F99

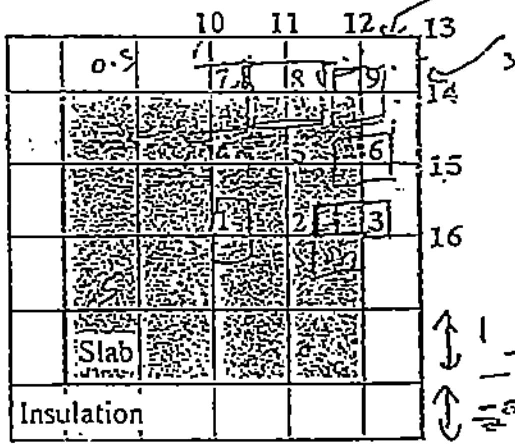
TEST– MEN310 (Heat Transfer)

November 17, 1999

1. A spherical storage tank (220-cm ID, 222-cm OD) made of stainless steel and containing hot water is covered with a glass wool insulation layer 9-cm thick. The water is maintained at 90°C by an internal source of heat and the corresponding convective heat transfer coefficient is 200 W/m^2°C (heat transfer between the hot water and tank wall). The whole system is exposed to cooler air at 10°C with a convective heat transfer coefficient of 20 W/m^2°C. Determine the heat lost to the surroundings and the temperature drops across the different materials. Show whether the system is safe or not and why.

2. The same storage tank (as in problem 1) with its content and a 0.1-cm thick protective covering (k=1 W/m°C) instead of the insulation is buried in the earth at a location where the conductivity is 1.6 W/m°C. The distance between the earth surface where the temperature is constant at 10 °C and the center of the tank is 2 m. Determine the amount of heat to be provided by the internal heat source in order to maintain the temperature of the hot water in the tank at 90°C. Justify the uniform temperature assumption used in the calculation by evaluating the temperature of the system outer surface. Is a buried protected tank better than a bare insulated one from a heat loss standpoint?

3. Consider the square cross-section (4 x 4cm) of an electric-conducting slab. (k=5 W/m°C) covered by a 0.75-cm thick insulative layer (k=0.5 W/m°C) as sketched on the following figure. The slab generates heat at a rate given by $C_0(1+0.05T)$ where C_0 is a constant equal to 100MW/m^3. The outer surface of the insulation is maintained at constant temperature of 30°C. Determine the temperature distribution across the cross-section. Assume a grid and node numbering as shown.



Points: 1 (25%), 2 (25%) and 3 (50%).

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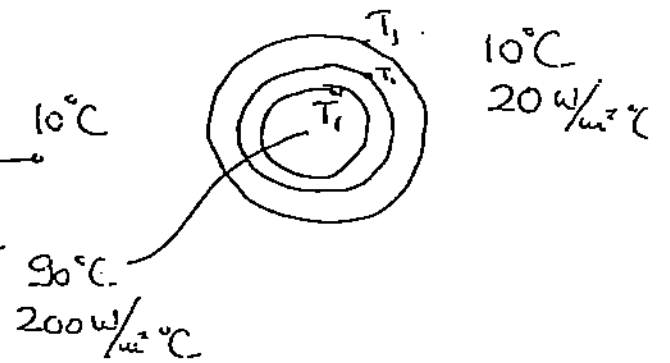
#1) ID = 220 cm
O.D = 222 cm

$K_{wool} = 0.038$

$K = 17 \ W/m°C$



90°C — T_1 — T_2 — T_3 — 10°C

$\frac{1}{h A_{inner}}$, $\frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K}$, $\frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K}$, $\frac{1}{h A_{outer}}$

$A \rightarrow 4\pi R^2$

$q = \frac{\Delta T}{\Sigma R} = \frac{90-10}{\Sigma R} = 551.9$ watts

$q = \frac{\Delta T_2}{K_2} \rightarrow \Delta T_2 =$ ___ $\rightarrow T_2 =$ ___

$q = \frac{\Delta T_1}{K_1} \rightarrow \Delta T_1 =$ ___ $\rightarrow T_1 =$ ___

$q = \frac{\Delta T_3}{K_3} \rightarrow \Delta T_3 =$ ___ $\rightarrow T_3 =$ ___

Check the temperature $T_3 = 13.164 °C$

$\longrightarrow$ safe system.

#2) $K = 1.6$ / $T_E = 10°C$ (isothermal)

$q = KS\Delta T$; $S = \frac{4\pi r}{1 - r/2D}$

Thus: 90°C — — — — 10°C.

$\frac{1}{hA}$, $\frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K}$, $\frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K}$, $(KS)^{-1} \longrightarrow \frac{1}{KS}$

$q = \frac{\Delta T}{\Sigma K} =$ ___ .

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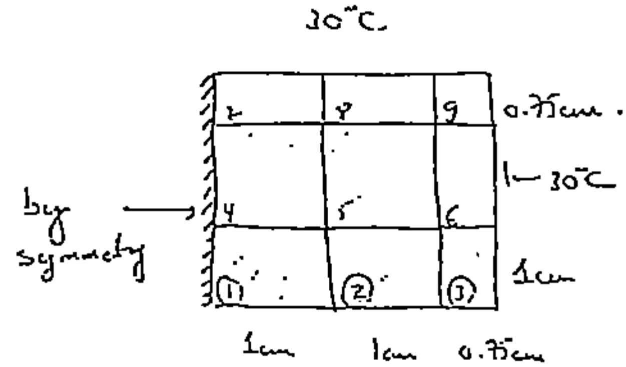
#3) $\dot{q} = C_o(1+0.05T)$

$C_o = 10^3$ W/m³



Node 1 $\frac{T_4 - T_1}{R_{14}} + \frac{T_2 - T_1}{R_{12}} + \dot{q}\Delta V = 0$ $R_{14} = \frac{0.01}{5 \times 0.005}$; $\Delta V = 2.5 \times 10^{-5}$

$\hookrightarrow$ $\boxed{-5T_1 + 2.5T_2 + 2.5T_4 + 125T + 2500 = 0}$

Node 2: $\frac{T_1 - T_2}{R_{12}} + \frac{T_5 - T_2}{R_{25}} + \frac{T_3 - T_2}{R_{23}} + \dot{q}\Delta V = 0$

$\frac{1}{R_{12}} = 2.5$; $R_{25} = \frac{0.01}{5 \times 0.01}$; $R_{23} = \frac{1}{2.5}$; $\Delta V = 5 \times 10^{-5}$

$\hookrightarrow$ $\boxed{2.5T_1 - 10T_2 + 2.5T_3 + 5T_5 + 250T + 5000 = 0}$

Node 3: $\frac{T_2 - T_3}{R_{23}} + \frac{30 - T_3}{R_{3,30}} + \frac{T_6 - T_3}{R_{36}} + \dot{q}\Delta V = 0.$

$\frac{1}{R_{23}} = 2.5$; $\frac{1}{R_{3,30}} = \frac{0.5 \times 0.005}{0.0075} = 0.34$; $\frac{1}{R_{36}} = \frac{5 \times 0.005}{0.01} + \frac{0.5 \times 0.00375}{0.01} = 2.6875$

$\Delta V = 0.01 \times 0.5 \times (0.005 + 0.00375)$ $\left[\left(\frac{1}{R_x} = \frac{1}{R_1} + \frac{1}{R_2}\right)\right]$

$\hookrightarrow \Delta V = 4.375 \times 10^{-5}$

$\hookrightarrow$ $\boxed{2.5T_2 - 5.5275T_3 + 2.6875T_6 + 218.75T + 4375.2 = 0}$

Node 4: $\rightarrow$ $\boxed{2.5T_1 - 10T_4 + 5T_5 + 2.5T_7 + 250T + 5000 = 0}$

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Cont'd #3

Node 5: — $5T_2 + 5T_4 - 20T_5 + 5T_6 + 5T_8 + 500T + 10000 = 0$

Node 6: $\frac{T_5 - T_6}{R_{56}} + \frac{T_9 - T_6}{R_{69}} + \frac{T_3 - T_6}{R_{36}} + \frac{30 - T_6}{R_{6,30}} + \dot{q}\,\Delta V = 0$

$\frac{1}{R_{56}} = 5$; $\frac{1}{R_{96}} = \frac{5 \times 0.005}{0.01} + \frac{0.5 \times 0.00775}{0.01} = 2.6875$

$\frac{1}{R_{36}} = 2.6875$; $\frac{1}{R_{6,30}} = 0.67$; $\Delta V = 0.01 \times (0.005 + 0.00775) = 8.75 \times 10$

↳ $2.6875\,T_3 + 5T_5 - 11.045\,T_6 + 2.6875\,T_9 + 437.5\,T + 8770.1 = 0$

Node 7: — — —

Node 8: ____

Node 9. ____

Assuming $T = \frac{T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9}{9}$

↳ Solve the system

↳ obtain the different nodal temperatures.