501

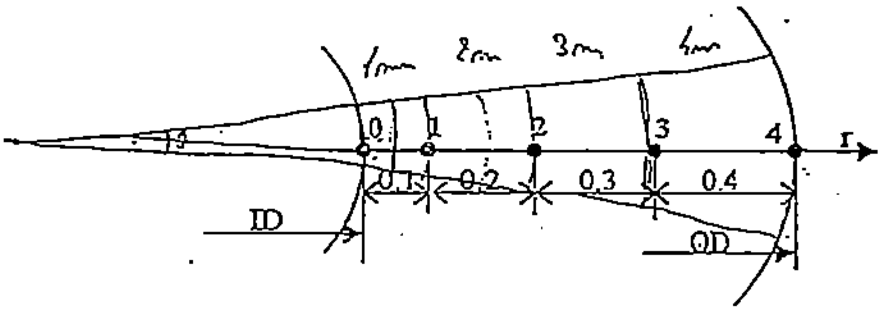
TEST– MEN310 (Heat Transfer)

March 28, 2001

1. Consider a pipe (8-cm ID, 10-cm OD) made of stainless steel in which energy is generated at a constant rate of $100 MW/m^3$. The outer side is insulated while the inner dissipates heat by convection with a heat transfer coefficient of $1000\ W/m^{2\circ}C$ into a fluid at 80°C. Find the temperature distribution as well as the heat rate along the thickness of the pipe wall. Determine the temperatures of the outer and inner sides as well as the heat lost on both sides of the pipe wall.

2. Steam in a heating system flows through tubes whose outer diameter is $D_1 = 3cm$ and whose walls are maintained at a temperature of 120°C. Circular copper fins of outer diameter $D_2 = 6cm$ and constant thickness t=2mm are attached to the tube. 30 The space between fins is 3mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surroundings at $T_\infty = 25°C$, with a heat transfer coefficient of $h = 60 W/m^{2\circ}C$. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

3. Solve problem 1 using the finite difference technique with the following mesh (dimensions in cm):



Compare the resulting numerical solution with the corresponding analytical solution of problem 1 and show whether such a numerical solution is good or not. What can be done in order to improve the accuracy of results (to get closer to the analytical one).

Points: 1 (30%), 2 (30%) and 3 (40%).

50

30

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#1

$k = 1000\ W/m^2\,°C$

$T_\infty = 80°C$

$q = 0 \iff \dfrac{dT}{dr} = 0$

$\dot{q} = 100\ MW/m^3$, $k = 17\ W/m\,°C$

0 insulated

- $q_{generated} = q_{convection\ in} + \cancel{q_{convection\ out}}$

$\hookrightarrow \quad \dot{q}V = hA(T_i - T_\infty)$

$L = 1m$

$\hookrightarrow \quad T_i = \dfrac{\dot{q}V + hAT_\infty}{hA} = \dfrac{100\times10^6 \times \pi(0.05^2 - 0.04^2)\times 1 + 1000\times 2\pi\times 0.04}{1000\times 2\pi\times 0.04}$

$\hookrightarrow \quad \boxed{T_i = 1205°C}$

- $T = -\dfrac{\dot{q}r^2}{4k} + C_1 \ln r + C_2$

boundary conditions $\rightarrow \left.\dfrac{dT}{dr}\right)_{r=5cm} = 0 \quad ; \quad \left.T_i\right)_{r=4cm} = 1205°C$

$\hookrightarrow \dfrac{dT}{dr} = -\dfrac{\dot{q}r}{2k} + \dfrac{C_1}{r} = 0 \longrightarrow C_1 = \dfrac{\dot{q}r^2}{2k} \longrightarrow C_1 = 7352.941176$

$\hookrightarrow C_2 = 1205 + \dfrac{\dot{q}r^2}{4k} - C_1 \ln r \longrightarrow C_2 = 27226.14577$

$$\boxed{T = -1470588.235\,r^2 + 7352.94 \ln r + 27226.14577}$$

- $\left.T_o\right)_{r=5cm} = 1522.236°C$

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$$q_{conv} = hA(T_i - T_\infty) = 1000 \times 2\pi \times 0.04 \times (1205 - 80) = 282743.3388 \text{ Watt}$$

Heat rate $\longrightarrow$ $q = -KA\dfrac{dT}{dr} = -KA\left(-\dfrac{\dot{q}r}{2K} + \dfrac{C_1}{r}\right)$

$\hookrightarrow$ $q = -17 \times 2\pi \times 0.04\left(-\dfrac{\dot{q}r}{2K} + \dfrac{C_1}{r}\right)$

$\hookrightarrow$ $\boxed{q = 12566370.61\, r - \dfrac{31415.92653}{r}}$

$r = 0.04 \longrightarrow q = -282743.3389$ Watts

$r = 0.05 \longrightarrow q = 0$

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#2

OD $\rightarrow$ 3cm

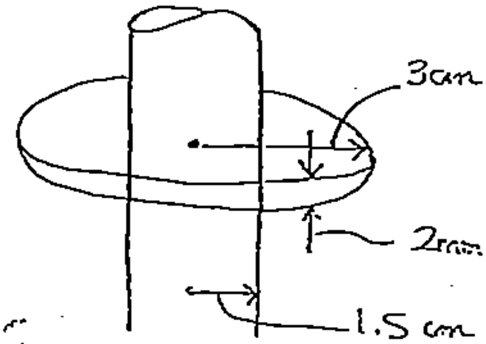
$T = 120\,^\circ C$

$k_{copper} = 386\ W/m\,^\circ C$

$T_\infty = 25\,^\circ C$

$h = 60\ W/m^2\,^\circ C$

200 fins/meter



- $q_{without} = hA(T - T_\infty) = 60 \times \pi \times 0.03 \times 1m \times (120 - 25) = 537.2123$ Watts

- $q_{with} = q_{fin} + q_{without\ fin}$ — for the space between the fins

where $\rightarrow$ $q_{without\ fin} = hA(T - T_\infty) = 60 \times \pi \times 0.03 \times 0.6 \times (120 - 25) = 322.32$ Watts ($0.6 = 200 \times 0.003$)

$L_c = 1.5 + \frac{0.2}{2} = 1.6$ cm

$r_{2c} = r_1 + L_c = 1.5 + 1.6 = 3.1$ cm

$A_m = 0.32\ cm^2$

$\rightarrow \frac{r_{2c}}{r_1} = 2.06667$; $L_c^{3/2}\left(\frac{h}{k A_m}\right)^{1/2} = 0.141$

$\rightarrow \eta = 93\%$

$q_{max} = hA(T - T_\infty) = 60 \times 2 \times \pi\,(0.031^2 - 0.015^2)(120 - 30)$

$= 24.97189168$ Watts

$\hookrightarrow q_{actual} = \eta\, q_{max} \times 200$ fin $= 4644.771853$ Watts

$\hookrightarrow q_{with} = q_{without} + q_{actual} = 4967.09926$ Watts

- $\frac{q_{with}}{q_{without}} = 9.24 \iff \Delta q = 4429.88511$ Watts

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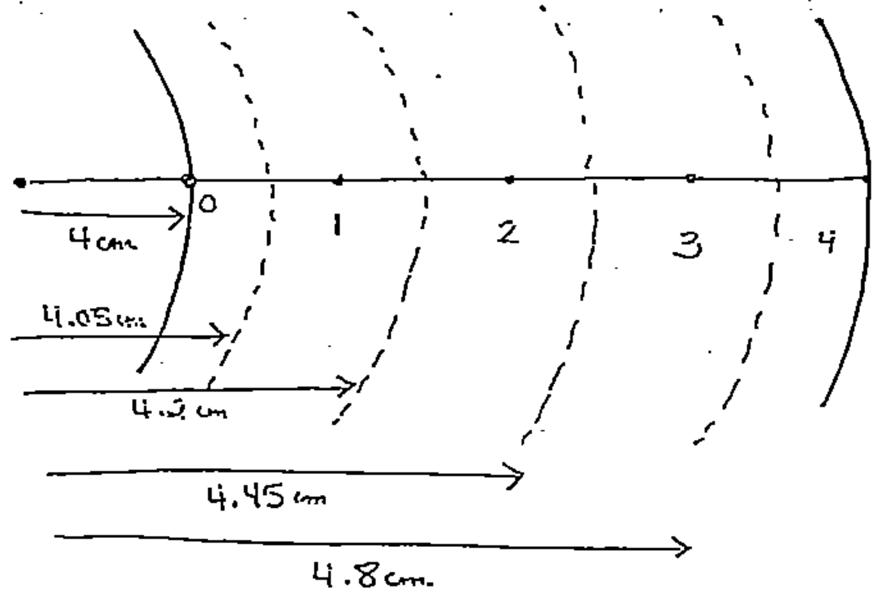
#3



Node 0:

$$\frac{T_1 - T_0}{R_{10}} + \frac{T_\infty - T_0}{R_{0\infty}} + \dot{q}\Delta V = 0$$

$$R_{10} = \frac{\ln(4.1/4)}{2\pi k L} \quad ; \quad R_{0\infty} = \frac{1}{h \times 2\pi \times 0.04}$$

$$\Delta V = \pi(0.0405^2 - 0.04^2)$$

$$\hookrightarrow \boxed{-4577.077\,T_0 + 4325.75\,T_1 + 32751.07 = 0}$$

Node 1:

$$\frac{T_0 - T_1}{R_{01}} + \frac{T_2 - T_1}{R_{21}} + \dot{q}\Delta V = 0 \qquad R_{21} = \frac{\ln(4.3/4.1)}{2\pi k L}$$

$$\Delta V = \pi(0.042^2 - 0.0405^2)$$

$$\boxed{4325.75\,T_0 - 6568.423\,T_1 + 2242.67\,T_2 + 38877.20909 = 0}$$

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Node 2:

$$\frac{T_1 - T_2}{R_{12}} + \frac{T_3 - T_2}{R_{32}} + \dot{q}\,\Delta V = 0 \qquad R_{23} = \frac{\ln(4.6/4.3)}{2\pi k L}$$

$$\Delta V = \pi\,(0.0445^2 - 0.042^2)$$

$$\boxed{2242.67\,T_1 - 3826.479\,T_2 + 1583.809\,T_3 + 67936.94113 = 0}$$

Node 3:

$$\frac{T_4 - T_3}{R_{34}} + \frac{T_2 - T_3}{R_{23}} + \dot{q}\,\Delta V = 0 \qquad R_{34} = \frac{\ln(5/4.6)}{2\pi k L}$$

$$\Delta V = \pi\,(0.048^2 - 0.0445^2)$$

$$\boxed{1583.809\,T_2 - 2864.84\,T_3 + 1281.0277\,T_4 + 101709.0622 = 0}$$

Node 4:

$$\frac{T_3 - T_4}{R_{34}} + \dot{q}\,\Delta V = 0 \qquad \Delta V = \pi\,(0.05^2 - 0.048^2)$$

$$\boxed{1281.0277\,T_3 - 1281.0277\,T_4 + 61575.21601 = 0}$$

Solving the above equations $\longrightarrow$

$T_0 = 1204.9\,°C$ $\quad T_1 = 1267.4\,°C$ $\quad T_2 = 1370.5\,°C$

$T_3 = 1473.5\,°C$ $\quad T_4 = 1521.6\,°C$