Problem 22-1

When a load of weight W_1 is suspended from a spring, the spring is stretched a distance d. Determine the natural frequency and the period of vibration for a load of weight W_2 attached to the same spring.

Given: $W_1 = 20\ \text{lb}$ $\quad W_2 = 10\ \text{lb}$ $\quad d = 4\ \text{in}$

Solution:

$$k = \frac{W_1}{d} \qquad k = 60.00\ \frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{k}{\frac{W_2}{g}}} \qquad \omega_n = 13.89\ \frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.45\ \text{s}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 2.21\ \frac{1}{\text{s}}$$

Problem 22-2

A spring has stiffness k. If a block of mass M is attached to the spring, pushed a distance d above its equilibrium position, and released from rest, determine the equation which describes the block's motion. Assume that positive displacement is measured downward.

Given: $k = 600\ \frac{\text{N}}{\text{m}}$ $\quad M = 4\ \text{kg}$ $\quad d = 50\ \text{mm}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 12.2\ \frac{\text{rad}}{\text{s}}$$

$v = 0 \quad x = -d \quad$ at $\quad t = 0$

$$x = A\sin(\omega_n t) + B\cos(\omega_n t) \qquad A = 0 \qquad B = -d$$

Thus,

$$x = B\cos(\omega_n t) \qquad B = -0.05\ \text{m} \qquad \omega_n = 12.2\ \frac{\text{rad}}{\text{s}}$$

Problem 22-3

When a block of mass m_1 is suspended from a spring, the spring is stretched a distance δ. Determine the natural frequency and the period of vibration for a block of mass m_2 attached to the same spring.

Given: $m_1 = 3\ \text{kg}$ $m_2 = 0.2\ \text{kg}$ $\delta = 60\ \text{mm}$ $g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$

Solution:

$$k = \frac{m_1 g}{\delta} \qquad k = 490.50\ \frac{\text{N}}{\text{m}}$$

$$\omega_n = \sqrt{\frac{k}{m_2}} \qquad \omega_n = 49.52\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 7.88\ \text{Hz}$$

$$r = \frac{1}{f} \qquad r = 0.127\ \text{s}$$

***Problem 22-4**

A block of mass M is suspended from a spring having a stiffness k. If the block is given an upward velocity v when it is distance d above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the block measured from the equilibrium position. Assume that the positive displacement is measured downward.

Given: $M = 8\ \text{kg}$ $k = 80\ \dfrac{\text{N}}{\text{m}}$ $v = 0.4\ \dfrac{\text{m}}{\text{s}}$ $d = 90\ \text{mm}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 3.16\ \frac{\text{rad}}{\text{s}}$$

$$x = A\sin(\omega_n)t + B\cos(\omega_n)t$$

$$B = -d \qquad A = \frac{-v}{\omega_n}$$

$$x = A\sin(\omega_n)t + B\cos(\omega_n)t \qquad A = -0.13\ \text{m} \qquad B = -0.09\ \text{m} \qquad \omega_n = 3.16\ \frac{\text{rad}}{\text{s}}$$

$x_{max} = \sqrt{A^2 + B^2}$ $\quad$ $x_{max} = 0.16\ \text{m}$

Problem 22-5

A weight *W* is suspended from a spring having a stiffness *k*. If the weight is pushed distance *d* upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

Given: $\quad W = 2\ \text{lb} \quad k = 2\ \dfrac{\text{lb}}{\text{in}} \quad d = 1\ \text{in} \quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$$\omega_n = \sqrt{\frac{k}{\frac{W}{g}}} \qquad \omega_n = 19.7\ \frac{\text{rad}}{\text{s}}$$

$$y = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$A = -d \qquad B = 0\ \text{in}$$

$$y = A\cos(\omega_n t) \qquad A = -0.08\ \text{ft} \qquad \omega_n = 19.7\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 3.13\ \text{Hz}$$

$$C = \sqrt{A^2 + B^2} \qquad C = 1.00\ \text{in}$$

Problem 22-6

A weight *W* is suspended from a spring having a stiffness *k*. If the weight is given an upward velocity of *v* when it is distance *d* above its equilibrium position, determine the equation which describes the motion and the maximum upward displacement of the weight, measured from the equilibrium position. Assume positive displacement is downward.

Given:

$W = 6\ \text{lb}$

$k = 3\ \dfrac{\text{lb}}{\text{in}}$

$$v = 20\ \frac{\text{ft}}{\text{s}}$$

$$d = 2\ \text{in}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad \omega_n = 13.90\ \frac{\text{rad}}{\text{s}} \qquad y = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$A = -d \qquad B = \frac{-v}{\omega_n}$$

$$y = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$A = -0.17\ \text{ft} \qquad B = -1.44\ \text{ft}$$

$$\omega_n = 13.90\ \frac{\text{rad}}{\text{s}}$$

$$C = \sqrt{A^2 + B^2} \qquad C = 1.45\ \text{ft}$$

Problem 22-7

A spring is stretched a distance d by a block of mass M. If the block is displaced a distance b downward from its equilibrium position and given a downward velocity v, determine the differential equation which describes the motion. Assume that positive displacement is measured downward. Use the Runge–Kutta method to determine the position of the block, measured from its unstretched position, at time t_1 (See Appendix B.) Use a time increment Δt.

Given:

$$M = 8\ \text{kg}$$

$$d = 175\ \text{mm}$$

$$b = 100\ \text{mm}$$

$$v = 1.50\ \frac{\text{m}}{\text{s}}$$

$$t_1 = 0.22\ \text{s}$$

$$\Delta t = 0.02\ \text{s}$$

$$g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

$$k = \frac{M g}{d} \qquad \omega_n = \sqrt{\frac{k}{M}}$$

$$y'' + \omega_n^2 y = 0 \qquad \omega_n^2 = 56.1\ \frac{\text{rad}}{\text{s}^2}$$

To numerically integrate in Mathcad we have to switch to nondimensional variables

$$\Omega_n = \omega_n \frac{\text{s}}{\text{rad}} \qquad B = \frac{b}{\text{mm}} \quad V = v\frac{\text{s}}{\text{mm}} \qquad T_1 = \frac{t_1}{\text{s}}$$

Given $\quad y''(t) + \Omega_n^2\, y(t) = 0 \qquad y(0) = B \quad y'(0) = V$

$$y = \text{Odesolve}(t, T_1) \qquad y(T_1) = 192\ \text{mm}$$

***Problem 22-8**

A spring is stretched a distance *d* by a block of mass *M*. If the block is displaced a distance *b* downward from its equilibrium position and given an upward velocity *v*, determine the differential equation which describes the motion. Assume that positive displacement is measured downward. What is the amplitude of the motion?

Given:

$$M = 8\ \text{kg}$$

$$d = 175\ \text{mm}$$

$$b = 60\ \text{mm}$$

$$v = 4\ \frac{\text{m}}{\text{s}}$$

$$t_1 = 0.22\ \text{s}$$

$$\Delta t = 0.02\ \text{s} \qquad g = 9.81\ \frac{\text{m}}{\text{s}^2}$$

Solution:

$$k = \frac{M g}{d} \qquad \omega_n = \sqrt{\frac{k}{M}} \qquad y'' + \omega_n^2 y = 0$$

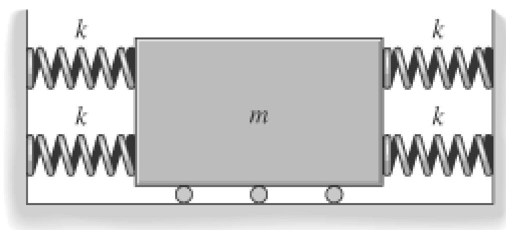
$$A = b \qquad B = \frac{-v}{\omega_n}$$

$$y(t) = A\cos(\omega_n t) + B\sin(\omega_n t) \qquad A = 0.06 \text{ m} \qquad B = -0.53 \text{ m} \qquad \omega_n = 7.49 \frac{\text{rad}}{\text{s}}$$

$$C = \sqrt{A^2 + B^2} \qquad C = 0.54 \text{ m}$$

Problem 22-9

Determine the frequency of vibration for the block. The springs are originally compressed Δ.



Solution:

$$m x'' + 4k x = 0$$

$$x'' + \frac{4k}{m} x = 0$$

$$f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$$

Problem 22-10

A pendulum has a cord of length L and is given a tangential velocity v toward the vertical from a position θ_0. Determine the equation which describes the angular motion.

Given:

$$L = 0.4 \text{ m} \qquad v = 0.2 \frac{\text{m}}{\text{s}}$$

$$\theta_0 = 0.3 \text{ rad} \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$$

Solution:

Since the motion remains small $\quad \omega_n = \sqrt{\dfrac{g}{L}}$

$$\theta = A\sin(\omega_n t) + B\cos(\omega_n t)$$

$A = \frac{-v}{\omega_n L} \qquad B = \theta_0$

$\theta = A\sin(\omega_n t) + B\cos(\omega_n t)$

$A = -0.101\,\text{rad} \qquad B = 0.30\,\text{rad} \qquad \omega_n = 4.95\,\frac{\text{rad}}{\text{s}}$

Problem 22-11

A platform, having an unknown mass, is supported by *four* springs, each having the same stiffness *k*. When nothing is on the platform, the period of vertical vibration is measured as t_1; whereas if a block of mass M_2 is supported on the platform, the period of vertical vibration is t_2. Determine the mass of a block placed on the (empty) platform which causes the platform to vibrate vertically with a period t_3. What is the stiffness *k* of each of the springs?

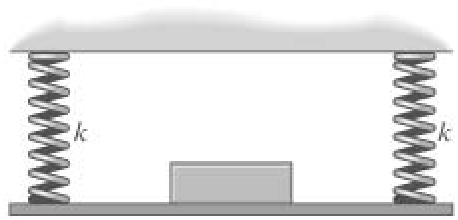
Given:

$M_2 = 3\ \text{kg}$

$t_1 = 2.35\ \text{s}$

$t_2 = 5.23\ \text{s}$

$t_3 = 5.62\ \text{s}$



Solution:

Guesses $M_1 = 1\ \text{kg}$

$M_3 = 1\ \text{kg}$

$k = 1\ \frac{\text{N}}{\text{m}}$

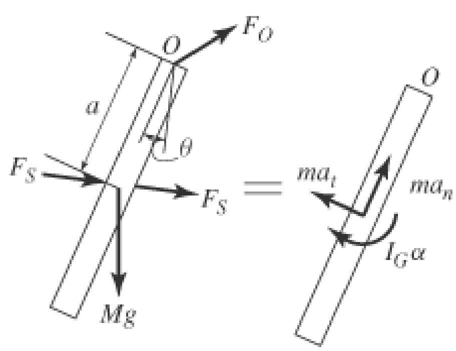
Given $t_1 = 2\pi\sqrt{\frac{M_1}{4k}} \qquad t_2 = 2\pi\sqrt{\frac{M_1 + M_2}{4k}} \qquad t_3 = 2\pi\sqrt{\frac{M_1 + M_3}{4k}}$

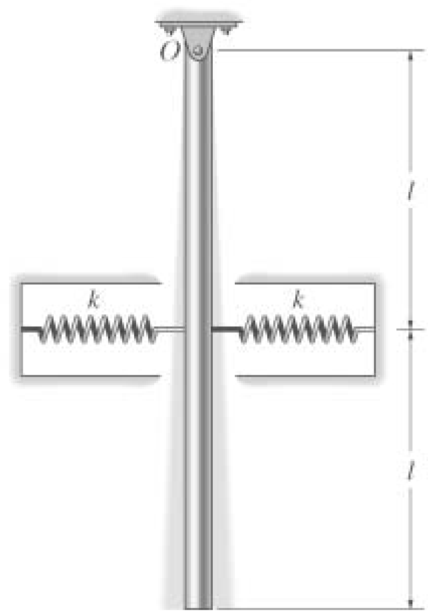
$\begin{pmatrix} M_1 \\ M_3 \\ k \end{pmatrix} = \text{Find}(M_1, M_3, k) \qquad M_1 = 0.759\ \text{kg} \qquad M_3 = 3.58\ \text{kg}$

$k = 1.36\,\frac{\text{N}}{\text{m}}$

***Problem 22-12**

If the lower end of the slender rod of mass M is displaced a small amount and released from rest, determine the natural frequency of vibration. Each spring has a stiffness k and is unstretched when the rod is hanging vertically.





Given:

$M = 30$ kg $\quad l = 1$ m

$k = 500\ \dfrac{\text{N}}{\text{m}}$

Solution:

$$M\,g\,l\sin(\theta) + 2k\,l\sin(\theta)l\cos(\theta) = -M\frac{(2l)^2}{3}\theta''$$

For small angles

$$\frac{4M\,l^2}{3}\theta'' + \left(M\,g\,l + 2k\,l^2\right)\theta = 0$$

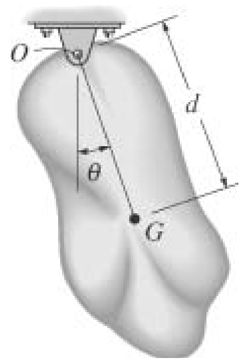
$$\theta'' + \left(\frac{3g}{4l} + \frac{3k}{2M}\right)\theta = 0$$

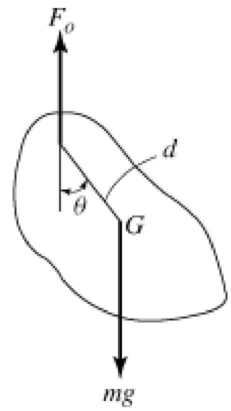
$$\omega_n = \sqrt{\frac{3g}{4l} + \frac{3k}{2M}} \qquad \omega_n = 5.69\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 0.91\ \frac{1}{\text{s}}$$

Problem 22-13

The body of arbitrary shape has a mass m, mass center at G, and a radius of gyration about G of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration.





Solution:

$\curvearrowright + \quad \Sigma M_O = I_O\alpha \qquad -m\,g\,d\sin(\theta) = \left(m\,k_G^2 + m\,d^2\right)\theta''$

$$\theta'' + \left(\frac{g\,d}{k_G^2 + d^2}\right)\sin(\theta) = 0$$

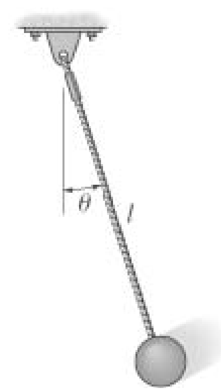
However, for small rotation $\sin(\theta) = \theta$. Hence $\qquad \theta'' + \left(\frac{g\,d}{k_G^2 + d^2}\right)\theta = 0$

From the above differential equation,

$$\omega_n = \sqrt{\frac{g\,d}{k_G^2 + d^2}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{k_G^2 + d^2}{g\,d}}$$

Problem 22-14

Determine to the nearest degree the maximum angular displacement of the bob if it is initially displaced θ_0 from the vertical and given a tangential velocity v away from the vertical.



Given:

$\theta_0 = 0.2 \text{ rad} \qquad v = 0.4 \frac{\text{m}}{\text{s}}$

$l = 0.4 \text{ m}$

Solution:

$$\omega_n = \sqrt{\frac{g}{l}} \qquad A = \theta_0 \qquad B = \frac{v}{l\omega_n}$$

$$\theta(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

$$C = \sqrt{A^2 + B^2} \qquad C = 16\,\text{deg}$$

Problem 22-15

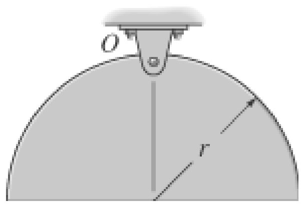
The semicircular disk has weight W. Determine the natural period of vibration if it is displaced a small amount and released.

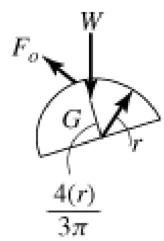
Given:

$W = 20$ lb

$r = 1$ ft

$g = 32.2 \, \frac{\text{ft}}{\text{s}^2}$





Solution:

$$I_O = \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2$$

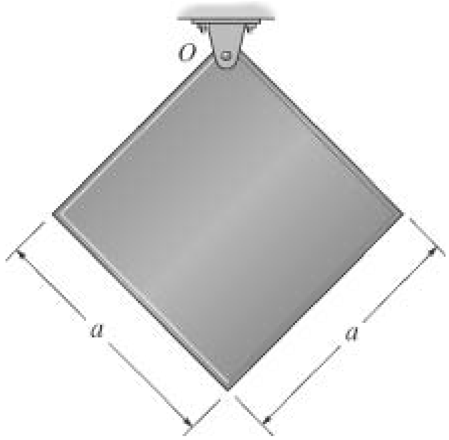
$$-W\left(r - \frac{4r}{3\pi}\right)\sin(\theta) = I_O\theta'' \qquad \theta'' + \left(r - \frac{4r}{3\pi}\right)\left(\frac{W}{I_O}\right)\theta = 0$$

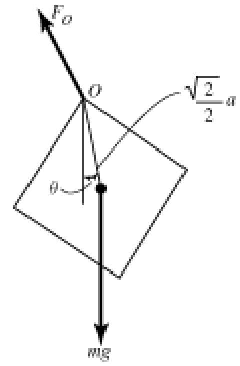
$$\omega_n = \sqrt{\left(r - \frac{4r}{3\pi}\right)\left(\frac{W}{I_O}\right)} \qquad \omega_n = 5.34\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.18\ \text{s}$$

***Problem 22-16**

The square plate has a mass m and is suspended at its corner by the pin O. Determine the natural period of vibration if it is displaced a small amount and released.





Solution:

$$I_O = \frac{2}{3} m a^2 \qquad -mg\left(\frac{\sqrt{2}}{2}a\right)\theta = \left(\frac{2}{3} m a^2\right)\theta'$$

$$\theta' + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0 \qquad \omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4}{3\sqrt{2}}}\sqrt{\frac{a}{g}}$$

$$b = 2\pi\sqrt{\frac{4}{3\sqrt{2}}} \qquad \tau = b\sqrt{\frac{a}{g}} \qquad b = 6.10$$

Problem 22-17

The disk has weight W and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced by rolling it counterclockwise through angle θ_0, determine the equation which describes its oscillatory motion when it is released.

Given:

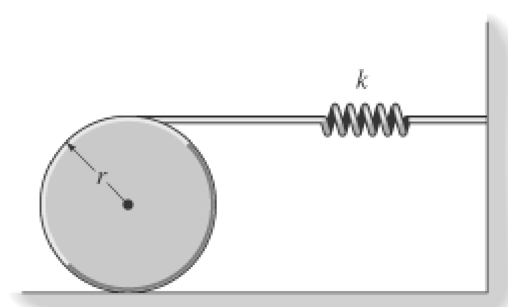
$W = 10$ lb

$\theta_0 = 0.4$ rad

$r = 1$ ft

$k = 100 \frac{\text{lb}}{\text{ft}}$

$g = 32.2 \frac{\text{ft}}{\text{s}^2}$



Solution:

$$-k(2r\theta)\,2r = \left(\frac{W}{g}\frac{r^2}{2} + \frac{W}{g}r^2\right)\theta'' \qquad \theta'' + \frac{8kg}{3W}\theta = 0 \qquad \omega_n = \sqrt{\frac{8kg}{3W}}$$

$$\theta = \theta_0 \cos(\omega_n t) \qquad \theta_0 = 0.40\,\text{rad} \qquad \omega_n = 29.3\,\frac{\text{rad}}{\text{s}}$$

Problem 22-18

The pointer on a metronome supports slider A of weight W, which is positioned at a fixed distance a from the pivot O of the pointer. When the pointer is displaced, a torsional spring at O exerts a restoring torque on the pointer having a magnitude $M = k\theta$ where θ represents the angle of displacement from the vertical. Determine the natural period of vibration when the pointer is displaced a small amount and released. Neglect the mass of the pointer.

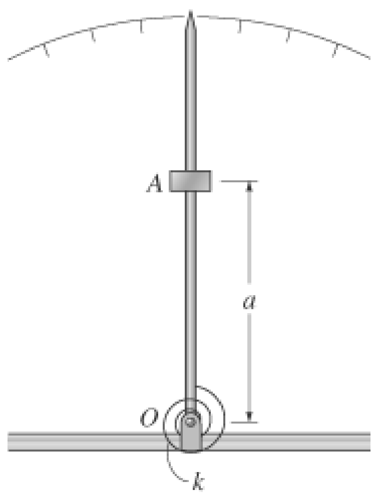
Given:

$W = 0.4$ lb

$k = 1.2\ \text{lb}\ \dfrac{\text{ft}}{\text{rad}}$

$a = 0.25$ ft

$g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$-Wa\theta + k\theta = \left(\frac{-W}{g}\right)a^2\theta'' \qquad \theta'' + \frac{g}{a}\left(\frac{k}{aW} - 1\right)\theta = 0$$

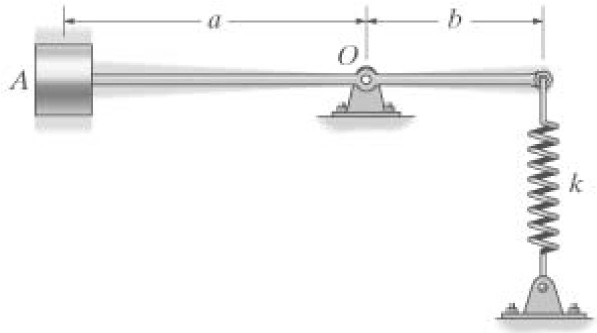
$$\omega_n = \sqrt{\frac{g}{a}\left(\frac{k}{aW} - 1\right)} \qquad \omega_n = 37.64\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.167\ \text{s}$$

Problem 22-19

The block has a mass *m* and is supported by a rigid bar of negligible mass. If the spring has a stiffness *k*, determine the natural period of vibration for the block.

Solution:



$$-k\,b\,\theta b = m\,a\,\theta'\,a$$

$$m\,a^2\,\theta' + k\,b^2\,\theta = 0$$

$$\theta' + \left(\frac{k\,b^2}{m\,a^2}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}\left(\frac{b}{a}\right)$$

$$\tau = 2\pi\left(\frac{a}{b}\right)\sqrt{\frac{m}{k}}$$

***Problem 22-20**

The disk, having weight *W*, is pinned at its center *O* and supports the block *A* that has weight W_A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.

Given:

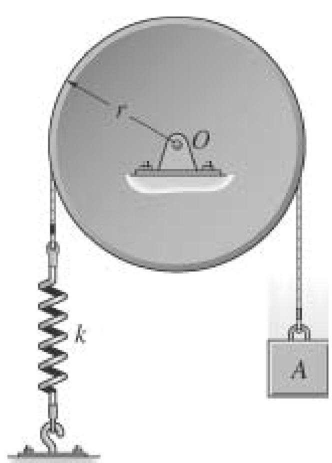
$W = 15\ \text{lb}$

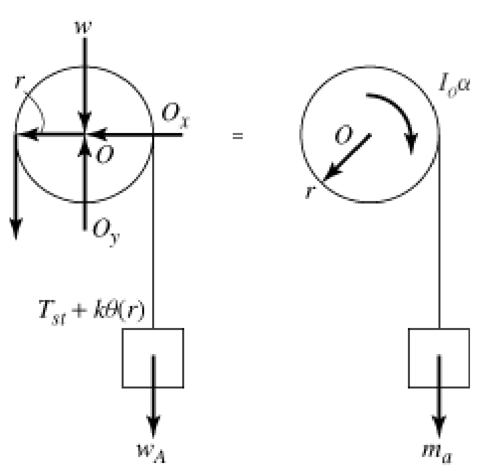
$W_A = 3\ \text{lb}$

$k = 80\ \frac{\text{lb}}{\text{ft}}$

$r = 0.75\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$k\,r\theta r = \left(\frac{-W}{g}\right)\left(\frac{r^2}{2}\right)\theta' - \left(\frac{W_A}{g}\right)r\theta'\,r$$

$$\frac{r^2}{g}\left(W_A + \frac{W}{2}\right)\theta' + k\,r^2\theta = 0$$

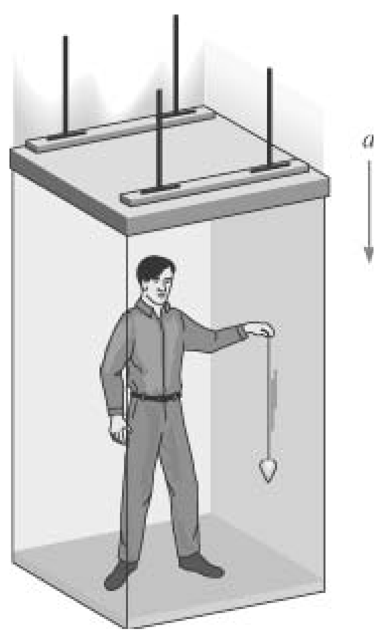
$$\theta' + \frac{k\,g}{W_A + \frac{W}{2}}\theta = 0$$

$$\omega_n = \sqrt{\frac{k\,g}{W_A + \frac{W}{2}}} \qquad \omega_n = 15.66\,\frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.401\ \text{s}$$

Problem 22-21

While standing in an elevator, the man holds a pendulum which consists of cord of length L and a bob of weight W. If the elevator is descending with an acceleration a, determine the natural period of vibration for small amplitudes of swing.



Given:

$$L = 18\ \text{in}$$

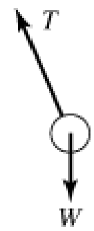
$$W = 0.5\ \text{lb}$$

$$a = 4\ \frac{\text{ft}}{\text{s}^2}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

Since the acceleration of the pendulum is



$$a' = (g - a) \qquad a' = 28.2\ \frac{\text{ft}}{\text{s}^2}$$

Using the result of Example 22-1, we have

$$\omega_n = \sqrt{\frac{a'}{L}} \qquad \omega_n = 4.34\ \frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.45\ \text{s}$$

Problem 22-22

The spool of weight W is attached to two springs. If the spool is displaced a small amount and released, determine the natural period of vibration. The radius of gyration of the spool is k_G. The spool rolls without slipping.

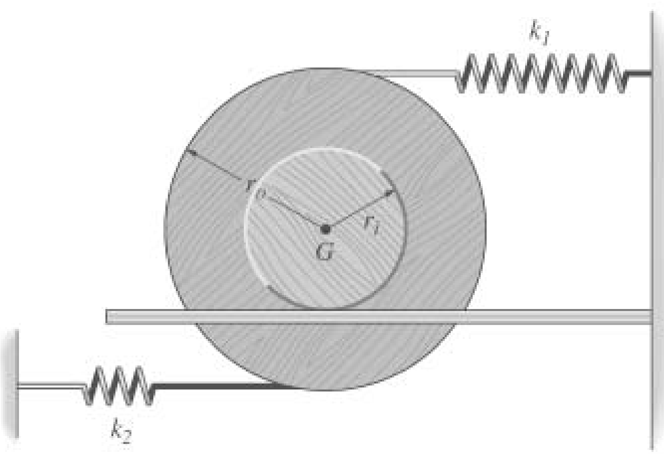
Given:

$W = 50\ \text{lb}$ $\qquad r_i = 1\ \text{ft}$

$k_G = 1.5\ \text{ft}$ $\qquad r_o = 2\ \text{ft}$

$k_1 = 3\ \frac{\text{lb}}{\text{ft}}$ $\qquad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

$k_2 = 1\ \frac{\text{lb}}{\text{ft}}$



Solution:

$$-k_1(r_o + r_i)\theta(r_o + r_i) - k_2(r_o - r_i)\theta(r_o - r_i) = \left(\frac{W}{g}k_G^2 + \frac{W}{g}r_i^2\right)\theta''$$

$$\left(\frac{W}{g}k_G^2 + \frac{W}{g}r_i^2\right)\theta'' + \left[k_1(r_o + r_i)^2 + k_2(r_o - r_i)^2\right]\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{W\left(k_G^2 + r_i^2\right)}\left[k_1(r_o + r_i)^2 + k_2(r_o - r_i)^2\right]} \qquad \omega_n = 2.36\ \frac{\text{rad}}{\text{s}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \boxed{\tau = 2.67\ \text{s}}$$

Problem 22-23

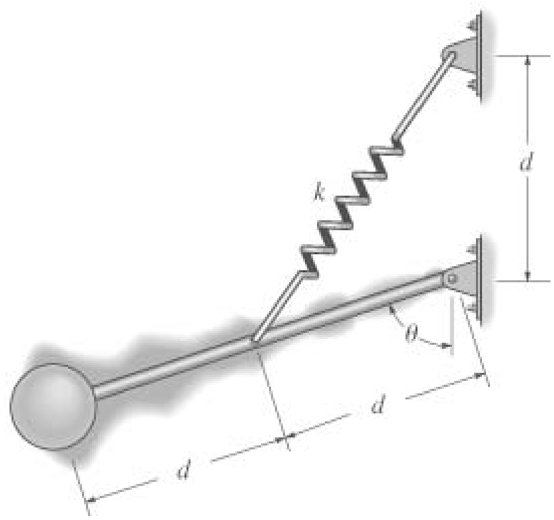
Determine the natural frequency for small oscillations of the sphere of weight W when the rod is displaced a slight distance and released. Neglect the size of the sphere and the mass of the rod. The spring has an unstretched length d.

Given:

$$W = 10\ \text{lb}$$

$$k = 5\ \frac{\text{lb}}{\text{ft}}$$

$$d = 1\ \text{ft}$$



Solution:

Geometry $\quad L = \sqrt{d^2 + d^2 + 2d\,d\cos(\theta)} = d\sqrt{2(1+\cos(\theta))} = 2d\cos\left(\frac{\theta}{2}\right)$

Dynamics $\quad W(2d)\sin(\theta) - k\left(2d\cos\left(\frac{\theta}{2}\right) - d\right)\sin\left(\frac{\theta}{2}\right)d = \left(\frac{-W}{g}\right)(2d)^2\theta''$

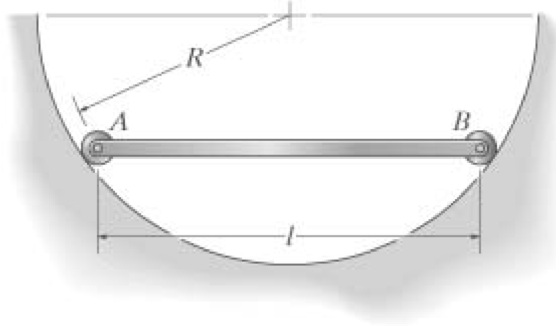
Linearize around $\theta = 0$.

$$\left(\frac{4Wd^2}{g}\right)\theta'' + \left[W(2d) - \frac{kd^2}{2}\right]\theta = 0$$

$$\theta'' + \left(\frac{g}{2d} - \frac{kg}{8W}\right)\theta = 0 \qquad \omega_n = \sqrt{\frac{g}{2d} - \frac{kg}{8W}} \qquad \omega_n = 3.75\ \frac{\text{rad}}{\text{s}}$$

$$f = \frac{\omega_n}{2\pi} \qquad \boxed{f = 0.60\ \frac{1}{\text{s}}}$$

***Problem 22-24**



The bar has length l and mass m. It is supported at its ends by rollers of negligible mass. If it is given a small displacement and released, determine the natural frequency of vibration.

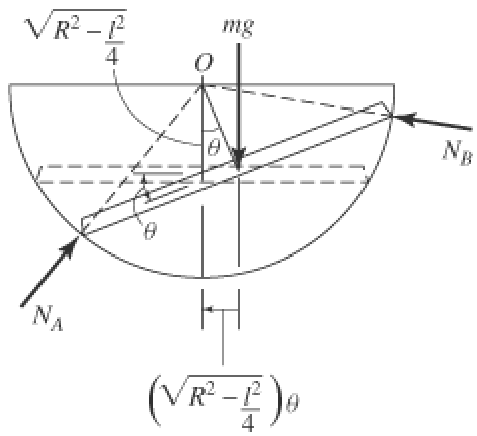
Solution:

Moment of inertia about point O:

$$I_O = \frac{1}{12} m l^2 + m\left(\sqrt{R^2 - \frac{l^2}{4}}\right)^2 = m\left(R^2 - \frac{1}{6}l^2\right)$$

$$m g\left(\sqrt{R^2 - \frac{l^2}{4}}\right)\theta = -m\left(R^2 - \frac{1}{6}l^2\right)\theta''$$

$$\theta'' + \frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}\theta = 0$$



From the above differential equation,

$$p = \sqrt{\frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}}$$

$$f = \frac{p}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3g\sqrt{4R^2 - l^2}}{6R^2 - l^2}}$$

Problem 22-25

The weight W is fixed to the end of the rod assembly. If both springs are unstretched when the assembly is in the position shown, determine the natural period of vibration for the weight when it is displaced slightly and released. Neglect the size of the block and the mass of the rods.

Given:

$$W = 25 \text{ lb}$$

$$k = 2 \frac{\text{lb}}{\text{in}}$$

$$l = 12 \text{ in}$$

$$d = 6 \text{ in}$$

Solution:

$$-Wl\theta - 2kd\theta d = \left(\frac{W}{g}\right) l^2 \theta'$$

$$\left(\frac{W}{g}\right) l^2 \theta' + \left(Wl + 2kd^2\right)\theta = 0$$

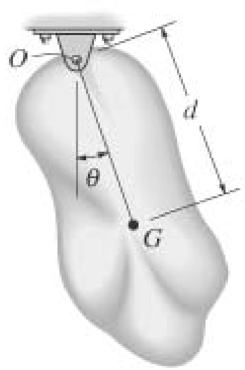
$$\theta' + \left(\frac{g}{l} + \frac{2kgd^2}{Wl^2}\right)\theta = 0$$

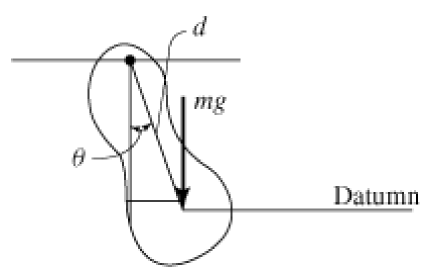
$$\omega_n = \sqrt{\frac{g}{l} + \frac{2kgd^2}{Wl^2}} \qquad T = \frac{2\pi}{\omega_n}$$

$$T = 0.91 \text{ s}$$

Problem 22-26

The body of arbitrary shape has a mass *m*, mass center at *G*, and a radius of gyration about *G* of k_G. If it is displaced a slight amount θ from its equilibrium position and released, determine the natural period of vibration. Solve using energy methods





Solution:

$$T + V = \frac{1}{2}\left(mk_G^{\,2} + md^2\right)\theta^2 + mgd\left(1 - \cos(\theta)\right)$$

$$m\left(k_G^{\,2} + d^2\right)\theta\,\theta' + mgd\left(\sin(\theta)\right)\theta = 0$$

$$\sin(\theta) \approx \theta$$

$$\theta'' + \left(\frac{g\,d}{k_G^{\,2} + d^2}\right)\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n}$$

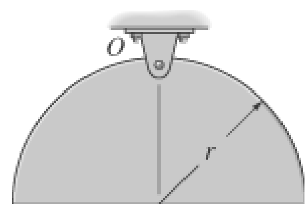
$$\tau = 2\pi\sqrt{\frac{k_G^{\,2} + d^2}{g\,d}}$$

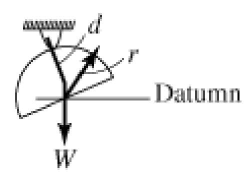
Problem 22-27

The semicircular disk has weight W. Determine the natural period of vibration if it is displaced a small amount and released. Solve using energy methods.

Given:

$W = 20$ lb





$r = 1$ ft

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$$I_O = \left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2$$

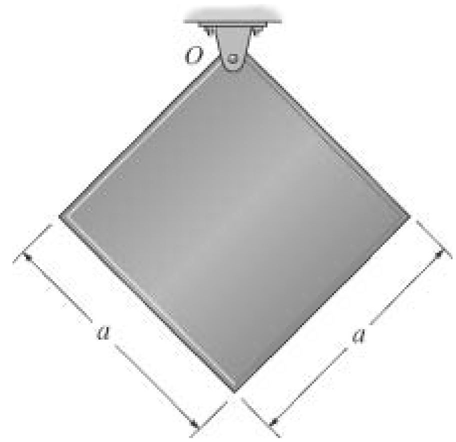
$$T + V = \frac{1}{2}I_O\theta'^2 - W\left(r - \frac{4r}{3\pi}\right)\cos(\theta)$$

$$I_O\theta'' + W\left(r - \frac{4r}{3\pi}\right)\theta = 0 \qquad \theta'' + \frac{W}{I_O}\left(r - \frac{4r}{3\pi}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{W}{I_O}\left(r - \frac{4r}{3\pi}\right)} \qquad \omega_n = 5.34\ \frac{\text{rad}}{\text{s}} \qquad \tau = \frac{2\pi}{\omega_n} \qquad \tau = 1.18\ \text{s}$$

***Problem 22-28**

The square plate has a mass *m* and is suspended at its corner by the pin *O*. Determine the natural period of vibration if it is displaced a small amount and released. Solve using energy methods.



Solution:

$$T + V = \frac{1}{2}\left[\frac{1}{12}m\left(a^2 + a^2\right) + m\left(\frac{a}{\sqrt{2}}\right)^2\right]\theta'^2 + mg\left(\frac{a}{\sqrt{2}}\right)(1 - \cos(\theta))$$

$$\frac{2}{3}ma^2\theta'\theta'' + mg\left(\frac{a}{\sqrt{2}}\right)(\sin(\theta))\theta' = 0$$

$$\theta'' + \left(\frac{3\sqrt{2}g}{4a}\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{3\sqrt{2}g}{4a}} \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4}{3\sqrt{2}}}\sqrt{\frac{a}{g}}$$

$$b = 2\pi\sqrt{\frac{4}{3\sqrt{2}}} \qquad \tau = b\sqrt{\frac{a}{g}}$$

$$b = 6.10$$

Problem 22-29

The disk, having weight *W*, is pinned at its center *O* and supports the block *A* that has weight W_A. If the belt which passes over the disk is not allowed to slip at its contacting surface, determine the natural period of vibration of the system.

Given:

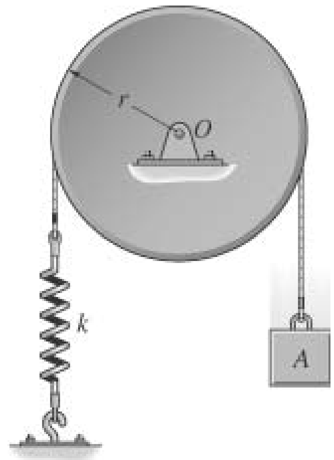
$W = 15\text{ lb}$

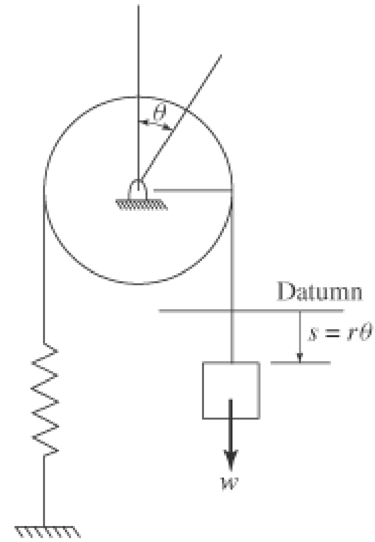
$W_A = 3\text{ lb}$

$k = 80\ \frac{\text{lb}}{\text{ft}}$

$r = 0.75\ \text{ft}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$





Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right)\theta^2 + \frac{1}{2}\left(\frac{W_A}{g}\right)r^2\,\theta^2 + \frac{1}{2}k(r\theta)^2$$

$$\left(\frac{W}{g}\frac{r^2}{2} + \frac{W_A}{g}r^2\right)\theta' + kr^2\theta = 0$$

$$\theta' + \frac{kg}{W_A + \frac{W}{2}}\theta = 0 \qquad \omega_n = \sqrt{\frac{kg}{W_A + \frac{W}{2}}} \qquad \omega_n = 15.66\ \frac{\text{rad}}{\text{s}}$$

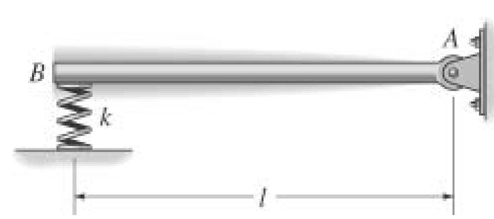
$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.401\ \text{s}$$

Problem 22-30

The uniform rod of mass m is supported by a pin at A and a spring at B. If the end B is given a small downward displacement and released, determine the natural period of vibration.

Solution:

$$T + V = \frac{1}{2}m\left(\frac{l^2}{3}\right)\theta^2 + \frac{1}{2}k(l\theta)^2$$



$$m\frac{l^2}{3}\theta' + kl^2\theta = 0 \qquad \theta' + \frac{3k}{m}\theta = 0$$

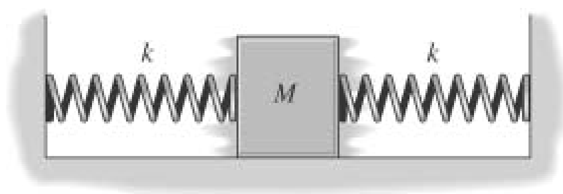
$$\omega_n = \sqrt{\frac{3k}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \boxed{\tau = 2\pi\sqrt{\frac{m}{3k}}}$$

Problem 22-31

Determine the differential equation of motion of the block of mass M when it is displaced slightly and released. The surface is smooth and the springs are originally unstretched.

Given:



$M = 3$ kg

$k = 500 \frac{\text{N}}{\text{m}}$

Solution:

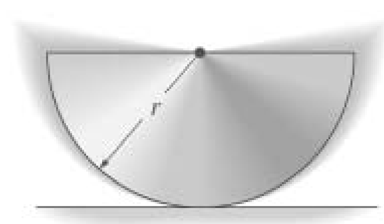
$$T + V = \frac{1}{2}Mx'^2 + 2\frac{1}{2}kx^2$$

$$Mx'' + 2kx = 0 \qquad x'' + \left(\frac{2k}{M}\right)x = 0$$

$$b = \frac{2k}{M} \qquad \boxed{x'' + bx = 0} \qquad \boxed{b = 333 \frac{\text{rad}}{\text{s}^2}}$$

***Problem 22-32**

Determine the natural period of vibration of the semicircular disk of weight W.



Given:

$W = 10$ lb $\qquad r = 0.5$ ft

Solution:

$$T + V = \frac{1}{2}\left[\left(\frac{W}{g}\right)\left(\frac{r^2}{2}\right) - \left(\frac{W}{g}\right)\left(\frac{4r}{3\pi}\right)^2 + \left(\frac{W}{g}\right)\left(r - \frac{4r}{3\pi}\right)^2\right]\theta^2 - W\left(\frac{4r}{3\pi}\right)(1 - \cos(\theta))$$

$$\frac{1}{2}\left(\frac{W}{g}\right)r^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\theta^2 - W\left(\frac{4r}{3\pi}\right)(1 - \cos(\theta)) = 0$$

$$\left(\frac{W}{g}\right)r^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\theta'' + W\left(\frac{4r}{3\pi}\right)\theta = 0$$

$$\theta'' + \frac{4g}{3r\pi\left(\frac{3}{2} - \frac{8}{3\pi}\right)}\theta = 0 \qquad \omega_n = \sqrt{\frac{4g}{3r\pi\left(\frac{3}{2} - \frac{8}{3\pi}\right)}}$$

$$\tau = \frac{2\pi}{\omega_n} \qquad \tau = 0.970 \text{ s}$$

Problem 22-33

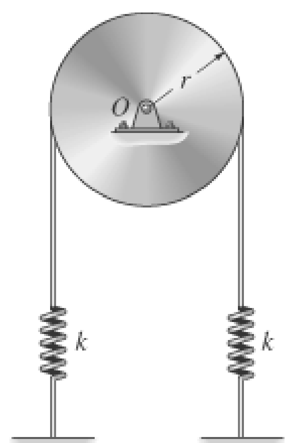
The disk of mass M is pin-connected at its midpoint. Determine the natural period of vibration of the disk if the springs have sufficient tension in them to prevent the cord from slipping on the disk as it oscillates. *Hint:* Assume that the initial stretch in each spring is δ_0. This term will cancel out after taking the time derivative of the energy equation.

Given:

$M = 7 \text{ kg}$

$k = 600 \frac{\text{N}}{\text{m}}$

$r = 100 \text{ mm}$



Solution:

$$T + V = \frac{1}{2}M\left(\frac{r^2}{2}\right)\theta'^2 + \frac{1}{2}k(r\theta + \delta_0)^2 + \frac{1}{2}k(r\theta - \delta_0)^2$$

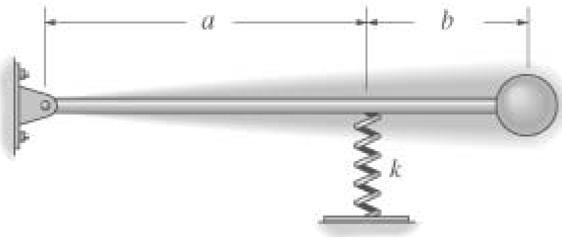
$$M\left(\frac{r^2}{2}\right)\theta'' + 2kr^2\theta = 0 \qquad \theta'' + \left(\frac{4k}{M}\right)\theta = 0$$

$$\tau = 2\pi\sqrt{\frac{M}{4k}} \qquad \tau = 0.339 \text{ s}$$

Problem 22-34

The sphere of weight W is attached to a rod of negligible mass and rests in the horizontal position. Determine the natural frequency of vibration. Neglect the size of the sphere.

Given:



$W = 5\ \text{lb}$ $\qquad a = 1\ \text{ft}$

$k = 10\ \dfrac{\text{lb}}{\text{ft}}$ $\qquad b = 0.5\ \text{ft}$

Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)(a+b)^2\,\theta'^2 + \frac{1}{2}k(a\theta)^2$$

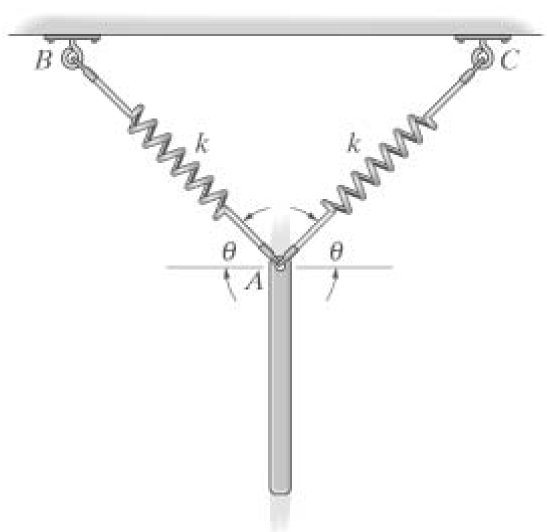
$$\left(\frac{W}{g}\right)(a+b)^2\theta'' + ka^2\theta = 0$$

$$\theta'' + \left[\frac{ka^2 g}{W(a+b)^2}\right]\theta = 0 \qquad \omega_n = \sqrt{\frac{ka^2 g}{W(a+b)^2}}$$

$$f = \frac{\omega_n}{2\pi} \qquad f = 0.85\ \frac{1}{\text{s}}$$

Problem 22-35

The bar has a mass M and is suspended from two springs such that when it is in equilibrium, the springs make an angle θ with the horizontal as shown. Determine the natural period of vibration if the bar is pulled down a short distance and released. Each spring has a stiffness k.



Given:

$M = 8\ \text{kg}$

$\theta = 45\ \text{deg}$

$k = 40\ \dfrac{\text{N}}{\text{m}}$

Solution:

Let $2b$ be the distance between B and C.

$$T + V = \frac{1}{2}My'^2 + \frac{1}{2}(2k)\,\delta^2$$

where

$$\delta = \sqrt{(b\tan(\theta) + y)^2 + b^2} - \sqrt{(b\tan(\theta))^2 + b^2} = \sin(\theta)\, y \qquad \text{for small } y$$

thus

$$\frac{1}{2}M\,y'^2 + k\sin(\theta)^2\,y^2 = 0 \qquad M\,y'' + 2k\sin(\theta)^2 y = 0$$

$$y'' + \left(\frac{2k\sin(\theta)^2}{M}\right)y = 0$$

$$\omega_n = \sqrt{\frac{2k\sin(\theta)^2}{M}} \qquad \tau = \frac{2\pi}{\omega_n} \qquad \tau = 2.81\ \text{s}$$

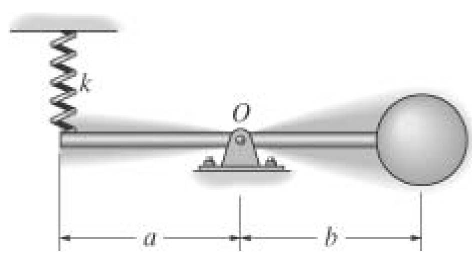
***Problem 22-36**

Determine the natural period of vibration of the sphere of mass M. Neglect the mass of the rod and the size of the sphere.

Given:

$M = 3\ \text{kg}$ $\quad a = 300\ \text{mm}$

$k = 500\ \frac{\text{N}}{\text{m}}$ $\quad b = 300\ \text{mm}$



Solution:

$$T + V = \frac{1}{2}M(b\theta')^2 + \frac{1}{2}k(a\theta)^2$$

$$M b^2\theta'' + k a^2\theta = 0 \qquad \theta'' + \frac{k a^2}{M b^2}\theta = 0$$

$$\tau = 2\pi\sqrt{\frac{M b^2}{k a^2}} \qquad \tau = 0.487\ \text{s}$$

Problem 22-37

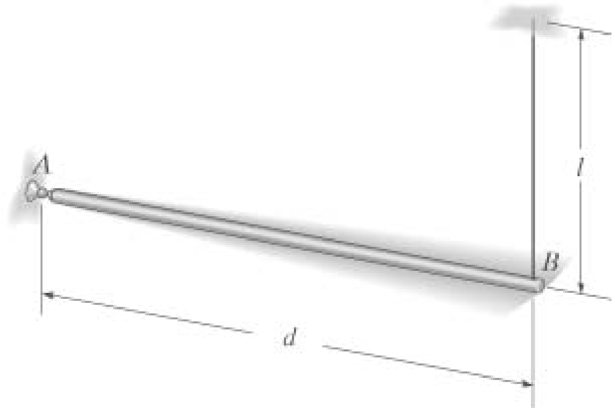
The slender rod has a weight *W*. If it is supported in the horizontal plane by a ball-and-socket joint at *A* and a cable at *B*, determine the natural frequency of vibration when the end *B* is given a small horizontal displacement and then released.

Given:

$W = 4\ \frac{\text{lb}}{\text{ft}}$

$d = 1.5\ \text{ft}$

$l = 0.75\ \text{ft}$



Solution:

$$d\theta = l\phi \qquad \phi = \frac{d\theta}{l}$$

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{d^2}{3}\right)\theta'^2 + \frac{W}{2}l\left(1 - \cos\left(\frac{d\theta}{l}\right)\right)$$

$$\left(\frac{Wd^2}{3g}\right)\theta'' + \left(\frac{W}{2}\right)l\sin\left(\frac{d\theta}{l}\right)\frac{d}{l} = 0$$

$$\left(\frac{Wd^2}{3g}\right)\theta'' + \left(\frac{Wd^2}{2l}\right)\theta = 0$$

$$\theta'' + \frac{3g}{2l}\theta = 0$$

$$f = \frac{1}{2\pi}\sqrt{\frac{3g}{2l}} \qquad f = 1.28\ \frac{1}{\text{s}}$$

Problem 22-38

Determine the natural frequency of vibration of the disk of weight *W*. Assume the disk does not slip on the inclined surface.

Given:

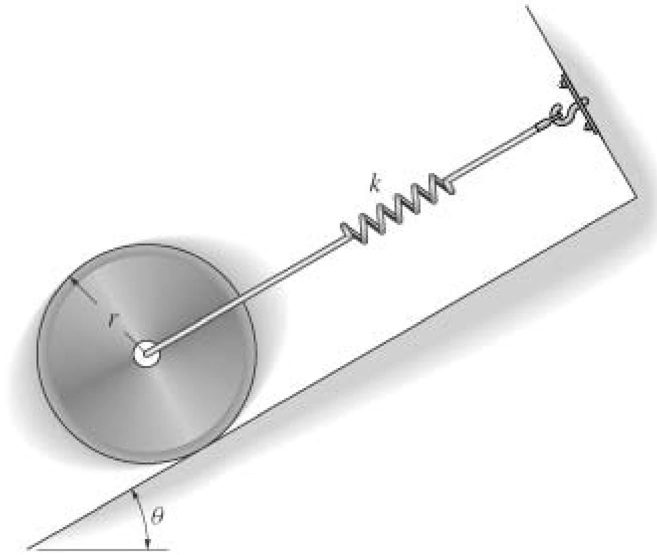
$W = 20\ \text{lb}$

$k = 10 \dfrac{\text{lb}}{\text{in}}$

$\theta = 30 \text{ deg}$

$r = 1 \text{ ft}$

$g = 32.2 \dfrac{\text{ft}}{\text{s}^2}$



Solution:

$$T + V = \frac{1}{2}\left(\frac{W}{g}\right)\left(\frac{3r^2}{2}\right)\theta'^2 + \frac{1}{2}k(r\theta)^2$$

$$\left(\frac{W}{g}\right)\left(\frac{3r^2}{2}\right)\theta'' + kr^2\theta = 0$$

$$\theta'' + \frac{2kg}{3W}\theta = 0 \qquad f = \frac{1}{2\pi}\sqrt{\frac{2kg}{3W}} \qquad f = 1.81 \frac{1}{\text{s}}$$

Problem 22-39

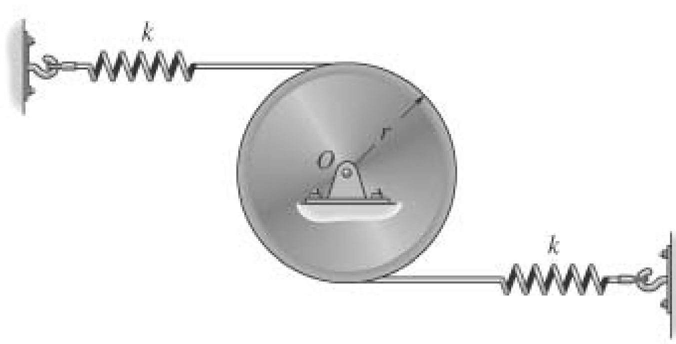
If the disk has mass M, determine the natural frequency of vibration. The springs are originally unstretched.

Given:

$M = 8 \text{ kg}$

$k = 400 \dfrac{\text{N}}{\text{m}}$

$r = 100 \text{ mm}$



Solution:

$$T + V = \frac{1}{2}\left(\frac{Mr^2}{2}\right)\theta'^2 + 2\frac{1}{2}k(r\theta)^2$$

$$M\left(\frac{r^2}{2}\right)\theta'' + 2kr^2\theta = 0$$

$$\theta'' + \frac{4k}{M}\theta = 0 \qquad f = \frac{1}{2\pi}\sqrt{\frac{4k}{M}} \qquad f = 2.25\ \frac{1}{s}$$

***Problem 22-40**

Determine the differential equation of motion of the spool of mass M. Assume that it does not slip at the surface of contact as it oscillates. The radius of gyration of the spool about its center of mass is k_G.

Given:

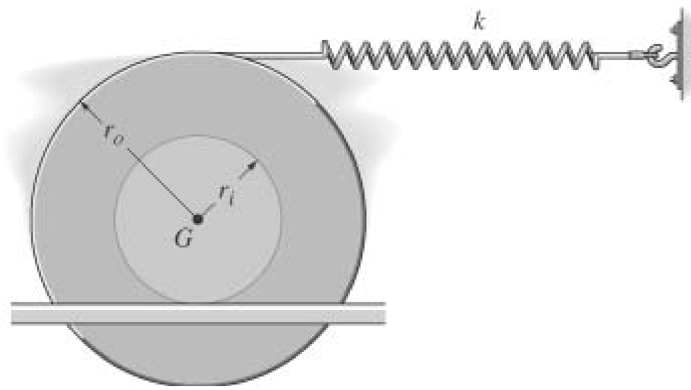
$M = 3\ \text{kg}$

$k_G = 125\ \text{mm}$

$r_i = 100\ \text{mm}$

$r_o = 200\ \text{mm}$

$k = 400\ \frac{\text{N}}{\text{m}}$



Solution:

$$T + V = \frac{1}{2}M\left(k_G^2 + r_i^2\right)\theta'^2 + \frac{1}{2}k\left[\left(r_o + r_i\right)\theta\right]^2$$

$$M\left(k_G^2 + r_i^2\right)\theta'' + k\left(r_o + r_i\right)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k\left(r_o + r_i\right)^2}{M\left(k_G^2 + r_i^2\right)}}$$

$$\theta'' + \omega_n^2\theta = 0$$

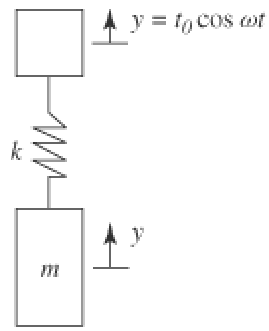
where $\omega_n^2 = 468\ \frac{\text{rad}}{\text{s}^2}$

Problem 22-41

Use a block-and-spring model like that shown in Fig. 22-14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \cos\omega t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement *y* measured from the static equilibrium position of the block when $t = 0$.

Solution:

For Static Equilibrium $\quad m g = k\delta_{st}$



Equation of Motion is then

$$k\left(\delta + \delta_{st} - y\right) - m g = m y''$$

$$m y'' + k y = k\delta_0 \cos(\omega t)$$

$$y'' + \frac{k}{m} y = \frac{k}{m}\delta_0 \cos(\omega t)$$

The solution consists of a homogeneous part and a particular part

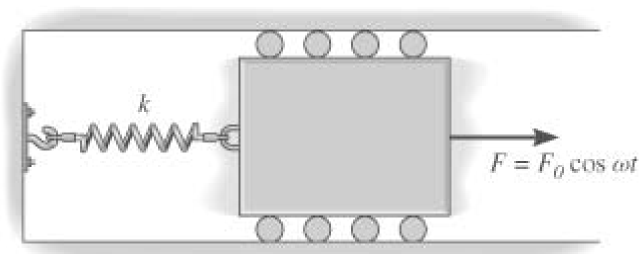
$$y(t) = A\cos\left(\sqrt{\frac{k}{m}}\, t\right) + B\sin\left(\sqrt{\frac{k}{m}}\, t\right) + \frac{\delta_0}{1 - \frac{m\omega^2}{k}} \cos(\omega t)$$

The constants *A* and *B* are determined from the initial conditions.

Problem 22-42

The block of weight *W* is attached to a spring having stiffness *k*. A force $F = F_0 \cos\omega t$ is applied to the block. Determine the maximum speed of the block after frictional forces cause the free vibrations to dampen out.

Given:



$$W = 20 \text{ lb}$$

$$k = 20 \ \frac{\text{lb}}{\text{ft}}$$

$$F_0 = 6 \text{ lb}$$

$$\omega = 2\ \frac{\text{rad}}{\text{s}}$$

$$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$$

Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad C = \frac{\frac{F_0 g}{W}}{\frac{k g}{W} - \omega^2}$$

$$x = C\cos(\omega t) \qquad v = -C\,\omega\sin(\omega t)$$

$$v_{max} = C\omega \qquad v_{max} = 0.685\ \frac{\text{ft}}{\text{s}}$$

Problem 22-43

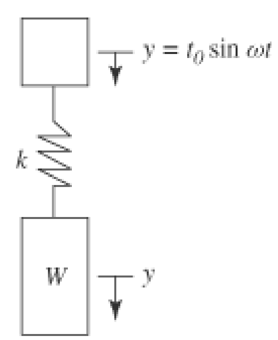
A weight *W* is attached to a spring having a stiffness *k*. The weight is drawn downward a distance *d* and released from rest. If the support moves with a vertical displacement $\delta = \delta_0 \sin\omega t$, determine the equation which describes the position of the weight as a function of time.

Given:

$$W = 4\ \text{lb} \qquad \delta_0 = 0.5\ \text{in}$$

$$k = 10\ \frac{\text{lb}}{\text{ft}} \qquad \omega = 4\ \frac{\text{rad}}{\text{s}}$$

$$d = 4\ \text{in}$$



Solution:

For Static Equilibrium $\quad W = k\delta_{st}$

Equation of Motion is then

$$k(y + \delta_{st} - \delta) - W = \left(\frac{-W}{g}\right)y''$$

$$\left(\frac{W}{g}\right)y'' + k\,y = k\delta_0 \sin(\omega t)$$

$$y'' + \left(\frac{kg}{W}\right) y = \left(\frac{kg}{W}\right) \delta_0 \sin(\omega t)$$

The solution consists of a homogeneous part and a particular part

$$y(t) = A\cos\left(\sqrt{\frac{kg}{W}}\, t\right) + B\sin\left(\sqrt{\frac{kg}{W}}\, t\right) + \frac{\delta_0}{1 - \frac{W\omega^2}{kg}} \sin(\omega t)$$

The constants A and B are determined from the initial conditions.

$$A = d \qquad B = \frac{-\delta_0\, \omega}{\left(1 - \frac{W\omega^2}{kg}\right)\sqrt{\frac{kg}{W}}} \qquad C = \frac{\delta_0}{1 - \frac{W\omega^2}{kg}} \qquad p = \sqrt{\frac{kg}{W}}$$

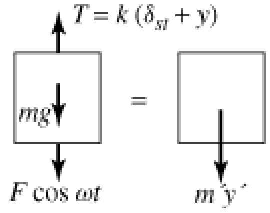
$$y = A\cos(p\,t) + B\sin(p\,t) + C\sin(\omega t)$$

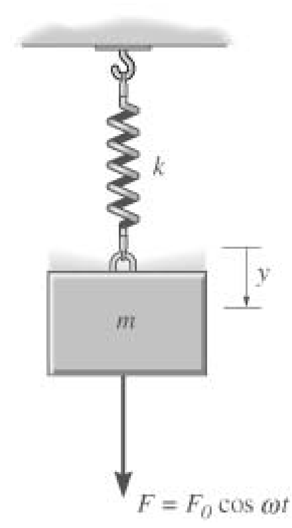
where $A = 0.33\ \text{ft}$ $\quad B = -0.0232\ \text{ft}$

$C = 0.05\ \text{ft}$ $\quad p = 8.97\ \frac{\text{rad}}{\text{s}}$ $\quad \omega = 4.00\ \frac{\text{rad}}{\text{s}}$

***Problem 22-44**

If the block is subjected to the impressed force $F = F_0 \cos(\omega t)$, show that the differential equation of motion is $y'' + (k/m)y = (F_0/m)\cos(\omega t)$, where y is measured from the equilibrium position of the block. What is the general solution of this equation ?





Solution:

$$W = k\delta_{st}$$

$$F_0 \cos(\omega t) + m g - k(\delta_{st} + y) = m y''$$

$$y'' + \left(\frac{k}{m}\right) y = \frac{F_0}{m} \cos(\omega t)$$ Q.E.D.

$$y = A \sin\left(\sqrt{\frac{k}{m}}\, t\right) + B \cos\left(\sqrt{\frac{k}{m}}\, t\right) + \left(\frac{F_o}{k - m\omega^2}\right) \cos(\omega t)$$

Problem 22-45

The light elastic rod supports the sphere of mass M. When a vertical force P is applied to the sphere, the rod deflects a distance d. If the wall oscillates with harmonic frequency f and has amplitude A, determine the amplitude of vibration for the sphere.

Given:

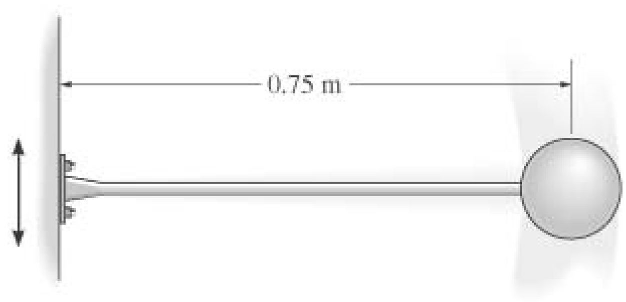
$M = 4$ kg

$P = 18$ N

$\delta = 14$ mm

$f = 2$ Hz

$A = 15$ mm



Solution:

$$k = \frac{P}{\delta}$$

$$\omega = 2\pi f \qquad \omega_n = \sqrt{\frac{k}{M}}$$

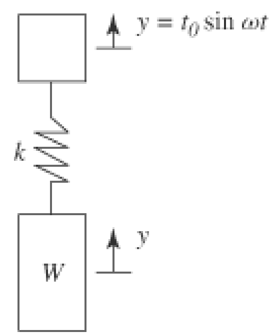
$$x_{pmax} = \left| \frac{A}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| \qquad x_{pmax} = 29.5 \text{ mm}$$

Problem 22-46

Use a block-and-spring model like that shown in Fig. 22-14*a* but suspended from a vertical position and subjected to a periodic support displacement of $\delta = \delta_0 \sin\omega t$, determine the equation of motion for the system, and obtain its general solution. Define the displacement y measured from the static equilibrium position of the block when $t = 0$.

Solution:

For Static Equilibrium $\quad mg = k\delta_{st}$



Equation of Motion is then

$$k(\delta + \delta_{st} - y) - mg = my''$$

$$my'' + ky = k\delta_0 \sin(\omega t)$$

$$y'' + \frac{k}{m}y = \frac{k}{m}\delta_0 \sin(\omega t)$$

The solution consists of a homogeneous part and a particular part

$$y(t) = A\sin\left(\sqrt{\frac{k}{m}}\,t\right) + B\cos\left(\sqrt{\frac{k}{m}}\,t\right) + \frac{\delta_0}{1 - \dfrac{m\omega^2}{k}}\sin(\omega t)$$

The constants A and B are determined from the initial conditions.

Problem 22-47

A block of mass M is suspended from a spring having a stiffness k. If the block is acted upon by a vertical force $F = F_0 \sin\omega t$, determine the equation which describes the motion of the block when it is pulled down a distance d from the equilibrium position and released from rest at $t = 0$. Assume that positive displacement is downward.

Given:

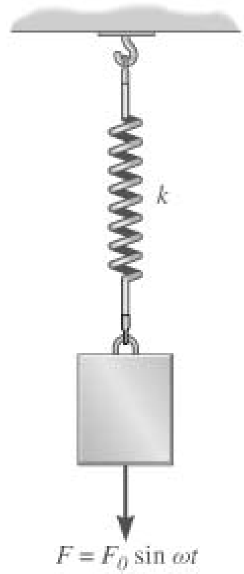
$$M = 5\ \text{kg}$$

$$k = 300\ \frac{\text{N}}{\text{m}}$$

$$F_0 = 7\ \text{N}$$

$\omega = 8\ \frac{rad}{s}$

$d = 100\ mm$



Solution:

$\omega_n = \sqrt{\frac{k}{M}}$ $\qquad C = \frac{F_0}{M(\omega_n^2 - \omega^2)}$

$y = A\sin(\omega_n t) + B\cos(\omega_n t) + C\sin(\omega t)$

$y' = A\omega_n \cos(\omega_n t) - B\omega_n \sin(\omega_n t) + C\omega\cos(\omega t)$

$y = d$ when $t = 0$ $\qquad B = d$

$y = y' = 0$ when $t = 0$ $\qquad A = -C\frac{\omega}{\omega_n}$

$y = A\sin(\omega_n t) + B\cos(\omega_n t) + C\sin(\omega t)$

$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 361 \\ 100 \\ -350 \end{pmatrix} mm$ $\qquad \begin{pmatrix} \omega_n \\ \omega \end{pmatrix} = \begin{pmatrix} 7.75 \\ 8.00 \end{pmatrix} \frac{rad}{s}$

***Problem 22-48**

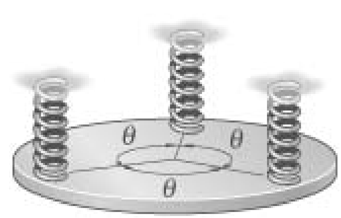
The circular disk of mass *M* is attached to three springs, each spring having a stiffness *k*. If the disk is immersed in a fluid and given a downward velocity *v* at the equilibrium position, determine the equation which describes the motion. Assume that positive displacement is measured downward, and that fluid resistance acting on the disk furnishes a damping force having a magnitude *F* = *cv*.

Given:

$M = 4\ kg$ $\qquad c = 60\ \frac{kg}{s}$

$k = 180\ \frac{N}{m}$ $\qquad \theta = 120\ deg$

$v = 0.3\ \frac{m}{s}$



Solution:

$$M\,y'' + c\,y' + 3k\,y = 0 \qquad y'' + \frac{c}{M}y' + \frac{3k}{M}y = 0$$

$$\omega_n = \sqrt{\frac{3k}{M}} \qquad \zeta = \frac{c}{2M\omega_n}$$ Since $\zeta = 0.65 < 1$ the system is underdamped

$$b = \zeta\,\omega_n \qquad \omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$y(t) = e^{-b\,t}\left(A\cos\left(\omega_d\,t\right) + B\sin\left(\omega_d\,t\right)\right)$$

Now find A and B from initial conditions. Guesses $A = 1\text{ m}$ $B = 1\text{ m}$

Given $0 = A$ $v = -A\,b + B\omega_d$ $\begin{pmatrix} A \\ B \end{pmatrix} = \text{Find}(A, B)$

$$y(t) = e^{-b\,t}\left(A\cos\left(\omega_d\,t\right) + B\sin\left(\omega_d\,t\right)\right)$$

where $b = 7.50\,\frac{\text{rad}}{\text{s}}$ $\omega_d = 8.87\,\frac{\text{rad}}{\text{s}}$ $A = 0.0\,\text{mm}$ $B = 33.8\,\text{mm}$

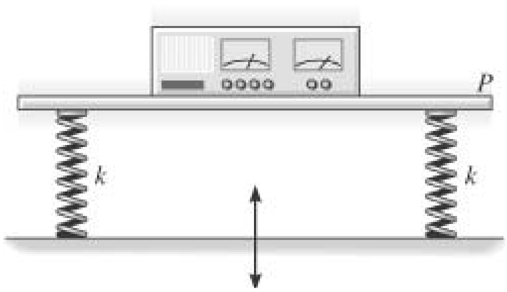
Problem 22-49

The instrument is centered uniformly on a platform P, which in turn is supported by *four* springs, each spring having stiffness k. If the floor is subjected to a vibration f, having a vertical displacement amplitude δ_0, determine the vertical displacement amplitude of the platform and instrument. The instrument and the platform have a total weight W.

Given:

$k = 130\,\frac{\text{lb}}{\text{ft}}$ $\delta_0 = 0.17\text{ ft}$

$f = 7\text{ Hz}$ $W = 18\text{ lb}$



Solution:

$$\omega_n = \sqrt{\frac{4kg}{W}} \qquad \omega_n = 30.50\,\frac{\text{rad}}{\text{s}}$$

$\omega = 2\pi f$ $\omega = 43.98\,\frac{\text{rad}}{\text{s}}$

Using Eq. 22-22, the amplitude is $x_{pmax} = \left|\dfrac{\delta_0}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right|$ $x_{pmax} = 1.89\text{ in}$

Problem 22-50

A trailer of mass M is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude a and wave length $2d$. If the two springs s which support the trailer each have a stiffness k, determine the speed v which will cause the greatest vibration (resonance) of the trailer. Neglect the weight of the wheels.

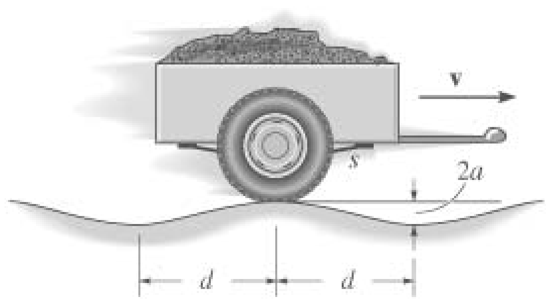
Given:

$M = 450\text{ kg}$

$k = 800\,\frac{\text{N}}{\text{m}}$

$d = 2\text{ m}$

$a = 50\text{ mm}$



Solution:

$p = \sqrt{\dfrac{2k}{M}}$ $\tau = \dfrac{2\pi}{p}$ $\tau = 3.33\text{ s}$

For maximum vibration of the trailer, resonance must occur, $\omega = p$

Thus the trailer must travel so that $v_R = \dfrac{2d}{\tau}$ $v_R = 1.20\,\frac{\text{m}}{\text{s}}$

Problem 22-51

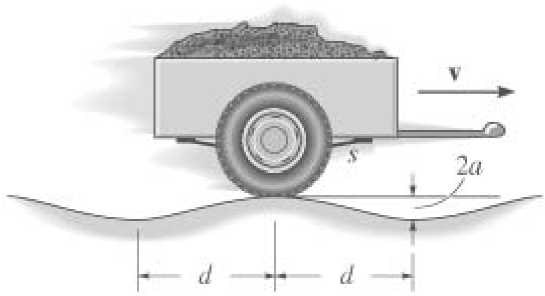
The trailer of mass M is pulled with a constant speed over the surface of a bumpy road, which may be approximated by a cosine curve having an amplitude a and wave length $2d$. If the two springs s which support the trailer each have a stiffness k, determine the amplitude of vibration of the trailer if the speed is v.

Given:

$M = 450\ \text{kg}$ $d = 2\ \text{m}$

$k = 800\ \dfrac{\text{N}}{\text{m}}$ $v = 15\ \dfrac{\text{km}}{\text{hr}}$

$a = 50\ \text{mm}$



Solution:

$$p = \sqrt{\frac{2k}{M}} \qquad \tau = \frac{2d}{v} \qquad \omega = \frac{2\pi}{\tau}$$

$$x_{max} = \left| \frac{a}{1 - \left(\frac{\omega}{p}\right)^2} \right| \qquad x_{max} = 4.53\,\text{mm}$$

***Problem 22-52**

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. If the static deflection of the beam is δ because of the weight of the motor, determine the angular velocity of the flywheel at which resonance will occur. The motor has weight W_m. Neglect the mass of the beam.

Given:

$W_b = 0.25\ \text{lb}$ $W_m = 150\ \text{lb}$

$d = 10\ \text{in}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\delta = 1\ \text{in}$

Solution:

$$k = \frac{W_m}{\delta} \qquad k = 1800\,\frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{kg}{W_m}} \qquad \omega_n = 19.66\,\frac{\text{rad}}{\text{s}}$$

Resonance occurs when $\omega = \omega_n$ $\omega = 19.7\,\dfrac{\text{rad}}{\text{s}}$

Problem 22-53

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. The static deflection of the beam is δ because of the weight of the motor. The motor has weight W_m. Neglect the mass of the beam. What will be the amplitude of steady-state vibration of the motor if the angular velocity of the flywheel is ω?

Given:

$W_b = 0.25\ \text{lb}$ $\quad W_m = 150\ \text{lb}$

$d = 10\ \text{in}$ $\quad g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

$\delta = 1\ \text{in}$ $\quad \omega = 20\ \dfrac{\text{rad}}{\text{s}}$

Solution:

$$k = \frac{W_m}{\delta} \qquad k = 1800\ \frac{\text{lb}}{\text{ft}}$$

$$\omega_n = \sqrt{\frac{kg}{W_m}} \qquad \omega_n = 19.66\ \frac{\text{rad}}{\text{s}}$$

$$F_0 = \frac{W_b}{g} d\omega^2 \qquad F_0 = 2.59\ \text{lb}$$

From Eq. 22-21, the amplitude of the steady state motion is

$$C = \left[\frac{\dfrac{F_0}{k}}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}\right] \qquad |C| = 0.490\ \text{in}$$

Problem 22-54

The electric motor turns an eccentric flywheel which is equivalent to an unbalanced weight W_b located distance d from the axis of rotation. The static deflection of the beam is δ because of the weight of the motor. The motor has weight W_m. Neglect the mass of the beam. Determine the angular velocity of the flywheel which will produce an amplitude of vibration C.

Given:

$W_b = 0.25\ \text{lb}$

$W_m = 150\ \text{lb}$

$d = 10\ \text{in}$

$\delta = 1\ \text{in}$

$C = 0.25\ \text{in}$

$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$k = \frac{W_m}{\delta}$ $\quad k = 1800\frac{\text{lb}}{\text{ft}}$ $\quad \omega_n = \sqrt{\frac{k}{\frac{W_m}{g}}}$ $\quad \omega_n = 19.657\frac{\text{rad}}{\text{s}}$

There are 2 correct answers to this problem. We can find these 2 answers by starting with different inital guesses.

$\omega = 25\frac{\text{rad}}{\text{s}}$ Given $C = \dfrac{\frac{W_b d\omega^2}{g\,k}}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$ $\quad \omega = \text{Find}(\omega)$ $\quad \omega = 20.3\ \frac{1}{\text{s}}$

$\omega = 18\frac{\text{rad}}{\text{s}}$ Given $C = \dfrac{\frac{W_b d\omega^2}{g\,k}}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$ $\quad \omega = \text{Find}(\omega)$ $\quad \omega = 19.0\ \frac{1}{\text{s}}$

Problem 22-55

The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have total weight W and the engine, when running, creates an impressed force $F = F_0 \sin\omega t$. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as k.

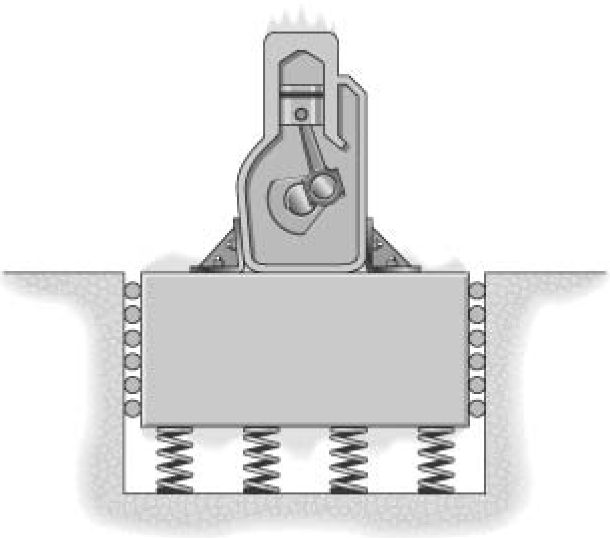
Given:

$W = 1500\ \text{lb}$

$F_0 = 50\ \text{lb}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$

$k = 2000\ \frac{\text{lb}}{\text{ft}}$



Solution:

$\omega_n = \sqrt{\frac{kg}{W}}$ $\omega_n = 6.55\ \frac{\text{rad}}{\text{s}}$

$$C = \frac{\frac{F_0}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$x_p = C\sin(\omega t)$ $\omega = 2.00\ \frac{\text{rad}}{\text{s}}$ $C = 0.0276\ \text{ft}$

***Problem 22-56**

The engine is mounted on a foundation block which is spring-supported. The block and engine have total weight W and the engine, when running, creates an impressed force $F = F_0 \sin\omega t$. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as k. What rotational speed ω will cause resonance?

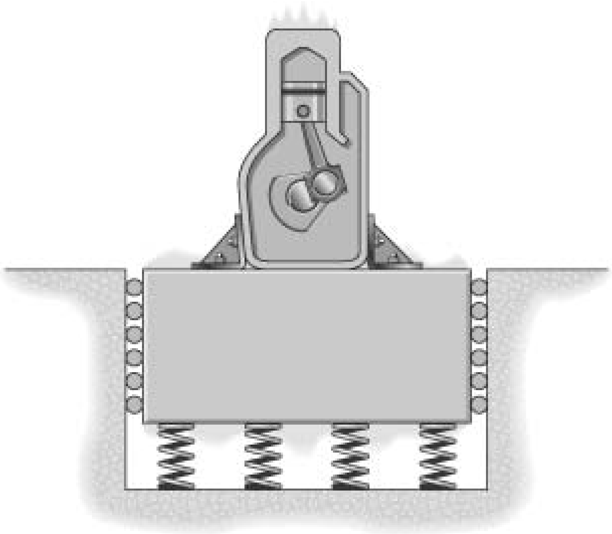
Given:

$W = 1500\ \text{lb}$

$F_0 = 50\ \text{lb}$

$\omega = 2\ \frac{\text{rad}}{\text{s}}$

$k = 2000\ \frac{\text{lb}}{\text{ft}}$

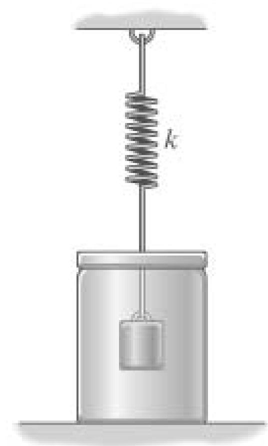


Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad \omega_n = 6.55\,\frac{\text{rad}}{\text{s}} \qquad \omega = \omega_n \qquad \omega = 6.55\,\frac{\text{rad}}{\text{s}}$$

Problem 22-57

The block, having weight W, is immersed in a liquid such that the damping force acting on the block has a magnitude of $F = c|v|$. If the block is pulled down at a distance d and released from rest, determine the position of the block as a function of time. The spring has a stiffness k. Assume that positive displacement is downward.



Given:

$W = 12\ \text{lb}$ $\qquad d = 0.62\ \text{ft}$

$c = 0.7\,\frac{\text{lb s}}{\text{ft}}$ $\qquad g = 32.2\,\frac{\text{ft}}{\text{s}^2}$

$k = 53\,\frac{\text{lb}}{\text{ft}}$

Solution:

$$\omega_n = \sqrt{\frac{k g}{W}} \qquad \zeta = \frac{c g}{2W\omega_n}$$ Since $\zeta = 0.08$ < 1 the system is underdamped

$$b = \zeta\omega_n \qquad \omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

We can write the solution as $\quad y(t) = B e^{-b t}\sin(\omega_d t + \phi)$

To solve for the constants B and ϕ

Guesses $\quad B = 1\ \text{ft} \qquad \phi = 1\ \text{rad}$

Given $\quad B\sin(\phi) = d \qquad B\omega_d\cos(\phi) - b B\sin(\phi) = 0 \qquad \begin{pmatrix} B \\ \phi \end{pmatrix} = \text{Find}(B, \phi)$

Thus

$$y(t) = B e^{-b t}\sin(\omega_d t + \phi)$$

where $B = 0.62\ \text{ft}$ $b = 0.94\ \dfrac{\text{rad}}{\text{s}}$

$\omega_d = 11.9\ \dfrac{\text{rad}}{\text{s}}$ $\phi = 1.49\ \text{rad}$

Problem 22-58

A block of weight W is suspended from a spring having stiffness k. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = \delta_0 \sin\omega t$. If the damping factor is C_{ratio}, determine the phase angle ϕ of the forced vibration.

Given:

$W = 7\ \text{lb}$ $\omega = 2\ \dfrac{\text{rad}}{\text{s}}$

$k = 75\ \dfrac{\text{lb}}{\text{ft}}$ $C_{ratio} = 0.8$

$\delta_0 = 0.15\ \text{ft}$ $g = 32.2\ \dfrac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_n = \sqrt{\dfrac{kg}{W}}$ $\omega_n = 18.57\ \dfrac{\text{rad}}{\text{s}}$

$$\phi = \operatorname{atan}\left[\frac{2C_{ratio}\left(\dfrac{\omega}{\omega_n}\right)}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2}\right] \qquad \phi = 9.89\ \text{deg}$$

Problem 22-59

A block of weight W is suspended from a spring having stiffness k. The support to which the spring is attached is given simple harmonic motion which may be expressed as $\delta = \delta_0 \sin\omega t$. If the damping factor is C_{ratio}, determine the magnification factor of the forced vibration.

Given:

$W = 7\ \text{lb}$ $\omega = 2\ \dfrac{\text{rad}}{\text{s}}$

$k = 75\ \frac{\text{lb}}{\text{ft}}$ $\quad C_{ratio} = 0.8$

$\delta_0 = 0.15\ \text{ft}$ $\quad g = 32.2\ \frac{\text{ft}}{\text{s}^2}$

Solution:

$\omega_n = \sqrt{\frac{kg}{W}}$ $\quad \omega_n = 18.57\ \frac{\text{rad}}{\text{s}}$

$$MF = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2(C_{ratio})\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$MF = 0.997$

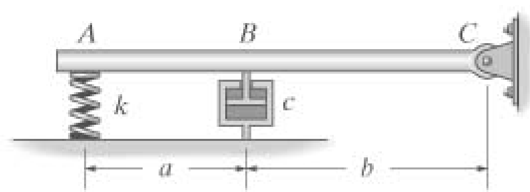
***Problem 22-60**

The bar has a weight W. If the stiffness of the spring is k and the dashpot has a damping coefficient c, determine the differential equation which describes the motion in terms of the angle θ of the bar's rotation. Also, what should be the damping coefficient of the dashpot if the bar is to be critically damped?

Given:

$W = 6\ \text{lb}$ $\quad b = 3\ \text{ft}$ $\quad c = 60\ \frac{\text{lb}\cdot\text{s}}{\text{ft}}$

$k = 8\ \frac{\text{lb}}{\text{ft}}$ $\quad a = 2\ \text{ft}$



Solution:

$$\left(\frac{W}{g}\right)\frac{(a+b)^2}{3}\theta'' + cb^2\theta' + k(a+b)^2\theta = 0$$

$M = \left(\frac{W}{g}\right)\frac{(a+b)^2}{3}$ $\quad C = cb^2$ $\quad K = k(a+b)^2$

$M\theta'' + C\theta' + K\theta = 0$

where $\quad M = 1.55\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$C = 540.00\ \text{lb}\cdot\text{ft}\cdot\text{s}$

$K = 200.00\ \text{lb}\cdot\text{ft}$

To find critical damping

$$\omega_n = \sqrt{\frac{K}{M}} \qquad C = 2M\omega_n$$

$$c = \frac{C}{b^2} \qquad c = 3.92\,\frac{\text{lb}\cdot\text{s}}{\text{ft}}$$

Problem 22-61

A block having mass *M* is suspended from a spring that has stiffness *k*. If the block is given an upward velocity *v* from its equilibrium position at *t* = 0, determine its position as a function of time. Assume that positive displacement of the block is downward and that motion takes place in a medium which furnishes a damping force *F* = *C* |*v*|.

Given: $M = 7\text{ kg} \qquad k = 600\,\frac{\text{N}}{\text{m}} \qquad C = 50\,\frac{\text{N}\cdot\text{s}}{\text{m}} \qquad v = 0.6\,\frac{\text{m}}{\text{s}}$

Solution:

$$\omega_n = \sqrt{\frac{k}{M}} \qquad \omega_n = 9.258\,\frac{\text{rad}}{\text{s}}$$

$$C_c = 2M\omega_n \qquad C_c = 129.6\,\frac{\text{N}\cdot\text{s}}{\text{m}}$$

If $C = 50.00\,\text{N}\cdot\frac{\text{s}}{\text{m}} < C_c = 129.61\,\text{N}\cdot\frac{\text{s}}{\text{m}}$ the system is underdamped

$$b = \frac{-C}{C_c}\omega_n \qquad \omega_d = \omega_n\sqrt{1-\left(\frac{C}{C_c}\right)^2}$$

$$y = 0\text{ m} \quad y' = -v \qquad A = 1\text{ m} \qquad \phi = 1\text{ rad} \qquad t = 0\text{ s}$$

Given $\qquad y = A e^{bt}\sin(\omega_d t + \phi)$

$$y' = A b e^{bt}\sin(\omega_d t + \phi) + A\omega_d e^{bt}\cos(\omega_d t + \phi)$$

$$\begin{pmatrix} A \\ \phi \end{pmatrix} = \text{Find}(A, \phi)$$

$$y = A e^{b t} \sin(\omega_d t + \phi)$$

$A = -0.0702\ \text{m}$ $\quad b = -3.57\ \frac{\text{rad}}{\text{s}}$ $\quad \omega_d = 8.54\ \frac{\text{rad}}{\text{s}}$ $\quad \phi = 0.00\ \text{rad}$

Problem 22-62

The damping factor C/C_c, may be determined experimentally by measuring the successive amplitudes of vibrating motion of a system. If two of these maximum displacements can be approximated by x_1 and x_2, as shown in Fig. 22-17, show that the ratio $\ln(x_1/x_2) = 2\pi(C/C_c)/(1-(C/C_e)^2)^{1/2}$. The quantity $\ln(x_1/x_2)$ is called the logarithmic decrement.

Solution:

$$x = D\left(e^{-\frac{C}{2m}t}\sin(\omega_d t + \phi)\right)$$

$$x_{max} = D e^{-\frac{C}{2m}t} \qquad x_1 = D e^{-\frac{C}{2m}t_1} \qquad x_2 = D e^{-\frac{C}{2m}t_2}$$

$$\frac{x_1}{x_2} = \frac{D e^{-\frac{C}{2m}t_1}}{D e^{-\frac{C}{2m}t_2}} = e^{\frac{C}{2m}(t_2 - t_1)}$$

Since $\omega_d t_2 - \omega_d t_1 = 2\pi$ then $t_2 - t_1 = \frac{2\pi}{\omega_d}$

so that $\ln\left(\frac{x_1}{x_2}\right) = \frac{C\pi}{m\omega_d}$

$$C_c = 2m\omega_n \qquad \omega_d = \omega_n\sqrt{1-\left(\frac{C}{C_c}\right)^2} = \frac{C_c}{2m}\sqrt{1-\left(\frac{C}{C_c}\right)^2}$$

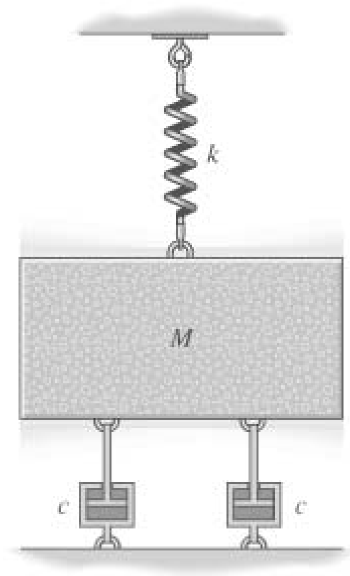
So that,

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\left(\frac{C}{C_c}\right)}{\sqrt{1-\left(\frac{C}{C_c}\right)^2}}$$

Q.E.D.

Problem 22-63

Determine the differential equation of motion for the damped vibratory system shown.What type of motion occurs?



Given:

$M = 25\ \text{kg} \qquad k = 100\ \frac{\text{N}}{\text{m}} \qquad c = 200\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

Solution:

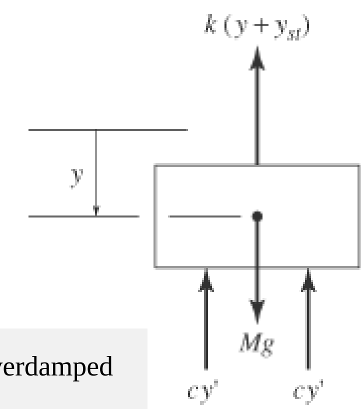
$$M g - k(y + y_{st}) - 2c\,y' = M\,y''$$

$$M\,y'' + k\,y + 2c\,y' + k\,y_{st} - M\,g = 0$$

Equilibrium $\quad k\,y_{st} - M\,g = 0$

$$M\,y'' + 2c\,y' + k\,y = 0 \qquad (1)$$

$$y'' + \frac{2c}{M}y' + \frac{k}{M}y = 0$$



By comparing Eq.(1) to Eq. 22-27

$p = \sqrt{\frac{k}{M}} \qquad p = 2.00\ \frac{\text{rad}}{\text{s}}$

$c_c = 2M\,p \qquad c_c = 100.00\ \text{N}\cdot\frac{\text{s}}{\text{m}}$

Since $c = 200.00\ \frac{\text{N}\cdot\text{s}}{\text{m}} > c_c = 100.00\ \frac{\text{N}\cdot\text{s}}{\text{m}}$, the system is overdamped and will not oscillate. The motion is an exponential decay.

***Problem 22-64**

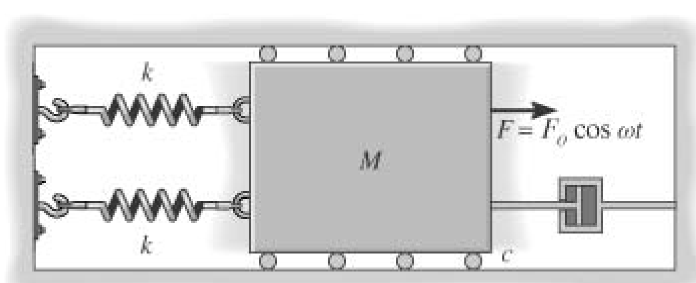
The block of mass M is subjected to the action of the harmonic force $F = F_0 \cos\omega\,t$. Write the equation which describes the steady-state motion.

Given:

$M = 20\ \text{kg} \qquad k = 400\ \frac{\text{N}}{\text{m}}$

$F_0 = 90\ \text{N} \qquad C = 125\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

$\omega = 6\ \frac{\text{rad}}{\text{s}}$



Solution:

$$\omega_n = \sqrt{\frac{2k}{M}} \qquad \omega_n = 6.32\,\frac{\text{rad}}{\text{s}}$$

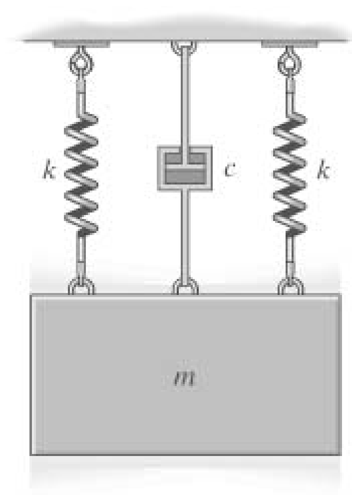
$$C_c = 2M\omega_n \qquad C_c = 253.0\,\text{N}\cdot\frac{\text{s}}{\text{m}}$$

$$A = \frac{\dfrac{F_0}{2k}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\dfrac{C}{C_c}\right)\left(\dfrac{\omega}{\omega_n}\right)\right]^2}} \qquad \phi = \operatorname{atan}\left[\frac{\dfrac{C\omega}{2k}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}\right]$$

$$x = A\cos(\omega t - \phi) \qquad A = 0.119\ \text{m} \qquad \omega = 6.00\,\frac{\text{rad}}{\text{s}} \qquad \phi = 83.9\,\text{deg}$$

Problem 22-65

Draw the electrical circuit that is equivalent to the mechanical system shown. What is the differential equation which describes the charge q in the circuit?



Solution:

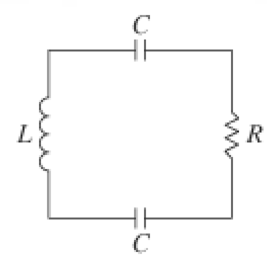
For the block,

$$m x'' + c x' + 2k x = 0$$

Let

$$m = L \qquad c = R \qquad x = q \qquad k = \frac{1}{C}$$

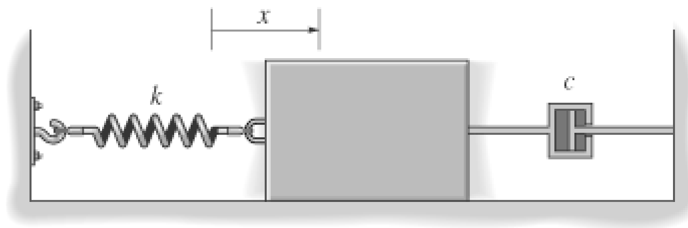
$$L q'' + R q' + \left(\frac{2}{C}\right) q = 0$$



Problem 22-66

The block of mass M is continually damped. If the block is displaced $x = x_1$ and released from rest, determine the time required for it to return to the position $x = x_2$.

Given:



$M = 10\ \text{kg}$ $\quad k = 60\ \frac{\text{N}}{\text{m}}$

$x_1 = 50\ \text{mm}$ $\quad C = 80\ \text{N}\cdot\frac{\text{s}}{\text{m}}$

$x_2 = 2\ \text{mm}$

Solution:

$\omega_n = \sqrt{\frac{k}{M}}$ $\qquad \omega_n = 2.45\,\frac{\text{rad}}{\text{s}}$ $\qquad C_c = 2M\omega_n$ $\qquad C_c = 48.99\,\text{N}\cdot\frac{\text{s}}{\text{m}}$

Since $C = 80.00\,\text{N}\cdot\frac{\text{s}}{\text{m}} > C_c = 48.99\,\text{N}\cdot\frac{\text{s}}{\text{m}}$ the system is overdamped

$$\lambda_1 = \frac{-C}{2M} + \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{k}{M}} \qquad \lambda_2 = \frac{-C}{2M} - \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{k}{M}}$$

$t = 0\ \text{s}$ $\quad x = x_1$ $\quad x' = 0\ \frac{\text{m}}{\text{s}}$ $\quad A_1 = 1\ \text{m}$ $\quad A_2 = 1\ \text{m}$

Given $\quad x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ $\qquad x' = A_1\lambda_1 e^{\lambda_1 t} + A_2\lambda_2 e^{\lambda_2 t}$

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \text{Find}(A_1, A_2) \qquad \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} -0.84 \\ -7.16 \end{pmatrix}\frac{\text{rad}}{\text{s}} \qquad \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0.06 \\ -0.01 \end{pmatrix}\text{m}$$

Given $\quad x_2 = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ $\qquad t = \text{Find}(t)$ $\qquad t = 3.99\,\text{s}$