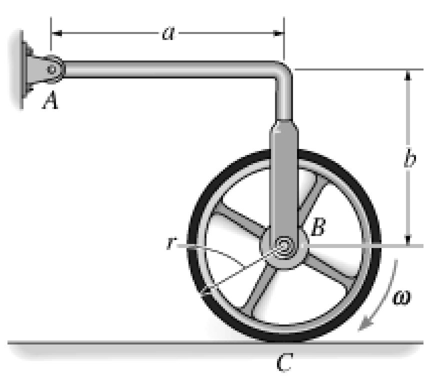
$\mu_C = 0.5$

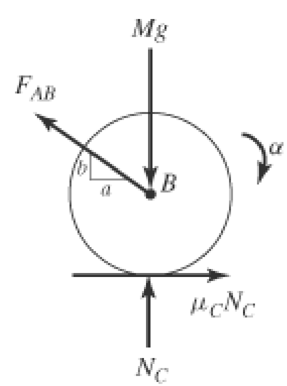
$a = 0.4\ \text{m}$

$b = 0.3\ \text{m}$

$r = 0.2\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guesses $F_{AB} = 1\ \text{N}$ $\quad N_C = 1\ \text{N}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given
$$\mu_C N_C - \left(\frac{a}{\sqrt{a^2 + b^2}}\right) F_{AB} = 0$$

$$N_C - M g + \left(\frac{b}{\sqrt{a^2 + b^2}}\right) F_{AB} = 0$$

$$\mu_C N_C r = -M k_B^2 \alpha$$

$$\begin{pmatrix} F_{AB} \\ N_C \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, N_C, \alpha) \qquad \begin{pmatrix} F_{AB} \\ N_C \end{pmatrix} = \begin{pmatrix} 111.477 \\ 178.364 \end{pmatrix} \text{N} \qquad \alpha = -31.709\ \frac{\text{rad}}{\text{s}^2}$$

$$t = \frac{\omega_1}{-\alpha} \qquad t = 1.261\ \text{s}$$

$$\mathbf{F_A} = \frac{F_{AB}}{\sqrt{a^2 + b^2}}\begin{pmatrix} a \\ b \end{pmatrix} \qquad \mathbf{F_A} = \begin{pmatrix} 89.2 \\ 66.9 \end{pmatrix} \text{N}$$

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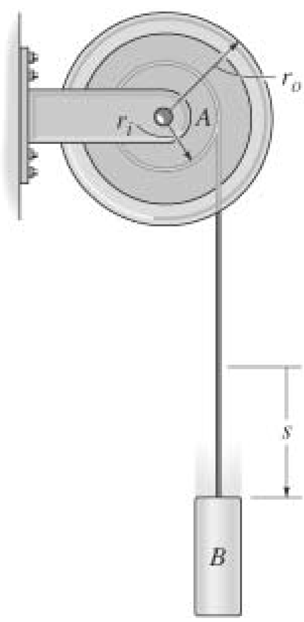
**Problem 17-80**




The cord is wrapped around the inner core of the spool. If block $B$ of weight $W_B$ is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight $W_S$ and the radius of gyration about the axle $A$ is $k_A$. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5\ \text{lb}$

$t_1 = 3\ \text{s}$

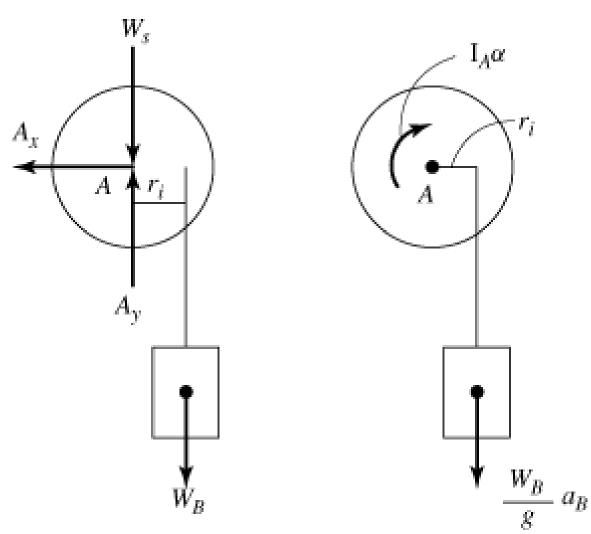
$W_S = 180\ \text{lb}$

$k_A = 1.25\ \text{ft}$

$r_i = 1.5\ \text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

(a) System as a whole

$$W_B r_i = \left(\frac{W_S}{g}\right) k_A^2 \alpha + \left(\frac{W_B}{g}\right)(r_i \alpha r_i) \qquad \alpha = \frac{W_B r_i g}{W_B r_i^2 + W_S k_A^2} \qquad \alpha = 0.826 \frac{\text{rad}}{\text{s}^2}$$

$$\omega = \alpha t_1 \qquad \omega = 2.477 \frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$

Given $$T r_i = \left(\frac{W_S}{g}\right) k_A^2 \alpha \qquad T - W_B = \left(\frac{-W_B}{g}\right)\alpha r_i \qquad \begin{pmatrix} T \\ \alpha \end{pmatrix} = \text{Find}(T, \alpha)$$

$$T = 4.808\ \text{lb} \qquad \alpha = 0.826 \frac{\text{rad}}{\text{s}^2} \qquad \omega = \alpha t_1 \qquad \omega = 2.477 \frac{\text{rad}}{\text{s}}$$

---

**Problem 17-81**

A boy of mass $m_b$ sits on top of the large wheel which has mass $m_w$ and a radius of gyration $k_G$. If the boy essentially starts from rest at $\theta = 0°$, and the wheel begins to rotate freely, determine the angle at which the boy begins to slip. The coefficient of static friction between the wheel and the boy is $\mu_s$. Neglect the size of the boy in the calculation.

Given:

$m_b = 40\ \text{kg}$

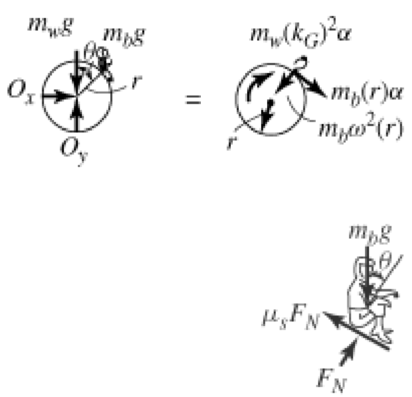
$m_w = 400\ \text{kg}$

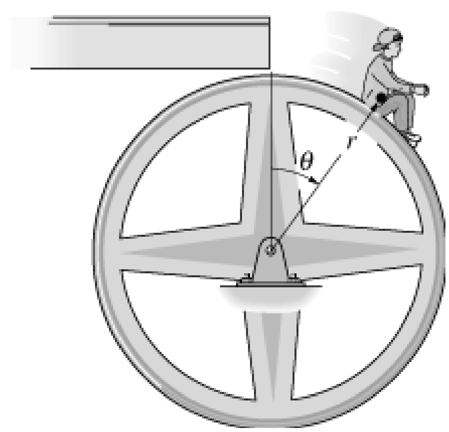
$k_G = 5.5\ \text{m}$

$\mu_s = 0.5$

$r = 8\ \text{m}$

$g = 9.81 \frac{\text{m}}{\text{s}^2}$




Solution: Assume slipping occurs before contact is lost

$$m_b g r \sin(\theta) = \left(m_b r^2 + m_w k_G^2\right)\alpha \qquad \alpha = \frac{m_b g r}{m_b r^2 + m_w k_G^2} \sin(\theta)$$

Guesses $\theta = 10\ \text{deg}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$ $\omega = 1 \frac{\text{rad}}{\text{s}}$ $F_N = 1\ \text{N}$

Given $\quad F_N - m_b g\cos(\theta) = -m_b r\omega^2 \qquad \mu_s F_N - m_b g\sin(\theta) = -m_b r\alpha$

$$\alpha = \frac{m_b g r}{m_b r^2 + m_w k_G^2}\sin(\theta) \qquad \frac{\omega^2}{2} = \frac{m_b g r}{m_b r^2 + m_w k_G^2}(1-\cos(\theta))$$

$$\begin{pmatrix}\theta\\ \alpha\\ \omega\\ F_N\end{pmatrix} = \text{Find}(\theta, \alpha, \omega, F_N)$$

Since $F_N = 322\ \text{N}$ > 0 our assumption is correct.

$$\alpha = 0.107\ \frac{\text{rad}}{\text{s}^2} \qquad \omega = 0.238\ \frac{\text{rad}}{\text{s}} \qquad \theta = 29.8\ \text{deg}$$

**Problem 17-82**

The “Catherine wheel” is a firework that consists of a coiled tube of powder which is pinned at its center. If the powder burns at a constant rate $m'$ such that the exhaust gases always exert a force having a constant magnitude of $F$, directed tangent to the wheel, determine the angular velocity of the wheel when $k$ of the mass is burned off. Initially, the wheel is at rest and has mass $m_0$ and radius $r_0$. For the calculation, consider the wheel to always be a thin disk.

Given:

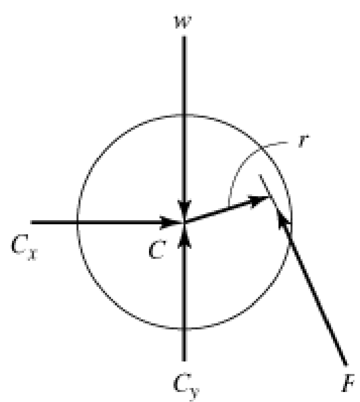
$m' = 20\ \frac{\text{gm}}{\text{s}}$

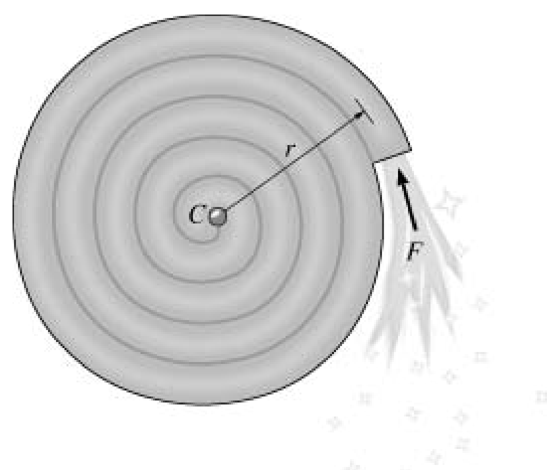
$F = 0.3\ \text{N}$

$m_0 = 100\ \text{gm}$

$r_0 = 75\ \text{mm}$

$k = 0.75$




Solution:

The density is $\quad \rho = \dfrac{m_0}{\pi r_0^2}$

The mass is $\quad m = m_0 - m' t = (1-k)m_0$

$$t_1 = \frac{k m_0}{m'} \qquad t_1 = 3.75\ \text{s}$$

Find the radius $\quad m_0 - m' t = \rho\,\pi r^2 \qquad r = \sqrt{\dfrac{m_0 - m' t}{\rho\,\pi}}$

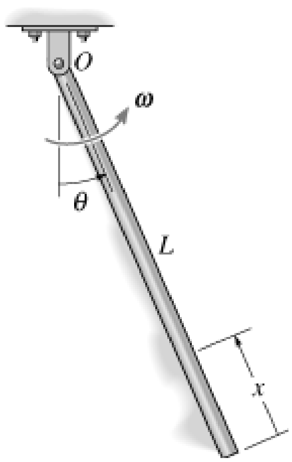
Dynamics $\quad F\,r = \frac{1}{2} m\,r^2 \alpha \qquad \alpha = \frac{2F\sqrt{\rho\,\pi}}{\sqrt{(m_0 - m'\,t)^3}}$

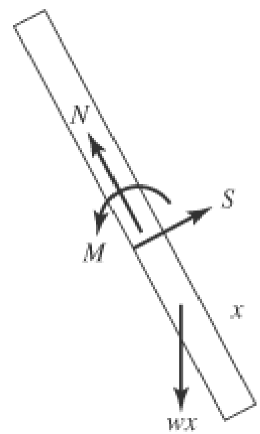
$$\omega = \int_0^{t_1} \frac{2F\sqrt{\rho\,\pi}}{\sqrt{(m_0 - m'\,t)^3}}\,dt \qquad \omega = 800\,\frac{\text{rad}}{\text{s}}$$

---

**Problem 17-83**

The bar has a weight per length of $w$. If it is rotating in the vertical plane at a constant rate $\omega$ about point $O$, determine the internal normal force, shear force, and moment as a function of $x$ and $\theta$.




Solution:

$$N - w\,x\cos(\theta) = \left(\frac{w\,x}{g}\right)\omega^2\left(L - \frac{x}{2}\right) \qquad N = w\,x\left[\cos(\theta) + \left(\frac{\omega^2}{g}\right)\left(L - \frac{x}{2}\right)\right]$$

$$S - w\,x\sin(\theta) = 0 \qquad S = w\,x\,\text{win}(\theta)$$

$$M - S\frac{x}{2} = 0 \qquad M = w\frac{x^2}{2}\sin(\theta)$$

---

**Problem 17-84**

A force $F$ is applied perpendicular to the axis of the rod of weight $W$ and moves from $O$ to $A$ at a constant rate $v$. If the rod is at rest when $\theta = 0°$ and $F$ is at $O$ when $t = 0$, determine the rod's angular velocity at the instant the force is at $A$. Through what angle has the rod rotated when this occurs? The rod rotates in the *horizontal plane*.

Given:

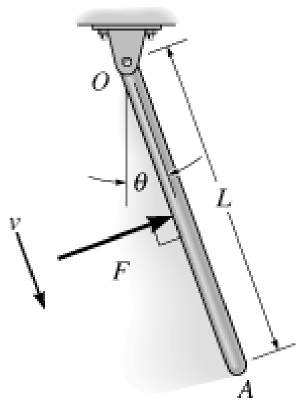
$F = 2$ lb

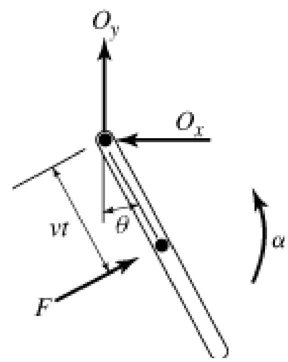
$W = 5\ \text{lb}$

$v = 4\dfrac{\text{ft}}{\text{s}}$

$L = 4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$




Solution:

$$F\,v\,t = \frac{1}{3}\frac{W}{g}L^2\alpha \qquad t = \frac{L}{v} \qquad t = 1\ \text{s}$$

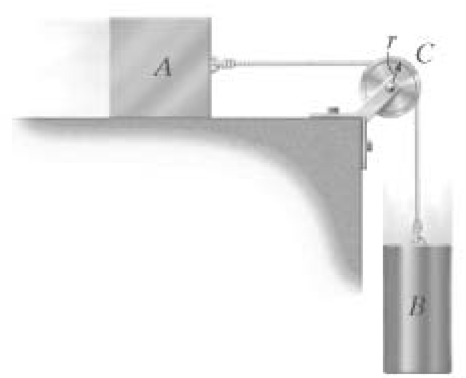
$$\alpha = \left(\frac{3F\,v\,g}{W L^2}\right)t \qquad \omega = \left(\frac{3F\,v\,g}{2W L^2}\right)t^2 \qquad \omega = 4.83\,\frac{\text{rad}}{\text{s}}$$

$$\theta = \left(\frac{F\,v\,g}{2W L^2}\right)t^3 \qquad \theta = 92.2\,\text{deg}$$

---

**Problem 17-85**

Block $A$ has a mass $m$ and rests on a surface having a coefficient of kinetic friction $\mu_k$. The cord attached to $A$ passes over a pulley at $C$ and is attached to a block $B$ having a mass $2m$. If $B$ is released, determine the acceleration of $A$. Assume that the cord does not slip over the pulley. The pulley can be approximated as a thin disk of radius $r$ and mass $m/4$. Neglect the mass of the cord.




Solution:

Given

$$T_1 - \mu_k m\,g = m\,a$$

$$T_2 - 2m\,g = -2m\,a$$

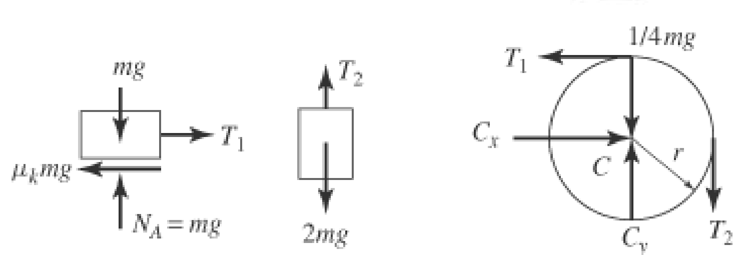
$$T_1\,r - T_2\,r = -\frac{1}{2}\left(\frac{m}{4}\right)r^2\alpha$$

$$a = \alpha r$$

Solving

$$T_1 = \frac{m\,g}{25}\left(16 + 17\mu_k\right)$$

$$T_2 = \frac{2m\,g}{25}\left(9 + 8\mu_k\right)$$



$$\alpha = \frac{8g}{25r}(2 - \mu_k)$$

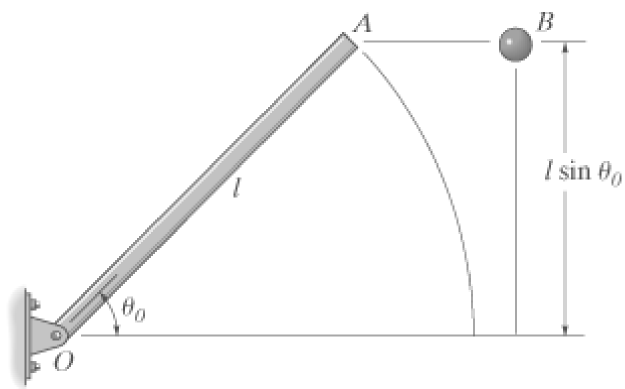
$$a = \frac{8g}{25}(2 - \mu_k)$$

**Problem 17-86**

The slender rod of mass $m$ is released from rest when $\theta = \theta_0$. At the same instant ball $B$ having the same mass $m$ is released.Will $B$ or the end $A$ of the rod have the greatest speed when they pass the horizontal? What is the difference in their speeds?

Given:

$$\theta_0 = 45 \text{ deg}$$




Solution: At horizontal $\theta_f = 0 \text{ deg}$

Rod

$$m g \frac{l}{2}\cos(\theta) = \frac{1}{3} m l^2 \alpha$$

$$\alpha = \frac{3g}{2l}\cos(\theta)$$

$$\frac{\omega^2}{2} = \frac{3g}{2l}(\sin(\theta_0) - \sin(\theta_f))$$

$$\omega = \sqrt{\frac{3g}{l}(\sin(\theta_0) - \sin(\theta_f))} \qquad v_A = \omega l = \sqrt{3g\, l(\sin(\theta_0) - \sin(\theta_f))}$$

Ball

$$m g = m a \qquad a = g$$

$$\frac{v^2}{2} = g\, l(\sin(\theta_0) - \sin(\theta_f))$$

$$v_B = \sqrt{2g\, l(\sin(\theta_0) - \sin(\theta_f))}$$

Define the constant $k = (\sqrt{3} - \sqrt{2})\sqrt{\sin(\theta_0) - \sin(\theta_f)}$

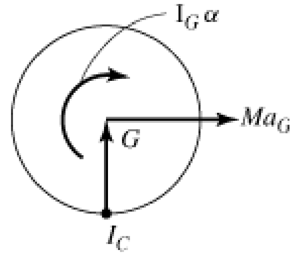
$A$ has the greater speed and the difference is given by $\Delta v = k\sqrt{g\, l}$

$k = 0.267$

**Problem 17-87**

If a disk *rolls without slipping*, show that when moments are summed about the instantaneous center of zero velocity, *IC*, it is possible to use the moment equation $\Sigma M_{IC} = I_{IC}\alpha$, where $I_{IC}$ represents the moment of inertia of the disk calculated about the instantaneous axis of zero velocity.

Solution:




$\Sigma M_{IC} = \Sigma(M_k)_{IC}$; $\quad \Sigma M_{IC} = I_G\alpha + ma_G r$

Since there is no slipping, $\quad a_G = \alpha r$

Thus, $\quad \Sigma M_{IC} = \left(I_G + m r^2\right)\alpha$

By the parallel - axis theorem, the term in parenthesis represents $I_{IC}$.

$\Sigma M_{IC} = I_{IC}\alpha \qquad$ Q.E.D

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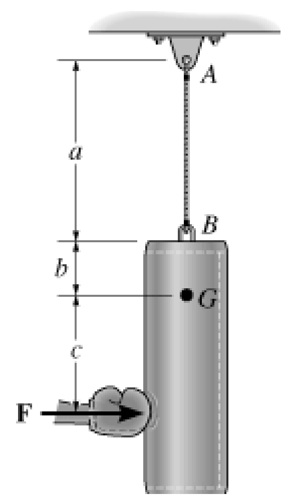
***Problem 17-88**

The punching bag of mass $M$ has a radius of gyration about its center of mass $G$ of $k_G$. If it is subjected to a horizontal force $F$, determine the initial angular acceleration of the bag and the tension in the supporting cable $AB$.




Given:

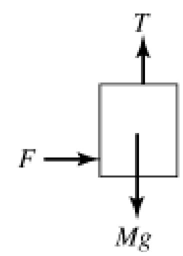
$M = 20\ \text{kg} \qquad b = 0.3\ \text{m}$

$k_G = 0.4\ \text{m} \qquad c = 0.6\ \text{m}$

$F = 30\ \text{N} \qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$a = 1\ \text{m}$

Solution:




$T - M g = 0 \qquad T = M g \qquad T = 196.2\ \text{N}$

$F c = M k_G^2\,\alpha \qquad \alpha = \dfrac{F c}{M k_G^2} \qquad \alpha = 5.625\ \dfrac{\text{rad}}{\text{s}^2}$

---

**Problem 17-89**

The trailer has mass $M_1$ and a mass center at $G$, whereas the spool has mass $M_2$, mass center at $O$, and radius of gyration about an axis passing through $O$ $k_O$. If a force $F$ is applied to the cable, determine the angular acceleration of the spool and the acceleration of the trailer. The wheels have negligible mass and are free to roll.

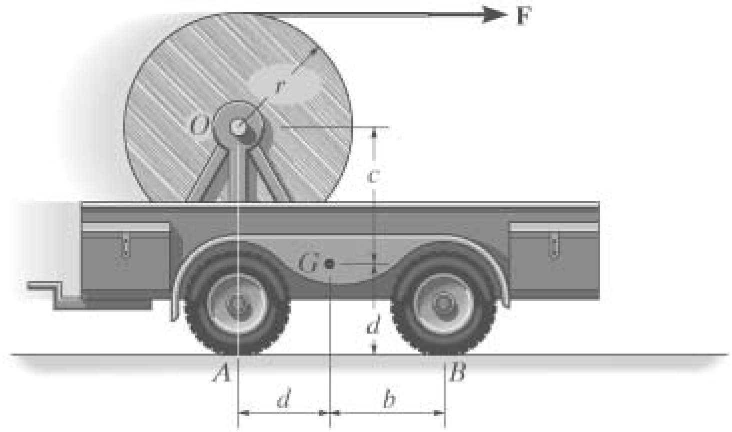
Given:

$M_1 = 580\ \text{kg}$  $b = 0.5\ \text{m}$

$M_2 = 200\ \text{kg}$  $c = 0.6\ \text{m}$

$k_O = 0.45\ \text{m}$  $d = 0.4\ \text{m}$

$F = 60\ \text{N}$  $r = 0.5\ \text{m}$

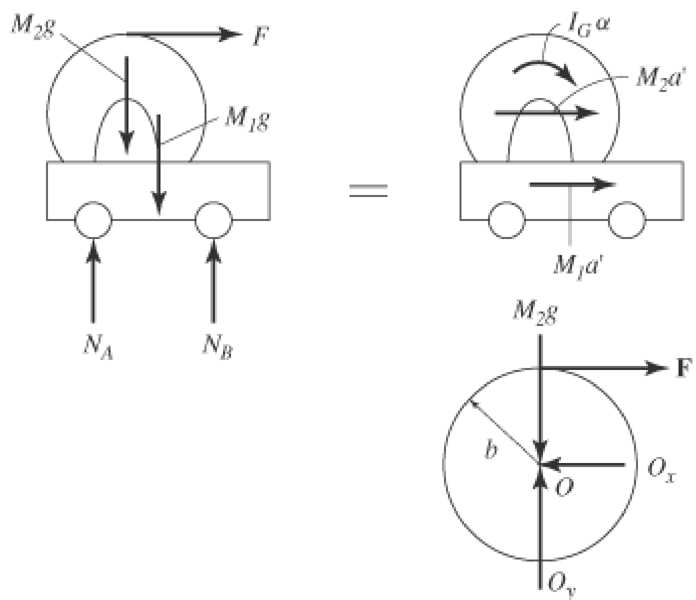



Solution:

$$F = (M_1 + M_2)a$$

$$a = \frac{F}{M_1 + M_2}$$

$$a = 0.0769\ \frac{\text{m}}{\text{s}^2}$$




$$F\,r = M_2\,k_O^{\,2}\,\alpha$$

$$\alpha = F\left(\frac{r}{M_2\,k_O^{\,2}}\right)$$

$$\alpha = 0.741\ \frac{\text{rad}}{\text{s}^2}$$

**Problem 17-90**

The rocket has weight $W$, mass center at $G$, and radius of gyration about the mass center $k_G$ when it is fired. Each of its two engines provides a thrust $T$. At a given instant, engine $A$ suddenly fails to operate. Determine the angular acceleration of the rocket and the acceleration of its nose $B$.

Given:

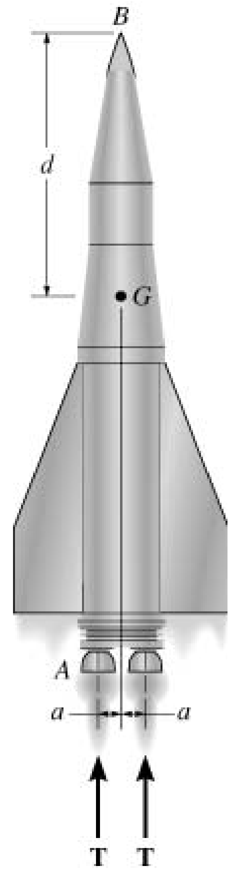
$W = 20000\ \text{lb}$ $T = 50000\ \text{lb}$

$k_G = 21\ \text{ft}$  $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$d = 30\ \text{ft}$

$a = 1.5$ ft

Solution:




$$T\,a = \left(\frac{W}{g}\right)k_G^{\,2}\alpha$$

$$\alpha = \frac{T\,a\,g}{W\,k_G^{\,2}} \qquad \alpha = 0.274\,\frac{\text{rad}}{\text{s}^2}$$

$$T - W = \left(\frac{W}{g}\right)a_{Gy}$$

$$a_{Gy} = \frac{(T-W)g}{W} \qquad a_{Gy} = 14.715\,\frac{\text{m}}{\text{s}^2}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} 0 \\ d \\ 0 \end{pmatrix} \qquad \mathbf{a_B} = \begin{pmatrix} -8.2 \\ 48.3 \\ 0.0 \end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

$$\left|\mathbf{a_B}\right| = 49.0\,\frac{\text{ft}}{\text{s}^2}$$

$$\theta = \operatorname{atan}\left(\frac{a_{Gy}}{\alpha d}\right) \qquad \theta = 80.3\,\text{deg}$$

**Problem 17-91**

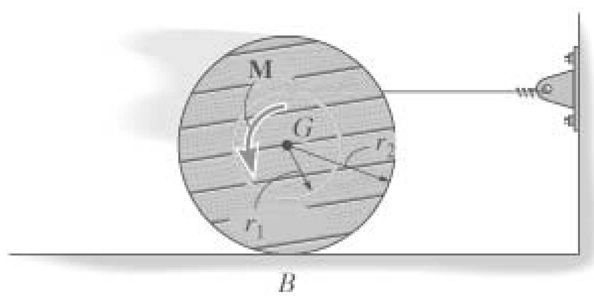
The spool and wire wrapped around its core have a mass $m_s$ and a centroidal radius of gyration $k_G$. If the coefficient of kinetic friction at the ground is $\mu_k$, determine the angular acceleration of the spool when the couple $M$ is applied.

Given:

$m_s = 20$ kg $\quad M = 30$ N m

$k_G = 250$ mm $\quad r_1 = 200$ mm

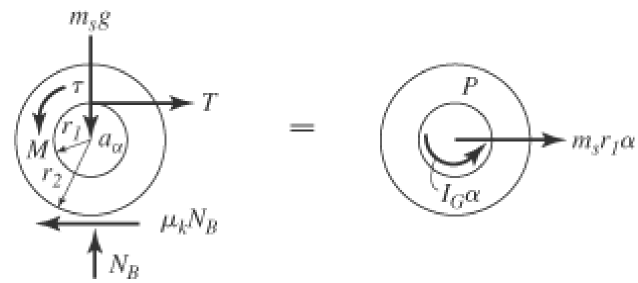
$\mu_k = 0.1 \quad r_2 = 400$ mm



Solution:

Guesses $T = 1\ \text{N}$ $N_B = 1\ \text{N}$

$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$




Given

$$T - \mu_k N_B = m_s r_1 \alpha$$

$$N_B - m_s g = 0$$

$$M - \mu_k N_B r_2 - T r_1 = m_s k_G^{\,2} \alpha$$

$$\begin{pmatrix} T \\ N_B \\ \alpha \end{pmatrix} = \text{Find}(T, N_B, \alpha) \qquad \begin{pmatrix} T \\ N_B \end{pmatrix} = \begin{pmatrix} 55.2 \\ 196.2 \end{pmatrix} \text{N} \qquad \alpha = 8.89\ \frac{\text{rad}}{\text{s}^2}$$

***Problem 17-92**

The uniform board of weight $W$ is suspended from cords at $C$ and $D$. If these cords are subjected to constant forces $F_A$ and $F_B$ respectively, determine the acceleration of the board's center and the board's angular acceleration. Assume the board is a thin plate. Neglect the mass of the pulleys at $E$ and $F$.

Given:

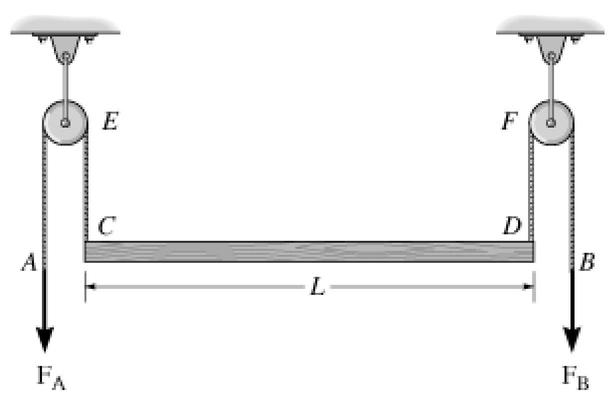
$W = 50\ \text{lb}$

$F_A = 30\ \text{lb}$

$F_B = 45\ \text{lb}$

$L = 10\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$




Solution:

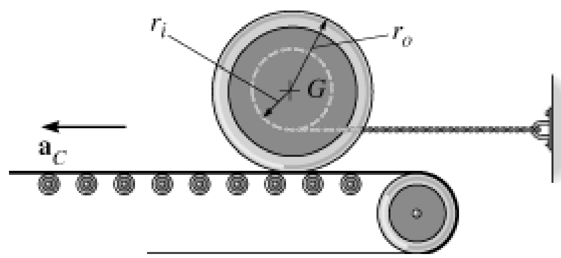
$$F_A + F_B - W = \left(\frac{W}{g}\right) a_{Gy} \qquad a_{Gy} = \left(\frac{F_A + F_B - W}{W}\right) g \qquad a_{Gy} = 16.1\ \frac{\text{ft}}{\text{s}^2}$$

$$F_B\left(\frac{L}{2}\right) - F_A\left(\frac{L}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right)L^2\alpha \qquad \alpha = \frac{6(F_B - F_A)g}{WL} \qquad \alpha = 5.796\,\frac{\text{rad}}{\text{s}^2}$$

**Problem 17-93**

The spool has mass $M$ and radius of gyration $k_G$. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s$ and the coefficient of kinetic friction is $\mu_k$. If the conveyor accelerates at rate $a_C$, determine the initial tension in the wire and the angular acceleration of the spool. The spool is originally at rest.

Units Used: $\text{kN} = 10^3\ \text{N}$




Given:

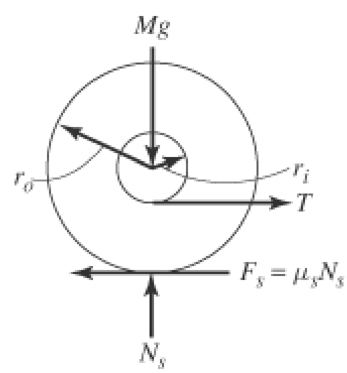
$M = 500\ \text{kg} \qquad a_C = 1\ \frac{\text{m}}{\text{s}^2}$

$k_G = 1.30\ \text{m} \qquad r_i = 0.8\ \text{m}$

$\mu_s = 0.5 \qquad r_o = 1.6\ \text{m}$

$\mu_k = 0.4$

Solution: Assume no slip




Guesses $\quad \alpha = 1\,\frac{\text{rad}}{\text{s}^2} \qquad a_x = 1\,\frac{\text{m}}{\text{s}^2} \qquad T = 1\text{N}$

$F_{max} = 1\text{N} \qquad N_s = 1\text{N} \qquad F_s = 1\text{N}$

Given $\quad T - F_s = M a_x \qquad N_s - Mg = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha \qquad F_{max} = \mu_s N_s$

$a_x = r_i \alpha \qquad a_C = (r_o - r_i)\alpha$

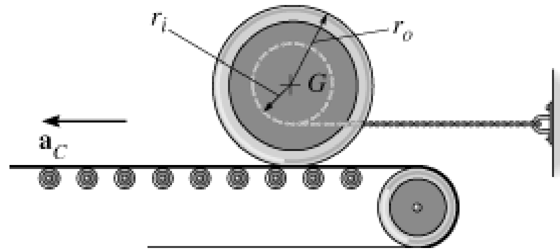
$$\begin{pmatrix} \alpha \\ a_x \\ F_{max} \\ N_s \\ F_s \\ T \end{pmatrix} = \text{Find}(\alpha, a_x, F_{max}, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \\ F_{max} \end{pmatrix} = \begin{pmatrix} 4.907 \\ 1.82 \\ 2.454 \end{pmatrix}\text{kN} \qquad \alpha = 1.25\,\frac{\text{rad}}{\text{s}^2}$$

$T = 2.32\,\text{kN}$

Since $F_s = 1.82\,\text{kN} < F_{max} = 2.454\,\text{kN}$ then our no-slip assumption is correct.

**Problem 17-94**

The spool has mass $M$ and radius of gyration $k_G$. It rests on the surface of a conveyor belt for which the coefficient of static friction is $\mu_s$. Determine the greatest acceleration of the conveyor so that the spool will not slip. Also, what are the initial tension in the wire and the angular acceleration of the spool? The spool is originally at rest.




Units Used: $\mathrm{kN} = 10^3\ \mathrm{N}$

Given: $M = 500\ \mathrm{kg}$

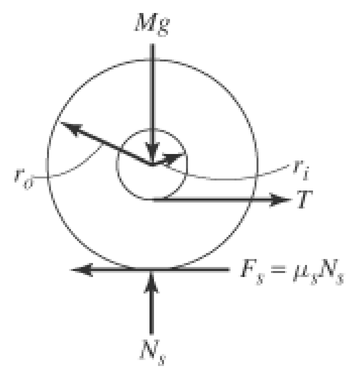
$k_G = 1.30\ \mathrm{m}$ $\quad r_i = 0.8\ \mathrm{m}$

$\mu_s = 0.5$ $\quad r_o = 1.6\ \mathrm{m}$

Solution:

Guesses $\quad \alpha = 1\ \dfrac{\mathrm{rad}}{\mathrm{s}^2}$ $\quad a_x = 1\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$ $\quad a_C = 1\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$

$N_s = 1\ \mathrm{N}$ $\quad F_s = 1\ \mathrm{N}$ $\quad T = 1\ \mathrm{N}$




Given $\quad T - F_s = M a_x$ $\quad N_s - M g = 0$

$T r_i - F_s r_o = -M k_G^2 \alpha$ $\quad F_s = \mu_s N_s$

$a_x = r_i \alpha$ $\quad a_C = (r_o - r_i)\alpha$

$$\begin{pmatrix} \alpha \\ a_x \\ a_C \\ N_s \\ F_s \\ T \end{pmatrix} = \mathrm{Find}(\alpha, a_x, a_C, N_s, F_s, T) \qquad \begin{pmatrix} N_s \\ F_s \end{pmatrix} = \begin{pmatrix} 4.907 \\ 2.454 \end{pmatrix} \mathrm{kN} \qquad a_C = 1.348\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\alpha = 1.685\ \frac{\mathrm{rad}}{\mathrm{s}^2} \qquad T = 3.128\ \mathrm{kN}$$

---

**Problem 17-95**

The wheel has weight $W$ and radius of gyration $k_G$. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s$ and $\mu_k$, determine the wheel's angular acceleration as it rolls down the incline.

Given:

$W = 30 \text{ lb}$

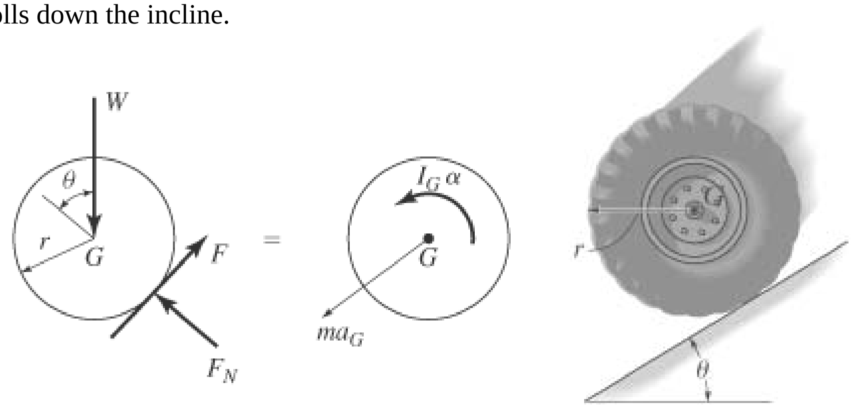
$k_G = 0.6 \text{ ft}$

$\mu_s = 0.2$

$\mu_k = 0.15$

$r = 1.25 \text{ ft}$

$\theta = 12 \text{ deg}$




Solution: Assume no slipping

Guesses $F_N = 1 \text{ lb}$ $F = 1 \text{ lb}$ $a_G = 1 \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$ $F_{max} = 1 \text{ lb}$

Given $F - W\sin(\theta) = \frac{-W}{g}a_G$ $F_N - W\cos(\theta) = 0$ $F_{max} = \mu_s F_N$

$F r = \frac{W}{g}k_G^2\alpha$ $a_G = r\alpha$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha)$$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \end{pmatrix} = \begin{pmatrix} 1.17 \\ 29.34 \\ 5.87 \end{pmatrix} \text{lb} \qquad a_G = 5.44 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 4.35 \frac{\text{rad}}{\text{s}^2}$$

Since $F = 1.17 \text{ lb} < F_{max} = 5.87 \text{ lb}$ then our no-slip assumption is correct.

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***Problem 17-96**

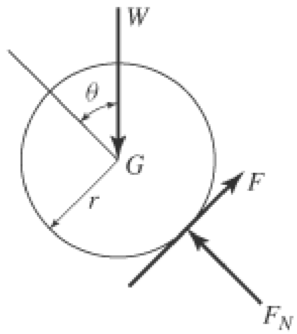
The wheel has a weight $W$ and a radius of gyration $k_G$. If the coefficients of static and kinetic friction between the wheel and the plane are $\mu_s$ and $\mu_k$, determine the maximum angle $\theta$ of the inclined plane so that the wheel rolls without slipping.

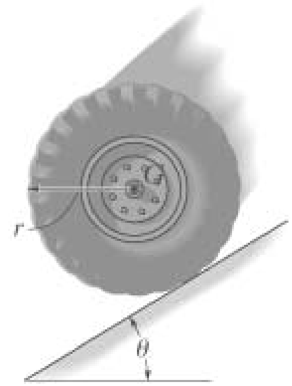
Given:

$W = 30\ \text{lb}$ $\quad r = 1.25\ \text{ft}$

$k_G = 0.6\ \text{ft}$ $\quad \theta = 12\ \text{deg}$

$\mu_s = 0.2$ $\quad \mu_k = 0.15$




Solution:

Guesses

$\theta = 1\ \text{deg}$ $\quad F_N = 1\ \text{lb}$ $\quad F = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F - W\sin(\theta) = \left(\frac{-W}{g}\right)a_G \qquad F\,r = \left(\frac{W}{g}\right)k_G{}^2\alpha$$

$$F_N - W\cos(\theta) = 0 \qquad F = \mu_s F_N$$

$$a_G = r\alpha$$

$$\begin{pmatrix} \theta \\ F_N \\ F \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(\theta, F_N, F, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} 4.10 \\ 20.50 \end{pmatrix}\text{lb} \qquad a_G = 19.1\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 15.3\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 46.9\ \text{deg}$$

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**Problem 17-97**

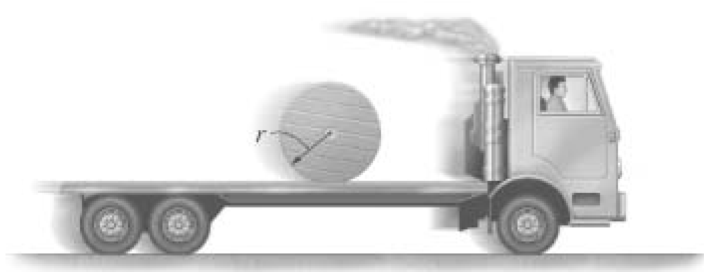
The truck carries the spool which has weight $W$ and radius of gyration $k_G$. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate $a_{At}$. Assume the spool does not slip on the bed of the truck.

Given:

$W = 500\ \text{lb}$ $\quad r = 3\ \text{ft}$

$k_G = 2\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

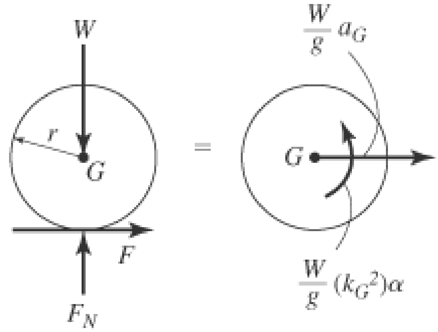
$a_{At} = 3\ \frac{\text{ft}}{\text{s}^2}$



Solution:

Guesses $a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given




$$a_G = a_{At} - \alpha r$$

$$0 = \left(\frac{-W}{g}\right) a_G r + \left(\frac{W}{g}\right) k_G^2 \alpha$$

$$\begin{pmatrix} a_G \\ \alpha \end{pmatrix} = \text{Find}(a_G, \alpha) \qquad a_G = 0.923\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 0.692\ \frac{\text{rad}}{\text{s}^2}$$

---

**Problem 17-98**

The truck carries the spool which has weight $W$ and radius of gyration $k_G$. Determine the angular acceleration of the spool if it is not tied down on the truck and the truck begins to accelerate at the rate $a_{At}$. The coefficients of static and kinetic friction between the spool and the truck bed are $\mu_s$ and $\mu_k$, respectively.

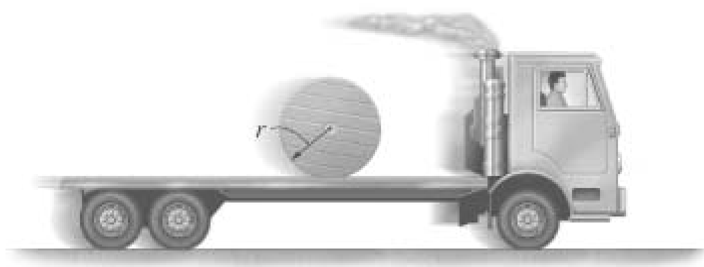
Given:

$W = 200\ \text{lb}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 2\ \text{ft}$ $\mu_s = 0.15$

$a_{At} = 5\ \frac{\text{ft}}{\text{s}^2}$ $\mu_k = 0.1$

$r = 3\ \text{ft}$

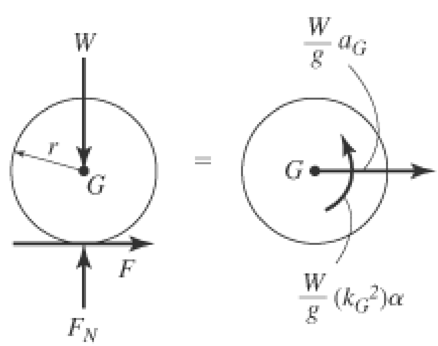



Solution: Assume no slip

Guesses

$F = 1\ \text{lb}$ $F_N = 1\ \text{lb}$ $F_{max} = 1\ \text{lb}$

$a_G = 1\ \frac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$




Given $F = \frac{W}{g} a_G$ $F r = \frac{W}{g} k_G^2 \alpha$ $F_N - W = 0$ $F_{max} = \mu_s F_N$

$$a_G = a_{At} - \alpha r$$

$$\begin{pmatrix} F \\ F_N \\ F_{max} \\ a_G \\ \alpha \end{pmatrix} = \text{Find}(F, F_N, F_{max}, a_G, \alpha) \qquad \begin{pmatrix} F \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 9.56 \\ 30.00 \\ 200.00 \end{pmatrix} \text{lb} \qquad a_G = 1.538 \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1.154 \frac{\text{rad}}{\text{s}^2}$$

Since $F = 9.56\,\text{lb} < F_{max} = 30\,\text{lb}$ then our no-slip assumption is correct.

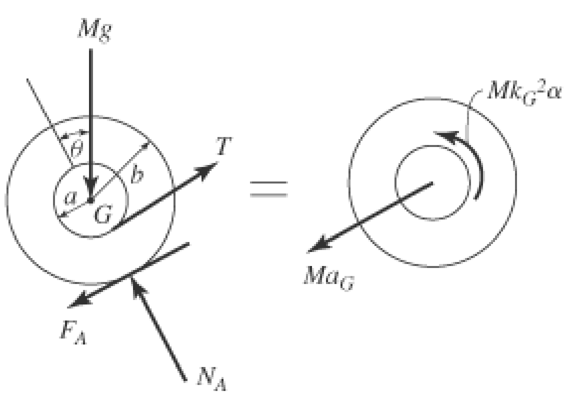
**Problem 17-99**

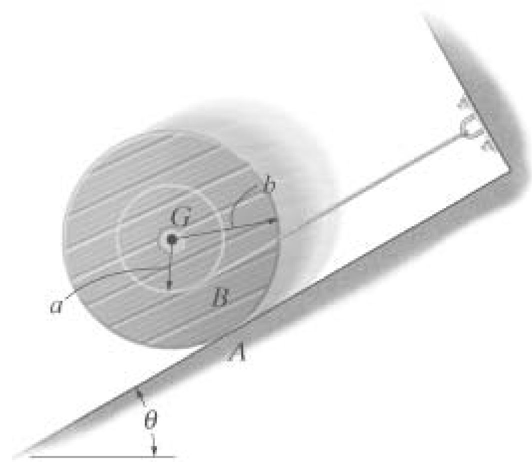
The spool has mass $M$ and radius of gyration $k_G$. It rests on the inclined surface for which the coefficient of kinetic friction is $\mu_k$. If the spool is released from rest and slips at $A$, determine the initial tension in the cord and the angular acceleration of the spool.

Given:

$M = 75\ \text{kg} \qquad k_G = 0.380\ \text{m} \qquad a = 0.3\ \text{m}$

$\mu_k = 0.15 \qquad \theta = 30\ \text{deg} \qquad b = 0.6\ \text{m}$




Solution:

Guesses $\qquad T = 1\ \text{N} \qquad N_A = 1\ \text{N} \qquad a_G = 1\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

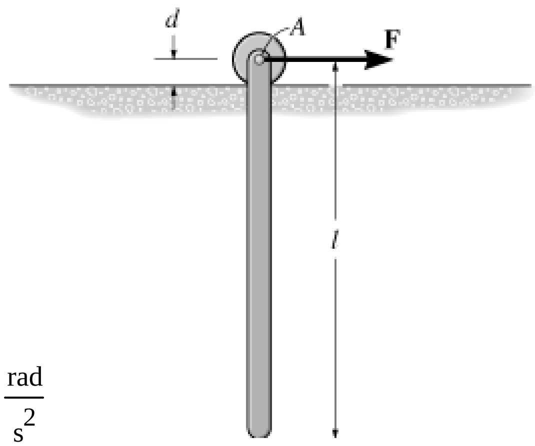
$$T - M g \sin(\theta) - \mu_k N_A = -M a_G \qquad N_A - M g \cos(\theta) = 0$$

$$T a - \mu_k N_A b = M k_G^2 \alpha \qquad a_G = \alpha a$$

$$\begin{pmatrix} T \\ N_A \\ \alpha \\ a_G \end{pmatrix} = \text{Find}(T, N_A, \alpha, a_G) \qquad a_G = 1.395\ \frac{\text{m}}{\text{s}^2} \qquad \alpha = 4.65\ \frac{\text{rad}}{\text{s}^2} \qquad T = 359\ \text{N}$$

***Problem 17-100**

A uniform rod having weight *W* is pin-supported at *A* from a roller which rides on horizontal track. If the rod is originally at rest, and horizontal force **F** is applied to the roller, determine the acceleration of the roller. Neglect the mass of the roller and its size *d* in the computations.




Given:

$W = 10\ \text{lb}$

$F = 15\ \text{lb}$

$l = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

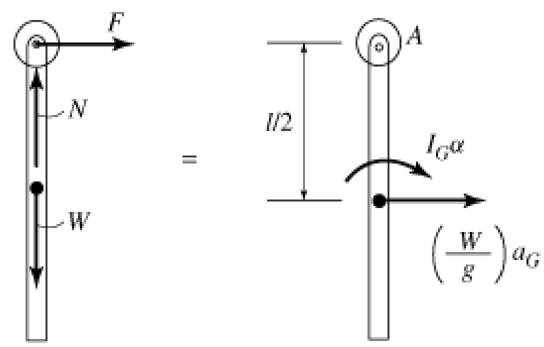
Guesses $\quad a_G = 1\ \frac{\text{ft}}{\text{s}^2} \quad a_A = 1\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$F = \left(\frac{W}{g}\right) a_G \qquad F\left(\frac{l}{2}\right) = \frac{1}{12}\left(\frac{W}{g}\right) l^2 \alpha$$

$$a_A = a_G + \alpha \frac{l}{2}$$

$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}(a_G, a_A, \alpha)$$

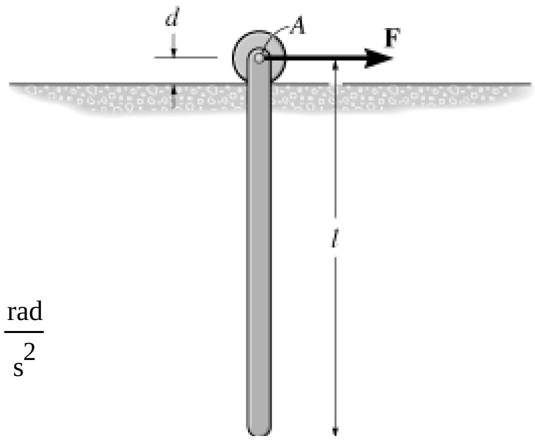



$$a_G = 48.3\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 145\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 193\ \frac{\text{ft}}{\text{s}^2}$$

**Problem 17-101**

A uniform rod having weight *W* is pin-supported at *A* from a roller which rides on horizontal track. Assume that the roller at *A* is replaced by a slider block having a negligible mass. If the rod is initially at rest, and a horizontal force **F** is applied to the slider, determine the slider's acceleration. The coefficient of kinetic friction between the block and the track is $\mu_k$. Neglect the dimension *d* and the size of the block in the computations.

Given:




$W = 10\ \text{lb} \quad \mu_k = 0.2$

$F = 15\ \text{lb} \quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$l = 2\ \text{ft}$

Solution:

Guesses $\quad a_G = 1\ \frac{\text{ft}}{\text{s}^2} \quad a_A = 1\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

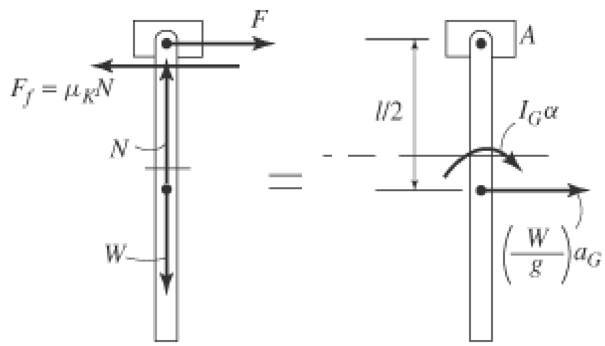
Given

$$F - \mu_k W = \left(\frac{W}{g}\right) a_G$$

$$\left(F - \mu_k W\right)\frac{l}{2} = \frac{1}{12}\left(\frac{W}{g}\right) l^2 \alpha$$

$$a_A = a_G + \alpha\left(\frac{l}{2}\right)$$

$$\begin{pmatrix} a_G \\ a_A \\ \alpha \end{pmatrix} = \text{Find}\left(a_G, a_A, \alpha\right)$$

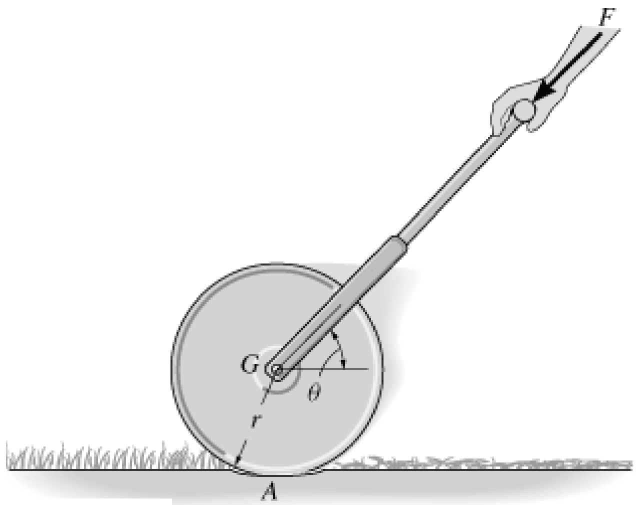



$a_G = 41.86\ \frac{\text{ft}}{\text{s}^2} \quad \alpha = 126\ \frac{\text{rad}}{\text{s}^2} \quad a_A = 167\ \frac{\text{ft}}{\text{s}^2}$

---

**Problem 17-102**

The lawn roller has mass $M$ and radius of gyration $k_G$. If it is pushed forward with a force **F** when the handle is in the position shown, determine its angular acceleration. The coefficients of static and kinetic friction between the ground and the roller are $\mu_s$ and $\mu_k$, respectively.




Given:

$M = 80 \text{ kg}$ $\mu_s = 0.12$

$k_G = 0.175 \text{ m}$ $\mu_k = 0.1$

$F = 200 \text{ N}$ $r = 200 \text{ mm}$

$\theta = 45 \text{ deg}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution: Assume no slipping,

Guesses

$F_f = 1 \text{ N}$ $F_N = 1 \text{ N}$ $a_x = 1 \frac{\text{m}}{\text{s}^2}$ $\alpha = 1 \frac{\text{rad}}{\text{s}^2}$ $F_{max} = 1 \text{ N}$

Given $-F\cos(\theta) + F_f = -M a_x$ $F_N - Mg - F\sin(\theta) = 0$

$F_f r = M k_G^2 \alpha$ $a_x = \alpha r$ $F_{max} = \mu_s F_N$

$$\begin{pmatrix} F_f \\ F_N \\ F_{max} \\ a_x \\ \alpha \end{pmatrix} = \text{Find}(F_f, F_N, F_{max}, a_x, \alpha) \qquad \begin{pmatrix} F_N \\ F_f \\ F_{max} \end{pmatrix} = \begin{pmatrix} 926.221 \\ 61.324 \\ 111.147 \end{pmatrix} \text{N} \qquad a_x = 1.001 \frac{\text{m}}{\text{s}^2}$$

$$\alpha = 5.01 \frac{\text{rad}}{\text{s}^2}$$

Since $F_f = 61.3 \text{ N} < F_{max} = 111.1 \text{ N}$ then our no-slip assumption is true.

---

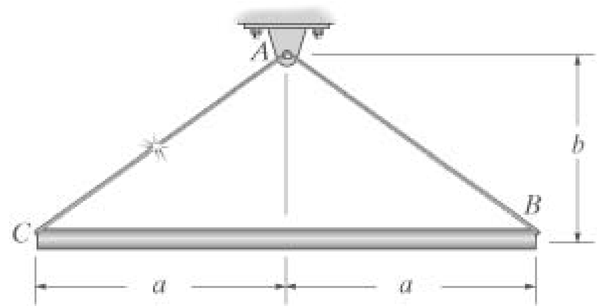
**Problem 17-103**

The slender bar of weight $W$ is supported by two cords $AB$ and $AC$. If cord $AC$ suddenly breaks, determine the initial angular acceleration of the bar and the tension in cord $AB$.

Given:

$W = 150\ \text{lb}$ $\quad a = 4\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad b = 3\ \text{ft}$

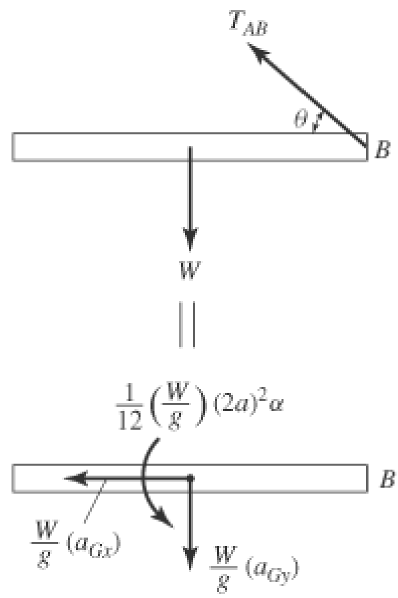



Solution: $\theta = \text{atan}\left(\dfrac{b}{a}\right)$

Guesses

$T_{AB} = 1\ \text{lb}$ $\quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

$a_{Gx} = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad a_{Gy} = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad a_B = 1\ \dfrac{\text{ft}}{\text{s}^2}$




Given

$$-T_{AB}\cos(\theta) = \left(\frac{-W}{g}\right)a_{Gx}$$

$$T_{AB}\sin(\theta) - W = \left(\frac{-W}{g}\right)a_{Gy} \qquad -a_{Gx} = -a_B\sin(\theta)$$

$$T_{AB}\sin(\theta)a = \frac{1}{12}\left(\frac{W}{g}\right)(2a)^2\alpha \qquad -a_{Gy} + \alpha a = -a_B\cos(\theta)$$

$$\begin{pmatrix} T_{AB} \\ \alpha \\ a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \text{Find}(T_{AB}, \alpha, a_{Gx}, a_{Gy}, a_B) \qquad \begin{pmatrix} a_{Gx} \\ a_{Gy} \\ a_B \end{pmatrix} = \begin{pmatrix} 7.43 \\ 26.63 \\ 12.38 \end{pmatrix}\frac{\text{ft}}{\text{s}^2}$$

$\alpha = 4.18\ \dfrac{\text{rad}}{\text{s}^2}$

$T_{AB} = 43.3\ \text{lb}$

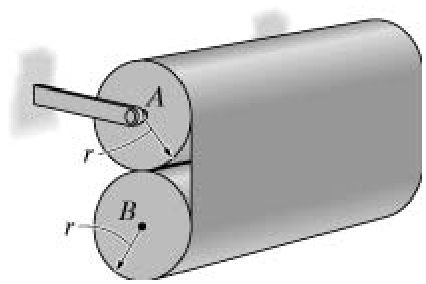
---

***Problem 17-104**

A long strip of paper is wrapped into two rolls, each having mass $M$. Roll $A$ is pin-supported about its center whereas roll $B$ is not centrally supported. If $B$ is brought into contact with $A$ and released from rest, determine the initial tension in the paper between the rolls and the angular acceleration of each roll. For the calculation, assume the rolls to be approximated by cylinders.

Given:

$M = 8\ \text{kg}$ $\quad r = 90\ \text{mm}$ $\quad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



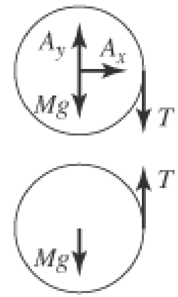

Solution: Guesses

$T = 1\ \text{N}$ $\quad a_{By} = 1\ \dfrac{\text{m}}{\text{s}^2}$ $\quad \alpha_A = 1\ \dfrac{\text{rad}}{\text{s}^2}$ $\quad \alpha_B = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$T\,r = \frac{1}{2}M\,r^2\alpha_A \qquad T\,r = \frac{1}{2}M\,r^2\alpha_B$$

$$T - M\,g = M\,a_{By} \qquad -\alpha_A\,r = a_{By} + \alpha_B\,r$$




$$\begin{pmatrix} T \\ a_{By} \\ \alpha_A \\ \alpha_B \end{pmatrix} = \text{Find}\left(T, a_{By}, \alpha_A, \alpha_B\right) \qquad T = 15.7\ \text{N}$$

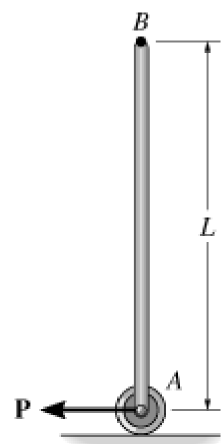
$$\begin{pmatrix} \alpha_A \\ \alpha_B \end{pmatrix} = \begin{pmatrix} 43.6 \\ 43.6 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$a_{By} = -7.848\ \frac{\text{m}}{\text{s}^2}$$

---

**Problem 17-105**

The uniform bar of mass $m$ and length $L$ is balanced in the vertical position when the horizontal force **P** is applied to the roller at $A$. Determine the bar's initial angular acceleration and the acceleration of its top point $B$.




Solution:

$$-P = m\,a_x \qquad a_x = \frac{-P}{m}$$

$$-P\left(\frac{L}{2}\right) = \frac{1}{12}m\,L^2\alpha \qquad \alpha = \frac{-6P}{m\,L}$$

$$a_B = a_x - \alpha\left(\frac{L}{2}\right) \qquad a_B = \frac{2P}{m}$$ positive means to the right

---

**Problem 17-106**

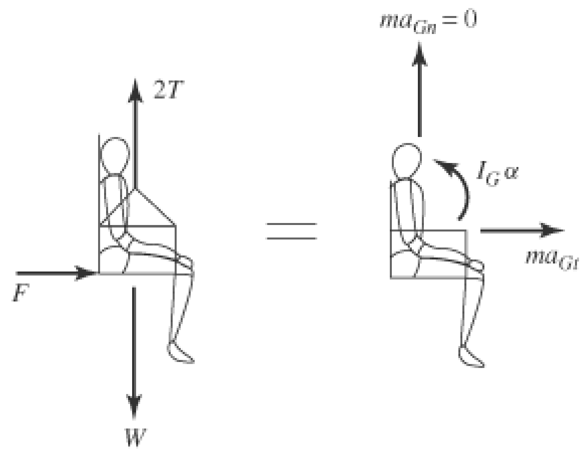
A woman sits in a rigid position in the middle of the swing. The combined weight of the woman and swing is $W$ and the radius of gyration about the center of mass $G$ is $k_G$. If a man pushes on the swing with a horizontal force **F** as shown, determine the initial angular acceleration and the tension in each of the two supporting chains $AB$. During the motion, assume that the chain segment $CAD$ remains rigid. The swing is originally at rest.

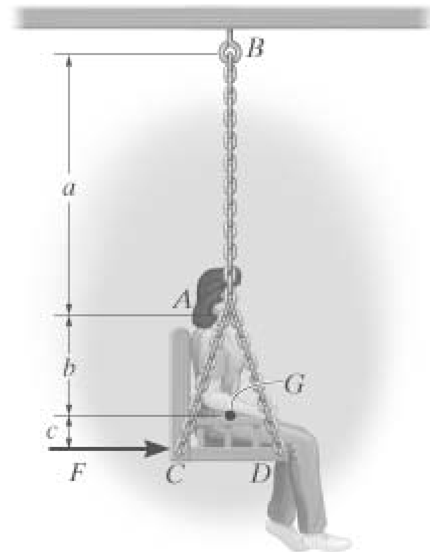
Given:

$W = 180\ \text{lb}$ $\quad a = 4\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 2.5\ \text{ft}$ $\quad b = 1.5\ \text{ft}$

$F = 20\ \text{lb}$ $\quad c = 0.4\ \text{ft}$




Solution:

Guesses $T = 1\ \text{lb}$ $a_{Gt} = 1\ \dfrac{\text{ft}}{\text{s}^2}$ $\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

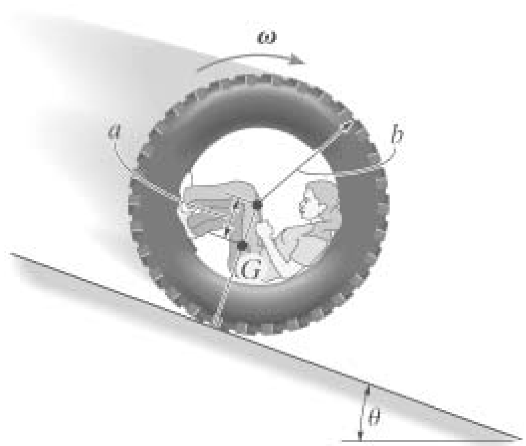
Given $F = \left(\dfrac{W}{g}\right)a_{Gt}$ $2T - W = 0$ $F\,c = \left(\dfrac{W}{g}\right)k_G^{\,2}\,\alpha$

$$\begin{pmatrix} T \\ a_{Gt} \\ \alpha \end{pmatrix} = \text{Find}(T, a_{Gt}, \alpha) \qquad a_{Gt} = 3.58\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha = 0.229\ \frac{\text{rad}}{\text{s}^2} \qquad T = 90.0\ \text{lb}$$

---

**Problem 17-107**

A girl sits snugly inside a large tire such that together the girl and tire have a total weight $W$, a center of mass at $G$, and a radius of gyration $k_G$ about $G$. If the tire rolls freely down the incline, determine the normal and frictional forces it exerts on the ground when it is in the position shown and has an angular velocity $\omega$. Assume that the tire does not slip as it rolls.




Given:

$W = 185\ \text{lb}$ $b = 2\ \text{ft}$

$k_G = 1.65\ \text{ft}$ $a = 0.75\ \text{ft}$

$\omega = 6\ \dfrac{\text{rad}}{\text{s}}$ $\theta = 20\ \text{deg}$

Solution:

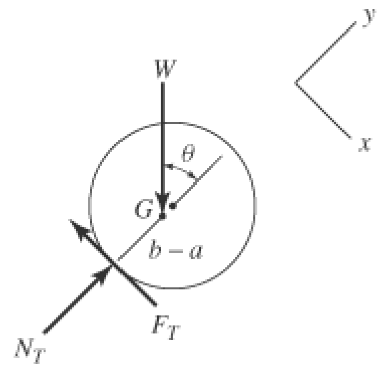
Guesses $N_T = 1\ \text{lb}$ $F_T = 1\ \text{lb}$

$\alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$N_T - W\cos(\theta) = \frac{W}{g}a\omega^2 \qquad F_T(b-a) = \frac{W}{g}k_G^{\,2}\alpha$$

$$F_T - W\sin(\theta) = \frac{-W}{g}(b-a)\alpha$$




$$\begin{pmatrix} N_T \\ F_T \\ \alpha \end{pmatrix} = \mathrm{Find}(N_T, F_T, \alpha) \qquad \alpha = 3.2\,\frac{\mathrm{rad}}{\mathrm{s}^2}$$

$$\begin{pmatrix} N_T \\ F_T \end{pmatrix} = \begin{pmatrix} 329.0 \\ 40.2 \end{pmatrix}\mathrm{lb}$$

***Problem 17-108**

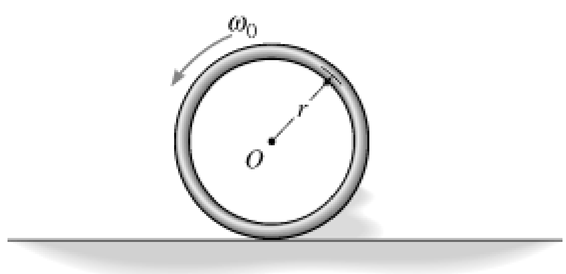
The hoop or thin ring of weight $W$ is given an initial angular velocity $\omega_0$ when it is placed on the surface. If the coefficient of kinetic friction between the hoop and the surface is $\mu_k$, determine the distance the hoop moves before it stops slipping.

Given:

$$W = 10\ \mathrm{lb} \qquad \mu_k = 0.3$$

$$\omega_0 = 6\,\frac{\mathrm{rad}}{\mathrm{s}} \qquad r = 6\ \mathrm{in}$$

$$g = 32.2\,\frac{\mathrm{ft}}{\mathrm{s}^2}$$

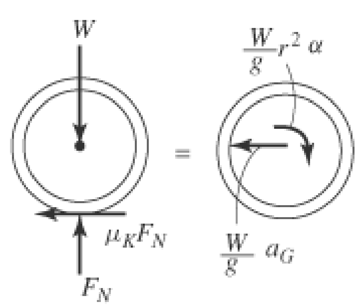



Solution:

$$F_N - W = 0 \qquad F_N = W \qquad F_N = 10\,\mathrm{lb}$$

$$\mu_k F_N = \left(\frac{W}{g}\right)a_G \qquad a_G = \mu_k g \qquad a_G = 9.66\,\frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$\mu_k F_N r = \left(\frac{W}{g}\right)r^2\alpha \qquad \alpha = \frac{\mu_k g}{r} \qquad \alpha = 19.32\,\frac{\mathrm{rad}}{\mathrm{s}^2}$$




When it stops slipping

$$v_G = \omega r$$

$$a_G t = (\omega_0 - \alpha t)r \qquad t = \frac{\omega_0 r}{a_G + \alpha r} \qquad t = 0.155\ \mathrm{s}$$

$$d = \frac{1}{2} a_G t^2 \qquad d = 1.398\ \text{in}$$

---

**Problem 17-109**

The circular plate of weight $W$ is suspended from a pin at $A$. If the pin is connected to a track which is given acceleration $a_A$, determine the horizontal and vertical components of reaction at $A$ and the acceleration of the plate's mass center $G$. The plate is originally at rest.

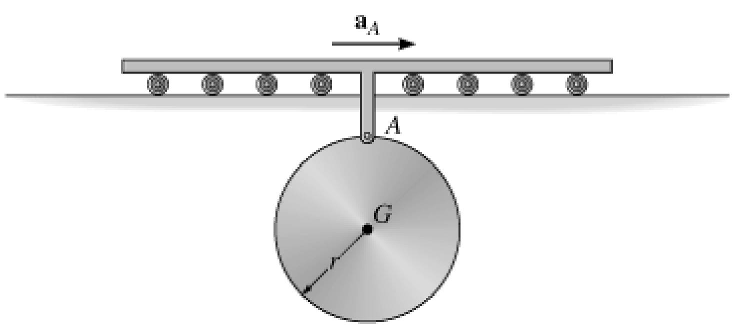
Given:

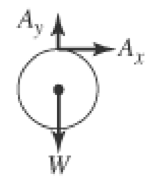
$W = 15\ \text{lb}$

$a_A = 3\ \dfrac{\text{ft}}{\text{s}^2}$

$r = 2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad a_{Gx} = 1\ \dfrac{\text{ft}}{\text{s}^2} \qquad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2} \qquad A_x = 1\ \text{lb} \qquad A_y = 1\ \text{lb}$

Given $\quad A_x = \dfrac{W}{g} a_{Gx} \qquad A_y - W = 0 \qquad -A_x r = \dfrac{-1}{2} \dfrac{W}{g} r^2 \alpha \qquad a_{Gx} + \alpha r = a_A$

$$\begin{pmatrix} a_{Gx} \\ \alpha \\ A_x \\ A_y \end{pmatrix} = \text{Find}(a_{Gx}, \alpha, A_x, A_y) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 0.466 \\ 15.000 \end{pmatrix} \text{lb} \qquad a_{Gx} = 1\ \frac{\text{ft}}{\text{s}^2}$$

$$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

---

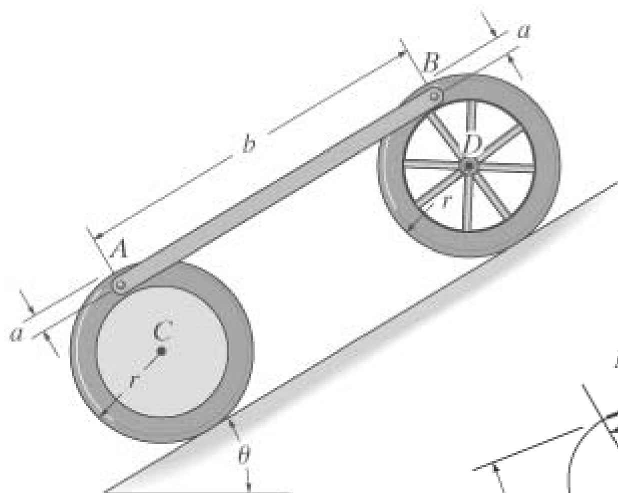
**Problem 17-110**

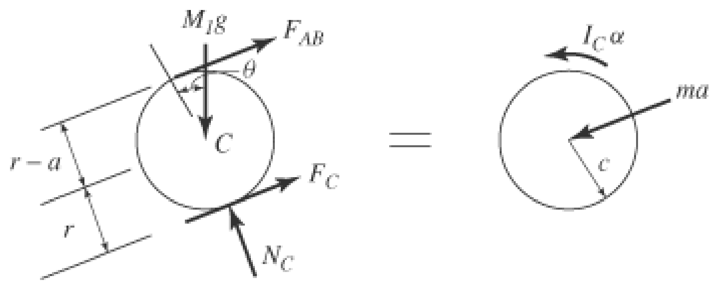
Wheel $C$ has a mass $M_1$ and a radius of gyration $k_C$, whereas wheel $D$ has a mass $M_2$ and a radius of gyration $k_D$. Determine the angular acceleration of each wheel at the instant shown. Neglect the mass of the link and assume that the assembly does not slip on the plane.

Given:

$M_1 = 60\ \text{kg} \quad k_C = 0.4\ \text{m} \quad r = 0.5\ \text{m} \quad b = 2\ \text{m}$

$M_2 = 40\ \text{kg} \quad k_D = 0.35\ \text{m} \quad a = 0.1\ \text{m} \quad \theta = 30\ \text{deg}$

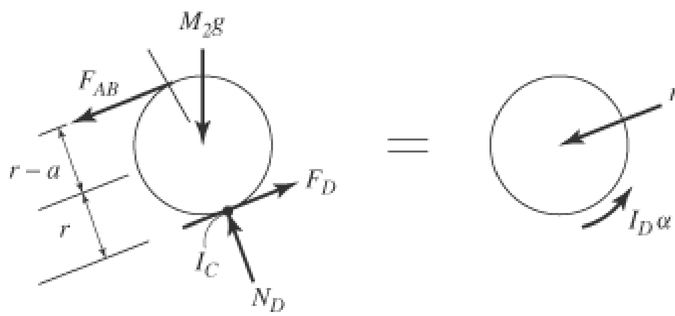



Solution:

Both wheels have the same angular acceleration.

Guesses




$$F_{AB} = 1\ \text{N}$$

$$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$$

Given

$$-F_{AB}(2r-a) + M_1\, g\sin(\theta)r = M_1\, k_C^{\,2}\alpha + M_1(r\alpha)r$$

$$F_{AB}(2r-a) + M_2\, g\sin(\theta)r = M_2\, k_D^{\,2}\alpha + M_2(r\alpha)r$$

$$\begin{pmatrix} F_{AB} \\ \alpha \end{pmatrix} = \text{Find}(F_{AB}, \alpha) \qquad F_{AB} = -6.21\ \text{N} \qquad \alpha = 6.21\ \frac{\text{rad}}{\text{s}^2}$$

---

**Problem 17-111**

The assembly consists of a disk of mass $m_D$ and a bar of mass $m_b$ which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s$ and $\mu_k$ respectively. Neglect friction at $B$.

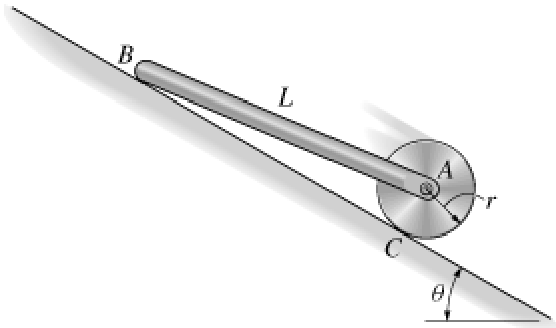
Given:

$m_D = 8\ \text{kg}$ $L = 1\ \text{m}$

$m_b = 10\ \text{kg}$ $r = 0.3\ \text{m}$

$\mu_s = 0.6$ $\theta = 30\ \text{deg}$

$\mu_k = 0.4$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$




Solution: $\phi = \text{asin}\left(\frac{r}{L}\right)$

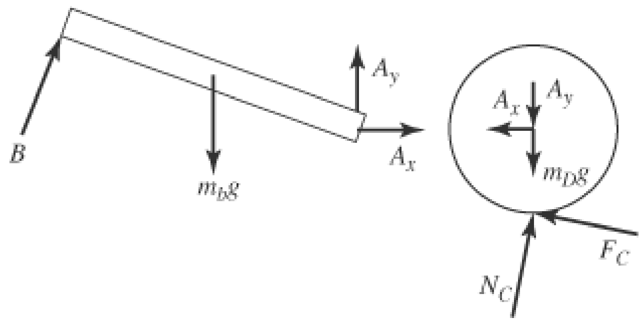
Assume no slip

Guesses

$N_C = 1\ \text{N}$ $F_C = 1\ \text{N}$

$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $a_A = 1\ \frac{\text{m}}{\text{s}^2}$

$F_{max} = 1\ \text{N}$




Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta-\phi) - m_b g\frac{L}{2}\cos(\theta-\phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 109.042 \\ 16.053 \\ 65.425 \end{pmatrix}\text{N} \qquad \alpha = 13.377\ \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 16.053\ \text{N} < F_{max} = 65.425\ \text{N}$ then our no-slip assumption is correct.

---

**Problem 17-112**

The assembly consists of a disk of mass $m_D$ and a bar of mass $m_b$ which is pin connected to the disk. If the system is released from rest, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and the inclined plane are $\mu_s$ and $\mu_k$ respectively. Neglect friction at $B$. Solve if the bar is removed.

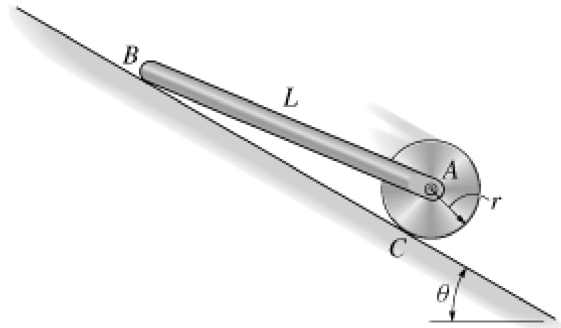
Given:

$m_D = 8\ \text{kg}$ $\quad L = 1\ \text{m}$

$m_b = 0\ \text{kg}$ $\quad r = 0.3\ \text{m}$

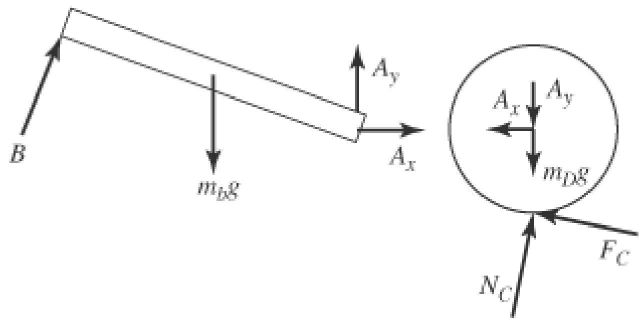
$\mu_s = 0.15$ $\quad \theta = 30\ \text{deg}$

$\mu_k = 0.1$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$




Solution: $\phi = \operatorname{asin}\left(\frac{r}{L}\right)$

Assume no slip




Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$

$\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1\ \frac{\text{m}}{\text{s}^2}$

$F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta-\phi) - m_b g\frac{L}{2}\cos(\theta-\phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D+m_b)g\sin(\theta) = (m_D+m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad a_A = r\alpha \qquad F_{max} = \mu_s N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \operatorname{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 13.08 \\ 10.195 \end{pmatrix}\text{N} \qquad \alpha = 10.9\ \frac{\text{rad}}{\text{s}^2}$$

Since $F_C = 13.08\ \text{N}$ $> F_{max} = 10.195\ \text{N}$ then our no-slip assumption is wrong and we know that slipping does occur.

Guesses

$N_C = 1\ \text{N}$ $\quad F_C = 1\ \text{N}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$ $\quad a_A = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad F_{max} = 1\ \text{N}$

Given

$$N_C L\cos(\phi) - m_D g L\cos(\theta - \phi) - m_b g\frac{L}{2}\cos(\theta - \phi) = \frac{-1}{2}m_D r^2\alpha - m_D a_A r - m_b a_A\frac{r}{2}$$

$$-F_C + (m_D + m_b)g\sin(\theta) = (m_D + m_b)a_A$$

$$F_C r = \frac{1}{2}m_D r^2\alpha \qquad F_{max} = \mu_s N_C \qquad F_C = \mu_k N_C$$

$$\begin{pmatrix} N_C \\ F_C \\ a_A \\ \alpha \\ F_{max} \end{pmatrix} = \text{Find}(N_C, F_C, a_A, \alpha, F_{max}) \qquad \begin{pmatrix} N_C \\ F_C \\ F_{max} \end{pmatrix} = \begin{pmatrix} 67.966 \\ 6.797 \\ 10.195 \end{pmatrix} \text{N} \qquad \alpha = 5.664\frac{\text{rad}}{\text{s}^2}$$

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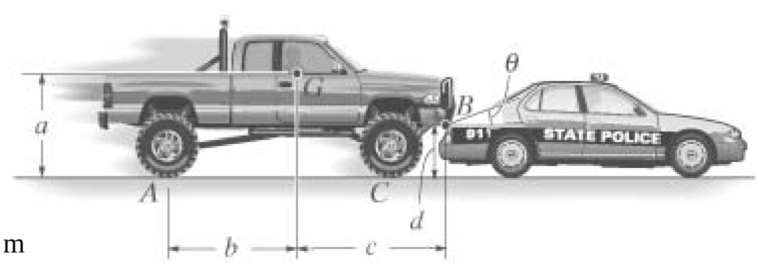
**Problem 17-113**

A “lifted” truck can become a road hazard since the bumper is high enough to ride up a standard car in the event the car is rear-ended. As a model of this case consider the truck to have a mass $M$, a mass center $G$, and a radius of gyration $k_G$ about $G$. Determine the horizontal and vertical components of acceleration of the mass center $G$, and the angular acceleration of the truck, at the moment its front wheels at $C$ have just left the ground and its smooth front bumper begins to ride up the back of the stopped car so that point $B$ has a velocity of $v_B$ at angle $\theta$ from the horizontal. Assume the wheels are free to roll, and neglect the size of the wheels and the deformation of the material.




Units Used:

$\text{Mg} = 10^3\ \text{kg}$

$\text{kN} = 10^3\ \text{N}$

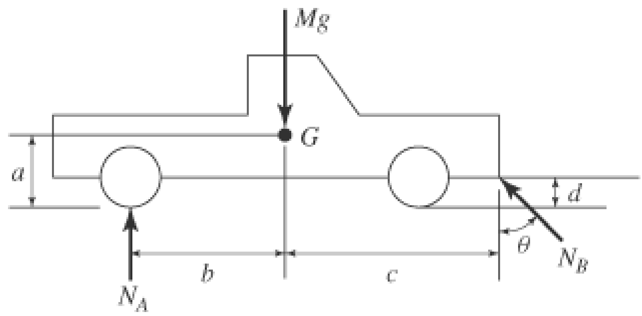
Given:

$M = 2.70\ \text{Mg}$ $\quad a = 1.3\ \text{m}$

$\theta = 20\ \text{deg}$ $\quad b = 1.6\ \text{m}$

$k_G = 1.45\ \text{m}$ $\quad c = 1.2\ \text{m}$

$v_B = 8\ \frac{\text{m}}{\text{s}}$ $\quad d = 0.4\ \text{m}$




Solution:

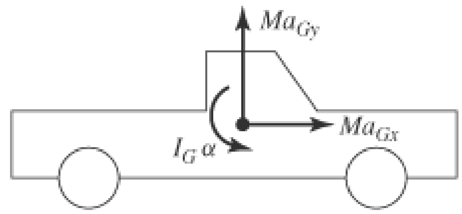
Guesses $v_A = 1\ \frac{\text{m}}{\text{s}}$ $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

$a_B = 1\ \frac{\text{m}}{\text{s}^2}$ $\quad \alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

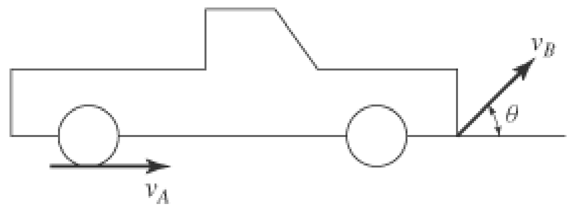
$a_{Gx} = 1\ \frac{m}{s^2} \quad a_{Gy} = 1\ \frac{m}{s^2}$

$N_A = 1\ N \quad N_B = 1\ N$

$a_A = 1\ \frac{m}{s^2}$




Given




$$N_A + N_B\cos(\theta) - Mg = M a_{Gy}$$

$$-N_B\sin(\theta) = M a_{Gx}$$

$$N_B\cos(\theta)c - N_B\sin(\theta)(a-d) - N_A b = M k_G^2 \alpha$$

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} v_B\cos(\theta) \\ v_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} - \omega^2 \begin{pmatrix} b+c \\ d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} \times \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} - \omega^2 \begin{pmatrix} c \\ -a+d \\ 0 \end{pmatrix} = \begin{pmatrix} a_B\cos(\theta) \\ a_B\sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ \omega \\ a_A \\ a_B \\ a_{Gx} \\ a_{Gy} \\ \alpha \\ N_A \\ N_B \end{pmatrix} = \text{Find}(v_A, \omega, a_A, a_B, a_{Gx}, a_{Gy}, \alpha, N_A, N_B)$$

$$\begin{pmatrix} a_{Gx} \\ a_{Gy} \end{pmatrix} = \begin{pmatrix} -1.82 \\ -1.69 \end{pmatrix} \frac{m}{s^2}$$

$$\omega = 0.977\ \frac{rad}{s} \qquad \alpha = -0.283\ \frac{rad}{s^2}$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = \begin{pmatrix} -0.664 \\ -3.431 \end{pmatrix} \frac{m}{s^2} \qquad \begin{pmatrix} N_A \\ N_B \end{pmatrix} = \begin{pmatrix} 8.38 \\ 14.40 \end{pmatrix} kN$$