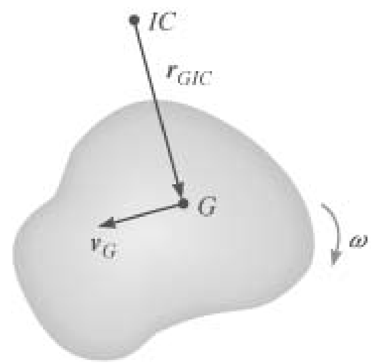
Problem 18-1



At a given instant the body of mass m has an angular velocity ω and its mass center has a velocity $\mathbf{v}_G$. Show that its kinetic energy can be represented as $T = 1/2\, I_{IC}\omega^2$, where I_{IC} is the moment of inertia of the body computed about the instantaneous axis of zero velocity, located a distance r_{GIC} from the mass center as shown.

Solution:

$$T = \left(\frac{1}{2}\right) m\, v_G^{\,2} + \left(\frac{1}{2}\right)\left(I_G\right)\omega^2 \qquad \text{where } v_G = \omega\, r_{GIC}$$

$$T = \left(\frac{1}{2}\right) m\left(\omega r_{GIC}\right)^2 + \frac{1}{2} I_G \omega^2$$

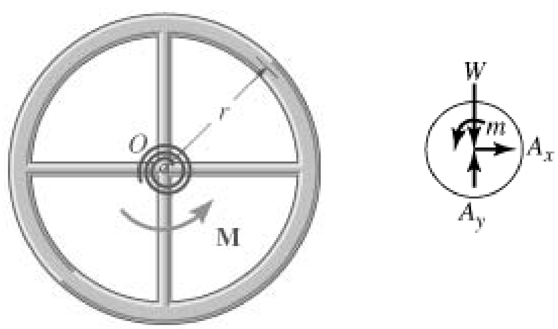
$$T = \left(\frac{1}{2}\right)\left(m\, r_{GIC}^{\,2} + I_G\right)\omega^2 \qquad \text{However } m(r_{GIC})^2 + I_G = I_{IC}$$

$$\boxed{T = \left(\frac{1}{2}\right) I_{IC}\omega^2}$$

Problem 18-2

The wheel is made from a thin ring of mass m_{ring} and two slender rods each of mass m_{rod}. If the torsional spring attached to the wheel's center has stiffness k, so that the torque on the center of the wheel is $M = -k\theta$, determine the maximum angular velocity of the wheel if it is rotated two revolutions and then released from rest.

Given:



$m_{ring} = 5\ \text{kg}$

$m_{rod} = 2\ \text{kg}$

$k = 2\ \dfrac{\text{N}\cdot\text{m}}{\text{rad}}$

$r = 0.5\ \text{m}$

Solution:

$$I_O = 2\left[\frac{1}{12} m_{rod}(2r)^2\right] + m_{ring} r^2$$

$$I_O = 1.583\ \text{kg}\cdot\text{m}^2$$

$$T_1 + \Sigma U_{12} = T_2$$

$$0 + \int_{4\pi}^{0} -k\,\theta\,d\theta = \frac{1}{2} I_O \omega^2 \qquad \omega = \sqrt{\frac{k}{I_O}}\,4\pi \qquad \omega = 14.1\,\frac{\text{rad}}{\text{s}}$$

Problem 18-3

At the instant shown, the disk of weight W has counterclockwise angular velocity ω when its center has velocity v. Determine the kinetic energy of the disk at this instant.

Given:

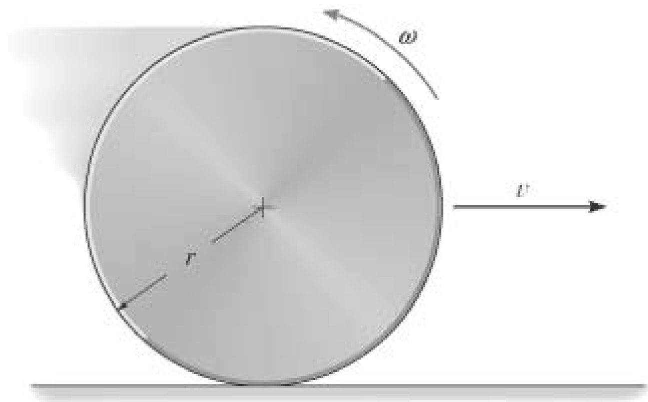
$W = 30\ \text{lb}$

$\omega = 5\ \frac{\text{rad}}{\text{s}}$

$v = 20\ \frac{\text{ft}}{\text{s}}$

$r = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

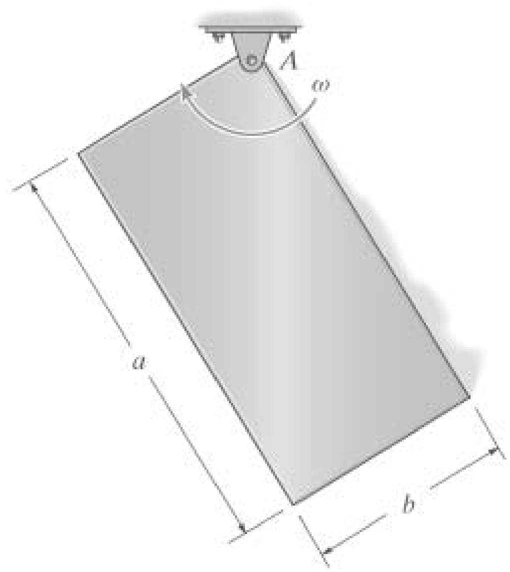


Solution:

$$T = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g} r^2\right)\omega^2 + \frac{1}{2}\left(\frac{W}{g}\right)v^2 \qquad T = 210\,\text{ft}\cdot\text{lb}$$

***Problem 18-4**

The uniform rectangular plate has weight W. If the plate is pinned at A and has an angular velocity ω, determine the kinetic energy of the plate.



Given:

$W = 30\ \text{lb}$

$\omega = 3\ \frac{\text{rad}}{\text{s}}$

$a = 2\ \text{ft}$

$b = 1\ \text{ft}$

Solution:

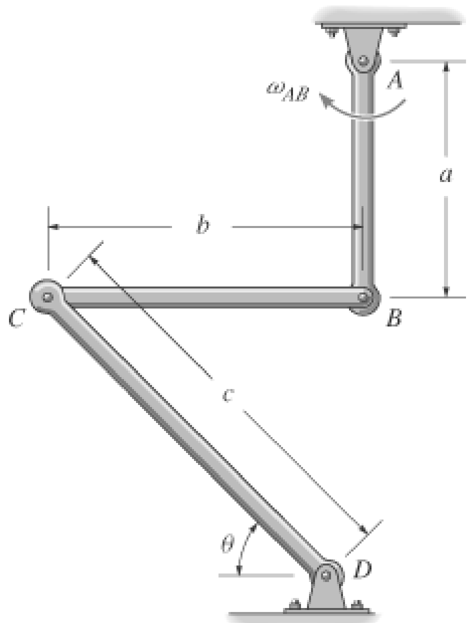
$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

$$T = \frac{1}{2}\left(\frac{W}{g}\right)\left(\omega\frac{\sqrt{b^2+a^2}}{2}\right)^2 + \frac{1}{2}\left[\frac{1}{12}\left(\frac{W}{g}\right)\left(b^2+a^2\right)\right]\omega^2$$

$T = 6.99\ \text{ft}\cdot\text{lb}$

Problem 18-5

At the instant shown, link AB has angular velocity ω_{AB}. If each link is considered as a uniform slender bar with weight density γ, determine the total kinetic energy of the system.



Given:

$\omega_{AB} = 2\ \frac{\text{rad}}{\text{s}}$ $\quad a = 3\ \text{in}$

$\gamma = 0.5\ \frac{\text{lb}}{\text{in}}$ $\quad b = 4\ \text{in}$

$\theta = 45\ \text{deg}$ $\quad c = 5\ \text{in}$

Solution: $\quad \rho = \frac{\gamma}{g}$

Guesses

$\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_{CD} = 1\ \frac{\text{rad}}{\text{s}}$

$v_{Gx} = 1\ \frac{\text{in}}{\text{s}}$ $\quad v_{Gy} = 1\ \frac{\text{in}}{\text{s}}$ $\quad T = 1\ \text{lb}\cdot\text{ft}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} -b \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{CD} \end{pmatrix} \times \begin{pmatrix} c\cos(\theta) \\ -c\sin(\theta) \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_{AB} \end{pmatrix} \times \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BC} \end{pmatrix} \times \begin{pmatrix} \frac{-b}{2} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} v_{Gx} \\ v_{Gy} \\ 0 \end{pmatrix}$$

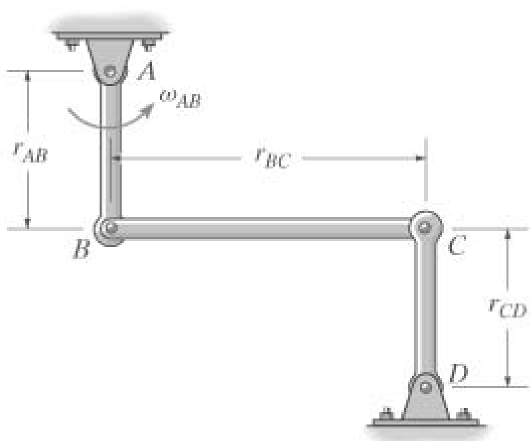
$$T = \frac{1}{2}\left(\frac{\rho a^3}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{\rho b^3}{12}\right)\omega_{BC}^2 + \frac{1}{2}\rho b\left(v_{Gx}^2 + v_{Gy}^2\right) + \frac{1}{2}\left(\frac{\rho c^3}{3}\right)\omega_{CD}^2$$

$$\begin{pmatrix} \omega_{BC} \\ \omega_{CD} \\ v_{Gx} \\ v_{Gy} \\ T \end{pmatrix} = \text{Find}\left(\omega_{BC}, \omega_{CD}, v_{Gx}, v_{Gy}, T\right) \qquad \begin{pmatrix} \omega_{BC} \\ \omega_{CD} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.697 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} v_{Gx} \\ v_{Gy} \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.25 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$T = 0.0188\ \text{ft}\cdot\text{lb}$

Problem 18-6

Determine the kinetic energy of the system of three links. Links *AB* and *CD* each have weight W_1, and link *BC* has weight W_2.



Given:

$W_1 = 10\ \text{lb}$

$W_2 = 20\ \text{lb}$

$\omega_{AB} = 5\ \frac{\text{rad}}{\text{s}}$

$r_{AB} = 1\ \text{ft}$

$r_{BC} = 2\ \text{ft}$

$r_{CD} = 1\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_{BC} = 0\ \frac{\text{rad}}{\text{s}} \qquad \omega_{CD} = \omega_{AB}\left(\frac{r_{AB}}{r_{CD}}\right)$

$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)\left(\omega_{AB} r_{AB}\right)^2 + \frac{1}{2}\left(\frac{W_1}{g}\right)\frac{r_{CD}^2}{3}\omega_{CD}^2$$

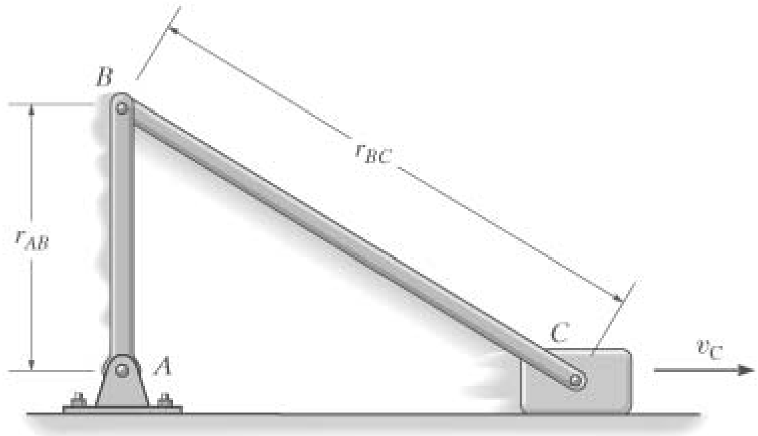
$T = 10.4\ \text{ft}\cdot\text{lb}$

Problem 18-7

The mechanism consists of two rods, *AB* and *BC*, which have weights W_1 and W_2, respectively, and a block at *C* of weight W_3. Determine the kinetic energy of the system at the instant shown, when the block is moving at speed v_C.

Given:

$W_1 = 10\ \text{lb}$



$W_2 = 20\ \text{lb}$

$W_3 = 4\ \text{lb}$

$r_{AB} = 2\ \text{ft}$

$r_{BC} = 4\ \text{ft}$

$v_C = 3\ \dfrac{\text{ft}}{\text{s}}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_{BC} = 0\ \dfrac{\text{rad}}{\text{s}}$ $\qquad \omega_{AB} = \dfrac{v_C}{r_{AB}}$

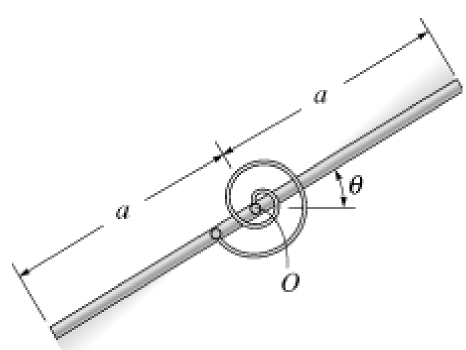
$$T = \frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{r_{AB}^2}{3}\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_3}{g}\right)v_C^2 \qquad T = 3.82\ \text{lb}\cdot\text{ft}$$

***Problem 18-8**

The bar of weight W is pinned at its center O and connected to a torsional spring. The spring has a stiffness k, so that the torque developed is $M = k\theta$. If the bar is released from rest when it is vertical at $\theta = 90°$, determine its angular velocity at the instant $\theta = 0°$. Use the prinicple of work and energy.

Given:

$W = 10\ \text{lb}$



$k = 5\ \dfrac{\text{lb}\cdot\text{ft}}{\text{rad}}$

$a = 1\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$\theta_0 = 90\ \text{deg} \qquad \theta_f = 0\ \text{deg}$

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\frac{1}{2}k\theta_0^{\,2} = \frac{1}{2}k\theta_f^{\,2} + \frac{1}{2}\left(\frac{W}{g}\right)\frac{(2a)^2}{12}\omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 10.9\ \frac{\text{rad}}{\text{s}}$

Problem 18-9

A force P is applied to the cable which causes the reel of mass M to turn since it is resting on the two rollers A and B of the dispenser. Determine the angular velocity of the reel after it has made two revolutions starting from rest. Neglect the mass of the rollers and the mass of the cable. The radius of gyration of the reel about its center axis is k_G.

Given:

$P = 20\ \text{N}$

$M = 175\ \text{kg}$

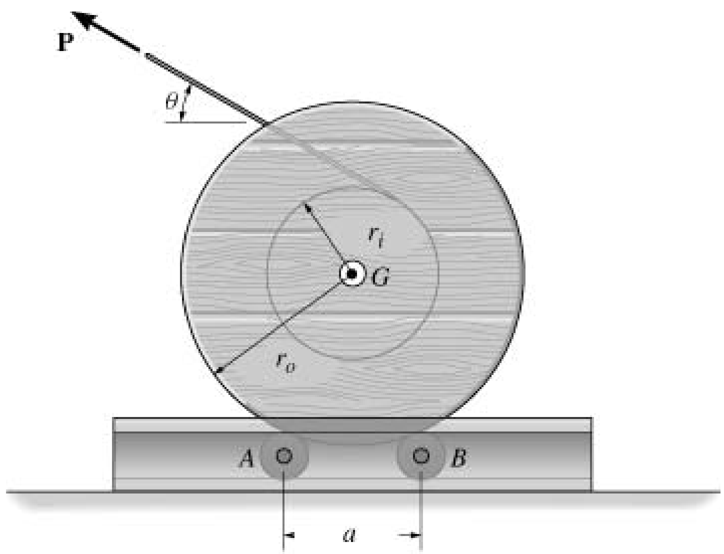
$k_G = 0.42\ \text{m}$

$\theta = 30\ \text{deg}$

$r_i = 250\ \text{mm}$

$r_0 = 500\ \text{mm}$

$a = 400\ \text{mm}$



Solution:

$$0 + P2(2\pi r_i) = \frac{1}{2}M k_G^{\,2}\,\omega^2$$

$$\omega = \sqrt{\frac{8\pi P r_i}{M k_G^{\,2}}} \qquad \omega = 2.02\ \frac{\text{rad}}{\text{s}}$$

Problem 18-10

The rotary screen S is used to wash limestone. When empty it has a mass M_1 and a radius of gyration k_G. Rotation is achieved by applying a torque M about the drive wheel A. If no slipping occurs at A and the supporting wheel at B is free to roll, determine the angular velocity of the screen after it has rotated n revolutions. Neglect the mass of A and B.

Unit Used:

$rev = 2\pi\ rad$

Given:

$M_1 = 800\ kg$

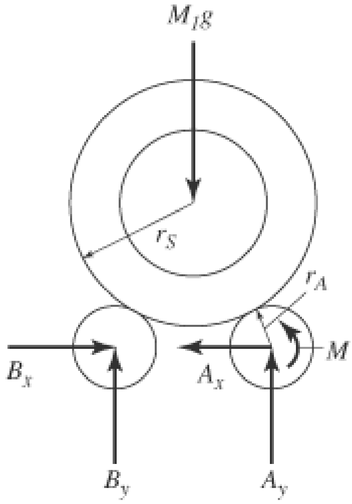
$k_G = 1.75\ m$

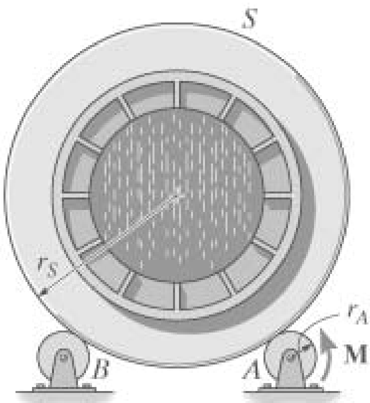
$M = 280\ N\cdot m$

$r_S = 2\ m$

$r_A = 0.3\ m$

$n = 5\ rev$





Solution:

$$M\left(\frac{r_S}{r_A}n\right) = \frac{1}{2}M_1 k_G^2 \omega^2$$

$$\omega = \sqrt{\frac{2M r_S n}{r_A M_1 k_G^2}} \qquad \omega = 6.92\ \frac{rad}{s}$$

Problem 18-11

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r.

Given:

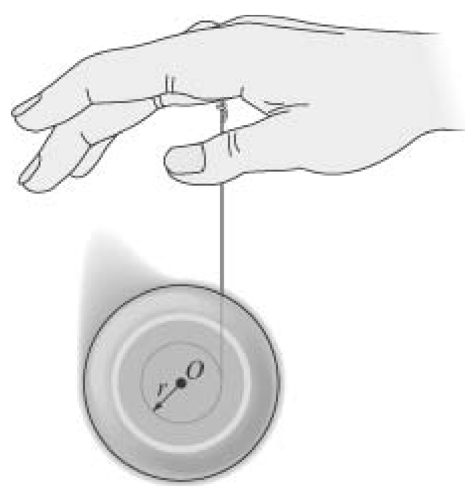
$W = 0.3\ lb$

$k_O = 0.06\ ft$

$\omega = 70\ \frac{rad}{s}$

$r = 0.02\ ft$

$g = 32.2\ \frac{ft}{s^2}$



Solution:

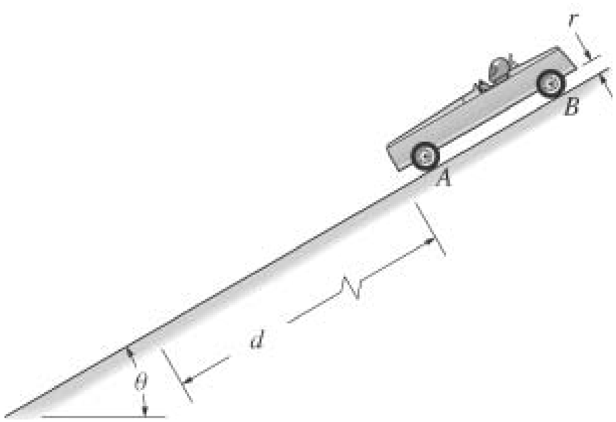
$$0 + W h = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^{2}\right)\omega^2$$

$$h = \frac{r^2 + k_O^{2}}{2g}\omega^2 \qquad h = 0.304\,\text{ft}$$

***Problem 18-12**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w, radius r, and radius of gyration k, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled a distance d starting from rest. The wheels roll without slipping. Neglect air resistance.

Given:



$W_c = 110\ \text{lb}$

$W_w = 5\ \text{lb}$

$r = 0.5\ \text{ft}$

$k = 0.3\ \text{ft}$

$d = 100\ \text{ft}$

$\theta = 30\ \text{deg}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

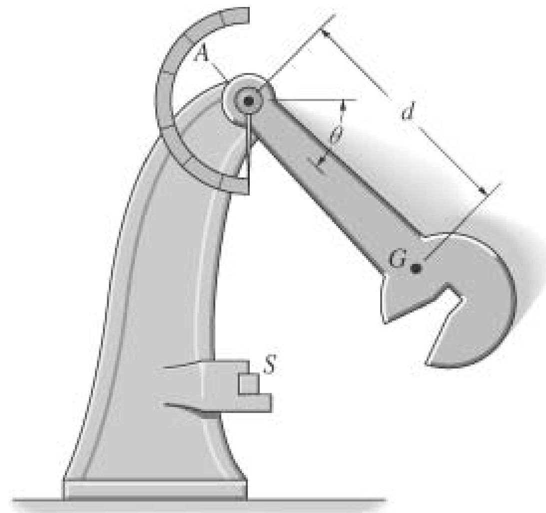
$$\left(W_c + 4W_w\right)d\sin(\theta) = \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right)v^2 + \frac{1}{2}4\left(\frac{W_w}{g}k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2\left(W_c + 4W_w\right)d\sin(\theta)g}{W_c + 4W_w + 4W_w\frac{k^2}{r^2}}} \qquad v = 55.2\,\frac{\text{ft}}{\text{s}}$$

Problem 18-13

The pendulum of the Charpy impact machine has mass M and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen S, $\theta = 90°$.

Given:



$M = 50\ \text{kg}$

$k_A = 1.75\ \text{m}$

$d = 1.25\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

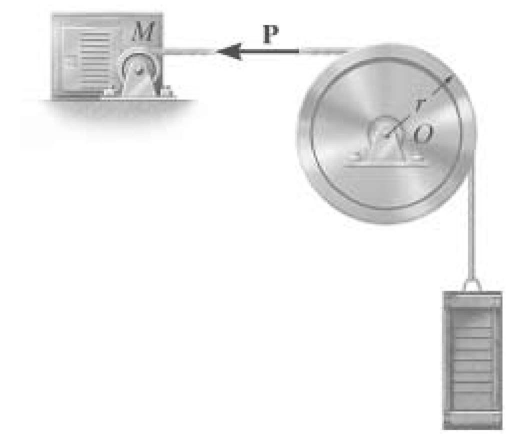
$$0 + M g d = \frac{1}{2} M k_A^2\ \omega_2^2$$

$$\omega_2 = \sqrt{\frac{2 g d}{k_A^2}} \qquad \omega_2 = 2.83\ \frac{\text{rad}}{\text{s}}$$

Problem 18-14

The pulley of mass M_p has a radius of gyration about O of k_O. If a motor M supplies a force to the cable of $P = a\,(b - ce^{-dx})$, where x is the amount of cable wound up, determine the speed of the crate of mass M_c when it has been hoisted a distance h starting from rest. Neglect the mass of the cable and assume the cable does not slip on the pulley.

Given:



$M_p = 10\ \text{kg}$ $\quad a = 800\ \text{N}$

$M_c = 50\ \text{kg}$ $\quad b = 3$

$k_O = 0.21\ \text{m}$ $\quad c = 2$

$r = 0.3\ \text{m}$ $\quad d = \dfrac{1}{\text{m}}$

$h = 2\ \text{m}$

Solution:

Guesses $\quad v_c = 1\ \dfrac{\text{m}}{\text{s}}$

Given $$\int_0^h a\left(b - c e^{-dx}\right) dx = \frac{1}{2} M_p k_O^2 \left(\frac{v_c}{r}\right)^2 + \frac{1}{2} M_c v_c^2 + M_c g h$$

$v_c = \mathrm{Find}(v_c)$ $\quad v_c = 9.419\ \frac{\mathrm{m}}{\mathrm{s}}$

Problem 18-15

The uniform pipe has a mass M and radius of gyration about the z axis of k_G. If the worker pushes on it with a horizontal force F, applied perpendicular to the pipe, determine the pipe's angular velocity when it has rotated through angle θ about the z axis, starting from rest. Assume the pipe does not swing.

Units Used:

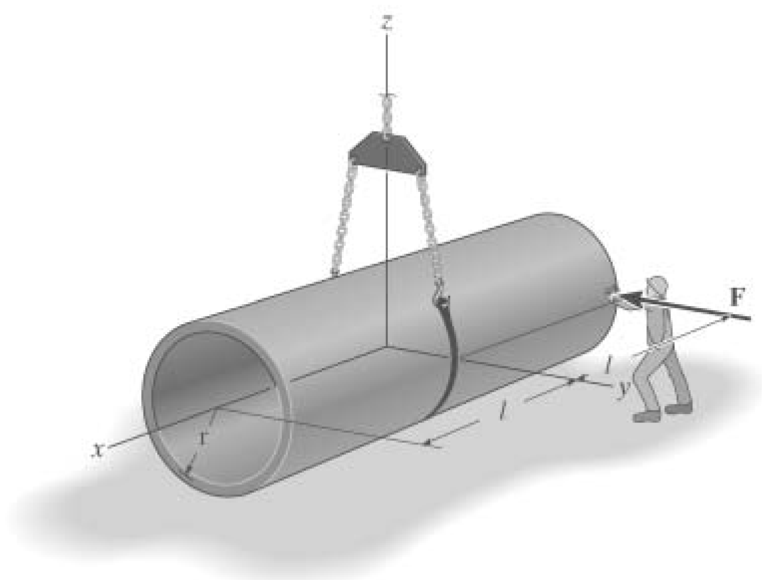
$\mathrm{Mg} = 10^3\ \mathrm{kg}$

Given:

$M = 16\ \mathrm{Mg}$ $\quad \theta = 90\ \mathrm{deg}$

$k_G = 2.7\ \mathrm{m}$ $\quad r = 0.75\ \mathrm{m}$

$F = 50\ \mathrm{N}$ $\quad l = 3\ \mathrm{m}$



Solution:

$$0 + F l \theta = \frac{1}{2} M k_G^2 \omega^2$$

$\omega = \frac{1}{M}\frac{\sqrt{M F l \pi}}{k_G}$ $\quad \omega = 0.0636\ \frac{\mathrm{rad}}{\mathrm{s}}$

***Problem 18-16**

The slender rod of mass m_{rod} is subjected to the force and couple moment. When it is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated downward 90°. The force is always applied perpendicular to the axis of the rod. Motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$

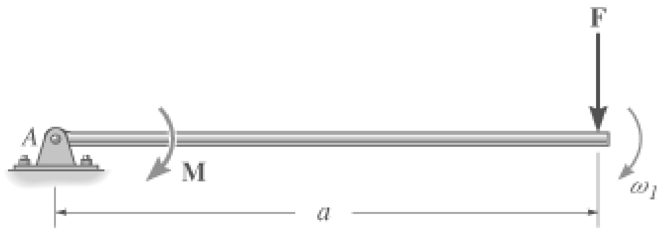
$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$

$F = 15\ \text{N}$

$M = 40\ \text{N}\cdot\text{m}$

$a = 3\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1^{\,2} + F a\left(\frac{\pi}{2}\right) + m_{rod}g\left(\frac{a}{2}\right) + M\frac{\pi}{2} = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2^{\,2}$$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = 8.25\ \frac{\text{rad}}{\text{s}}$

Problem 18-17

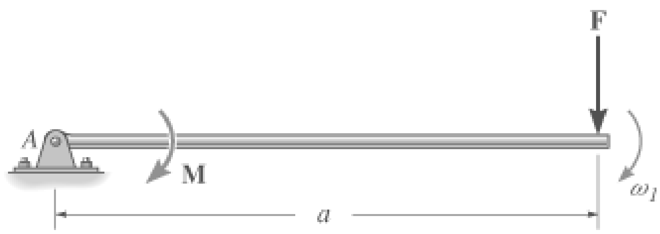
The slender rod of mass M is subjected to the force and couple moment. When the rod is in the position shown it has angular velocity ω_1. Determine its angular velocity at the instant it has rotated 360°. The force is always applied perpendicular to the axis of the rod and motion occurs in the vertical plane.

Given:

$m_{rod} = 4\ \text{kg}$ $M = 40\ \text{N}\cdot\text{m}$

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}}$ $a = 3\ \text{m}$

$F = 15\ \text{N}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\omega_2 = 1\ \frac{\text{rad}}{\text{s}}$

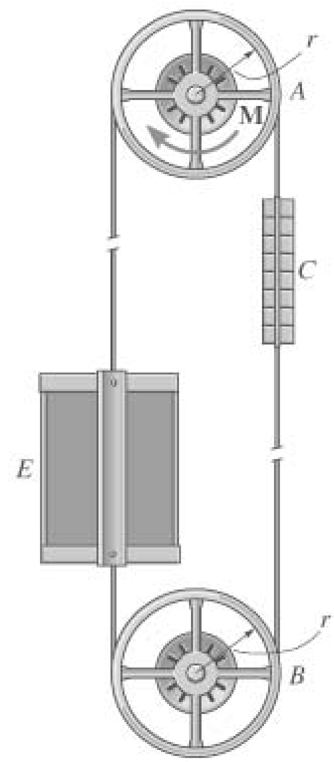
Given

$$\frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_1{}^2 + F\,a2\pi + M2\pi = \frac{1}{2}\left(\frac{m_{rod}a^2}{3}\right)\omega_2{}^2$$

$\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = 11.2\ \frac{\text{rad}}{\text{s}}$

Problem 18-18

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with constant torque M, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.



Units Used: $\text{Mg} = 1000\ \text{kg}$

Given:

$m_E = 1.80\ \text{Mg}$ $d = 10\ \text{m}$

$m_C = 2.30\ \text{Mg}$ $r = 0.35\ \text{m}$

$m_S = 150\ \text{kg}$ $k = 0.2\ \text{m}$

$M = 100\ \text{N}\cdot\text{m}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$M\frac{d}{r} - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$$

$$v = \sqrt{\frac{2\left[M\frac{d}{r} - (m_E - m_C)g\,d\right]}{m_E + m_C + 2m_S\frac{k^2}{r^2}}}$$ $v = 4.973\ \frac{\text{m}}{\text{s}}$

Problem 18-19

The elevator car E has mass m_E and the counterweight C has mass m_C. If a motor turns the driving sheave A with torque $a\theta^2 + b$, determine the speed of the elevator when it has ascended a distance d starting from rest. Each sheave A and B has mass m_S and radius of gyration k about its mass center or pinned axis. Neglect the mass of the cable and assume the cable does not slip on the sheaves.

Units Used:

$Mg = 1000\ \text{kg}$

Given:

$m_E = 1.80\ \text{Mg}$

$m_C = 2.30\ \text{Mg}$

$m_S = 150\ \text{kg}$

$a = 0.06\ \text{N}\cdot\text{m}$

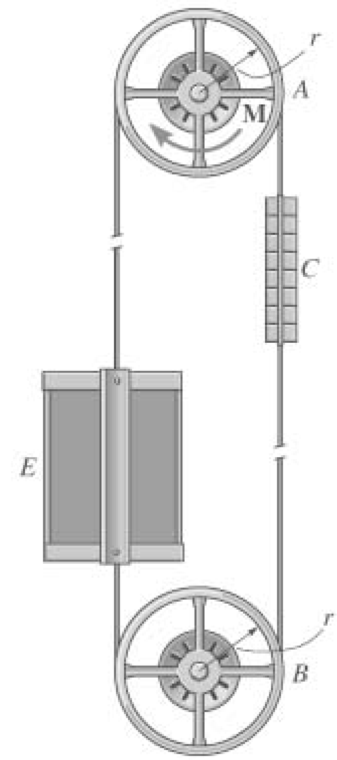
$b = 7.5\ \text{N}\cdot\text{m}$

$d = 12\ \text{m}$

$r = 0.35\ \text{m}$

$k = 0.2\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $v = 1\ \frac{\text{m}}{\text{s}}$

Given $$\int_0^{\frac{d}{r}} a\theta^2 + b\,d\theta - (m_E - m_C)g\,d = \frac{1}{2}\left(m_E + m_C + 2m_S\frac{k^2}{r^2}\right)v^2$$

$v = \text{Find}(v)$ $\quad v = 5.343\ \frac{\text{m}}{\text{s}}$

***Problem 18-20**

The wheel has a mass M_1 and a radius of gyration k_O. A motor supplies a torque **M** = $(a\theta + b)$, about the drive shaft at O. Determine the speed of the loading car, which has a mass M_2, after it travels a distance $s = d$. Initially the car is at rest when $s = 0$ and $\theta = 0°$. Neglect the mass of the attached cable and the mass of the car's wheels.

Given:

$M_1 = 100$ kg

$M_2 = 300$ kg

$d = 4$ m

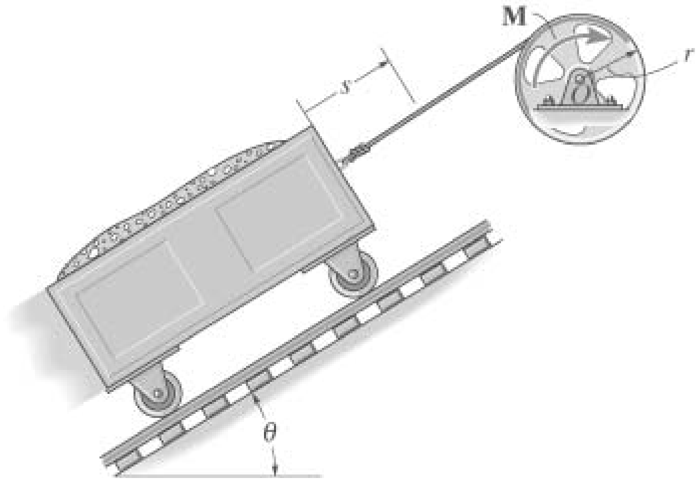
$a = 40$ N·m

$b = 900$ N·m

$r = 0.3$ m

$k_O = 0.2$ m

$\theta = 30$ deg



Solution:

Guess $v = 1 \frac{\text{m}}{\text{s}}$

Given $$\int_0^{\frac{d}{r}} (a\theta + b)\, d\theta = \frac{1}{2} M_2 v^2 + \frac{1}{2} M_1 k_O{}^2 \left(\frac{v}{r}\right)^2 + M_2\, g\, d \sin(\theta)$$

$v = \text{Find}(v)$ $v = 7.49 \frac{\text{m}}{\text{s}}$

Problem 18-21

The gear has a weight W and a radius of gyration k_G. If the spring is unstretched when the torque M is applied, determine the gear's angular velocity after its mass center G has moved to the left a distance d.

Given:

$W = 15$ lb

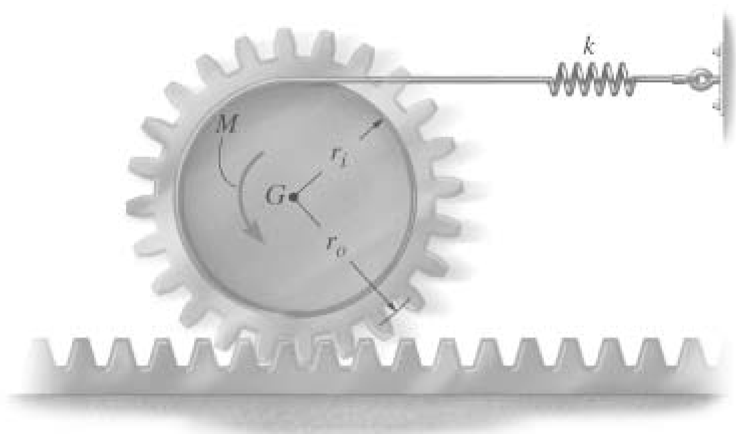
$M = 6$ lb·ft

$r_o = 0.5$ ft

$r_i = 0.4$ ft

$d = 2$ ft

$k = 3 \frac{\text{lb}}{\text{ft}}$



$k_G = 0.375\ \text{ft}$

Solution:

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $M\left(\frac{d}{r_o}\right) = \frac{1}{2}\left(\frac{W}{g}\right)(\omega r_o)^2 + \frac{1}{2}\left(\frac{W}{g}\right)k_G{}^2\,\omega^2 + \frac{1}{2}k\left(\frac{r_i + r_o}{r_o}d\right)^2$

$\omega = \text{Find}(\omega)$ $\omega = 7.08\ \frac{\text{rad}}{\text{s}}$

Problem 18-22

The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine its angular velocity at the instant its mass center G has moved distance d down along the inclined plane. The disk rolls without slipping.

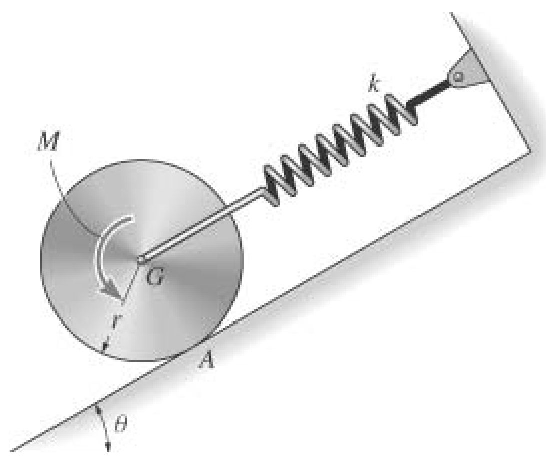
Given:

$m_d = 20\ \text{kg}$ $\theta = 30\ \text{deg}$

$M = 30\ \text{N}\cdot\text{m}$ $r = 0.2\ \text{m}$

$d = 0.8\ \text{m}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$k = 150\ \frac{\text{N}}{\text{m}}$



Solution: Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Initial stretch in the spring $k\,d_0 = m_d\,g\sin(\theta)$

$d_0 = \frac{m_d\,g\sin(\theta)}{k}$ $d_0 = 0.654\ \text{m}$

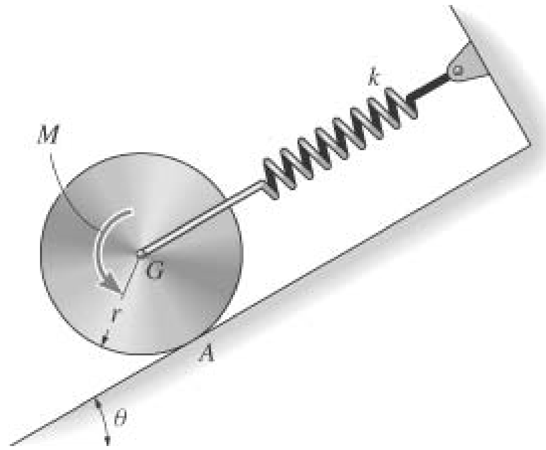
Given

$$M\frac{d}{r} + m_d\,g\,d\sin(\theta) - \frac{k}{2}\left[(d + d_0)^2 - d_0{}^2\right] = \frac{1}{2}m_d(\omega r)^2 + \frac{1}{2}\left(\frac{1}{2}m_d\,r^2\right)\omega^2$$

$\omega = \text{Find}(\omega)$ $\qquad \omega = 11.0\,\frac{\text{rad}}{\text{s}}$

Problem 18-23

The disk of mass m_d is originally at rest, and the spring holds it in equilibrium. A couple moment M is then applied to the disk as shown. Determine how far the center of mass of the disk travels down along the incline, measured from the equilibrium position, before it stops. The disk rolls without slipping.



Given:

$m_d = 20\ \text{kg}$

$M = 30\ \text{N}\cdot\text{m}$

$k = 150\ \frac{\text{N}}{\text{m}}$

$\theta = 30\ \text{deg}$

$r = 0.2\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $d = 3\ \text{m}$

Initial stretch in the spring $k\,d_0 = m_d\,g\,\sin(\theta)$

$$d_0 = \frac{m_d\,g\,\sin(\theta)}{k} \qquad d_0 = 0.654\ \text{m}$$

Given $$M\frac{d}{r} + m_d\,g\,d\,\sin(\theta) - \frac{k}{2}\left[\left(d + d_0\right)^2 - d_0^{\,2}\right] = 0$$

$d = \text{Find}(d)$ $\qquad d = 2\ \text{m}$

***Problem 18-24**

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M

and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.

Given:

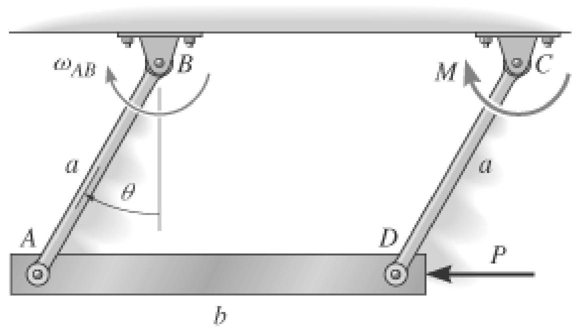
$W_1 = 8\ \text{lb}$ $\quad a = 2\ \text{ft}$

$W_2 = 10\ \text{lb}$ $\quad b = 3\ \text{ft}$

$\omega_0 = 2\ \frac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 90\ \text{deg}$

$P = 20\ \text{lb}$ $\quad M = 15\ \text{lb}\cdot\text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$U = P a \sin(\theta_1) + M\theta_1 - 2W_1 \frac{a}{2}(1 - \cos(\theta_1)) - W_2 a(1 - \cos(\theta_1))$$

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

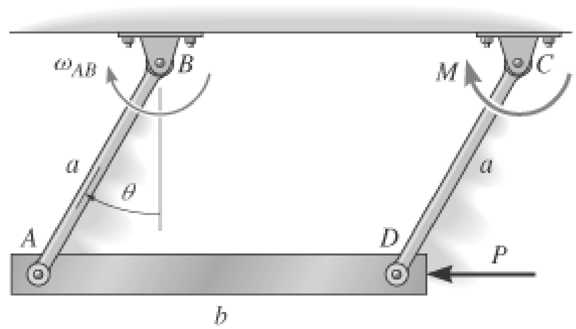
Given

$$\frac{1}{2} 2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega_0)^2 + U = \frac{1}{2} 2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

$\omega = \text{Find}(\omega)$ $\quad \omega = 5.739\ \frac{\text{rad}}{\text{s}}$

Problem 18-25

The linkage consists of two rods AB and CD each of weight W_1 and bar AD of weight W_2. When $\theta = 0$, rod AB is rotating with angular velocity ω_O. If rod CD is subjected to a couple moment M and bar AD is subjected to a horizontal force P as shown, determine ω_{AB} at the instant $\theta = \theta_1$.



Given:

$W_1 = 8\ \text{lb}$ $\quad a = 2\ \text{ft}$

$W_2 = 10\ \text{lb}$ $\quad b = 3\ \text{ft}$

$\omega_0 = 2\ \dfrac{\text{rad}}{\text{s}}$ $\quad \theta_1 = 45\ \text{deg}$

$M = 15\ \text{lb}\cdot\text{ft}$ $\quad P = 20\ \text{lb}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$U = P\,a\sin(\theta_1) + M\theta_1 - 2W_1\frac{a}{2}(1-\cos(\theta_1)) - W_2\,a(1-\cos(\theta_1))$$

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given
$$\frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega_0^{\,2} + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega_0)^2 + U = \frac{1}{2}2\left(\frac{W_1}{g}\frac{a^2}{3}\right)\omega^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)(a\omega)^2$$

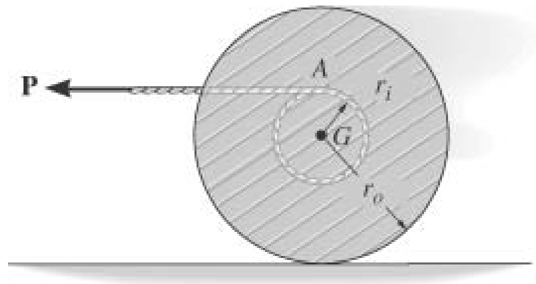
$\omega = \text{Find}(\omega)$ $\qquad \omega = 5.916\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-26

The spool has weight W and radius of gyration k_G. A horizontal force P is applied to a cable wrapped around its inner core. If the spool is originally at rest, determine its angular velocity after the mass center G has moved distance d to the left. The spool rolls without slipping. Neglect the mass of the cable.

Given:

$W = 500\ \text{lb}$ $\quad d = 6\ \text{ft}$



$k_G = 1.75$ ft $\quad r_i = 0.8$ ft

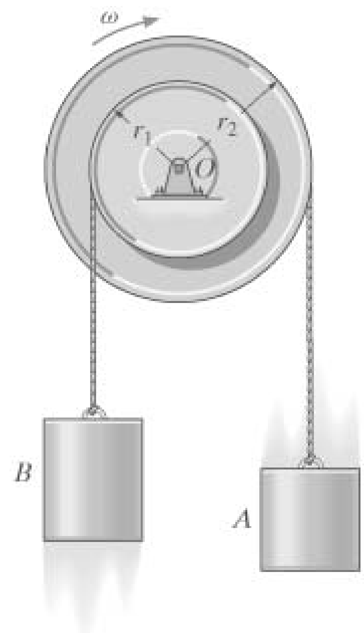
$P = 15$ lb $\quad r_o = 2.4$ ft

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution: Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $P\left(\frac{r_o + r_i}{r_o}\right)d = \frac{1}{2}\left(\frac{W}{g}\right)k_G^2\,\omega^2 + \frac{1}{2}\left(\frac{W}{g}\right)(r_o\omega)^2 \qquad \omega = \text{Find}(\omega) \qquad \omega = 1.324\ \frac{\text{rad}}{\text{s}}$

Problem 18-27



The double pulley consists of two parts that are attached to one another. It has a weight W_p and a centroidal radius of gyration k_O and is turning with an angular velocity ω clockwise. Determine the kinetic energy of the system. Assume that neither cable slips on the pulley.

Given:

$W_P = 50$ lb $\quad r_1 = 0.5$ ft

$W_A = 20$ lb $\quad r_2 = 1$ ft

$W_B = 30$ lb $\quad k_O = 0.6$ ft

$\omega = 20\ \frac{\text{rad}}{\text{s}}$

Solution:

$$K_E = \frac{1}{2}I\omega^2 + \frac{1}{2}W_A v_A^2 + \frac{1}{2}W_B v_B^2$$

$$K_E = \frac{1}{2}\left(\frac{W_P}{g}\right)k_O^2\,\omega^2 + \frac{1}{2}\left(\frac{W_A}{g}\right)(r_2\omega)^2 + \frac{1}{2}\left(\frac{W_B}{g}\right)(r_1\omega)^2$$

$K_E = 283\ \text{ft}\cdot\text{lb}$

***Problem 18-28**

The system consists of disk *A* of weight W_A, slender rod *BC* of weight W_{BC}, and smooth collar *C* of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$.

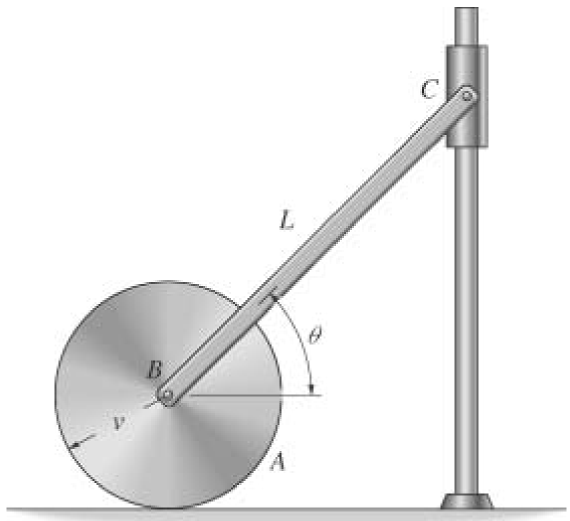
Given:

$W_A = 20 \text{ lb}$ $\quad L = 3 \text{ ft}$

$W_{BC} = 4 \text{ lb}$ $\quad r = 0.8 \text{ ft}$

$W_C = 1 \text{ lb}$ $\quad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45 \text{ deg}$



Solution:

Guess $\quad v_C = 1 \frac{\text{ft}}{\text{s}}$

Given

$$W_{BC}\frac{L}{2}\cos(\theta_0) + W_C L\cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2$$

$v_C = \text{Find}(v_C) \qquad v_C = 13.3 \frac{\text{ft}}{\text{s}}$

Problem 18-29

The cement bucket of weight W_1 is hoisted using a motor that supplies a torque **M** to the axle of the wheel. If the wheel has a weight W_2 and a radius of gyration about *O* of k_O, determine the speed of the bucket when it has been hoisted a distance *h* starting from rest.

Given:

$W_1 = 1500 \text{ lb}$

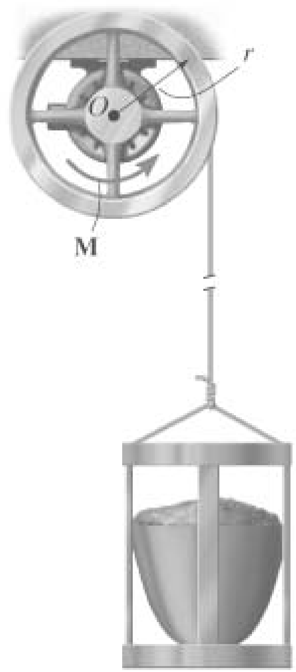
$W_2 = 115 \text{ lb}$

$M = 2000 \text{ lb}\cdot\text{ft}$

$k_O = 0.95 \text{ ft}$

$h = 10 \text{ ft}$

$r = 1.25 \text{ ft}$



Solution:

Guess $v = 1 \frac{\text{ft}}{\text{s}}$

Given $$M\frac{h}{r} = \frac{1}{2}\left(\frac{W_1}{g}\right)v^2 + \frac{1}{2}\left(\frac{W_2}{g}\right)k_O^2\left(\frac{v}{r}\right)^2 + W_1 h$$

$v = \text{Find}(v)$ $\quad v = 6.41 \frac{\text{ft}}{\text{s}}$

Problem 18-30

The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0$. The disk rolls without slipping.

Given:

$W_r = 15 \text{ lb}$

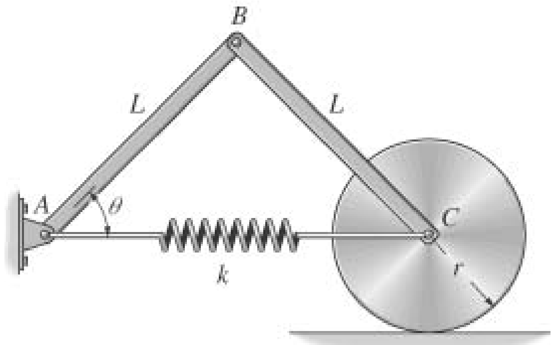
$W_d = 20 \text{ lb}$

$\theta_1 = 45 \text{ deg}$

$k = 4 \frac{\text{lb}}{\text{ft}}$

$L = 3 \text{ ft}$

$r = 1 \text{ ft}$



Solution:

Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

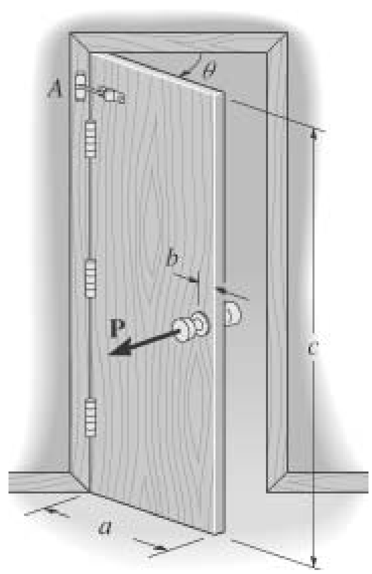
Given $$2W_r\left(\frac{L}{2}\right)\sin\left(\theta_1\right) - \frac{1}{2}k\left(2L - 2L\cos\left(\theta_1\right)\right)^2 = 2\frac{1}{2}\left(\frac{1}{3}\frac{W_r}{g}L^2\right)\omega^2$$

$\omega = \mathrm{Find}(\omega)$ $\omega = 4.284\ \dfrac{\mathrm{rad}}{\mathrm{s}}$

Problem 18-31

The uniform door has mass *M* and can be treated as a thin plate having the dimensions shown. If it is connected to a torsional spring at *A*, which has stiffness *k*, determine the required initial twist of the spring in radians so that the door has an angular velocity ω when it closes at $\theta = 0°$ after being opened at $\theta = 90°$ and released from rest. *Hint:* For a torsional spring $M = k\theta$, where *k* is the stiffness and θ is the angle of twist.

Given:



$M = 20\ \mathrm{kg}$ $a = 0.8\ \mathrm{m}$

$k = 80\ \dfrac{\mathrm{N\cdot m}}{\mathrm{rad}}$ $b = 0.1\ \mathrm{m}$

$\omega = 12\ \dfrac{\mathrm{rad}}{\mathrm{s}}$ $c = 2\ \mathrm{m}$

$P = 0\ \mathrm{N}$

Solution:

Guess $\theta_0 = 1\ \mathrm{rad}$

Given

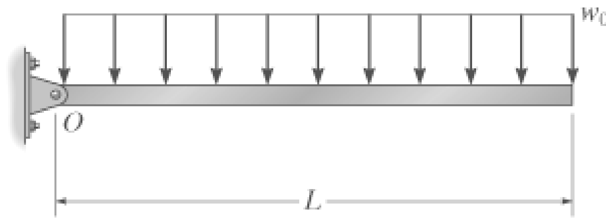
$$\int_{\theta_0+90\ \mathrm{deg}}^{\theta_0} -k\,\theta\,d\theta = \frac{1}{2}\frac{1}{3}M\,a^2\,\omega^2$$

$\theta_0 = \mathrm{Find}(\theta_0)$ $\theta_0 = 1.659\ \mathrm{rad}$

***Problem 18-32**

The uniform slender bar has a mass *m* and a length *L*. It is subjected to a uniform distributed load w_0 which is always directed perpendicular to the axis of the bar. If it is released from the position shown, determine its angular velocity at the instant it has rotated 90°. Solve the problem for rotation in (a) the horizontal plane, and (b) the vertical plane.

Solution:



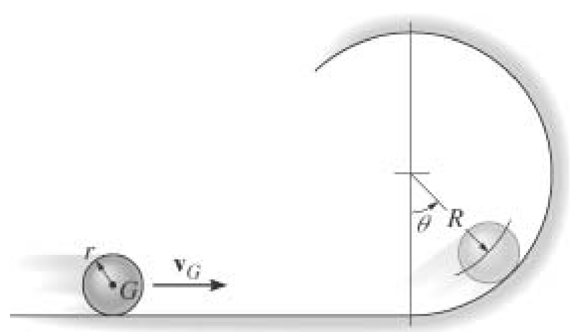
$$\int_0^{\frac{\pi}{2}} \int_0^L x w_0 \, dx \, d\theta = \frac{1}{4}\pi L^2 w_0$$

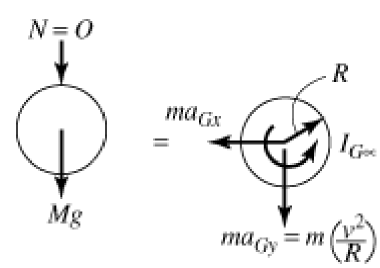
(a) $$\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2 \qquad \omega = \sqrt{\frac{3\pi w_0}{2m}}$$

(b) $$\frac{1}{4}\pi L^2 w_0 = \frac{1}{2}\left(\frac{1}{3}m L^2\right)\omega^2 - m g \frac{L}{2} \qquad \omega = \sqrt{\frac{3\pi w_0}{2m} + \frac{3g}{L}}$$

Problem 18-33

A ball of mass m and radius r is cast onto the horizontal surface such that it rolls without slipping. Determine the minimum speed v_G of its mass center G so that it rolls completely around the loop of radius $R + r$ without leaving the track.





Solution:

$$m g = m\left(\frac{v^2}{R}\right) \qquad v^2 = g R$$

$$\frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v_G}{r}\right)^2 + \frac{1}{2}m v_G^2 - m g 2R = \frac{1}{2}\left(\frac{2}{5}m r^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}m v^2$$

$$\frac{1}{5}v_G^2 + \frac{1}{2}v_G^2 = 2g R + \frac{1}{5}g R + \frac{1}{2}g R \qquad v_G = 3\sqrt{\frac{3}{7}g R}$$

Problem 18-34

The beam has weight *W* and is being raised to a vertical position by pulling very slowly on its bottom end *A*. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of *A* at the instant cord *BC* becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod.

Given:

$W = 1500\ \text{lb}$

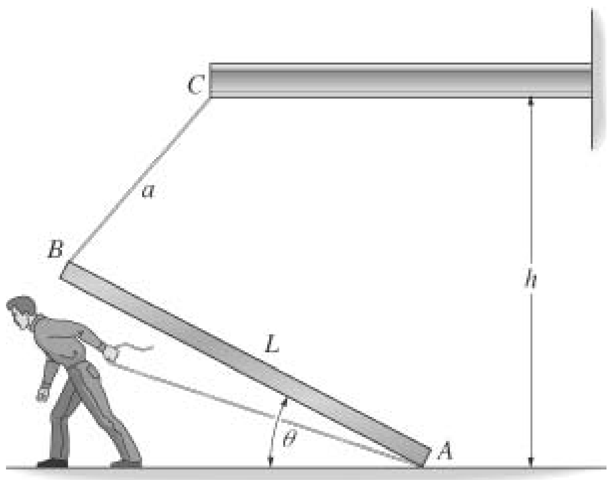
$\theta_1 = 60\ \text{deg}$

$L = 13\ \text{ft}$

$h = 12\ \text{ft}$

$a = 7\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



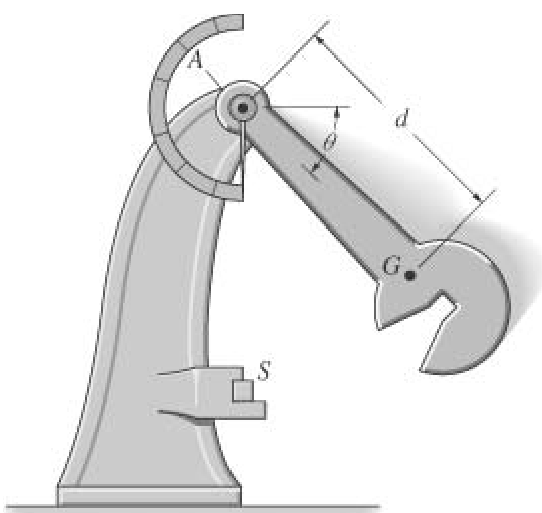
Solution:

$$W\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right] = \frac{1}{2}\left(\frac{W}{g}\right)v_A^2$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2\ \frac{\text{ft}}{\text{s}}$$

Problem 18-35

The pendulum of the Charpy impact machine has mass *M* and radius of gyration k_A. If it is released from rest when $\theta = 0°$, determine its angular velocity just before it strikes the specimen *S*, $\theta = 90°$, using the conservation of energy equation.



Given:

$M = 50\ \text{kg}$

$k_A = 1.75\ \text{m}$

$d = 1.25\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$0 + M\,g\,d = 0 + \frac{1}{2}M\,k_A^{\,2}\,\omega_2^{\,2} \qquad \omega_2 = \sqrt{\frac{2g\,d}{k_A^{\,2}}}$$

$$\omega_2 = 2.83\,\frac{\text{rad}}{\text{s}}$$

***Problem 18-36**

The soap-box car has weight W_c including the passenger but *excluding* its four wheels. Each wheel has weight W_w radius r, and radius of gyration k, computed about an axis passing through the wheel's axle. Determine the car's speed after it has traveled distance d starting from rest. The wheels roll without slipping. Neglect air resistance. Solve using conservation of energy.

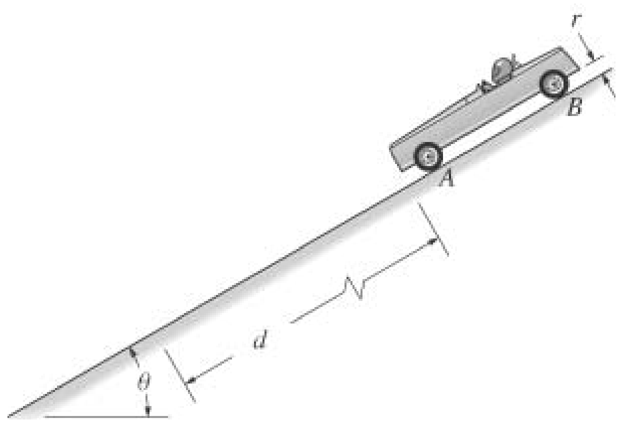
Given:

$W_c = 110\ \text{lb}$ $d = 100\ \text{ft}$

$W_w = 5\ \text{lb}$ $\theta = 30\ \text{deg}$

$r = 0.5\ \text{ft}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k = 0.3\ \text{ft}$



Solution:

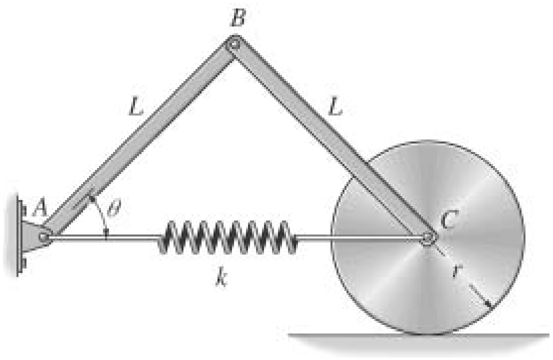
$$0 + (W_c + 4W_w)d\sin(\theta) = 0 + \frac{1}{2}\left(\frac{W_c + 4W_w}{g}\right)v^2 + \frac{1}{2}4\left(\frac{W_w}{g}k^2\right)\left(\frac{v}{r}\right)^2$$

$$v = \sqrt{\frac{2(W_c + 4W_w)d\sin(\theta)g}{W_c + 4W_w + 4W_w\left(\frac{k^2}{r^2}\right)}} \qquad v = 55.2\,\frac{\text{ft}}{\text{s}}$$

Problem 18-37

The assembly consists of two slender rods each of weight W_r and a disk of weight W_d. If the spring is unstretched when $\theta = \theta_1$ and the assembly is released from rest at this position, determine the angular velocity of rod AB at the instant $\theta = 0$. The disk rolls without slipping. Solve using the conservation of energy.

Given:



$W_r = 15\ \text{lb}$ $\quad k = 4\ \dfrac{\text{lb}}{\text{ft}}$

$W_d = 20\ \text{lb}$ $\quad L = 3\ \text{ft}$

$\theta_1 = 45\ \text{deg}$ $\quad r = 1\ \text{ft}$

Solution: Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

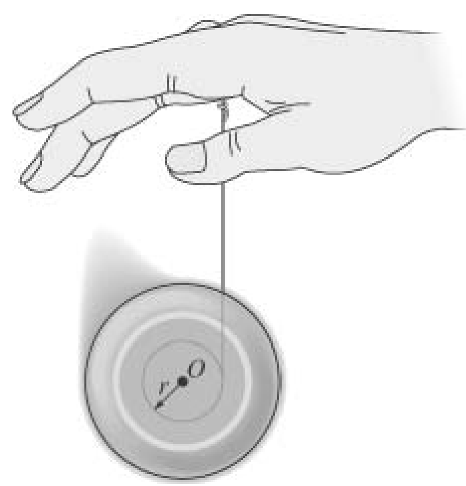
$$0 + 2W_r\left(\frac{L}{2}\right)\sin(\theta_1) = 2\frac{1}{2}\left[\frac{1}{3}\left(\frac{W_r}{g}\right)L^2\right]\omega^2 + \frac{1}{2}k\left(2L - 2L\cos(\theta_1)\right)^2$$

$\omega = \text{Find}(\omega)$ $\qquad \omega = 4.284\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-38

A yo-yo has weight W and radius of gyration k_O. If it is released from rest, determine how far it must descend in order to attain angular velocity ω. Neglect the mass of the string and assume that the string is wound around the central peg such that the mean radius at which it unravels is r. Solve using the conservation of energy.

Given:



$W = 0.3\ \text{lb}$

$k_O = 0.06\ \text{ft}$

$\omega = 70\ \dfrac{\text{rad}}{\text{s}}$

$r = 0.02\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

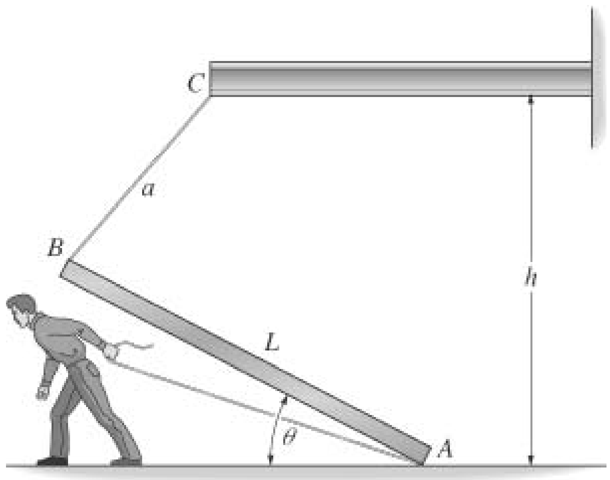
$$0 + W s = \frac{1}{2}\left(\frac{W}{g}\right)(r\omega)^2 + \frac{1}{2}\left(\frac{W}{g}k_O^2\right)\omega^2 + 0$$

$s = \left(\dfrac{r^2 + k_O^2}{2g}\right)\omega^2$ $\qquad s = 0.304\ \text{ft}$

Problem 18-39

The beam has weight W and is being raised to a vertical position by pulling very slowly on its bottom end *A*. If the cord fails when $\theta = \theta_1$ and the beam is essentially at rest, determine the speed of *A* at the instant cord *BC* becomes vertical. Neglect friction and the mass of the cords, and treat the beam as a slender rod. Solve using the conservation of energy.

Given:



$W = 1500\ \text{lb}$

$\theta_1 = 60\ \text{deg}$

$L = 13\ \text{ft}$

$h = 12\ \text{ft}$

$a = 7\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

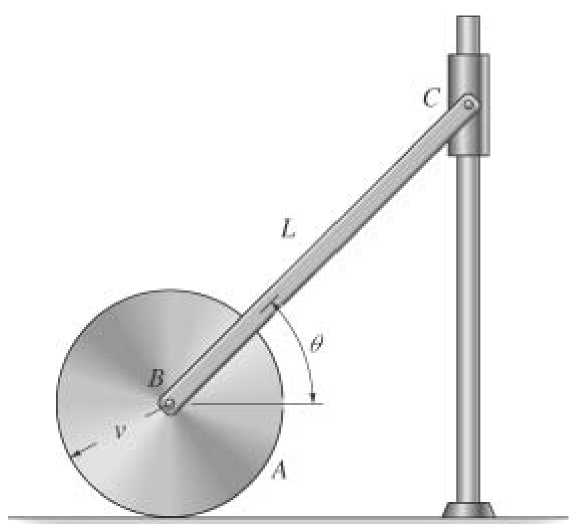
Solution:

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right)v_A^{\ 2} + W\left(\frac{h-a}{2}\right)$$

$$v_A = \sqrt{2g\left[\frac{L}{2}\sin(\theta_1) - \left(\frac{h-a}{2}\right)\right]} \qquad v_A = 14.2\ \frac{\text{ft}}{\text{s}}$$

***Problem 18-40**

The system consists of disk *A* of weight W_A, slender rod *BC* of weight W_{BC}, and smooth collar *C* of weight W_C. If the disk rolls without slipping, determine the velocity of the collar at the instant the rod becomes horizontal, i.e. $\theta = 0°$. The system is released from rest when $\theta = \theta_0$. Solve using the conservation of energy.



Given:

$W_A = 20\ \text{lb} \qquad L = 3\ \text{ft}$

$W_{BC} = 4\ \text{lb} \qquad r = 0.8\ \text{ft}$

$W_C = 1 \text{ lb} \qquad g = 32.2 \frac{\text{ft}}{\text{s}^2}$

$\theta_0 = 45 \text{ deg}$

Solution:

Guess $\quad v_C = 1 \frac{\text{ft}}{\text{s}}$

Given

$$0 + W_{BC}\left(\frac{L}{2}\right)\cos(\theta_0) + W_C L\cos(\theta_0) = \frac{1}{2}\left(\frac{W_C}{g}\right)v_C^{\,2} + \frac{1}{2}\left(\frac{W_{BC}}{g}\frac{L^2}{3}\right)\left(\frac{v_C}{L}\right)^2 + 0$$

$v_C = \text{Find}(v_C) \qquad v_C = 13.3 \frac{\text{ft}}{\text{s}}$

Problem 18-41

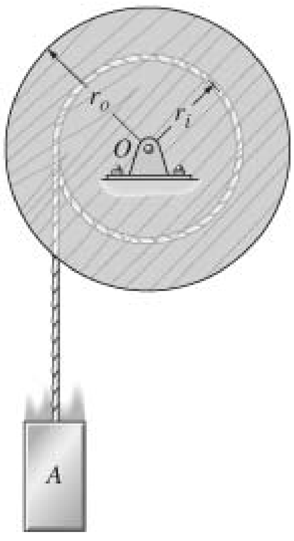
The spool has mass m_S and radius of gyration k_O. If block A of mass m_A is released from rest, determine the distance the block must fall in order for the spool to have angular velocity ω. Also, what is the tension in the cord while the block is in motion? Neglect the mass of the cord.

Given:

$m_S = 50 \text{ kg} \qquad r_i = 0.2 \text{ m} \qquad g = 9.81 \frac{\text{m}}{\text{s}^2}$

$m_A = 20 \text{ kg} \qquad r_o = 0.3 \text{ m}$

$\omega = 5 \frac{\text{rad}}{\text{s}} \qquad k_O = 0.280 \text{ m}$



Solution:

Guesses $\quad d = 1 \text{ m} \qquad T = 1 \text{ N}$

Given

$$0 + 0 = \frac{1}{2}m_S k_O^{\,2}\,\omega^2 + \frac{1}{2}m_A(r_i\omega)^2 - m_A g d$$

$$0 + 0 - T d = \frac{1}{2}m_A(r_i\omega)^2 - m_A g d$$

$\begin{pmatrix} d \\ T \end{pmatrix} = \text{Find}(d, T) \qquad d = 0.301 \text{ m} \qquad T = 163 \text{ N}$

Problem 18-42

When slender bar AB of mass M is horizontal it is at rest and the spring is unstretched. Determine the stiffness k of the spring so that the motion of the bar is momentarily stopped when it has rotated downward 90°.

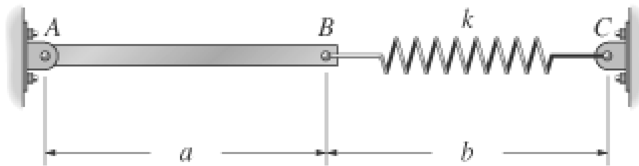
Given:

$M = 10\ \text{kg}$

$a = 1.5\ \text{m}$

$b = 1.5\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$



Solution:

$$0 + 0 = 0 + \frac{1}{2}k\left[\sqrt{(a+b)^2 + a^2} - b\right]^2 - Mg\frac{a}{2}$$

$$k = \frac{Mga}{\left[\sqrt{(a+b)^2 + a^2} - b\right]^2} \qquad \boxed{k = 42.8\ \frac{\text{N}}{\text{m}}}$$

Problem 18-43

The disk of weight W is rotating about pin A in the vertical plane with an angular velocity ω_1 when $\theta = 0°$. Determine its angular velocity at the instant shown, $\theta = 90$ deg. Also, compute the horizontal and vertical components of reaction at A at this instant.

Given:

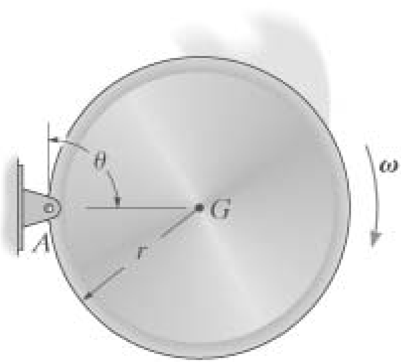
$W = 15\ \text{lb}$

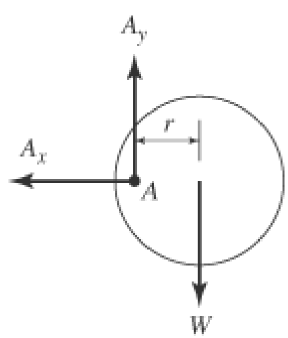
$\omega_1 = 2\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 90\ \text{deg}$

$r = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

Guesses $\quad A_x = 1\ \text{lb} \quad A_y = 1\ \text{lb} \quad \omega_2 = 1\ \dfrac{\text{rad}}{\text{s}} \quad \alpha = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given $\frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_1{}^2 + W r = \frac{1}{2}\left(\frac{3}{2}\frac{W}{g}r^2\right)\omega_2{}^2$

$$-A_x = \left(\frac{-W}{g}\right)r\omega_2{}^2 \qquad A_y - W = \left(\frac{-W}{g}\right)\alpha r \qquad -W r = \frac{-3}{2}\left(\frac{W}{g}\right)r^2\alpha$$

$$\begin{pmatrix} A_x \\ A_y \\ \omega_2 \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_y, \omega_2, \alpha) \qquad \alpha = 42.9\,\frac{\text{rad}}{\text{s}^2} \qquad \omega_2 = 9.48\,\frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} 20.9 \\ 5.0 \end{pmatrix}\text{lb}$$

***Problem 18-44**

The door is made from one piece, whose ends move along the horizontal and vertical tracks. If the door is in the open position $\theta = 0°$, and then released, determine the speed at which its end A strikes the stop at C. Assume the door is a thin plate of weight W having width c.

Given:

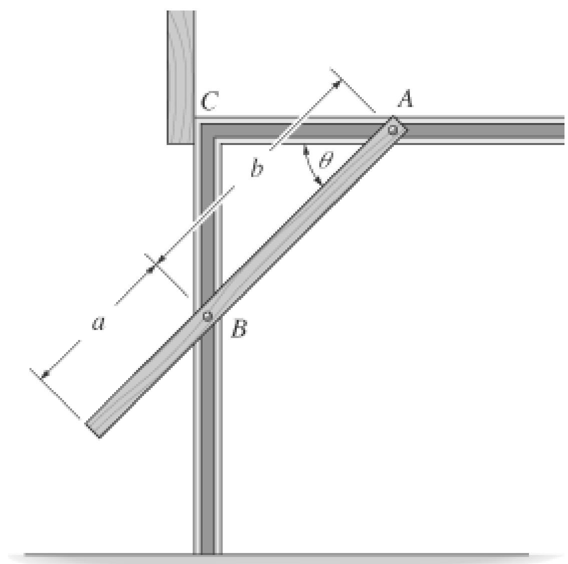
$W = 180$ lb

$a = 3$ ft

$b = 5$ ft

$c = 10$ ft

$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$



Solution:

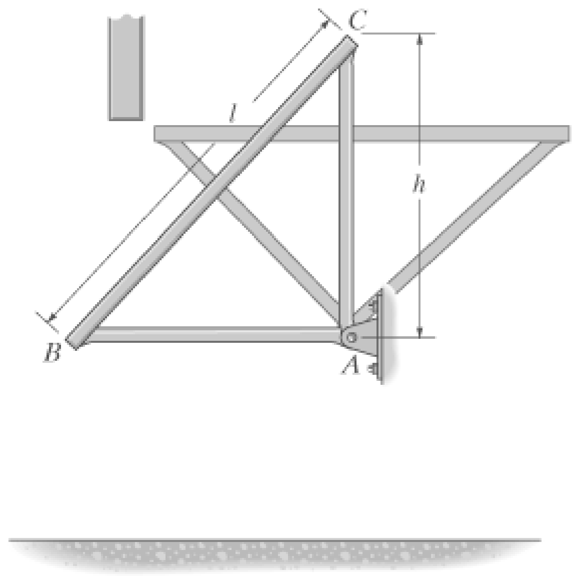
Guesses $v_A = 1\,\frac{\text{ft}}{\text{s}}$ $\qquad \omega = 1\,\frac{\text{rad}}{\text{s}}$

Given

$$0 + 0 = \frac{1}{2}\frac{W}{g}\left[\frac{(a+b)^2}{12} + \left(b - \frac{a+b}{2}\right)^2\right]\omega^2 - W\left(\frac{a+b}{2}\right) \qquad v_A = \omega b$$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \text{Find}(\omega, v_A) \qquad \omega = 6.378\,\frac{\text{rad}}{\text{s}} \qquad v_A = 31.9\,\frac{\text{ft}}{\text{s}}$$

Problem 18-45



The overhead door *BC* is pushed slightly from its open position and then rotates downward about the pin at *A*. Determine its angular velocity just before its end *B* strikes the floor. Assume the door is a thin plate having a mass *M* and length *l*. Neglect the mass of the supporting frame *AB* and *AC*.

Given:

$M = 180\ \text{kg}$

$l = 6\ \text{m}$

$h = 5\ \text{m}$

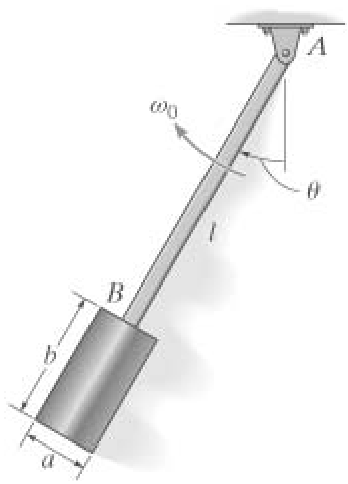
Solution: $d = \sqrt{h^2 - \left(\frac{l}{2}\right)^2}$

Guess $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $M g d = \frac{1}{2}\left(\frac{M l^2}{12} + M d^2\right)\omega^2$ $\omega = \text{Find}(\omega)$ $\omega = 2.03\ \frac{\text{rad}}{\text{s}}$

Problem 18-46

The cylinder of weight W_1 is attached to the slender rod of weight W_2 which is pinned at point *A*. At the instant $\theta = \theta_0$ the rod has an angular velocity ω_0 as shown. Determine the angle θ_f to which the rod swings before it momentarily stops.



Given:

$W_1 = 80\ \text{lb}$ $a = 1\ \text{ft}$

$W_2 = 10\ \text{lb}$ $b = 2\ \text{ft}$

$\omega_0 = 1\ \frac{\text{rad}}{\text{s}}$ $l = 5\ \text{ft}$

$\theta_0 = 30\ \text{deg}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_A = \frac{W_1}{g}\left[\frac{1}{4}\left(\frac{a}{2}\right)^2 + \frac{b^2}{12}\right] + \frac{W_1}{g}\left(l + \frac{b}{2}\right)^2 + \left(\frac{W_2}{g}\right)\frac{l^2}{3}$$

$$d = \frac{W_1\left(l + \frac{b}{2}\right) + W_2\left(\frac{l}{2}\right)}{W_1 + W_2}$$

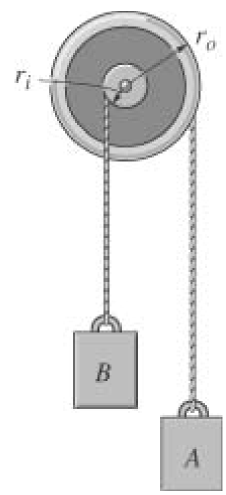
Guess $\theta_f = 1 \text{ deg}$

Given $\frac{1}{2}I_A\omega_0{}^2 - (W_1 + W_2)d\cos(\theta_0) = -(W_1 + W_2)d\cos(\theta_f)$

$\theta_f = \text{Find}(\theta_f)$ $\theta_f = 39.3\,\text{deg}$

Problem 18-47

The compound disk pulley consists of a hub and attached outer rim. If it has mass m_P and radius of gyration k_G, determine the speed of block A after A descends distance d from rest. Blocks A and B each have a mass m_b. Neglect the mass of the cords.



Given:

$m_p = 3 \text{ kg}$ $r_i = 30 \text{ mm}$ $m_b = 2 \text{ kg}$

$k_G = 45 \text{ mm}$ $r_o = 100 \text{ mm}$

$d = 0.2 \text{ m}$ $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Solution:

Guess $v_A = 1 \frac{\text{m}}{\text{s}}$

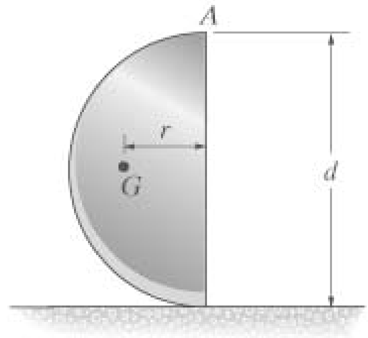
Given

$$0 + 0 = \frac{1}{2}m_b v_A{}^2 + \frac{1}{2}m_b\left(\frac{r_i}{r_o}v_A\right)^2 + \frac{1}{2}\left(m_p k_G{}^2\right)\left(\frac{v_A}{r_o}\right)^2 - m_b g d + m_b g\left(\frac{r_i}{r_o}\right)d$$

$v_A = \text{Find}(v_A)$ $v_A = 1.404 \frac{\text{m}}{\text{s}}$

***Problem 18-48**

The semicircular segment of mass M is released from rest in the position shown. Determine the velocity of point A when it has rotated counterclockwise 90°. Assume that the segment rolls without slipping on the surface. The moment of inertia about its mass center is I_G.



Given:

$M = 15 \text{ kg}$ $\quad r = 0.15 \text{ m}$

$I_G = 0.25 \text{ kg}\cdot\text{m}^2$ $\quad d = 0.4 \text{ m}$

Solution:

Guesses $\quad \omega = 1 \dfrac{\text{rad}}{\text{s}} \quad v_G = 1 \dfrac{\text{m}}{\text{s}}$

Given $\quad M g d = \dfrac{1}{2} M v_G^2 + \dfrac{1}{2} I_G \omega^2 + M g (d - r) \qquad v_G = \omega\left(\dfrac{d}{2} - r\right)$

$\begin{pmatrix} \omega \\ v_G \end{pmatrix} = \text{Find}(\omega, v_G) \qquad \omega = 12.4 \dfrac{\text{rad}}{\text{s}} \qquad v_G = 0.62 \dfrac{\text{m}}{\text{s}}$

$$\mathbf{v_A} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} \dfrac{-d}{2} \\ \dfrac{d}{2} \\ 0 \end{pmatrix} \qquad \mathbf{v_A} = \begin{pmatrix} -2.48 \\ -2.48 \\ 0.00 \end{pmatrix} \dfrac{\text{m}}{\text{s}} \qquad |\mathbf{v_A}| = 3.50 \dfrac{\text{m}}{\text{s}}$$

Problem 18-49

The uniform stone (rectangular block) of weight W is being turned over on its side by pulling the vertical cable *slowly* upward until the stone begins to tip. If it then falls freely (**T** = 0) from an essentially balanced at-rest position, determine the speed at which the corner A strikes the pad at B. The stone does not slip at its corner C as it falls.

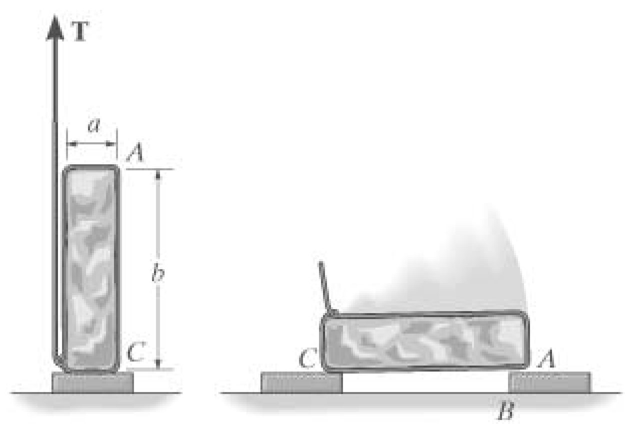
Given:

$W = 150 \text{ lb}$

$a = 0.5 \text{ ft}$

$b = 2 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1 \dfrac{\text{rad}}{\text{s}}$

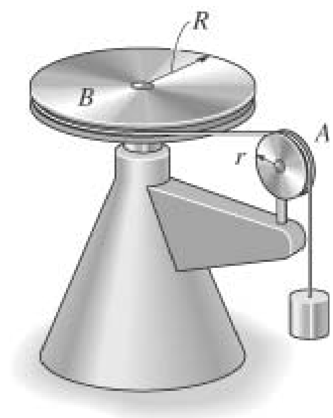
Given $$W\left(\frac{\sqrt{a^2+b^2}}{2}\right) = \frac{1}{2}\frac{W}{g}\left(\frac{a^2+b^2}{3}\right)\omega^2 + W\frac{a}{2} \qquad \omega = \text{Find}(\omega)$$

$$v_A = \omega b \qquad v_A = 11.9\ \frac{\text{ft}}{\text{s}}$$

Problem 18-50

The assembly consists of pulley A of mass m_A and pulley B of mass m_B. If a block of mass m_b is suspended from the cord, determine the block's speed after it descends a distance d starting from rest. Neglect the mass of the cord and treat the pulleys as thin disks. No slipping occurs.

Given:



$m_A = 3\ \text{kg}$

$m_B = 10\ \text{kg}$

$m_b = 2\ \text{kg}$

$d = 0.5\ \text{m}$

$r = 30\ \text{mm}$

$R = 100\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Guess $v_b = 1\ \frac{\text{m}}{\text{s}}$

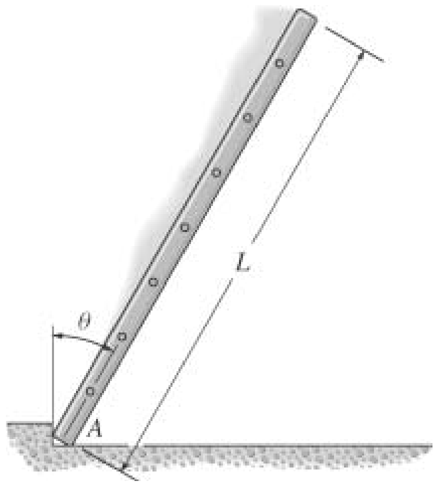
Given

$$0 + 0 = \frac{1}{2}\left(\frac{m_A r^2}{2}\right)\left(\frac{v_b}{r}\right)^2 + \frac{1}{2}\left(\frac{m_B R^2}{2}\right)\left(\frac{v_b}{R}\right)^2 + \frac{1}{2}m_b v_b^2 - m_b g d$$

$$v_b = \text{Find}(v_b) \qquad v_b = 1.519\ \frac{\text{m}}{\text{s}}$$

Problem 18-51

A uniform ladder having weight W is released from rest when it is in the vertical position. If it is allowed to fall freely, determine the angle at which the bottom end A starts to lift off the ground. For the calculation, assume the ladder to be a slender rod and neglect friction at A.



Given:

$W = 30\ \text{lb}$

$L = 10\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

(a) The rod will rotate around point A until it loses contact with the horizontal constraint ($A_x = 0$). We will find this point first

Guesses

$\theta_1 = 30\ \text{deg} \quad \omega_1 = 1\ \dfrac{\text{rad}}{\text{s}} \quad \alpha_1 = 1\ \dfrac{\text{rad}}{\text{s}^2}$

Given

$$0 + W\left(\frac{L}{2}\right) = \frac{1}{2}\left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\omega_1{}^2 + W\left(\frac{L}{2}\right)\cos(\theta_1)$$

$$W\left(\frac{L}{2}\right)\sin(\theta_1) = \left[\frac{1}{3}\left(\frac{W}{g}\right)L^2\right]\alpha_1$$

$$\alpha_1\left(\frac{L}{2}\right)\cos(\theta_1) - \omega_1{}^2\left(\frac{L}{2}\right)\sin(\theta_1) = 0$$

$$\begin{pmatrix}\omega_1\\ \alpha_1\\ \theta_1\end{pmatrix} = \text{Find}(\omega_1, \alpha_1, \theta_1) \qquad \theta_1 = 48.19\,\text{deg} \qquad \omega_1 = 1.794\,\frac{\text{rad}}{\text{s}} \qquad \alpha_1 = 3.6\,\frac{\text{rad}}{\text{s}^2}$$

(b) Now the rod moves without any horizontal constraint. If we look for the point at which it loses contact with the floor ($A_y = 0$) we will find that this condition never occurs.

***Problem 18-52**

The slender rod *AB* of weight *W* is attached to a spring *BC* which, has unstretched length *L*. If the rod is released from rest when $\theta = \theta_1$,determine its angular velocity at the instant $\theta = \theta_2$.

Given:

$W = 25\ \text{lb}$

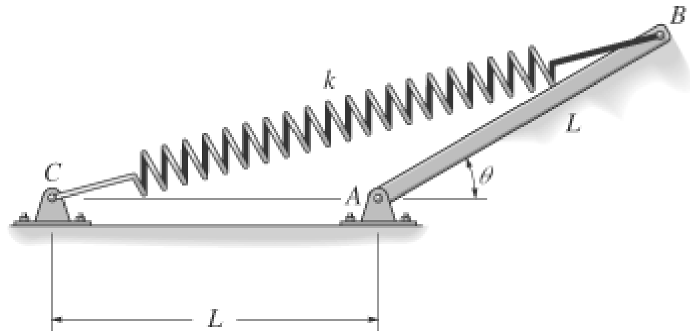
$L = 4\ \text{ft}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$

$\theta_1 = 30\ \text{deg}$

$\theta_2 = 90\ \text{deg}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) + \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_1))} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin(\theta_2) \ldots$$
$$+ \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_2))} - 1\right]^2$$

$\omega = \text{Find}(\omega)$ $\omega = 1.178\ \dfrac{\text{rad}}{\text{s}}$

Problem 18-53

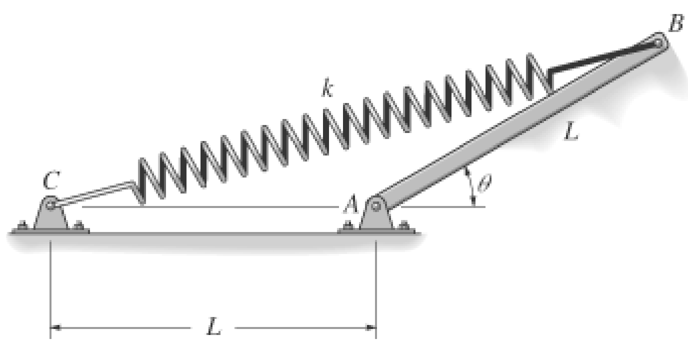
The slender rod *AB* of weight *w* is attached to a spring *BC* which has an unstretched length *L*. If the rod is released from rest when $\theta = \theta_1$, determine the angular velocity of the rod the instant the spring becomes unstretched.

Given:

$W = 25\ \text{lb}$ $\theta_1 = 30\ \text{deg}$

$L = 4\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$k = 5\ \dfrac{\text{lb}}{\text{ft}}$



Solution:

When the spring is unstretched $\theta_2 = 120\ \text{deg}$

Guess $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

$$0 + W\left(\frac{L}{2}\right)\sin(\theta_1) + \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_1))} - 1\right]^2 = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{3}\right)\omega^2 + W\left(\frac{L}{2}\right)\sin(\theta_2) \ ...$$
$$+ \frac{1}{2}kL^2\left[\sqrt{2(1+\cos(\theta_2))} - 1\right]^2$$

$\omega = \text{Find}(\omega)$ $\quad$ $\omega = 2.817\ \frac{\text{rad}}{\text{s}}$

Problem 18-54

A chain that has a negligible mass is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$.

Given:

$m_S = 2\ \text{kg}$

$k_O = 50\ \text{mm}$

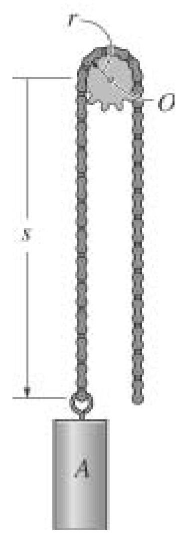
$m_A = 4\ \text{kg}$

$s_1 = 1\ \text{m}$

$s_2 = 2\ \text{m}$

$r = 0.1\ \text{m}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution:

Guess $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

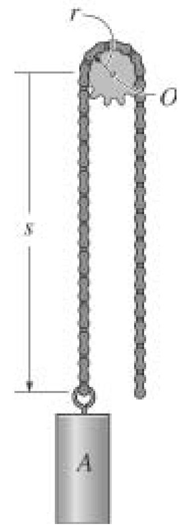
Given

$$0 - m_A g s_1 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_S k_O^2\omega^2 - m_A g s_2$$

$\omega = \text{Find}(\omega)$ $\quad$ $\omega = 41.8\ \frac{\text{rad}}{\text{s}}$

Problem 18-55

A chain that has a mass density ρ is draped over the sprocket which has mass m_s and radius of gyration k_O. If block A of mass m_A is released from rest in the position $s = s_1$, determine the angular velocity of the sprocket at the instant $s = s_2$. When released there is an equal amount of chain on each side. Neglect the portion of the chain that wraps over the sprocket.



Given:

$$m_s = 2\ \text{kg} \qquad s_1 = 1\ \text{m}$$

$$k_O = 50\ \text{mm} \qquad s_2 = 2\ \text{m}$$

$$m_A = 4\ \text{kg} \qquad r = 0.1\ \text{m}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2} \qquad \rho = 0.8\ \frac{\text{kg}}{\text{m}}$$

Solution:

Guess $\quad T_1 = 1\ \text{N m} \quad T_2 = 1\ \text{N m} \quad \omega = 10\ \frac{\text{rad}}{\text{s}}$

$\quad V_1 = 1\ \text{N m} \quad V_2 = 1\ \text{N m}$

Given

$$T_1 = 0$$

$$V_1 = -m_A g s_1 - 2\rho s_1 g\left(\frac{s_1}{2}\right)$$

$$T_2 = \frac{1}{2}m_A(r\omega)^2 + \frac{1}{2}m_s k_O^2\,\omega^2 + \frac{1}{2}\rho(2s_1)(r\omega)^2$$

$$V_2 = -m_A g s_2 - \rho s_2 g\left(\frac{s_2}{2}\right) - \rho(2s_1 - s_2)g\left(\frac{2s_1 - s_2}{2}\right)$$

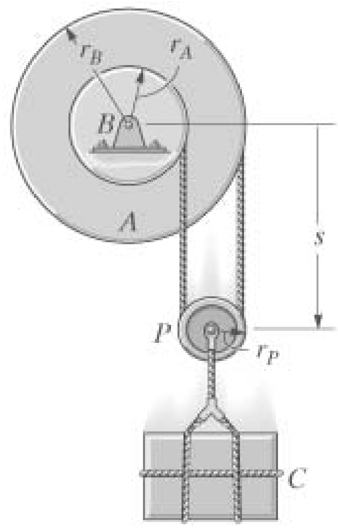
$$T_1 + V_1 = T_2 + V_2$$

$$\begin{pmatrix} T_1 \\ V_1 \\ T_2 \\ V_2 \\ \omega \end{pmatrix} = \text{Find}(T_1, V_1, T_2, V_2, \omega) \qquad \omega = 39.3\ \frac{\text{rad}}{\text{s}}$$

***Problem 18-56**

Pulley A has weight W_A and centroidal radius of gyration k_B. Determine the speed of the crate C of weight W_C at the instant $s = s_2$. Initially, the crate is released from rest when $s = s_1$. The pulley at P "rolls" downward on the cord without slipping. For the calculation, neglect the mass of this pulley and the cord as it unwinds from the inner and outer hubs of pulley A.

Given:



$W_A = 30\ \text{lb}$ $\quad r_A = 0.4\ \text{ft}$

$W_C = 20\ \text{lb}$ $\quad r_B = 0.8\ \text{ft}$

$k_B = 0.6\ \text{ft}$ $\quad r_P = \dfrac{r_B - r_A}{2}$

$s_1 = 5\ \text{ft}$ $\quad s_2 = 10\ \text{ft}$

Solution:

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}} \quad v_C = 1\ \dfrac{\text{ft}}{\text{s}}$

Given

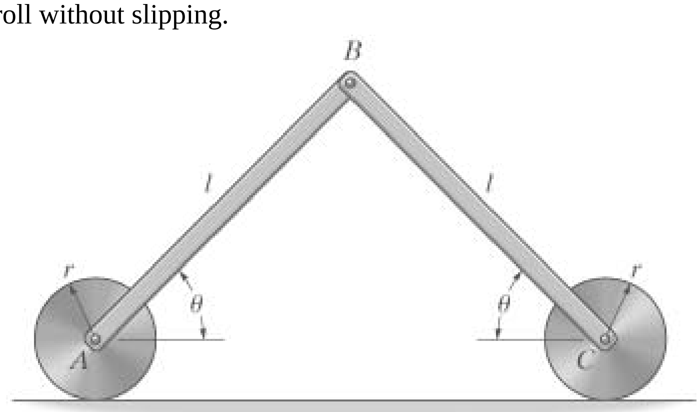
$$-W_C s_1 = \frac{1}{2}\left(\frac{W_A}{g}\right) k_B^2\,\omega^2 + \frac{1}{2}\left(\frac{W_C}{g}\right) v_C^2 - W_C s_2 \qquad v_C = \omega\left(\frac{r_A + r_B}{2}\right)$$

$$\begin{pmatrix} \omega \\ v_C \end{pmatrix} = \text{Find}(\omega, v_C) \qquad \omega = 18.9\ \frac{\text{rad}}{\text{s}} \qquad v_C = 11.3\ \frac{\text{ft}}{\text{s}}$$

Problem 18-57

The assembly consists of two bars of weight W_1 which are pinconnected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_0$, determine their angular velocities at the instant $\theta = 0°$. Assume the disks roll without slipping.

Given:



$W_1 = 8\ \text{lb}$

$W_2 = 10\ \text{lb}$

$r = 0.5\ \text{ft}$

$l = 3\ \text{ft}$

$\theta_0 = 60\ \text{deg}$

Solution:

Guess $\omega = 1\,\frac{\text{rad}}{\text{s}}$

Given $2W_1\left(\frac{l}{2}\right)\sin(\theta_0) = 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2$

$\omega = \text{Find}(\omega)$ $\omega = 5.28\,\frac{\text{rad}}{\text{s}}$

Problem 18-58

The assembly consists of two bars of weight W_1 which are pin-connected to the two disks of weight W_2. If the bars are released from rest at $\theta = \theta_1$, determine their angular velocities at the instant $\theta = \theta_2$. Assume the disks roll without slipping.

Given:

$W_1 = 8$ lb

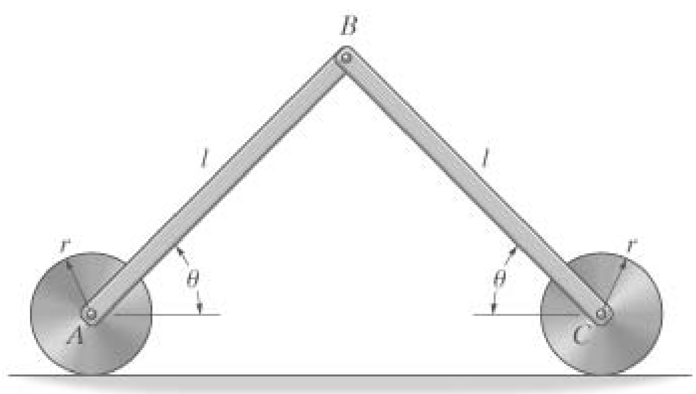
$W_2 = 10$ lb

$\theta_1 = 60$ deg

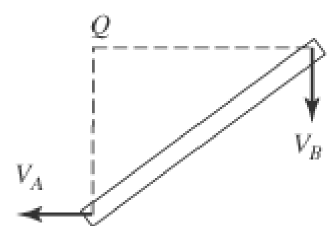
$\theta_2 = 30$ deg

$r = 0.5$ ft

$l = 3$ ft



Solution:



Guesses

$\omega = 1\,\frac{\text{rad}}{\text{s}}$ $v_A = 1\,\frac{\text{ft}}{\text{s}}$

Given

$$2W_1\left(\frac{l}{2}\right)\sin(\theta_1) = 2W_1\left(\frac{l}{2}\right)\sin(\theta_2) + 2\frac{1}{2}\left(\frac{W_1}{g}\right)\left(\frac{l^2}{3}\right)\omega^2 + 2\frac{1}{2}\left[\frac{3}{2}\left(\frac{W_2}{g}\right)r^2\right]\left(\frac{v_A}{r}\right)^2$$

$v_A = \omega l \sin(\theta_2)$

$$\begin{pmatrix} \omega \\ v_A \end{pmatrix} = \text{Find}(\omega, v_A) \qquad v_A = 3.32\,\frac{\text{ft}}{\text{s}} \qquad \omega = 2.21\,\frac{\text{rad}}{\text{s}}$$

Problem 18-59

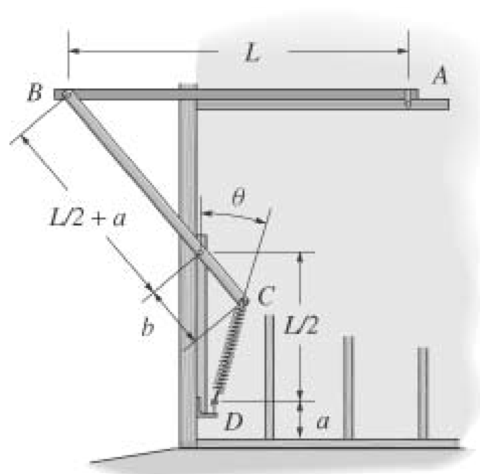
The end *A* of the garage door *AB* travels along the horizontal track, and the end of member *BC* is attached to a spring at *C*. If the spring is originally unstretched, determine the stiffness *k* so that when the door falls downward from rest in the position shown, it will have zero angular velocity the moment it closes, i.e., when it and *BC* become vertical. Neglect the mass of member *BC* and assume the door is a thin plate having weight *W* and a width and height of length *L*. There is a similar connection and spring on the other side of the door.

Given:

$W = 200\ \text{lb}$ $\qquad b = 2\ \text{ft}$

$L = 12\ \text{ft}$ $\qquad \theta = 15\ \text{deg}$

$a = 1\ \text{ft}$



Solution:

Guess $\quad k = 1\,\frac{\text{lb}}{\text{ft}} \qquad d = 1\ \text{ft}$

Given

$$b^2 = \left(\frac{L}{2}\right)^2 + d^2 - 2d\left(\frac{L}{2}\right)\cos(\theta)$$

$$0 = -W\left(\frac{L}{2}\right) + 2\,\frac{1}{2}k\left(\frac{L}{2} + b - d\right)^2$$

$$\begin{pmatrix} k \\ d \end{pmatrix} = \text{Find}(k, d) \qquad d = 4.535\ \text{ft} \qquad k = 100.0\,\frac{\text{lb}}{\text{ft}}$$