TEST (Make-Up)– MEN310 (Heat Transfer)

January 05, 2000

1. A thick piece of ceramic material ($k = 0.8$ W/m°C, $\rho = 2700$ kg/m^3, $c = 0.8$ kJ/kg°C) is initially at a uniform temperature of 20 °C. The surface of the material is suddenly exposed to a convection surroundings at 400 °C with $h = 150$W/m^2°C. Determine the time required for a point located 25 mm under the skin of the material to reach steady state conditions.

2. Air at T_∞=20 °C flows with a velocity u_∞=10 m/s along a flat plate L=1 m long which is heated to a constant temperature of 100 °C starting at a distance of 35 cm from the leading edge. Determine the thickness of the thermal boundary layer at the end of the plate and the average heat transfer coefficient over its entire length. Assume $Re_{crit}=10^6$.

3. A rectangular carbon-steel bar of 8 cm by 4 cm cross section ($\alpha=1\times10^{-5}$ m^2/s, k=35 W/m°C) is initially at uniform temperature T_i=425 °C. Suddenly all its surfaces are exposed to cooling by an air stream at T_∞=25 °C with a heat transfer coefficient h=100 W/m^2°C. By using an explicit finite difference scheme and mesh sizes 2 cm by 1 cm (Δx and Δy respectively), calculate the center temperature 10 s after the start of cooling.

Points: 1 (25%), 2 (25%) and 3 (50%).

Jan. 05, 2000

1) $k = 0.8\ W/m°C$, $\rho = 2700\ Kg/m^3$, $c = 0.8\ KJ/Kg°C = 800\ J/Kg°C$

$T_i = 20°C$

$T_\infty = 400°C$ $\quad h = 150\ W/m^2°C$.

$x = 0.025\ m$.

$\alpha = \frac{k}{\rho c} = \frac{0.8}{2700 \times 800} = 3.704 \times 10$ m^2/sec

Using eq. (4.15) with a steady temperature of $T = \frac{20+400}{2} = 210°C$

check ????

with $T(x, \tau)$ as steady temperature.

→ iterate, find τ.

~~set $\tau \to \infty$, a steady temp. should be obtained~~

$$\cancel{\rightarrow \ T = (T_\infty - T_i)\left[1 - \underbrace{erf\left(\frac{x}{2\sqrt{\alpha\tau}}\right)}_{0} - \underbrace{\left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha\tau}{k^2}\right)\right]}_{\infty}\left[1 - \underbrace{erf\left(\frac{x}{2\sqrt{\alpha\tau}} + \frac{h\sqrt{\alpha\tau}}{k}\right)}_{\infty}\right]\right]}$$

$$\cancel{T = (T_\infty - T_i)(1 - L)(1 - L) + T_i = (T_\infty - T_i)(1 - L^2) + T_i}$$

~~Assuming $L = 10^{10} \rightarrow T \rightarrow \infty$ (illogical) where $L >>>>$~~

Jan. 05, 2000

2) $T_\infty = 20°C$ $\qquad$ $Re_{cr} = 10^6$

$U_\infty = 10 m/sec$

$L = 1 m$

$T_p = 100°C$, $X_o = 0.35$

$$\rightarrow T_f = \frac{100+20}{2} = \frac{120}{2} = 60°C = 333 K \longrightarrow \begin{cases} \rho = 1.059 \ Kg/m^3 \\ \mu = 1.997 \times 10^{-5} \ Kg/m.sec \\ k = 0.0287414 \ w/m.°C \\ P_r = 0.70074 . \end{cases}$$

$$Re_L = \frac{\rho U L}{\mu} = \frac{1.059 \times 10 \times 1}{1.997} \cdot 10^5$$

$Re_L = 5.3029 \times 10^5 < Re_{cr} \longrightarrow$ Laminar

$$\text{eq. } 5.36 \longrightarrow \delta_t = \frac{\delta}{1.026} P_r^{-\frac{1}{3}} \left[1 - \left(\frac{x_o}{x}\right)^{3/4} \right]^{1/3}$$

we need $\delta = \dfrac{4.64 x}{Re^{1/2}}$

$$\rightarrow \delta_t = \frac{4.64 x}{Re^{1/2}} \cdot P_r^{-\frac{1}{3}} \left[1 - \left(\frac{x_o}{x}\right)^{3/4} \right]^{\frac{1}{3}}$$

$$\delta_{t_{x=L}} = 5.8595 \times 10^{-3} \ m = \boxed{5.8595 \ mm. = \delta_{t_{x=L}}}$$

Jan. 05, 2000

using eq. 5.43

$$\longrightarrow Nu_x = 0.332\, Re_x^{\frac{1}{2}}\, Pr^{\frac{1}{3}} \left[1 - \left(\frac{x_0}{x} \right)^{3/4} \right]^{-\frac{1}{3}}$$

$$Nu_{x=L} = 262.9012 = \frac{h x}{k} \rightarrow h_{x=L} = 7.556149 \ W/m^2 {}^\circ C$$

$$\longrightarrow \bar{h} = 2h_{x=L} \longrightarrow \boxed{\bar{h} = 15.1123 \ W/m^2 {}^\circ C}$$

$\bar{h} = 2h_L$ for $x_0 = 0$

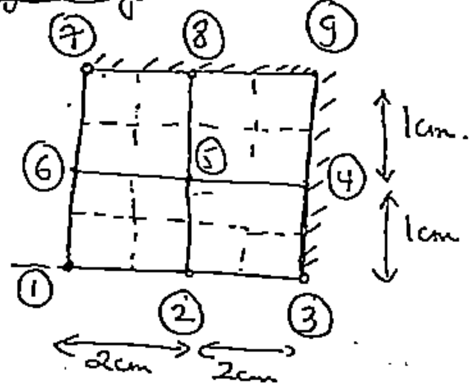
here, like Ex 5-6)
we must do integration:

$$\bar{h} = \frac{\int_{x_0}^{L} h_x\, dx}{L - x_0}$$

$L = 1$, $x_0 = 0.35$

Jan. 05, 2000

3) By symmetry:



$\alpha = 10^{-5}\, m^2/sec$
$k = 35\ W/m^\circ C$
$T_i = 425^\circ C$
$T_\infty = 25^\circ C$
$h = 100\ W/m^2\,^\circ C.$

$\alpha = \frac{k}{\rho c} \rightarrow \rho c = \frac{k}{\alpha}$

Node 1:

$$\frac{T_2 - T_1}{R_{12}} + \frac{T_6 - T_1}{R_{16}} + \frac{T_\infty - T_1}{R_{1\infty\leftarrow}} + \frac{T_\infty - T_1}{R_{1\infty\downarrow}} = C_1 \cdot \frac{T_1^{p+1} - T_1}{\Delta t}$$

where $R_{12} = \frac{0.02}{35 \times 0.005}$, $R_{16} = \frac{0.01}{35 \times 0.01}$, $R_{1\infty\leftarrow} = \frac{1}{100 \times 0.005}$, $R_{1\infty\downarrow} = \frac{1}{100 \times 0}$

$$C_1 = \rho c V = \frac{k}{\alpha} V = 35 \times 10^5 \times 0.01 \times 0.005$$

$$\Delta t = \frac{C_i}{\Sigma 1/R} = \text{---}$$

Node 2 — easy $(1, 5, 3, \infty\downarrow)$

Node 3

$$\frac{T_2 - T_3}{R_{23}} + \frac{T_4 - T_3}{R_{34}} + \frac{T_\infty - T_3}{R_{3\infty}} = C_3 \frac{T_3^{p+1} - T_3^{p}}{\Delta t}$$

$R_{3\infty\downarrow} = \frac{1}{100 \times 0.01}$, $\Delta t = \frac{C_i}{\Sigma 1/R}$

Jan. 05, 2000

Node 4 $\longrightarrow$ $(9, 3, 5)$

Node 5 $\longrightarrow$ $(8, 6, 2, 4)$

Node 6 $\longrightarrow$ $(7, 5, 1, \infty)$

Node 7 $\longrightarrow$ $(6, 8, \infty)$

Node 8 $\longrightarrow$ $(7, 9, 5)$

Node 9 $\longrightarrow$ $(8, 4)$

choose ΔT_{min} and iterate.