TEST– MEN310 (Heat Transfer)
December 11, 2000

1. In areas where the air temperature remains below 0°C for prolonged periods of time, the freezing of water in underground pipes is a major concern. Fortunately, the soil remains relatively warm during those periods, and it takes weeks for the subfreezing temperatures to reach the water mains in the ground. Thus, the soil effectively serves as an insulator to protect the water from subfreezing temperatures in winter.

   The ground at a particular location is covered with snow pack at -10°C for a continuous period of three months (90 days), and the average soil properties at that location are k=0.4W/m°C and α=0.15x10^{-6}m^2/s. Assuming an initial uniform temperature of 15°C for the ground, determine the minimum burial depth to prevent the water pipes from freezing.

2. Consider a large uranium plate of thickness L=4cm, thermal conductivity k=20W/m°C, and thermal diffusivity α=1.0x10^{-5}m^2/s that is initially at a uniform temperature of 200°C. Heat is generated uniformly in the plate at a rate of $\dot{q}$=5x10^{6}W/m^3. At time t=0s, one side of the plate if brought into contact with iced water and is maintained at 0°C at all times, while the other side is subjected to convection environment at T_∞=30°C with a heat transfer coefficient h=50W/m^2°C. Considering a total of three equally spaced nodes in the medium, two at the boundaries and one at the middle, estimate the exposed surface temperature of the plate 1 minute after the start of cooling using the explicit method.

3. The forming section of a plastics plant puts out a continuous sheet of plastic that is 1-m wide and 1-cm thick at a rate of 10m/min. The temperature of the plastic sheet is 95°C when it is exposed in the cooling zone to an air flow at a temperature of 25°C and a velocity of 3m/s on both sides along its surfaces normal to the direction of motion of the sheet (fig.P3). Determine (a) the rate of heat transfer from the plastic sheet to the air and (b) the temperature of the plastic sheet at its exit from the cooling zone. Take the density and specific heat of the plastic sheet to be 5 kg/m^3 and 1675 J/kg°C. The cross section of the cooling duct is assumed to be very large in order for a boundary layer to develop unaffected on both sides of the sheet.

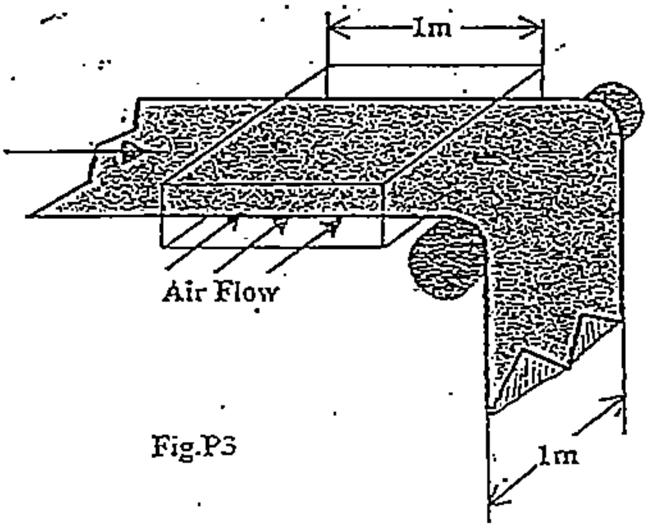

Fig.P3

Points: 1 (20%), 2 (40%) and 3 (40%).

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4.1) $T_o = -10°C$

$\tau = 90$ days $= 90 \times 24 \times 3600 = 7776000$ sec

$k = 0.4 W/m°C$

$\alpha = 0.15 \times 10^{-6} m^2/sec$

$T_i = 15°C$.

freezing occurs @ $T = 0°C$

eq. 4.8 $\rightarrow \dfrac{T(x,\tau) - T_o}{T_i - T_o} = \text{erf} \dfrac{x}{2\sqrt{\alpha\tau}} \rightarrow \dfrac{0+10}{15+10} = \text{erf} \dfrac{x}{2\sqrt{\alpha\tau}} = 0.4$

$\hookrightarrow \dfrac{x}{2\sqrt{\alpha\tau}} = 0.3708 \longrightarrow x = 2 \times 0.3708 \times \sqrt{0.15 \times 10^{-6} \times 7776000}$

$\hookrightarrow \boxed{x = 0.8 m}$

4.2) $L = 4 cm$
$K = 20 W/m°C$

$\alpha = 10^{-5} m^2/sec$

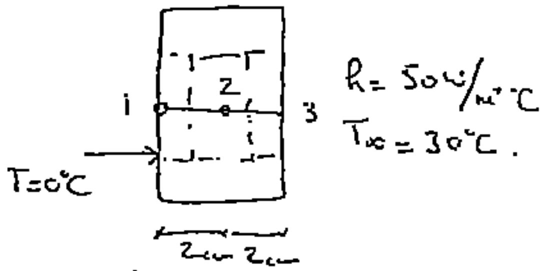
$\dot{q} = 5 \times 10^6 W/m^3$

$T_i = 200°C$

$\tau = 1 min = 60 sec$.

$\alpha = \dfrac{k}{\rho c} \rightarrow \rho c = \dfrac{k}{\alpha}$

$t = 0$ sec $\rightarrow T_o = 0°C$



$$q_i + \sum_j \frac{T_j^p - T_i^p}{R_{ij}} = C_i \frac{T_i^{p+1} - T_i^p}{\Delta\tau}$$

At node 1 $\rightarrow$ we know that $T_1 = 0°C$ for all times.

Node 2: $\dfrac{T_1^p - T_2^p}{R_{12}} + \dfrac{T_3 - T_2^p}{R_{23}} + \dot{q}\Delta V = C_2 \cdot \dfrac{T_2^{p+1} - T_2^p}{\Delta\tau}$

where $R_{12} = \dfrac{\Delta x}{kA} = \dfrac{0.02}{20 \times 1} \longrightarrow \dfrac{1}{R_{12}} = 1000$; $\dfrac{1}{R_{23}} = 1000$; $\Delta V = 0.02$

$C_2 = \rho c V = 20 \times 10^5 \times 0.02$

$\hookrightarrow C_2 = 40000 J/°C$.

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Cont'd #2)

$$\hookrightarrow \quad T_2^{p+1} = \frac{\Delta\tau}{40000}\left[\cancel{1000\,T_1^p} - 2000\,T_2^p + 1000\,T_3^p + 100\,000\right] + T_2^p$$

$$\& \quad \Delta\tau < \frac{C_i}{\Sigma 1/R} = \frac{40\,000}{2000} = 20\,\text{sec}.$$

Node 3

$$\frac{T_2 - T_3}{R_{23}} + \frac{T_\infty - T_3}{R_{3\infty}} + \dot{q}\,\Delta V = C_3 \cdot \frac{T_3^{p+1} - T_3^p}{\Delta\tau}$$

$$\frac{1}{R_{23}} = 1000 \quad ; \quad R_{3\infty} = \frac{1}{hA} \rightarrow \frac{1}{R_{3\infty}} = hA = 50 \times 1 = 50 \quad ; \quad \Delta V = 0.01$$

$$C_3 = 20 \times 10^5 \times 0.01 = 20\,000$$

$$T_3^{p+1} = \frac{\Delta\tau}{C_3}\left[\frac{T_2 - T_3}{R_{23}} + \frac{T_\infty - T_3}{R_{3\infty}} + \dot{q}\,\Delta V\right] + T_3^p$$

$$\hookrightarrow \quad T_3^{p+1} = \frac{\Delta\tau}{20\,000}\left[1000\,T_2^p - 1050\,T_3^p + 1500 + 50000\right] + T_3^p$$

$$\& \quad \Delta\tau < \frac{20000}{1050} = 19.047\,\text{sec}.$$

Choosing $\Delta\tau = 15$ sec $\xrightarrow{\text{T=...}}$ itterating 4 times, we get:

$\tau = 0 \rightarrow T_1 = 0°C \quad ; T_2 = 200 \; ; T_3 = 200$

iterating:

① $T_2 = 162.5°C \longrightarrow T_3 = 203°C$

② $T_2 = \ldots$ $\quad T_3 = \ldots$

③ $T_2 = \ldots$ $\quad T_3 = \ldots$

④ $T_2 = \ldots$ $\quad \boxed{T_3 = \ldots}$

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#3) $T_p = 95°C$ $\quad V = \frac{dl}{dt} = 10 m/min$

$T_a = 25°C$ $\quad V = 3 m/sec$

$\rho_p = 5 Kg/m^3$

$c = 1675 J/Kg°C$

At $T_f = \frac{95+25}{2} = 60°C = 333 K \longrightarrow \begin{cases} \rho_{air} = 1.059 \ Kg/m^3 \\ \mu = 1.997208 \times 10^{-5} \ Kg/m\,sec \\ k = 0.0287414 \ W/m°C \\ Pr = 0.70074 \end{cases}$

$Re_L = \frac{\rho V L}{\mu} = \frac{1.059 \times 3 \times 1 \times 10^5}{1.997208}$

$\hookrightarrow Re_L = 1.59072 \times 10^5 < 5 \times 10^5 \longrightarrow$ laminar $\longrightarrow$ find h.

indication:

$w = 1m$, $t = 1cm$

$T_w = 55°C$, $T_\infty = 25°C$, $U_\infty = 3 m/s$

$\rho = 5 Kg/m^3$

$c = 1675 J/Kg°C$

$\bar{h}$ from Eq. (5-46b) ✓

and then $q = \bar{h} A (T_w - T_\infty) = - - -$

$q = \dot{m} c_p (T_f - T_i)$, with $A = 2 \times 1 \times 1$, $c_p = 1675$, $T_i = 55°C$

$\dot{m} = \rho V A_c = \rho V w t$, with $\rho = 5$, $V = 10 m/min$, $t = 0.01$

$\longrightarrow T_f = - - -$

$q = \dot{m} c (T_{pl_1} - T_{pl_2}) = \bar{h} A \left(\frac{T_{pl_1} + T_{pl_2}}{2} - T_\infty \right)$

solve - - -

for more accuracy, $T_{w_2} = T_f$

$\bar{h} A \left(\frac{T_{w_1} + T_{w_2}}{2} - T_\infty \right) = \dot{m} c_p (T_{w_1} - T_{w_2})$, with $T_\infty = 25$

$\longrightarrow T_{w_2} = - - -$