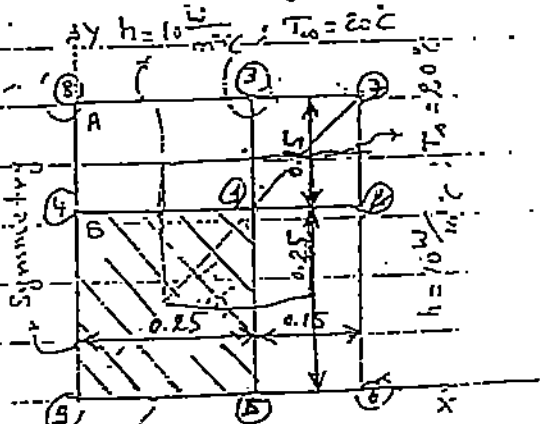
Test MEN 310 (Heat Transfer)

January 7, 1999

1) In a certain application the critical Reynolds number for flow over a flat plate is 10^6. Air at 1 atm, 300 K, and 10 m/s flows across an isothermal plate with this critical Reynolds number, and with a plate temperature of 400 K. The Reynolds number at the end of the plate is 5×10^6. How long is the plate? What will the average heat transfer coefficient be for this system? What is the heat lost from the plate?

2) Consider the following cross-section of an infinite slab (B) covered with an insulation layer (A). The grid and the boundary conditions are as shown and the physical properties are:

	A	B	Units
k	0.1	10.0	W/m°C
ρ	1700	8160	Kg./m³
c_p	1100	460	J/kg°C



The slab is a heat generator at a constant rate of 10 kW/m³ and is initially at 20°C. Determine the temperature of nodes (1) & (2) after 1 s using an explicit technique with a time step of 1 s. What can be done in order to improve the accuracy? (* Mesh sizes are in m).

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$Re_{xi} = 10^6$

$P = 1 atm \rightarrow \rho = \frac{1.0132 \times 10^5}{287 \times 350} = 1.00866\ Kg/m^3$

$T = 300K$

$U = 10 m/sec$

$T_f = \frac{T_w + T_\infty}{2} = 350K$

$\mu = 2.075 \times 10^{-5}\ kg/m.s$

$k = 0.03003$

$Pr = 0.697$

$T = 400K$

$Re_{x=L} = 5 \times 10^6 = \frac{1.00866 \times 10 \times L}{2.075 \times 10^{-5}} \rightarrow L = 10.2859\ m.$

$\rightarrow$ turbulent.

eq. 5-85 $\rightarrow$ $\overline{Nu} = \frac{\overline{h} L}{k} = Pr^{\frac{1}{3}} (0.037\, Re_L^{0.8} - 1670)$

$\rightarrow \overline{h} = 17.5766\ W/m^2 {}^\circ C \rightarrow q = \overline{h} A (T_w - T_\infty) = 18079.17221\ watts$

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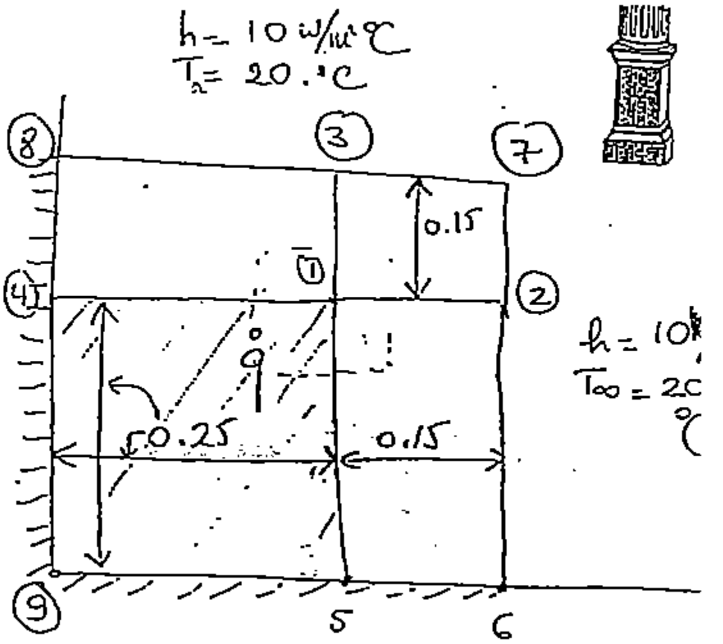
2) $\Delta\tau = 1\,sec.$ $\tau = 1\,sec$ → 1 iteration

$\dot{q} = \ldots kW/m^3$ only in (1)



implicit: $\dot{q}_i + \sum \frac{T_j^{p+1} - T_i^{p+1}}{R_{ij}} = C_i \frac{T_i^{p+1} - T_i^{p}}{\Delta\tau}$

Node 1

$$\frac{T_4 - T_1}{R_{14}} + \frac{T_3 - T_1}{R_{13}} + \frac{T_2 - T_1}{R_{12}} + \frac{T_5 - T_1}{R_{15}} + \dot{q}\,\Delta A = C_1 \frac{T_1^{p+1} - T_1^{p}}{\Delta\tau = 1}$$

$$\frac{1}{R_{14}} = \frac{0.1\left(\ldots + \frac{0.15}{2}\right)}{0.25} + \frac{10.\left(\frac{0.25}{2}\right)}{0.25} = 5.03$$

$$R_{13} = \frac{0.15}{0.1 \times \left(\frac{0.25}{2} + \frac{0.15}{2}\right)} \Rightarrow \frac{1}{R_{13}} = 0.1334$$

$$R_{12} = \frac{0.15}{0.1 \times \left(\frac{0.25}{2} + \frac{0.15}{2}\right)} \Rightarrow \frac{1}{R_{12}} = 0.1334$$

$$\frac{1}{R_{15}} = \frac{0.1 \times \frac{0.15}{2}}{0.25} + \frac{10.\ \frac{0.25}{2}}{0.25} = 5.03$$

$\frac{0.25 \times 0.15}{4}$

$$\Delta A = \frac{0.25}{2} \times \frac{0.25}{2}$$

$$C_1 = 1700 \times 1100 \times \left(\frac{0.25}{2} \cdot \frac{0.15}{2} + \frac{0.15^2}{4} + \ldots\right) + 8160 \times 460\left(\frac{0.25 \cdot 0.25}{4}\right)$$

$$C_1 = 104231.25$$

$\dot{q}_1 = \ldots = 156.25$

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$$5.03T_4 + 0.1334T_3 + 0.1334T_2 + 5.03T_5 - 104241.5768T_1 = -2084781.25 \quad (1)$$

Node 2

$$\frac{T_1-T_2}{R_{12}} + \frac{T_7-T_2}{R_{27}} + \frac{T_6-T_2}{R_{26}} + \frac{20-T_2}{R_{200}} = C_2 \frac{T_2 - 20}{\Delta T}$$

$1/R_{12} = 0.1334$

$R_{27} = \frac{0.15}{0.1 \times 0.15} \times 2 \rightarrow \frac{1}{R_{27}} = 0.05$

$R_{26} = \frac{0.25}{0.1 \times 0.15} \times 2 \rightarrow \frac{1}{R_{26}} = 0.03$

$R_{200} = \frac{1}{10 \times \left(\frac{0.15}{2} + \frac{0.25}{2}\right)} \rightarrow \frac{1}{R_{200}} = 2$

$C_2 = 1700 \times 1100 \times \left[\frac{0.15}{2} + \frac{0.25}{2}\right]\left[\frac{0.15}{2}\right] = 28050$

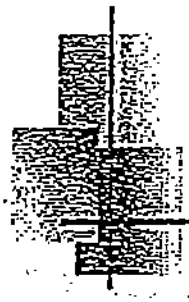
$$\rightarrow \quad 0.1334T_1 + 0.05T_7 + 0.03T_6 - 28052.2134T_2 = -561040 \quad (2)$$

Node 3

$$\frac{T_8-T_3}{R_{38}} + \frac{T_7-T_3}{R_{37}} + \frac{T_1-T_3}{R_{13}} + \frac{20-T_3}{R_{300}} = C_3 \frac{T_3 - 20}{\Delta T}$$

$R_{38} = \frac{0.25}{0.1 \times 0.15} \times 2 \rightarrow \frac{1}{R_{38}} = 0.03$ / $R_{37} = \frac{0.15}{0.1 \times 0.15} \times 2 \Rightarrow \frac{1}{R_{37}} = 0.05$ / $\frac{1}{R_{13}} = 0.1334$ / $\frac{1}{R_{300}} = 10 \times \left(\frac{0.25}{2} + \right) = 2$

$C_3 = 1700 \times 1100 \times \left[\frac{0.15}{2}\right]\left[\frac{0.15}{2} + \frac{0.25}{2}\right] = 28$



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$$0.03T_8 + 0.05T_7 + 0.1334T_1 - 28052.2134T_1 = -561040 \quad (3)$$

Node 4

$$\frac{T_8-T_4}{R_{48}} + \frac{T_1-T_4}{R_{14}} + \frac{T_9-T_4}{R_{94}} + \dot{q}\Delta A = C_4 \cdot \frac{T-20}{\Delta\tau}$$

$R_{48} = \frac{0.15 \times 2}{0.1 \times 0.25} \Rightarrow \frac{1}{R_{48}} = 0.0834$

$\frac{1}{R_{14}} = 5.03$ / $R_{94} = \frac{0.25 \times 2}{10 \times 0.25} \rightarrow \frac{1}{R_{94}} = 5$

$\Delta A = \frac{0.25}{2} \cdot \left[\frac{0.15}{2} + \frac{0.25}{2}\right]$

$= 0.025$

$C_4 = 1700 \times 1100 \times \frac{0.25}{2} \times \frac{0.15}{2} + \text{[illegible]}$

$+ 8160 \times 460 \times \frac{0.25^2}{4} = 76181.$

$$0.0834T_8 + 5.03T_1 + 5T_9 - 76191.3634T_4 = -1523875 \quad (4)$$

Node 5

$$\frac{T_1-T_5}{R_{15}} + \frac{T_6-T_5}{R_{65}} + \frac{T_9-T_5}{R_{59}} + \dot{q}\Delta A = C_5 \cdot \frac{T_5-20}{\Delta\tau}$$

$\frac{1}{R_{15}} = \frac{10 \times \frac{0.25}{2}}{0.25} + \frac{0.1 \times \frac{0.15}{2}}{0.25} = 5.03$

$R_{65} = \frac{0.15 \times 2}{0.1 \times 0.25} \Rightarrow \frac{1}{R_{65}} = 0.0834$

/ $R_{59} = \frac{0.25 \times 2}{10 \times 0.25}$

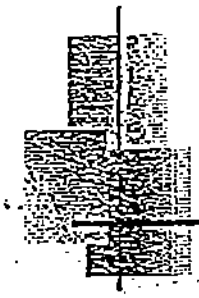
$\frac{1}{R_{59}} = 5$

/ $\Delta A = \frac{0.25}{2} \cdot \frac{0.25}{2}$

$C = 1700 \times 1100 \times \frac{0.25}{2} \times \frac{0.15}{2} + 8160 \times 0$

$C_5 = 76181.25$

$$5.03T_1 + 0.0834T_6 + 5T_9 - 76191.3634T_5 = -1523781.25 \quad (5)$$



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Solving:

$T_1 = 20.001\,49918\,^\circ C$

$T_2 = 20.000000001\,^\circ C$

$T_3 = 20.000000001\,^\circ C$

$T_4 = 20.00328149\,^\circ C$

$T_5 = 20.00205103\,^\circ C$

$T_6 = 20.000000001\,^\circ C$

$T_7 = 20\,^\circ C$

$T_8 = 20.000000002\,^\circ C$

$T_9 = 20.00266411\,^\circ C$.

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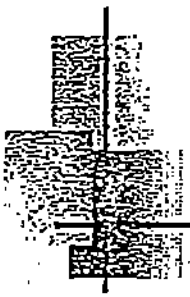

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Node 6: $\frac{T_5-T_6}{R_{56}} + \frac{T_2-T_6}{R_{26}} + \frac{20-T_6}{R_{600}} = C_6 \cdot \frac{T_6-20}{\Delta\tau}$; $\frac{1}{R_{60}} = 10 \times \frac{0.25}{\tau} = 1.25$

$C_6 = 1700 \times 1100 \times 0.15 \times \frac{0.25}{\tau} = 17531.25$

$$0.0834T_5 + 0.03T_2 - 17532.6134T_6 = -350650 \quad (6)$$

Node 7:

$\frac{1}{R_{700}} = 10 \times \frac{0.15}{\tau} = 1.5$

$\frac{T_3-T_7}{R_{37}} + \frac{T_2-T_7}{R_{27}} + \frac{20-T_7}{R_{720}} + \frac{20-T_7}{R_{700}} = C_7 \cdot \frac{T_7-20}{\Delta\tau}$; $C_7 = 1700 \times 1100 \times \frac{0.15}{4}\tau = 10$

$$0.05T_3 + 0.05T_2 - 10520.35T_7 = -210405 \quad (7)$$

Node 8:

$\frac{1}{R_{80}} = 10 \times \frac{0.25}{\tau} = 1.25$

$C_8 = 17531.25$

$\frac{T_4-T_8}{R_{48}} + \frac{T_3-T_8}{R_{38}} + \frac{20-T_8}{R_{80}} = C_8 \cdot \frac{T_8-20}{\Delta\tau}$

$$0.0834T_4 + 0.03T_3 - 17532.6134T_8 = -350650 \quad (8)$$

Node 9:

$\frac{T_4-T_9}{R_{49}} + \frac{T_5-T_9}{R_{59}} + \dot{q}\Delta A = C_9 \frac{T_9-20}{\Delta\tau}$

$\Delta A = \frac{0.25^2}{4} = 0.015625$

$C_9 = 8160 \times 460 \times \frac{0.25^2}{4} = 58650$

$$5T_4 + 5T_5 - 58660T_9 = -1173156.25 \quad (9)$$