MEN 102
Exam 1
Spring 2006

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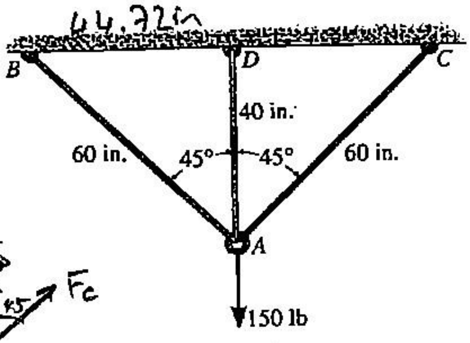
1) (15 pts) Wires AB and AC are made of steel ($E_{st} = 29000 ksi, \alpha_{st} = 8\times10^{-6}$ /° F), and wire AD is made of copper ($E_{cu} = 17000 ksi, \alpha_{cu} = 9.6\times10^{-6}$ /° F). Before the 150-*lb* is applied, AB and AC are each 60 *in* long and AD is 40 *in* long. If the temperature is increased by $80°F$, determine the force in each wire needed to support the load. Each wire has a cross-sectional area of $0.0123\, in^2$.



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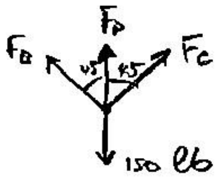
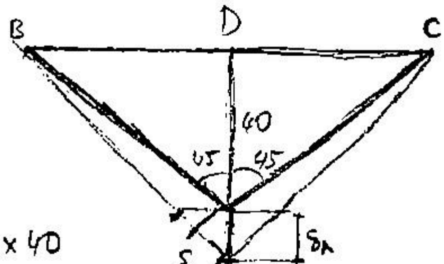
At pt A :



$\Sigma F_y = 0 \Rightarrow$ $F_D + F_B \cos 45 + F_C \cos 45 = 150$ ... (1)

$\Sigma F_x = 0 \Rightarrow$ $F_B = F_C$ ... (2)



$\delta_{A/D} = \delta_A - \delta_{T_A}$ X

$$\frac{F_D \times 40}{0.0123 \times 17000 \times 10^{+3}} = \delta_A - 9.6\times10^{-6} \times 80 \times 40$$

$$\delta_A = \frac{40 F_D}{209.1\times10^3} + 0.03072$$

$\delta_{A/B} = \delta_B - \delta_{T_B}$ X

$$\frac{F_B \times 60}{0.0123 \times 29000 \times 10^3} = \delta_B - 8\times10^{-6} \times 80 \times 60$$

$$\delta_B = \frac{60 F_B}{356.7 \times 10^3} + 0.0384$$

$$(60 + \delta_B)^2 = (40 + \delta_A)^2 + 2000 \quad \ldots \text{Pythag. th.}$$

$$3600 + 120\,\delta_B + \delta_B^2 = 1600 + 80\,\delta_A + \delta_A^2 + 2000$$

$$120\,\delta_B + \delta_B^2 = 80\,\delta_A + \delta_A^2$$

(4) ...

$$0.02\,F_B + 4.608 + 28.29 \times 10^{-9} F_B^2 + 6.45 \times 10^{-6} F_B + 1.47 \times 10^{-3}$$
$$= 0.015\,F_D + 36.59 \times 10^{-9} F_D^2 + 11.75\,F_D + 0.94 \times 10^{-3}$$

from (1) & (2)

$$F_D + 2 F_B \cos 45 = 150$$

$$F_D = 180 - 2 F_B \cos 45$$

$$0.02\,F_B + 4.608 + 28.29 \times 10^{-9} F_B^2 + 6.15 \times 10^{-6} F_B + 1.47 \times 10^{-3}$$

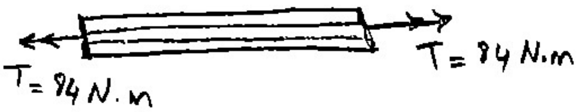
$$= 0.015\,(180$$

$$\frac{1}{x} = \frac{180}{\pi} \Rightarrow 180x = \pi$$

$$x = \frac{\pi}{180}$$

2) (15 pts)A hollow shaft $150mm$ long is used as a torsional spring. The ratio of inside to outside radii is $\frac{1}{2}$. The required stiffness for this spring is $84\,N.m$ of torque per one degree of twist. Let $G = 84000MPa$, calculate the outside radius of the shaft.

$$r_{out} = 2\,r_{in}$$



$$\phi = \frac{TL}{JG} \quad \text{and} \quad \phi = 1^\circ = \quad \frac{1^\circ}{x_{rad}} = \frac{180}{\pi}$$

$$\Rightarrow x = 0.017\ rad$$

✓

$$\phi = \frac{84 \times 150 \times 10^{-3}}{\frac{\pi}{2}\left(r_{out}^4 - \left(\frac{r_{out}}{2}\right)^4\right) \times 84000 \times 10^6}$$

$$0.017 = \frac{12.6}{1.319 \times 10^{11}\left(r_{out}^4 - \frac{r_{out}^4}{16}\right)}$$ ✓

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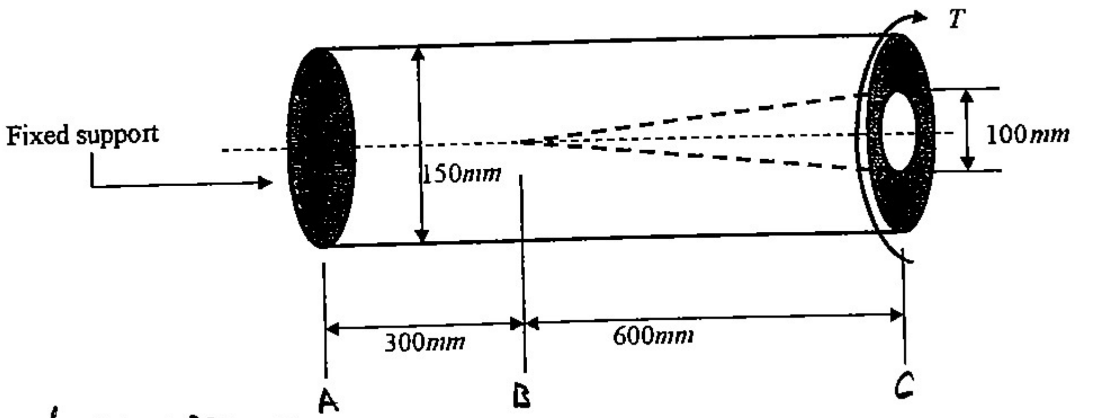
$$2.2423 \times 10^9 \left(\frac{15 r_{out}^4}{16}\right) = 12.6$$

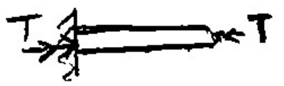
$$r_{out}^4 = 5.99 \times 10^{-9}$$

$$\boxed{r_{out} = 8.79\ mm}$$ ✓

3) (20 pts) A 150 *mm* diameter shaft of an elastic material of known shear modulus of elasticity *G* has in it a conical bore (hole) that is 600 *mm* long, as shown. The shaft is fixed at its left end and free at the right end. The free end is under a torque T as shown. Derive an expression for the maximum angle of twist in the shaft. Do not actually carry-out any integration, rather, set up the expression fully.



for $0 < x < 300$ mm



$$\phi_{B/A} = \frac{T\,(300\times10^{-3})}{G\left(\frac{\pi}{2}\left(\frac{150}{2}\right)^4\right)}$$ ✓

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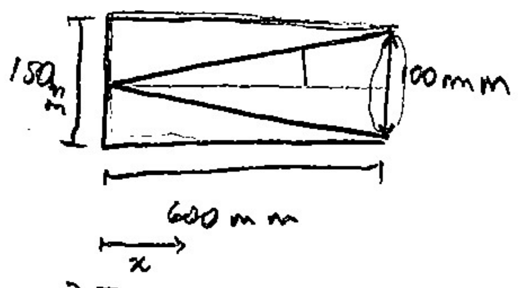
for $300 < x < 600$



$r_{out} = 75$ mm $\qquad r_{in} = \frac{100}{600}\,x$ ✓

$$\phi_{C/B} = \int_0^{600\times10^{-3}} \frac{T\,dx}{G\left(\frac{\pi}{2}\left((75\times10^{-3})^4 - \left(\frac{100}{200}\,x\right)^4\right)\right)}$$

$$\phi_{C/B} = \frac{2T}{\pi G}\int_0^{600\times10^{-3}} \frac{dx}{(75\times10^{-3})^4 - \left(\frac{1}{6}\,x\right)^4}$$

$$\phi_{max} = \phi_C = \phi_{C/A} = \phi_{B/A} + \phi_{C/B}$$

$$\Rightarrow \phi_{max} = \frac{T \times 0.3}{G \times \frac{\pi}{2}\left(\frac{150\times10^{-3}}{2}\right)^4} + \frac{2T}{\pi G}\int_0^{600\times10^{-3}} \frac{dx}{(75\times10^{-3})^4 - \left(\frac{x}{6}\right)^4}$$ ✓

good