American University of Beirut

MATH 201

Calculus and Analytic Geometry III

Fall 2009-2010

quiz # 2

1. *(14 points)* for each of the following functions, find the domain and the range

$$a)\ f(x,y,z)=\sqrt{9-x^2-y^2-z^2} \qquad b)\ g(x,y)=\frac{x}{x^2-y}$$

determine the boundary of the domain, is the domain open or closed, also determine if the domain is bounded or unbounded

2. *(12 points)* use power series to find

$$\lim_{t\to 0}\left(\frac{1}{2-2\cos t}-\frac{1}{t^2}\right)$$

3. *(16 points)* give the Taylor series expansion of $f(x)=\frac{2+x}{(1-x)(1+2x)}$ at $x=-1$, then find $f^{(n)}(-1)$

4. *(20 points)* find the area inside the circle $r=-2\cos\theta$ and outside the circle $r=1$

5. **a.** *(10 points)* find the limit of $f(x,y)=\dfrac{x^2y}{2x^2+y^2}$ at $(0,0)$

b. *(10 points)* use the two path test to show that $f(x,y)=\dfrac{xy-y}{x^2-2x+4y^2+1}$ has no limit at $(1,0)$

6. the Fourier series expansion of the function $f(x)=\begin{cases} x & 0\le x\le \pi \\ 1 & \pi < x\le 2\pi \end{cases}$ is

$$a_0+\sum_{n=1}^{+\infty}\frac{1}{\pi}\,\frac{(-1)^n-1}{n^2}\,\cos(nx)+\sum_{n=1}^{+\infty}b_n\sin(nx)$$

a. *(8 points)* find **ONLY** the coefficient a_0

b. *(10 points)* use the series in part a) to show that $\displaystyle\sum_{k=0}^{+\infty}\frac{1}{(2k+1)^2}=\frac{\pi^2}{8}$