Problem 20-1

The ladder of the fire truck rotates around the *z* axis with angular velocity ω_1 which is increasing at rate α_1. At the same instant it is rotating upwards at the constant rate ω_2. Determine the velocity and acceleration of point *A* located at the top of the ladder at this instant.

Given:

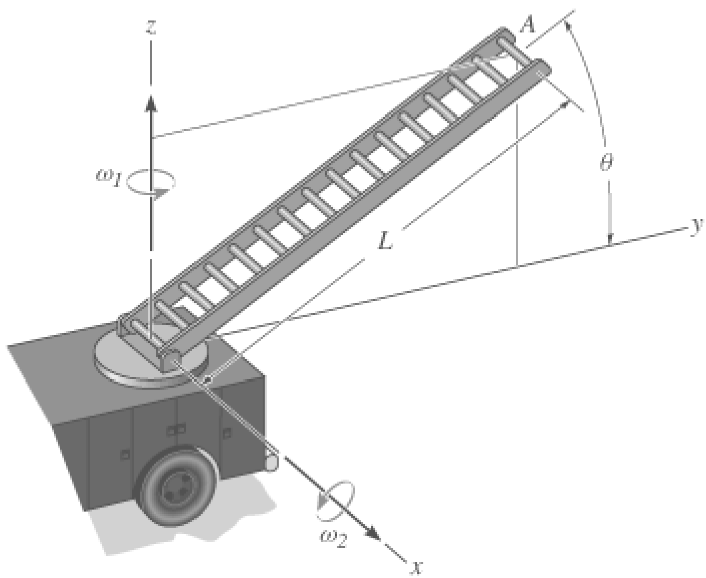
$$\omega_1 = 0.15 \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 0.8 \frac{\text{rad}}{\text{s}^2}$$

$$\omega_2 = 0.6 \frac{\text{rad}}{\text{s}}$$

$$\theta = 30 \text{ deg}$$

$$L = 40 \text{ ft}$$



Solution:

$$\mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) \\ L\sin(\theta) \end{pmatrix}$$

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega_1\omega_2 \\ \alpha_1 \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -5.20 \\ -12.00 \\ 20.78 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 24.6 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -24.11 \\ -13.25 \\ -7.20 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 28.4 \frac{\text{ft}}{\text{s}^2}$$

Problem 20-2

The ladder of the fire truck rotates around the z axis with angular velocity ω_1 which is increasing at rate α_1. At the same instant it is rotating upwards at rate ω_2 while increasing at rate α_2. Determine the velocity and acceleration of point A located at the top of the ladder at this instant.

Given:

$$\omega_1 = 0.15 \frac{\text{rad}}{\text{s}}$$

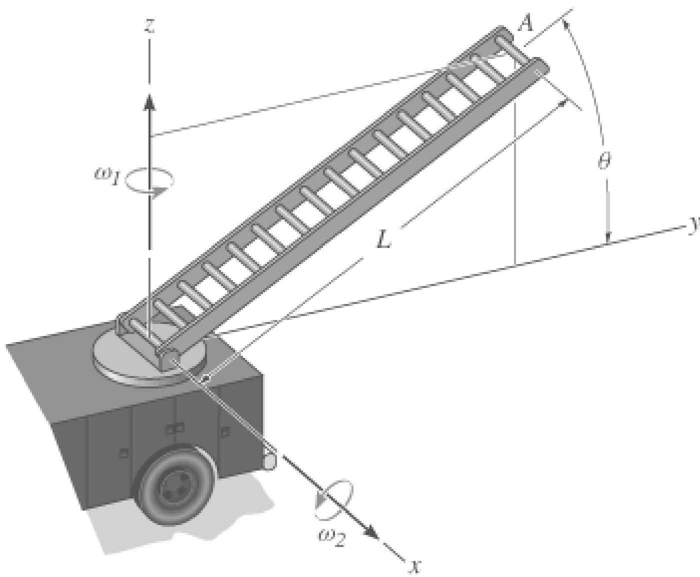
$$\alpha_1 = 0.2 \frac{\text{rad}}{\text{s}^2}$$

$$\omega_2 = 0.6 \frac{\text{rad}}{\text{s}}$$

$$\alpha_2 = 0.4 \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 30 \text{ deg}$$

$$L = 40 \text{ ft}$$



Solution:

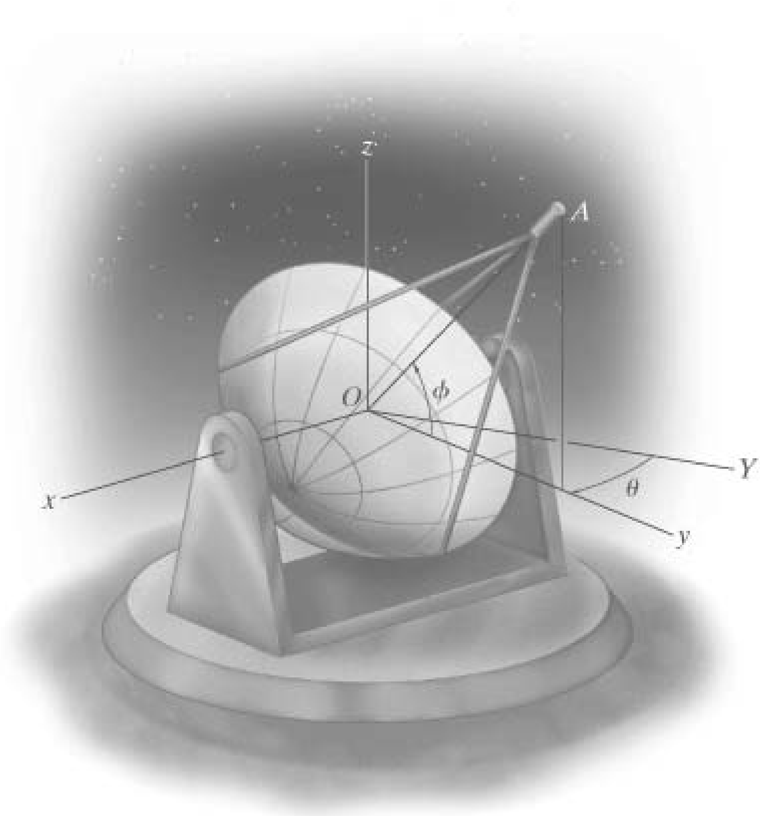
$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_2 \\ \omega_1 \omega_2 \\ \alpha_1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) \\ L\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -5.20 \\ -12.00 \\ 20.78 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 24.6 \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -3.33 \\ -21.25 \\ 6.66 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 22.5 \frac{\text{ft}}{\text{s}^2}$$

Problem 20-3

The antenna is following the motion of a jet plane. At the instant shown, the constant angular rates of change are θ' and ϕ'. Determine the velocity and acceleration of the signal horn A at this instant. The distance OA is d.



Given:

$\theta = 25\ \text{deg}$

$\theta' = 0.4\ \frac{\text{rad}}{\text{s}}$

$\phi = 75\ \text{deg}$

$\phi' = 0.6\ \frac{\text{rad}}{\text{s}}$

$d = 0.8\ \text{m}$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \phi' \\ 0 \\ -\theta' \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\theta'\phi' \\ 0 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} 0 \\ d\cos(\phi) \\ d\sin(\phi) \end{pmatrix}$$

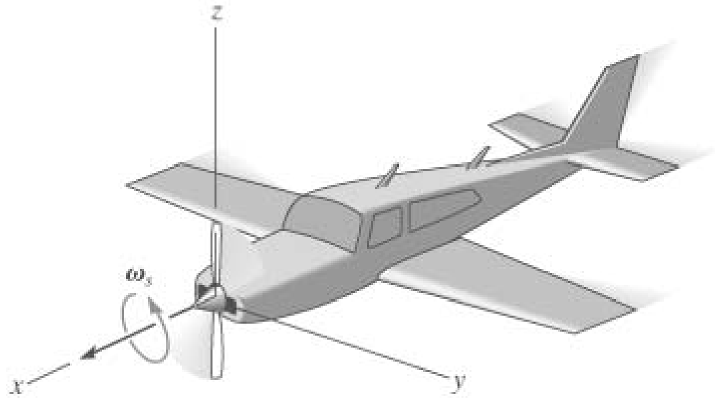
$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} 0.083 \\ -0.464 \\ 0.124 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -0.371 \\ -0.108 \\ -0.278 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-4**

The propeller of an airplane is rotating at a constant speed $\omega_s\mathbf{i}$, while the plane is undergoing a turn at a constant rate ω_t. Determine the angular acceleration of the propeller if (a) the turn is horizontal, i.e., $\omega_t\mathbf{k}$, and (b) the turn is vertical, downward, i.e., $\omega_t\mathbf{j}$.

Solution:



(a) $\boldsymbol{\alpha} = \left(\omega_t\vec{\mathbf{k}}\right) \times \left(\omega_s\vec{\mathbf{i}}\right)$

$\boldsymbol{\alpha} = \omega_s\omega_t\vec{\mathbf{j}}$

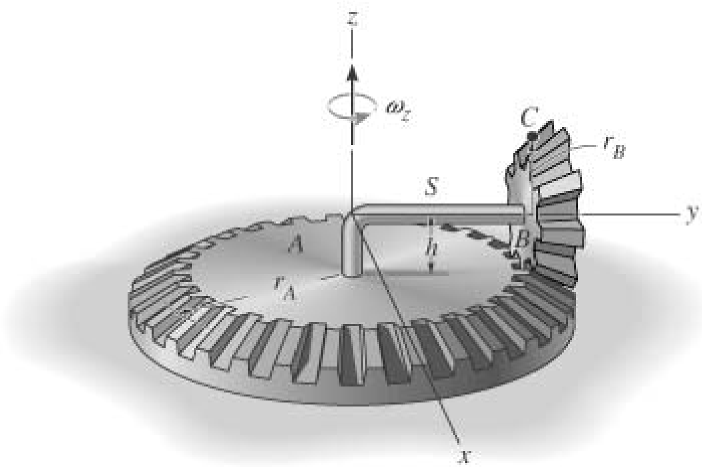
(b) $\boldsymbol{\alpha} = \left(\omega_t\vec{\mathbf{j}}\right) \times \left(\omega_s\vec{\mathbf{i}}\right)$

$\boldsymbol{\alpha} = -\omega_s\,\omega_t\vec{\mathbf{k}}$

Problem 20-5

Gear A is fixed while gear B is free to rotate on the shaft S. If the shaft is turning about the z axis with angular velocity ω_z, while increasing at rate α_z, determine the velocity and acceleration of point C at the instant shown. The face of gear B lies in a vertical plane.

Given:



$\omega_z = 5\ \frac{\text{rad}}{\text{s}}$

$\alpha_z = 2\ \frac{\text{rad}}{\text{s}^2}$

$r_A = 160\ \text{mm}$

$r_B = 80\ \text{mm}$

$h = 80\ \text{mm}$

Solution:

$\omega_z r_A = \omega_B r_B \qquad \omega_B = \omega_z\frac{r_A}{r_B} \qquad \omega_B = 10\,\frac{\text{rad}}{\text{s}}$

$\alpha_z r_A = \alpha_B r_B \qquad \alpha_B = \alpha_z\frac{r_A}{r_B} \qquad \alpha_B = 4\,\frac{\text{rad}}{\text{s}^2}$

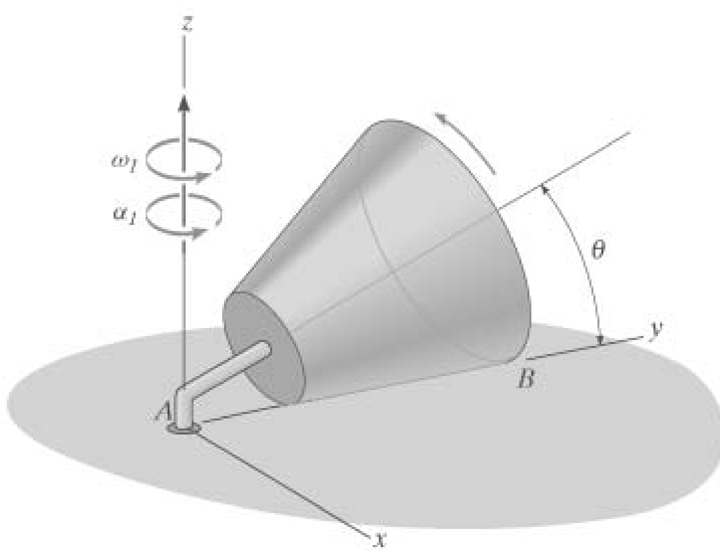
$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_B \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\alpha_B \\ \alpha_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ r_A \\ r_B \end{pmatrix}$$

$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} -1.6 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \left|\mathbf{v_C}\right| = 1.6\ \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_C} = \begin{pmatrix} -0.64 \\ -12 \\ -8 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \left|\mathbf{a_C}\right| = 14.436\ \frac{\text{m}}{\text{s}^2}$$

Problem 20-6

The conical spool rolls on the plane without slipping. If the axle has an angular velocity ω_1 and an angular acceleration α_1 at the instant shown, determine the angular velocity and angular acceleration of the spool at this instant. Neglect the small vertical part of the rod at A.



Given:

$$\omega_1 = 3\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 2\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 20\ \text{deg}$$

Solution:

$$L = 1\ \text{m}$$

$$R = L\tan(\theta)$$

$$\mathbf{r} = \begin{pmatrix} 0 \\ L\cos(\theta) + R\sin(\theta) \\ L\sin(\theta) - R\cos(\theta) \end{pmatrix}$$

Guesses $\omega_2 = 1\,\frac{rad}{s}$ $\alpha_2 = 1\,\frac{rad}{s^2}$ $a_y = 1\,\frac{m}{s^2}$ $a_z = 1\,\frac{m}{s^2}$

Given

Enforce the no-slip constraint

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \times \mathbf{r} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_1 + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix}\right] \times \mathbf{r} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 0 \\ 26.3 \end{pmatrix}\frac{m}{s^2} \qquad \omega_2 = -8.77\,\frac{rad}{s}$$

$$\alpha_2 = -5.85\,\frac{rad}{s^2}$$

Now construct the angular velocity and angular acceleration.

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.00 \\ -8.24 \\ 0.00 \end{pmatrix}\frac{rad}{s}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_1 + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_1 + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 24.73 \\ -5.49 \\ 0.00 \end{pmatrix}\frac{rad}{s^2}$$

Problem 20-7

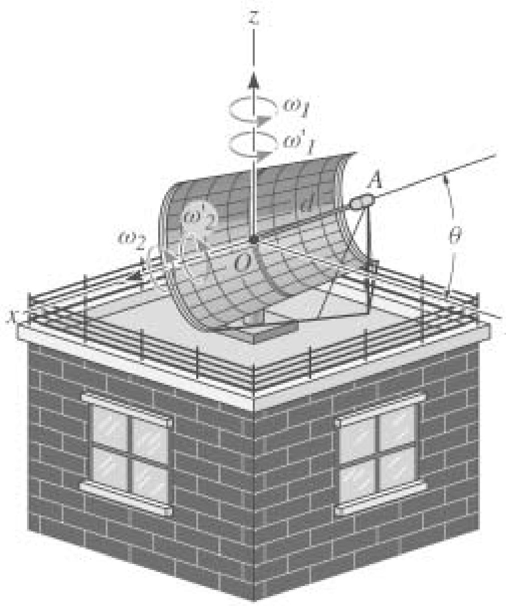
At a given instant, the antenna has an angular motion ω_1 and ω'_1 about the *z* axis. At the same instant $\theta = \theta_1$, the angular motion about the *x* axis is ω_2 and ω'_2. Determine the velocity and acceleration of the signal horn A at this instant. The distance from *O* to *A* is *d*.

Given:

$\omega_1 = 3\,\frac{\text{rad}}{\text{s}}$ $\omega_2 = 1.5\,\frac{\text{rad}}{\text{s}}$

$\omega'_1 = 2\,\frac{\text{rad}}{\text{s}^2}$ $\omega'_2 = 4\,\frac{\text{rad}}{\text{s}^2}$

$\theta_1 = 30\text{ deg}$ $d = 3\text{ ft}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} \omega'_2 \\ 0 \\ \omega'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{r_A} = \begin{pmatrix} 0 \\ d\cos(\theta_1) \\ d\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -7.79 \\ -2.25 \\ 3.90 \end{pmatrix}\frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_A}| = 9\,\frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} 8.30 \\ -35.23 \\ 7.02 \end{pmatrix}\frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_A}| = 36.868\,\frac{\text{ft}}{\text{s}^2}$$

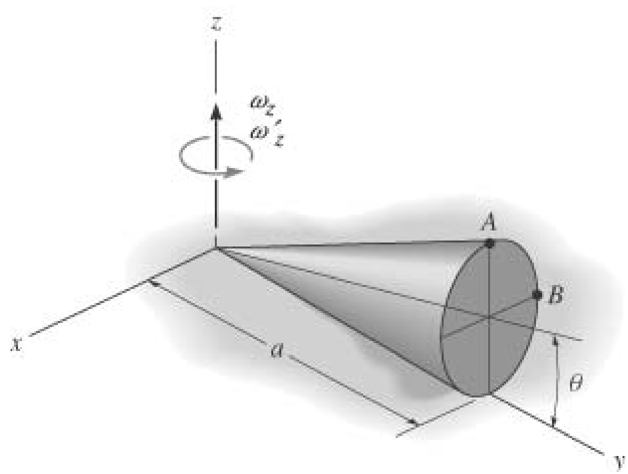
***Problem 20-8**

The cone rolls without slipping such that at the instant shown ω_z and ω'_z are as given. Determine the velocity and acceleration of point *A* at this instant.

Given:

$\omega_z = 4\,\frac{\text{rad}}{\text{s}}$

$\omega'_z = 3\,\frac{\text{rad}}{\text{s}^2}$



$\theta = 20\ \text{deg}$

$a = 2\ \text{ft}$

Solution:

$b = a\sin(\theta)$

$\omega_z - \omega_2 \sin(\theta) = 0 \qquad \omega_2 = \dfrac{\omega_z}{\sin(\theta)} \qquad \omega_2 = 11.695\ \dfrac{\text{rad}}{\text{s}}$

$\omega'_z - \omega'_2 \sin(\theta) = 0 \qquad \omega'_2 = \dfrac{\omega'_z}{\sin(\theta)} \qquad \omega'_2 = 8.771\ \dfrac{\text{rad}}{\text{s}^2}$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_2 \cos(\theta) \\ -\omega_2 \sin(\theta) + \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\omega'_2 \cos(\theta) \\ -\omega'_2 \sin(\theta) + \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

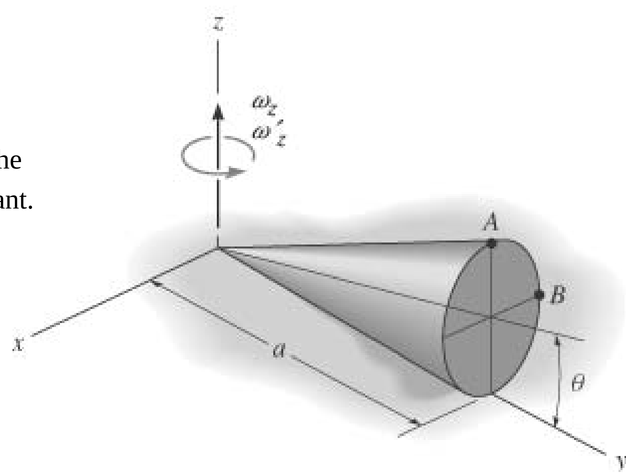
$$\mathbf{r_A} = \begin{pmatrix} 0 \\ a - 2b\sin(\theta) \\ 2b\cos(\theta) \end{pmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ 1.532 \\ 1.286 \end{pmatrix} \text{ft}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -14.1 \\ 0.0 \\ 0.0 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \left|\mathbf{v_A}\right| = 14.128\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} -10.6 \\ -56.5 \\ -87.9 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad \left|\mathbf{a_A}\right| = 105.052\ \frac{\text{ft}}{\text{s}^2}$$

Problem 20-9

The cone rolls without slipping such that at the instant shown ω_z and ω'_z are given. Determine the velocity and acceleration of point *B* at this instant.



Given:

$\omega_z = 4\ \dfrac{\text{rad}}{\text{s}}$

$\omega'_z = 3\ \dfrac{\text{rad}}{\text{s}^2}$

$\theta = 20\ \text{deg}$

$a = 2\ \text{ft}$

Solution:

$b = a\sin(\theta)$

$\omega_z - \omega_2 \sin(\theta) = 0$ $\quad \omega_2 = \dfrac{\omega_z}{\sin(\theta)}$ $\quad \omega_2 = 11.695\ \dfrac{\text{rad}}{\text{s}}$

$\omega'_z - \omega'_2 \sin(\theta) = 0$ $\quad \omega'_2 = \dfrac{\omega'_z}{\sin(\theta)}$ $\quad \omega'_2 = 8.771\ \dfrac{\text{rad}}{\text{s}^2}$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_2 \cos(\theta) \\ -\omega_2 \sin(\theta) + \omega_z \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ -\omega'_2 \cos(\theta) \\ -\omega'_2 \sin(\theta) + \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{r_B} = \begin{pmatrix} -b \\ a - b\sin(\theta) \\ b\cos(\theta) \end{pmatrix} \qquad \mathbf{r_B} = \begin{pmatrix} -0.684 \\ 1.766 \\ 0.643 \end{pmatrix} \text{ft}$$

$$\mathbf{v_B} = \boldsymbol{\omega} \times \mathbf{r_B} \qquad \mathbf{v_B} = \begin{pmatrix} -7.064 \\ 0 \\ -7.518 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad |\mathbf{v_B}| = 10.316\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_B} = \boldsymbol{\alpha} \times \mathbf{r_B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_B}) \qquad \mathbf{a_B} = \begin{pmatrix} 77.319 \\ -28.257 \\ -5.638 \end{pmatrix} \frac{\text{ft}}{\text{s}^2} \qquad |\mathbf{a_B}| = 82.513\ \frac{\text{ft}}{\text{s}^2}$$

Problem 20-10

If the plate gears *A* and *B* are rotating with the angular velocities shown, determine the angular velocity of gear *C* about the shaft *DE*. What is the angular velocity of *DE* about the *y* axis?

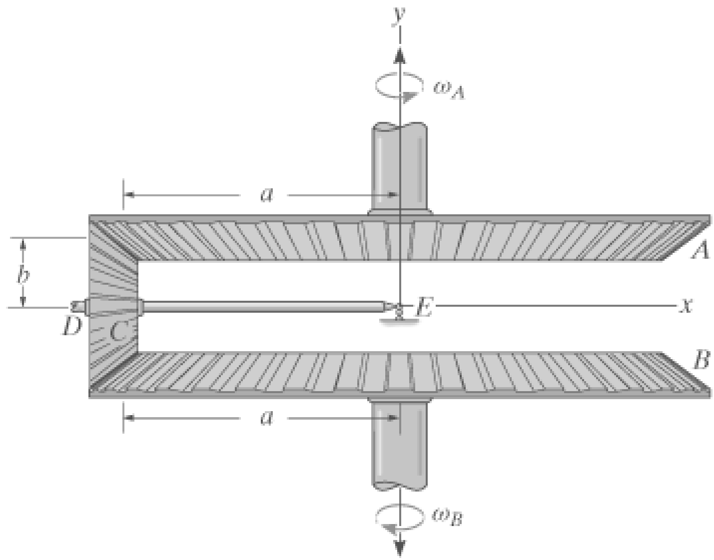
Given:

$\omega_A = 5\ \dfrac{\text{rad}}{\text{s}}$

$\omega_B = 15\ \dfrac{\text{rad}}{\text{s}}$

$a = 100\ \text{mm}$

$b = 25\ \text{mm}$



Solution:

Guesses

$\omega_{DE} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_A a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\omega_B a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 2\ b \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega_B a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \omega_{DE} \\ 0 \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix}$$

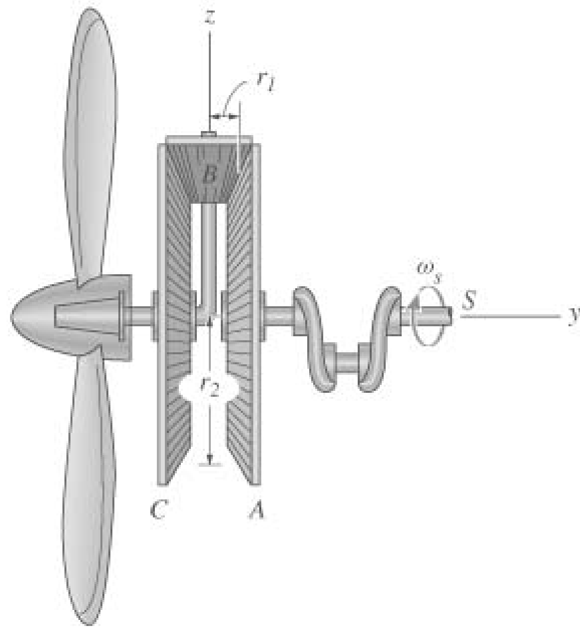
$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \omega_{DE} \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \omega_{DE}) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \omega_{DE} = -5\,\frac{\text{rad}}{\text{s}}$$

Problem 20-11

Gear A is fixed to the crankshaft S, while gear C is fixed and gear B and the propeller are free to rotate. The crankshaft is turning with angular velocity ω_s about its axis. Determine the magnitudes of the angular velocity of the propeller and the angular acceleration of gear B.

Given:

$\omega_s = 80\ \frac{\text{rad}}{\text{s}}$



$r_2 = 0.4$ ft

$r_1 = 0.1$ ft

Solution:

$v_P = \omega_s r_2 = \omega_B (2r_1)$

$\omega_B = \dfrac{\omega_s r_2}{2r_1}$ $\qquad \omega_B = 160\ \dfrac{\text{rad}}{\text{s}}$

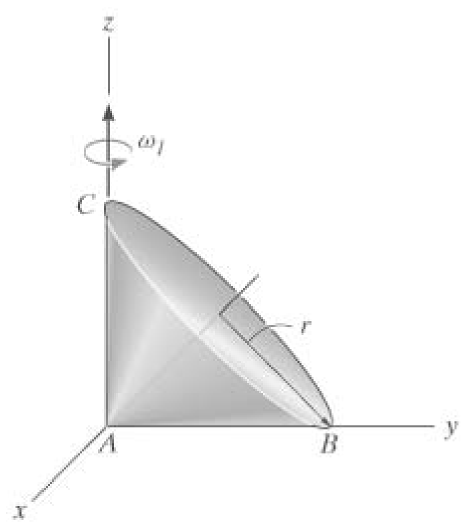
$v_B = \omega_B r_1$ $\qquad v_B = 16\ \dfrac{\text{ft}}{\text{s}}$

$\omega_{prop} = \dfrac{v_B}{r_2}$ $\qquad \boldsymbol{\omega} = \begin{pmatrix} 0 \\ -\omega_{prop} \\ 0 \end{pmatrix}$ $\qquad \boldsymbol{\omega} = \begin{pmatrix} 0.0 \\ -40.0 \\ 0.0 \end{pmatrix} \dfrac{\text{rad}}{\text{s}}$

$\boldsymbol{\alpha_B} = \begin{pmatrix} 0 \\ -\omega_{prop} \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \omega_B \end{pmatrix}$ $\qquad \boldsymbol{\alpha_B} = \begin{pmatrix} -6400 \\ 0 \\ 0 \end{pmatrix} \dfrac{\text{rad}}{\text{s}^2}$

***Problem 20-12**

The right circular cone rotates about the *z* axis at a constant rate ω_1 without slipping on the horizontal plane. Determine the magnitudes of the velocity and acceleration of points *B* and *C*.



Given:

$\omega_1 = 4\ \dfrac{\text{rad}}{\text{s}}$ $\qquad r = 50\ \text{mm}$ $\qquad \theta = 45\ \text{deg}$

Solution:

Enforce no-slip condition $\qquad$ Guess $\qquad \omega_2 = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta)+\omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \sqrt{2}r \\ 0 \end{pmatrix} = 0$ $\omega_2 = \text{Find}(\omega_2)$ $\omega_2 = -5.66\ \frac{\text{rad}}{\text{s}}$

Define terms

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta)+\omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta)+\omega_1 \end{pmatrix}$$

$$\mathbf{r_B} = \begin{pmatrix} 0 \\ \sqrt{2}r \\ 0 \end{pmatrix} \qquad \mathbf{r_C} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}r \end{pmatrix}$$

Find velocities and accelerations

$$\mathbf{v_B} = \boldsymbol{\omega}\times\mathbf{r_B} \qquad \mathbf{v_C} = \boldsymbol{\omega}\times\mathbf{r_C}$$

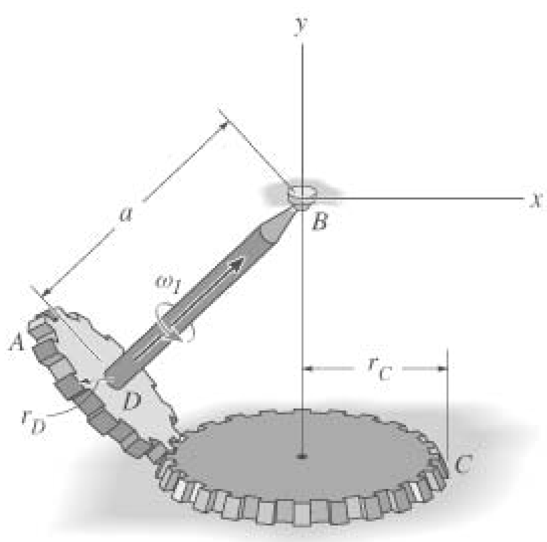
$$\mathbf{a_B} = \boldsymbol{\alpha}\times\mathbf{r_B} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r_B}) \qquad \mathbf{a_C} = \boldsymbol{\alpha}\times\mathbf{r_C} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r_C})$$

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} 0 \\ 0 \\ 1.131 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \mathbf{v_C} = \begin{pmatrix} -0.283 \\ 0 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_C} = \begin{pmatrix} 0 \\ -1.131 \\ -1.131 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$|\mathbf{v_B}| = 0\ \frac{\text{m}}{\text{s}} \qquad |\mathbf{a_B}| = 1.131\ \frac{\text{m}}{\text{s}^2} \qquad |\mathbf{v_C}| = 0.283\ \frac{\text{m}}{\text{s}} \qquad |\mathbf{a_C}| = 1.6\ \frac{\text{m}}{\text{s}^2}$$

Problem 20-13

Shaft *BD* is connected to a ball-and-socket joint at *B*, and a beveled gear *A* is attached to its other end. The gear is in mesh with a fixed gear *C*. If the shaft and gear *A* are *spinning* with a constant angular velocity ω_1, determine the angular velocity and angular acceleration of gear *A*.



Given:

$$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$$

$$a = 300\ \text{mm}$$

$r_D = 75\ \text{mm}$

$r_C = 100\ \text{mm}$

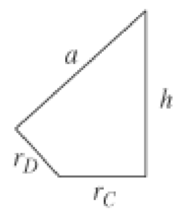
Solution:

Guesses $\theta = 10\ \text{deg}$ $h = 10\ \text{mm}$

Given

$a\cos(\theta) + r_D\sin(\theta) = h$ $a\sin(\theta) = r_C + r_D\cos(\theta)$

$\begin{pmatrix} h \\ \theta \end{pmatrix} = \text{Find}(h, \theta)$ $h = 0.293\ \text{m}$ $\theta = 32.904\,\text{deg}$



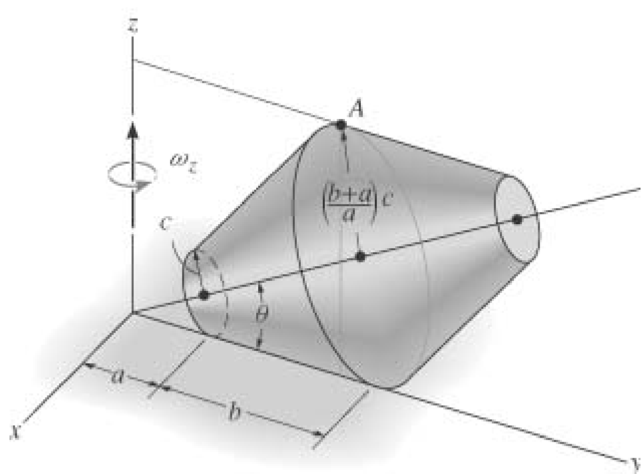
$\omega_y = \dfrac{\omega_1 r_D}{a\sin(\theta) - r_D\cos(\theta)}$ $\omega_y = 6\dfrac{\text{rad}}{\text{s}}$

$\boldsymbol{\omega} = \begin{pmatrix} \omega_1\sin(\theta) \\ \omega_1\cos(\theta) + \omega_y \\ 0 \end{pmatrix}$ $\boldsymbol{\omega} = \begin{pmatrix} 4.346 \\ 12.717 \\ 0 \end{pmatrix}\dfrac{\text{rad}}{\text{s}}$

$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega_y \\ 0 \end{pmatrix} \times \boldsymbol{\omega}$ $\boldsymbol{\alpha} = \begin{pmatrix} 0.0 \\ 0.0 \\ -26.1 \end{pmatrix}\dfrac{\text{rad}}{\text{s}^2}$

Problem 20-14

The truncated cone rotates about the z axis at a constant rate ω_z without slipping on the horizontal plane. Determine the velocity and acceleration of point A on the cone.



Given:

$\omega_z = 0.4\dfrac{\text{rad}}{\text{s}}$

$a = 1\ \text{ft}$

$b = 2\ \text{ft}$

$c = 0.5\ \text{ft}$

Solution: $\theta = \text{asin}\left(\frac{c}{a}\right)$

$\omega_Z + \omega_S \sin(\theta) = 0 \qquad \omega_S = \frac{-\omega_Z}{\sin(\theta)}$

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_S \cos(\theta) \\ \omega_Z + \omega_S \sin(\theta) \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0 \\ -0.693 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_Z \end{pmatrix} \times \boldsymbol{\omega} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0.277 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

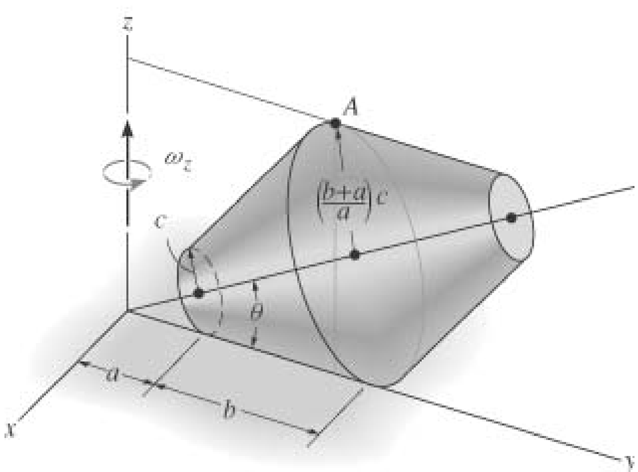
$$\mathbf{r_A} = \begin{bmatrix} 0 \\ a + b - 2\left(\frac{b+a}{a}\right) c \sin(\theta) \\ 2\left(\frac{b+a}{a}\right) c \cos(\theta) \end{bmatrix} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ 1.5 \\ 2.598 \end{pmatrix} \text{ft}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -1.8 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} 0.000 \\ -0.720 \\ -0.831 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-15

The truncated cone rotates about the z axis at ω_z without slipping on the horizontal plane. If at this same instant ω_z is increasing at ω'_z, determine the velocity and acceleration of point A on the cone.



Given:

$\omega_Z = 0.4 \frac{\text{rad}}{\text{s}} \qquad a = 1 \text{ ft}$

$\omega'_Z = 0.5 \frac{\text{rad}}{\text{s}^2} \qquad b = 2 \text{ ft}$

$\theta = 30\ \text{deg} \qquad c = 0.5\ \text{ft}$

Solution: $r = \left(\frac{b+a}{a}\right)c \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ a+b-2r\sin(\theta) \\ 2r\cos(\theta) \end{pmatrix}$

Guesses $\omega_2 = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_2 = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_y = 1\ \frac{\text{ft}}{\text{s}^2} \qquad a_z = 1\ \frac{\text{ft}}{\text{s}^2}$

Given Enforce the no-slip constraints

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix} \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \omega'_z + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \omega_2 = -0.8\ \frac{\text{rad}}{\text{s}} \qquad \alpha_2 = -1\ \frac{\text{rad}}{\text{s}^2}$$

Define terms

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \omega'_z + \alpha_2\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_z + \omega_2\sin(\theta) \end{pmatrix}$$

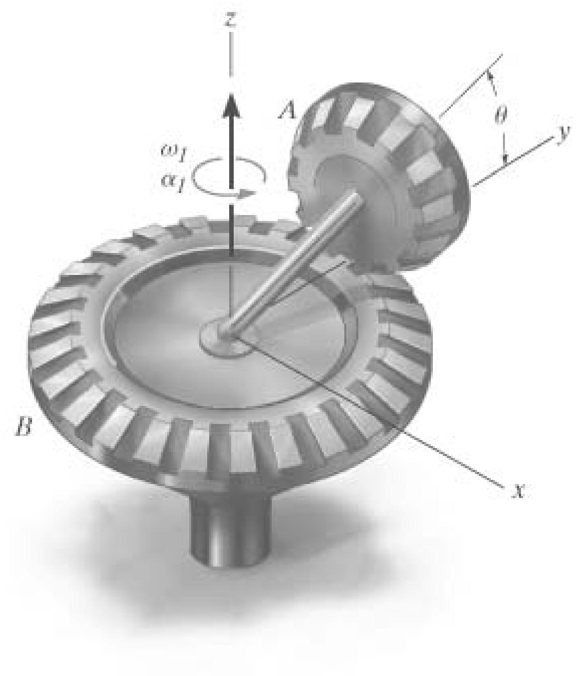
Calculate velocity and acceleration.

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A} \qquad \mathbf{v_A} = \begin{pmatrix} -1.80 \\ 0.00 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A}) \qquad \mathbf{a_A} = \begin{pmatrix} -2.25 \\ -0.72 \\ -0.831 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

***Problem 20-16**

The bevel gear *A* rolls on the fixed gear *B*. If at the instant shown the shaft to which *A* is attached is rotating with angular velocity ω_1 and has angular acceleration α_1, determine the angular velocity and angular acceleration of gear *A*.



Given:

$$\omega_1 = 2\ \frac{\text{rad}}{\text{s}}$$

$$\alpha_1 = 4\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta = 30\ \text{deg}$$

Solution:

$$L = 1\ \text{m}$$

$$R = L\tan(\theta) \qquad b = L\sec(\theta)$$

Guesses

$$\omega_2 = 1\ \frac{\text{rad}}{\text{s}} \qquad \alpha_2 = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$a_y = 1\ \frac{\text{m}}{\text{s}^2} \qquad a_z = 1\ \frac{\text{m}}{\text{s}^2}$$

Given Enforce the no-slip constraints.

$$\begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta) + \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = 0$$

$$\left[\begin{pmatrix} 0 \\ \alpha_2\cos(\theta) \\ \alpha_2\sin(\theta) + \alpha_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2\cos(\theta) \\ \omega_2\sin(\theta) + \omega_1 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ a_y \\ a_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_2 \\ \alpha_2 \\ a_y \\ a_z \end{pmatrix} = \text{Find}(\omega_2, \alpha_2, a_y, a_z) \qquad \begin{pmatrix} a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} \frac{\text{m}}{\text{s}^2} \qquad \omega_2 = -4\,\frac{\text{rad}}{\text{s}} \qquad \alpha_2 = -8\,\frac{\text{rad}}{\text{s}^2}$$

Build the angular velocity and angular acceleration.

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.00 \\ -3.46 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \alpha_2 \cos(\theta) \\ \alpha_2 \sin(\theta) + \alpha_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ \omega_2 \cos(\theta) \\ \omega_2 \sin(\theta) + \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 6.93 \\ -6.93 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

Problem 20-17

The differential of an automobile allows the two rear wheels to rotate at different speeds when the automobile travels along a curve. For operation, the rear axles are attached to the wheels at one end and have beveled gears *A* and *B* on their other ends. The differential case *D* is placed over the left axle but can rotate about *C* independent of the axle. The case supports a pinion gear *E* on a shaft, which meshes with gears *A* and *B*. Finally, a ring gear *G* is *fixed* to the differential case so that the case rotates with the ring gear when the latter is driven by the drive pinion H. This gear, like the differential case, is free to rotate about the left wheel axle. If the drive pinion is turning with angular velocity ω_H and the pinion gear *E* is spinning about its shaft with angular velocity ω_E, determine the angular velocity ω_A and ω_B of each axle.

Given:

$$\omega_H = 100\ \frac{\text{rad}}{\text{s}}$$

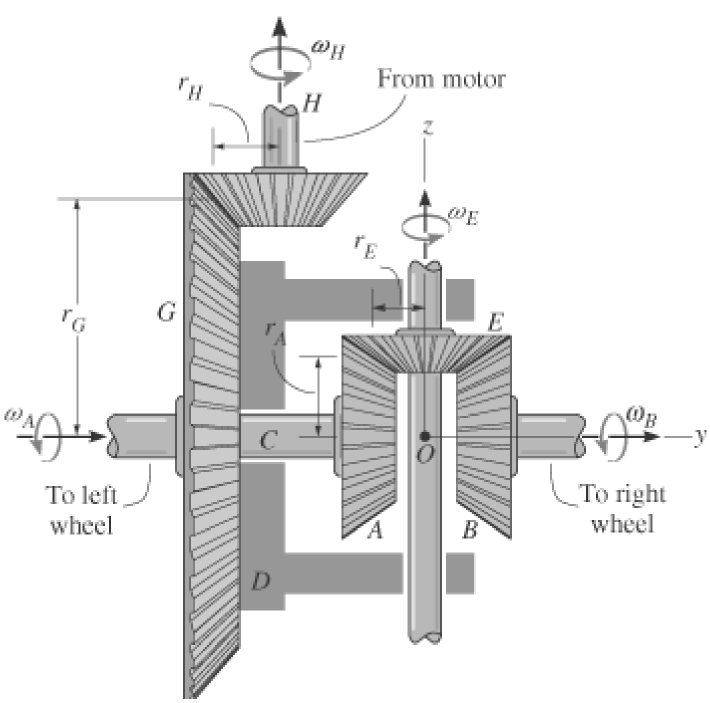
$$\omega_E = 30\ \frac{\text{rad}}{\text{s}}$$

$$r_G = 180\ \text{mm}$$

$$r_H = 50\ \text{mm}$$

$$r_E = 40\ \text{mm}$$

$$r_A = 60\ \text{mm}$$



Solution:

$$\omega_H r_H = \omega_G r_G$$

$$\omega_G = \omega_H \frac{r_H}{r_G}$$

$$\omega_G = 27.778 \frac{\text{rad}}{\text{s}}$$

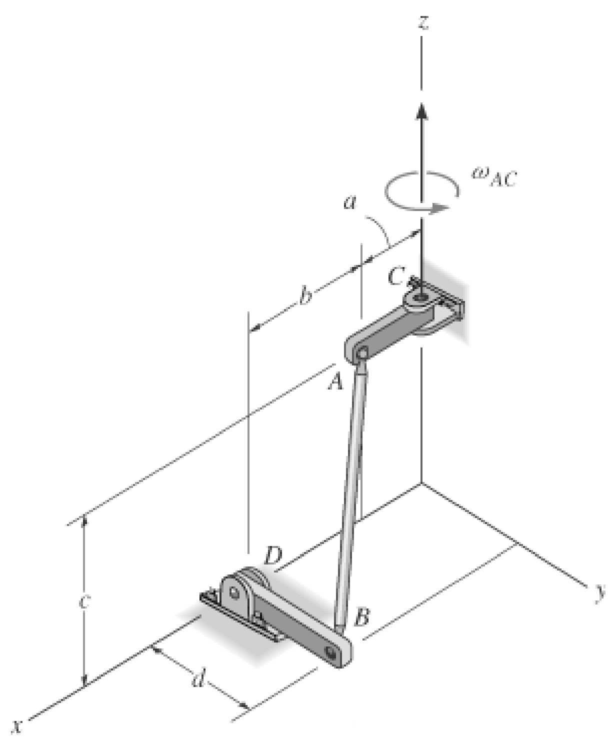
$$v_E = \omega_G r_A \qquad v_E = 1.667 \frac{\text{m}}{\text{s}}$$

$$v_E - \omega_E r_E = \omega_B r_A \qquad \omega_B = \frac{v_E - \omega_E r_E}{r_A} \qquad \omega_B = 7.778 \frac{\text{rad}}{\text{s}}$$

$$v_E + \omega_E r_E = \omega_A r_A \qquad \omega_A = \frac{v_E + \omega_E r_E}{r_A} \qquad \omega_A = 47.8 \frac{\text{rad}}{\text{s}}$$

Problem 20-18

Rod AB is attached to the rotating arm using ball-and-socket joints. If AC is rotating with constant angular velocity ω_{AC} about the pin at C, determine the angular velocity of link BD at the instant shown.



Given:

$a = 1.5 \text{ ft}$ $\qquad d = 2 \text{ ft}$

$b = 3 \text{ ft}$ $\qquad \omega_{AC} = 8 \frac{\text{rad}}{\text{s}}$

$c = 6 \text{ ft}$

Solution:

Guesses

$\omega_{BD} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \omega_{ABx} = 1 \frac{\text{rad}}{\text{s}}$

$\omega_{ABy} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \omega_{ABz} = 1 \frac{\text{rad}}{\text{s}}$

Given

Note that $\boldsymbol{\omega_{AB}}$ is perpendicular to $\mathbf{r_{AB}}$.

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{BD} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0 \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_{BD} \\ \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \text{Find}(\omega_{BD}, \omega_{ABx}, \omega_{ABy}, \omega_{ABz}) \qquad \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \begin{pmatrix} -1.633 \\ 0.245 \\ -0.735 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\omega_{BD} = -2 \frac{\text{rad}}{\text{s}}$$

Problem 20-19

Rod *AB* is attached to the rotating arm using ball-and-socket joints. If *AC* is rotating about the pin at *C* with angular velocity ω_{AC} and angular acceleration α_{AC}, determine the angular velocity and angular acceleration of link *BD* at the instant shown.

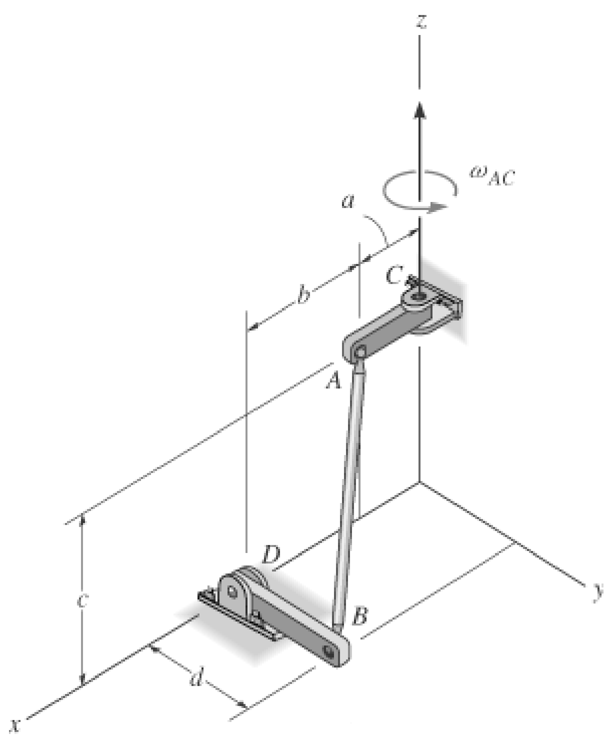
Given:

$a = 1.5 \text{ ft}$

$\omega_{AC} = 8 \frac{\text{rad}}{\text{s}}$ $\qquad b = 3 \text{ ft}$

$\alpha_{AC} = 6 \frac{\text{rad}}{\text{s}^2}$ $\qquad c = 6 \text{ ft}$

$d = 2 \text{ ft}$



Solution:

Guesses

$\omega_{BD} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \alpha_{BD} = 1 \frac{\text{rad}}{\text{s}^2}$

$\omega_{ABx} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \alpha_{ABx} = 1 \frac{\text{rad}}{\text{s}^2}$

$\omega_{ABy} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \alpha_{ABy} = 1 \frac{\text{rad}}{\text{s}^2}$

$\omega_{ABz} = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \alpha_{ABz} = 1 \frac{\text{rad}}{\text{s}^2}$

Given Note that ω_{AB} and α_{AB} are perpendicular to r_{AB}.

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{AC} \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{BD} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} -a\,\omega_{AC}^2 \\ a\alpha_{AC} \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} + \begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABy} \end{pmatrix} \times \left[\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \times \begin{pmatrix} b \\ d \\ -c \end{pmatrix} \right] = \begin{pmatrix} 0 \\ -d\,\omega_{BD}^2 \\ d\alpha_{BD} \end{pmatrix}$$

$$\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} \begin{pmatrix} b \\ d \\ -c \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_{BD} \\ \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \\ \alpha_{BD} \\ \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} = \text{Find}\left(\omega_{BD}, \omega_{ABx}, \omega_{ABy}, \omega_{ABz}, \alpha_{BD}, \alpha_{ABx}, \alpha_{ABy}, \alpha_{ABz}\right)$$

$$\begin{pmatrix} \omega_{ABx} \\ \omega_{ABy} \\ \omega_{ABz} \end{pmatrix} = \begin{pmatrix} -1.633 \\ 0.245 \\ -0.735 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_{ABx} \\ \alpha_{ABy} \\ \alpha_{ABz} \end{pmatrix} = \begin{pmatrix} 0.495 \\ -14.18 \\ -4.479 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$\omega_{BD} = -2\,\frac{\text{rad}}{\text{s}}$$

$$\alpha_{BD} = 31.6\,\frac{\text{rad}}{\text{s}^2}$$

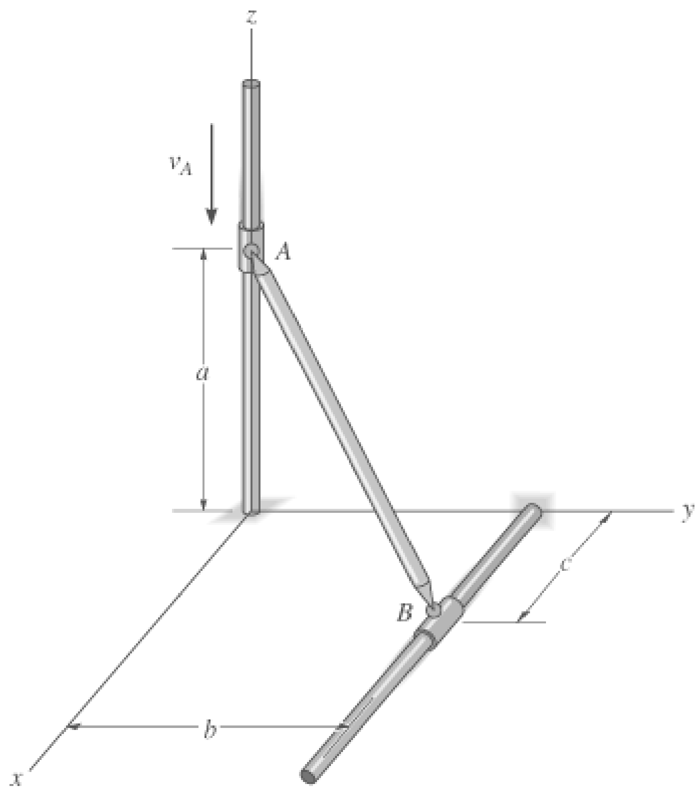
***Problem 20-20**

If the rod is attached with ball-and-socket joints to smooth collars A and B at its end points, determine the speed of B at the instant shown if A is moving downward at constant speed v_A. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.

Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}}$ $\quad b = 6\ \text{ft}$

$a = 3\ \text{ft}$ $\quad c = 2\ \text{ft}$



Solution:

Guesses

$v_B = 1\ \frac{\text{ft}}{\text{s}}$

$\omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.980 \\ -1.061 \\ -1.469 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12\ \frac{\text{ft}}{\text{s}}$$

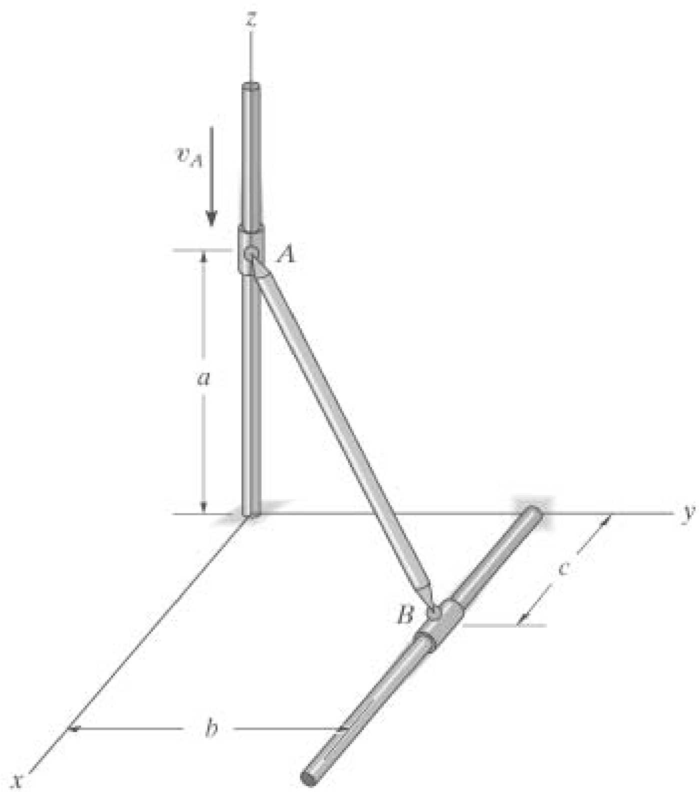
Problem 20-21

If the collar at A is moving downward with an acceleration a_A, at the instant its speed is v_A, determine the acceleration of the collar at B at this instant.

Given:

$v_A = 8\ \frac{\text{ft}}{\text{s}}$ $\quad a = 3\ \text{ft}$ $\quad c = 2\ \text{ft}$

$a_A = 5\ \frac{\text{ft}}{\text{s}^2} \qquad b = 6\ \text{ft}$



Solution:

Guesses

$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

$a_B = 1\ \frac{\text{ft}}{\text{s}^2} \qquad \alpha_x = 1\ \frac{\text{rad}}{\text{s}^2}$

$\alpha_y = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_z = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -a_A \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} \right] = \begin{pmatrix} a_B \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} 0.61 \\ 5.70 \\ 11.82 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

$$v_B = 12\,\frac{\text{ft}}{\text{s}}$$

$$a_B = -96.5\,\frac{\text{ft}}{\text{s}^2}$$

Problem 20-22

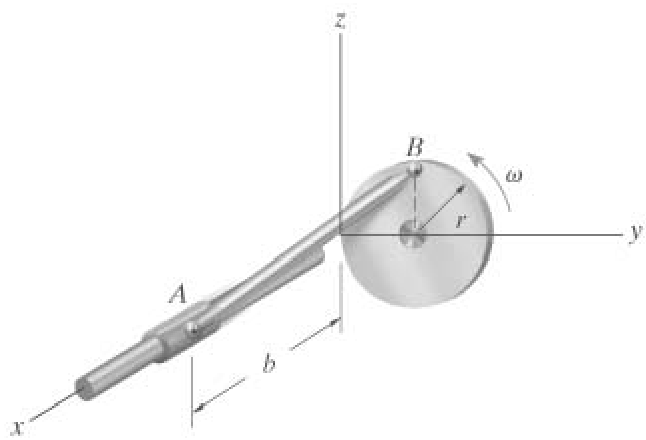
Rod *AB* is attached to a disk and a collar by ball-and-socket joints. If the disk is rotating at a constant angular velocity ω, determine the velocity and acceleration of the collar at *A* at the instant shown. Assume the angular velocity is directed perpendicular to the rod.

Given:

$$\omega = 2\,\frac{\text{rad}}{\text{s}}$$

$$r = 1\text{ ft}$$

$$b = 3\text{ ft}$$



Solution:

Guesses

$$\omega_x = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_x = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$\omega_y = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_y = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$\omega_z = 1\,\frac{\text{rad}}{\text{s}} \qquad \alpha_z = 1\,\frac{\text{rad}}{\text{s}^2}$$

$$v_A = 1\,\frac{\text{ft}}{\text{s}} \qquad a_A = 1\,\frac{\text{ft}}{\text{s}^2}$$

Given

$$\begin{pmatrix} 0 \\ -\omega r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ 0 \\ -\omega^2 r \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} \right] = \begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_A, a_A) \qquad v_A = 0.667 \frac{\text{ft}}{\text{s}} \qquad a_A = -0.148 \frac{\text{ft}}{\text{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.182 \\ -0.061 \\ 0.606 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -0.364 \\ -1.077 \\ -0.013 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

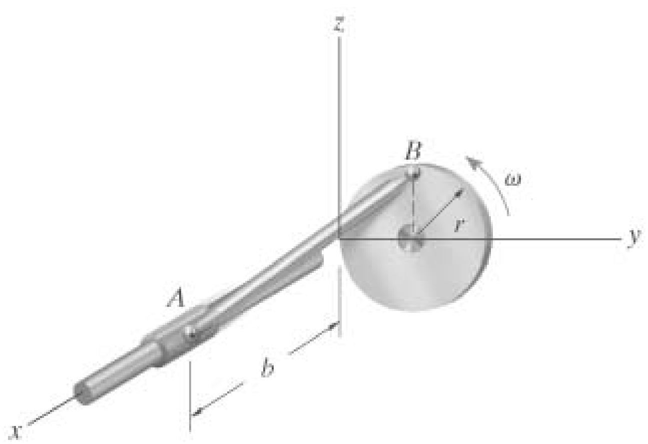
Problem 20-23

Rod *AB* is attached to a disk and a collar by ball and-socket joints. If the disk is rotating with an angular acceleration α, and at the instant shown has an angular velocity ω, determine the velocity and acceleration of the collar at *A* at the instant shown.

Given:

$\omega = 2 \frac{\text{rad}}{\text{s}}$ $\qquad r = 1 \text{ ft}$

$\alpha = 4 \frac{\text{rad}}{\text{s}^2}$ $\qquad b = 3 \text{ ft}$



Solution:

Guesses $\omega_x = 1 \frac{\text{rad}}{\text{s}}$ $\qquad \omega_y = 1 \frac{\text{rad}}{\text{s}}$

$\omega_z = 1 \frac{\text{rad}}{\text{s}}$ $\qquad v_A = 1 \frac{\text{ft}}{\text{s}}$

$$\alpha_x = 1\ \frac{\text{rad}}{\text{s}^2} \qquad \alpha_y = 1\ \frac{\text{rad}}{\text{s}^2}$$

$$\alpha_z = 1\ \frac{\text{rad}}{\text{s}^2} \qquad a_A = 1\ \frac{\text{ft}}{\text{s}^2}$$

Given

$$\begin{pmatrix} 0 \\ -\omega r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = \begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ -\alpha r \\ -\omega^2 r \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -r \\ -r \end{pmatrix} \right] = \begin{pmatrix} a_A \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_A \\ a_A \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_A, a_A) \qquad v_A = 0.667\ \frac{\text{ft}}{\text{s}} \qquad a_A = 1.185\ \frac{\text{ft}}{\text{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.182 \\ -0.061 \\ 0.606 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -3.636 \times 10^{-7} \\ -1.199 \\ 1.199 \end{pmatrix} \frac{\text{rad}}{\text{s}^2}$$

***Problem 20-24**

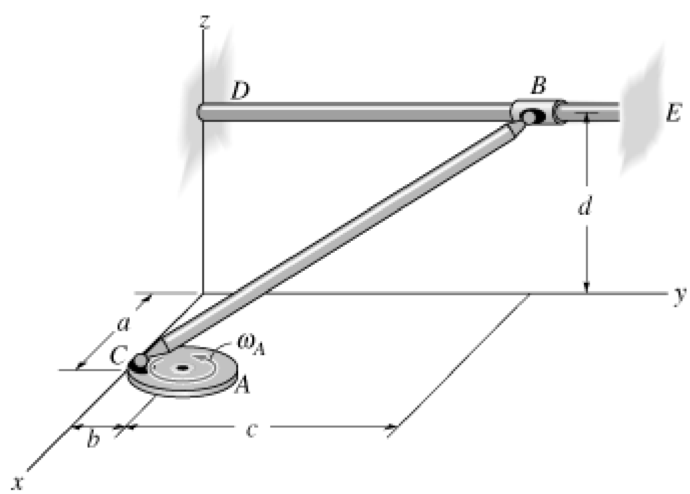
The rod *BC* is attached to collars at its ends by ball-and-socket joints. If disk *A* has angular velocity ω_A, determine the angular velocity of the rod and the velocity of collar *B* at the instant shown. Assume the angular velocity of the rod is directed perpendicular to the rod.

Given:

$a = 200\ \text{mm}$

$b = 100\text{ mm} \qquad \omega_A = 10\ \frac{\text{rad}}{\text{s}}$

$c = 500\text{ mm} \qquad d = 300\text{ mm}$



Solution:

Guesses

$v_B = 1\ \frac{\text{m}}{\text{s}} \qquad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} \omega_A b \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \cdot \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = 0\ \frac{\text{m}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.204 \\ -0.612 \\ 1.361 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = -0.333\ \frac{\text{m}}{\text{s}}$$

Problem 20-25

The rod *BC* is attached to collars at its ends. There is a ball-and-socket at *C*. The connection at *B* now consists of a pin as shown in the bottom figure. If disk *A* has angular velocity ω_A, determine the angular velocity of the rod and the velocity of collar *B* at the instant shown. *Hint*: The constraint allows rotation of the rod both along the bar *DE* (**j** direction) and along the axis of the pin (**n** direction). Since there is no rotational component in the **u** direction, i.e., perpendicular to **n** and **j** where $\mathbf{u} = \mathbf{j} \times \mathbf{n}$, an additional equation for solution can be obtained from $\boldsymbol{\omega} \cdot \mathbf{u} = 0$. The vector **n** is in the same direction as $\mathbf{r_{BC}} \times \mathbf{r_{DC}}$.

Given:

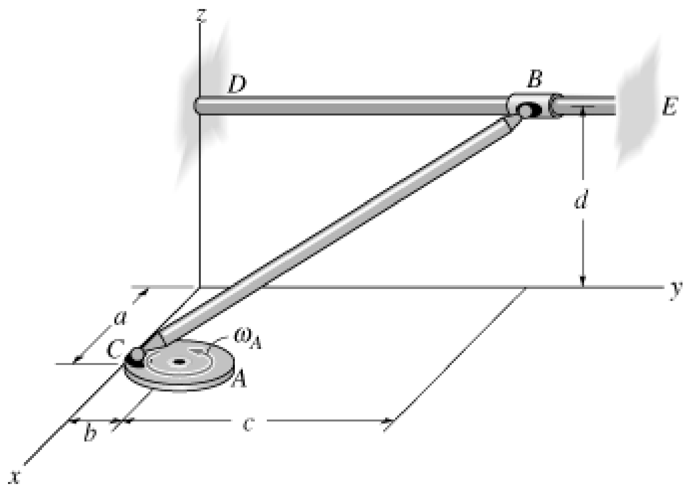
$\omega_A = 10\ \frac{\text{rad}}{\text{s}}$

$a = 200\text{ mm}$

$b = 100\text{ mm}$

$c = 500\ \text{mm}$

$d = 300\ \text{mm}$

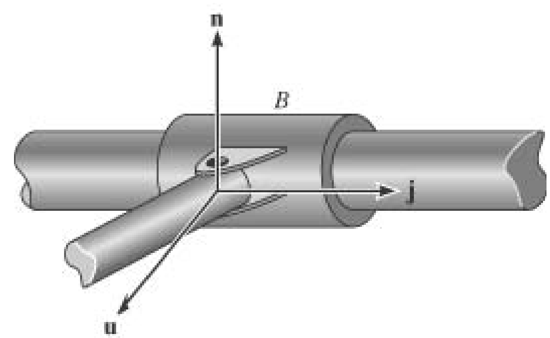


Solution:

$$\mathbf{r_{BC}} = \begin{pmatrix} a \\ -b-c \\ d \end{pmatrix}$$

$$\mathbf{r_{DC}} = \begin{pmatrix} a \\ 0 \\ -d \end{pmatrix}$$

$$\mathbf{n} = \frac{\mathbf{r_{BC}} \times \mathbf{r_{DC}}}{\left|\mathbf{r_{BC}} \times \mathbf{r_{DC}}\right|} \qquad \mathbf{n} = \begin{pmatrix} 0.728 \\ 0.485 \\ 0.485 \end{pmatrix}$$



$$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \mathbf{n} \qquad \mathbf{u} = \begin{pmatrix} 0.485 \\ 0 \\ -0.728 \end{pmatrix}$$

Guesses $\quad v_B = 1\ \frac{\text{m}}{\text{s}} \quad \omega_x = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_y = 1\ \frac{\text{rad}}{\text{s}} \quad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given

$$\begin{pmatrix} \omega_A b \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ b+c \\ d \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \mathbf{u} = 0\ \frac{\text{rad}}{\text{s}}$$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.769 \\ -2.308 \\ 0.513 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = -0.333\ \frac{\text{m}}{\text{s}}$$

Problem 20-26

The rod AB is attached to collars at its ends by ball-and-socket joints. If collar A has a speed v_A, determine the speed of collar B at the instant shown.

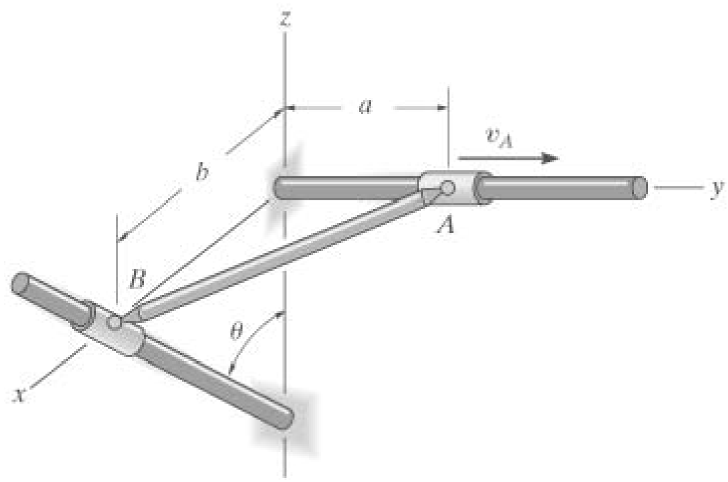
Given:

$v_A = 20\ \frac{ft}{s}$

$a = 2\ ft$

$b = 6\ ft$

$\theta = 45\ deg$



Solution:

Guesses $\omega_x = 1\ \frac{rad}{s}$ $\omega_y = 1\ \frac{rad}{s}$ $\omega_z = 1\ \frac{rad}{s}$ $v_B = 1\ \frac{ft}{s}$

Given

$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} = v_B \begin{pmatrix} -\sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 0.333 \\ 1 \\ -3.333 \end{pmatrix} \frac{rad}{s} \qquad v_B = 9.43\ \frac{ft}{s}$$

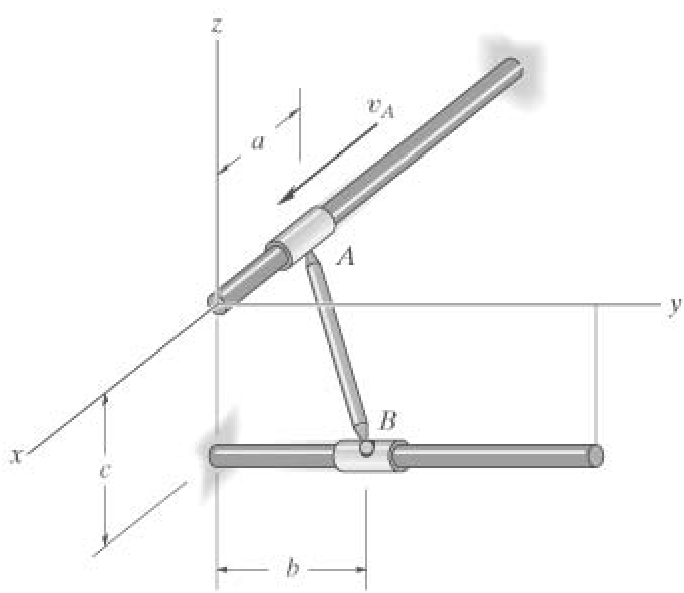
Problem 20-27

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. Determine the speed of B at the instant shown if A is moving with speed v_A. Also, determine the angular velocity of the rod if it is directed perpendicular to the axis of the rod.

Given:

$v_A = 6\ \frac{m}{s}$ $b = 1\ m$

$a = 0.5\ m$ $c = 1\ m$



Solution:

Guesses

$\omega_x = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_z = 1\ \frac{\text{rad}}{\text{s}}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$

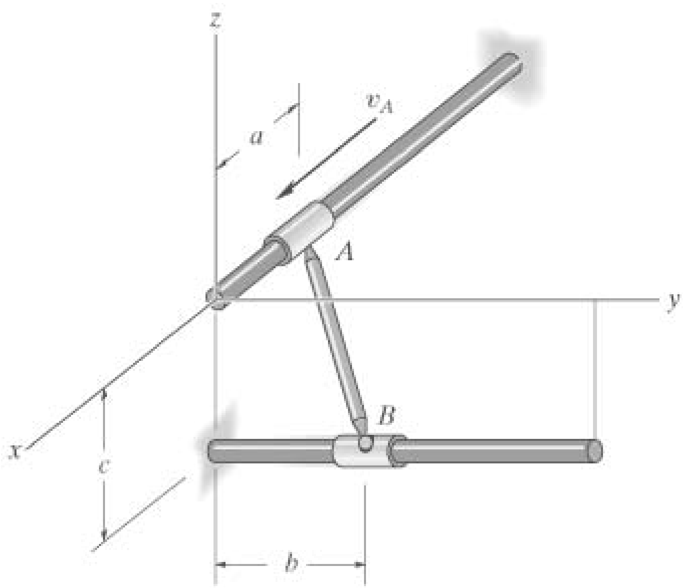
Given

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1.33 \\ 2.67 \\ 3.33 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 3.00\ \frac{\text{m}}{\text{s}}$$

***Problem 20-28**

The rod is attached to smooth collars A and B at its ends using ball-and-socket joints. At the instant shown, A is moving with speed v_A and is decelerating at the rate a_A. Determine the acceleration of collar B at this instant.



Given:

$v_A = 6\ \frac{\text{m}}{\text{s}}$ $a = 0.5\ \text{m}$

$a_A = 5\ \frac{\text{m}}{\text{s}^2}$ $b = 1\ \text{m}$

$c = 1\ \text{m}$

Solution:

Guesses $\omega_x = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_z = 1\ \frac{\text{rad}}{\text{s}}$ $v_B = 1\ \frac{\text{m}}{\text{s}}$

$\alpha_x = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_y = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_z = 1\ \frac{\text{rad}}{\text{s}^2}$ $a_B = 1\ \frac{\text{m}}{\text{s}^2}$

Given

$$\begin{pmatrix} v_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = \begin{pmatrix} 0 \\ v_B \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} a \\ b \\ -c \end{pmatrix} = 0$$

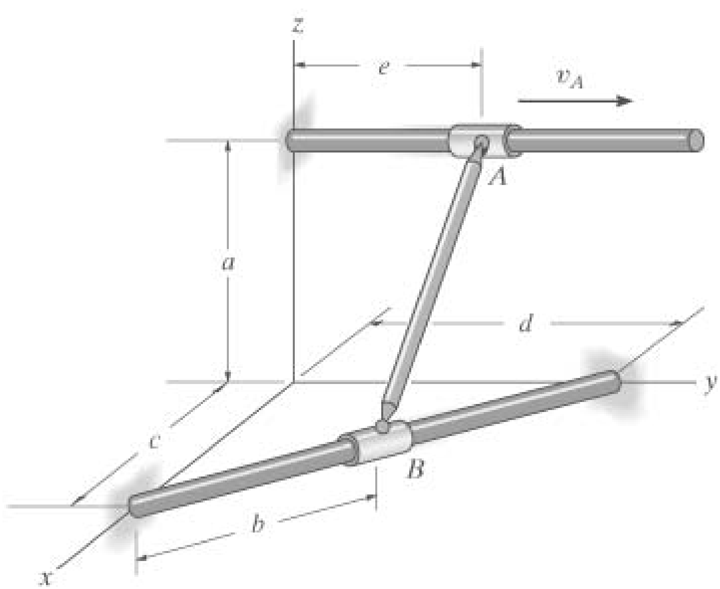
$$\begin{pmatrix} -a_A \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -c \end{pmatrix} \right] = \begin{pmatrix} 0 \\ a_B \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \text{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1.33 \\ 2.67 \\ 3.33 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} -21.11 \\ -2.22 \\ -12.78 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} \qquad v_B = 3.00 \ \frac{\text{m}}{\text{s}} \qquad a_B = -47.5 \ \frac{\text{m}}{\text{s}^2}$$

Problem 20-29

Rod *AB* is attached to collars at its ends by using ball-and-socket joints. If collar *A* moves along the fixed rod with speed v_A, determine the angular velocity of the rod and the velocity of collar *B* at the instant shown. Assume that the rod's angular velocity is directed perpendicular to the axis of the rod.



Given:

$v_A = 8 \ \frac{\text{ft}}{\text{s}}$ $\quad c = 6$ ft

$a = 8$ ft $\quad d = 8$ ft

$b = 5 \text{ ft} \qquad e = 6 \text{ ft}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{d}{c}\right)$$

Guesses $\omega_x = 1 \frac{\text{rad}}{\text{s}} \qquad \omega_y = 1 \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1 \frac{\text{rad}}{\text{s}} \qquad v_B = 1 \frac{\text{ft}}{\text{s}}$

Given

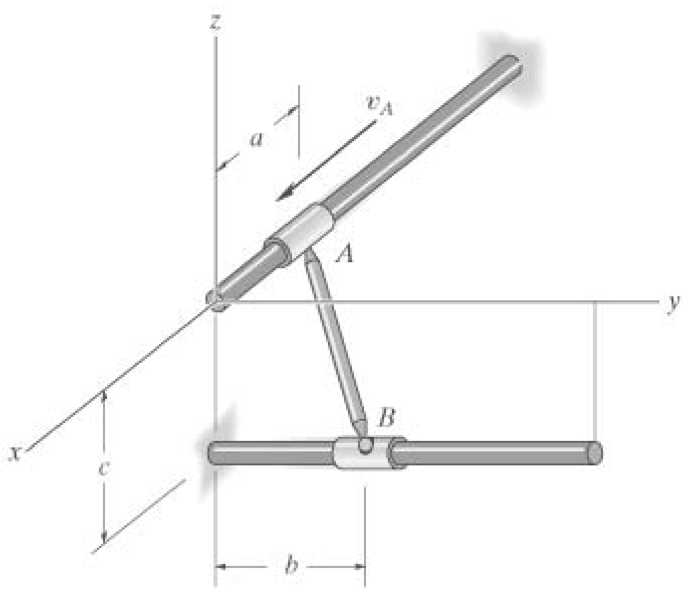
$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_B \end{pmatrix} = \operatorname{Find}(\omega_x, \omega_y, \omega_z, v_B) \qquad \mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -2.82 \\ 3.76 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.440 \\ 0.293 \\ -0.238 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

Problem 20-30

Rod *AB* is attached to collars at its ends by using ball-and-socket joints. If collar *A* moves along the fixed rod with a velocity v_A and has an acceleration a_A at the instant shown, determine the angular acceleration of the rod and the acceleration of collar *B* at this instant. Assume that the rod's angular velocity and angular acceleration are directed perpendicular to the axis of the rod.



Given:

$v_A = 8 \frac{\text{ft}}{\text{s}} \qquad a_A = 4 \frac{\text{ft}}{\text{s}^2}$

$a = 8 \text{ ft} \qquad b = 5 \text{ ft} \qquad c = 6 \text{ ft} \qquad d = 8 \text{ ft} \qquad e = 6 \text{ ft}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{d}{c}\right)$$

Guesses $\omega_x = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_z = 1\ \frac{\text{rad}}{\text{s}}$ $v_B = 1\ \frac{\text{ft}}{\text{s}}$

$\alpha_x = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_y = 1\ \frac{\text{rad}}{\text{s}^2}$ $\alpha_z = 1\ \frac{\text{rad}}{\text{s}^2}$ $a_B = 1\ \frac{\text{ft}}{\text{s}^2}$

Given

$$\begin{pmatrix} 0 \\ v_A \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ a_A \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix}\right] = a_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0 \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} \begin{pmatrix} c - b\cos(\theta) \\ -e + b\sin(\theta) \\ -a \end{pmatrix} = 0$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \\ \alpha_x \\ \alpha_y \\ \alpha_z \\ v_B \\ a_B \end{pmatrix} = \operatorname{Find}(\omega_x, \omega_y, \omega_z, \alpha_x, \alpha_y, \alpha_z, v_B, a_B)$$

$$\mathbf{v_{Bv}} = v_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Bv}} = a_B \begin{pmatrix} -\cos(\theta) \\ \sin(\theta) \\ 0 \end{pmatrix} \qquad \mathbf{a_{Bv}} = \begin{pmatrix} -5.98 \\ 7.98 \\ 0.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.440 \\ 0.293 \\ -0.238 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix} = \begin{pmatrix} 0.413 \\ 0.622 \\ -0.000 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} \qquad \mathbf{v_{Bv}} = \begin{pmatrix} -2.824 \\ 3.765 \\ 0 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

Problem 20-31

Consider again Example 20.5. The pendulum consists of two rods: *AB* is pin supported at *A* and swings only in the *y*-*z* plane, whereas a bearing at *B* allows the attached rod *BD* to spin about rod *AB*. At a given instant, the rods have the angular motions shown. Also, a collar *C* has velocity v_C and acceleration a_C along the rod. Determine the velocity and acceleration of the collar at this instant. Solve such that the *x*, *y*, *z* axes move with curvilinear translation, $\Omega = 0$, in which case the collar appears to have both an angular velocity $\mathbf{\Omega_{xyz}} = \mathbf{\omega_1} + \mathbf{\omega_2}$ and radial motion.

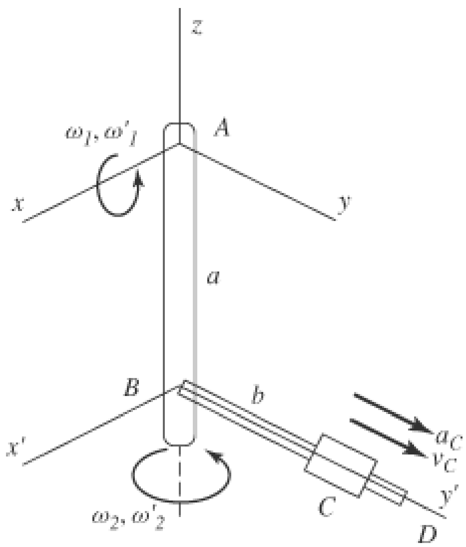
Given:

$\omega_1 = 4\ \frac{\text{rad}}{\text{s}}$ $\quad v_{CB} = 3\ \frac{\text{m}}{\text{s}}$

$\omega_2 = 5\ \frac{\text{rad}}{\text{s}}$ $\quad a_{CB} = 2\ \frac{\text{m}}{\text{s}^2}$

$\omega'_1 = 1.5\ \frac{\text{rad}}{\text{s}^2}$ $\quad a = 0.5\ \text{m}$

$\omega'_2 = -6\ \frac{\text{rad}}{\text{s}^2}$ $\quad b = 0.2\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} \omega'_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \right] \qquad \mathbf{a_B} = \begin{pmatrix} 0 \\ 0.75 \\ 8 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

$$\mathbf{v_C} = \mathbf{v_B} + \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -1.00 \\ 5.00 \\ 0.80 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

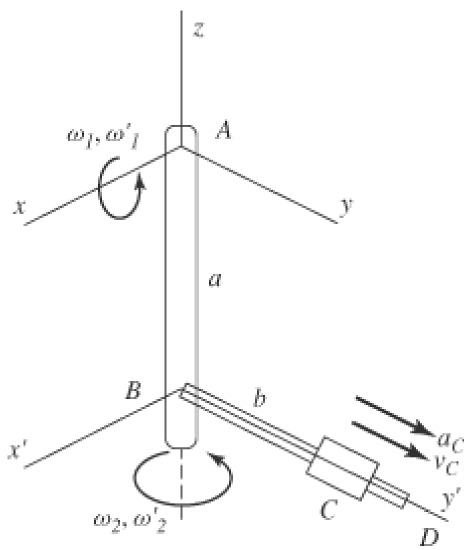
$$\mathbf{a_C} = \mathbf{a_B} + \begin{pmatrix} 0 \\ a_{CB} \\ 0 \end{pmatrix} + \left[\begin{pmatrix} \omega'_1 \\ 0 \\ \omega'_2 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \right] \dots$$

$$+ 2 \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix}$$

$$\mathbf{a_C} = \begin{pmatrix} -28.8 \\ -5.45 \\ 32.3 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-32

Consider again Example 20.5. The pendulum consists of two rods: *AB* is pin supported at *A* and swings only in the *y-z* plane, whereas a bearing at *B* allows the attached rod *BD* to spin about rod *AB*. At a given instant, the rods have the angular motions shown. Also, a collar *C* has velocity v_C and acceleration a_C along the rod. Determine the velocity and acceleration of the collar at this instant. Solve by fixing the *x, y, z* axes to rod *BD* in which case the collar appears only to have radial motion.



Given:

$\omega_1 = 4 \frac{\text{rad}}{\text{s}}$ $\omega'_1 = 1.5 \frac{\text{rad}}{\text{s}^2}$

$\omega_2 = 5 \frac{\text{rad}}{\text{s}}$ $\omega'_2 = -6 \frac{\text{rad}}{\text{s}^2}$

$a = 0.5 \text{ m}$ $b = 0.2 \text{ m}$

$v_{CB} = 3 \frac{\text{m}}{\text{s}}$ $a_{CB} = 2 \frac{\text{m}}{\text{s}^2}$

Solution:

$$\mathbf{v_B} = \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} \omega'_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \right]$$

$$\mathbf{v_C} = \mathbf{v_B} + \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -1.00 \\ 5.00 \\ 0.80 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \mathbf{a_B} + \begin{pmatrix} 0 \\ a_{CB} \\ 0 \end{pmatrix} + \left[\begin{pmatrix} \omega'_1 \\ 0 \\ \omega'_2 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \left[\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}\right] \ldots$$

$$+ 2\begin{pmatrix} \omega_1 \\ 0 \\ \omega_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_{CB} \\ 0 \end{pmatrix}$$

$$\mathbf{a_C} = \begin{pmatrix} -28.8 \\ -5.45 \\ 32.3 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-33

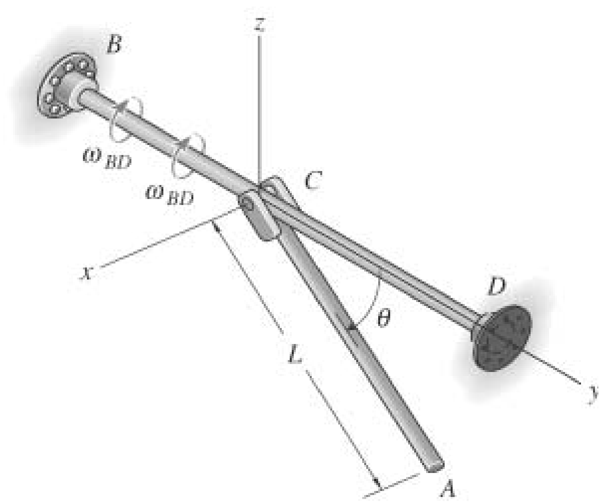
At a given instant, rod *BD* is rotating about the *y* axis with angular velocity ω_{BD} and angular acceleration ω'_{BD}. Also, when $\theta = \theta_1$, link *AC* is rotating downward such that $\theta = \omega_2$ and $\theta' = \alpha_2$. Determine the velocity and acceleration of point *A* on the link at this instant.

Given:

$\omega_{BD} = 2 \ \frac{\text{rad}}{\text{s}}$ $\qquad \theta_1 = 60 \ \text{deg}$

$\omega'_{BD} = 5 \ \frac{\text{rad}}{\text{s}^2}$ $\qquad \omega_2 = 2 \ \frac{\text{rad}}{\text{s}}$

$L = 3 \ \text{ft}$ $\qquad \alpha_2 = 8 \ \frac{\text{rad}}{\text{s}^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} -\omega_2 \\ -\omega_{BD} \\ 0 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\alpha_2 \\ -\omega'_{BD} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega_{BD} \\ 0 \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r_A} = \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ -L\sin(\theta_1) \end{pmatrix}$$

$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r_A}$ $\qquad$ $\mathbf{v_A} = \begin{pmatrix} 5.196 \\ -5.196 \\ -3 \end{pmatrix} \dfrac{\text{ft}}{\text{s}}$

$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r_A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_A})$ $\qquad$ $\mathbf{a_A} = \begin{pmatrix} 24.99 \\ -26.785 \\ 8.785 \end{pmatrix} \dfrac{\text{ft}}{\text{s}^2}$

Problem 20-34

During the instant shown the frame of the X-ray camera is rotating about the vertical axis at ω_z and ω'_z. Relative to the frame the arm is rotating at ω_{rel} and ω'_{rel}. Determine the velocity and acceleration of the center of the camera C at this instant.

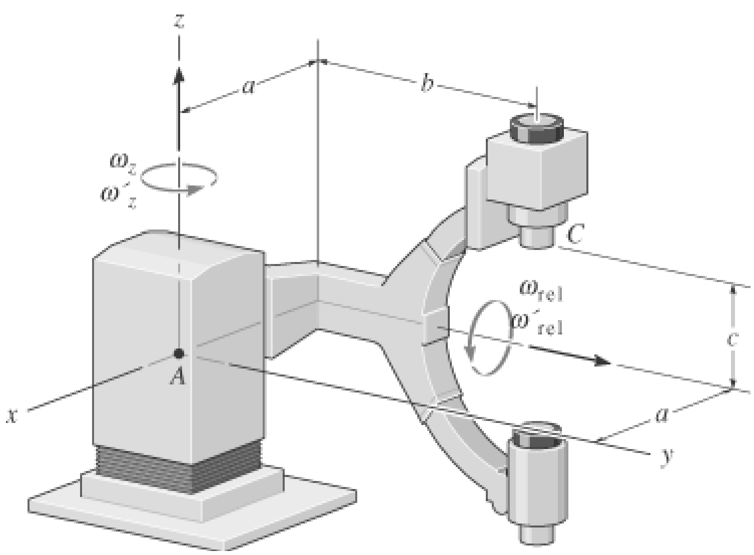
Given:

$\omega_z = 5 \dfrac{\text{rad}}{\text{s}}$ $\qquad$ $a = 1.25\ \text{m}$

$\omega'_z = 2 \dfrac{\text{rad}}{\text{s}^2}$ $\qquad$ $b = 1.75\ \text{m}$

$\omega_{rel} = 2 \dfrac{\text{rad}}{\text{s}}$ $\qquad$ $c = 1\ \text{m}$

$\omega'_{rel} = 1 \dfrac{\text{rad}}{\text{s}^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{rel} \\ \omega_z \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 \\ \omega'_{rel} \\ \omega'_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ \omega_z \end{pmatrix} \times \begin{pmatrix} -a \\ 0 \\ 0 \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} \qquad \mathbf{v_C} = \begin{pmatrix} -6.75 \\ -6.25 \\ 0 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \begin{pmatrix} a\omega_z^2 \\ -a\,\omega'_z \\ 0 \end{pmatrix} + \boldsymbol{\alpha} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} + \boldsymbol{\omega} \times \left[\boldsymbol{\omega} \times \begin{pmatrix} 0 \\ b \\ c \end{pmatrix} \right] \qquad \mathbf{a_C} = \begin{pmatrix} 28.75 \\ -26.25 \\ -4 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-35

At the instant shown, the frame of the brush cutter is traveling forward in the *x* direction with a constant velocity *v*, and the cab is rotating about the vertical axis with a constant angular velocity ω_1. At the same instant the boom *AB* has a constant angular velocity θ', in the direction shown. Determine the velocity and acceleration of point *B* at the connection to the mower at this instant.

Given:

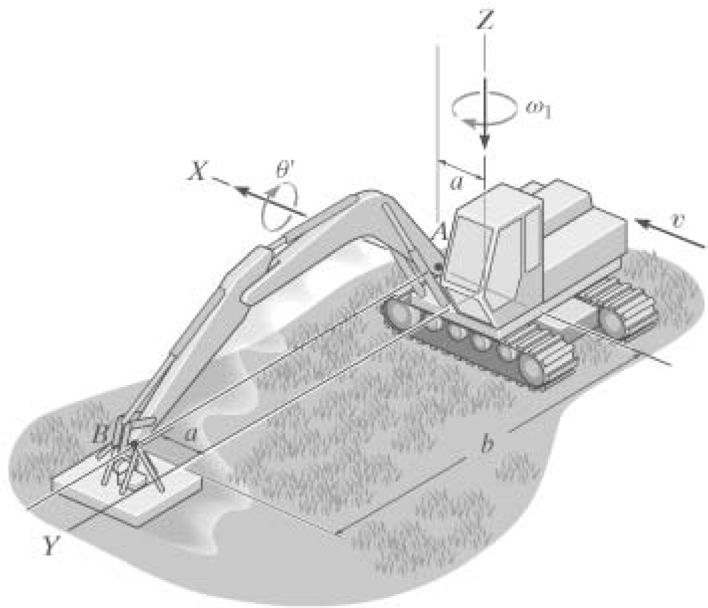
$$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$\theta' = 0.8\ \frac{\text{rad}}{\text{s}}$$

$$v = 1\ \frac{\text{m}}{\text{s}}$$

$$a = 1\ \text{m}$$

$$b = 8\ \text{m}$$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 5 \\ -0.5 \\ 6.4 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

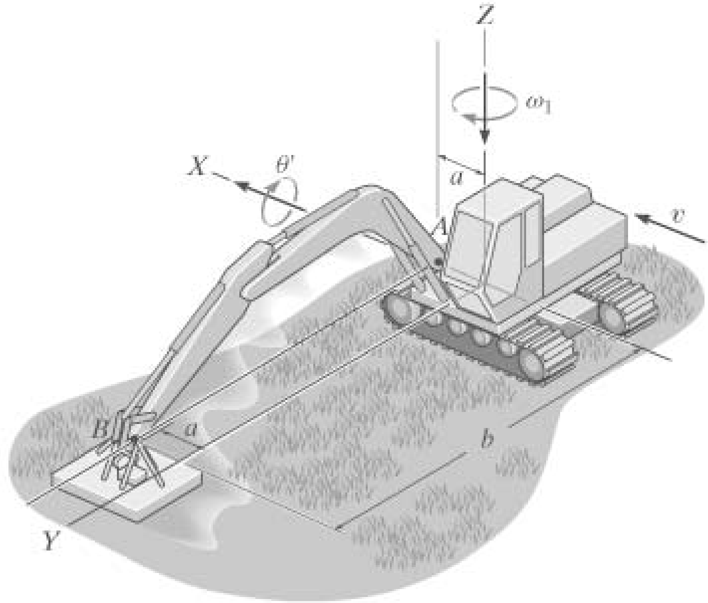
$$\mathbf{a_B} = \begin{pmatrix} 0 \\ -\omega_1\,\theta' \\ 0 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \right]$$

$$\mathbf{a_B} = \begin{pmatrix} -0.25 \\ -7.12 \\ 0.00 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-36**

At the instant shown, the frame of the brush cutter is traveling forward in the x direction with a constant velocity v, and the cab is rotating about the vertical axis with an angular velocity ω_1, which is increasing at ω'_1. At the same instant the boom AB has an angular velocity θ', which is increasing at θ''. Determine the velocity and acceleration of point B at the connection to the mower at this instant.

Given:



$$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$$

$$\omega'_1 = 0.4\ \frac{\text{rad}}{\text{s}^2}$$

$$\theta' = 0.8\ \frac{\text{rad}}{\text{s}}$$

$$\theta'' = 0.9\ \frac{\text{rad}}{\text{s}^2}$$

$$v = 1\ \frac{\text{m}}{\text{s}}$$

$$a = 1\ \text{m}$$

$$b = 8\ \text{m}$$

Solution:

$$\mathbf{v_B} = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \mathbf{v_B} = \begin{pmatrix} 5 \\ -0.5 \\ 6.4 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_B} = \begin{pmatrix} \theta'' \\ -\omega_1\theta' \\ -\omega'_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \theta' \\ 0 \\ -\omega_1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix}\right] \qquad \mathbf{a_B} = \begin{pmatrix} 2.95 \\ -7.52 \\ 7.20 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-37

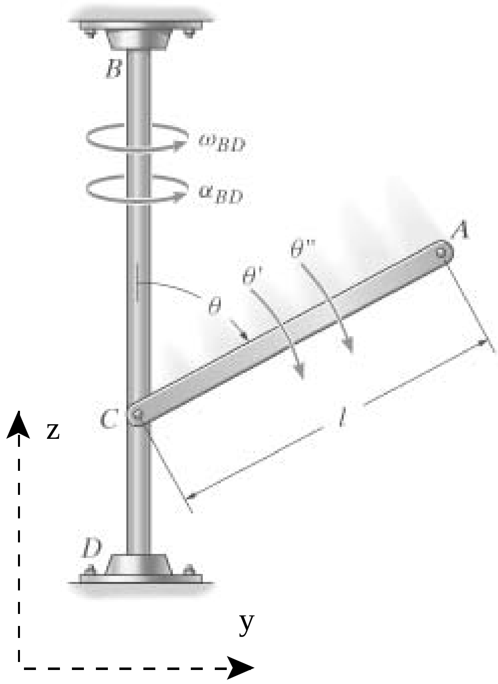
At the instant shown, rod BD is rotating about the vertical axis with an angular velocity ω_{BD} and an angular acceleration α_{BD}. Link AC is rotating downward. Determine the velocity and acceleration of point A on the link at this instant.

Given:

$\omega_{BD} = 7\ \frac{rad}{s}$ $\theta = 60\ deg$

$\alpha_{BD} = 4\ \frac{rad}{s^2}$ $\theta' = 2\ \frac{rad}{s}$

$l = 0.8\ m$ $\theta'' = 3\ \frac{rad}{s^2}$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} -\theta' \\ 0 \\ \omega_{BD} \end{pmatrix} \qquad \mathbf{r} = l\begin{pmatrix} 0 \\ \sin(\theta) \\ \cos(\theta) \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} -\theta'' \\ 0 \\ \alpha_{BD} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{BD} \end{pmatrix} \times \boldsymbol{\omega}$$

$$\mathbf{v_A} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_A} = \begin{pmatrix} -4.85 \\ 0.80 \\ -1.39 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_A} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_A} = \begin{pmatrix} -13.97 \\ -35.52 \\ -3.68 \end{pmatrix} \frac{m}{s^2}$$

Problem 20-38

The boom *AB* of the locomotive crane is rotating about the *Z* axis with angular velocity ω_1 which is increasing at ω'_1. At this same instant, $\theta = \theta_1$ and the boom is rotating upward at a constant rate of $\theta' = \omega_2$. Determine the velocity and acceleration of the tip *B* of the boom at this instant.

Given:

$\omega_1 = 0.5\ \frac{rad}{s}$

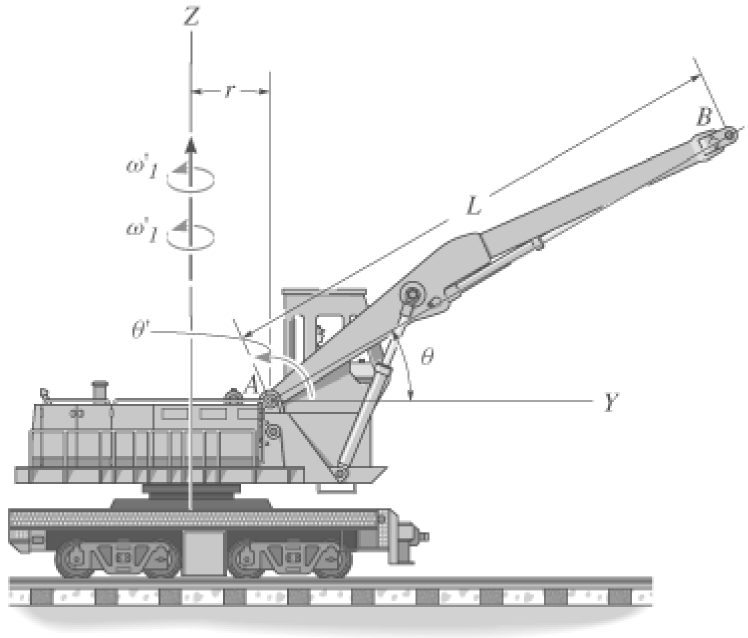
$\omega'_1 = 3\ \frac{rad}{s^2}$

$\theta_1 = 30\ deg$

$\omega_2 = 3\ \frac{\text{rad}}{\text{s}}$

$L = 20\ \text{m}$

$r = 3\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} -\omega'_1 r \\ -\omega_1^2 r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}\right]$$

$$\mathbf{v_B} = \begin{pmatrix} -10.2 \\ -30.0 \\ 52.0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} -31.0 \\ -161.0 \\ -90.0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-39

The locomotive crane is traveling to the right with speed v and acceleration a. The boom AB is rotating about the Z axis with angular velocity ω_1 which is increasing at ω'_1. At this same instant, $\theta = \theta_1$ and the boom is rotating upward at a constant rate of $\theta' = \omega_2$. Determine the velocity and acceleration of the tip B of the boom at this instant.

Given:

$v = 2\ \frac{\text{m}}{\text{s}}$

$a = 1.5\ \frac{\text{m}}{\text{s}^2}$

$\omega_1 = 0.5\ \frac{\text{rad}}{\text{s}}$

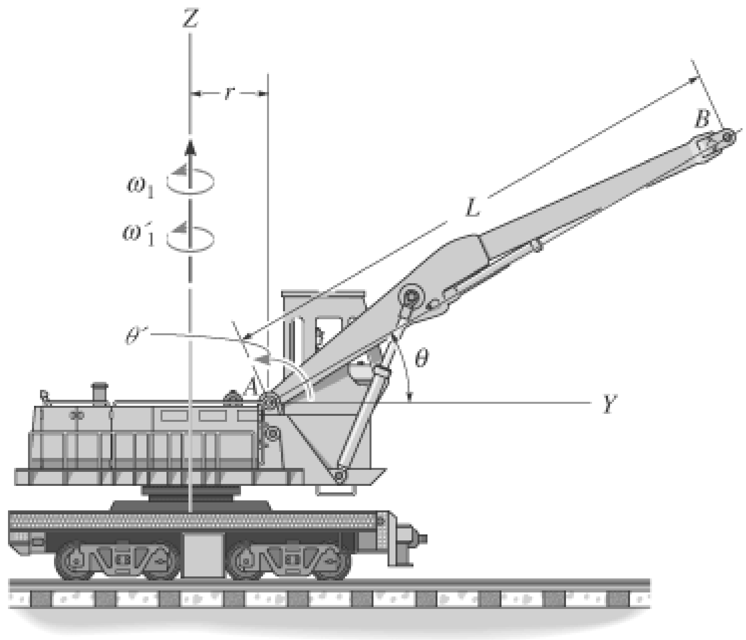
$\omega'_1 = 3\ \frac{\text{rad}}{\text{s}^2}$

$\theta_1 = 30\ \text{deg}$

$\omega_2 = 3\ \frac{\text{rad}}{\text{s}}$

$L = 20\ \text{m}$

$r = 3\ \text{m}$



Solution:

$$\mathbf{v_B} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix}$$

$$\mathbf{a_B} = \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} -\omega'_1 r \\ -\omega_1^2 r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ L\cos(\theta_1) \\ L\sin(\theta_1) \end{pmatrix} \right]$$

$$\mathbf{v_B} = \begin{pmatrix} -10.2 \\ -28.0 \\ 52.0 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \mathbf{a_B} = \begin{pmatrix} -31.0 \\ -159.5 \\ -90.0 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

***Problem 20-40**

At a given instant, the rod has the angular motions shown, while the collar C is moving down *relative* to the rod with a velocity v and an acceleration a. Determine the collar's velocity and acceleration at this instant.

Given:

$v = 6\ \frac{\text{ft}}{\text{s}}$

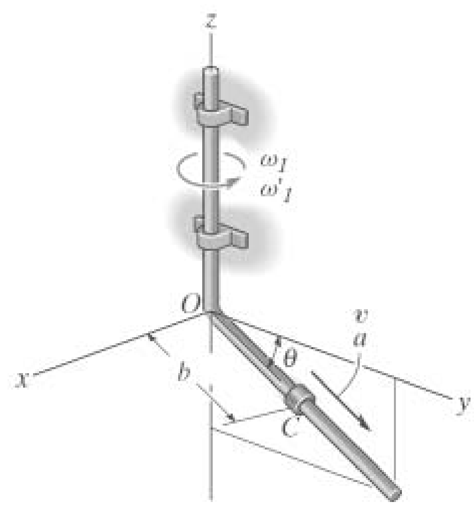
$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$\omega'_1 = 12\ \frac{\text{rad}}{\text{s}^2}$

$\theta = 30\ \text{deg}$

$a = 2\ \frac{\text{ft}}{\text{s}^2}$

$b = 0.8\ \text{ft}$



Solution:

$$\mathbf{v_C} = v\begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix}$$

$$\mathbf{a_C} = a\begin{pmatrix} 0 \\ \cos(\theta) \\ -\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ b\cos(\theta) \\ -b\sin(\theta) \end{pmatrix}\right] \ldots$$
$$+ 2\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ v\cos(\theta) \\ -v\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_C} = \begin{pmatrix} -5.54 \\ 5.20 \\ -3.00 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \mathbf{a_C} = \begin{pmatrix} -91.45 \\ -42.61 \\ -1.00 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-41

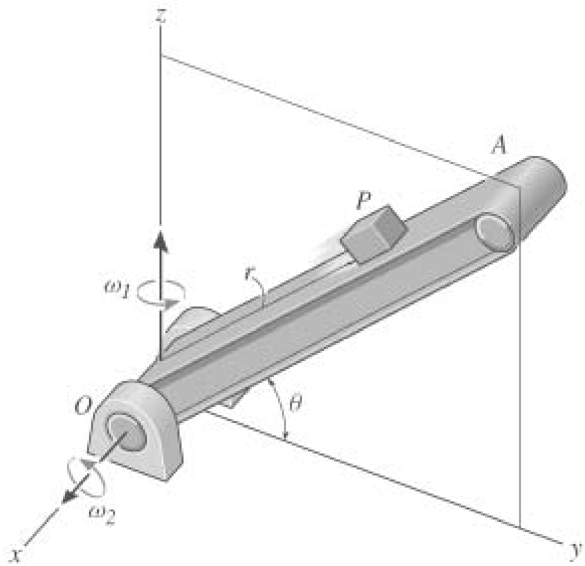
At the instant shown, the arm *OA* of the conveyor belt is rotating about the z axis with a constant angular velocity ω_1, while at the same instant the arm is rotating upward at a constant rate ω_2. If the conveyor is running at a constant rate $r' = v$, determine the velocity and acceleration of the package *P* at the instant shown. Neglect the size of the package.

Given:

$\omega_1 = 6\ \frac{\text{rad}}{\text{s}} \qquad v = 5\frac{\text{ft}}{\text{s}}$

$\omega_2 = 4\ \frac{rad}{s}$ $\quad r = 6\ ft$

$\theta = 45\ deg$



Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega}$$

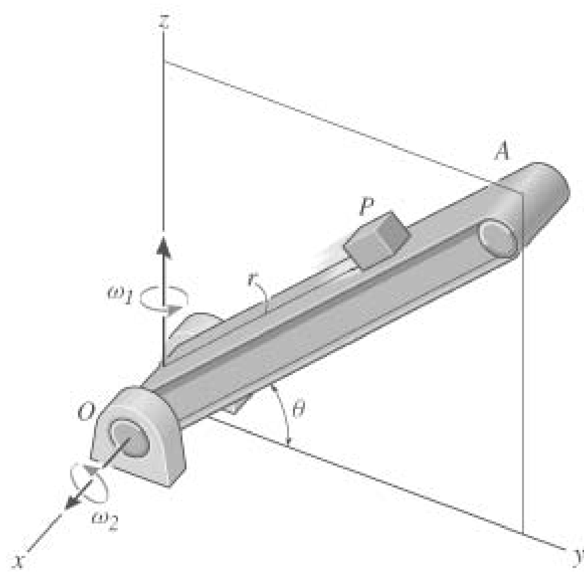
$$\mathbf{r_P} = \begin{pmatrix} 0 \\ r\cos(\theta) \\ r\sin(\theta) \end{pmatrix} \qquad \mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v\cos(\theta) \\ v\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_P} = \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r_P} \qquad \mathbf{a_P} = \boldsymbol{\alpha} \times \mathbf{r_P} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_P}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}}$$

$$\mathbf{v_P} = \begin{pmatrix} -25.5 \\ -13.4 \\ 20.5 \end{pmatrix} \frac{ft}{s} \qquad \mathbf{a_P} = \begin{pmatrix} 161.2 \\ -248.9 \\ -39.6 \end{pmatrix} \frac{ft}{s^2}$$

Problem 20-42

At the instant shown, the arm OA of the conveyor belt is rotating about the z axis with a constant angular velocity ω_1, while at the same instant the arm is rotating upward at a constant rate ω_2. If the conveyor is running at the rate $r' = v$ which is increasing at the rate $r'' = a$, determine the velocity and acceleration of the package P at the instant shown. Neglect the size of the package.



Given:

$\omega_1 = 6\ \frac{rad}{s}$ $\quad \omega_2 = 4\ \frac{rad}{s}$

$v = 5\ \frac{ft}{s}$ $\quad a = 8\ \frac{ft}{s^2}$

$r = 6\ ft$ $\quad \theta = 45\ deg$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \boldsymbol{\omega} \qquad \mathbf{r_P} = \begin{pmatrix} 0 \\ r\cos(\theta) \\ r\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v\cos(\theta) \\ v\sin(\theta) \end{pmatrix} \qquad \mathbf{a_{rel}} = \begin{pmatrix} 0 \\ a\cos(\theta) \\ a\sin(\theta) \end{pmatrix}$$

$$\mathbf{v_P} = \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r_P} \qquad \mathbf{a_P} = \mathbf{a_{rel}} + \boldsymbol{\alpha} \times \mathbf{r_P} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r_P}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}}$$

$$\mathbf{v_P} = \begin{pmatrix} -25.5 \\ -13.4 \\ 20.5 \end{pmatrix} \frac{\text{ft}}{\text{s}} \qquad \mathbf{a_P} = \begin{pmatrix} 161.2 \\ -243.2 \\ -33.9 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

Problem 20-43

At the given instant, the rod is spinning about the z axis with an angular velocity ω_1 and angular acceleration ω'_1. At this same instant, the disk is spinning, with ω_2 and ω'_2 both measured *relative* to the rod. Determine the velocity and acceleration of point P on the disk at this instant.

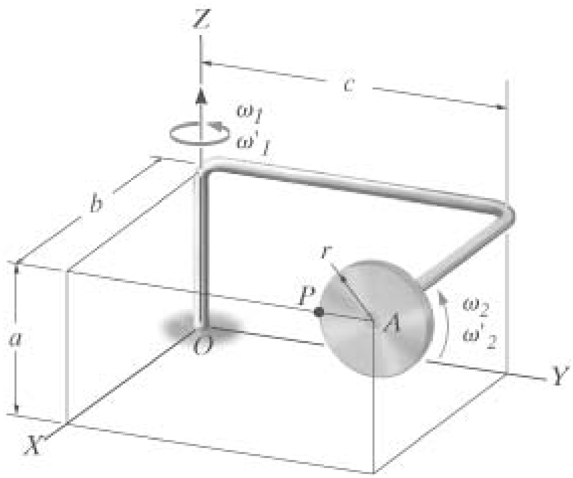
Given:

$$\omega_1 = 3 \frac{\text{rad}}{\text{s}} \qquad a = 2 \text{ ft}$$

$$\omega'_1 = 4 \frac{\text{rad}}{\text{s}^2} \qquad b = 3 \text{ ft}$$

$$\omega_2 = 2 \frac{\text{rad}}{\text{s}} \qquad c = 4 \text{ ft}$$

$$\omega'_2 = 1 \frac{\text{rad}}{\text{s}^2} \qquad r = 0.5 \text{ ft}$$



Solution:

$$\mathbf{v_P} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}$$

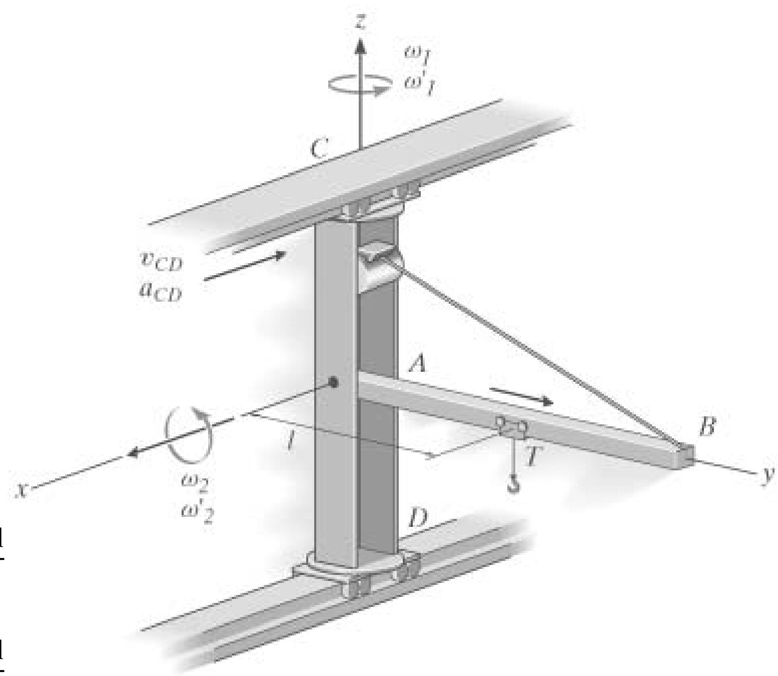
$$\mathbf{v_P} = \begin{pmatrix} -10.50 \\ 9.00 \\ -1.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$\mathbf{ap} = \begin{pmatrix} 0 \\ 0 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} b \\ c \\ 0 \end{pmatrix}\right] \dots$$
$$+ \begin{pmatrix} \omega'_2 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ -r \\ 0 \end{pmatrix}\right]$$

$$\mathbf{ap} = \begin{pmatrix} -41.00 \\ -17.50 \\ -0.50 \end{pmatrix} \frac{\text{ft}}{\text{s}^2}$$

***Problem 20-44**

At a given instant, the crane is moving along the track with a velocity v_{CD} and acceleration a_{CD}. Simultaneously, it has the angular motions shown. If the trolley T is moving outwards along the boom AB with a relative speed v_r and relative acceleration a_r, determine the velocity and acceleration of the trolley.



Given:

$\omega_1 = 0.5 \frac{\text{rad}}{\text{s}}$ $\quad \omega'_1 = 0.8 \frac{\text{rad}}{\text{s}^2}$

$\omega_2 = 0.4 \frac{\text{rad}}{\text{s}}$ $\quad \omega'_2 = 0.6 \frac{\text{rad}}{\text{s}^2}$

$v_{CD} = 8 \frac{\text{m}}{\text{s}}$ $\quad a_{CD} = 9 \frac{\text{m}}{\text{s}^2}$

$v_r = 3 \frac{\text{m}}{\text{s}}$ $\quad a_r = 5 \frac{\text{m}}{\text{s}^2}$ $\quad l = 3 \text{ m}$

Solution:

$$\mathbf{v_A} = \begin{pmatrix} -v_{CD} \\ 0 \\ 0 \end{pmatrix} \quad \mathbf{a_A} = \begin{pmatrix} -a_{CD} \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{\omega} = \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \quad \boldsymbol{\alpha} = \begin{pmatrix} \omega'_2 \\ \omega_1\omega_2 \\ \omega'_1 \end{pmatrix}$$

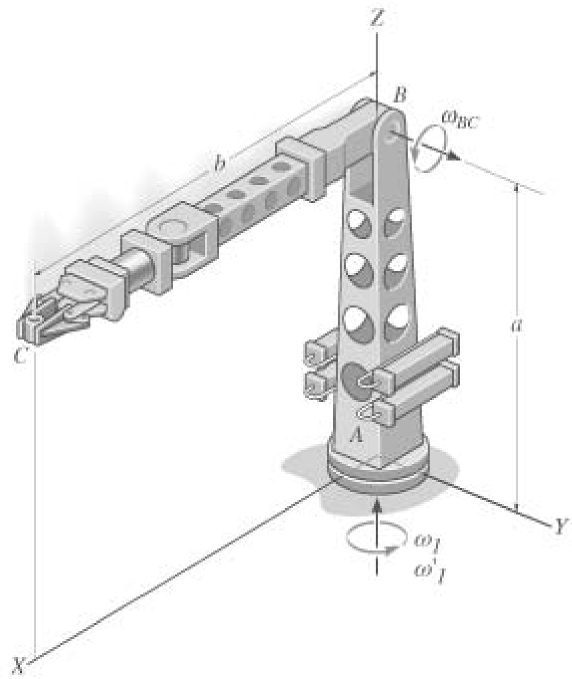
$$\mathbf{r} = \begin{pmatrix} 0 \\ l \\ 0 \end{pmatrix} \quad \mathbf{v_{rel}} = \begin{pmatrix} 0 \\ v_r \\ 0 \end{pmatrix} \quad \mathbf{a_{rel}} = \begin{pmatrix} 0 \\ a_r \\ 0 \end{pmatrix}$$

$$\mathbf{v_T} = \mathbf{v_A} + \mathbf{v_{rel}} + \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_T} = \begin{pmatrix} -9.50 \\ 3.00 \\ 1.20 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_T} = \mathbf{a_A} + \mathbf{a_{rel}} + \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v_{rel}} \qquad \mathbf{a_T} = \begin{pmatrix} -14.40 \\ 3.77 \\ 4.20 \end{pmatrix} \frac{m}{s^2}$$

Problem 20-45

At the instant shown, the base of the robotic arm is turning about the z axis with angular velocity ω_1, which is increasing at ω'_1. Also, the boom segment BC is rotating at constant rate ω_{BC}. Determine the velocity and acceleration of the part C held in its grip at this instant.



Given:

$$\omega_1 = 4\ \frac{rad}{s} \qquad a = 0.5\ m$$

$$\omega'_1 = 3\ \frac{rad}{s^2} \qquad b = 0.7\ m$$

$$\omega_{BC} = 8\ \frac{rad}{s}$$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{BC} \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\omega_1\,\omega_{BC} \\ 0 \\ \omega'_1 \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

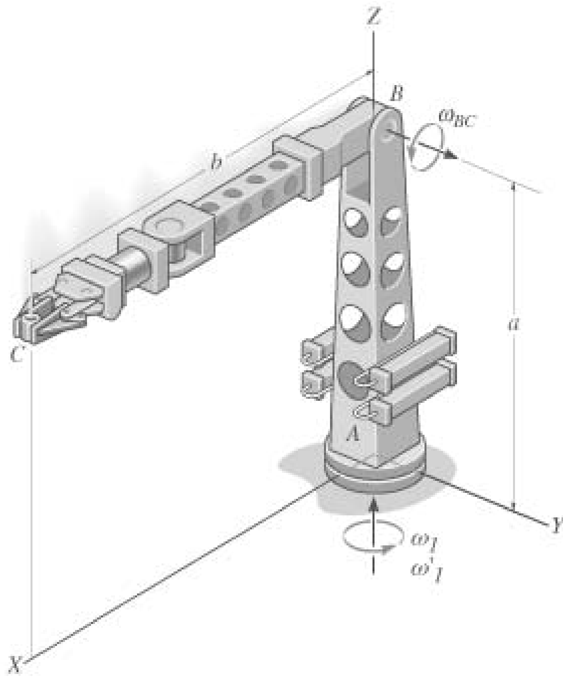
$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} 0 \\ 2.8 \\ -5.6 \end{pmatrix} \frac{m}{s}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad \mathbf{a_C} = \begin{pmatrix} -56 \\ 2.1 \\ 0 \end{pmatrix} \frac{m}{s^2}$$

Problem 20-46

At the instant shown, the base of the robotic arm is turning about the z axis with angular velocity ω_1, which is increasing at ω'_1. Also, the boom segment BC is rotating with angular velociy ω_{BC} which is incrasing at ω'_{BC}. Determine the velocity and acceleration of the part C held in its grip at this instant.

Given:



$\omega_1 = 4\ \frac{\text{rad}}{\text{s}}$ $\quad \omega'_1 = 3\ \frac{\text{rad}}{\text{s}^2}$

$\omega_{BC} = 8\ \frac{\text{rad}}{\text{s}}$ $\quad \omega'_{BC} = 2\ \frac{\text{rad}}{\text{s}^2}$

$a = 0.5\ \text{m}$ $\quad b = 0.7\ \text{m}$

Solution:

$$\boldsymbol{\omega} = \begin{pmatrix} 0 \\ \omega_{BC} \\ \omega_1 \end{pmatrix} \qquad \boldsymbol{\alpha} = \begin{pmatrix} -\omega_1\,\omega_{BC} \\ \omega'_{BC} \\ \omega'_1 \end{pmatrix}$$

$$\mathbf{r} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{v_C} = \boldsymbol{\omega} \times \mathbf{r} \qquad \mathbf{v_C} = \begin{pmatrix} 0 \\ 2.8 \\ -5.6 \end{pmatrix} \frac{\text{m}}{\text{s}}$$

$$\mathbf{a_C} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

$$\mathbf{a_C} = \begin{pmatrix} -56 \\ 2.1 \\ -1.4 \end{pmatrix} \frac{\text{m}}{\text{s}^2}$$

Problem 20-47

The load is being lifted upward at a constant rate v relative to the crane boom AB. At the instant shown, the boom is rotating about the vertical axis at a constant rate ω_1, and the trolley T is moving outward along the boom at a constant rate v_t. Furthermore, at this same instant the rectractable arm supporting the load is vertical and is swinging in the y-z plane at an angular rate ω_2, with an increase in the rate of swing α_2. Determine the velocity and acceleration of the center G of the load at this instant.

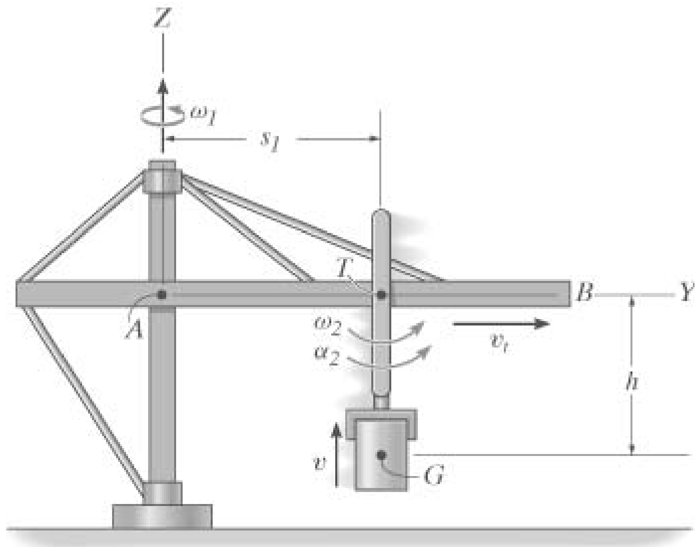
Given:

$\omega_1 = 4\ \frac{rad}{s}$ $\alpha_2 = 7\ \frac{rad}{s^2}$

$\omega_2 = 5\ \frac{rad}{s}$ $h = 3\ m$

$v_t = 2\ \frac{m}{s}$ $s_1 = 4\ m$

$v = 9\ \frac{m}{s}$



Solution:

$$\mathbf{v_T} = \begin{pmatrix} 0 \\ v_t \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ s_1 \\ 0 \end{pmatrix} \qquad \mathbf{a_T} = \begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ s_1 \\ 0 \end{pmatrix}\right] + 2\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ v_t \\ 0 \end{pmatrix}$$

$$\mathbf{v_G} = \mathbf{v_T} + \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}$$

$$\mathbf{a_G} = \mathbf{a_T} + \begin{pmatrix} \alpha_2 \\ \omega_1\omega_2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix} + \begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \left[\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -h \end{pmatrix}\right] + 2\begin{pmatrix} \omega_2 \\ 0 \\ \omega_1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

$$\mathbf{v_G} = \begin{pmatrix} -16.0 \\ 17.0 \\ 9.0 \end{pmatrix} \frac{m}{s} \qquad \mathbf{a_G} = \begin{pmatrix} -136.0 \\ -133.0 \\ 75.0 \end{pmatrix} \frac{m}{s^2}$$