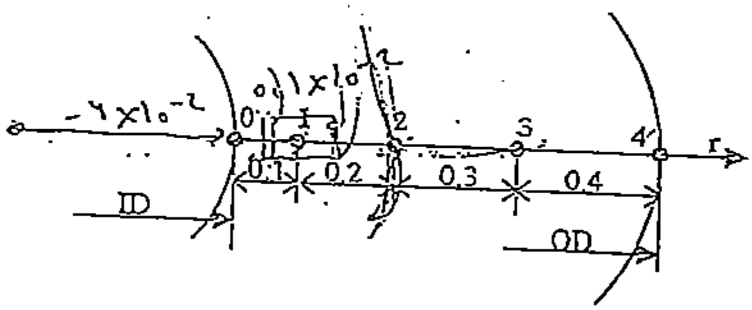
TEST– MEN310 (Heat Transfer)

April 07, 2000

1. Consider a pipe (8-cm ID, 10-cm OD) made of stainless steel in which energy is generated at a constant rate of $80 MW/m^3$. The outer side is insulated while the inner dissipates heat by convection with a heat transfer coefficient of 4000 $W/m^{2\circ}C$ into a fluid at 100°C. Find the temperature distribution as well as the heat rate along the thickness of the pipe wall. Determine the temperatures of the outer and inner sides as well as the heat lost on both sides of the pipe wall.

2. Consider a steel rod of length L=50cm, diameter D=2cm and thermal conductivity k=55 $W/m^\circ C$. One end of the rod is thermally attached to a hot surface maintained at T_1=150°C, and the other is attached to a cold surface at 50°C. The rod dissipates heat into ambient air at T_∞=20°C with a heat transfer coefficient of 15 $W/m^{2\circ}C$. Determine the heat lost by the rod (to the ambient air). What fraction of this heat loss is from the surface maintained at 150°C?

3. Solve problem 1 using the finite difference technique with the following mesh (dimensions in cm)



Compare the resulting numerical solution with the corresponding analytical solution of problem 1 and show whether such a numerical solution is good or not. What can be done in order to improve the accuracy of results (to get closer to the analytical one).

Points: 1 (30%), 2 (30%) and 3 (40%).

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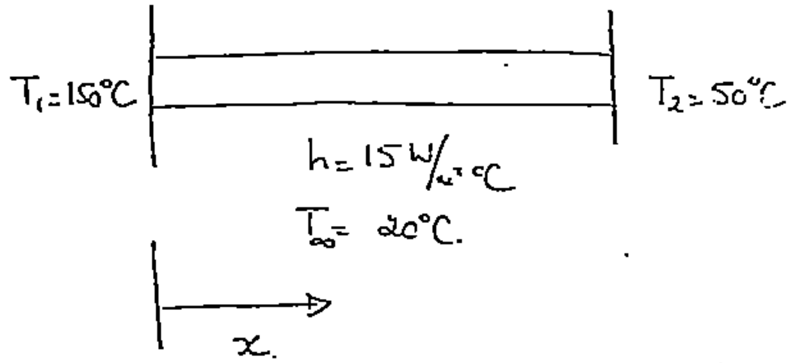
#1 Refer to March 28, 2001

#2



$q = ??$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad \text{where } m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{15 \times 2\pi r}{55\pi r^2}} = \sqrt{\frac{30}{55 \times 0.01}}$$

$\hookrightarrow \theta = C_1 e^{-mx} + C_2 e^{mx}$ $\qquad \hookrightarrow m = 7.385$

$x = 0 \rightarrow T_1 = T = 150°C \rightarrow \theta = T - T_\infty = T_1 - T_\infty$

$x = L \rightarrow \theta = T_2 - T_\infty$

Solving for C_1 & C_2 $\rightarrow C_1 = 129.33297$; $C_2 = 0.66703$

Replacing C_1, C_2, and $\theta = T - T_\infty$

we find that the temperature distribution is

$$\boxed{T = 129.33297 e^{-7.385x} + 0.66703 e^{7.355x} + 20}$$

To find the location of the minimum temperature $\rightarrow \frac{dT}{dx} = 0$

$\hookrightarrow -955.1239835 e^{-7.385x} + 4.92601655 e^{7.385x} = 0$

$\longrightarrow \boxed{x = 0.3566222409\ m}$

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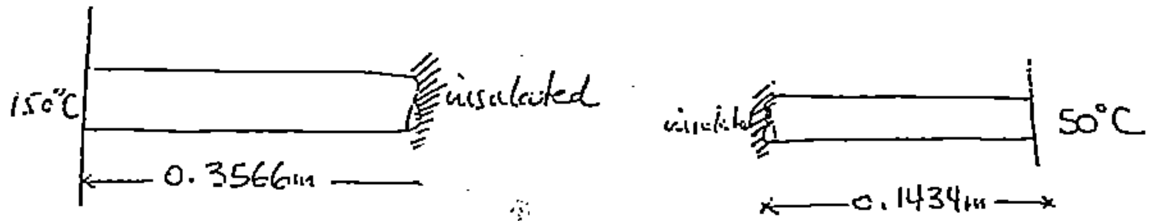
@ $x = 0.3566222409 \longrightarrow T = 38.576°C$.

We can now cut the original bar in two parts as:



$q_{Total} = q_1 + q_2$

• $q_1 = \sqrt{hPkA} \cdot \theta_0 \tanh mL$ $L_c = L + \frac{d}{4}$

$h = 15$ / $P = 2\pi r = \pi \times 0.5$ / $k = 55$ / $A = \pi r^2$

$\theta_0 = T_0 - T_\infty = 150 - 20 = 130$

$m = 7.385$

↳ $q_1 = 82.09637375$ Watts 16.419

15.03176596

• Similarly for $q_2 =$ ~~[illegible]~~ Watts where $\theta_0 = 50 - 20 = 3$

↳ $\boxed{q_T = 97.12813971 \text{ Watts}}$

$\frac{q_1}{q_T} \times 100 = 84.523\%$

43| Refer to March 28, 2001.