Problem 21-1

Show that the sum of the moments of inertia of a body, $I_{xx}+I_{yy}+I_{zz}$, is independent of the orientation of the x, y, z axes and thus depends only on the location of the origin.

Solution:

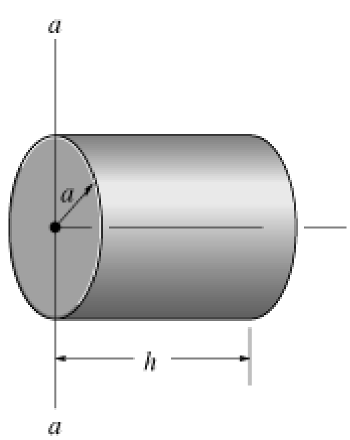
$$I_{xx} + I_{yy} + I_{zz} = \int_m y^2 + z^2\ \mathrm{d}m + \int_m x^2 + z^2\ \mathrm{d}m + \int_m x^2 + y^2\ \mathrm{d}m$$

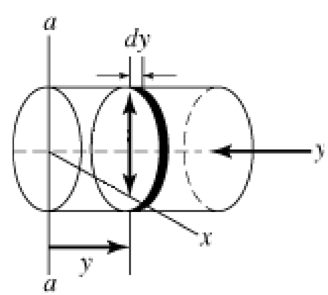
$$I_{xx} + I_{yy} + I_{zz} = 2\int_m x^2 + y^2 + z^2\ \mathrm{d}m$$

However, $x^2 + y^2 + z^2 = r^2$, where r is the distance from the origin O to dm. The magnitude $|\ r\ |$ does not depend on the orientation of the x, y, z axes. Consequently, $I_{xx}+I_{yy}+I_{zz}$ is also independent of the x, y, z axes.

Problem 21-2

Determine the moment of inertia of the cylinder with respect to the a-a axis of the cylinder. The cylinder has a mass m.





Solution:

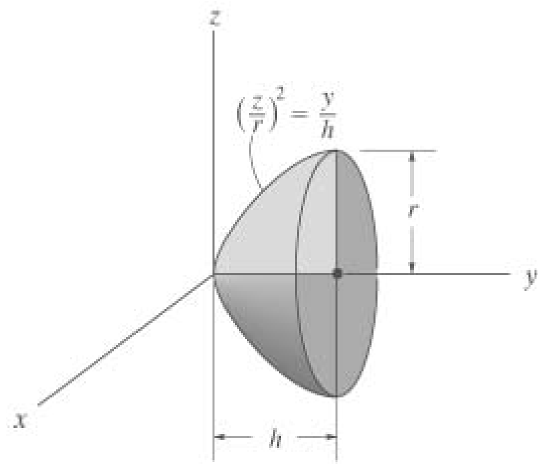
$$m = \int_0^h \rho\,\pi a^2\ \mathrm{d}y = h\,\rho\,\pi a^2 \qquad \rho = \frac{m}{h\pi a^2}$$

$$I_{aa} = \frac{m}{h\pi a^2}\left[\int_0^h \left(\frac{a^2}{4} + y^2\right)\pi a^2\ \mathrm{d}y\right] = \frac{m}{h\,\pi a^2}\left(\frac{1}{4}\,h\,\pi a^4 + \frac{1}{3}\,\pi\,a^2\,h^3\right)$$

$$I_{aa} = m\left(\frac{a^2}{4} + \frac{h^2}{3}\right)$$

Problem 21-3

Determine the moments of inerta I_x and I_y of the paraboloid of revolution. The mass of the paraboloid is M.



Given:

$M = 20$ slug

$r = 2$ ft

$h = 2$ ft

Solution:

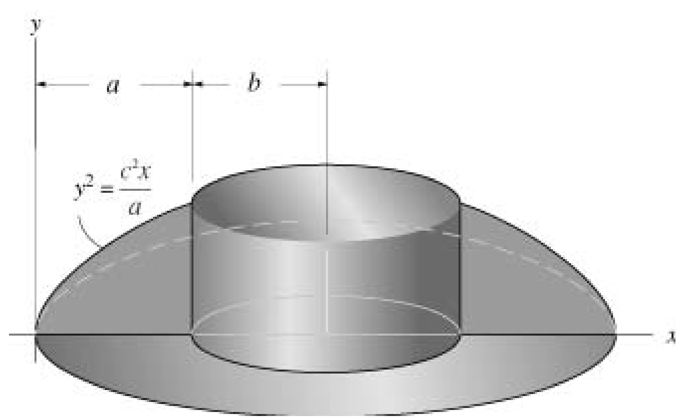
$$V = \int_0^h \pi r^2 \frac{y}{h}\, dy \qquad \rho = \frac{M}{V}$$

$$I_x = \int_0^h \rho \frac{1}{2}\left(\pi r^2 \frac{y}{h}\right)\left(r^2 \frac{y}{h}\right) dy \qquad I_x = 26.7\ \text{slug}\cdot\text{ft}^2$$

$$I_y = \int_0^h \rho\left(\pi r^2 \frac{y}{h}\right)\left[\frac{1}{4}\left(r^2 \frac{y}{h}\right) + y^2\right] dy \qquad I_y = 53.3\ \text{slug}\cdot\text{ft}^2$$

***Problem 21-4**

Determine the product of inertia I_{xy} of the body formed by revolving the shaded area about the line $x = a + b$. Express your answer in terms of the density ρ.



Given:

$a = 3$ ft

$b = 2$ ft

$c = 3$ ft

Solution:

$$k = \int_0^a \left[2\pi c \sqrt{\frac{x}{a}}(a+b-x)\right](a+b)\,\frac{c}{2}\sqrt{\frac{x}{a}}\,\mathrm{d}x$$

$I_{xy} = k\rho$ $\qquad$ $k = 636\,\mathrm{ft}^5$

Problem 21-5

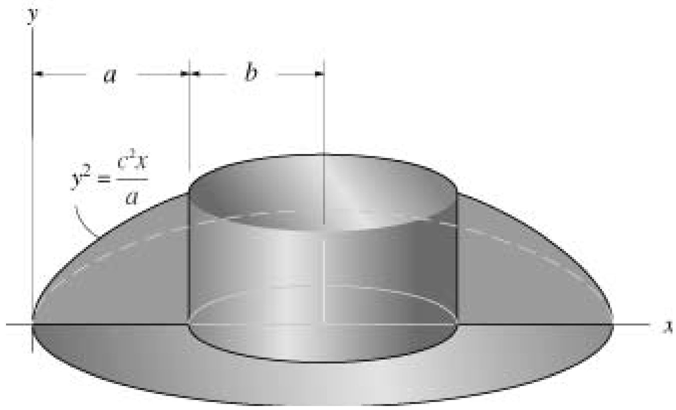
Determine the moment of inertia I_y of the body formed by revolving the shaded area about the line $x = a + b$. Express your answer in terms of the density ρ.

Given:

$a = 3\ \mathrm{ft}$

$b = 2\ \mathrm{ft}$

$c = 3\ \mathrm{ft}$



Solution:

$$k = \int_0^a \left[2\pi c \sqrt{\frac{x}{a}}(a+b-x)\right]\left[(a+b-x)^2 + (a+b)^2\right]\mathrm{d}x$$

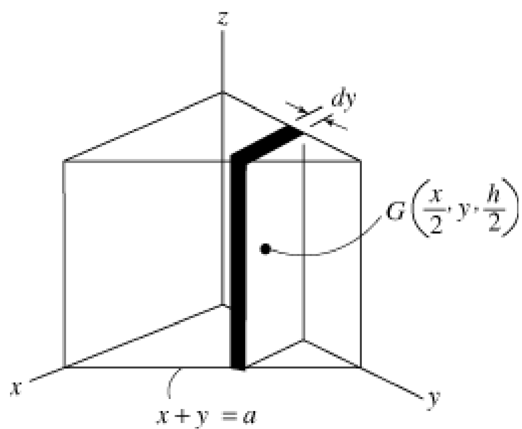
$I_y = k\rho$ $\qquad$ $k = 4481\,\mathrm{ft}^5$

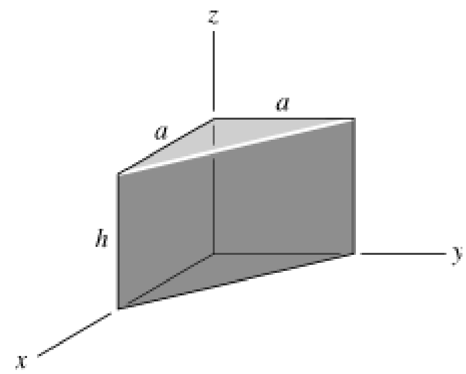
Problem 21-6

Determine by direct integration the product of inertia I_{yz} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.

Solution:

$$\rho = \frac{2m}{a^2 h}$$



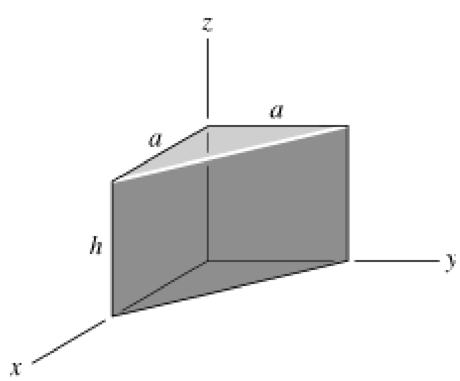


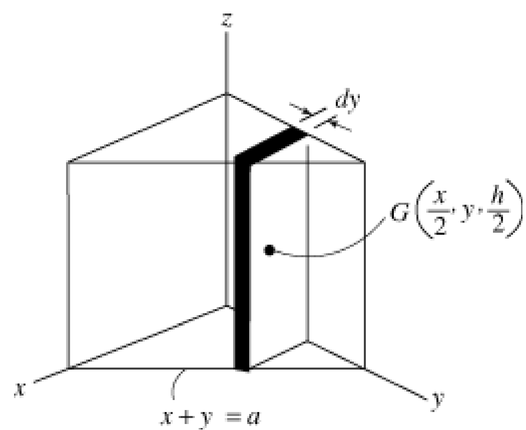
$$I_{yz} = \int_0^a \left(\frac{2m}{a^2 h}\right) h(a-y)\,\frac{h}{2}\left(\frac{a-y}{2}\right) \mathrm{d}y = \frac{1}{6} m\,a\,h$$

$$I_{yz} = \frac{1}{6} m\,a\,h$$

Problem 21-7

Determine by direct integration the product of inertia I_{xy} for the homogeneous prism. The density of the material is ρ. Express the result in terms of the mass m of the prism.





Solution:

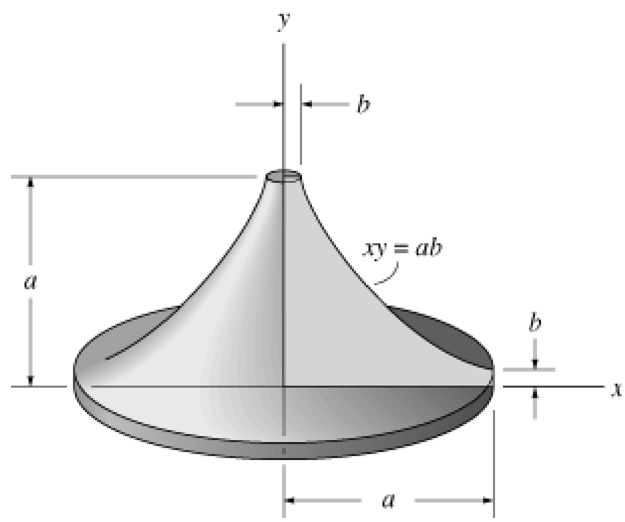
$$\rho = \frac{2m}{a^2 h}$$

$$I_{xy} = \int_0^a \left(\frac{2m}{a^2 h}\right) h(a-y)\, y\left(\frac{a-y}{2}\right) \mathrm{d}y = \frac{1}{12} a^4 \left(\frac{m}{a^2}\right)$$

$$I_{xy} = \frac{1}{12} a^2 m$$

***Problem 21-8**

Determine the radii of gyration k_x and k_y for the solid formed by revolving the shaded area about the y axis. The density of the material is ρ.



Given:

$a = 4\ \text{ft}$

$b = 0.25\ \text{ft}$

$\rho = 12\ \frac{\text{slug}}{\text{ft}^3}$

Solution:

$$M = \int_0^b \rho\,\pi a^2\, \mathrm{d}y + \int_b^a \rho\,\pi \frac{a^2 b^2}{y^2}\, \mathrm{d}y \qquad M = 292.17\ \text{slug}$$

$$I_x = \int_0^b \rho\left(\frac{a^2}{4} + y^2\right)\pi a^2\, \mathrm{d}y + \int_b^a \rho\left(\frac{a^2 b^2}{4y^2} + y^2\right)\pi \frac{a^2 b^2}{y^2}\, \mathrm{d}y \qquad I_x = 948.71\ \text{slug}\cdot\text{ft}^2$$

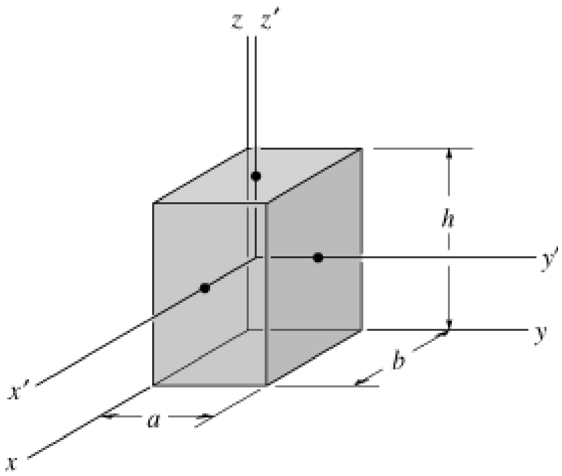
$$I_y = \int_0^b \rho\left(\frac{a^2}{2}\right)\pi a^2\, \mathrm{d}y + \int_b^a \rho\left(\frac{a^2 b^2}{2y^2}\right)\pi \frac{a^2 b^2}{y^2}\, \mathrm{d}y \qquad I_y = 1608.40\ \text{slug}\cdot\text{ft}^2$$

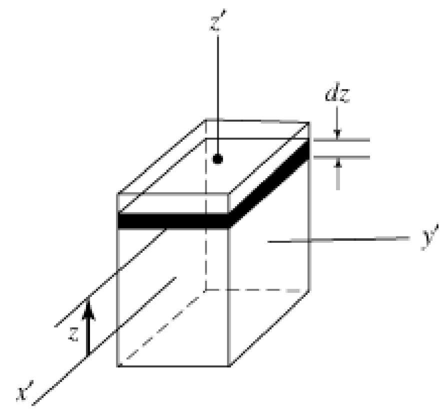
$$k_x = \sqrt{\frac{I_x}{M}} \qquad k_x = 1.80\ \text{ft}$$

$$k_y = \sqrt{\frac{I_y}{M}} \qquad k_y = 2.35\ \text{ft}$$

Problem 21-9

Determine the mass moment of inertia of the homogeneous block with respect to its centroidal x' axis. The mass of the block is m.





Solution:

$$m = \rho a b h \qquad \rho = \frac{m}{a b h}$$

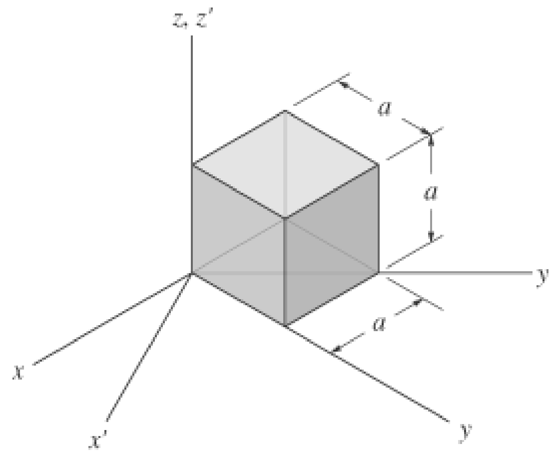
$$I_{x'} = \frac{m}{a b h}\int_{\frac{-h}{2}}^{\frac{h}{2}} \left(\frac{1}{12}a^2 + z^2\right) a b\ \mathrm{dz} = \frac{m}{a b h}\left(\frac{1}{12}a^3 b h + \frac{1}{12}a b h^3\right)$$

$$I_{x'} = \frac{m}{12}\left(a^2 + h^2\right)$$

Problem 21-10

Determine the elements of the inertia tensor for the cube with respect to the x, y, z coordinate system. The mass of the cube is m.

Solution:



$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} m a^2$$

$$I_{xy} = I_{xz} = -m\left(\frac{a}{2}\right)^2 = -m\frac{a^2}{4}$$

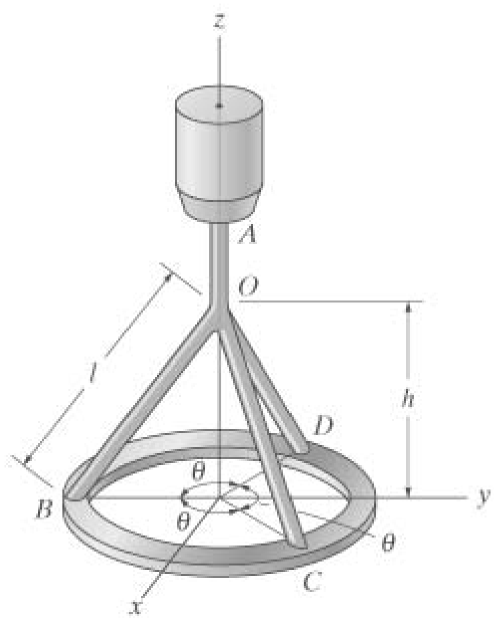
$$I_{yz} = m\left(\frac{a}{2}\right)^2 = m\frac{a^2}{4}$$

$$\mathbf{I} = \frac{m a^2}{12}\begin{pmatrix} 8 & 3 & 3 \\ 3 & 8 & -3 \\ 3 & -3 & 8 \end{pmatrix}$$

Remember to change the signs of the products of inertia to put them in the inertia tensor

Problem 21-11

Compute the moment of inertia of the rod-and-thin-ring assembly about the z axis. The rods and ring have a mass density ρ.



Given:

$$\rho = 2\ \frac{\text{kg}}{\text{m}}$$

$$l = 500\ \text{mm}$$

$$h = 400\ \text{mm}$$

$$\theta = 120\ \text{deg}$$

Solution:

$$r = \sqrt{l^2 - h^2}$$

$$\phi = \text{acos}\left(\frac{h}{l}\right)$$

$$I_z = 3\left(\rho l \frac{l^2}{3} \sin(\phi)^2\right) + \rho(2\pi r) r^2 \qquad I_z = 0.43\ \text{kg}\cdot\text{m}^2$$

***Problem 21-12**

Determine the moment of inertia of the cone about the z' axis. The weight of the cone is W, the *height* is h, and the radius is r.

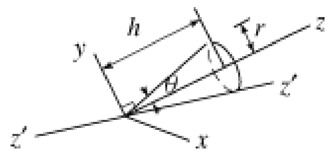
Given:

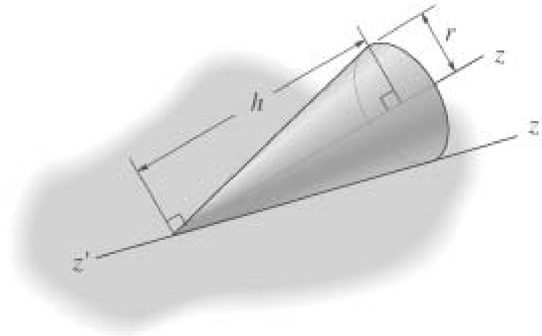
$W = 15\ \text{lb}$

$h = 1.5\ \text{ft}$

$r = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$





Solution:

$$\theta = \text{atan}\left(\frac{r}{h}\right)$$

$$I_x = \frac{3}{80} W\left(4r^2 + h^2\right) + W\left(\frac{3h}{4}\right)^2$$

$$I_y = I_x \qquad I_z = \frac{3}{10} W r^2$$

$$I_{z'} = I_x \sin(\theta)^2 + I_z \cos(\theta)^2 \qquad I_{z'} = 0.0962\ \text{slug}\cdot\text{ft}^2$$

Problem 21-13

The bent rod has weight density γ. Locate the center of gravity $G(x', y')$ and determine the principal moments of inertia $I_{x'}$, $I_{y'}$, and $I_{z'}$ of the rod with respect to the x', y', z' axes.

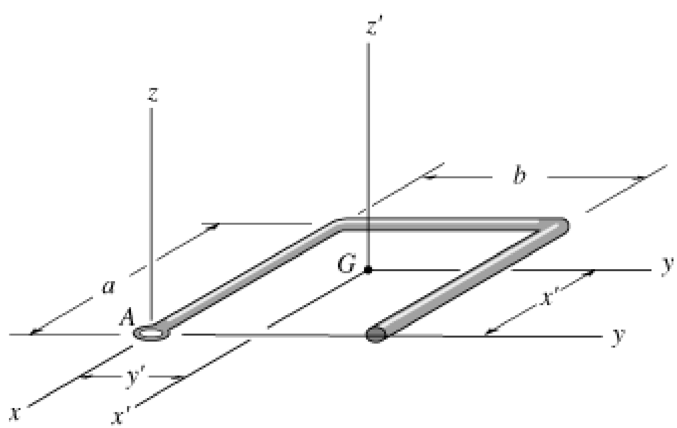
Given:

$\gamma = 1.5\ \dfrac{\text{lb}}{\text{ft}}$

$a = 1\ \text{ft}$

$b = 1\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$x' = \frac{2a\frac{a}{2} + b\,a}{2a + b} \qquad x' = 0.667\,\text{ft}$$

$$y' = \frac{a\,b + b\frac{b}{2}}{2a + b} \qquad y' = 0.50\,\text{ft}$$

$$I_{x'} = \gamma a\,y'^2 + \gamma a\,(b - y')^2 + \frac{1}{12}\gamma b\,b^2 + \gamma b\left(\frac{b}{2} - y'\right)^2 \qquad I_{x'} = 0.0272\,\text{slug}\cdot\text{ft}^2$$

$$I_{y'} = \frac{2}{12}\gamma a\,a^2 + 2\gamma a\left(\frac{a}{2} - x'\right)^2 + \gamma b\,(a - x')^2 \qquad I_{y'} = 0.0155\,\text{slug}\cdot\text{ft}^2$$

$$I_{z'} = I_{x'} + I_{y'} \qquad I_{z'} = 0.0427\,\text{slug}\cdot\text{ft}^2$$

Problem 21-14

The assembly consists of two square plates *A* and *B* which have a mass M_A each and a rectangular plate *C* which has a mass M_C. Determine the moments of inertia I_x, I_y and I_z.

Given:

$M_A = 3\ \text{kg}$

$M_C = 4.5\ \text{kg}$

$\theta = 60\ \text{deg}$

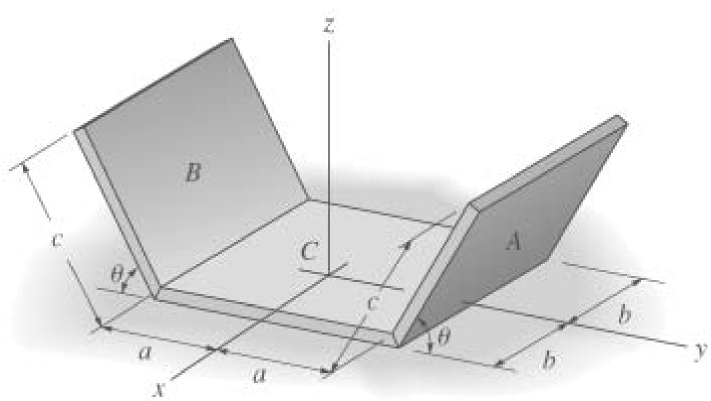
$\theta_1 = 90\ \text{deg}$

$\theta_2 = 30\ \text{deg}$

$a = 0.3\ \text{m}$

$b = 0.2\ \text{m}$

$c = 0.4\ \text{m}$



Solution:

$$\rho_A = \frac{M_A}{c(2b)}$$

$$I_x = \frac{1}{12} M_C (2a)^2 + 2 \int_0^c \rho_A (2b) \left[(a + \xi \cos(\theta))^2 + (\xi \sin(\theta))^2 \right] d\xi$$

$$I_y = \frac{1}{12} M_C (2b)^2 + 2 \int_{-b}^{b} \int_0^c \rho_A \left(x^2 + \xi^2 \sin(\theta)^2 \right) d\xi \, dx$$

$$I_z = \frac{1}{12} M_C \left[(2b)^2 + (2a)^2 \right] + 2 \int_{-b}^{b} \int_0^c \rho_A \left[x^2 + (a + \xi \cos(\theta))^2 \right] d\xi \, dx$$

$I_x = 1.36\ \text{kg}\cdot\text{m}^2$ $I_y = 0.380\ \text{kg}\cdot\text{m}^2$ $I_z = 1.25\ \text{kg}\cdot\text{m}^2$

Problem 21-15

Determine the moment of inertia I_x of the composite plate assembly. The plates have a specific weight γ.

Given:

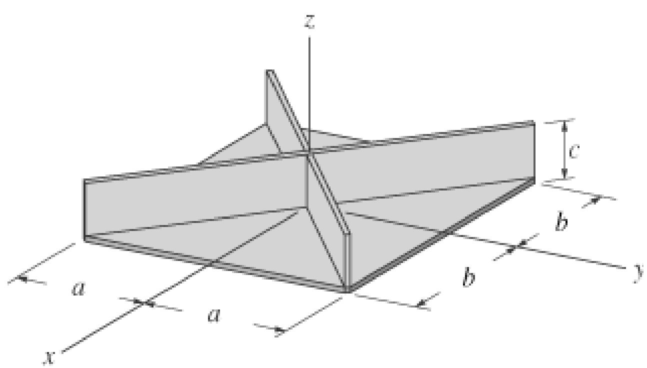
$\gamma = 6\ \frac{\text{lb}}{\text{ft}^2}$

$a = 0.5\ \text{ft}$

$b = 0.5\ \text{ft}$

$c = 0.25\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$$\theta = \operatorname{atan}\left(\frac{a}{b}\right)$$

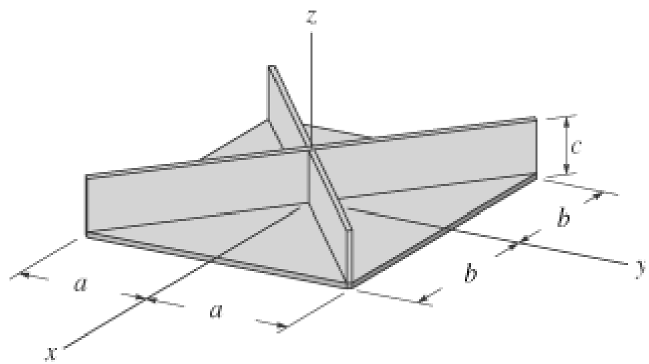
$$I_1 = \gamma c 2 \sqrt{a^2 + b^2} \left[\frac{c^2}{3} + \frac{(2a)^2 + (2b)^2}{12} \right]$$

$$I_2 = \gamma c 2\sqrt{a^2+b^2}\,\frac{c^2}{3}$$

$$I_x = 2\left(I_1 \sin(\theta)^2 + I_2 \cos(\theta)^2\right) + \gamma(2a)(2b)\frac{(2a)^2}{12} \qquad I_x = 0.0293\ \text{slug}\cdot\text{ft}^2$$

***Problem 21-16**

Determine the product of inertia I_{yz} of the composite plate assembly. The plates have a specific weight γ.



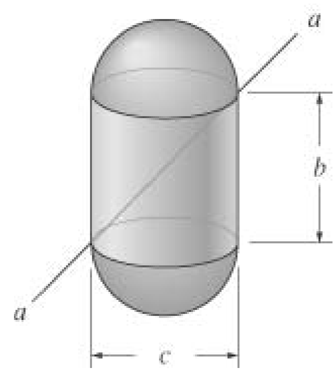
Solution:

Due to symmetry,

$I_{yz} = 0$

Problem 21-17

Determine the moment of inertia of the composite body about the *aa* axis. The cylinder has weight W_c and each hemisphere has weight W_h.



Given:

$W_c = 20\ \text{lb}$

$W_h = 10\ \text{lb}$

$b = 2\ \text{ft}$

$c = 2\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$\theta = \text{atan}\left(\frac{c}{b}\right)$$

$$I_z = 2\frac{2}{5}W_h\left(\frac{c}{2}\right)^2 + \frac{1}{2}W_c\left(\frac{c}{2}\right)^2 \qquad I_z = 0.56\ \text{slug}\cdot\text{ft}^2$$

$$I_y = 2\frac{83}{320}W_h\left(\frac{c}{2}\right)^2 + 2W_h\left(\frac{b}{2} + \frac{3}{8}\frac{c}{2}\right)^2 + W_c\left[\frac{b^2}{12} + \left(\frac{c}{2}\right)^2\frac{1}{4}\right]$$

$$I_y = 1.70\,\text{slug}\cdot\text{ft}^2$$

$$I_{aa} = I_z\cos(\theta)^2 + I_y\sin(\theta)^2 \qquad I_{aa} = 1.13\,\text{slug}\cdot\text{ft}^2$$

Problem 21-18

Determine the moment of inertia about the *z* axis of the assembly which consists of the rod *CD* of mass M_R and disk of mass M_D.

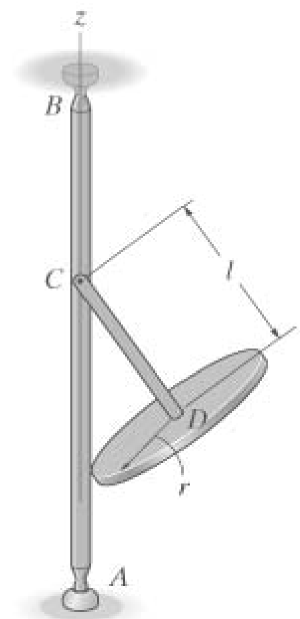
Given:

$M_R = 1.5\ \text{kg}$

$M_D = 7\ \text{kg}$

$r = 100\ \text{mm}$

$l = 200\ \text{mm}$



Solution:

$$\theta = \text{atan}\left(\frac{r}{l}\right)$$

$$I_1 = \frac{1}{3}M_R l^2 + \frac{1}{4}M_D r^2 + M_D l^2$$

$$I_2 = I_1$$

$$I_3 = \frac{1}{2}M_D r^2$$

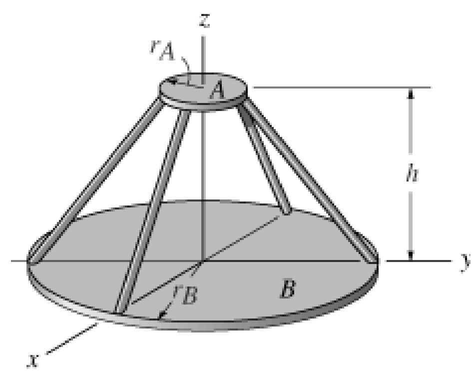
$$\mathbf{I_{mat}} = \left[\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta) & \sin(\theta)\\ 0 & -\sin(\theta) & \cos(\theta)\end{pmatrix}\begin{pmatrix}I_1 & 0 & 0\\ 0 & I_2 & 0\\ 0 & 0 & I_3\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos(\theta) & -\sin(\theta)\\ 0 & \sin(\theta) & \cos(\theta)\end{pmatrix}\right]$$

$$I_z = \mathbf{I_{mat}}_{2,2} \qquad I_z = 0.0915\,\text{kg}\cdot\text{m}^2$$

Problem 21-19

The assembly consists of a plate A of weight W_A, plate B of weight W_B, and four rods each of weight W_r. Determine the moments of inertia of the assembly with respect to the principal x, y, z axes.

Given:



$W_A = 15\ \text{lb}$

$W_B = 40\ \text{lb}$

$W_r = 7\ \text{lb}$

$r_A = 1\ \text{ft}$

$r_B = 4\ \text{ft}$

$h = 4\ \text{ft}$

Solution:

$$L = \sqrt{(r_B - r_A)^2 + h^2} \qquad L = 5.00\,\text{ft}$$

$$\theta = \text{asin}\left(\frac{h}{L}\right) \qquad \theta = 53.13\,\text{deg}$$

$$I_x = 2W_r\left(\frac{L^2}{3}\right)\sin(\theta)^2 + 2\left[W_r\left(\frac{L^2}{12}\right) + W_r\left[\left(\frac{h}{2}\right)^2 + \left(\frac{r_A + r_B}{2}\right)^2\right]\right] + W_B\frac{r_B^2}{4} \ \dots$$
$$+ W_A\left(\frac{r_A^2}{4}\right) + W_A h^2$$

$I_y = I_x$ by symmetry

$$I_z = 4\left[W_r\frac{L^2}{12}\cos(\theta)^2 + W_r\left(\frac{r_B + r_A}{2}\right)^2\right] + W_A\left(\frac{r_A^2}{2}\right) + W_B\left(\frac{r_B^2}{2}\right)$$

$$\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} 20.2 \\ 20.2 \\ 16.3 \end{pmatrix} \text{slug}\cdot\text{ft}^2$$

***Problem 21-20**

The thin plate has a weight W_p and each of the four rods has weight W_r. Determine the moment of inertia of the assembly about the z axis.

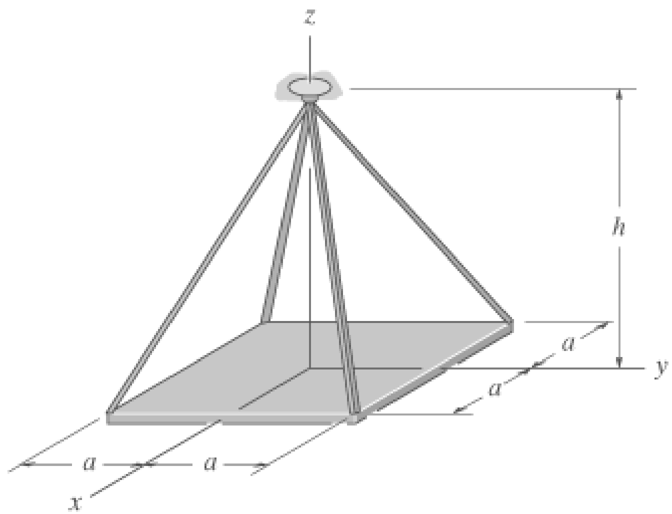
Given:

$W_p = 5\ \text{lb}$

$W_r = 3\ \text{lb}$

$h = 1.5\ \text{ft}$

$a = 0.5\ \text{ft}$



Solution:

$$L = \sqrt{h^2 + a^2 + a^2}$$

$$\theta = \operatorname{acos}\left(\frac{h}{L}\right)$$

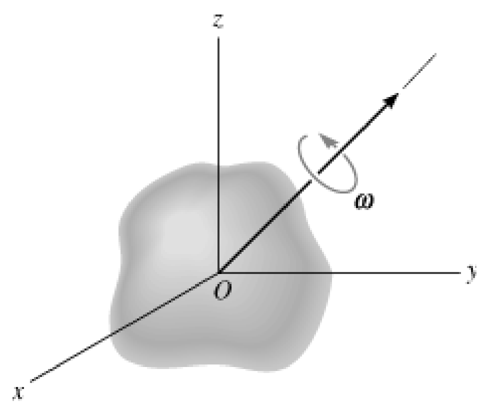
$$I_z = 4\frac{1}{3}W_r L^2 \sin(\theta)^2 + \frac{1}{12}W_p\left[(2a)^2 + (2a)^2\right] \qquad I_z = 0.0881\ \text{slug}\cdot\text{ft}^2$$

Problem 21-21

If a body contains *no planes of symmetry*, the principal moments of inertia can be determined mathematically. To show how this is done, consider the rigid body which is spinning with an angular velocity ω, directed along one of its principal axes of inertia. If the principal moment of inertia about this axis is I, the angular momentum can be expressed as $\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$. The components of $\mathbf{H}$ may also be expressed by Eqs. 21-10, where the inertia tensor is assumed to be known. Equate the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components of both expressions for $\mathbf{H}$ and consider ω_x, ω_y, and ω_z to be unknown. The solution of these three equations is obtained provided the determinant of the coefficients is zero. Show that this determinant, when expanded, yields the cubic equation $I^3 - (I_{xx} + I_{yy} + I_{zz})I^2 + (I_{xx}I_{yy} + I_{yy}I_{zz} + I_{zz}I_{xx} - I^2_{xy} - I^2_{yz} - I^2_{zx})I - (I_{xx}I_{yy}I_{zz} - 2I_{xy}I_{yz}I_{zx} - I_{xx}I^2_{yz} - I_{yy}I^2_{zx} - I_{zz}I^2_{xy}) = 0$. The three positive roots of I, obtained from the solution of this equation, represent the principal moments of inertia I_x, I_y, and I_z.

Solution:

$$\mathbf{H} = I\boldsymbol{\omega} = I\omega_x\mathbf{i} + I\omega_y\mathbf{j} + I\omega_z\mathbf{k}$$



Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components to the scalar (Eq. 21 - 10) yields

$$\left(I_{xx} - I\right)\omega_x - I_{xy}\omega_y - I_{xz}\omega_z = 0$$

$$-I_{yx}\,\omega_x + \left(I_{yy} - I\right)\omega_y - I_{yz}\omega_z = 0$$

$$-I_{zx}\,\omega_x - I_{zy}\omega_y + (I_{zz} - I)\omega_z = 0$$

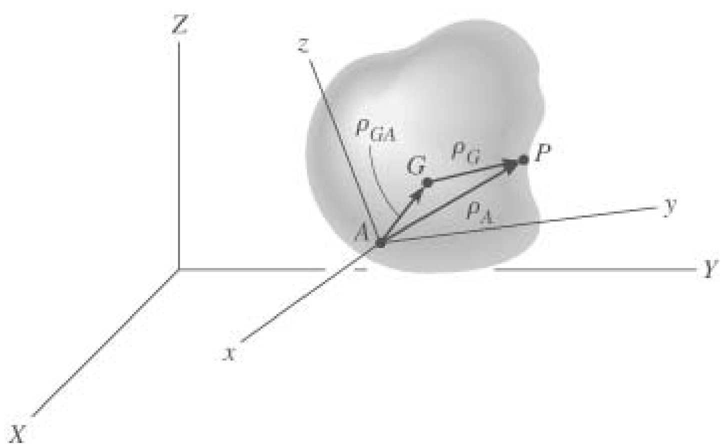
Solution for nontrivial ω_x, ω_y, and ω_z requires

$$\begin{bmatrix} (I_{xx} - I) & -I_{xy} & -I_{xz} \\ -Iyx & (I_{yy} - I) & -I_{yz} \\ -I_{zx} & -I_{zy} & (I_{zz} - I) \end{bmatrix} = 0$$

Expanding the determinant produces the required equation QED

Problem 21-22

Show that if the angular momentum of a body is determined with respect to an arbitrary point A, then $\mathbf{H_A}$ can be expressed by Eq. 21-9. This requires substituting $\boldsymbol{\rho}_\mathbf{A} = \boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}$ into Eq. 21-6 and expanding, noting that $\int \boldsymbol{\rho}_\mathbf{G} dm = 0$ by definition of the mass center and $\mathbf{v_G} = \mathbf{v_A} + \boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA}$.



Solution:

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{A}\, dm \times \mathbf{v_A} + \int \boldsymbol{\rho}_\mathbf{A} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{A})\, dm$$

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}\, dm \times \mathbf{v_A} + \int (\boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA}) \times [\boldsymbol{\omega} \times (\boldsymbol{\rho}_\mathbf{G} + \boldsymbol{\rho}_\mathbf{GA})]\, dm$$

$$\mathbf{H_A} = \int \boldsymbol{\rho}_\mathbf{G}\, dm \times \mathbf{v_A} + \boldsymbol{\rho}_\mathbf{GA} \times m\mathbf{v_A} + \int \boldsymbol{\rho}_\mathbf{G} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{G})\, dm + \int \boldsymbol{\rho}_\mathbf{G}\, dm \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA}) \ldots$$

$$+ \boldsymbol{\rho}_\mathbf{GA} \times \left(\boldsymbol{\omega} \times \int \boldsymbol{\rho}_\mathbf{G}\, dm\right) + \boldsymbol{\rho}_\mathbf{GA} \times (\boldsymbol{\omega} \times \boldsymbol{\rho}_\mathbf{GA})m$$

Since $\int \rho_G \, dm = 0$ and $H_G = \int \rho_G \times (\omega \times \rho_G) \, dm$

$$H_A = \rho_{GA} \times m v_A + H_G + \rho_{GA} \times \omega \times (\rho_{GA}) m = \rho_{GA} \times m(v_A + \omega \times \rho_{GA}) + H_G$$

$$H_A = \rho_G \times m v_G + H_G$$ Q.E.D

Problem 21-23

The thin plate of mass M is suspended at O using a ball-and-socket joint. It is rotating with a constant angular velocity $\omega = \omega_1 \mathbf{k}$ when the corner A strikes the hook at S, which provides a permanent connection. Determine the angular velocity of the plate immediately after impact.

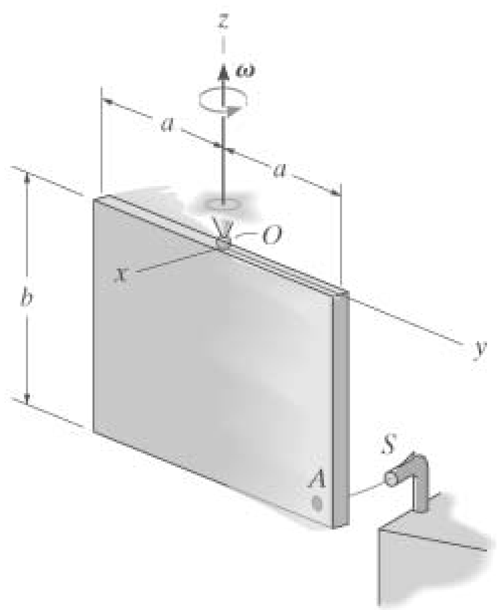
Given:

$M = 5 \text{ kg}$

$\omega_1 = 2 \frac{\text{rad}}{\text{s}}$

$a = 300 \text{ mm}$

$b = 400 \text{ mm}$



Solution:

Angular Momentum is conserved about the line OA.

$$\mathbf{OA} = \begin{pmatrix} 0 \\ a \\ -b \end{pmatrix} \qquad \mathbf{oa} = \frac{\mathbf{OA}}{|\mathbf{OA}|}$$

$$I_2 = \frac{1}{3} M b^2 \qquad I_3 = \frac{1}{12} M (2a)^2 \qquad I_1 = I_2 + I_3$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

$$I_{oa} = \mathbf{oa}^T \mathbf{I_{mat}} \mathbf{oa}$$

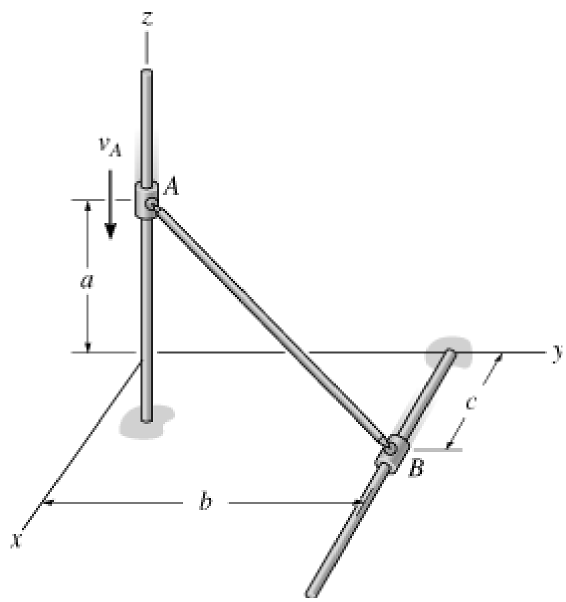
Guess $\omega_2 = 1 \frac{\text{rad}}{\text{s}}$

Given $\quad \mathbf{I_{mat}}\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix}\mathbf{oa} = I_{oa}\omega_2$

$\omega_2 = \text{Find}(\omega_2) \qquad \omega_2\mathbf{oa} = \begin{pmatrix} 0.00 \\ -0.75 \\ 1.00 \end{pmatrix}\frac{\text{rad}}{\text{s}}$

***Problem 21-24**

Rod *AB* has weight *W* and is attached to two smooth collars at its end points by ball-and-socket joints. If collar *A* is moving downward at speed v_A, determine the kinetic energy of the rod at the instant shown. Assume that at this instant the angular velocity of the rod is directed perpendicular to the rod's axis.



Given:

$W = 6\ \text{lb}$

$v_A = 8\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$

$b = 6\ \text{ft}$

$c = 2\ \text{ft}$

Solution: $\quad L = \sqrt{a^2 + b^2 + c^2}$

Guesses

$v_B = 1\ \frac{\text{ft}}{\text{s}} \qquad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}} \qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given $\quad \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}\begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z) \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12.00\ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_G} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} \frac{c}{2} \\ \frac{b}{2} \\ \frac{-a}{2} \end{pmatrix} \qquad T = \frac{1}{2}\left(\frac{W}{g}\right)(\mathbf{v_G}\,\mathbf{v_G}) + \frac{1}{2}\left(\frac{W}{g}\right)\frac{L^2}{12}(\boldsymbol{\omega}\,\boldsymbol{\omega}) \qquad T = 6.46\ \text{lb}\cdot\text{ft}$$

Problem 21-25

At the instant shown the collar at *A* on rod *AB* of weight *W* has velocity v_A. Determine the kinetic energy of the rod after the collar has descended a distance *d*. Neglect friction and the thickness of the rod. Neglect the mass of the collar and the collar is attached to the rod using ball-and-socket joints.

Given:

$W = 6\ \text{lb}$

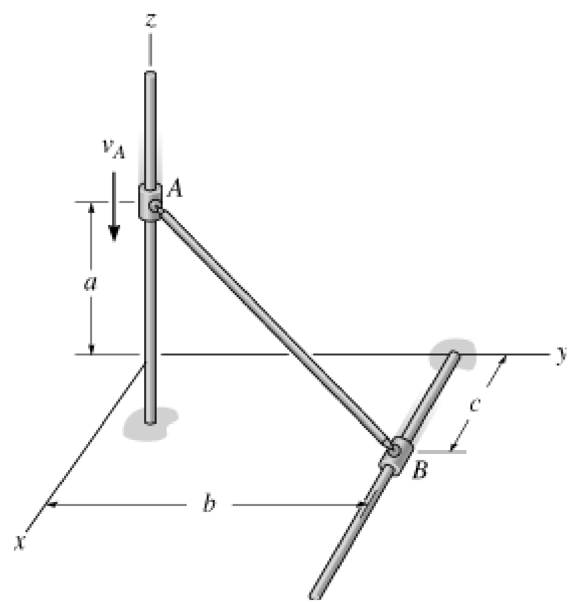
$v_A = 8\ \frac{\text{ft}}{\text{s}}$

$a = 3\ \text{ft}$

$b = 6\ \text{ft}$

$c = 2\ \text{ft}$

$d = 3\ \text{ft}$



Solution:

$$L = \sqrt{a^2 + b^2 + c^2}$$

Guesses

$v_B = 1\ \frac{\text{ft}}{\text{s}}$ $\qquad \omega_x = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_y = 1\ \frac{\text{rad}}{\text{s}}$ $\qquad \omega_z = 1\ \frac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_B \\ 0 \\ 0 \end{pmatrix}$ $\qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0 \ \frac{\text{ft}}{\text{s}}$

$$\begin{pmatrix} v_B \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \text{Find}(v_B, \omega_x, \omega_y, \omega_z)$$

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_B = 12.00 \ \frac{\text{ft}}{\text{s}}$$

$$\mathbf{v_G} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega} \times \begin{pmatrix} \frac{c}{2} \\ \frac{b}{2} \\ \frac{-a}{2} \end{pmatrix}$$

$$T_1 = \frac{1}{2}\left(\frac{W}{g}\right)(\mathbf{v_G}\cdot\mathbf{v_G}) + \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{L^2}{12}\right)(\boldsymbol{\omega}\cdot\boldsymbol{\omega}) \qquad T_1 = 6.46 \ \text{lb}\cdot\text{ft}$$

In position 2 the center of mass has fallen a distance $d/2$

$$T_1 + 0 = T_2 - W\left(\frac{d}{2}\right)$$

$$T_2 = T_1 + W\left(\frac{d}{2}\right) \qquad T_2 = 15.5 \ \text{lb}\cdot\text{ft}$$

Problem 21-26

The rod AB of mass M_{AB} is attached to the collar of mass M_A at A and a link BC of mass M_{BC} using ball-and-socket joints. If the rod is released from rest in the position shown, determine the angular velocity of the link after it has rotated 180°.

Given:

$M_{AB} = 4 \ \text{kg}$

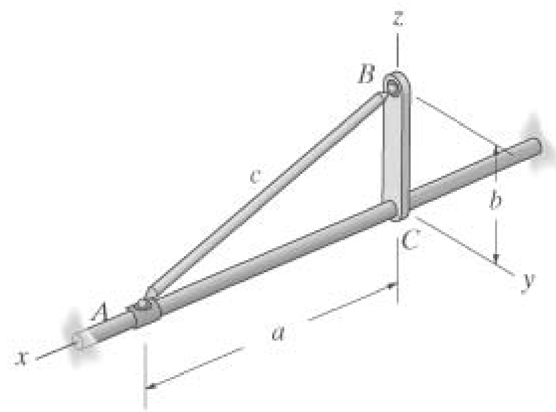
$M_A = 1 \ \text{kg}$

$M_{BC} = 2$ kg

$a = 1.2$ m

$b = 0.5$ m

$c = \sqrt{a^2 + b^2}$



Solution:

$\theta = \operatorname{atan}\left(\frac{b}{a}\right)$

$I = \frac{1}{3} M_{AB} c^2 \sin(\theta)^2 + \frac{1}{3} M_{BC} b^2$

Guess $\omega = 1 \frac{\text{rad}}{\text{s}}$

Given $(M_{AB} + M_{BC}) g\, b = \frac{1}{2} I \omega^2$

$\omega = \operatorname{Find}(\omega)$ $\omega = 10.85 \frac{\text{rad}}{\text{s}}$

Problem 21-27

The rod has weight density γ and is suspended from parallel cords at A and B. If the rod has angular velocity ω about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.

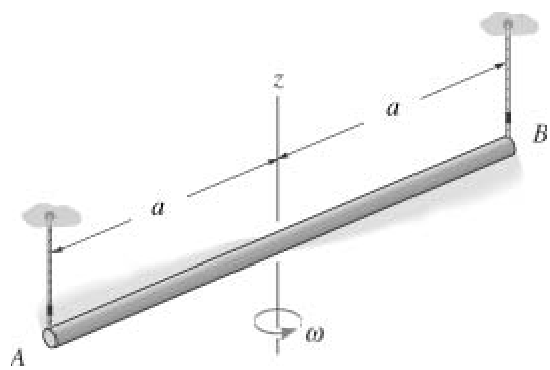
Given:

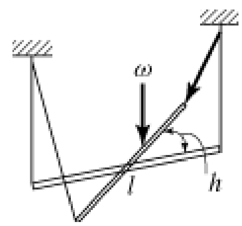
$\gamma = 3 \frac{\text{lb}}{\text{ft}}$

$\omega = 2 \frac{\text{rad}}{\text{s}}$

$a = 3$ ft

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$





Solution:

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}\left[\frac{1}{12}\,\frac{\gamma(2a)}{g}\right](2a)^2\,\omega^2 = \gamma 2a\,h$$

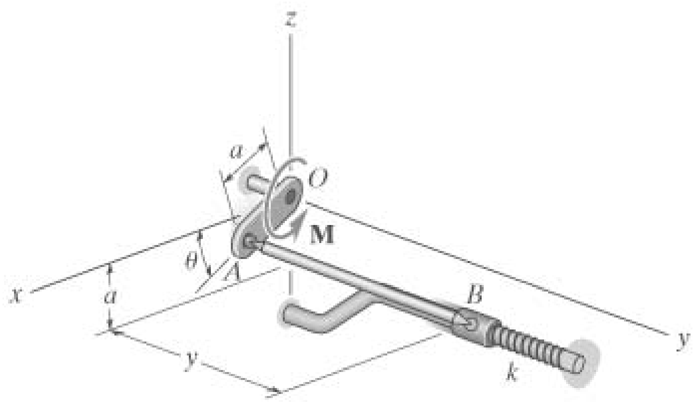
$$h = \frac{1}{6}a^2\left(\frac{\omega^2}{g}\right) \qquad h = 2.24\,\text{in}$$

***Problem 21-28**

The assembly consists of a rod AB of mass m_{AB} which is connected to link OA and the collar at B by ball-and-socket joints. When $\theta = 0$ and $y = y_1$, the system is at rest, the spring is unstretched, and a couple moment M, is applied to the link at O. Determine the angular velocity of the link at the instant $\theta = 90°$. Neglect the mass of the link.

Units Used:

$$\text{kN} = 10^3\ \text{N}$$



Given:

$$m_{AB} = 4\ \text{kg}$$

$$M = 7\ \text{N m}$$

$$a = 200\ \text{mm}$$

$$y_1 = 600\ \text{mm}$$

$$k = 2\ \frac{\text{kN}}{\text{m}}$$

Solution:

$$L = \sqrt{a^2 + a^2 + y_1{}^2}$$

$$I = \frac{1}{3}m_{AB}L^2$$

$$\delta = L - y_1$$

Guess $\qquad \omega = 1\ \frac{\text{rad}}{\text{s}}$

Given $\qquad m_{AB}\,g\,\frac{a}{2} + M(90\ \text{deg}) = \frac{1}{2}I\omega^2 + \frac{1}{2}k\delta^2$

$\omega = \text{Find}(\omega) \qquad \omega = 6.10\,\frac{\text{rad}}{\text{s}}$

$\omega_{OA} = \omega\frac{L}{a} \qquad \omega_{OA} = 20.2\,\frac{\text{rad}}{\text{s}}$

Problem 21-29

The assembly consists of a rod AB of mass m_{AB} which is connected to link OA and the collar at B by ball-and-socket joints. When $\theta = 0$ and $y = y_1$, the system is at rest, the spring is unstretched, and a couple moment $M = M_0(b\theta + c)$, is applied to the link at O. Determine the angular velocity of the link at the instant $\theta = 90°$. Neglect the mass of the link.

Units Used: $\quad \text{kN} = 10^3\ \text{N}$

Given:

$m_{AB} = 4\ \text{kg}$

$M_0 = 1\ \text{N m}$

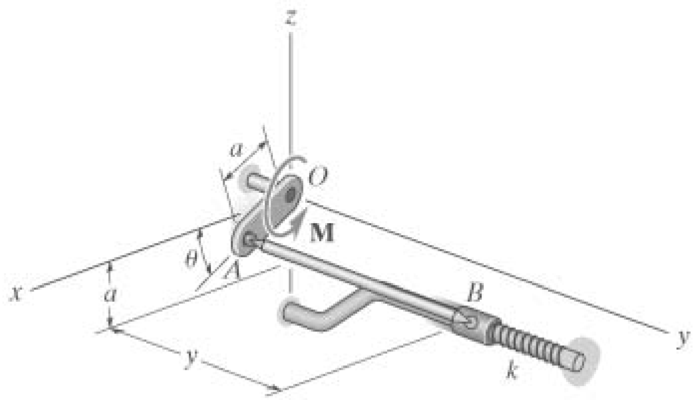
$y_1 = 600\ \text{mm}$

$a = 200\ \text{mm}$

$b = 4$

$c = 2$

$k = 2\,\frac{\text{kN}}{\text{m}}$



Solution:

$L = \sqrt{a^2 + a^2 + {y_1}^2}$

$I = \frac{1}{3}m_{AB}L^2$

$\delta = L - y_1$

Guess $\quad \omega = 1\,\frac{\text{rad}}{\text{s}}$

Given $\quad m_{AB}\,g\,\frac{a}{2} + \int_0^{90\,\text{deg}} M_0(b\theta + c)\,d\theta = \frac{1}{2}I\omega^2 + \frac{1}{2}k\delta^2$

$\omega = \text{Find}(\omega)$

$\omega = 5.22\,\frac{\text{rad}}{\text{s}}$ $\quad \omega_{OA} = \omega\frac{L}{a}$ $\quad \omega_{OA} = 17.3\,\frac{\text{rad}}{\text{s}}$

Problem 21-30

The circular plate has weight W and diameter d. If it is released from rest and falls horizontally a distance h onto the hook at S, which provides a permanent connection, determine the velocity of the mass center of the plate just after the connection with the hook is made.

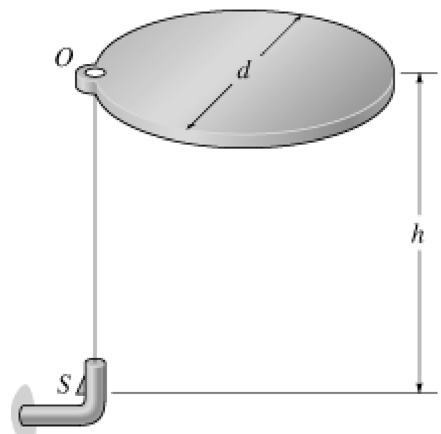
Given:

$W = 19\ \text{lb}$

$d = 1.5\ \text{ft}$

$h = 2.5\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



Solution:

$v_{G1} = \sqrt{2gh}$ $\quad v_{G1} = 12.69\,\frac{\text{ft}}{\text{s}}$

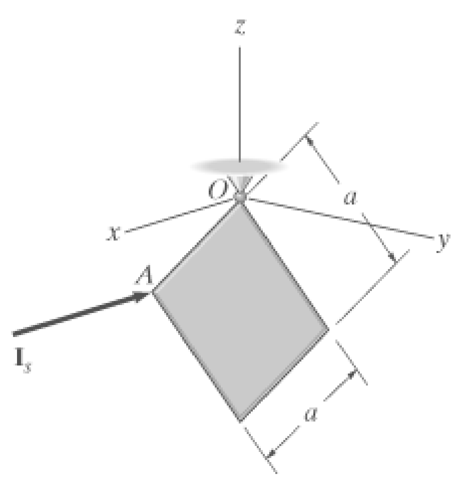
$$\left(\frac{W}{g}\right)v_{G1}\left(\frac{d}{2}\right) = \frac{5}{4}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2\omega_2$$

$\omega_2 = \frac{8v_{G1}}{5d}$ $\quad \omega_2 = 13.53\,\frac{\text{rad}}{\text{s}}$

$v_{G2} = \omega_2\frac{d}{2}$ $\quad v_{G2} = 10.2\,\frac{\text{ft}}{\text{s}}$

Problem 21-31

A thin plate, having mass M, is suspended from one of its corners by a ball-and-socket joint O. If a stone strikes the plate perpendicular to its surface at an adjacent corner A with an impulse $\mathbf{I_s}$, determine the instantaneous axis of rotation for the plate and the impulse created at O.



Given:

$M = 4\ \text{kg}$

$a = 200\ \text{mm}$

$$\theta = -45\ \text{deg}$$

$$\mathbf{I_s} = \begin{pmatrix} -60 \\ 0 \\ 0 \end{pmatrix} \text{N}\cdot\text{s}$$

Solution:

$$I_1 = \frac{2}{3} M a^2 \qquad I_2 = \frac{1}{3} M a^2$$

$$I_3 = I_2 \qquad I_{23} = M \frac{a^2}{4}$$

$$\mathbf{C_{mat}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

$$\mathbf{I_{mat}} = \mathbf{C_{mat}} \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & -I_{23} \\ 0 & -I_{23} & I_3 \end{pmatrix} \mathbf{C_{mat}}^T$$

Guesses $\begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{N s} \qquad \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{\text{m}}{\text{s}} \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{\text{rad}}{\text{s}}$

Given

$$\mathbf{I_s} + \begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} = M \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\frac{a}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \times \mathbf{I_s} = \mathbf{I_{mat}} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \frac{-a}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \\ \omega_x \\ \omega_y \\ \omega_z \\ v_x \\ v_y \\ v_z \end{pmatrix} = \text{Find}(I_{Ox}, I_{Oy}, I_{Oz}, \omega_x, \omega_y, \omega_z, v_x, v_y, v_z)$$

$$\textbf{Impulse} = \begin{pmatrix} I_{Ox} \\ I_{Oy} \\ I_{Oz} \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \textbf{axis} = \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|}$$

$$\textbf{Impulse} = \begin{pmatrix} 8.57 \\ 0.00 \\ 0.00 \end{pmatrix} \text{N}\cdot\text{s}$$

$$\textbf{axis} = \begin{pmatrix} 0.00 \\ 0.14 \\ -0.99 \end{pmatrix}$$

***Problem 21-32**

Rod AB has weight W and is attached to two smooth collars at its ends by ball-and-socket joints. If collar A is moving downward with speed v_A when $z = a$, determine the speed of A at the instant $z = 0$. The spring has unstretched length c. Neglect the mass of the collars. Assume the angular velocity of rod AB is perpendicular to its axis.

Given:

$W = 6 \text{ lb}$

$v_A = 8 \dfrac{\text{ft}}{\text{s}}$

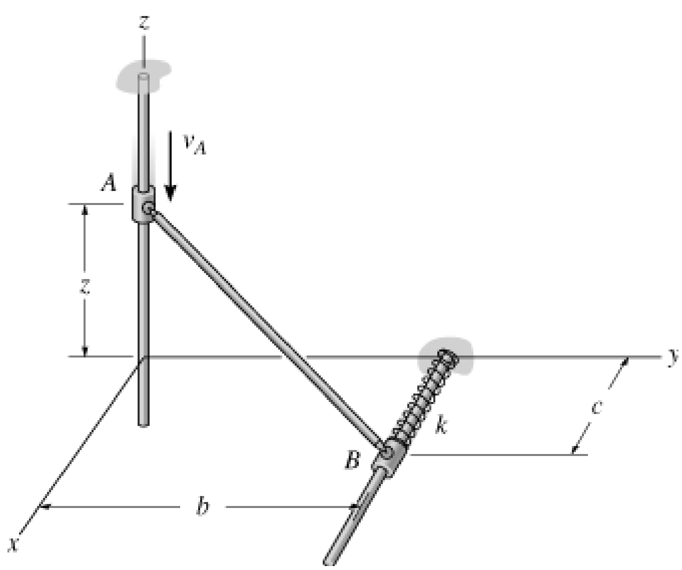
$a = 3 \text{ ft}$

$b = 6 \text{ ft}$

$c = 2 \text{ ft}$

$k = 4 \dfrac{\text{lb}}{\text{ft}}$

$\delta = 2 \text{ ft}$



$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

Solution: $L = \sqrt{a^2 + b^2 + c^2}$

First Position

Guesses

$v_{B1} = 1\,\frac{\text{ft}}{\text{s}}$ $\omega_{x1} = 1\,\frac{\text{rad}}{\text{s}}$

$\omega_{y1} = 1\,\frac{\text{rad}}{\text{s}}$ $\omega_{z1} = 1\,\frac{\text{rad}}{\text{s}}$

Given $\begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \times \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = \begin{pmatrix} v_{B1} \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} = 0\,\frac{\text{ft}}{\text{s}}$$

$$\begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \\ v_{B1} \end{pmatrix} = \text{Find}(\omega_{x1}, \omega_{y1}, \omega_{z1}, v_{B1}) \qquad \boldsymbol{\omega_1} = \begin{pmatrix} \omega_{x1} \\ \omega_{y1} \\ \omega_{z1} \end{pmatrix} \qquad \boldsymbol{\omega_1} = \begin{pmatrix} 0.98 \\ -1.06 \\ -1.47 \end{pmatrix} \frac{\text{rad}}{\text{s}}$$

$$\mathbf{v_{G1}} = \begin{pmatrix} 0 \\ 0 \\ -v_A \end{pmatrix} + \boldsymbol{\omega_1} \times \left[\frac{1}{2} \begin{pmatrix} c \\ b \\ -a \end{pmatrix} \right]$$

$$\mathbf{v_{G1}} = \begin{pmatrix} 6.00 \\ 0.00 \\ -4.00 \end{pmatrix} \frac{\text{ft}}{\text{s}}$$

$$T_1 = \frac{1}{2}\frac{W}{g}(\mathbf{v_{G1}} \cdot \mathbf{v_{G1}}) + \frac{1}{2}\frac{W}{g}\frac{L^2}{12}(\boldsymbol{\omega_1} \cdot \boldsymbol{\omega_1}) \qquad T_1 = 6.46\,\text{lb}\cdot\text{ft}$$

Work - Energy $T_1 + V_1 = T_2 + V_2$

$$T_2 = T_1 + W\frac{a}{2} - \frac{1}{2}k\left(\sqrt{L^2 - b^2} - c\right)^2 \qquad T_2 = 10.30\ \text{lb}\cdot\text{ft}$$

Second Position Note that B becomes the instantaneous center

Guesses $v_{A2} = 1\ \frac{\text{ft}}{\text{s}}$ $v_{B2} = 1\ \frac{\text{ft}}{\text{s}}$

$\omega_{x2} = 1\ \frac{\text{rad}}{\text{s}}$ $\omega_{y2} = 1\ \frac{\text{rad}}{\text{s}}$

$\omega_{z2} = 1\ \frac{\text{rad}}{\text{s}}$

Given
$$\begin{pmatrix} 0 \\ 0 \\ -v_{A2} \end{pmatrix} + \begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} \times \begin{pmatrix} \sqrt{L^2 - b^2} \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} v_{B2} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} \begin{pmatrix} \sqrt{L^2 - b^2} \\ b \\ 0 \end{pmatrix} = 0\ \frac{\text{ft}}{\text{s}}$$

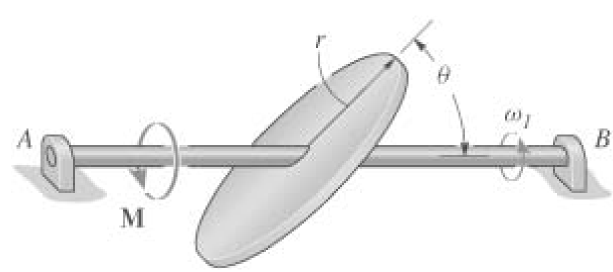
$$T_2 = \frac{1}{2}\frac{W}{g}\frac{L^2}{3}\left(\omega_{x2}^2 + \omega_{y2}^2 + \omega_{z2}^2\right)$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \\ \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} = \text{Find}\left(v_{A2}, v_{B2}, \omega_{x2}, \omega_{y2}, \omega_{z2}\right) \qquad \begin{pmatrix} \omega_{x2} \\ \omega_{y2} \\ \omega_{z2} \end{pmatrix} = \begin{pmatrix} 2.23 \\ -1.34 \\ 0.00 \end{pmatrix} \frac{\text{rad}}{\text{s}} \qquad v_{B2} = 0.00\ \frac{\text{ft}}{\text{s}}$$

$$v_{A2} = 18.2\ \frac{\text{ft}}{\text{s}}$$

Problem 21-33

The circular disk has weight W and is mounted on the shaft AB at angle θ with the horizontal. Determine the angular velocity of the shaft when $t = t_1$ if a constant torque **M** is applied to the shaft. The shaft is originally spinning with angular velocity ω_1 when the torque is applied.



Given:

$W = 15\ \text{lb}$

$\theta = 45\ \text{deg}$

$t_1 = 3\ \text{s}$

$M = 2\ \text{lb}\cdot\text{ft}$

$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$r = 0.8\text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\sin(\theta)^2 \qquad I_{AB} = 0.11\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\alpha = \frac{M}{I_{AB}} \qquad \omega_2 = \omega_1 + \alpha t_1 \qquad \omega_2 = 61.7\ \frac{\text{rad}}{\text{s}}$$

Problem 21-34

The circular disk has weight W and is mounted on the shaft AB at angle of θ with the horizontal. Determine the angular velocity of the shaft when $t = t_1$ if a torque $\mathbf{M} = \mathbf{M_0}\, e^{bt}$ applied to the shaft. The shaft is originally spinning at ω_1 when the torque is applied.

Given:

$W = 15\ \text{lb}$

$\theta = 45\ \text{deg}$

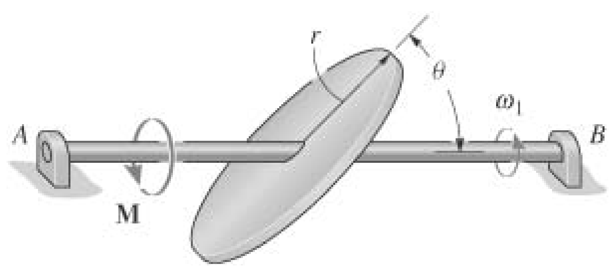
$t_1 = 2\ \text{s}$

$M_0 = 4\ \text{lb}\cdot\text{ft}$

$\omega_1 = 8\ \frac{\text{rad}}{\text{s}}$

$r = 0.8\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$



$$b = 0.1\ \mathrm{s}^{-1}$$

Solution:

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\sin(\theta)^2 \qquad I_{AB} = 0.11\ \mathrm{lb\cdot ft\cdot s^2}$$

$$\omega_2 = \omega_1 + \frac{1}{I_{AB}}\left(\int_0^{t_1} M_0\, e^{b\,t}\, dt\right) \qquad \omega_2 = 87.2\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 21-35

The rectangular plate of mass m_p is free to rotate about the y axis because of the bearing supports at A and B. When the plate is balanced in the vertical plane, a bullet of mass m_b is fired into it, perpendicular to its surface, with a velocity v. Compute the angular velocity of the plate at the instant it has rotated 180°. If the bullet strikes corner D with the same velocity v, instead of at C, does the angular velocity remain the same? Why or why not?

Given:

$m_p = 15\ \mathrm{kg}$

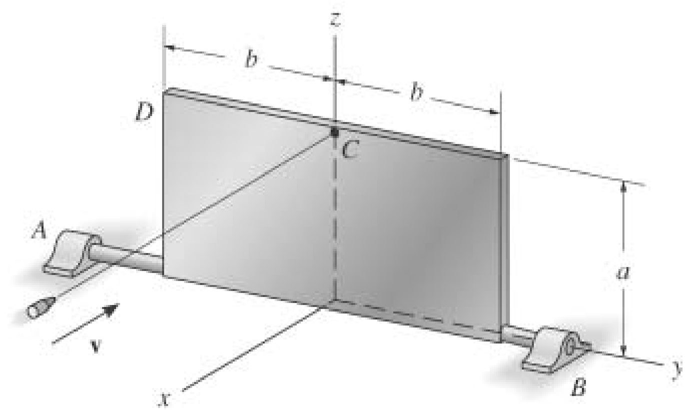
$m_b = 0.003\ \mathrm{kg}$

$v = 2000\ \dfrac{\mathrm{m}}{\mathrm{s}}$

$a = 150\ \mathrm{mm}$

$b = 150\ \mathrm{mm}$

$g = 9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}$



Solution: Guesses $\omega_2 = 1\ \dfrac{\mathrm{rad}}{\mathrm{s}}$ $\omega_3 = 1\ \dfrac{\mathrm{rad}}{\mathrm{s}}$

Given $m_b\, v\, a = \dfrac{1}{3} m_p\, a^2 \omega_2$

$$\frac{1}{2} m_p \frac{a^2}{3}\omega_2^{\,2} + m_p\, g\, \frac{a}{2} = \frac{1}{2} m_p \frac{a^2}{3}\omega_3^{\,2} - m_p\, g\, \frac{a}{2}$$

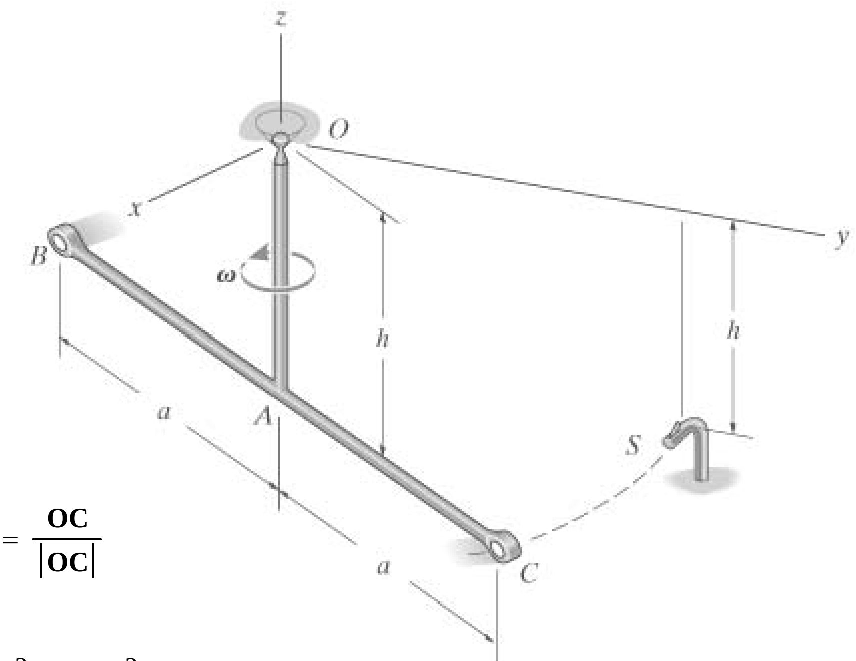
$$\begin{pmatrix} \omega_2 \\ \omega_3 \end{pmatrix} = \text{Find}(\omega_2, \omega_3) \qquad \omega_2 = 8.00 \frac{\text{rad}}{\text{s}} \qquad \omega_3 = 21.4 \frac{\text{rad}}{\text{s}}$$

If the bullet strikes at *D*, the result will be the same.

***Problem 21-36**

The rod assembly has a mass density ρ and is rotating with a constant angular velocity $\omega = \omega_1 \mathbf{k}$ when the loop end at *C* encounters a hook at *S*, which provides a permanent connection. Determine the angular velocity of the assembly immediately after impact.

Given:



$$\rho = 2.5 \frac{\text{kg}}{\text{m}}$$

$$a = 0.5 \text{ m}$$

$$\omega_1 = 2 \frac{\text{rad}}{\text{s}}$$

$$h = 0.5 \text{ m}$$

Solution:

$$\mathbf{OC} = \begin{pmatrix} 0 \\ a \\ -h \end{pmatrix} \qquad \mathbf{oc} = \frac{\mathbf{OC}}{|\mathbf{OC}|}$$

$$I_1 = \frac{1}{3}\rho h^3 + \frac{1}{12}\rho(2a)^3 + \rho 2a h^2$$

$$I_2 = \frac{1}{3}\rho h^3 + \rho 2a h^2 \qquad I_3 = I_1 - I_2$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \qquad \mathbf{I_{OC}} = \mathbf{oc}^T \mathbf{I_{mat}} \, \mathbf{oc}$$

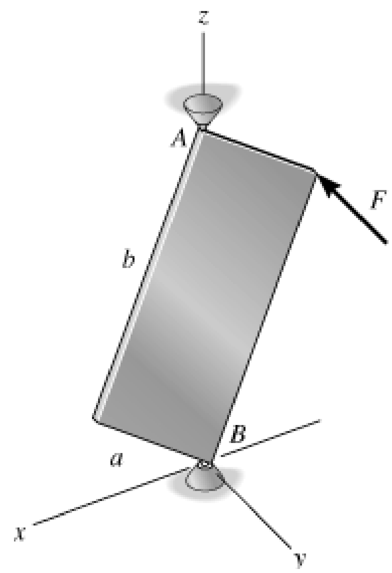
Guess $\qquad \omega_2 = 1 \frac{\text{rad}}{\text{s}}$

Given $\quad \mathbf{I_{mat}}\begin{pmatrix} 0 \\ 0 \\ \omega_1 \end{pmatrix}\mathbf{oc} = \mathbf{I_{OC}}\,\omega_2 \qquad \omega_2 = \text{Find}(\omega_2) \qquad \omega_2 = -0.63\,\dfrac{\text{rad}}{\text{s}}$

$$\omega_2\mathbf{oc} = \begin{pmatrix} 0.00 \\ -0.44 \\ 0.44 \end{pmatrix}\frac{\text{rad}}{\text{s}}$$

Problem 21-37

The plate of weight W is subjected to force F which is always directed perpendicular to the face of the plate. If the plate is originally at rest, determine its angular velocity after it has rotated one revolution (360°). The plate is supported by ball-and-socket joints at A and B.



Given:

$W = 15\ \text{lb}$

$F = 8\ \text{lb}$

$a = 0.4\ \text{ft}$

$b = 1.2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$\theta = \text{atan}\left(\dfrac{a}{b}\right) \qquad \theta = 18.43\,\text{deg}$

$$I_{AB} = \left(\frac{W}{g}\right)\left(\frac{a^2}{12}\right)\cos(\theta)^2 + \left(\frac{W}{g}\right)\left(\frac{b^2}{12}\right)\sin(\theta)^2$$

$I_{AB} = 0.0112\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$

Guess $\quad \omega = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad F\,a\cos(\theta)(2\pi) = \dfrac{1}{2}I_{AB}\omega^2$

$\omega = \text{Find}(\omega) \qquad \omega = 58.4\ \dfrac{\text{rad}}{\text{s}}$

Problem 21-38

The space capsule has mass m_c and the radii of gyration are $k_x = k_z$ and k_y. If it is traveling with a velocity v_G, compute its angular velocity just after it is struck by a meteoroid having mass m_m and a velocity $\mathbf{v_m} = (v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$. Assume that the meteoroid embeds itself into the capsule at point A and that the capsule initially has no angular velocity.

Units Used:

$$\mathrm{Mg} = 1000\ \mathrm{kg}$$

Given:

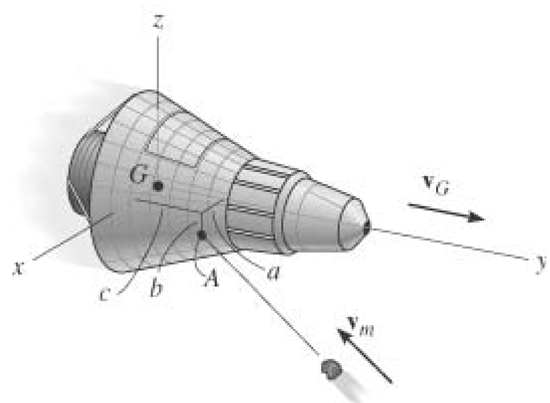
$$m_c = 3.5\ \mathrm{Mg} \qquad m_m = 0.60\ \mathrm{kg}$$

$$k_x = 0.8\ \mathrm{m} \qquad v_x = -200\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$k_y = 0.5\ \mathrm{m} \qquad v_y = -400\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$v_G = 600\ \frac{\mathrm{m}}{\mathrm{s}} \qquad v_z = 200\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$a = 1\ \mathrm{m} \quad b = 1\ \mathrm{m} \quad c = 3\ \mathrm{m}$$



Solution:

Guesses

$$\omega_x = 1\ \frac{\mathrm{rad}}{\mathrm{s}} \qquad \omega_y = 1\ \frac{\mathrm{rad}}{\mathrm{s}} \qquad \omega_z = 1\ \frac{\mathrm{rad}}{\mathrm{s}}$$

Given

$$\begin{pmatrix} a \\ c \\ -b \end{pmatrix} \times \left[m_m \begin{pmatrix} v_x \\ v_y - v_G \\ v_z \end{pmatrix} \right] = m_c \begin{pmatrix} k_x^2 & 0 & 0 \\ 0 & k_y^2 & 0 \\ 0 & 0 & k_x^2 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \mathrm{Find}(\omega_x, \omega_y, \omega_z) \qquad \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} -0.107 \\ 0.000 \\ -0.107 \end{pmatrix} \frac{\mathrm{rad}}{\mathrm{s}}$$

Problem 21-39

Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *not constant* with respect to time.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \; ... \\ + \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \; ... \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

One can obtain y and z components in a similar manner.

***Problem 21-40**

Derive the scalar form of the rotational equation of motion along the x axis when $\mathbf{\Omega} \neq \boldsymbol{\omega}$ and the moments and products of inertia of the body are *constant* with respect to time.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \; ... \\ + \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \; ... \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

For constant inertia, expanding the time derivative of the above equation yields

$$\Sigma M_x = \left(I_x\omega'_x - I_{xy}\omega'_y - I_{xz}\omega'_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ldots$$
$$+ \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)$$

One can obtain y and z components in a similar manner.

Problem 21-41

Derive the Euler equations of motion for $\mathbf{\Omega} \neq \boldsymbol{\omega}$ i.e., Eqs. 21-26.

Solution:

In general

$$\mathbf{M} = \frac{d}{dt}\left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

$$\mathbf{M} = \left(H'_x\mathbf{i} + H'_y\mathbf{j} + H'_z\mathbf{k}\right) + \mathbf{\Omega} \times \left(H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}\right)$$

Substitute $\mathbf{\Omega} = \Omega_x\mathbf{i} + \Omega_y\mathbf{j} + \Omega_z\mathbf{k}$ and expanding the cross product yields

$$\mathbf{M} = \left(H'_x - \Omega_z H_y + \Omega_y H_z\right)\mathbf{i} + \left(H'_y - \Omega_x H_z + \Omega_z H_x\right)\mathbf{j} \ldots$$
$$+ \left(H'_z - \Omega_y H_x + \Omega_x H_y\right)\mathbf{k}$$

Substitute H_x, H_y, and H_z using Eq. 21 - 10. For the i component

$$\Sigma M_x = \frac{d}{dt}\begin{bmatrix}\left(I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z\right) - \Omega_z\left(I_y\omega_y - I_{yz}\omega_z - I_{yx}\omega_x\right) \ldots \\ + \Omega_y\left(I_z\omega_z - I_{zx}\omega_x - I_{zy}\omega_y\right)\end{bmatrix}$$

Set $I_{xy} = I_{yz} = I_{zx} = 0$ and require I_x, I_y, I_z to be constant. This yields

$$\Sigma M_x = I_x\omega'_x - I_y\Omega_z\omega_y + I_z\Omega_y\omega_z$$

One can obtain y and z components in a similar manner.

Problem 21-42

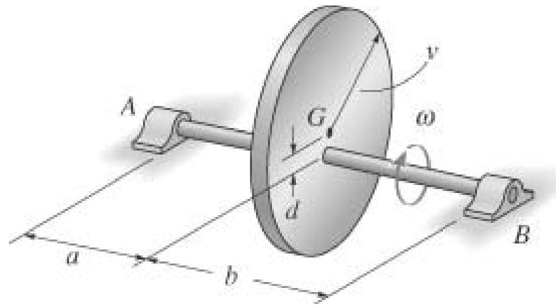
The flywheel (disk of mass M) is mounted a distance d off its true center at G. If the shaft is rotating at constant speed ω, determine the maximum reactions exerted on the journal bearings at A and B.

Given:

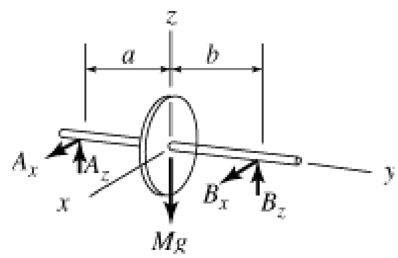
$M = 40 \text{ kg}$ $\quad a = 0.75 \text{ m}$

$d = 20 \text{ mm}$ $\quad b = 1.25 \text{ m}$

$\omega = 8 \dfrac{\text{rad}}{\text{s}}$ $\quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$



Solution: Check both up and down positions



Guesses $\quad A_{up} = 1 \text{ N} \quad B_{up} = 1 \text{ N}$

Given $\quad A_{up} + B_{up} - M g = -M d \omega^2$

$$-A_{up} a + B_{up} b = 0$$

$$\begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \text{Find}(A_{up}, B_{up}) \qquad \begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \begin{pmatrix} 213.25 \\ 127.95 \end{pmatrix} \text{N}$$

Guesses $\quad A_{down} = 1 \text{ N} \quad B_{down} = 1 \text{ N}$

Given $\quad A_{down} + B_{down} - M g = M d \omega^2$

$$-A_{down} a + B_{down} b = 0$$

$$\begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \text{Find}(A_{down}, B_{down}) \qquad \begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \begin{pmatrix} 277.25 \\ 166.35 \end{pmatrix} \text{N}$$

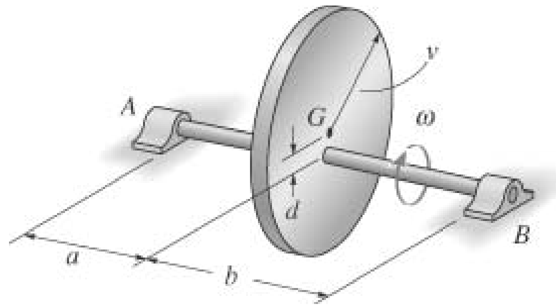
Thus $\quad A_{max} = \max(A_{up}, A_{down}) \qquad A_{max} = 277 \text{ N}$

$\quad B_{max} = \max(B_{up}, B_{down}) \qquad B_{max} = 166 \text{ N}$

Problem 21-43

The flywheel (disk of mass *M*) is mounted a distance *d* off its true center at *G*. If the shaft is rotating at constant speed ω, determine the minimum reactions exerted on the journal bearings at *A* and *B* during the motion.

Given:

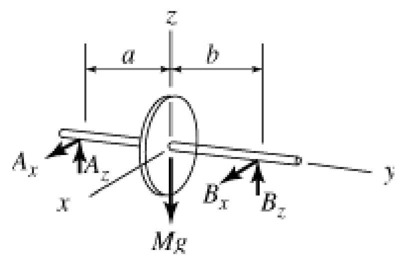


$M = 40\ \text{kg}$ $a = 0.75\ \text{m}$

$d = 20\ \text{mm}$ $b = 1.25\ \text{m}$

$\omega = 8\ \frac{\text{rad}}{\text{s}}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution: Check both up and down positions



Guesses $A_{up} = 1\ \text{N}$ $B_{up} = 1\ \text{N}$

Given $A_{up} + B_{up} - M g = -M d \omega^2$

$-A_{up} a + B_{up} b = 0$

$$\begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \text{Find}(A_{up}, B_{up}) \qquad \begin{pmatrix} A_{up} \\ B_{up} \end{pmatrix} = \begin{pmatrix} 213.25 \\ 127.95 \end{pmatrix} \text{N}$$

Guesses $A_{down} = 1\ \text{N}$ $B_{down} = 1\ \text{N}$

Given $A_{down} + B_{down} - M g = M d \omega^2$

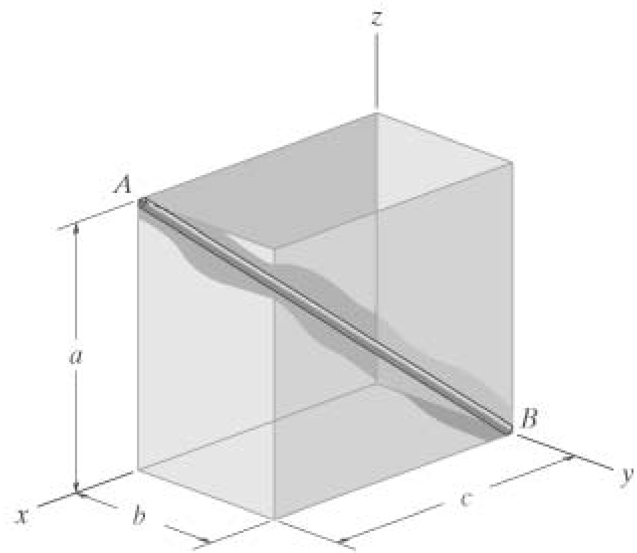
$-A_{down} a + B_{down} b = 0$

$$\begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \text{Find}(A_{down}, B_{down}) \qquad \begin{pmatrix} A_{down} \\ B_{down} \end{pmatrix} = \begin{pmatrix} 277.25 \\ 166.35 \end{pmatrix} \text{N}$$

Thus $A_{min} = \min(A_{up}, A_{down})$ $A_{min} = 213\ \text{N}$

$B_{min} = \min(B_{up}, B_{down})$ $B_{min} = 128\ \text{N}$

***Problem 21-44**



The bar of weight W rests along the smooth corners of an open box. At the instant shown, the box has a velocity $v = v_1\mathbf{k}$ and an acceleration $a = a_1\mathbf{k}$. Determine the x, y, z components of force which the corners exert on the bar.

Given:

$W = 4\ \text{lb}$ $\quad a = 2\ \text{ft}$

$v_1 = 5\ \dfrac{\text{ft}}{\text{s}}$ $\quad b = 1\ \text{ft}$

$a_1 = 2\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad c = 2\ \text{ft}$

Solution:

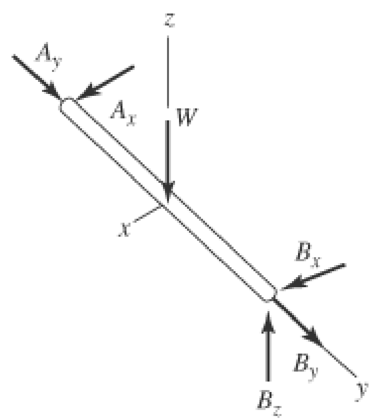
Guesses

$A_x = 1\ \text{lb}$ $\quad B_x = 1\ \text{lb}$

$A_y = 1\ \text{lb}$ $\quad B_y = 1\ \text{lb}$

$B_z = 1\ \text{lb}$

Given



$$A_x + B_x = 0$$

$$A_y + B_y = 0$$

$$B_z - W = \frac{W}{g}a_1$$

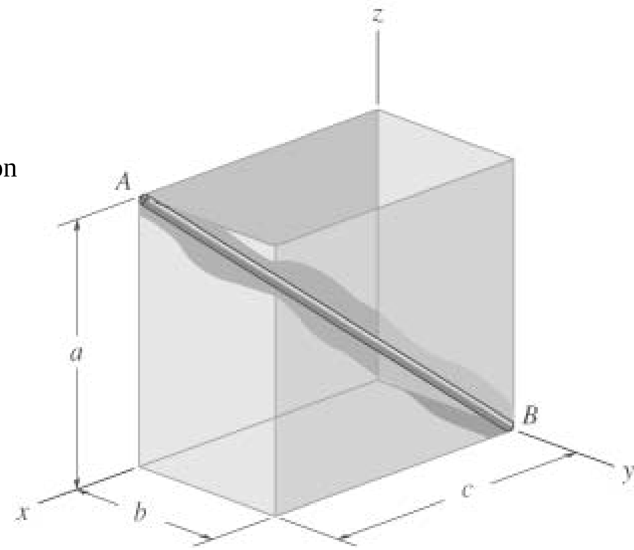
$$\frac{1}{2}\begin{pmatrix} -c \\ b \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ B_z \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, B_z)$$

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} -2.12 \\ 1.06 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 2.12 \\ -1.06 \\ 4.25 \end{pmatrix}\text{lb}$$

Problem 21-45

The bar of weight W rests along the smooth corners of an open box. At the instant shown, the box has a velocity $v = v_1\mathbf{j}$ and an acceleration $a = a_1\mathbf{j}$. Determine the x, y, z components of force which the corners exert on the bar.



Given:

$W = 4\ \text{lb}$ $\quad a = 2\ \text{ft}$

$v_1 = 3\ \dfrac{\text{ft}}{\text{s}}$ $\quad b = 1\ \text{ft}$

$a_1 = -6\ \dfrac{\text{ft}}{\text{s}^2}$ $\quad c = 2\ \text{ft}$

Solution:

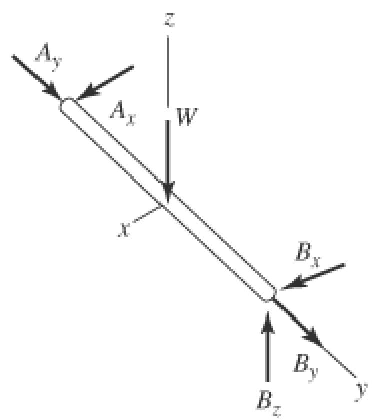
Guesses

$A_x = 1\ \text{lb}$ $\quad B_x = 1\ \text{lb}$

$A_y = 1\ \text{lb}$ $\quad B_y = 1\ \text{lb}$

$B_z = 1\ \text{lb}$

Given



$$A_x + B_x = 0$$

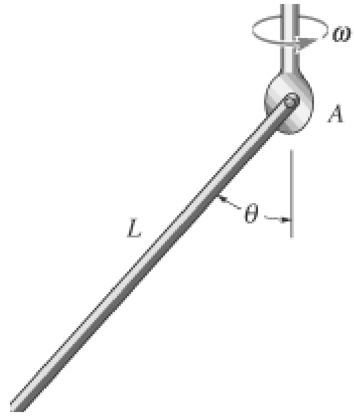
$$A_y + B_y = \frac{W}{g}a_1$$

$$B_z - W = 0$$

$$\frac{1}{2}\begin{pmatrix} -c \\ b \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ -b \\ a \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} = 0$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \\ B_z \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y, B_z) \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} -2.00 \\ 0.63 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 2.00 \\ -1.37 \\ 4.00 \end{pmatrix} \text{lb}$$

Problem 21-46



The conical pendulum consists of a bar of mass *m* and length *L* that is supported by the pin at its end *A*. If the pin is subjected to a rotation ω, determine the angle θ that the bar makes with the vertical as it rotates.

Solution:

$$I_x = I_z = \frac{1}{3} m L^2$$

$$I_y = 0 \qquad \omega_x = 0$$

$$\omega_y = -\omega \cos(\theta)$$

$$\omega_z = \omega \sin(\theta)$$

$$\omega'_x = 0 \qquad \omega'_y = 0 \qquad \omega'_z = 0$$

$$\Sigma M_x = I_x \omega'_x - (I_y - I_z)\omega_y \omega_x$$

$$-m g\left(\frac{L}{2}\right)\sin(\theta) = 0 - \left(0 - \frac{1}{3} m L^2\right)(-\omega \cos(\theta))(\omega \sin(\theta))$$

$$\frac{g}{2} = \frac{1}{3} L \omega^2 \cos(\theta) \qquad \theta = a\cos\left(\frac{3g}{2L\omega^2}\right)$$

Problem 21-47

The plate of weight *W* is mounted on the shaft *AB* so that the plane of the plate makes an angle θ with the vertical. If the shaft is turning in the direction shown with angular velocity ω, determine the vertical reactions at the bearing supports *A* and *B* when the plate is in the position shown.

Given:

$W = 20\ \text{lb}$

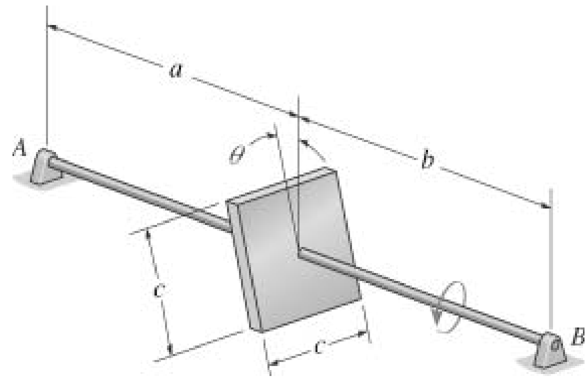
$\theta = 30\ \text{deg}$

$\omega = 25\ \frac{\text{rad}}{\text{s}}$

$a = 18\ \text{in}$

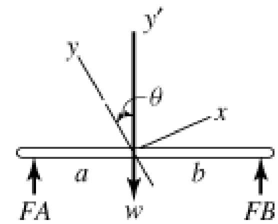
$b = 18\ \text{in}$

$c = 6\ \text{in}$



Solution:

$I_x = \left(\frac{W}{g}\right)\left(\frac{c^2}{6}\right)$



$I_z = \frac{I_x}{2} \qquad I_y = I_z$

$\omega_x = \omega \sin(\theta) \qquad \omega_y = -\omega \cos(\theta)$

$\omega_z = 0\ \frac{\text{rad}}{\text{s}}$

Guesses $\quad F_A = 1\ \text{lb} \qquad F_B = 1\ \text{lb}$

Given $\quad F_A + F_B - W = 0$

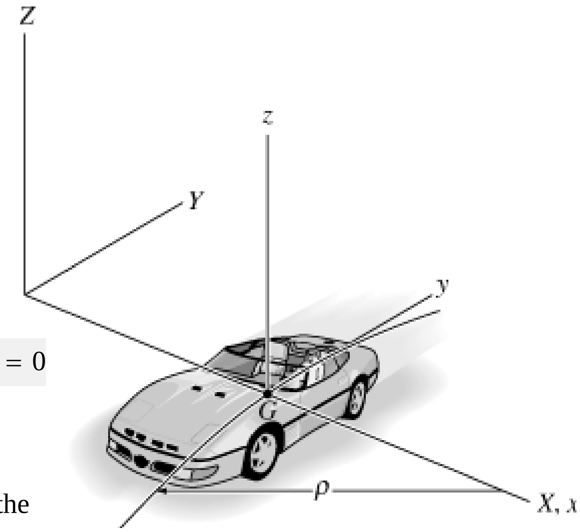
$$\begin{pmatrix} 0 \\ 0 \\ F_B b - F_A a \end{pmatrix} = \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \times \left[\begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}\right]$$

$\begin{pmatrix} F_A \\ F_B \end{pmatrix} = \text{Find}(F_A, F_B) \qquad \begin{pmatrix} F_A \\ F_B \end{pmatrix} = \begin{pmatrix} 8.83 \\ 11.17 \end{pmatrix} \text{lb}$

***Problem 21-48**

The car is traveling around the curved road of radius ρ such that its mass center has a constant speed v_G. Write the equations of rotational motion with respect to the x, y, z axes. Assume that the car's six moments and products of inertia with respect to these axes are known.

Solution:



Applying Eq. 21-24 with $\omega_x = 0$ $\omega_y = 0$

$\omega_z = \frac{v_G}{\rho}$ $\omega'_x = \omega'_y = \omega'_z = 0$

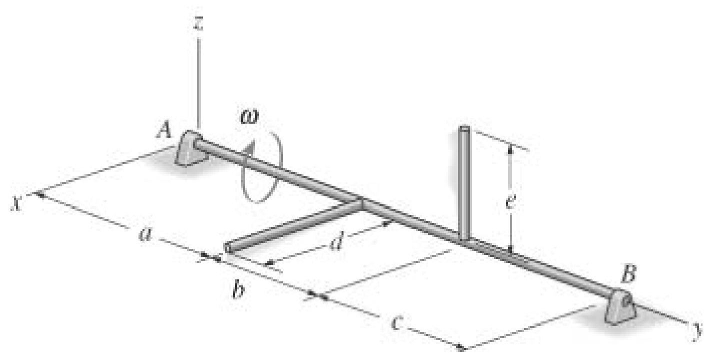
$\Sigma M_x = I_{yz}\left(\frac{v_G}{\rho}\right)^2$ $\Sigma M_y = I_{zx}\left(\frac{v_G}{\rho}\right)^2$ $\Sigma M_z = 0$

Note: This result indicates the normal reactions of the tires on the ground are not all necessarily equal. Instead, they depend upon the speed of the car, radius of curvature, and the products of inertia, I_{yz} and I_{zx}. (See Example 13-6.)

Problem 21-49

The rod assembly is supported by journal bearings at *A* and *B*, which develops only *x* and *z* force reactions on the shaft. If the shaft *AB* is rotating in the direction shown with angular velocity ω, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass density of each rod is ρ.

Given:



$\omega = -5\ \frac{\text{rad}}{\text{s}}$

$\rho = 1.5\ \frac{\text{kg}}{\text{m}}$

$a = 500\ \text{mm}$

$b = 300\ \text{mm}$

$c = 500\ \text{mm}$

$d = 400\ \text{mm}$

$e = 300\ \text{mm}$

$g = 9.81\ \frac{\text{m}}{\text{s}^2}$

Solution:

$$I_{xx} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d a^2 + \rho e\frac{e^2}{12} + \rho e\left[(a+b)^2 + \left(\frac{e}{2}\right)^2\right]$$

$$I_{zz} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d\frac{d^2}{12} + \rho d\left[a^2 + \left(\frac{d}{2}\right)^2\right] + \rho e(a+b)^2$$

$$I_{yy} = \rho d\frac{d^2}{3} + \rho e\frac{e^2}{3} \qquad I_{xy} = \rho d a\frac{d}{2} \qquad I_{yz} = \rho e(a+b)\frac{e}{2}$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 1.5500 & -0.0600 & 0.0000 \\ -0.0600 & 0.0455 & -0.0540 \\ 0.0000 & -0.0540 & 1.5685 \end{pmatrix} \text{kg}\cdot\text{m}^2$$

Guesses $A_x = 1\ \text{N}$ $A_z = 1\ \text{N}$ $B_x = 1\ \text{N}$ $B_z = 1\ \text{N}$ $\omega'_y = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d+e)g \end{bmatrix} = \begin{pmatrix} -\rho d\frac{d}{2}\omega^2 + \rho e\frac{e}{2}\omega'_y \\ 0 \\ -\rho d\frac{d}{2}\omega'_y - \rho e\frac{e}{2}\omega^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \frac{a+b+c}{2} \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -\rho(a+b+c)g \end{bmatrix} + \begin{pmatrix} \frac{d}{2} \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho d g \end{pmatrix} \dots = \mathbf{I_{mat}}\begin{pmatrix} 0 \\ \omega'_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_{mat}}\begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix}\right]$$

$$+ \begin{pmatrix} 0 \\ a+b \\ \frac{e}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho e g \end{pmatrix} + \begin{pmatrix} 0 \\ a+b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \omega'_y) \qquad \begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} -1.17 \\ 12.33 \end{pmatrix} \text{N} \qquad \begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} -0.0791 \\ 12.3126 \end{pmatrix} \text{N}$$

$$\omega'_y = 25.9\ \frac{\text{rad}}{\text{s}^2}$$

Problem 21-50

The rod assembly is supported by journal bearings at *A* and *B*, which develops only *x* and *z* force reactions on the shaft. If the shaft *AB* is subjected to a couple moment M_0 **j** and at the instant shown the shaft has an angular velocity ω **j**, determine the reactions at the bearings when the assembly is in the position shown. Also, what is the shaft's angular acceleration? The mass density of each rod is ρ.

Given:

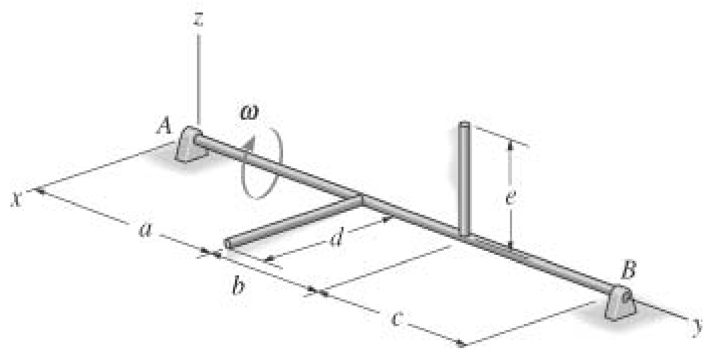
$\omega = -5\ \frac{\text{rad}}{\text{s}}$ $\quad c = 500\ \text{mm}$

$\rho = 1.5\ \frac{\text{kg}}{\text{m}}$ $\quad d = 400\ \text{mm}$

$a = 500\ \text{mm}$ $\quad e = 300\ \text{mm}$

$b = 300\ \text{mm}$ $\quad g = 9.81\ \frac{\text{m}}{\text{s}^2}$

$M_0 = 8\ \text{N}\cdot\text{m}$



Solution:

$$I_{xx} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d a^2 + \rho e\frac{e^2}{12} + \rho e\left[(a+b)^2 + \left(\frac{e}{2}\right)^2\right]$$

$$I_{zz} = \rho(a+b+c)\frac{(a+b+c)^2}{3} + \rho d\frac{d^2}{12} + \rho d\left[a^2 + \left(\frac{d}{2}\right)^2\right] + \rho e(a+b)^2$$

$$I_{yy} = \rho d\frac{d^2}{3} + \rho e\frac{e^2}{3} \qquad I_{xy} = \rho d a\frac{d}{2} \qquad I_{yz} = \rho e(a+b)\frac{e}{2}$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 1.5500 & -0.0600 & 0.0000 \\ -0.0600 & 0.0455 & -0.0540 \\ 0.0000 & -0.0540 & 1.5685 \end{pmatrix}\text{kg}\cdot\text{m}^2$$

Guesses $\quad A_x = 1\ \text{N} \quad A_z = 1\ \text{N} \quad B_x = 1\ \text{N} \quad B_z = 1\ \text{N} \quad \omega'_y = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d+e)g \end{bmatrix} = \begin{pmatrix} -\rho d \frac{d}{2} \omega^2 + \rho e \frac{e}{2} \omega'_y \\ 0 \\ -\rho d \frac{d}{2} \omega'_y - \rho e \frac{e}{2} \omega^2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ \frac{a+b+c}{2} \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -\rho(a+b+c)g \end{bmatrix} + \begin{pmatrix} \frac{d}{2} \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho d g \end{pmatrix} \dots = \mathbf{I_{mat}} \begin{pmatrix} 0 \\ \omega'_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_{mat}} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$+ \begin{pmatrix} 0 \\ a+b \\ \frac{e}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho e g \end{pmatrix} + \begin{pmatrix} 0 \\ a+b+c \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \omega'_y)$$

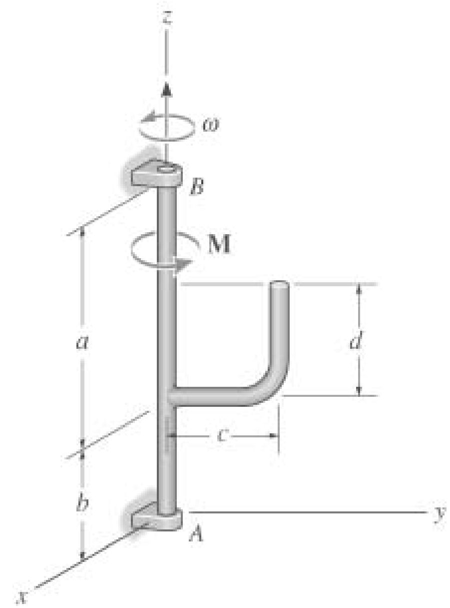
$$\begin{pmatrix} A_x \\ A_z \end{pmatrix} = \begin{pmatrix} 3.39 \\ -0.66 \end{pmatrix} \text{N} \qquad \begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} 7.2243 \\ 4.1977 \end{pmatrix} \text{N}$$

$$\omega'_y = 201.7 \frac{\text{rad}}{\text{s}^2}$$

Problem 21-51

The rod assembly has a weight density *r*. It is supported at *B* by a smooth journal bearing, which develops *x* and *y* force reactions, and at *A* by a smooth thrust bearing, which develops *x*, *y*, and *z* force reactions. If torque **M** is applied along rod *AB*, determine the components of reaction at the bearings when the assembly has angular velocity ω at the instant shown.

Given:



$$\gamma = 5\ \frac{\text{lb}}{\text{ft}} \qquad a = 4\ \text{ft} \qquad d = 2\ \text{ft}$$

$$M = 50\ \text{lb}\cdot\text{ft} \qquad b = 2\ \text{ft} \qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

$$\omega = 10\ \frac{\text{rad}}{\text{s}} \qquad c = 2\ \text{ft}$$

Solution:

$$\rho = \frac{\gamma}{g}$$

$$I_{yz} = \rho c b \frac{c}{2} + \rho d c\left(b + \frac{d}{2}\right)$$

$$I_{zz} = \rho c \frac{c^2}{3} + \rho d c^2$$

$$I_{xx} = \rho(a+b)\frac{(a+b)^2}{3} + \rho c \frac{c^2}{12} + \rho c\left[b^2 + \left(\frac{c}{2}\right)^2\right] + \rho d \frac{d^2}{12} + \rho d\left[c^2 + \left(b + \frac{d}{2}\right)^2\right]$$

$$I_{yy} = \rho(a+b)\frac{(a+b)^2}{3} + \rho c b^2 + \rho d \frac{d^2}{12} + \rho d\left(b + \frac{d}{2}\right)^2$$

$$\mathbf{I_{mat}} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix} \qquad \mathbf{I_{mat}} = \begin{pmatrix} 16.98 & 0.00 & 0.00 \\ 0.00 & 15.32 & -2.48 \\ 0.00 & -2.48 & 1.66 \end{pmatrix} \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Guesses $A_x = 1\ \text{lb}$ $A_y = 1\ \text{lb}$ $A_z = 1\ \text{lb}$ $B_x = 1\ \text{lb}$ $B_y = 1\ \text{lb}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} - \begin{bmatrix} 0 \\ 0 \\ \rho(a+b+c+d)g \end{bmatrix} = \begin{pmatrix} -\rho c \frac{c}{2}\alpha - \rho d c \alpha \\ -\rho c \frac{c}{2}\omega^2 - \rho d c \omega^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ a+b \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{c}{2} \\ b \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho c g \end{pmatrix} \ldots = \mathbf{I_{mat}} \begin{pmatrix} 0 \\ 0 \\ \alpha \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_{mat}} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \right]$$
$$+ \begin{pmatrix} 0 \\ c \\ b + \frac{d}{2} \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -\rho d g \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ B_x \\ B_y \\ \alpha \end{pmatrix} = \mathrm{Find}(A_x, A_y, A_z, B_x, B_y, \alpha)$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} -15.6 \\ -46.8 \\ 50.0 \end{pmatrix} \mathrm{lb} \qquad \begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} -12.5 \\ -46.4 \end{pmatrix} \mathrm{lb}$$

$$\alpha = 30.19 \frac{\mathrm{rad}}{\mathrm{s}^2}$$

***Problem 21-52**

The rod *AB* supports the sphere of weight *W*. If the rod is pinned at *A* to the vertical shaft which is rotating at a constant rate $\omega\mathbf{k}$, determine the angle θ of the rod during the motion. Neglect the mass of the rod in the calculation.

Given:

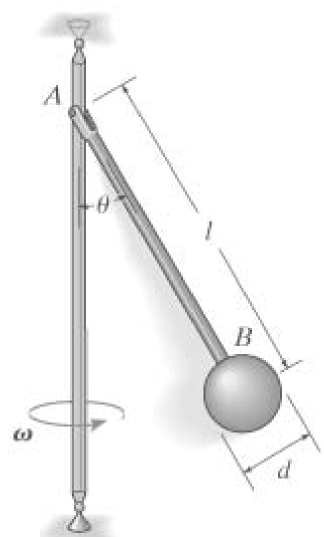
$W = 10\ \mathrm{lb}$

$\omega = 7\ \frac{\mathrm{rad}}{\mathrm{s}}$

$d = 0.5\ \mathrm{ft}$

$l = 2\ \mathrm{ft}$

$g = 32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}$



Solution:

$$I_3 = \frac{2}{5}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2$$

$$I_1 = I_3 + \left(\frac{W}{g}\right) l^2$$

Guess $\theta = 50\ \text{deg}$

Given $W\,l\sin(\theta) = -(I_3 - I_1)\,\omega\cos(\theta)\,\omega\sin(\theta)$

$\theta = \text{Find}(\theta)$ $\theta = 70.8\,\text{deg}$

Problem 21-53

The rod *AB* supports the sphere of weight *W*. If the rod is pinned at *A* to the vertical shaft which is rotating with angular acceleration $\alpha\,\mathbf{k}$, and at the instant shown the shaft has an angular velocity $\omega\mathbf{k}$, determine the angle θ of the rod during the motion. Neglect the mass of the rod in the calculation.

Given:

$W = 10\ \text{lb}$

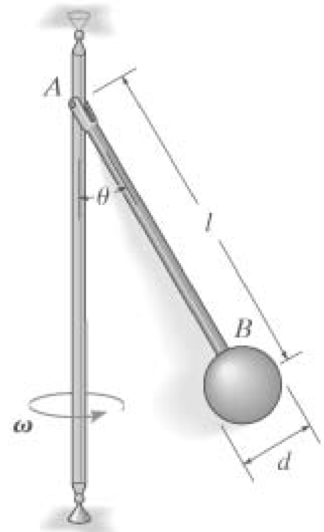
$\alpha = 2\ \dfrac{\text{rad}}{\text{s}^2}$

$\omega = 7\ \dfrac{\text{rad}}{\text{s}}$

$d = 0.5\ \text{ft}$

$l = 2\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$I_3 = \frac{2}{5}\left(\frac{W}{g}\right)\left(\frac{d}{2}\right)^2$$

$$I_1 = I_3 + \left(\frac{W}{g}\right)l^2$$

Guess $\theta = 50\ \text{deg}$

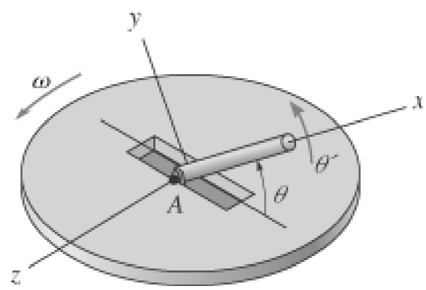
Given $W\,l\sin(\theta) = -(I_3 - I_1)\,\omega\cos(\theta)\,\omega\sin(\theta)$

$\theta = \text{Find}(\theta)$ $\theta = 70.8\,\text{deg}$

Problem 21-54

The *thin* rod has mass m_{rod} and total length L. Only half of the rod is visible in the figure. It is rotating about its midpoint at a constant rate θ', while the table to which its axle A is fastened is rotating at angular velocity ω. Determine the x, y, z moment components which the axle exerts on the rod when the rod is in position θ.

Given:



$$m_{rod} = 0.8 \text{ kg}$$

$$L = 150 \text{ mm}$$

$$\theta' = 6 \frac{\text{rad}}{\text{s}}$$

$$\omega = 2 \frac{\text{rad}}{\text{s}}$$

Solution:

$$I_A = m_{rod}\frac{L^2}{12}$$

$$\boldsymbol{\omega_v} = \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ 0 \end{pmatrix} \times \boldsymbol{\omega_v} = \begin{pmatrix} \omega\theta'\cos(\theta) \\ -\omega\theta'\sin(\theta) \\ 0 \end{pmatrix}$$

$$\mathbf{M} = \mathbf{I_{mat}}\boldsymbol{\alpha} + \boldsymbol{\omega_v} \times \left(\mathbf{I_{mat}}\boldsymbol{\omega_v}\right)$$

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & I_A & 0 \\ 0 & 0 & I_A \end{pmatrix} \times \begin{pmatrix} \omega\theta'\cos(\theta) \\ -\omega\theta'\sin(\theta) \\ 0 \end{pmatrix} + \begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix} \times \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & I_A & 0 \\ 0 & 0 & I_A \end{pmatrix}\begin{pmatrix} \omega\sin(\theta) \\ \omega\cos(\theta) \\ \theta' \end{pmatrix}\right]$$

$$\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 \\ -2I_A\theta'\omega\sin(\theta) \\ \frac{1}{2}I_A\omega^2\sin(2\theta) \end{pmatrix} \qquad \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = \begin{pmatrix} 0 \\ -2I_A\theta'\omega \\ \frac{1}{2}I_A\omega^2 \end{pmatrix}$$

$M_x = 0$

$M_y = k_y \sin(\theta)$ $\quad$ $k_y = -0.036\ \text{N}\cdot\text{m}$

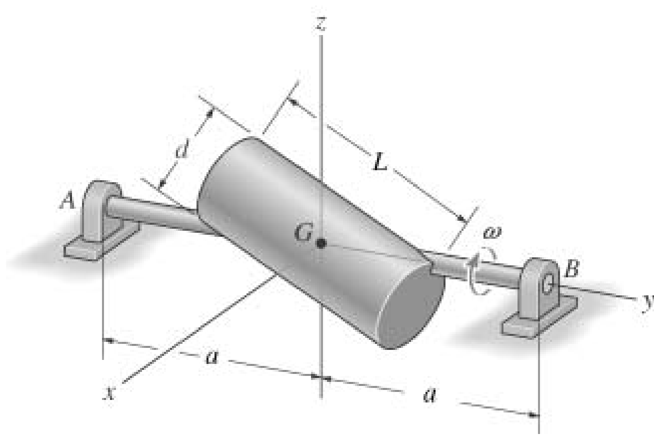
$M_z = k_z \sin(2\ \theta)$ $\quad$ $k_z = 0.0030\ \text{N}\cdot\text{m}$

Problem 21-55

The cylinder has mass m_c and is mounted on an axle that is supported by bearings at A and B. If the axle is turning at $\omega\,\mathbf{j}$, determine the vertical components of force acting at the bearings at this instant.

Units Used:

$\text{kN} = 10^3\ \text{N}$



Given:

$m_C = 30\ \text{kg}$

$a = 1\ \text{m}$

$\omega = -40\ \dfrac{\text{rad}}{\text{s}}$

$d = 0.5\ \text{m}$

$L = 1.5\ \text{m}$

$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$\theta = \operatorname{atan}\left(\frac{d}{L}\right)$$

$$I_{x'} = m_C\frac{L^2}{12} + \frac{m_C}{4}\left(\frac{d}{2}\right)^2 \qquad I_{z'} = I_{x'} \qquad I_{y'} = \frac{m_C}{2}\left(\frac{d}{2}\right)^2$$

$$\mathbf{I_G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix}\begin{pmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Guesses $A_x = 1\ \text{N}$ $A_z = 1\ \text{N}$ $B_x = 1\ \text{N}$ $B_z = 1\ \text{N}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ m_c g \end{pmatrix} = 0 \qquad \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{N} \qquad \begin{pmatrix} A_z \\ B_z \end{pmatrix} = \begin{pmatrix} 1.38 \\ -1.09 \end{pmatrix} \text{kN}$$

***Problem 21-56**

The cylinder has mass m_c and is mounted on an axle that is supported by bearings at A and B. If the axle is subjected to a couple moment M **j** and at the instant shown has an angular velocity ω **j**, determine the vertical components of force acting at the bearings at this instant.

Units Used: $\text{kN} = 10^3\ \text{N}$

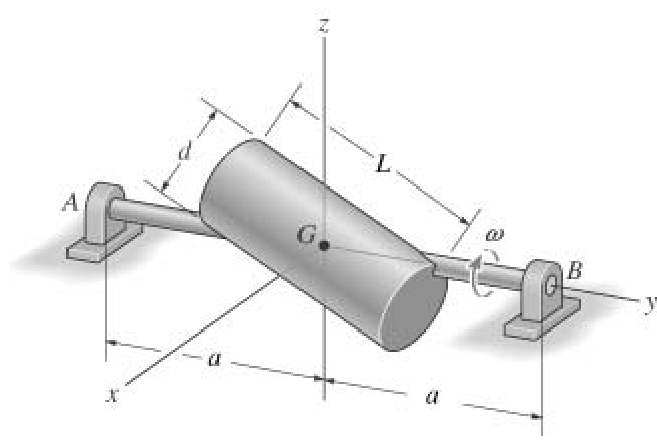
Given:

$m_c = 30\ \text{kg}$ $d = 0.5\ \text{m}$

$a = 1\ \text{m}$ $L = 1.5\ \text{m}$

$\omega = -40\ \dfrac{\text{rad}}{\text{s}}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$M = -30\ \text{N}\cdot\text{m}$



Solution:

$$\theta = \text{atan}\left(\frac{d}{L}\right)$$

$$I_{x'} = m_c \frac{L^2}{12} + \frac{m_c}{4}\left(\frac{d}{2}\right)^2$$

$$I_{z'} = I_{x'} \qquad I_{y'} = \frac{m_c}{2}\left(\frac{d}{2}\right)^2$$

$$\mathbf{I_G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{pmatrix}$$

Guesses $A_x = 1\ \text{N}$ $A_z = 1\ \text{N}$ $B_x = 1\ \text{N}$ $B_z = 1\ \text{N}$ $\alpha = 1\ \frac{\text{rad}}{\text{s}^2}$

Given

$$\begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ m_c g \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 \\ M \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ -a \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ 0 \\ A_z \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ -\omega \\ 0 \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_z \\ B_x \\ B_z \\ \alpha \end{pmatrix} = \text{Find}(A_x, A_z, B_x, B_z, \alpha)$$

$$\begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 15.97 \\ -15.97 \end{pmatrix} \text{N} \qquad \begin{pmatrix} A_z \\ B_z \end{pmatrix} = \begin{pmatrix} 1.38 \\ -1.09 \end{pmatrix} \text{kN}$$

$$\alpha = -20.65\ \frac{\text{rad}}{\text{s}^2}$$

Problem 21-57

The uniform hatch door, having mass M and mass center G, is supported in the horizontal plane by bearings at A and B. If a vertical force **F** is applied to the door as shown, determine the components of reaction at the bearings and the angular acceleration of the door. The bearing at A will resist a component of force in the y direction, whereas the bearing at B will not. For the calculation, assume the door to be a thin plate and neglect the size of each bearing. The door is originally at rest.

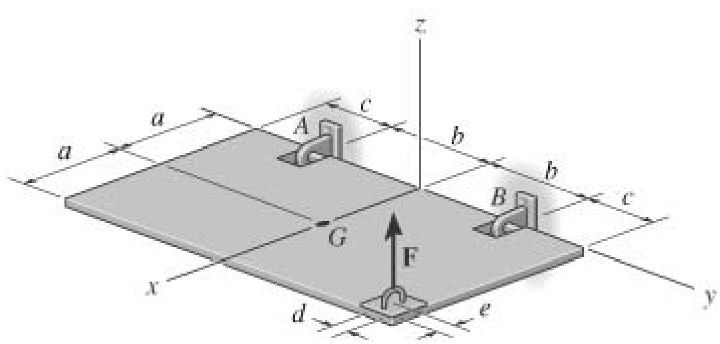
Given:

$M = 15\ \text{kg}$ $c = 100\ \text{mm}$

$F = 300\ \text{N}$ $d = 30\ \text{mm}$

$a = 200\ \text{mm}$ $e = 30\ \text{mm}$

$b = 150\ \text{mm}$ $g = 9.81\ \frac{\text{m}}{\text{s}^2}$



Solution: Guesses

$A_x = 1\ \text{N} \quad A_y = 1\ \text{N} \quad A_z = 1\ \text{N} \quad B_x = 1\ \text{N} \quad B_z = 1\ \text{N} \quad \omega'_y = 1\ \frac{\text{rad}}{\text{s}^2}$

Given $\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F - Mg \end{pmatrix} = M \begin{pmatrix} 0 \\ 0 \\ -\omega'_y\, a \end{pmatrix}$

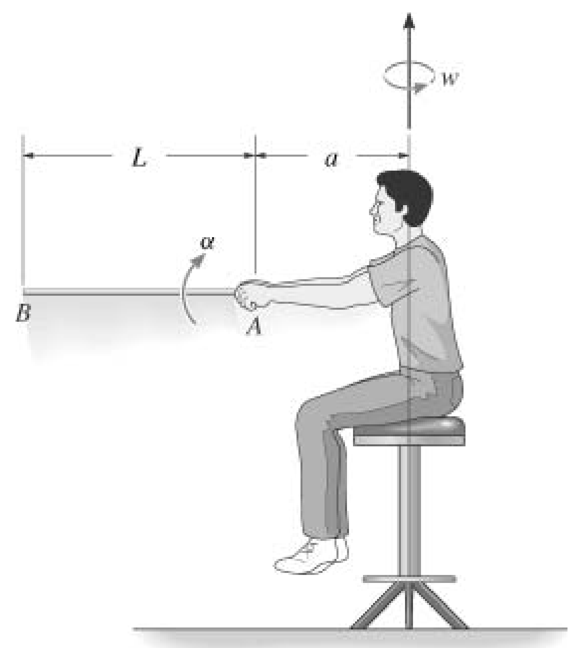
$$\begin{pmatrix} -a \\ b \\ 0 \end{pmatrix} \times \begin{pmatrix} B_x \\ 0 \\ B_z \end{pmatrix} + \begin{pmatrix} -a \\ -b \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} a - e \\ b + c - d \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ F \end{pmatrix} = \begin{bmatrix} 0 \\ \frac{M(2a)^2}{12}\omega'_y \\ 0 \end{bmatrix}$$

$\begin{pmatrix} A_x \\ A_y \\ A_z \\ B_x \\ B_z \\ \omega'_y \end{pmatrix} = \text{Find}(A_x, A_y, A_z, B_x, B_z, \omega'_y)$ $\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 297 \end{pmatrix} \text{N}$ $\begin{pmatrix} B_x \\ B_z \end{pmatrix} = \begin{pmatrix} 0 \\ -143 \end{pmatrix} \text{N}$

$\omega'_y = -102\ \frac{\text{rad}}{\text{s}^2}$

Problem 21-58

The man sits on a swivel chair which is rotating with constant angular velocity ω. He holds the uniform rod AB of weight W horizontal. He suddenly gives it an angular acceleration α measured relative to him, as shown. Determine the required force and moment components at the grip, A, necessary to do this. Establish axes at the rod's center of mass G, with $+z$ upward, and $+y$ directed along the axis of the rod towards A.



Given:

$\omega = 3\ \frac{\text{rad}}{\text{s}}$

$W = 5\ \text{lb}$

$L = 3\ \text{ft}$

$a = 2\ \text{ft}$

$\alpha = 2\ \dfrac{\text{rad}}{\text{s}^2}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution: $I_G = \dfrac{W}{g}\dfrac{L^2}{12}$

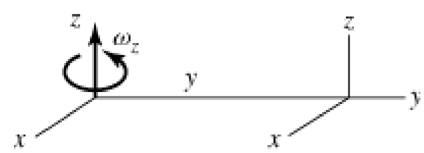
Guesses

$A_x = 1\ \text{lb}$ $A_y = 1\ \text{lb}$ $A_z = 1\ \text{lb}$

$M_x = 1\ \text{lb}\cdot\text{ft}$ $M_y = 1\ \text{lb}\cdot\text{ft}$ $M_z = 1\ \text{lb}\cdot\text{ft}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -W \end{pmatrix} = \frac{W}{g}\begin{bmatrix} 0 \\ \left(a + \frac{L}{2}\right)\omega^2 \\ \frac{L}{2}\alpha \end{bmatrix}$$



$$\begin{pmatrix} 0 \\ \frac{L}{2} \\ 0 \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} I_G & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_G \end{pmatrix}\begin{pmatrix} -\alpha \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\begin{pmatrix} I_G & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_G \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ M_x \\ M_y \\ M_z \end{pmatrix} = \text{Find}\left(A_x, A_y, A_z, M_x, M_y, M_z\right) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 4.89 \\ 5.47 \end{pmatrix}\text{lb} \qquad \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -8.43 \\ 0.00 \\ 0.00 \end{pmatrix}\text{lb}\cdot\text{ft}$$

Problem 21-59

Four spheres are connected to shaft *AB*. If you know m_C and m_E, determine the mass of *D* and *F* and the angles of the rods, θ_D and θ_F so that the shaft is dynamically balanced, that is, so that the bearings at *A* and *B* exert only vertical reactions on the shaft as it rotates. Neglect the mass of the rods.

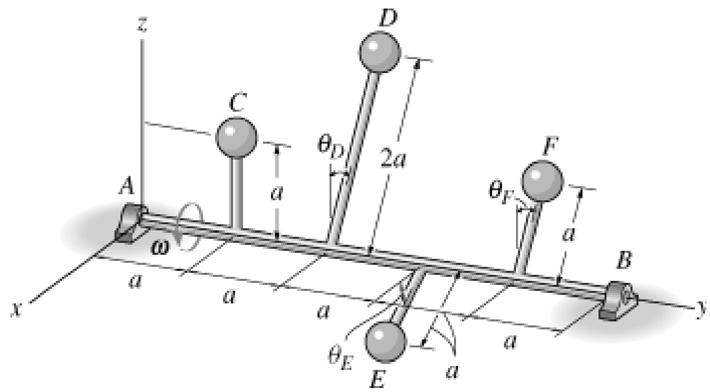
Given:

$m_C = 1\ \text{kg}$

$m_E = 2\ \text{kg}$

$\theta_E = 30\ \text{deg}$

$a = 0.1\ \text{m}$



Solution: We need to put the center of mass along AB and to make the product of inetia go to zero.

Guesses $m_D = 1\ \text{kg}$ $m_F = 1\ \text{kg}$ $\theta_D = 40\ \text{deg}$ $\theta_F = 10\ \text{deg}$

Given

$$m_E a \cos(\theta_E) - m_D(2a)\sin(\theta_D) - m_F a \sin(\theta_F) = 0$$

$$m_C a + m_D(2a)\cos(\theta_D) - m_E a \sin(\theta_E) + m_F a \cos(\theta_F) = 0$$

$$m_C a\, a + m_D(2a)(2a)\cos(\theta_D) - m_E(3a)a\sin(\theta_E) + m_F(4a)a\cos(\theta_F) = 0$$

$$-m_D(2a)(2a)\sin(\theta_D) + m_E(3a)a\cos(\theta_E) - m_F(4a)a\sin(\theta_F) = 0$$

$$\begin{pmatrix} m_D \\ m_F \\ \theta_D \\ \theta_F \end{pmatrix} = \text{Find}(m_D, m_F, \theta_D, \theta_F) \qquad \begin{pmatrix} m_D \\ m_F \end{pmatrix} = \begin{pmatrix} 0.661 \\ 1.323 \end{pmatrix} \text{kg} \qquad \begin{pmatrix} \theta_D \\ \theta_F \end{pmatrix} = \begin{pmatrix} 139.1 \\ 40.9 \end{pmatrix} \text{deg}$$

***Problem 21-60**

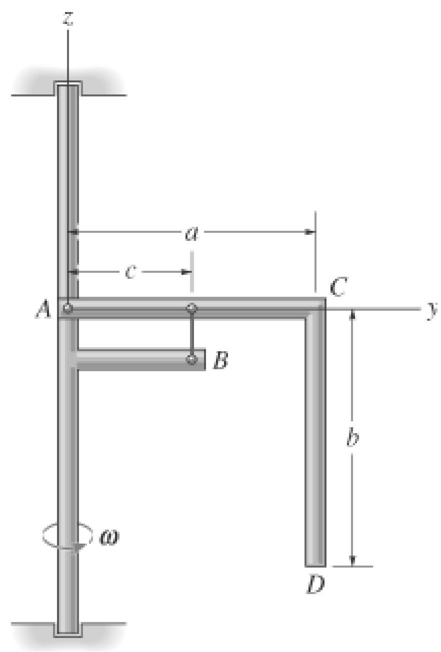
The bent uniform rod ACD has a weight density γ, and is supported at A by a pin and at B by a cord. If the vertical shaft rotates with a constant angular velocity ω, determine the x, y, z components of force and moment developed at A and the tension of the cord.

Given:

$\gamma = 5\ \frac{\text{lb}}{\text{ft}}$

$a = 1\ \text{ft}$

$b = 1\ \text{ft}$



$c = 0.5 \text{ ft}$

$\omega = 20 \frac{\text{rad}}{\text{s}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$

Solution: $\rho = \frac{\gamma}{g}$

$$I_{xx} = \rho a\left(\frac{a^2}{3}\right) + \rho b\left(\frac{b^2}{12}\right) + \rho b\left[a^2 + \left(\frac{b}{2}\right)^2\right]$$

$$I_{yy} = \rho b\left(\frac{b^2}{3}\right)$$

$$I_{zz} = \rho a\left(\frac{a^2}{3}\right) + \rho b a^2$$

$$I_{yz} = -\rho b a \frac{b}{2}$$

$$\mathbf{I_A} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & -I_{yz} \\ 0 & -I_{yz} & I_{zz} \end{pmatrix}$$

Guesses $M_y = 1 \text{ lb}\cdot\text{ft}$ $A_x = 1 \text{ lb}$ $A_z = 1 \text{ lb}$

$M_z = 1 \text{ lb}\cdot\text{ft}$ $A_y = 1 \text{ lb}$ $T = 1 \text{ lb}$

Given

$$\begin{pmatrix} A_x \\ A_y \\ A_z - T \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\gamma(a+b) \end{bmatrix} = \begin{pmatrix} 0 \\ -\rho a \frac{a}{2} \omega^2 - \rho b a \omega^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -\gamma a \frac{a}{2} - \gamma b a - T c \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_A} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}\right]$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ T \\ M_y \\ M_z \end{pmatrix} = \text{Find}(A_x, A_y, A_z, T, M_y, M_z) \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \\ T \end{pmatrix} = \begin{pmatrix} 0.0 \\ -93.2 \\ 57.1 \\ 47.1 \end{pmatrix} \text{lb} \qquad \begin{pmatrix} M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{lb}\cdot\text{ft}$$

Problem 21-61

Show that the angular velocity of a body, in terms of Euler angles ϕ, θ and ψ may be expressed as $\boldsymbol{\omega} = (\phi' \sin\theta \sin\psi + \theta' \cos\psi)\, \mathbf{i} + (\phi' \sin\theta \cos\psi - \theta' \sin\psi)\, \mathbf{j} + (\phi' \cos\theta + \psi')\, \mathbf{k}$, where **i**, **j**, and **k** are directed along the *x, y, z* axes as shown in Fig. 21-15*d*.

Solution:

From Fig. 21 - 15*b*, due to rotation ϕ, the *x, y, z* components of ϕ' are simply ϕ' along *z* axis

From Fig. 21 - 15*c*, due to rotation θ, the *x, y, z* components of ϕ' and θ' are $\phi' \sin\theta$ in the *y* direction, $\phi' \cos\theta$ in the *z* direction, and θ' in the *x* direction.

Lastly, rotation ψ, Fig 21 - 15*d*, produces the final components which yields

$$\boldsymbol{\omega} = (\phi' \sin(\theta)\sin(\psi) + \theta' \cos(\psi))\mathbf{i} + (\phi' \sin(\theta)\cos(\psi) - \theta' \sin(\psi))\mathbf{j} + (\phi' \cos(\theta) + \psi')\mathbf{k}$$

Q.E.D

Problem 21-62

A thin rod is initially coincident with the Z axis when it is given three rotations defined by the Euler angles ϕ, θ, and ψ. If these rotations are given in the order stated, determine the coordinate direction angles α, β, γ of the axis of the rod with respect to the *X*, *Y*, and *Z* axes. Are these directions the same for any order of the rotations? Why?

Given:

$\phi = 30$ deg

$\theta = 45$ deg

$\psi = 60$ deg

Solution:

$$\mathbf{u} = \begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

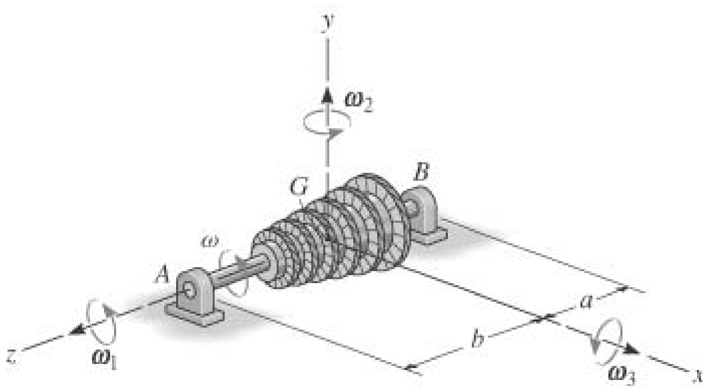
$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \text{acos}(\mathbf{u}) \qquad \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 69.3 \\ 127.8 \\ 45.0 \end{pmatrix} \text{deg}$$

The last rotation (ψ) does not affect the result because the rod just spins around its own axis.

The order of application of the rotations does affect the final result since rotational position is not a vector quantity.

Problem 21-63

The turbine on a ship has mass M and is mounted on bearings A and B as shown. Its center of mass is at G, its radius of gyration is k_z, and $k_x = k_y$. If it is spinning at angular velocity ω, determine the vertical reactions at the bearings when the ship undergoes each of the following motions: (a) rolling ω_1, (b) turning ω_2, (c) pitching ω_3.



Units Used:

$\text{kN} = 1000 \text{ N}$

Given:

$M = 400 \text{ kg}$ $\qquad k_x = 0.5 \text{ m}$

$\omega = 200 \dfrac{\text{rad}}{\text{s}}$ $\qquad k_z = 0.3 \text{ m}$

$\omega_1 = 0.2 \dfrac{\text{rad}}{\text{s}}$ $\qquad a = 0.8 \text{ m}$

$\omega_2 = 0.8 \dfrac{\text{rad}}{\text{s}}$ $\qquad b = 1.3 \text{ m}$

$\omega_3 = 1.4 \dfrac{\text{rad}}{\text{s}}$

Solution:

$$\mathbf{I_G} = M\begin{pmatrix} k_x^2 & 0 & 0 \\ 0 & k_x^2 & 0 \\ 0 & 0 & k_z^2 \end{pmatrix}$$

Guesses $\quad A_x = 1 \text{ N} \quad A_y = 1 \text{ N}$

$B_x = 1 \text{ N} \quad B_y = 1 \text{ N}$

(a) Rolling Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \frac{\text{rad}}{\text{s}^2} + \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

(b) Turning Given

$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} \omega \omega_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} 0 \\ \omega_2 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.00 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} -1.25 \\ 5.17 \end{pmatrix} \text{kN}$$

(c) Pitching Given

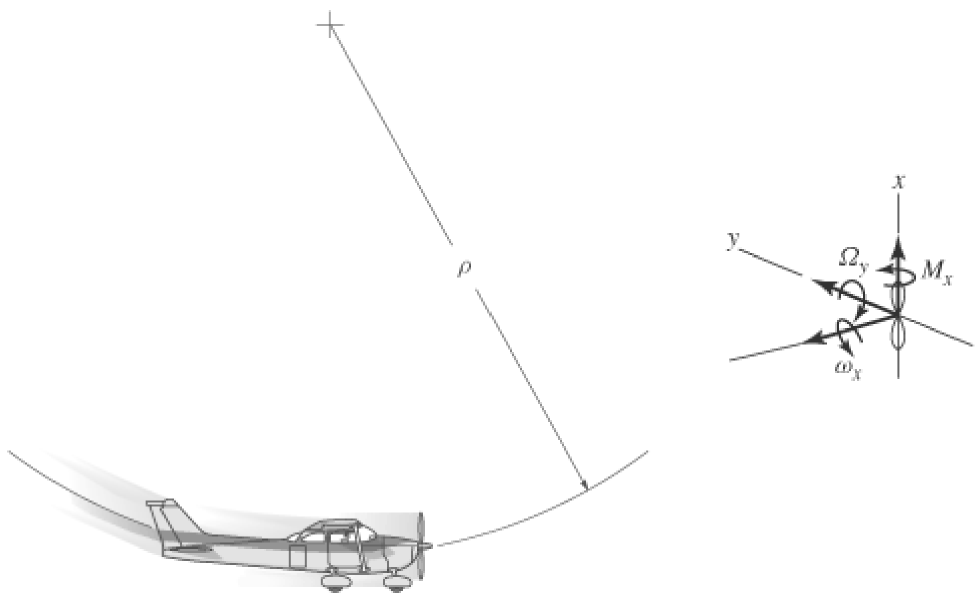
$$\begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -Mg \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{N}$$

$$\begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a \end{pmatrix} \times \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \mathbf{I_G} \begin{pmatrix} 0 \\ -\omega\,\omega_3 \\ 0 \end{pmatrix} + \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \times \left[\mathbf{I_G} \begin{pmatrix} \omega_3 \\ 0 \\ \omega \end{pmatrix} \right]$$

$$\begin{pmatrix} A_x \\ A_y \\ B_x \\ B_y \end{pmatrix} = \text{Find}(A_x, A_y, B_x, B_y) \qquad \begin{pmatrix} A_x \\ B_x \end{pmatrix} = \begin{pmatrix} -4.80 \\ 4.80 \end{pmatrix} \text{kN} \qquad \begin{pmatrix} A_y \\ B_y \end{pmatrix} = \begin{pmatrix} 1.50 \\ 2.43 \end{pmatrix} \text{kN}$$

***Problem 21-64**

An airplane descends at a steep angle and then levels off horizontally to land. If the propeller is turning clockwise when observed from the rear of the plane, determine the direction in which the plane tends to turn as caused by the gyroscopic effect as it levels off.



Solution:

As noted on the diagram M_x represents the effect of the plane on the propeller. The opposite effect occurs on the plane. Hence, the plane tends to **turn to the right when viewed from above.**

Problem 21-65

The propeller on a single-engine airplane has a mass M and a centroidal radius of gyration k_G computed about the axis of spin. When viewed from the front of the airplane, the propeller is turning clockwise at ω_s about the spin axis. If the airplane enters a vertical curve having a radius ρ and is traveling at speed v, determine the gyroscopic bending moment which the propeller exerts on the bearings of the engine when the airplane is in its lowest position.

Given:

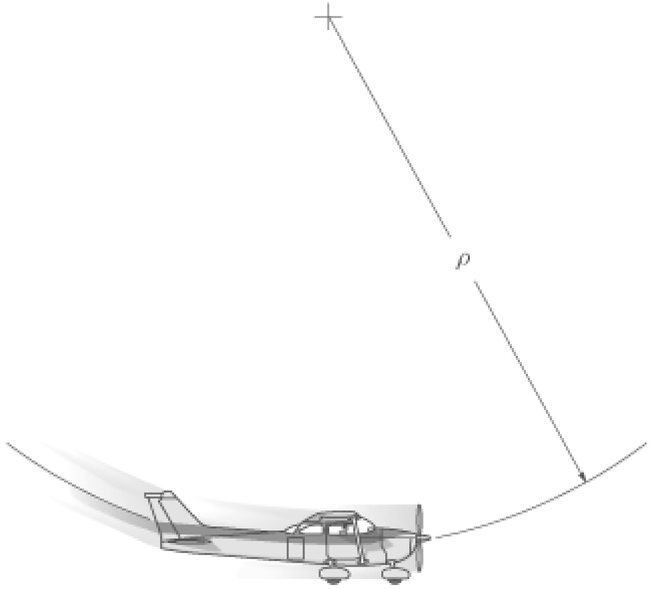
$M = 15\ \text{kg}$

$k_G = 0.3\ \text{m}$

$v = 200\ \dfrac{\text{km}}{\text{hr}}$

$\omega_s = 350\ \dfrac{\text{rad}}{\text{s}}$

$\rho = 80\ \text{m}$



Solution:

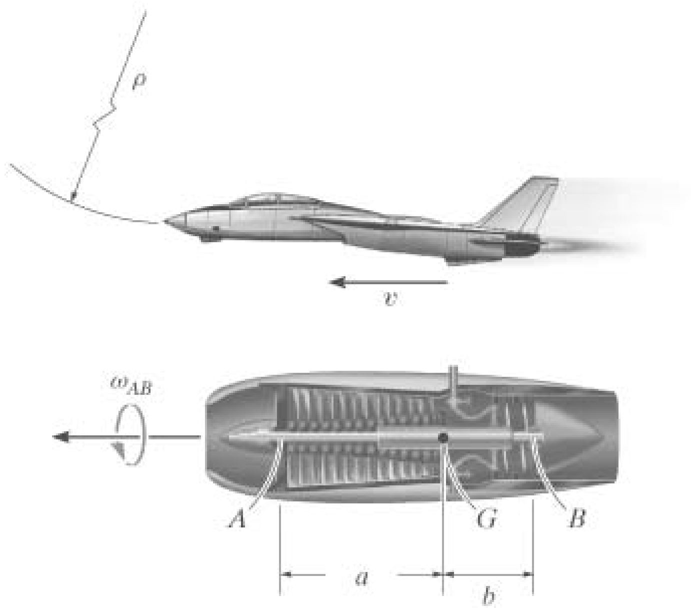
$$\Omega_y = \frac{v}{\rho}$$

$$M_z = \left(M k_G^{\,2}\right)\Omega_y\omega_s$$

$$\boxed{M_z = 328\,\text{N}\cdot\text{m}}$$

Problem 21-66

The rotor assembly on the engine of a jet airplane consists of the turbine, drive shaft, and compressor. The total mass is m_r, the radius of gyration about the shaft axis is k_{AB}, and the mass center is at G. If the rotor has an angular velocity ω_{AB}, and the plane is pulling out of a vertical curve while traveling at speed v, determine the components of reaction at the bearings A and B due to the gyroscopic effect.



Units Used:

$\text{kN} = 10^3\ \text{N}$

Given:

$m_r = 700\ \text{kg}$

$k_{AB} = 0.35\ \text{m}$

$\omega_{AB} = 1000\ \dfrac{\text{rad}}{\text{s}}$

$\rho = 1.30\ \text{km}$

$a = 0.8\ \text{m}$

$b = 0.4\ \text{m}$

$v = 250\ \dfrac{\text{m}}{\text{s}}$

Solution: $M = m_r k_{AB}{}^2 \omega_{AB} \dfrac{v}{\rho}$

Guesses $A = 1\ \text{N}$ $B = 1\ \text{N}$

Given $A a - B b = M$ $A + B = 0$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$ $\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 13.7 \\ -13.7 \end{pmatrix} \text{kN}$

Problem 21-67

A motor has weight W and has radius of gyration k_z about the z axis. The shaft of the motor is supported by bearings at A and B, and is turning at a constant rate $\omega_s = \omega_z\mathbf{k}$, while the frame has an angular velocity of $\omega_y = \omega_y\mathbf{j}$. Determine the moment which the bearing forces at A and B exert on the shaft due to this motion.

Given:

$W = 50\ \text{lb}$

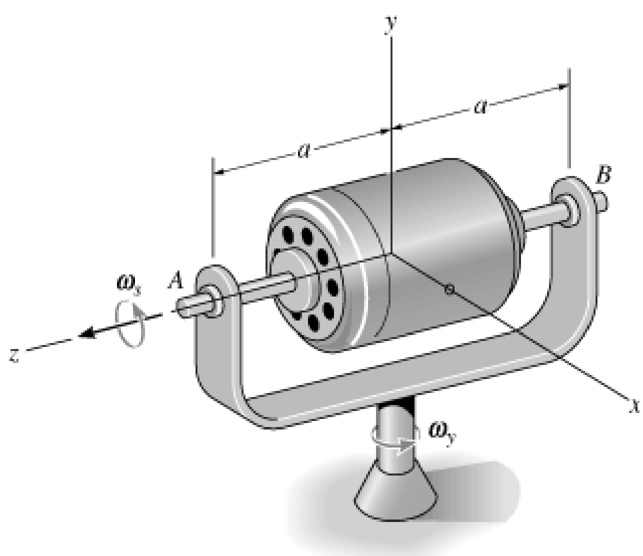
$k_z = 0.2\ \text{ft}$

$\omega_z = 100\ \dfrac{\text{rad}}{\text{s}}$

$\omega_y = 2\ \dfrac{\text{rad}}{\text{s}}$

$a = 0.5\ \text{ft}$

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

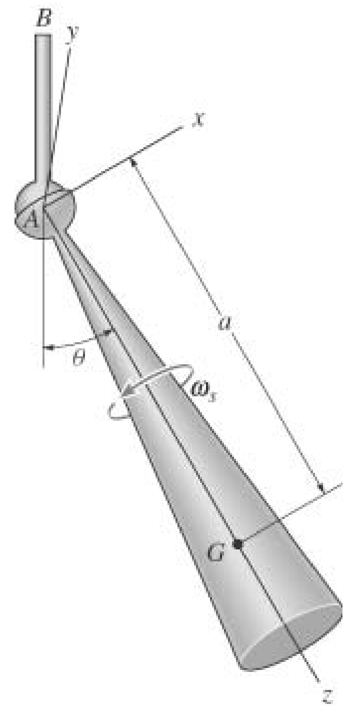


Solution:

$$\mathbf{M} = \begin{pmatrix} 0 \\ \omega_y \\ 0 \end{pmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{W}{g}\right) k_z{}^2 \omega_z \end{bmatrix} \qquad \mathbf{M} = \begin{pmatrix} 12.4 \\ 0.0 \\ 0.0 \end{pmatrix} \text{lb}\cdot\text{ft}$$

***Problem 21-68**

The conical top has mass M, and the moments of inertia are $I_x = I_y$ and I_z. If it spins freely in the ball-and-socket joint at A with angular velocity ω_s compute the precession of the top about the axis of the shaft AB.



Given:

$M = 0.8\ \text{kg}$ $\qquad a = 100\ \text{mm}$

$I_x = 3.5\ 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\qquad \theta = 30\ \text{deg}$

$I_z = 0.8 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ $\qquad g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

$\omega_s = 750\ \dfrac{\text{rad}}{\text{s}}$

Solution: Using Eq. 21-30.

$$\Sigma M_x = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)\left(\phi' \cos(\theta) + \psi'\right)$$

Guess $\quad \phi' = 1\ \dfrac{\text{rad}}{\text{s}}$

Given $\quad M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)\left(\phi' \cos(\theta) + \omega_s\right)$

$\phi' = \text{Find}(\phi')$ $\qquad \phi' = 1.31\ \dfrac{\text{rad}}{\text{s}}$ $\qquad$ low precession

Guess $\quad \phi' = 200\ \dfrac{\text{rad}}{\text{s}}$

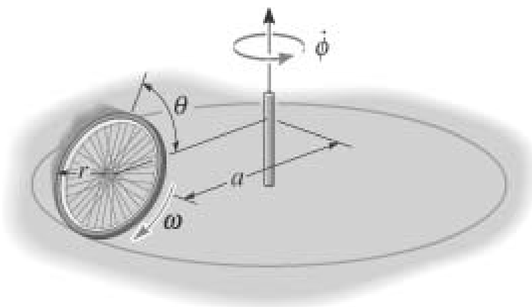
Given $\quad M g \sin(\theta) a = -I_x \phi'^2 \sin(\theta)\cos(\theta) + I_z \phi' \sin(\theta)\left(\phi' \cos(\theta) + \omega_s\right)$

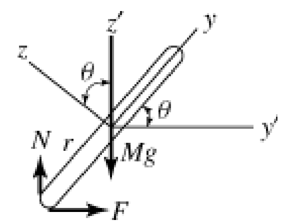
$\phi' = \text{Find}(\phi')$ $\qquad \phi' = 255\ \dfrac{\text{rad}}{\text{s}}$ $\qquad$ high precession

---

Problem 21-69

A wheel of mass m and radius r rolls with constant spin ω about a circular path having a radius a. If the angle of inclination is θ, determine the rate of precession. Treat the wheel as a thin ring. No slipping occurs.





Solution:

Since no sipping occurs, $r\psi' = a + r\cos(\theta)\phi'$ $\psi' = \left(\frac{a + r\cos(\theta)}{r}\right)\phi'$

Also, $\omega = \phi' + \psi'$ $F = m\left(a\phi'^2\right)$ $N - mg = 0$

$$I_x = I_y = \frac{mr^2}{2} \qquad I_z = mr^2$$

$$\omega = \phi' \sin(\theta)\mathbf{j} + (-\psi' + \phi' \cos(\theta)\mathbf{k}$$

Thus, $\omega_x = 0$ $\omega_y = \phi' \sin(\theta)$ $\omega_z = -\psi' + \phi' \cos(\theta)$

$$\omega' = \phi' \times \psi' = -\phi' \psi' \sin(\theta)$$

$$\omega'_x = -\phi' \psi' \sin(\theta) \qquad \omega'_y = \omega'_z = 0$$

Applying

$$\Sigma M_x = I_x\omega'_x + (I_z - I_y)\omega_z\omega_y$$

$$F r\sin(\theta) - N r\cos(\theta) = \frac{mr^2}{2}(-\phi' \psi' \sin(\theta)) + \left(mr^2 - \frac{mr^2}{2}\right)(-\psi' + \phi' \cos(\theta))(\phi' \sin(\theta))$$

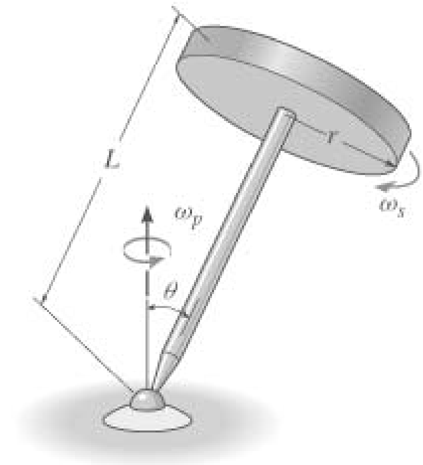
Solving we find

$$m a\phi'^2 r\sin(\theta) - m g r\cos(\theta) = \left(\frac{-mr^2}{2}\right)\phi'^2 \sin(\theta)\left(\frac{a + r\cos(\theta)}{r}\right) - \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right)\phi'^2 \sin(\theta)$$

$$2g\cos(\theta) = a\phi'^2 \sin(\theta) + r\phi'^2 \sin(\theta)\cos(\theta) \qquad \phi' = \sqrt{\frac{2g\cot(\theta)}{a + r\cos(\theta)}}$$

Problem 21-70

The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.



Given:

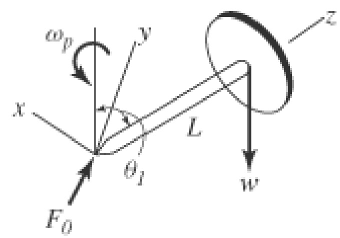
$W = 8$ lb $\quad \theta = 40$ deg

$r = 0.3$ ft $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5$ ft $\quad \omega_s = 300\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\quad \omega_p = 1\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p{}^2 \sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)\left(\omega_p \cos(\theta) + \omega_s\right)$$

$\omega_p = \text{Find}(\omega_p)$ $\quad \omega_p = 1.21\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ low precession

Guess $\quad \omega_p = 70\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ Given

$$W L \sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p{}^2 \sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)\left(\omega_p \cos(\theta) + \omega_s\right)$$

$\omega_p = \text{Find}(\omega_p)$ $\quad \omega_p = 76.3\ \dfrac{\text{rad}}{\text{s}}$ $\quad$ high precession

---

Problem 21-71

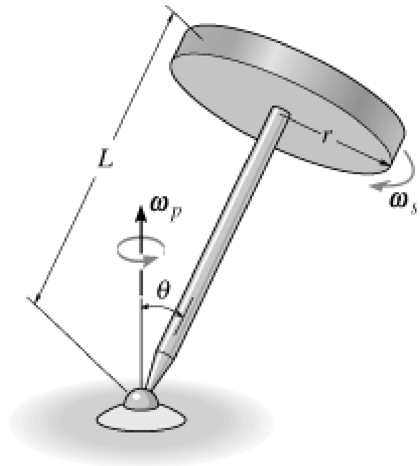
The top consists of a thin disk that has weight W and radius r. The rod has a negligible mass and length L. If the top is spinning with an angular velocity ω_s, determine the steady-state precessional angular velocity ω_p.

Given:

$W = 8$ lb $\quad \theta = 90$ deg

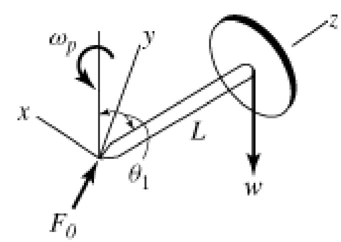
$r = 0.3$ ft $\quad g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$

$L = 0.5$ ft $\quad \omega_s = 300 \dfrac{\text{rad}}{\text{s}}$



Solution:

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)(\phi' \cos(\theta) + \psi')$$



Guess $\quad \omega_p = 1 \dfrac{\text{rad}}{\text{s}} \quad$ Given

$$W\,L\sin(\theta) = -\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{4}\right) + \left(\frac{W}{g}\right)L^2\right]\omega_p^{\,2}\sin(\theta)\cos(\theta) + \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\omega_p \sin(\theta)(\omega_p \cos(\theta) + \omega_s)$$

$\omega_p = \text{Find}(\omega_p) \qquad \omega_p = 1.19 \dfrac{\text{rad}}{\text{s}}$

***Problem 21-72**

The top has weight *W* and can be considered as a solid cone. If it is observed to precess about the vertical axis at a constant rate of ω_y, determine its spin ω_s.

Given:

$W = 3$ lb

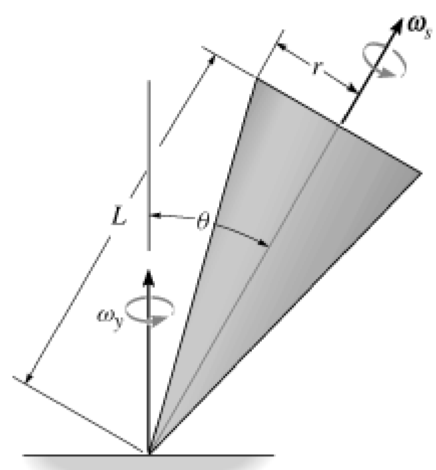
$\omega_y = 5 \dfrac{\text{rad}}{\text{s}}$

$\theta = 30$ deg

$L = 6$ in

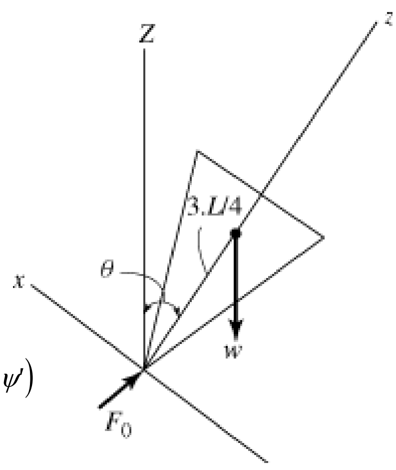
$r = 1.5$ in

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$I = \frac{3}{80}\left(\frac{W}{g}\right)\left(4r^2 + L^2\right) + \left(\frac{W}{g}\right)\left(\frac{3L}{4}\right)^2$$



$$I_z = \frac{3}{10}\left(\frac{W}{g}\right)r^2$$

$$\Sigma M_x = -I\,\phi'^2 \sin(\theta)\cos(\theta) + I_z\phi' \sin(\theta)\left(\phi' \cos(\theta) + \psi'\right)$$

$$W\frac{3L}{4}\sin(\theta) = -I\,\omega_y^2 \sin(\theta)\cos(\theta) + I_z\omega_y \sin(\theta)\left(\omega_y \cos(\theta) + \psi'\right)$$

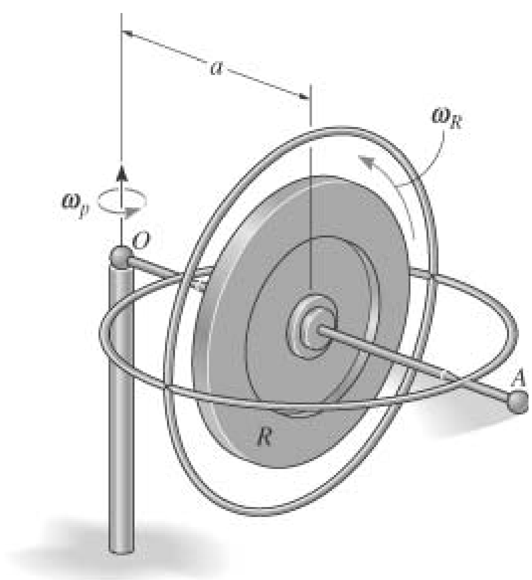
$$\psi' = \frac{1}{4}\left(\frac{3\,WL + 4I\,\omega_y^2\cos(\theta) - 4I_z\,\omega_y^2\cos(\theta)}{I_z\omega_y}\right)$$

$$\psi' = 652\,\frac{\text{rad}}{\text{s}}$$

Problem 21-73

The toy gyroscope consists of a rotor R which is attached to the frame of negligible mass. If it is observed that the frame is precessing about the pivot point O at rate ω_p determine the angular velocity ω_R of the rotor. The stem OA moves in the horizontal plane. The rotor has mass M and a radius of gyration k_{OA} about OA.

Given:



$$\omega_p = 2\,\frac{\text{rad}}{\text{s}}$$

$$M = 200\text{ gm}$$

$$k_{OA} = 20\text{ mm}$$

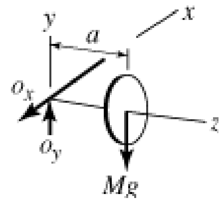
$$a = 30\text{ mm}$$

$$g = 9.81\,\frac{\text{m}}{\text{s}^2}$$

Solution:

$$\Sigma M_x = I_z\Omega_y\omega_z$$

$$M g a = M k_{OA}{}^2 \omega_p \omega_R$$



$$\omega_R = \frac{g a}{k_{OA}{}^2 \omega_p}$$

$$\omega_R = 368 \frac{\text{rad}}{\text{s}}$$

Problem 21-74

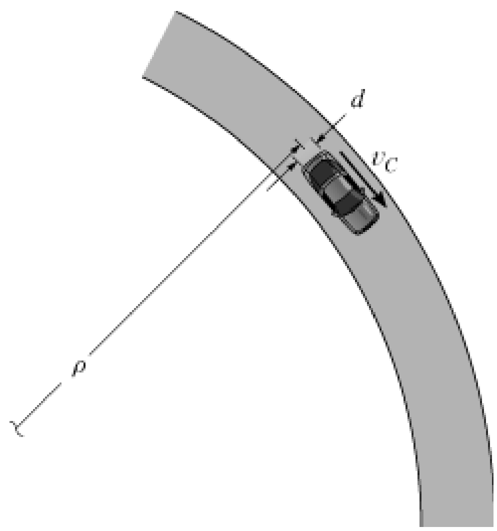
The car is traveling at velocity v_c around the horizontal curve having radius ρ. If each wheel has mass M, radius of gyration k_G about its spinning axis, and radius r, determine the difference between the normal forces of the rear wheels, caused by the gyroscopic effect. The distance between the wheels is d.

Given:

$v_c = 100 \frac{\text{km}}{\text{hr}}$ $\quad k_G = 300 \text{ mm}$

$\rho = 80 \text{ m}$ $\quad r = 400 \text{ mm}$

$M = 16 \text{ kg}$ $\quad d = 1.3 \text{ m}$



Solution:

$I = 2M k_G{}^2$ $\quad I = 2.88 \text{ kg}\cdot\text{m}^2$

$\omega_s = \frac{v_c}{r}$ $\quad \omega_s = 69.44 \frac{\text{rad}}{\text{s}}$

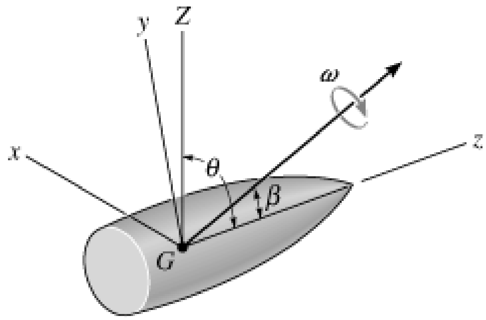
$\omega_p = \frac{v_c}{\rho}$ $\quad \omega_p = 0.35 \frac{\text{rad}}{\text{s}}$

$M = I\omega_s\omega_p$

$\Delta F\, d = I\omega_s\omega_p$ $\quad \Delta F = I\,\omega_s \frac{\omega_p}{d}$ $\quad \Delta F = 53.4 \text{ N}$

Problem 21-75

The projectile shown is subjected to torque-free motion. The transverse and axial moments of inertia are I and I_z respectively. If θ represents the angle between the precessional axis Z and the axis of symmetry z, and β is the angle between the angular velocity ω and the z axis, show that β and θ are related by the equation $\tan\theta = (I/I_z)\tan\beta$.



Solution:

From Eq. 21-34 $\omega_y = \dfrac{H_G \sin(\theta)}{I}$ and $\omega_z = \dfrac{H_G \cos(\theta)}{I_z}$

Hence $\dfrac{\omega_y}{\omega_z} = \dfrac{I_z}{I}\tan(\theta)$

However, $\omega_y = \omega \sin(\beta)$ and $\omega_z = \omega\cos(\beta)$

$$\frac{\omega_y}{\omega_z} = \tan(\beta) = \frac{I_z}{I}\tan(\theta)$$

$$\tan(\theta) = \frac{I}{I_z}\tan(\beta)$$ Q.E.D

***Problem 21-76**

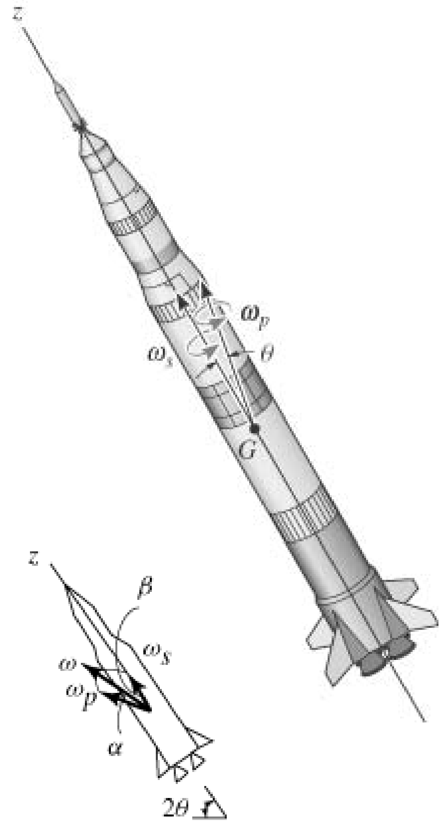
While the rocket is in free flight, it has a spin ω_s and precesses about an axis measured angle θ from the axis of spin. If the ratio of the axial to transverse moments of inertia of the rocket is r, computed about axes which pass through the mass center G, determine the angle which the resultant angular velocity makes with the spin axis. Construct the body and space cones used to describe the motion. Is the precession regular or retrograde?

Given:

$\omega_s = 3\ \dfrac{\text{rad}}{\text{s}}$

$\theta = 10\ \text{deg}$

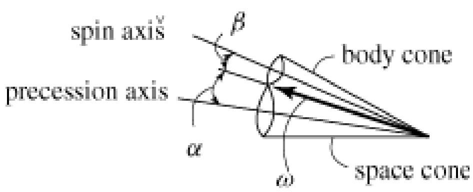
$r = \frac{1}{15}$



Solution:

Determine the angle β from the result of prob.21-75

$$\tan(\theta) = \frac{\tan(\beta)}{r}$$



$\beta = \operatorname{atan}(r\tan(\theta))$ $\quad\quad$ $\beta = 0.673\,\text{deg}$

Thus,

$\alpha = \theta - \beta$ $\quad\quad$ $\alpha = 9.33\,\text{deg}$

Regular Precession $\quad$ Since $\quad I_z < I$

Problem 21-77

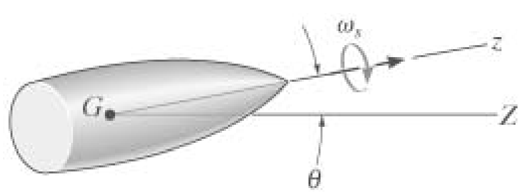
The projectile has a mass M and axial and transverse radii of gyration k_z and k_t, respectively. If it is spinning at ω_s when it leaves the barrel of a gun, determine its angular momentum. Precession occurs about the Z axis.

Given:

$M = 0.9\ \text{kg}$ $\quad\quad$ $\omega_s = 6\ \frac{\text{rad}}{\text{s}}$

$k_z = 20\ \text{mm}$ $\quad\quad$ $\theta = 10\ \text{deg}$

$k_t = 25\ \text{mm}$



Solution:

$I = M k_t^2$ $\quad\quad$ $I = 5.625 \times 10^{-4}\,\text{kg}\cdot\text{m}^2$

$I_z = M k_z^2$ $\quad\quad$ $I_z = 3.600 \times 10^{-4}\,\text{kg}\cdot\text{m}^2$

$\psi = \omega_s$

$$\psi = \left(\frac{I - I_z}{I\,I_z}\right) H_G \cos(\theta)$$

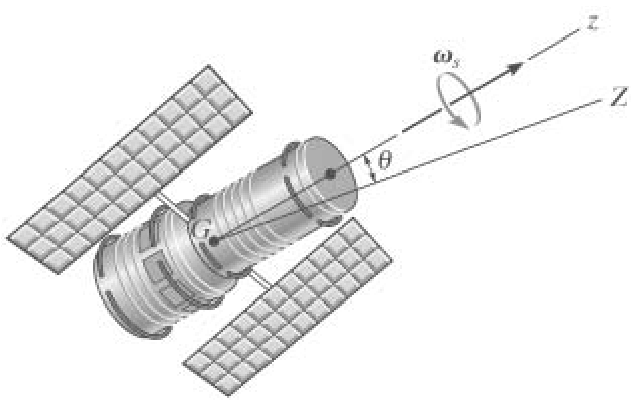
$$H_G = \psi I\left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 6.09 \times 10^{-3}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

Problem 21-78

The satellite has mass M, and about axes passing through the mass center G the axial and transverse radii of gyration are k_z and k_t, respectively. If it is spinning at ω_s when it is launched, determine its angular momentum. Precession occurs about the Z axis.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



Given:

$M = 1.8\ \text{Mg}$ $\qquad \omega_s = 6\ \frac{\text{rad}}{\text{s}}$

$k_z = 0.8\ \text{m}$

$k_t = 1.2\ \text{m}$ $\qquad \theta = 5\ \text{deg}$

Solution:

$I = M k_t^2 \qquad I = 2592\,\text{kg}\cdot\text{m}^2$

$I_z = M k_z^2 \qquad I_z = 1152\,\text{kg}\cdot\text{m}^2$

$\psi' = \omega_s$

$$\psi' = \left(\frac{I - I_z}{I\,I_z}\right) H_G \cos(\theta)$$

$$H_G = \psi' I\left[\frac{I_z}{\cos(\theta)(I - I_z)}\right] \qquad H_G = 12.5\,\text{Mg}\cdot\frac{\text{m}^2}{\text{s}}$$

Problem 21-79

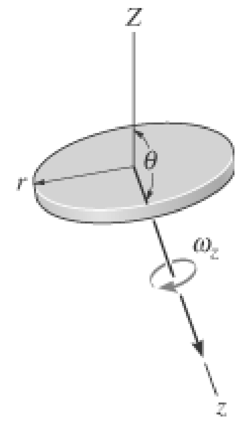
The disk of mass M is thrown with a spin ω_z. The angle θ is measured as shown. Determine the precession about the Z axis.

Given: $M = 4\ \text{kg}$

$\theta = 160\ \text{deg}$

$r = 125\ \text{mm}$

$\omega_z = 6\ \dfrac{\text{rad}}{\text{s}}$



Solution:

$$I = \frac{1}{4}M r^2 \qquad I_z = \frac{1}{2}M r^2$$

Applying Eq.21 - 36

$$\psi' = \omega_z = \frac{I - I_z}{I I_z} H_G \cos(\theta)$$

$$H_G = \omega_z \frac{I I_z}{\cos(\theta)(I - I_z)} \qquad H_G = 0.1995\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\phi' = \frac{H_G}{I} \qquad \phi' = 12.8\ \frac{\text{rad}}{\text{s}}$$

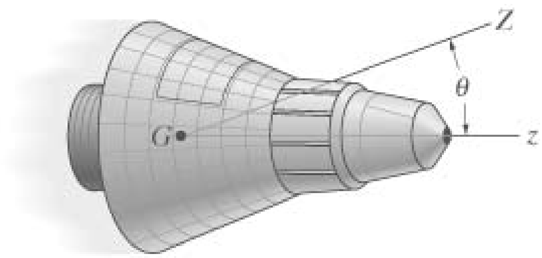
Note that this is a case of retrograde precession since $I_z > I$

***Problem 21-80**

The radius of gyration about an axis passing through the axis of symmetry of the space capsule of mass M is k_z, and about any transverse axis passing through the center of mass G, is k_t. If the capsule has a known steady-state precession of two revolutions per hour about the Z axis, determine the rate of spin about the z axis.

Units Used:

$\text{Mg} = 10^3\ \text{kg}$



Given:

$M = 1.6\ \text{Mg}$

$k_z = 1.2\ \text{m}$

$k_t = 1.8 \text{ m}$

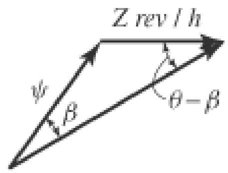
$\theta = 20 \text{ deg}$

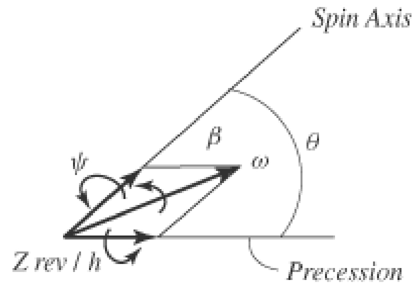
Solution:

$I = M k_t^2$

$I_z = M k_z^2$

Using the Eqn.





$$\tan(\theta) = \left(\frac{I}{I_z}\right)\tan(\beta)$$

$$\beta = \operatorname{atan}\left(\tan(\theta)\frac{I_z}{I}\right)$$

$$\beta = 9.19 \text{ deg}$$