American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2008-2009

quiz # 2

1. *(15 points)* Find the domain and the range of the function $f(x,y) = \sqrt{xy}$. Determine the boundary of the domain, is the domain open or closed, also determine if the domain is bounded or unbounded. Find the equation of the level curve passing through $(1,1)$. Determine all the level curves of f.

2. **a.** *(8 points)* Give the first four terms of the Maclaurin series of $f(x) = \sqrt{1-x^2}$, then deduce $f^{(4)}(0)$.

b. *(10 points)* Find the values of a for which

$$\lim_{x\to 0}\left(\frac{\sin x}{x^3} - \frac{1}{x^2} + a\right) = 0$$

3. **a.** *(10 points)* Let $w = xe^{2y}\cos(z)$. Use the chain rule to find the value of dw/dt at the point $(1, \ln 2, 0)$ on the curve $x = \cos t, y = \ln(t+2), z = t$.

b. *(8 points)* Find the equation of the tangent plane to the surface $z = y - x^2$ at $(1,2,1)$.

4. *(16 points)* Find the absolute minimum and maximum of the function $f(x,y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0, y = 4, y = x$.

5. **a.** *(8 points)* Find the direction of maximum increase of $f(x,y) = x^2 - 3xy + 4y^2$ at $P(1,2)$. Is there is a direction $\mathbf{u}$ in which the rate of change of f at $P(1,2)$ equals 14? Justify your answer.

b. *(10 points)* Find the points on the surface $(y+z)^2 + (z-x)^2 = 16$ where the normal line is parallel to the yz-plane.

6. **a.** *(8 points)* Show that the limit of $f(x,y) = \dfrac{3x^2y}{x^2+4y^2}$ exists at $(0,0)$.

b. *(7 points)* Show that $f(x,y) = \dfrac{\sin(x-y)}{|x|+|y|}$ has no limit at $(0,0)$.