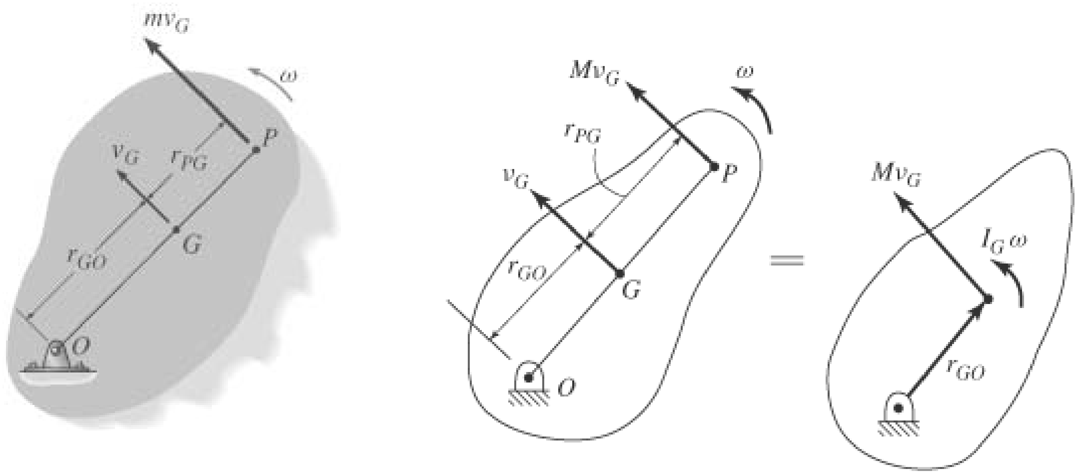
Problem 19-1

The rigid body (slab) has a mass m and is rotating with an angular velocity ω about an axis passing through the fixed point O. Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude mv_G and acting through point P, called the center of *percussion*, which lies at a distance $r_{PG} = k^2_G / r_{GO}$ from the mass center G. Here k_G is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through G.



Solution:

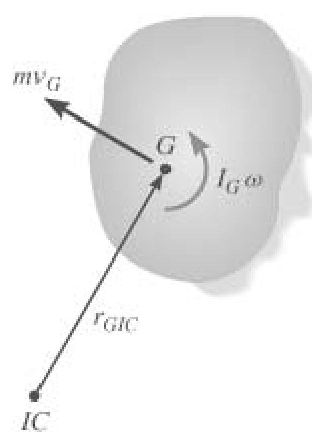
$$H_o = (r_{GO} + r_{PG}) m v_G = r_{GO} m v_G + I_G \omega \qquad \text{Where} \qquad I_G = m k_G^2$$

$$r_{GO} m v_G + r_{PG} m v_G = r_{GO} m v_G + m k_G^2 \omega$$

$$r_{PG} = \frac{k_G^2 \omega}{v_G} = \frac{k_G^2}{v_G}\left(\frac{v_G}{r_{GO}}\right) = \frac{k_G^2}{r_{GO}} \qquad \text{Q.E.D}$$

Problem 19-2

At a given instant, the body has a linear momentum $L = mv_G$ and an angular momentum $H_G = I_G\omega$ computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity IC can be expressed as $H_{IC} = I_{IC}\omega$ where I_{IC} represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the IC is located at a distance r_{GIC} away from the mass center G.



Solution:

$$H_{IC} = r_{GIC} m v_G + I_G \omega \qquad \text{Where} \qquad v_G = \omega r_{GIC}$$

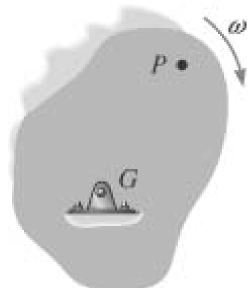
$$H_{IC} = r_{GIC} m\omega r_{GIC} + I_G\omega$$

$$H_{IC} = \left(I_G + m r_{GIC}^2\right)\omega$$

$$H_{IC} = I_{IC}\omega \qquad \text{Q.E.D.}$$

Problem 19-3

Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center *G*, the angular momentum is the same when computed about any other point *P* on the slab.



Solution:

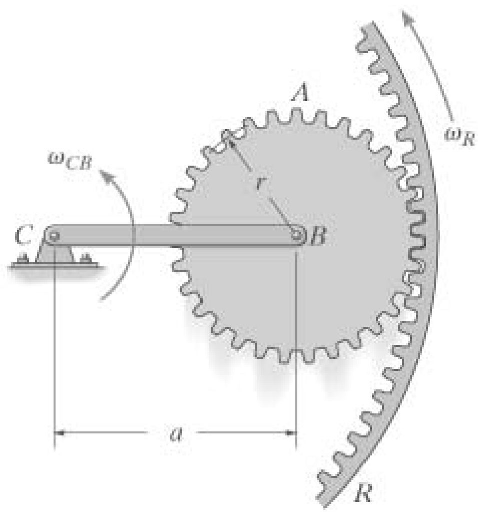
Since $v_G = 0$, the linear momentum $L = mv_G = 0$. Hence the angular momentum about any point *P* is

$$H_P = I_G\omega$$

Since ω is a free vector , so is H_P. Q.E.D.

***Problem 19-4**

Gear *A* rotates along the inside of the circular gear rack *R*. If *A* has weight *W* and radius of gyration k_B, determine its angular momentum about point *C* when (a) $\omega_R = 0$, (b) $\omega_R = \omega$.



Given:

$W = 4$ lbf $\qquad r = 0.75$ ft

$\omega_{CB} = 30 \frac{\text{rad}}{\text{s}} \qquad a = 1.5$ ft

$\omega = 20 \frac{\text{rad}}{\text{s}} \qquad k_B = 0.5$ ft

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution:

(a) $\omega_R = 0 \frac{\text{rad}}{\text{s}}$

$$v_B = a\omega_{CB} \qquad \omega_A = \frac{\omega_R(a+r) - \omega_{CB}a}{r}$$

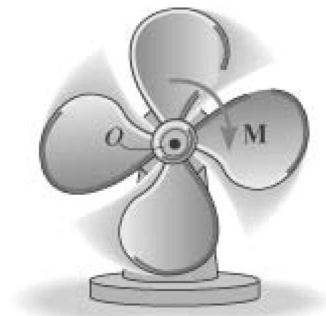
$$H_c = \left(\frac{W}{g}\right)v_B a + \left(\frac{W}{g}\right)k_B^2\omega_A \qquad H_c = 6.52\,\text{slug}\cdot\frac{\text{ft}^2}{\text{s}}$$

(b) $\omega_R = \omega$

$v_B = a\omega_{CB}$ $\qquad \omega_A = \dfrac{\omega_R(a+r) - \omega_{CB}\,a}{r}$

$H_c = \left(\dfrac{W}{g}\right)v_B\,a + \left(\dfrac{W}{g}\right)k_B^{\,2}\omega_A$ $\qquad H_c = 8.39\,\text{slug}\cdot\dfrac{\text{ft}^2}{\text{s}}$

Problem 19-5



The fan blade has mass m_b and a moment of inertia I_0 about an axis passing through its center O. If it is subjected to moment $M = A(1 - e^{bt})$ determine its angular velocity when $t = t_1$ starting from rest.

Given:

$m_b = 2\text{ kg}$ $\qquad A = 3\text{ N}\cdot\text{m}$ $\qquad t_1 = 4\text{ s}$

$I_O = 0.18\text{ kg}\cdot\text{m}^2$ $\qquad b = -0.2\text{ s}^{-1}$

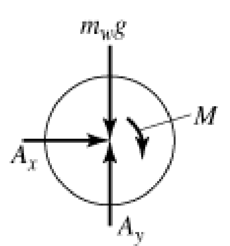
Solution:

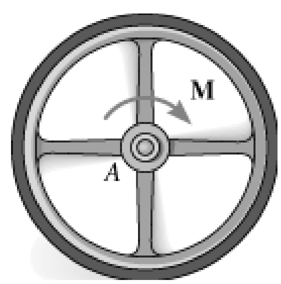
$0 + \displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\text{d}t = I_O\omega_1$ $\qquad \omega_1 = \dfrac{1}{I_O}\displaystyle\int_0^{t_1} A\left(1 - e^{b\,t}\right)\text{d}t$ $\qquad \omega_1 = 20.8\,\dfrac{\text{rad}}{\text{s}}$

Problem 19-6

The wheel of mass m_w has a radius of gyration k_A. If the wheel is subjected to a moment $M = bt$, determine its angular velocity at time t_1 starting from rest. Also, compute the reactions which the fixed pin A exerts on the wheel during the motion.

Given:





$m_w = 10\text{ kg}$ $\qquad t_1 = 3\text{ s}$

$k_A = 200\text{ mm}$ $\qquad g = 9.81\,\dfrac{\text{m}}{\text{s}^2}$

$b = 5\text{ N}\cdot\dfrac{\text{m}}{\text{s}}$

Solution:

$0 + \displaystyle\int_0^{t_1} b\,t\,\text{d}t = m_w k_A^{\,2}\omega_1$ $\qquad \omega_1 = \dfrac{1}{m_w k_A^{\,2}}\displaystyle\int_0^{t_1} b\,t\,\text{d}t$ $\qquad \omega_1 = 56.25\,\dfrac{\text{rad}}{\text{s}}$

$0 + Ax\,t_1 = 0$ $\qquad A_x = 0$ $\qquad A_x = 0.00$

$0 + A_y t_1 - m_w g t_1 = 0$ $A_y = m_w g$ $A_y = 98.10 \text{ N}$

Problem 19-7

Disk D of weight W is subjected to counterclockwise moment $M = bt$. Determine the angular velocity of the disk at time t_2 after the moment is applied. Due to the spring the plate P exerts constant force P on the disk. The coefficients of static and kinetic friction between the disk and the plate are μ_s and μ_k respectively. *Hint*: First find the time needed to start the disk rotating.

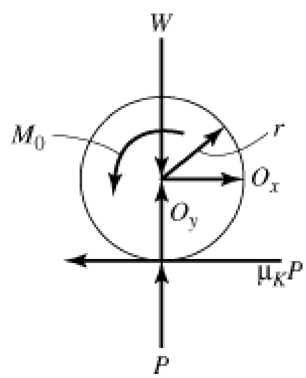
Given:

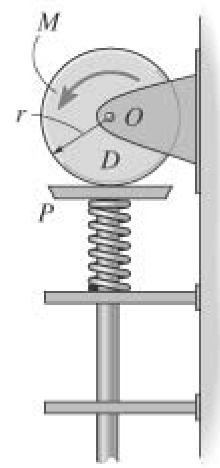
$W = 10 \text{ lb}$ $\mu_s = 0.3$

$b = 10 \text{ lb}\cdot\frac{\text{ft}}{\text{s}}$ $\mu_k = 0.2$

$t_2 = 2 \text{ s}$ $r = 0.5 \text{ ft}$

$P = 100\text{lb}$ $g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution: When motion begins

$b t_1 = \mu_s P r$ $t_1 = \frac{\mu_s P r}{b}$ $t_1 = 1.50 \text{ s}$

At a later time we have

$$0 + \int_{t_1}^{t_2} (b t - \mu_k P r) \, dt = \left(\frac{W}{g}\right)\frac{r^2}{2}\omega_2$$

$$\omega_2 = \frac{2g}{W r^2}\int_{t_1}^{t_2} (b t - \mu_k P r) \, dt \qquad \omega_2 = 96.6 \frac{\text{rad}}{\text{s}}$$

***Problem 19-8**

The cord is wrapped around the inner core of the spool. If block B of weight W_B is suspended from the cord and released from rest, determine the spool's angular velocity when $t = t_1$. Neglect the mass of the cord. The spool has weight W_S and the radius of gyration about the axle A is k_A. Solve the problem in two ways, first by considering the "system" consisting of the block and spool, and then by considering the block and spool separately.

Given:

$W_B = 5 \text{ lb}$

$t_1 = 3\ \text{s}$

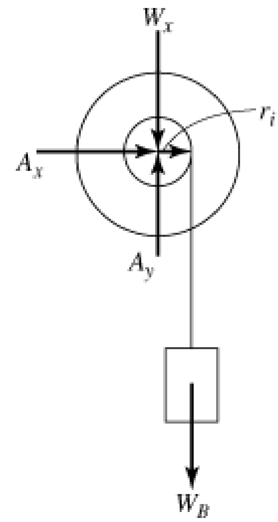
$W_S = 180\ \text{lb}$

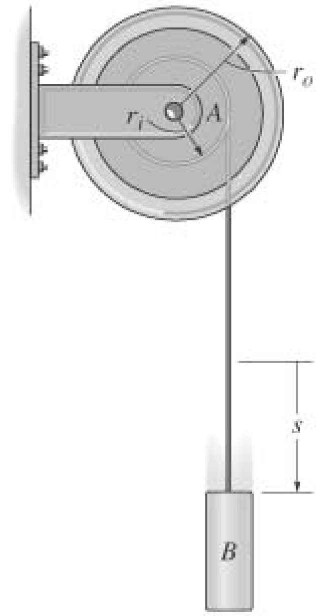
$k_A = 1.25\ \text{ft}$

$r_i = 1.5\text{ft}$

$r_o = 2.75\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

(a) System as a whole

$$0 + W_B r_i t_1 = \left(\frac{W_S}{g}\right) k_A^2 \omega + \left(\frac{W_B}{g}\right)(r_i \omega) r_i \qquad \omega = \frac{W_B r_i t_1 g}{W_S k_A^2 + W_B r_i^2} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

(b) Parts separately Guesses $T = 1\ \text{lb}$ $\omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $$0 + T t_1 r_i = \left(\frac{W_S}{g}\right) k_A^2 \omega \qquad T t_1 - W_B t_1 = \left(\frac{-W_B}{g}\right)(r_i \omega)$$

$$\begin{pmatrix} T \\ \omega \end{pmatrix} = \text{Find}(T, \omega) \qquad T = 4.81\ \text{lb} \qquad \omega = 2.48\ \frac{\text{rad}}{\text{s}}$$

Problem 19-9

The disk has mass M and is originally spinning at the end of the strut with angular velocity ω. If it is then placed against the wall, for which the coefficient of kinetic friction is μ_k determine the time required for the motion to stop. What is the force in strut BC during this time?

Given:

$M = 20\ \text{kg}$ $\theta = 60\ \text{deg}$

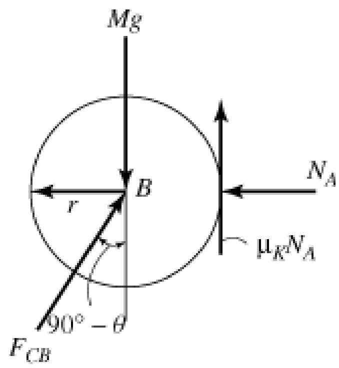
$\omega = 60\ \dfrac{\text{rad}}{\text{s}}$ $r = 150\ \text{mm}$

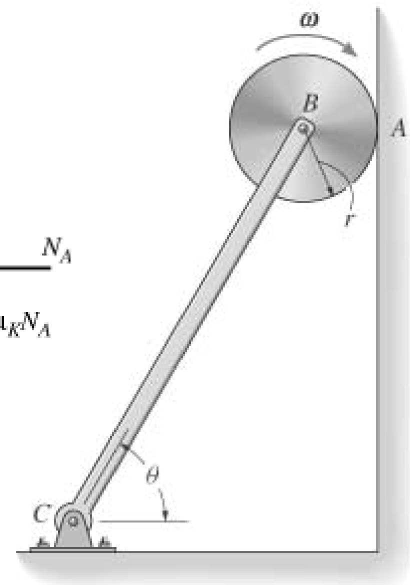
$\mu_k = 0.3$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

Initial Guess:

$F_{CB} = 1\ \text{N} \qquad t = 1\ \text{s} \qquad N_A = 1\ \text{N}$





Given

$$-M\left(\frac{r^2}{2}\right)\omega + \mu_k N_A r t = 0$$

$$F_{CB}\cos(\theta)t - N_A t = 0$$

$$F_{CB}\sin(\theta)t - M g t + \mu_k N_A t = 0$$

$$\begin{pmatrix} F_{CB} \\ N_A \\ t \end{pmatrix} = \text{Find}(F_{CB}, N_A, t) \qquad N_A = 96.55\ \text{N} \qquad F_{CB} = 193\ \text{N} \qquad t = 3.11\ \text{s}$$

Problem 19-10

A flywheel has a mass M and radius of gyration k_G about an axis of rotation passing through its mass center. If a motor supplies a clockwise torque having a magnitude $M = kt$, determine the flywheel's angular veliocity at time t_1. Initially the flywheel is rotating clockwise at angular velocity ω_0.

Given:

$M = 60\ \text{kg} \qquad k_G = 150\ \text{mm} \qquad k = 5\ \frac{\text{N}\cdot\text{m}}{\text{s}} \qquad t_1 = 3\ \text{s} \qquad \omega_0 = 2\ \frac{\text{rad}}{\text{s}}$

Solution:

$$M k_G^2 \omega_0 + \int_0^{t_1} k t\ dt = M k_G^2 \omega_1$$

$$\omega_1 = \omega_0 + \frac{1}{M k_G^2}\int_0^{t_1} k t\ dt \qquad \omega_1 = 18.7\ \frac{\text{rad}}{\text{s}}$$

Problem 19-11

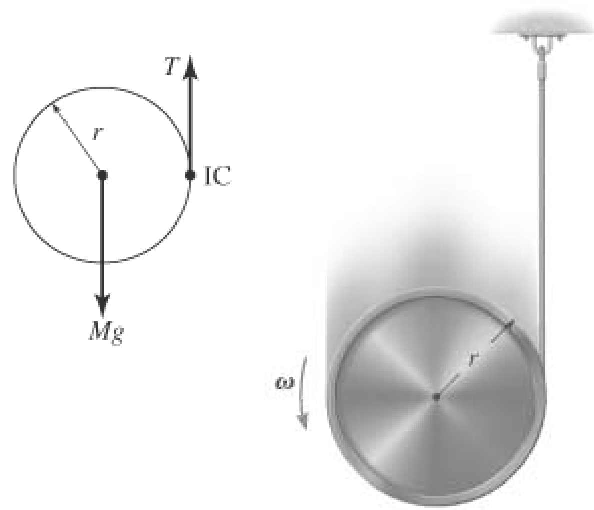
A wire of negligible mass is wrapped around the outer surface of the disk of mass M. If the disk is released from rest, determine its angular velocity at time t.

Given:

$M = 2\ \text{kg}$

$t = 3\ \text{s}$

$r = 80\ \text{mm}$



Solution:

$$0 + M g r t = \frac{3}{2} M r^2 \omega$$

$$\omega = \frac{2}{3}\left(\frac{g}{r}\right) t \qquad \omega = 245\ \frac{\text{rad}}{\text{s}}$$

***Problem 19-12**

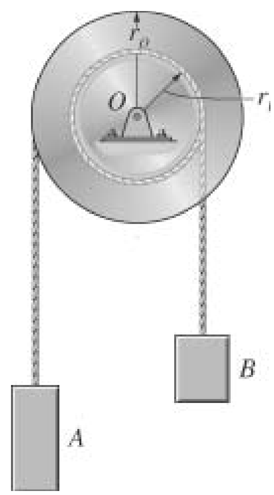
The spool has mass m_S and radius of gyration k_O. Block A has mass m_A, and block B has mass m_B. If they are released from rest, determine the time required for block A to attain speed v_A. Neglect the mass of the ropes.

Given:

$m_S = 30\ \text{kg}$ $\quad m_B = 10\ \text{kg}$ $\quad r_o = 0.3\ \text{m}$

$k_O = 0.25\ \text{m}$ $\quad v_A = 2\ \frac{\text{m}}{\text{s}}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$m_A = 25\ \text{kg}$ $\quad r_i = 0.18\ \text{m}$



Solution:

Guesses $\quad t = 1\ \text{s}$ $\quad v_B = 1\ \frac{\text{m}}{\text{s}}$ $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\quad v_A = \omega r_o$ $\quad v_B = \omega r_i$

$$0 + m_A g t r_o - m_B g t r_i = m_A v_A r_o + m_B v_B r_i + m_S k_O^2 \omega$$

$$\begin{pmatrix} t \\ v_B \\ \omega \end{pmatrix} = \text{Find}(t, v_B, \omega) \qquad v_B = 1.20\ \frac{\text{m}}{\text{s}} \qquad \omega = 6.67\ \frac{\text{rad}}{\text{s}} \qquad t = 0.530\ \text{s}$$

Problem 19-13

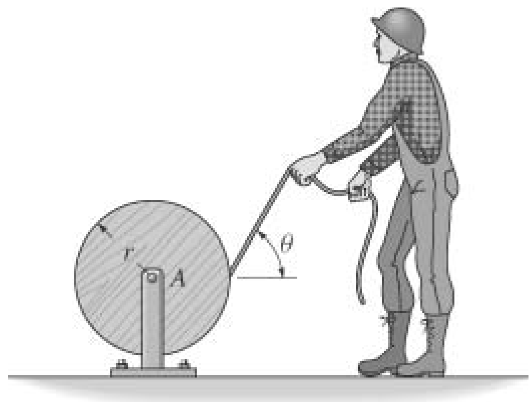
The man pulls the rope off the reel with a constant force P in the direction shown. If the reel has weight W and radius of gyration k_G about the trunnion (pin) at A, determine the angular velocity of the reel at time t starting from rest. Neglect friction and the weight of rope that is removed.

Given:

$P = 8\ \text{lb}$ $\quad t = 3\ \text{s}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$W = 250\ \text{lb}$ $\quad \theta = 60\ \text{deg}$

$k_G = 0.8\ \text{ft}$ $\quad r = 1.25\ \text{ft}$



Solution:

$$0 + P r t = \left(\frac{W}{g}\right) k_G^2 \omega$$

$$\omega = \frac{P r t g}{W k_G^2} \qquad \omega = 6.04\ \frac{\text{rad}}{\text{s}}$$

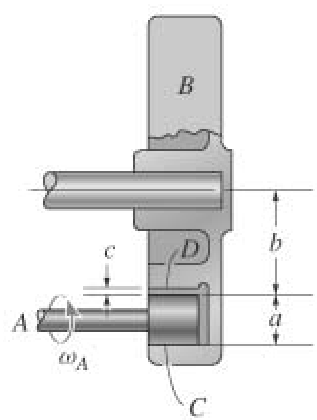
Problem 19-14

Angular motion is transmitted from a driver wheel A to the driven wheel B by friction between the wheels at C. If A always rotates at constant rate ω_A and the coefficient of kinetic friction between the wheels is μ_k, determine the time required for B to reach a constant angular velocity once the wheels make contact with a normal force F_N. What is the final angular velocity of wheel B? Wheel B has mass m_B and radius of gyration about its axis of rotation k_G.

Given:

$\omega_A = 16\ \dfrac{\text{rad}}{\text{s}}$ $\quad m_B = 90\ \text{kg}$ $\quad a = 40\ \text{mm}$ $\quad c = 4\ \text{mm}$

$\mu_k = 0.2$ $\quad k_G = 120\ \text{mm}$ $\quad b = 50\ \text{mm}$ $\quad F_N = 50\ \text{N}$



Solution: Guesses $t = 1\ \text{s}$ $\quad \omega_B = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad \mu_k F_N (a + b) t = m_B k_G^2 \omega_B$

$$\omega_B (a + b) = \omega_A \left(\frac{a}{2}\right)$$

$$\begin{pmatrix} t \\ \omega_B \end{pmatrix} = \text{Find}(t, \omega_B) \qquad \omega_B = 3.56\ \frac{\text{rad}}{\text{s}} \qquad t = 5.12\ \text{s}$$

Problem 19-15

The slender rod of mass M rests on a smooth floor. If it is kicked so as to receive a horizontal impulse I at point A as shown, determine its angular velocity and the speed of its mass center.

Given:

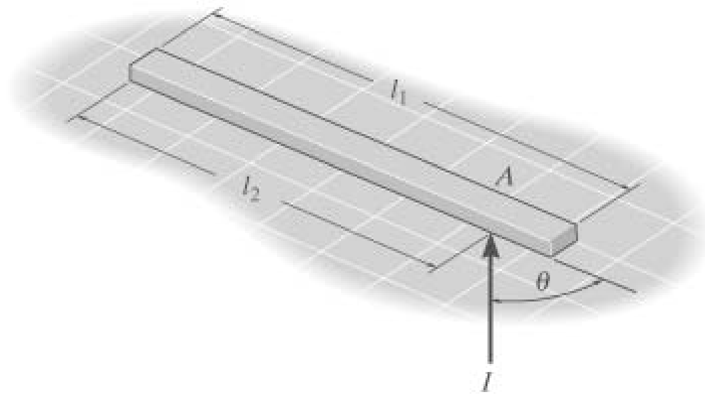
$M = 4$ kg

$l_1 = 2$ m

$l_2 = 1.75$ m

$I = 8$ N s

$\theta = 60$ deg



Solution:

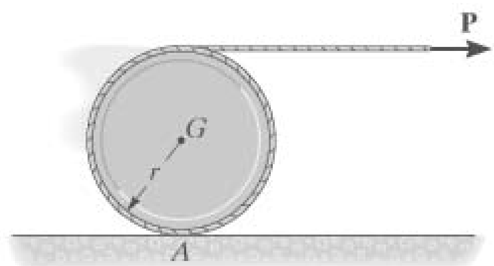
Guesses $v = 1 \frac{m}{s}$ $\omega = 1 \frac{rad}{s}$

Given $I \sin(\theta)\left(l_2 - \frac{l_1}{2}\right) = \frac{1}{12} M l_1^2 \omega$ $I = M v$

$\begin{pmatrix} \omega \\ v \end{pmatrix} = \text{Find}(\omega, v)$ $\omega = 3.90 \frac{rad}{s}$ $v = 2.00 \frac{m}{s}$

***Problem 19-16**

A cord of negligible mass is wrapped around the outer surface of the cylinder of weight W and its end is subjected to a constant horizontal force **P**. If the cylinder rolls without slipping at A, determine its angular velocity in time t starting from rest. Neglect the thickness of the cord.



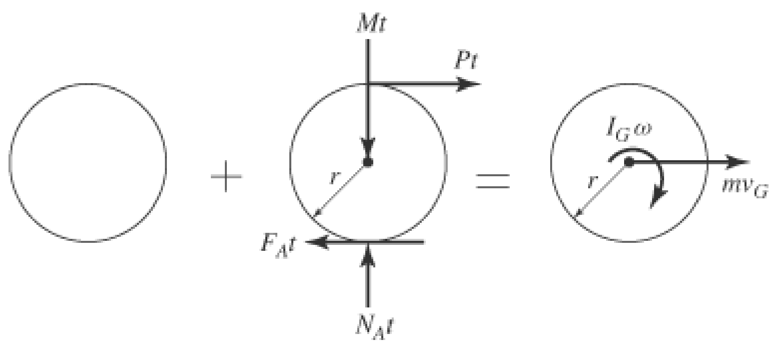
Given:

$W = 50$ lb

$P = 2$ lb

$t = 4$ s

$r = 0.6$ ft



Solution:

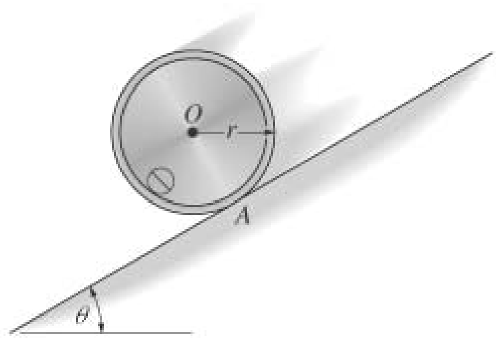
$$0 + P\,t(2r) = \left[\frac{1}{2}\left(\frac{W}{g}\right)r^2\right]\omega + \left(\frac{W}{g}\right)(r\omega)r$$

$$\omega = \frac{4P\,t\,g}{3r\,W} \qquad \omega = 11.4\ \frac{\text{rad}}{\text{s}}$$

Problem 19-17

The drum has mass M, radius r, and radius of gyration k_O. If the coefficients of static and kinetic friction at A are μ_s and μ_k respectively, determine the drum's angular velocity at time t after it is released from rest.

Given:



$M = 70\ \text{kg}$ $\quad \mu_s = 0.4$ $\quad \theta = 30\ \text{deg}$

$r = 300\text{mm}$ $\quad \mu_k = 0.3$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$k_O = 125\ \text{mm}$ $\quad t = 2\ \text{s}$

Solution: Assume no slip

Guesses $\quad F_f = 1\ \text{N} \quad F_N = 1\ \text{N}$

$\omega = 1\ \frac{\text{rad}}{\text{s}} \quad v = 1\ \frac{\text{m}}{\text{s}} \quad F_{max} = 1\ \text{N}$

Given $\quad 0 + F_f r t = M k_O^2 \omega \qquad v = \omega r \qquad F_{max} = \mu_s F_N$

$M g \sin(\theta)t - F_f t = M v \qquad F_N t - M g \cos(\theta)t = 0$

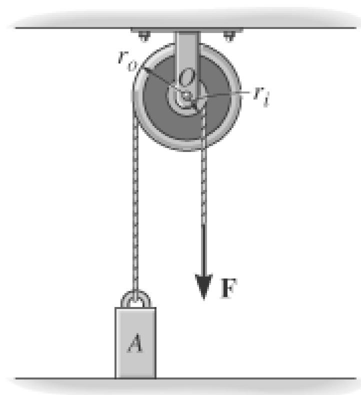
$$\begin{pmatrix} F_f \\ F_{max} \\ F_N \\ \omega \\ v \end{pmatrix} = \text{Find}(F_f, F_{max}, F_N, \omega, v) \qquad \begin{pmatrix} F_f \\ F_{max} \\ F_N \end{pmatrix} = \begin{pmatrix} 51 \\ 238 \\ 595 \end{pmatrix} \text{N} \qquad v = 8.36\ \frac{\text{m}}{\text{s}}$$

$$\omega = 27.9\ \frac{\text{rad}}{\text{s}}$$

Since $F_f = 51\ \text{N} < F_{max} = 238\ \text{N}$ then our no-slip assumption is good.

Problem 19-18

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has mass m_p and radius of gyration k_O. If the block at A has mass m_A, determine the speed of the block at time t after a constant force **F** is applied to the rope wrapped around the inner hub of the pulley. The block is originally at rest. Neglect the mass of the rope.



Units Used: $kN = 10^3\ N$

Given:

$m_p = 15\ kg$ $F = 2\ kN$

$k_O = 110\ mm$ $r_i = 75\ mm$

$m_A = 40\ kg$ $r_o = 200\ mm$

$t = 3\ s$

Solution: Guess $v_A = 1\ \frac{m}{s}$ $\omega = 1\ \frac{rad}{s}$

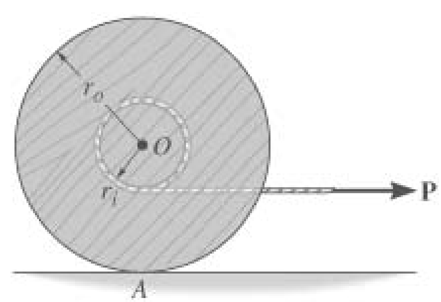
Given $-F\,r_i\,t + m_A\,g\,r_o\,t = -m_p\,k_O^{\ 2}\,\omega - m_A\,v_A\,r_o$

$v_A = \omega r_o$

$\begin{pmatrix} v_A \\ \omega \end{pmatrix} = \text{Find}(v_A, \omega)$ $\omega = 120.44\ \frac{rad}{s}$ $v_A = 24.1\ \frac{m}{s}$

Problem 19-19

The spool has weight W and radius of gyration k_O. A cord is wrapped around its inner hub and the end subjected to a horizontal force **P**. Determine the spool's angular velocity at time t starting from rest. Assume the spool rolls without slipping.



Given:

$W = 30\ lb$ $t = 4\ s$

$k_O = 0.45\ ft$ $r_i = 0.3\ ft$

$P = 5\ lb$ $r_o = 0.9\ ft$

Solution: Guesses $\omega = 1\ \frac{rad}{s}$ $v_O = 1\ \frac{ft}{s}$

Given $-P(r_o - r_i)t = \left(\frac{-W}{g}\right)v_O\,r_o - \left(\frac{W}{g}\right)k_O^{\ 2}\,\omega$ $v_O = \omega r_o$

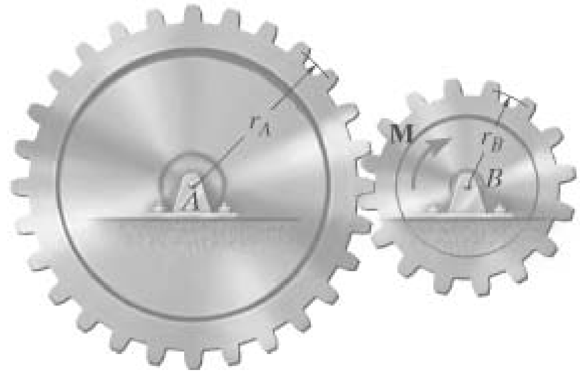
$\begin{pmatrix} v_O \\ \omega \end{pmatrix} = \text{Find}(v_O, \omega)$ $v_O = 3.49\ \frac{m}{s}$ $\omega = 12.7\ \frac{rad}{s}$

***Problem 19-20**

The two gears A and B have weights W_A, W_B and radii of gyration k_A and k_B respectively. If a motor transmits a couple moment to gear B of $M = M_0\,(1 - e^{-bt})$, determine the angular velocity of gear A

at time t, starting from rest.

Given:



$W_A = 15 \text{ lb}$ $W_B = 10 \text{ lb}$

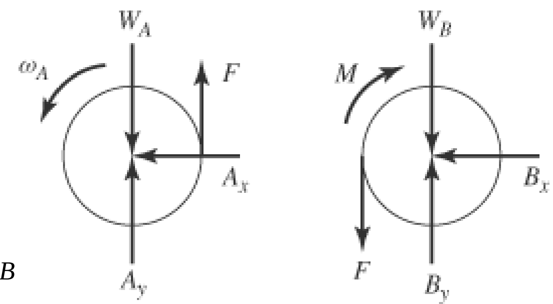
$r_A = 0.8 \text{ ft}$ $r_B = 0.5 \text{ ft}$

$k_A = 0.5 \text{ ft}$ $k_B = 0.35 \text{ ft}$

$M_0 = 2 \text{ lb}\cdot\text{ft}$ $b = 0.5 \text{ s}^{-1}$

$t = 5 \text{ s}$

Solution:



Guesses

$\omega_A = 1 \frac{\text{rad}}{\text{s}}$ $\omega_B = 1 \frac{\text{rad}}{\text{s}}$ $ImpF = 1 \text{ lb}\cdot\text{s}$

Given $\int_0^t M_0\left(1 - e^{-b\,t}\right) dt - ImpF\, r_B = \left(\frac{W_B}{g}\right) k_B^{\,2}\,\omega_B$

$ImpF\, r_A = \left(\frac{W_A}{g}\right) k_A^{\,2}\,\omega_A$ $\omega_A r_A = \omega_B r_B$

$\begin{pmatrix} \omega_A \\ \omega_B \\ ImpF \end{pmatrix} = \text{Find}\left(\omega_A, \omega_B, ImpF\right)$ $ImpF = 6.89 \text{ lb s}$ $\omega_B = 75.7 \frac{\text{rad}}{\text{s}}$

$\omega_A = 47.3 \frac{\text{rad}}{\text{s}}$

Problem 19-21

Spool B is at rest and spool A is rotating at ω when the slack in the cord connecting them is taken up. Determine the angular velocity of each spool immediately after the cord is jerked tight by the spinning of spool A. The weights and radii of gyration of A and B are W_A, k_A, and W_B, k_A, respectively.

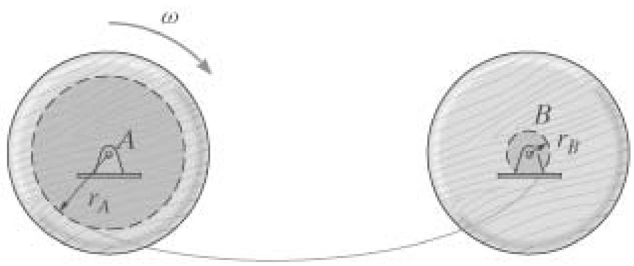
Given:

$W_A = 30\ \text{lb}$ $\quad W_B = 15\ \text{lb}$

$k_A = 0.8\ \text{ft}$ $\quad k_B = 0.6\ \text{ft}$

$r_A = 1.2\ \text{ft}$ $\quad r_B = 0.4\ \text{ft}$

$\omega = 6\ \frac{\text{rad}}{\text{s}}$

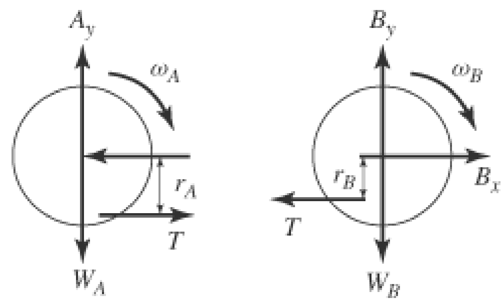


Solution:

Guesses

$\omega_A = 1\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_B = 1\ \frac{\text{rad}}{\text{s}}$

$Imp = 1\ \text{lb}\cdot\text{s}$



Given

$$\left(\frac{W_A}{g}\right)k_A^2\omega - Imp\,r_A = \left(\frac{W_A}{g}\right)k_A^2\omega_A \qquad Imp\,r_B = \left(\frac{W_B}{g}\right)k_B^2\omega_B \qquad \omega_A r_A = \omega_B r_B$$

$$\begin{pmatrix}\omega_A\\ \omega_B\\ Imp\end{pmatrix} = \text{Find}(\omega_A, \omega_B, Imp) \qquad Imp = 2.14\,\text{lb}\cdot\text{s} \qquad \begin{pmatrix}\omega_A\\ \omega_B\end{pmatrix} = \begin{pmatrix}1.70\\ 5.10\end{pmatrix}\frac{\text{rad}}{\text{s}}$$

Problem 19-22

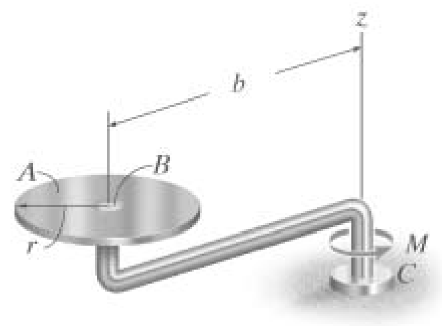
Disk A of mass m_A is mounted on arm BC, which has a negligible mass. If a torque of $M = M_0e^{at}$ is applied to the arm at C, determine the angular velocity of BC at time t starting from rest. Solve the problem assuming that (a) the disk is set in a smooth bearing at B so that it rotates with curvilinear translation, (b) the disk is fixed to the shaft BC, and (c) the disk is given an initial freely spinning angular velocity $\omega_D\mathbf{k}$ prior to application of the torque.

Given:

$m_A = 4\ \text{kg}$ $\quad M_0 = 5\ \text{N}\cdot\text{m}$ $\quad \omega_D = -80\ \frac{\text{rad}}{\text{s}}$

$r = 60\ \text{mm}$ $\quad a = 0.5\ \text{s}^{-1}$

$b = 250\ \text{mm}$ $\quad t = 2\ \text{s}$



Solution: Guess $\omega_{BC} = 1\ \frac{\text{rad}}{\text{s}}$

(a) Given $\displaystyle\int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 68.7\ \frac{\text{rad}}{\text{s}}$

(b) Given $\displaystyle\int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2 + m_A\left(\frac{r^2}{2}\right)\omega_{BC}$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 66.8\ \frac{\text{rad}}{\text{s}}$

(c) Given $\displaystyle -m_A\left(\frac{r^2}{2}\right)\omega_D + \int_0^t M_0 e^{a t}\, dt = m_A \omega_{BC} b^2 - m_A\left(\frac{r^2}{2}\right)\omega_D$

$\omega_{BC} = \text{Find}(\omega_{BC})$ $\quad \omega_{BC} = 68.7\ \frac{\text{rad}}{\text{s}}$

Problem 19-23

The inner hub of the wheel rests on the horizontal track. If it does not slip at A, determine the speed of the block of weight W_b at time t after the block is released from rest. The wheel has weight W_w and radius of gyration k_G. Neglect the mass of the pulley and cord.

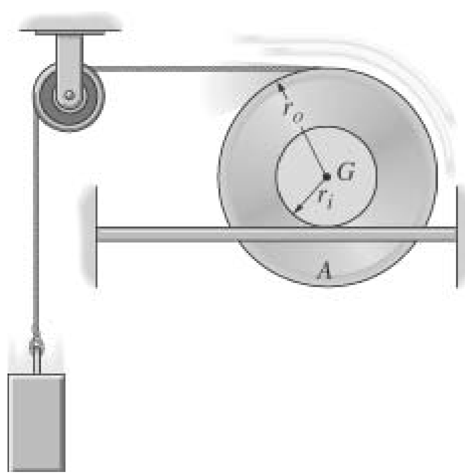
Given:

$W_b = 10\ \text{lb}$ $\quad r_i = 1\ \text{ft}$

$t = 2\ \text{s}$ $\quad r_o = 2\ \text{ft}$

$W_w = 30\ \text{lb}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_G = 1.30\ \text{ft}$



Solution: Guesses $v_G = 1\ \frac{\text{ft}}{\text{s}}$

$v_B = 1\ \frac{\text{ft}}{\text{s}}$ $\quad T = 1\ \text{lb}$ $\quad \omega = 1\ \frac{\text{rad}}{\text{s}}$

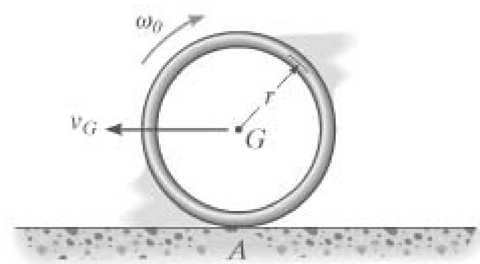
Given $\quad T\,t - W_b\,t = \left(\frac{-W_b}{g}\right) v_B \qquad T\left(r_o + r_i\right)t = \left(\frac{W_w}{g}\right) v_G\,r_i + \left(\frac{W_w}{g}\right) k_G^{\,2}\,\omega$

$v_G = \omega r_i \qquad v_B = \omega\left(r_i + r_o\right)$

$$\begin{pmatrix} v_G \\ v_B \\ \omega \\ T \end{pmatrix} = \text{Find}\left(v_G, v_B, \omega, T\right) \qquad T = 4.73\,\text{lb} \quad \omega = 11.3\,\frac{\text{rad}}{\text{s}} \qquad v_G = 11.3\,\frac{\text{ft}}{\text{s}} \qquad v_B = 34.0\,\frac{\text{ft}}{\text{s}}$$

***Problem 19-24**

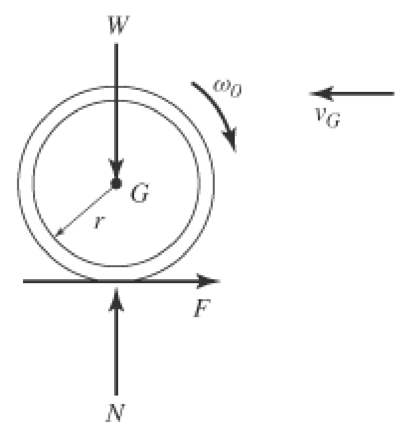
If the hoop has a weight *W* and radius *r* and is thrown onto a *rough surface* with a velocity v_G parallel to the surface, determine the amount of backspin, ω_0, it must be given so that it stops spinning at the same instant that its forward velocity is zero. It is not necessary to know the coefficient of kinetic friction at *A* for the calculation.



Solution:

$$\left(\frac{W}{g}\right) v_G\,r - \left(\frac{W}{g}\right) r^2\,\omega_0 = 0$$

$$\omega_0 = \frac{v_G}{r}$$



Problem 19-25

The rectangular plate of weight *W* is at rest on a smooth *horizontal* floor. If it is given the horizontal impulses shown, determine its angular velocity and the velocity of the mass center.

Given:

$W = 10\ \text{lb}$ $d = 0.5\ \text{ft}$

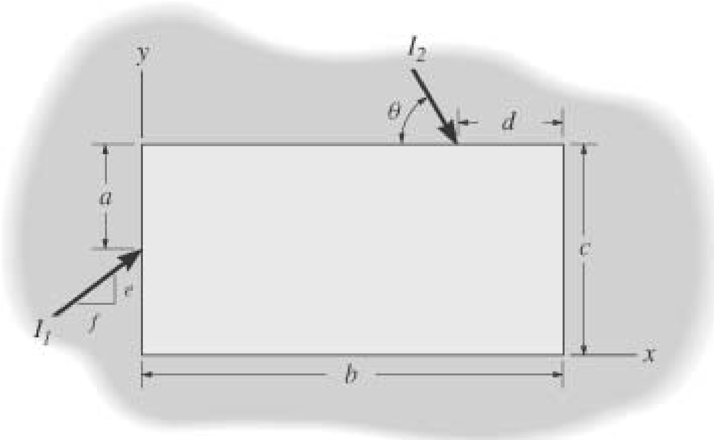
$I_1 = 20\ \text{lb}\cdot\text{s}$ $\theta = 60\ \text{deg}$

$I_2 = 5\ \text{lb}\cdot\text{s}$ $e = 3$

$a = 0.5\ \text{ft}$ $f = 4$

$b = 2\ \text{ft}$ $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$c = 1\ \text{ft}$



Solution: Guesses $v_x = 1\ \frac{\text{ft}}{\text{s}}$ $v_y = 1\ \frac{\text{ft}}{\text{s}}$ $\omega = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)I_1 + I_2\cos(\theta) = \left(\frac{W}{g}\right)v_x$$

$$\left(\frac{e}{\sqrt{e^2+f^2}}\right)I_1 - I_2\sin(\theta) = \left(\frac{W}{g}\right)v_y$$

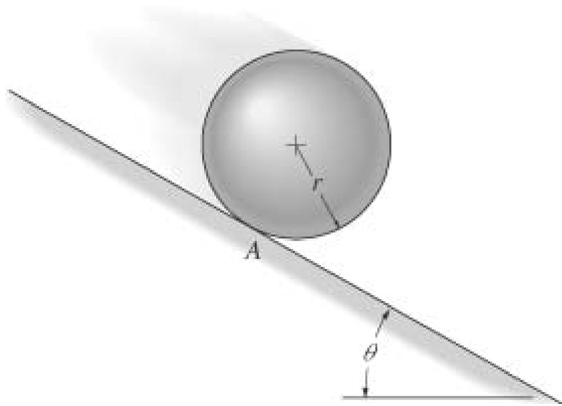
$$\left(\frac{f}{\sqrt{e^2+f^2}}\right)I_1\,a - I_2\sin(\theta)(b-d) = \frac{1}{12}\left(\frac{W}{g}\right)\left(b^2+c^2\right)\omega + \left(\frac{W}{g}\right)v_x\left(\frac{c}{2}\right) + \left(\frac{W}{g}\right)v_y\left(\frac{b}{2}\right)$$

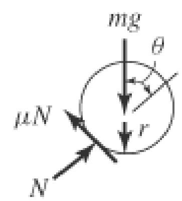
$$\begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} = \text{Find}(v_x, v_y, \omega) \qquad \mathbf{v_G} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \omega = -119\ \frac{\text{rad}}{\text{s}} \qquad \mathbf{v_G} = \begin{pmatrix} 59.6 \\ 24.7 \end{pmatrix}\frac{\text{ft}}{\text{s}}$$

$$\left|\mathbf{v_G}\right| = 64.5\ \frac{\text{ft}}{\text{s}}$$

Problem 19-26

The ball of mass *m* and radius *r* rolls along an inclined plane for which the coefficient of static friction is μ. If the ball is released from rest, determine the maximum angle θ for the incline so that it rolls without slipping at *A*.





Solution:

$N t - m g \cos(\theta) t = 0$ $\qquad$ $N = m g \cos(\theta)$

$\mu N r t = \frac{2}{5} m r^2 \omega$ $\qquad$ $t = \frac{2\omega r}{5\mu g \cos(\theta)}$

$m g \sin(\theta) t - \mu N t = m r \omega$ $\qquad$ $t = \frac{r\omega}{g(\sin(\theta) - \mu\cos(\theta))}$

Thus

$$\frac{2\omega r}{5\mu g \cos(\theta)} = \frac{r\omega}{g(\sin(\theta) - \mu\cos(\theta))} \qquad \theta = \operatorname{atan}\left(\frac{7\mu}{2}\right)$$

Problem 19-27

The spool has weight W_s and radius of gyration $k_{O.}$ If the block B has weight W_b and a force **P** is applied to the cord, determine the speed of the block at time t starting from rest. Neglect the mass of the cord.

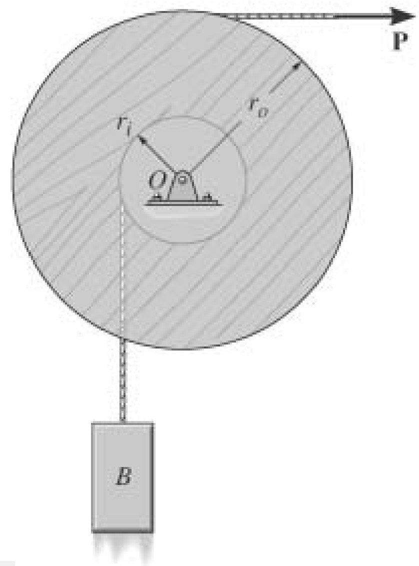
Given:

$W_S = 75\text{lb} \qquad t = 5\text{s}$

$k_O = 1.2\text{ft}$ $\quad r_o = 2\text{ft}$

$W_b = 60\text{lb}$ $\quad r_i = 0.75\text{ft}$

$P = 25\text{lb}$



Solution:

Guess $\quad v_B = 1\frac{\text{ft}}{\text{s}} \qquad \omega = 1\frac{\text{rad}}{\text{s}}$

Given

$$-P\,r_o\,t + W_b\,r_i\,t = \left(\frac{-W_b}{g}\right)v_B\,r_i - \left(\frac{W_s}{g}\right)k_O^{\,2}\,\omega$$

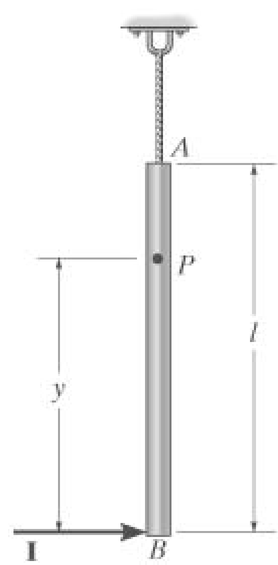
$$v_B = \omega r_i$$

$$\begin{pmatrix} v_B \\ \omega \end{pmatrix} = \text{Find}(v_B, \omega) \qquad \omega = 5.68\frac{\text{rad}}{\text{s}} \qquad v_B = 4.26\frac{\text{ft}}{\text{s}}$$

***Problem 19-28**

The slender rod has a mass *m* and is suspended at its end *A* by a cord. If the rod receives a horizontal blow giving it an impulse **I** at its bottom *B*, determine the location *y* of the point *P* about which the rod appears to rotate during the impact.

Solution:



$$F\Delta t = m\,v_{cm}$$

$$m\Delta t = I\omega$$

$$I\,y = I'\,\omega$$

$$I\,y = m\left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right]\omega \qquad (1)$$

$$I = m\,v_{cm}$$

$$I = m\frac{l}{2}\omega \qquad (2)$$

Divide Eqs (1) by (2)

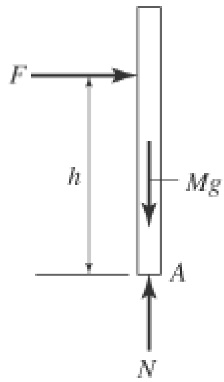
$$\frac{y\,l}{2} = \left[\frac{l^2}{12} + \left(y - \frac{l}{2}\right)^2\right] \qquad y = \left(\frac{3}{4} + \frac{\sqrt{33}}{12}\right)l$$

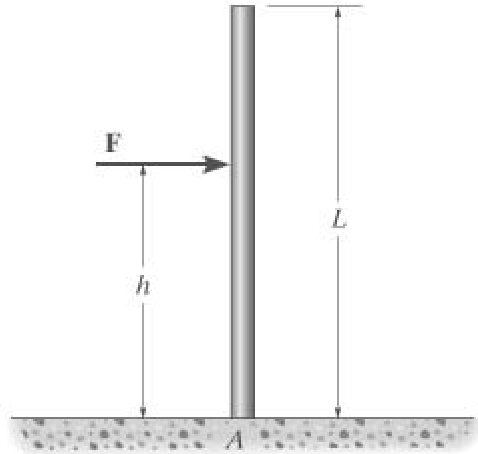
Problem 19-29

A thin rod having mass M is balanced vertically as shown. Determine the height h at which it can be struck with a horizontal force **F** and not slip on the floor. This requires that the frictional force at A be essentially zero.

Given:

$M = 4\ \text{kg} \qquad L = 0.8\ \text{m}$





Solution:

$$F\,t = M\omega\frac{L}{2}$$

$$F\,t\,h = M\frac{L^2}{3}\omega$$

Thus

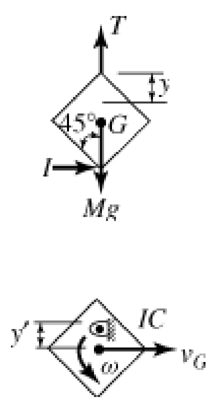
$$M\omega\frac{L}{2}h = M\frac{L^2}{3}\omega$$

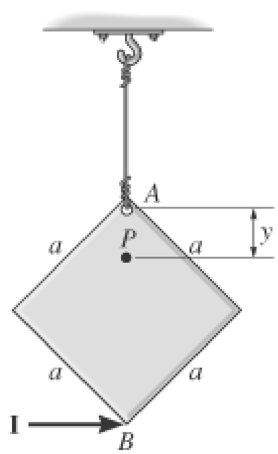
$$h = \frac{2}{3}L \qquad h = 0.53\ \text{m}$$

Problem 19-30

The square plate has a mass M and is suspended at its corner A by a cord. If it receives a horizontal impulse **I** at corner B, determine the location y' of the point P about which the plate appears to rotate during the impact.

Solution:





$$I\frac{a}{\sqrt{2}} = \frac{1}{6}Ma^2\omega \qquad \omega = \frac{6}{\sqrt{2}}\frac{I}{Ma}$$

$$I = Mv_G \qquad v_G = \frac{I}{M}$$

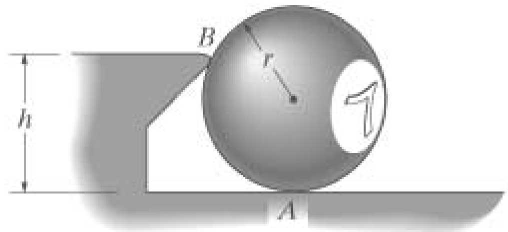
$$y' = \frac{v_G}{\omega} \qquad y' = \frac{\sqrt{2}}{6}a$$

$$y = \frac{a}{\sqrt{2}} - y' \qquad y = \frac{\sqrt{2}}{3}a$$

Problem 19-31

Determine the height h of the bumper of the pool table, so that when the pool ball of mass m strikes it, no frictional force will be developed between the ball and the table at A. Assume the bumper exerts only a horizontal force on the ball.

Solution:



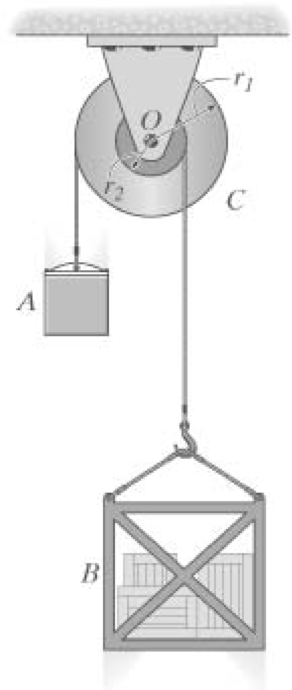
$F\Delta t = M\Delta v \quad F\Delta t\,h = \frac{7}{5}Mr^2\Delta\omega \quad \Delta v = r\Delta\omega$

Thus

$Mr\Delta\omega h = \frac{7}{5}Mr^2\Delta\omega \qquad h = \frac{7}{5}r$

***Problem 19-32**

The double pulley consists of two wheels which are attached to one another and turn at the same rate. The pulley has a mass M_C and a radius of gyration k_O. If the block at A has a mass M_A and the container at B has a mass M_B, including its contents, determine the speed of the container at time t after it is released from rest.



Given:

$M_C = 15\text{ kg} \quad k_O = 110\text{ mm}$

$M_A = 40\text{ kg} \quad r_1 = 200\text{ mm}$

$M_B = 85\text{ kg} \quad r_2 = 75\text{ mm}$

$t = 3\text{ s}$

Solution:

Guess $\quad v_A = 1\frac{\text{m}}{\text{s}} \quad v_B = 1\frac{\text{m}}{\text{s}} \quad \omega = 1\frac{\text{rad}}{\text{s}}$

Given $\quad M_A g t r_1 - M_B g t r_2 = M_A v_A r_1 + M_B v_B r_2 + M_C k_O^2 \omega$

$v_A = \omega r_1 \qquad v_B = \omega r_2$

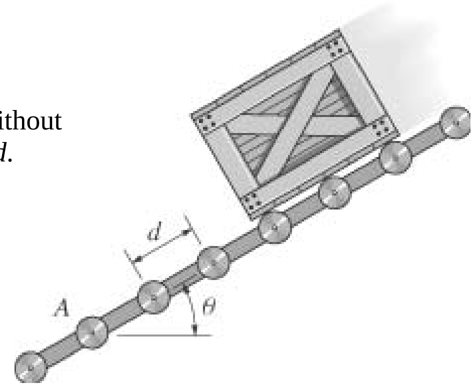
$\begin{pmatrix} v_A \\ v_B \\ \omega \end{pmatrix} = \text{Find}(v_A, v_B, \omega) \qquad \omega = 21.2\frac{\text{rad}}{\text{s}} \qquad v_A = 4.23\ \frac{\text{m}}{\text{s}} \qquad v_B = 1.59\ \frac{\text{m}}{\text{s}}$

Problem 19-33

The crate has a mass M_c. Determine the constant speed v_0 it acquires as it moves down the conveyor. The rollers each have radius r, mass M, and are spaced distance d apart. Note that friction causes each roller to rotate when the crate comes in contact with it.

Solution:

Assume each roller is brought to the condition of roll without slipping. In time t, the number of rollers affected is v_0t/d.

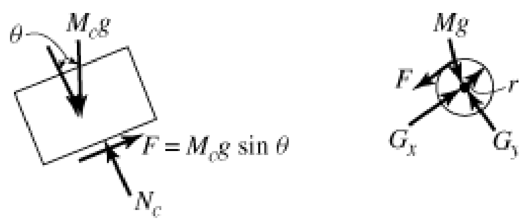


$$M_C\,g\sin(\theta)t - F\,t = 0$$

$$F = M_C\,g\sin(\theta)$$

$$F\,t\,r = \left(\frac{1}{2}M\,r^2\right)\frac{v_0}{r}\left(\frac{v_0}{d}t\right)$$

$$v_0 = \sqrt{2g\sin(\theta)d\frac{M_C}{M}}$$



Problem 19-34

Two wheels A and B have masses m_A and m_B and radii of gyration about their central vertical axes of k_A and k_B respectively. If they are freely rotating in the same direction at ω_A and ω_B about the same vertical axis, determine their common angular velocity after they are brought into contact and slipping between them stops.

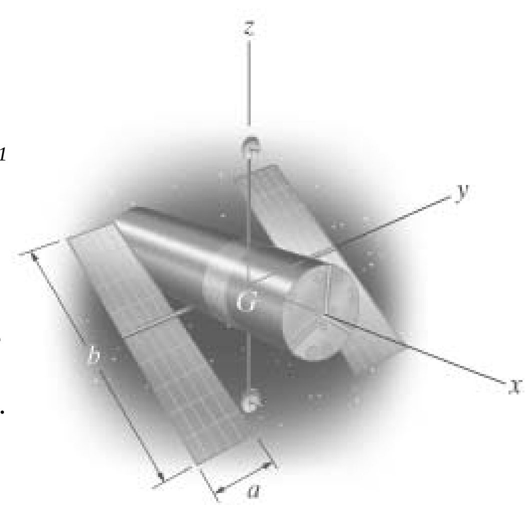
Solution:

$$m_A k_A^2 \omega_A + m_B k_B^2 \omega_B = \left(m_A k_A^2 + m_B k_B^2\right)\omega$$

$$\omega = \frac{m_A k_A^2 \omega_A + m_B k_B^2 \omega_B}{m_A k_A^2 + m_B k_B^2}$$

Problem 19-35

The Hubble Space Telescope is powered by two solar panels as shown. The body of the telescope has a mass M_1 and radii of gyration k_x and k_y, whereas the solar panels can be considered as thin plates, each having a mass M_2. Due to an internal drive, the panels are given an angular velocity of $\omega_0\mathbf{j}$, measured relative to the telescope. Determine the angular velocity of the telescope due to the rotation of the panels. Prior to rotating the panels, the telescope was originally traveling at $\mathbf{v_G} = (v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$. Neglect its orbital rotation.



Units Used: $\text{Mg} = 10^3\text{kg}$

Given:

$M_1 = 11\ \text{Mg}$ $\quad \omega_0 = 0.6\ \frac{\text{rad}}{\text{s}}$ $\quad v_x = -400\ \frac{\text{m}}{\text{s}}$

$M_2 = 54\ \text{kg}$ $\quad a = 1.5\ \text{m}$ $\quad v_y = 250\ \frac{\text{m}}{\text{s}}$

$k_x = 1.64\ \text{m}$

$k_y = 3.85\ \text{m}$ $\quad b = 6\ \text{m}$ $\quad v_z = 175\ \frac{\text{m}}{\text{s}}$

Solution: Angular momentum is conserved.

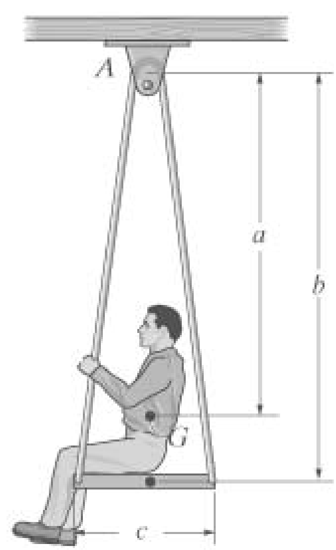
Guess $\omega_T = 1\frac{\text{rad}}{\text{s}}$

Given $0 = 2\left(\frac{1}{12}M_2\,b^2\right)\left(\omega_0 - \omega_T\right) - \left(M_1\,k_y^{\,2}\right)\omega_T \qquad \omega_T = \text{Find}\left(\omega_T\right)$

$\omega_T = 0.00\ \frac{\text{rad}}{\text{s}}$

***Problem 19-36**

The platform swing consists of a flat plate of weight W_p suspended by four rods of negligible weight. When the swing is at rest, the man of weight W_m jumps off the platform when his center of gravity *G* is at distance *a* from the pin at *A*. This is done with a horizontal velocity *v*, measured relative to the swing at the level of *G*. Determine the angular velocity he imparts to the swing just after jumping off.



Given:

$W_p = 200\text{lb}$ $\quad a = 10\text{ft}$

$W_m = 150\text{lb}$ $\quad b = 11\text{ft}$

$v = 5\frac{\text{ft}}{\text{s}}$ $\quad c = 4\text{ft}$

Solution:

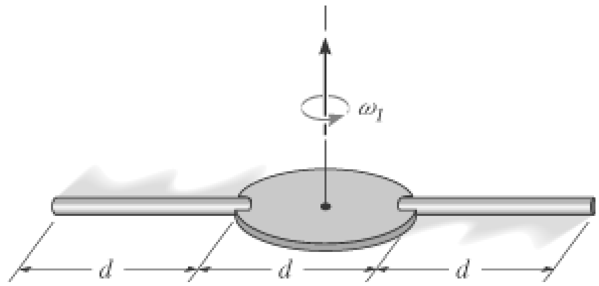
Guess $\omega = 1\frac{\text{rad}}{\text{s}}$

Given $0 = \frac{-W_m}{g}\left(v - \omega a\right)a + \frac{W_p}{g}\left(\frac{c^2}{12} + b^2\right)\omega$

$\omega = \text{Find}(\omega) \qquad \omega = 0.190\ \frac{\text{rad}}{\text{s}}$

Problem 19-37

Each of the two slender rods and the disk have the same mass *m*. Also, the length of each rod is equal to the diameter *d* of the disk. If the assembly is rotating with an angular velocity ω_1 when the rods are directed outward, determine the angular velocity of the assembly if by internal means the rods are brought to an upright vertical position.



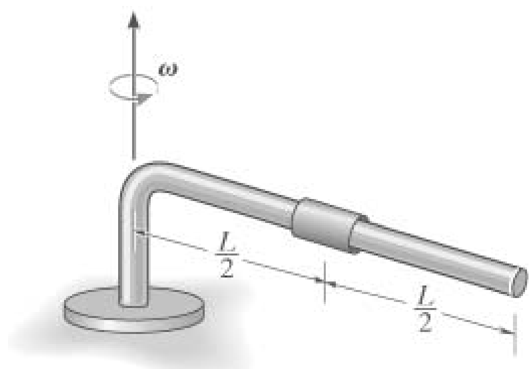
Solution:

$$H_1 = H_2$$

$$\left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2\frac{1}{12}md^2 + 2md^2\right]\omega_1 = \left[\frac{1}{2}m\left(\frac{d}{2}\right)^2 + 2m\left(\frac{d}{2}\right)^2\right]\omega_2 \qquad \omega_2 = \frac{11}{3}\omega_1$$

Problem 19-38

The rod has a length *L* and mass *m*. A smooth collar having a negligible size and one-fourth the mass of the rod is placed on the rod at its midpoint. If the rod is freely rotating with angular velocity ω about its end and the collar is released, determine the rod's angular velocity just before the collar flies off the rod. Also, what is the speed of the collar as it leaves the rod?



Solution:

$$H_1 = H_2$$

$$\frac{1}{3}mL^2\omega + \left(\frac{m}{4}\right)\left(\frac{L}{2}\right)\omega\left(\frac{L}{2}\right) = \frac{1}{3}mL^2\omega' + \left(\frac{m}{4}\right)L\omega' L \qquad \omega' = \frac{19}{28}\omega$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{4}\right)\left(\frac{L}{2}\omega\right)^2 = \frac{1}{2}\left(\frac{m}{4}\right)v'^2 + \frac{1}{2}\left(\frac{m}{4}\right)(L\omega')^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega'^2$$

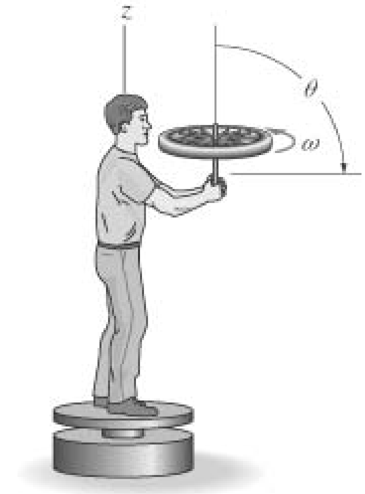
$$v'^2 = \frac{57}{112}L^2\omega^2$$

$$v'' = \sqrt{\frac{57}{112}L^2\omega^2 + \left[L\left(\frac{19}{28}\omega\right)\right]^2} \qquad v'' = \sqrt{\frac{95}{98}}\,\omega L \qquad v'' = 0.985\,\omega L$$

Problem 19-39

A man has a moment of inertia I_z about the z axis. He is originally at rest and standing on a small platform which can turn freely. If he is handed a wheel which is rotating at angular velocity ω and has a moment of inertia I about its spinning axis, determine his angular velocity if (a) he holds the wheel upright as shown, (b) turns the wheel out $\theta = 90°$, and (c) turns the wheel downward, $\theta = 180°$.

Solution:



(a) $0 + I\omega = I_z\omega_M + I\omega$ $\qquad \omega_M = 0$

(b) $0 + I\omega = I_z\omega_M + 0$ $\qquad \omega_M = \dfrac{I}{I_z}\omega$

(c) $0 + I\omega = I_z\omega_M - I\omega$ $\qquad \omega_M = \dfrac{2I}{I_z}\omega$

*** Problem 19-40**

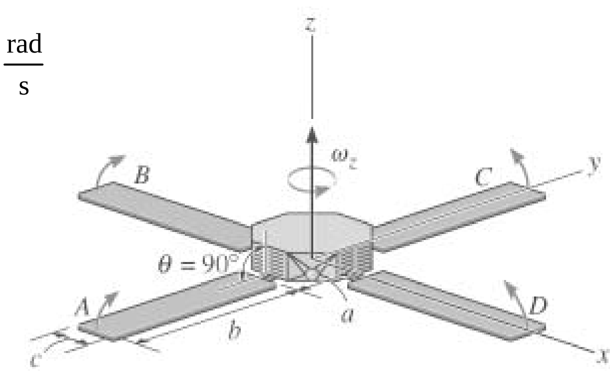
The space satellite has mass m_{ss} and moment of inertia I_z, excluding the four solar panels A,B,C, and D. Each solar panel has mass m_p and can be approximated as a thin plate.If the satellite is originally spinning about the z axis at aconstant rate ω_z, when $\theta = 90°$, determine the rate of spin if all the panels are raised and reach the upward position, $\theta = 0°$, at the same instsnt.

Given:

$m_{SS} = 125\ \text{kg}$ $\qquad a = 0.2\ \text{m}$ $\qquad \omega_z = 0.5\ \dfrac{\text{rad}}{\text{s}}$

$I_z = 0.940\ \text{kg}\cdot\text{m}^2$ $\qquad b = 0.75\ \text{m}$

$m_{sp} = 20\ \text{kg}$ $\qquad c = 0.2\ \text{m}$



Solution: Guess $\omega_{z2} = 1\ \dfrac{\text{rad}}{\text{s}}$

Given

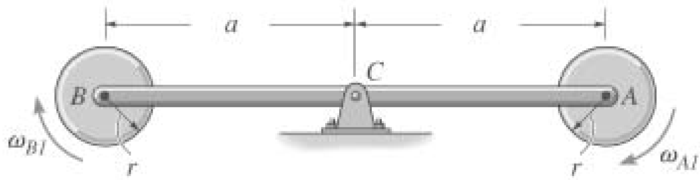
$$\left[I_z + 4\left[\frac{m_{sp}}{12}\left(b^2 + c^2\right) + m_{sp}\left(a + \frac{b}{2}\right)^2\right]\right]\omega_z = \left[I_z + 4\left(\frac{m_{sp}}{12}c^2 + m_{sp}\,a^2\right)\right]\omega_{z2}$$

$\omega_{z2} = \text{Find}\left(\omega_{z2}\right)$ $\qquad \omega_{z2} = 3.56\ \dfrac{\text{rad}}{\text{s}}$

Problem 19-41

Rod *ACB* of mass m_r supports the two disks each of mass m_d at its ends. If both disks are given a clockwise angular velocity $\omega_{A1} = \omega_{B1} = \omega_0$ while the rod is held stationary and then released, determine the angular velocity of the rod after both disks have stopped spinning relative to the rod due to frictional resistance at the pins *A* and *B*. Motion is in the *horizontal plane.* Neglect friction at pin *C.*

Given:



$m_r = 2\ \text{kg}$

$m_d = 4\ \text{kg}$

$\omega_0 = 5\ \frac{\text{rad}}{\text{s}}$

$a = 0.75\ \text{m}$

$r = 0.15\ \text{m}$

Solution:

$$2\left(\frac{1}{2}m_d\right)r^2\omega_0 = \left[2\left(\frac{1}{2}m_d\right)r^2 + 2m_d a^2 + \frac{m_r}{12}(2a)^2\right]\omega_2$$

$$\omega_2 = \frac{m_d r^2}{m_d r^2 + 2m_d a^2 + \left(\frac{m_r}{3}\right)a^2}\omega_0 \qquad \omega_2 = 0.0906\ \frac{\text{rad}}{\text{s}}$$

Problem 19-42

Disk *A* has a weight W_A. An inextensible cable is attached to the weight *W* and wrapped around the disk.The weight is dropped distance *h* before the slack is taken up. If the impact is perfectly elastic, i.e., $e = 1$, determine the angular velocity of the disk just after impact.

Given:

$W_A = 20\ \text{lb} \quad h = 2\ \text{ft}$

$W = 10\ \text{lb} \quad r = 0.5\ \text{ft}$

Solution:

$v_1 = \sqrt{2gh}$

Guess $\quad \omega_2 = 1\ \frac{\text{rad}}{\text{s}} \quad v_2 = 1\ \frac{\text{ft}}{\text{s}}$

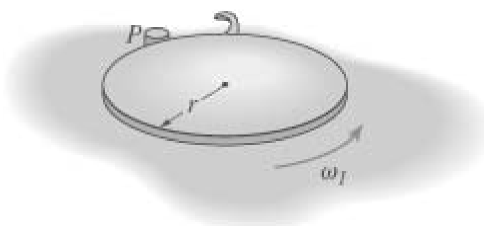
Given $\left(\frac{W}{g}\right)v_1 r = \left(\frac{W}{g}\right)v_2 r + \left(\frac{W_A}{g}\right)\frac{r^2}{2}\omega_2$ $\quad v_2 = \omega_2 r$

$\begin{pmatrix} v_2 \\ \omega_2 \end{pmatrix} = \text{Find}(v_2, \omega_2)$ $\quad v_2 = 5.67\,\frac{\text{ft}}{\text{s}}$ $\quad \omega_2 = 11.3\,\frac{\text{rad}}{\text{s}}$

Problem 19-43

A thin disk of mass m has an angular velocity ω_1 while rotating on a smooth surface. Determine its new angular velocity just after the hook at its edge strikes the peg P and the disk starts to rotate about P without rebounding.

Solution:

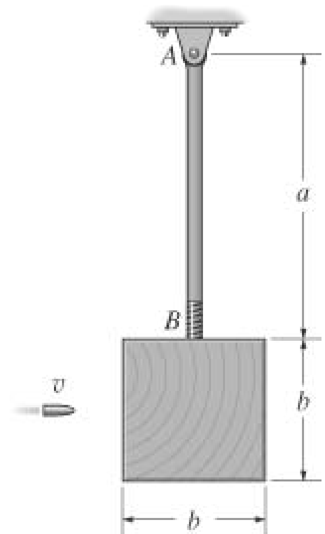


$H_1 = H_2$

$$\left(\frac{1}{2}m r^2\right)\omega_1 = \left(\frac{1}{2}m r^2 + m r^2\right)\omega_2$$

$$\omega_2 = \frac{1}{3}\omega_1$$

***Problem 19-44**



The pendulum consists of a slender rod AB of weight W_r and a wooden block of weight W_b. A projectile of weight W_p is fired into the center of the block with velocity v. If the pendulum is initially at rest, and the projectile embeds itself into the block, determine the angular velocity of the pendulum just after the impact.

Given:

$W_r = 5\text{ lb}$ $\quad W_p = 0.2\text{ lb}$ $\quad v = 1000\,\frac{\text{ft}}{\text{s}}$ $\quad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$W_b = 10\text{ lb}$ $\quad a = 2\text{ ft}$ $\quad b = 1\text{ ft}$

Solution:

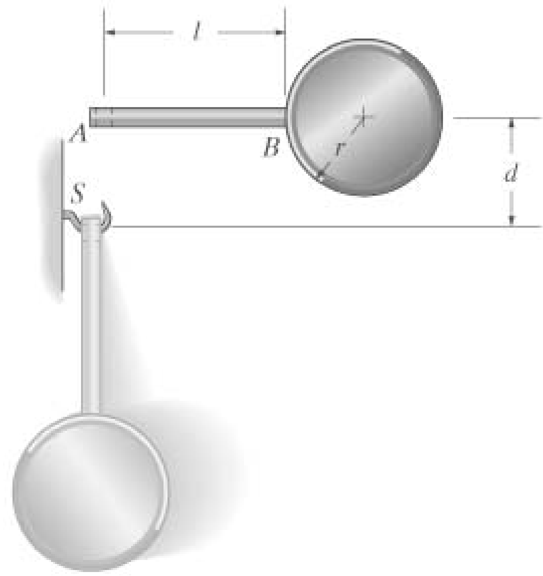
$$\left(\frac{W_p}{g}\right)v\left(a+\frac{b}{2}\right) = \left[\left(\frac{W_r}{g}\right)\left(\frac{a^2}{3}\right) + \left(\frac{W_b}{g}\right)\left(\frac{b^2}{6}\right) + \left(\frac{W_b}{g}\right)\left(a+\frac{b}{2}\right)^2 + \left(\frac{W_p}{g}\right)\left(a+\frac{b}{2}\right)^2\right]\omega_2$$

$$\omega_2 = \frac{W_p v\left(a+\frac{b}{2}\right)}{W_r\frac{a^2}{3} + W_b\frac{b^2}{6} + W_b\left(a+\frac{b}{2}\right)^2 + W_p\left(a+\frac{b}{2}\right)^2}$$

$$\omega_2 = 6.94\,\frac{\text{rad}}{\text{s}}$$

Problem 19-45

The pendulum consists of a slender rod *AB* of mass M_1 and a disk of mass M_2. It is released from rest without rotating. When it falls a distance *d*, the end *A* strikes the hook *S*, which provides a permanent connection. Determine the angular velocity of the pendulum after it has rotated 90°. Treat the pendulum's weight during impact as a nonimpulsive force.



Given:

$M_1 = 2\text{kg}$ $\quad r = 0.2\text{m}$

$M_2 = 5\text{kg}$ $\quad l = 0.5\text{m}$

$d = 0.3\text{m}$

Solution:

$$v_1 = \sqrt{2g\,d}$$

$$I_A = M_1\frac{l^2}{3} + M_2\frac{r^2}{2} + M_2(l+r)^2$$

Guesses

$$\omega_2 = 1\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 1\frac{\text{rad}}{\text{s}}$$

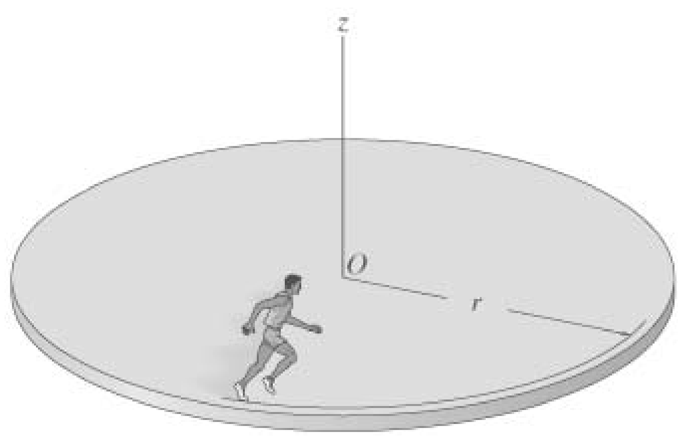
Given

$$M_1 v_1\frac{l}{2} + M_2 v_1(l+r) = I_A\omega_2 \qquad \frac{1}{2}I_A\omega_2^{\,2} = \frac{1}{2}I_A\omega_3^{\,2} - M_1 g\frac{l}{2} - M_2 g(l+r)$$

$$\begin{pmatrix}\omega_2\\ \omega_3\end{pmatrix} = \text{Find}(\omega_2, \omega_3) \qquad \omega_2 = 3.57\frac{\text{rad}}{\text{s}} \qquad \omega_3 = 6.46\frac{\text{rad}}{\text{s}}$$

Problem 19-46

A horizontal circular platform has a weight W_1 and a radius of gyration k_z about the *z* axis passing through its center *O*. The platform is free to rotate about the *z* axis and is initially at rest. A man having a weight W_2 begins to run along the edge in a circular path of radius *r*. If he has a speed *v* and maintains this speed relative to the platform, determine the angular velocity of the platform. Neglect friction.



Given:

$W_1 = 300 \text{ lb} \quad v = 4 \frac{\text{ft}}{\text{s}}$

$W_2 = 150 \text{ lb}$

$r = 10 \text{ ft} \qquad k_z = 8 \text{ ft}$

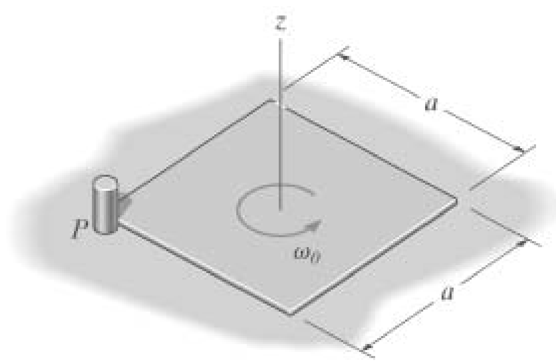
Solution:

$M v r = I\omega$

$\frac{W_2}{g} v r = \frac{W_1}{g} k_z^2 \omega \qquad \omega = W_2 v \frac{r}{W_1 k_z^2} \qquad \omega = 0.312 \frac{\text{rad}}{\text{s}}$

Problem 19-47

The square plate has a weight W and is rotating on the smooth surface with a constant angular velocity ω_0. Determine the new angular velocity of the plate just after its corner strikes the peg P and the plate starts to rotate about P without rebounding.

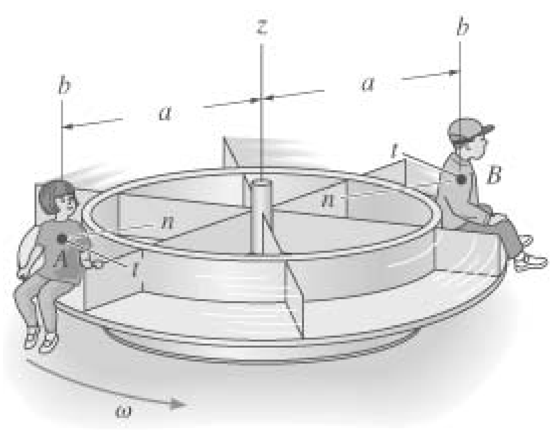


Solution:

$\left(\frac{W}{g}\right)\left(\frac{a^2}{6}\right)\omega_0 = \left(\frac{W}{g}\right)\left(\frac{2a^2}{3}\right)\omega \qquad \omega = \frac{1}{4}\omega_0$

***Problem 19-48**

Two children A and B, each having a mass M_1, sit at the edge of the merry-go-round which is rotating with angular velocity ω. Excluding the children, the merry-go-round has a mass M_2 and a radius of gyration k_z. Determine the angular velocity of the merry-go-round if A jumps off horizontally in the $-n$ direction with a speed v, measured with respect to the merry-go-round. What is the merry-go-round's angular velocity if B then jumps off horizontally in the $+t$ direction with a speed v, measured with respect to the merry-go-round? Neglect friction and the size of each child.



Given:

$M_1 = 30\ \text{kg}$ $\quad k_z = 0.6\ \text{m}$

$M_2 = 180\ \text{kg}$ $\quad a = 0.75\ \text{m}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$ $\quad v = 2\ \frac{\text{m}}{\text{s}}$

Solution:

(a) Guess $\omega_2 = 1\frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + 2M_1 a^2\right)\omega = \left(M_2 k_z^2 + M_1 a^2\right)\omega_2$

$\omega_2 = \text{Find}(\omega_2)$ $\qquad \omega_2 = 2.41\frac{\text{rad}}{\text{s}}$

(b) Guess $\omega_3 = 1\frac{\text{rad}}{\text{s}}$

Given $\left(M_2 k_z^2 + M_1 a^2\right)\omega_2 = M_2 k_z^2\omega_3 + M_1(v + \omega_3 a)a$

$\omega_3 = \text{Find}(\omega_3)$ $\qquad \omega_3 = 1.86\frac{\text{rad}}{\text{s}}$

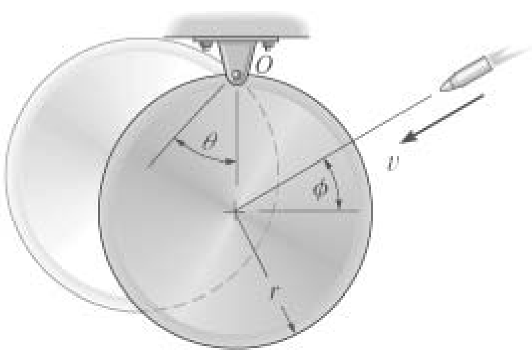
Problem 19-49

A bullet of mass m_b having velocity v is fired into the edge of the disk of mass m_d as shown. Determine the angular velocity of the disk of mass m_d just after the bullet becomes embedded in it. Also, calculate how far θ the disk will swing until it stops. The disk is originally at rest.

Given:

$m_b = 7\ \text{gm}$ $\quad m_d = 5\ \text{kg}$ $\quad \phi = 30\ \text{deg}$

$v = 800\ \frac{\text{m}}{\text{s}}$ $\quad r = 0.2\ \text{m}$



Solution:

Guesses $\omega = 1\ \frac{\text{rad}}{\text{s}}$ $\qquad \theta = 10\ \text{deg}$

Given $m_b v\cos(\phi)r = \frac{3}{2}m_d r^2\omega$ $\qquad -m_d g r + \frac{1}{2}\left(\frac{3}{2}m_d r^2\right)\omega^2 = -m_d g r\cos(\theta)$

$\begin{pmatrix}\omega \\ \theta\end{pmatrix} = \text{Find}(\omega, \theta)$ $\qquad \omega = 3.23\frac{\text{rad}}{\text{s}}$ $\qquad \theta = 32.8\,\text{deg}$

Problem 19-50

The two disks each have weight W. If they are released from rest when $\theta = \theta_1$, determine the maximum angle θ_2 after they collide and rebound from each other. The coefficient of restitution is e. When $\theta = 0°$ the disks hang so that they just touch one another.

Given:

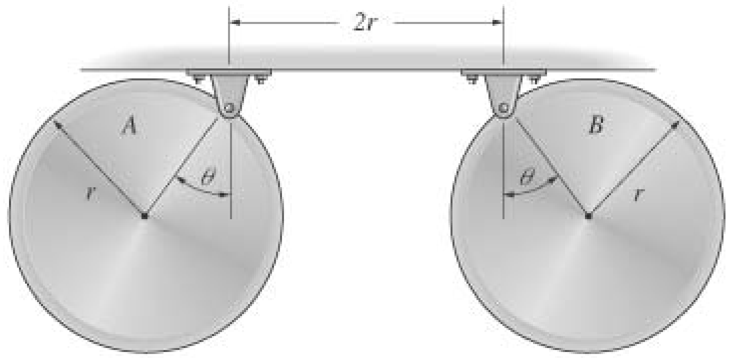
$g = 32.2 \frac{ft}{s^2}$

$W = 10\ lb$

$\theta_1 = 30\ deg$

$e = 0.75$

$r = 1\ ft$



Solution:

Guesses $\omega_1 = 1 \frac{rad}{s}$ $\omega_2 = 1 \frac{rad}{s}$ $\theta_2 = 10deg$

Given $-W r \cos(\theta_1) = \frac{1}{2}\left(\frac{3}{2}\frac{W}{g} r^2\right)\omega_1^2 - W r$

$e r \omega_1 = r \omega_2$

$-W r + \frac{1}{2}\left(\frac{3}{2}\frac{W}{g} r^2\right)\omega_2^2 = -W r \cos(\theta_2)$

$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \theta_2 \end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2)$ $\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} = \begin{pmatrix} 2.40 \\ 1.80 \end{pmatrix} \frac{rad}{s}$ $\theta_2 = 22.4\ deg$

Problem 19-51

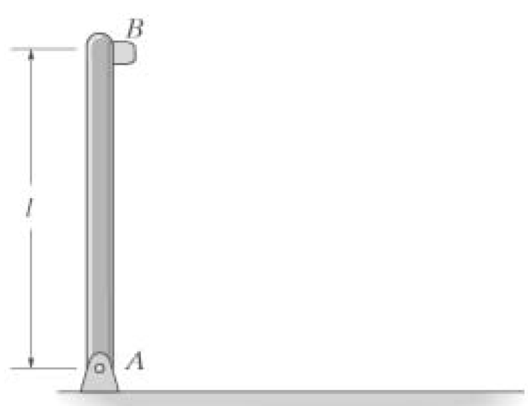
The rod AB of weight W is released from rest in the vertical position. If the coefficient of restitution between the floor and the cushion at B is e, determine how high the end of the rod rebounds after impact with the floor.

Given:

$W = 15\ lb$

$l = 2\ ft$

$e = 0.7$



Solution:

Guesses $\omega_1 = 1\frac{rad}{s}$ $\omega_2 = 1\frac{rad}{s}$ $\theta = 1deg$

Given

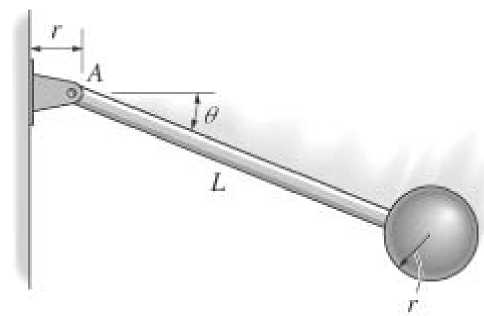
$$W\left(\frac{l}{2}\right) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_1^{\,2} \qquad e\omega_1 l = \omega_2 l \qquad W\left(\frac{l}{2}\right)\sin(\theta) = \frac{1}{2}\left(\frac{W}{g}\right)\frac{l^2}{3}\omega_2^{\,2}$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}6.95\\ 4.86\end{pmatrix}\frac{rad}{s} \qquad \theta = 29.34\,deg$$

$$h = l\sin(\theta) \qquad h = 0.980\,ft$$

***Problem 19-52**

The pendulum consists of a solid ball of weight W_b and a rod of weight W_r. If it is released from rest when $\theta_1 = 0°$, determine the angle θ_2 after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest.



Given:

$W_b = 10lb$ $e = 0.6$ $L = 2ft$

$W_r = 4lb$ $r = 0.3ft$ $g = 32.2\frac{ft}{s^2}$

Solution:

$$I_A = \left(\frac{W_r}{g}\right)\left(\frac{L^2}{3}\right) + \frac{2}{5}\left(\frac{W_b}{g}\right)r^2 + \frac{W_b}{g}(L+r)^2$$

Guesses $\omega_1 = 1\frac{rad}{s}$ $\omega_2 = 1\frac{rad}{s}$ $\theta_2 = 10deg$

Given $0 = -W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_1^{\,2}$

$$e(L+r)\omega_1 = (L+r)\omega_2$$

$$-W_b(L+r) - W_r\left(\frac{L}{2}\right) + \frac{1}{2}I_A\omega_2^{\,2} = -\left[W_b(L+r) + W_r\left(\frac{L}{2}\right)\right]\sin(\theta_2)$$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta_2\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta_2) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}5.45\\ 3.27\end{pmatrix}\frac{rad}{s} \qquad \theta_2 = 39.8\,deg$$

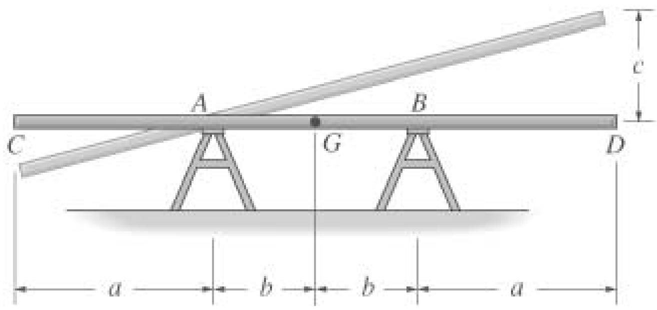
Problem 19-53

The plank has a weight W, center of gravity at G, and it rests on the two sawhorses at A and B. If the end D is raised a distance c above the top of the sawhorses and is released from rest, determine how high end C will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about A, strikes and pivots on the sawhorses at B, and rotates clockwise off the sawhorse at A.

Given:

$W = 30\text{lb}$ $b = 1.5\text{ft}$

$a = 3\text{ft}$ $c = 2\text{ft}$



Solution:

$$I_G = \frac{1}{12}\left(\frac{W}{g}\right)4(a+b)^2 \qquad I_A = I_G + \left(\frac{W}{g}\right)b^2$$

Guesses $\omega_1 = 1\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1\frac{\text{rad}}{\text{s}}$ $\theta = 1\text{deg}$ $h = 1\text{ft}$

Given $W\left(\frac{b}{2b+a}\right)c = \frac{1}{2}I_A\omega_1^{\,2}$ $-I_G\,\omega_1 + \left(\frac{W}{g}\right)\omega_1\,b\,b = -I_A\,\omega_2$

$W\,b\sin(\theta) = \frac{1}{2}I_A\omega_2^{\,2}$ $h = (a+2b)\sin(\theta)$

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \theta\\ h\end{pmatrix} = \text{Find}(\omega_1, \omega_2, \theta, h) \qquad \begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix} = \begin{pmatrix}1.89\\ 0.95\end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \theta = 4.78\,\text{deg} \qquad h = 0.500\,\text{ft}$$

Problem 19-54

Tests of impact on the fixed crash dummy are conducted using the ram of weight W that is released from rest at $\theta = \theta_1$ and allowed to fall and strike the dummy at $\theta = \theta_2$. If the coefficient of restitution between the dummy and the ram is e, determine the angle θ_3 to which the ram will rebound before momentarily coming to rest.

Given:

$W = 300\text{ lb}$ $e = 0.4$

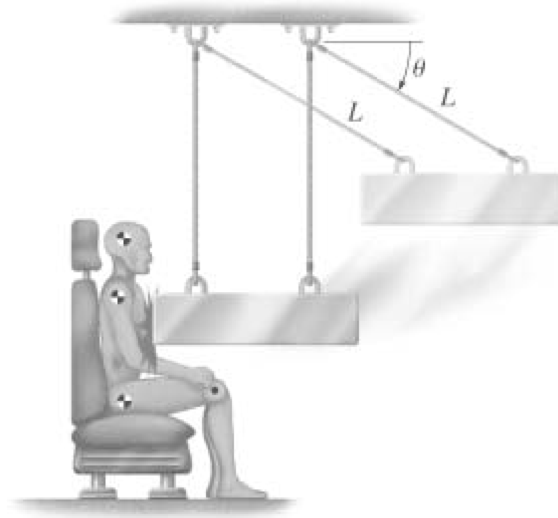
$\theta_1 = 30\text{ deg}$ $L = 10\text{ ft}$

$\theta_2 = 90\text{ deg}$ $g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

Solution:

Guesses $v_1 = 1\frac{\text{ft}}{\text{s}}$ $v_2 = 1\frac{\text{ft}}{\text{s}}$

$\theta_3 = 1 \text{ deg}$



Given

$$-W L \sin(\theta_1) = \frac{1}{2}\left(\frac{W}{g}\right) v_1^2 - W L \sin(\theta_2)$$

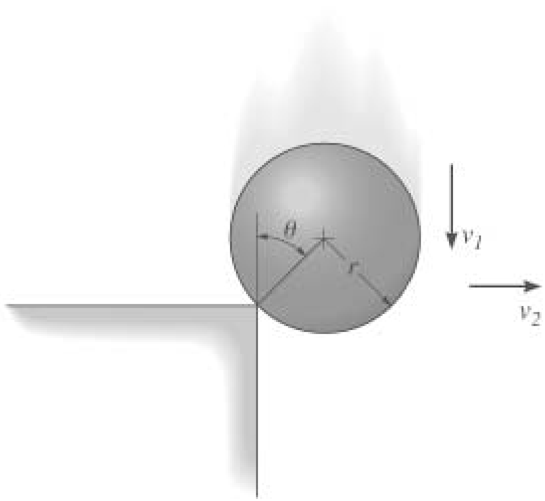
$$-W L \sin(\theta_2) + \frac{1}{2}\left(\frac{W}{g}\right) v_2^2 = -W L \sin(\theta_3)$$

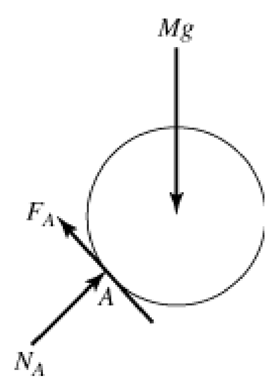
$$e v_1 = v_2$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \theta_3 \end{pmatrix} = \text{Find}(v_1, v_2, \theta_3) \qquad \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 17.94 \\ 7.18 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad \theta_3 = 66.9 \text{ deg}$$

Problem 19-55

The solid ball of mass *m* is dropped with a velocity v_1 onto the edge of the rough step. If it rebounds horizontally off the step with a velocity v_2, determine the angle θ at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is e.





Solution:

No slip $\omega_2 r = v_2 \cos(\theta)$

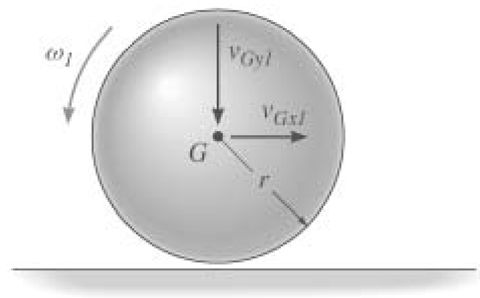
Angular Momentum about *A* $m v_1 r \sin(\theta) = m v_2 r \cos(\theta) + \frac{2}{5} m r^2 \omega_2$

Restitution $e v_1 \cos(\theta) = v_2 \sin(\theta)$

Combining we find $\theta = \operatorname{atan}\left(\sqrt{\frac{7e}{5}}\right)$

***Problem 19-56**

A solid ball with a mass *m* is thrown on the ground such that at the instant of contact it has an angular velocity ω_1 and velocity components v_{Gx1} and v_{Gy1} as shown. If the ground is rough so no slipping occurs, determine the components of the velocity of its mass center just after impact. The coefficient of restitution is *e*.



Solution:

Restitution $e v_{Gy1} = v_{Gy2}$

Angular Momentum $\frac{2}{5} m r^2 \omega_1{}^2 - m v_{Gx1} r = \frac{2}{5} m r^2 \omega_2 + m v_{Gx2} r$

No slip $v_{Gx2} = \omega_2 r$

Combining $v_{G2} = \begin{pmatrix} \frac{5}{7} v_{Gx1} - \frac{2}{7} r\omega_1 \\ e v_{Gy1} \end{pmatrix}$
