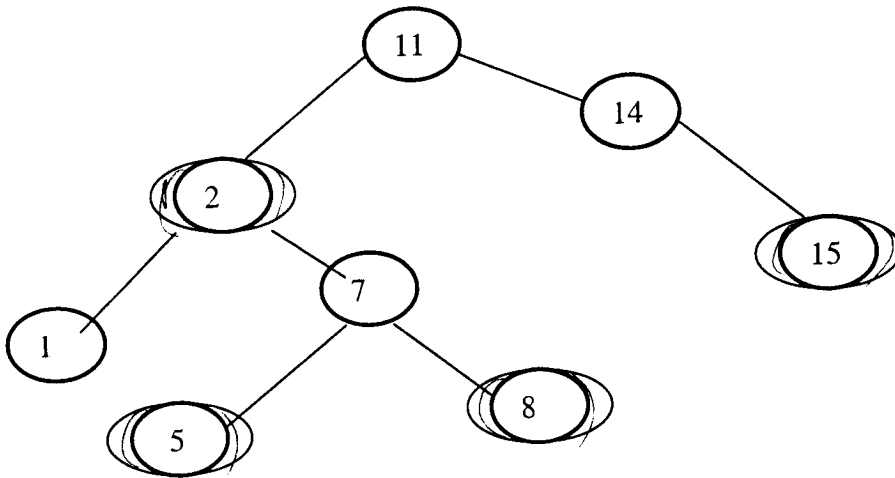
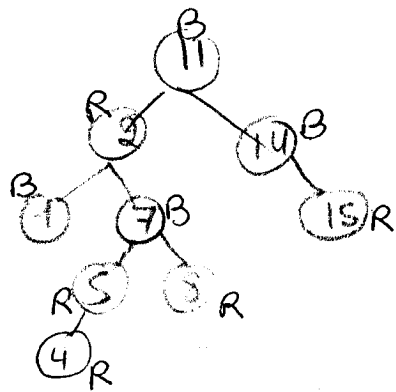


Name: _____

1. In the following binary search tree, let the simple circled nodes be black, and the double-circled nodes are red. (15 points)



Insert 4

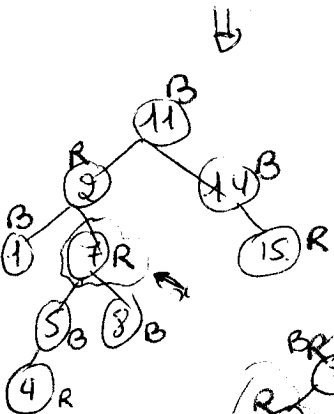


Violation at 4.

Red \rightarrow Red child!!

uncle Red (8)

\Rightarrow Case 1: uncle & parent \rightarrow Black
parent of parent \rightarrow Red



Violation at 7

uncle Black, Right child

Case 2

$\{x = p \rightarrow x$
Rotate left(x)

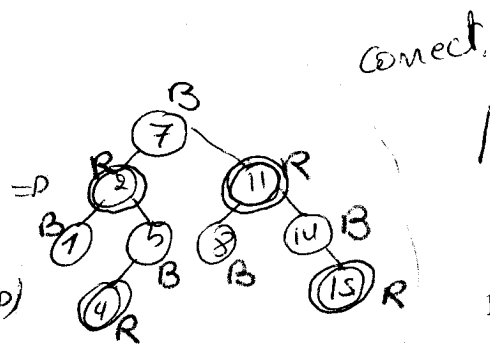
\Rightarrow Case 3 (left child)

$\Rightarrow p \rightarrow p =$ Black

$x \rightarrow p \rightarrow p =$ Red.

Right Rotate ($x \rightarrow p \rightarrow p$)

\Rightarrow 11

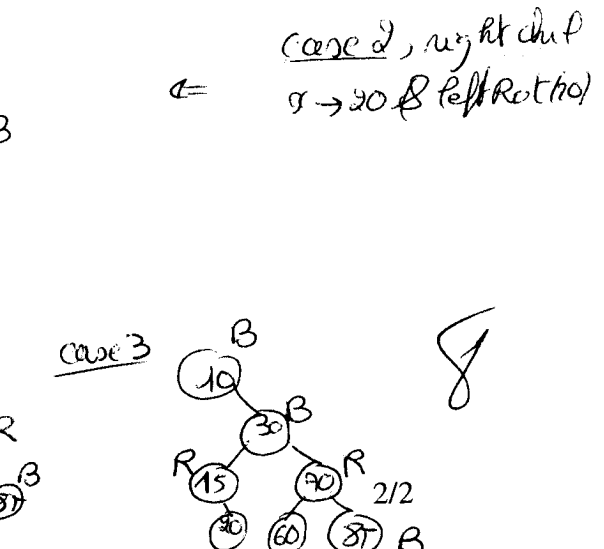
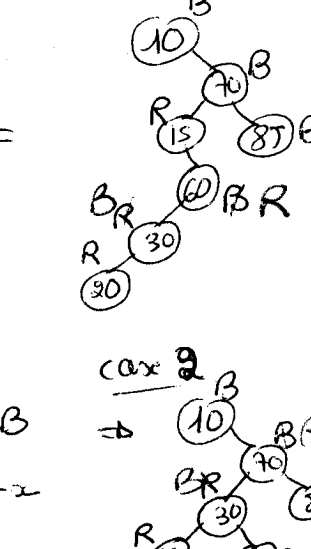
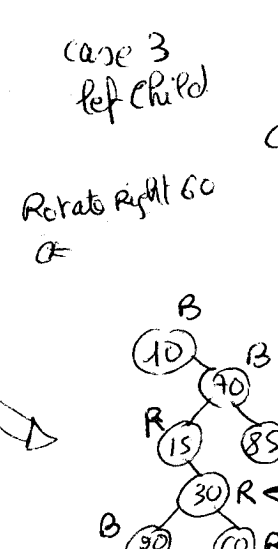
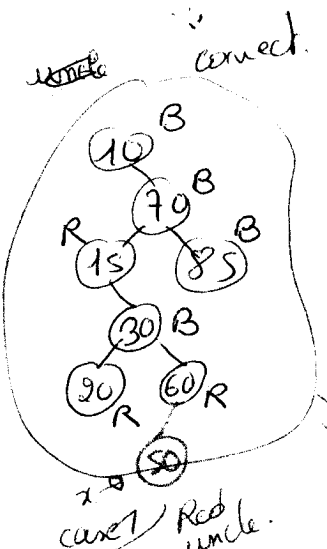
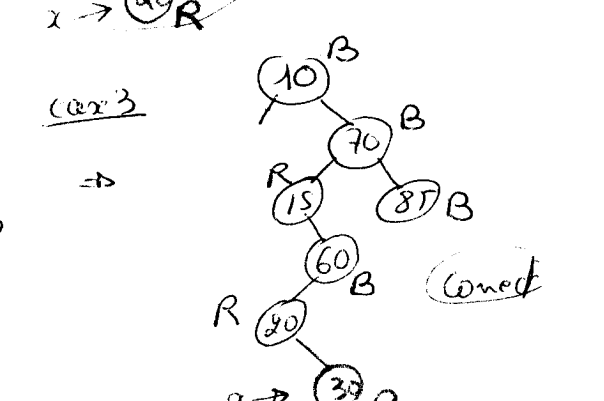
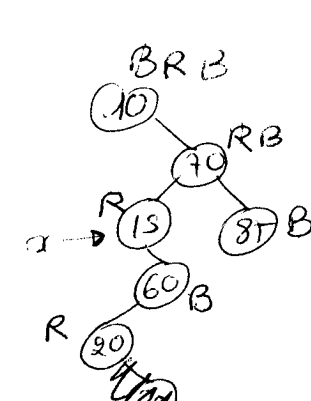
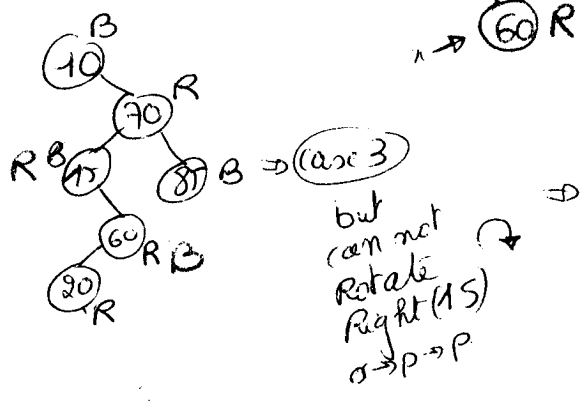
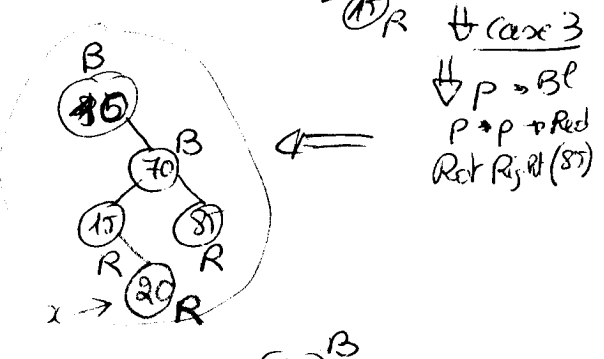
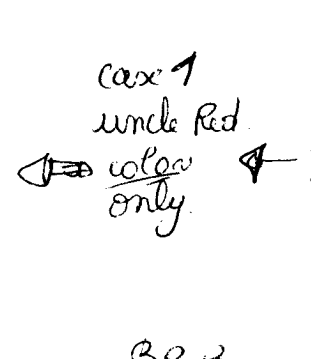
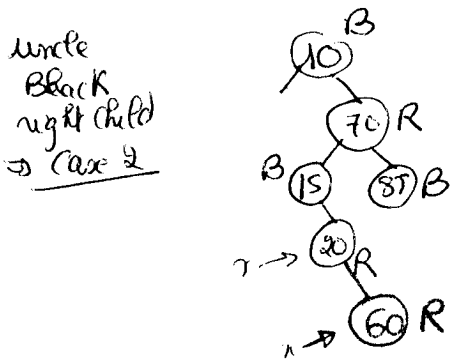
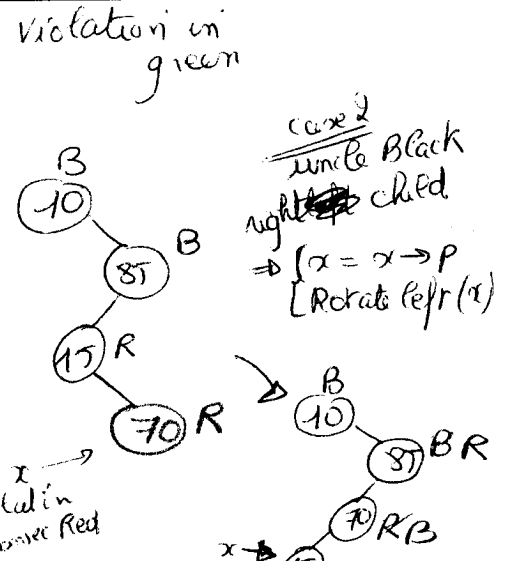
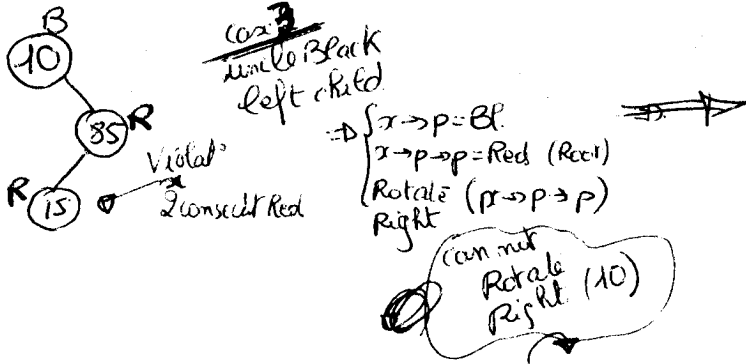


Correct.

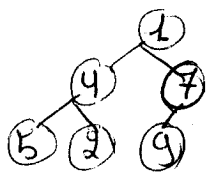
15

2- Draw all the steps to build a Red-Black tree (15 points)

10 85 15 70 20 60 30 50 65 80 90 40 5 55



3. Show the result of applying Build-Max-Heap to the array, 1; 4; 7; 5; 2; 9. Your answer should give the final contents of the array. (15 points)



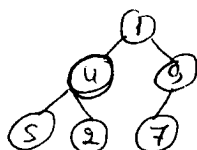
$N=6 \Rightarrow$ start heapifying from $\frac{N}{2} \Rightarrow$ (7)

steps in heapify \rightarrow compare both children, take Max child

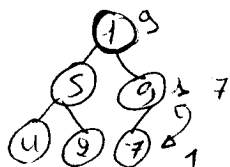
(1) ~~compare~~ if $\text{Max(child)} > x \Rightarrow$ swap

(2) keep on till the ~~Root is~~ ^{end} (leaf)

① $9 > 7 \Rightarrow$



② $5 > 2 \Rightarrow \text{Max(child)} = 5. 5 > 4 \Rightarrow$ swap

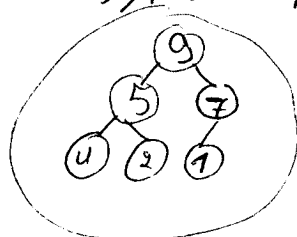


③ $9 > 5 \Rightarrow \text{Max(child)} = 9$

$9 > 1 \Rightarrow$ swap $9 \ 8 \ 1$

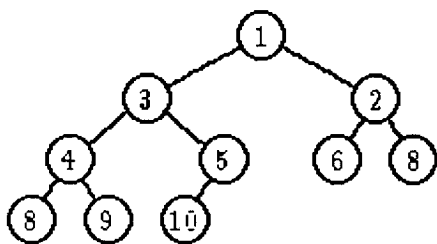
at (1) Max(child) is 7

$8 < 7 \Rightarrow$ swap (1 & 7)



Max Heap

4. Consider the priority queue (heap) represented by the given height-biased leftist tree. Show the modified tree under each of the following operations. (Note: The two operations are independent. Each of them starts from the above tree.) (15 points)



Insertion of the key 7.

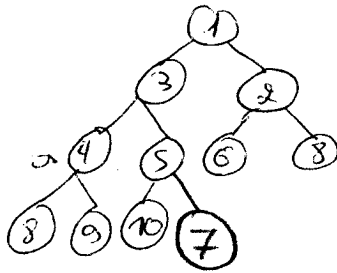
Deletion of a key.

15

Insertion

Step (7)

- ① create a hole in the next available location
- ② percolate up all the way to the root if necessary



① 7 min child is 7
 $7 < 5 \Rightarrow$ No swap

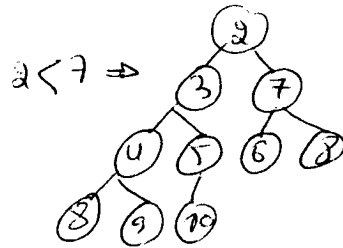
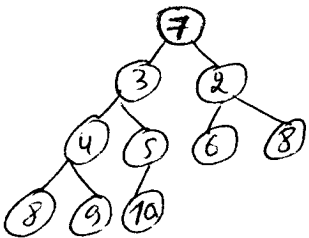
percolate up (then Heap)

take min child
if min child < parent
 \Rightarrow swap
Repeat till Root.

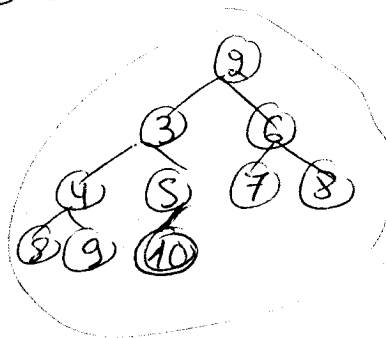
Deletion In heap deletion is done at the Root.

Delete Root:
put last element in Root's place
percolate Down all the way to a leaf

percolate Down
- find smallest child
- min child < parent \Rightarrow swap.



$6 < 7 \Rightarrow$

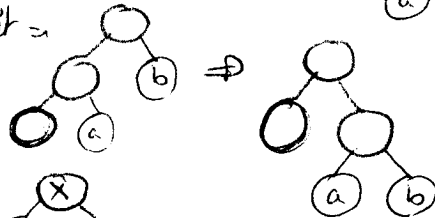
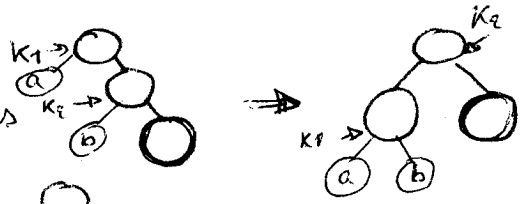


15

5. AVL: Explain single and double rotations used in AVL tree support your explanation by giving examples (10 points)

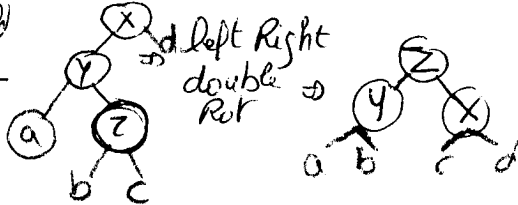
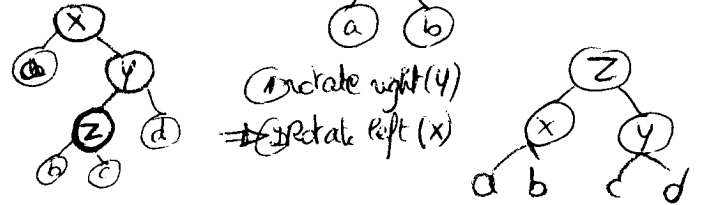
Single Rotation is done in 2 cases:

- adding right child to right subtree \Rightarrow Rotate left \Rightarrow
- adding left child to left subtree \Rightarrow Rotate right \Rightarrow



Double Rotation, also 2 cases:

- adding left child to right subtree \Rightarrow right-left double Rot
- adding right child to left subtree \Rightarrow left-right double Rot



6. SHELL SORT using the sequences : 9, 5 and 1 (15 points)

-9, 25, -3, 0, 14, 55, 12, 7, 4, -7, -8, 45, 3, 9, 5, 1

-9 25 -3 0 14 55 12 7 4 -7 -8 45 3 9 5 1

seq 4) -9 -8 -3 0 9 5 1 7 4 -7 25 45 3 14 55 12

seq 5) -9 -8 -3 0 9 5 1 7 4 -7 25 45 3 14 55 12
-9 -8 -3 0 -7 5 1 3 4 9 12 45 7 14 55 25

seq 1) -9 -8 -3 -7 0 \rightarrow [-9 -8 -7 -3] 0 5 1 3 4 9 12 45 7 14 55 25
-7 go left

7 < 12 swap \rightarrow go left
7 < 9 swap \rightarrow stop
45 > 14 swap \rightarrow stop
14 > 12 stop

[-9 -8 -7 -3 0 1 3 4 5] 9 12 45 7 14 55 25
[7 9 12] 45 14 55 25
] 7 9 12 14 45 55 25

55 < 25 swap
go left \rightarrow 25 < 45 swap, 25 > 14 stop \rightarrow

[-9 -8 -7 -3 0 1 3 4 5 7 9 12 14 25 45 55]

7. QUICK SORT (15 points)

6, 5, 4, 8, 4, 3, 1, 9

Choose Always The First Element As A Pivot.

$$\begin{array}{cccccccc} \text{pivot} & & & & & & & \\ \underline{6} & 5 & 4 & 8 & 4 & 3 & 1 & 9 \\ & & & & & & & \left[\begin{array}{l} 8 > 6 \\ 1 < 6 \end{array} \right] \\ \underline{6} & 5 & 4 & 1 & 4 & \textcircled{3} & 8 & 9 \quad \text{crossing} \\ \underline{3} & \textcircled{5} & 4 & 1 & 4 & \boxed{6} & 8 & 9 \quad \text{2 subarrays} \\ \text{pivot} & & & & & & & \\ & & & & & & & \text{at 8} \\ & & & & & & & \text{crossing} \\ 5 > 3 \Rightarrow i \\ 1 < 3 \Rightarrow j \\ \underline{3} & \textcircled{5} & 4 & 5 & 4 & \boxed{6} & 8 & 9 \quad \text{one element} \Rightarrow \text{No swap} \\ & \text{crossing} & \text{swap} & & & & & \\ 4 & \boxed{3} & 4 & 5 & 4 & \boxed{6} & 8 & 9 \quad \begin{array}{l} 5 > 4 \Rightarrow i \\ 4 \leq 4 \Rightarrow j \end{array} \\ \boxed{1} & \boxed{3} & 4 & \textcircled{4} & 5 & \boxed{6} & 8 & 9 \quad \text{crossing} \Rightarrow \text{swap } 4 \& 4 \\ & & & & & & & \text{finished.} \\ \boxed{1} & 3 & 4 & 4 & 5 & 6 & 8 & 9 \end{array}$$

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Good Luck