NDU - Faculty of Engineering
EEN 344
Communication Systems I

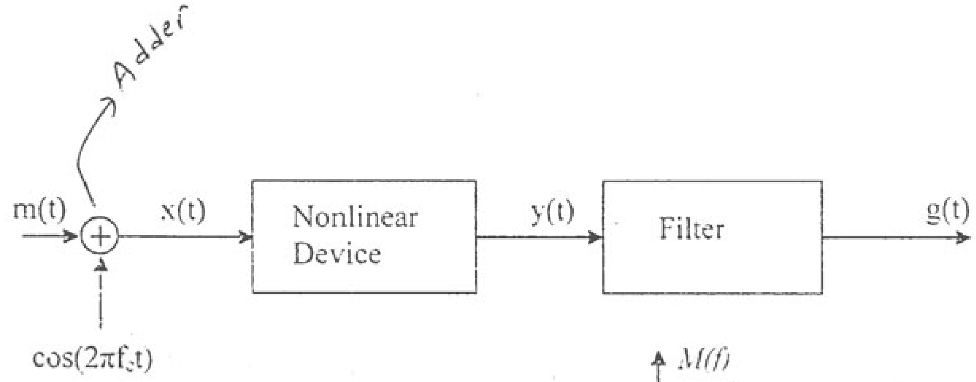
Exam 1
Time: 1 hr 20 min

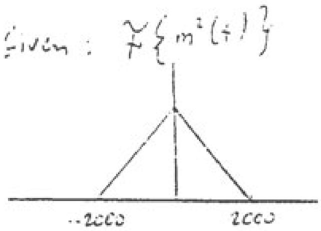
Name: ______________________ **ID #:** ______________

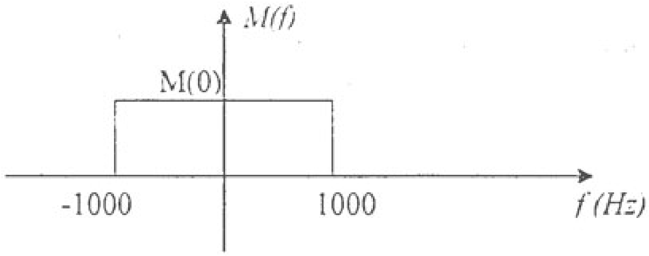
Problem I (20 Points)

Consider the spectrum of the baseband signal and the communication system shown in the figure below. Assume that the output of the nonlinear device is defined as:

$$y(t) = 8x(t) + 2x^2(t)$$







1. Evaluate y(t). $x(t) = m(t) + \cos(2\pi f_c t)$

$$y(t) = 8x(t) + 2x^2(t)$$

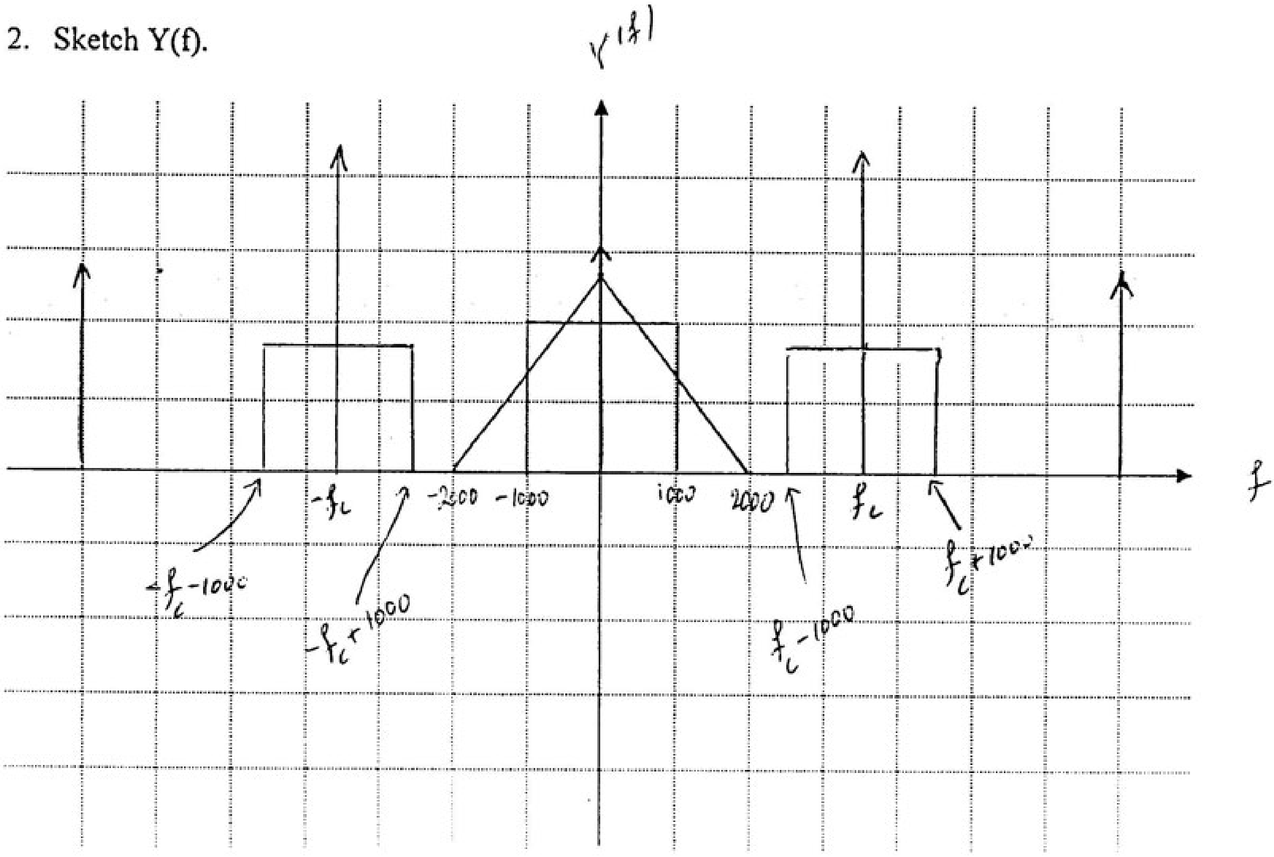
$$= 8\left[m(t) + \cos 2\pi f_c t\right] + 2\left[m(t) + \cos 2\pi f_c t\right]^2$$

$$= 8m(t) + 8\cos 2\pi f_c t + 2m^2(t) + 2\cos^2 2\pi f_c t$$
$$+ 4m(t)\cos 2\pi f_c t$$

$$= 8\left[1 + \frac{1}{2}m(t)\right]\cos 2\pi f_c t + 8m(t) + 2m^2(t)$$

$$+ 1 + \cos 4\pi f_c t$$

2. Sketch Y(f).



3. Find the AM wave component contained in the output y(t). What is the amplitude sensitivity of this AM signal?

$$8\left[1+\frac{1}{2}m(t)\right]\cos 2\pi f_c t$$

$$k_a = \frac{1}{2}$$

4. Suppose that this system is to be used as a square-law modulator. Describe the filter that would yield an AM signal for g(t). State the necessary filter type and the cutoff frequency of interest.

Bandpass filter: $f_c \geq 3000$ Hz

Bandwidth $= B = 2000$ Hz

5. Assume now for the sake of this question that this system is to be used as a square-law detector. Describe the filter that would yield m(t) at its output. State the necessary filter type and the cutoff frequency of interest. Will there be distortion in the output? Find the signal to distortion ratio and discuss the conditions that would reduce distortion.

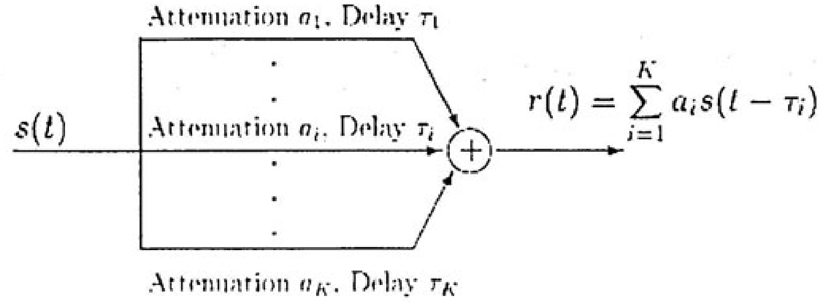
Lowpass filter: Bandwidth $= W = 1000$ Hz

Signal: $8m(t)$ Distortion: $2m^2(t)$

$\therefore$ SDR $= \frac{8m(t)}{\ldots} = \frac{4}{\ldots}$ The smaller $m(t)$ ...

Problem 2 (20 Points)

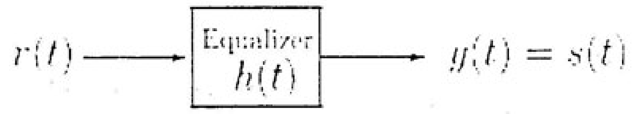
In wireless communications, the signal arriving at the receiver frequently suffers what is called a multipath distortion. More specifically, the received signal r(t) is the superposition of multiple versions of the transmitted signal s(t), each of which is attenuated and delayed by a different amount.



For the purpose of this problem assume that the received signal r(t) is given by:

$$r(t) = s(t) + \frac{1}{4} s(t - T)$$

where s(t) is the transmitted signal and T > 0 is a positive delay. The signal r(t) is processed by a linear time-invariant (LTI) system with impulse response h(t), so that its output y(t) is the transmitted signal s(t). This system is called an equalizer.



1. Find the frequency response *H(f)* of the equalizer and determine its phase as a function of *f*.

$$y(t) = r(t) * h(t) \qquad \text{But } y(t) = s(t)$$

$$s(t) = r(t) * h(t)$$

$$\Rightarrow S(f) = R(f)\, H(f)$$

$$r(t) = s(t) + \frac{1}{4} s(t-T)$$

$$R(f) = S(f) + \frac{1}{4} S(f) e^{-j2\pi f T} = S(f)\left[1 + \frac{1}{4} e^{-j2\pi f T}\right]$$

$$H(f) = \frac{S(f)}{R(f)} = \frac{S(f)}{S(f)\left[1 + \frac{1}{4} e^{-j2\pi f T}\right]} = \frac{1}{1 + \frac{1}{4} e^{-j2\pi f T}}$$

$$H(f) = \frac{1}{\ldots} \qquad = -\tan^{-1}\left[-\frac{\sin 2\pi f T}{4 + \cos 2\pi f T}\right]$$

2. The impulse response of the equalizer can be expressed in the form:

$$h(t) = \sum_{k=0}^{+\infty} h_k \delta(t - kT)$$

Find h_o, h_1, and h_2.
Hint: Recall that for $|x| < 1$, the following relationship holds:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4 - \cdots + \cdots$$

$$h(t) = h_0 \delta(t) + h_1 \delta(t - T) + h_2 \delta(t - 2T) + \cdots$$

$$H(f) = \frac{1}{1 + \frac{1}{4} e^{-j2\pi f T}} = 1 - \frac{1}{4} e^{-j2\pi f T} + \frac{1}{16} e^{-j4\pi f T} - \cdots + \cdots$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = \delta(t) - \frac{1}{4}\delta(t-T) + \frac{1}{16}\delta(t-2T) - \cdots + \cdots$$

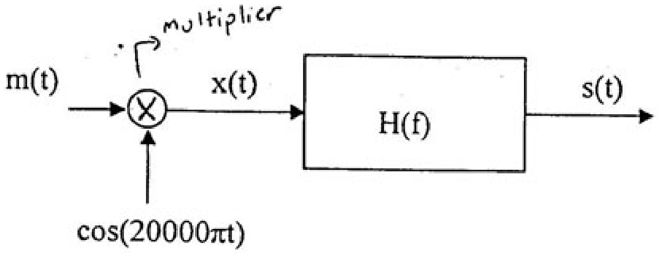
Comparing: $h_0 = 1$

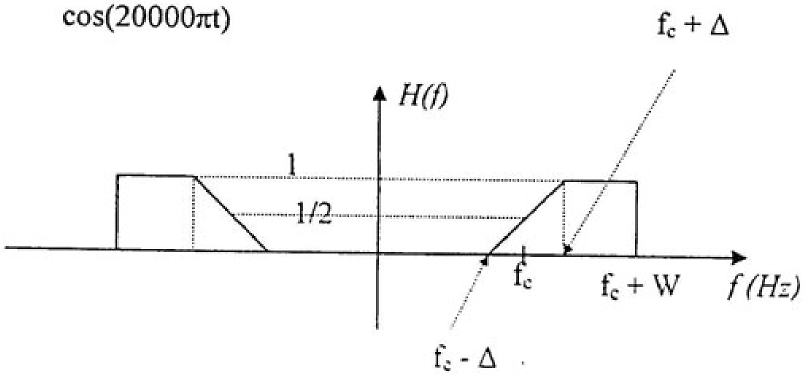
$h_1 = -\frac{1}{4}$

$h_2 = \frac{1}{16}$

Problem 3 (25 Points)

Consider the VSB modulation scheme shown in the figure below. Also shown is the frequency response of the VSB filter H(f). Given: $f_c = 10000$ Hz, $W = 250$ Hz, and $\Delta = 100$ Hz.





Let $m(t) = \cos(300\pi t) + \cos(100\pi t)$.

1. Evaluate and simplify x(t).

Given: $\cos(A)\cos(B) = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$$x(t) = m(t)\cos(20000\pi t)$$

$$= [\cos(300\pi t) + \cos(100\pi t)]\cos(20000\pi t)$$

$$= \cos(300\pi t)\cos(20000\pi t) + \cos(100\pi t)\cos(20000\pi t)$$

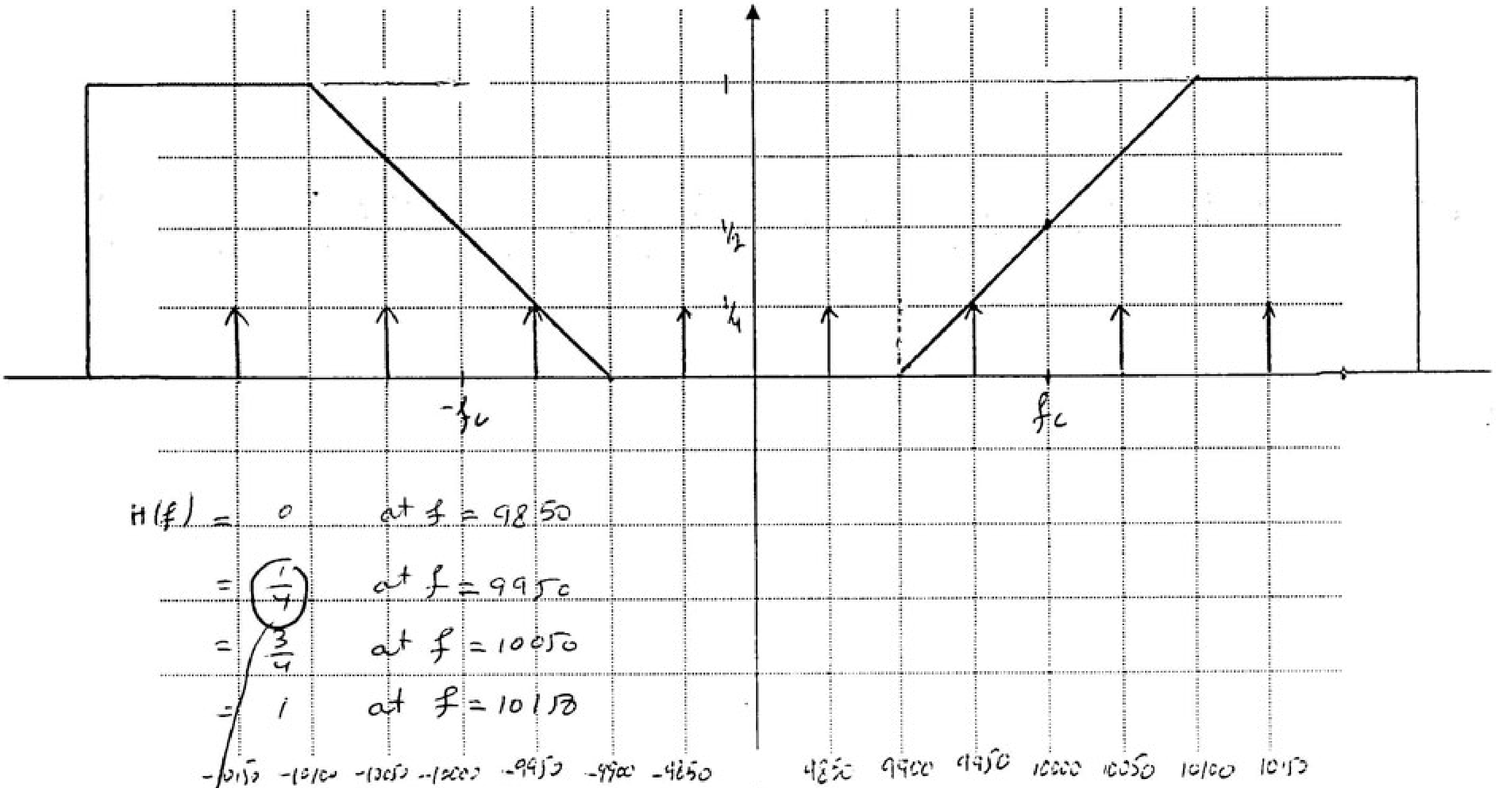
$$= \frac{1}{2}\cos(19700\pi t) + \frac{1}{2}\cos(20300\pi t) + \frac{1}{2}\cos(19900\pi t)$$

$$+ \frac{1}{2}\cos(20100\pi t).$$

$$X(f) = \frac{1}{4}\delta(f - 9850) + \frac{1}{4}\delta(f + 9850) + \frac{1}{4}\delta(f - 10150)$$

$$+ \frac{1}{4}\delta(f + 10150) + \frac{1}{4}\delta(f - 9950) + \frac{1}{4}\delta(f + 9950)$$

2. Sketch X(f).

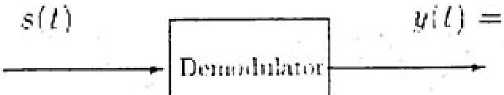


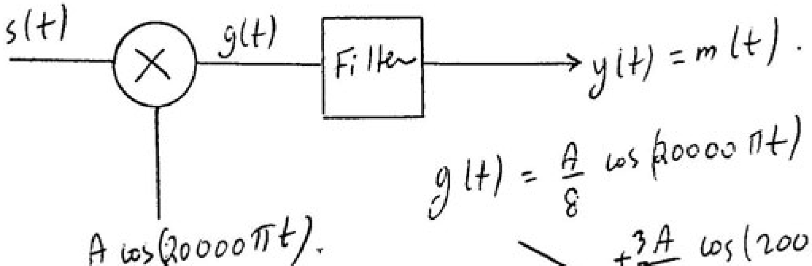
3. Find the transmitted VSB-modulated waveform s(t).

$$S(f) = \frac{1}{16}\delta(f-9950) + \frac{1}{16}\delta(f+9950) + \frac{3}{16}\delta(f-10050) + \frac{3}{16}\delta(f+10050) + \frac{1}{4}\delta(f-1015 \quad + \frac{1}{4}\delta(f+101$$

$$\Rightarrow s(t) = \frac{1}{8}\cos(19900\pi t) + \frac{3}{8}\cos(20100\pi t) + \frac{1}{2}\cos(20300\pi t)$$

4. Describe a coherent demodulator for s(t) so that its output y(t) = m(t).





$$g(t) = \frac{A}{8}\cos(20000\pi t)\cos(19900\pi t) + \frac{3A}{8}\cos(20000\pi t)\cos(20100\pi t) + \frac{A}{2}\cos(20000\pi t)\cos(20300\pi t)$$

$$g(t) = \frac{A}{16}\cos 100\pi t + \frac{A}{16}\cos 39900\pi t$$

Problem 4 (15 Points)

Let $y(t) = x(t)\sin(2\pi f_c t)$.

1. Using the Fourier transform technique, determine the Hilbert transform of $y(t)$.

$$y(t) = x(t)\sin 2\pi f_c t$$

$$Y(f) = X(f) * \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

$$= \frac{1}{2j} X(f-f_c) - \frac{1}{2j} X(f+f_c)$$

$$\hat{Y}(f) = -j\,\mathrm{sgn}(f)\,Y(f)$$

$$= -j\,(1)\left(\frac{1}{2j}\right)X(f-f_c) - \left(\frac{1}{2j}\right)(-j)(-1)\,X(f+f_c)$$

$$= -\frac{1}{2}X(f-f_c) - \frac{1}{2}X(f+f_c) = -\frac{1}{2}\left[X(f-f_c)+X(f+f_c)\right]$$

$$\Rightarrow \hat{y}(t) = -\,x(t)\cos 2\pi f_c t$$

2. Evaluate the pre-envelope of $y(t)$.

$$y_+(t) = y(t) + j\hat{y}(t)$$

$$= x(t)\sin 2\pi f_c t - j\,x(t)\cos 2\pi f_c t$$

$$= -j\,x(t)\left[\cos 2\pi f_c t + j\sin 2\pi f_c t\right]$$

$$= -j\,x(t)\,e^{j2\pi f_c t}$$

3. Find the in-phase and quadrature components of $y(t)$. What is the complex envelope of $y(t)$?

$$y(t) = x(t)\sin 2\pi f_c t$$

$$y_I(t) = 0 \qquad y_Q(t) = -x(t)$$

$$\tilde{y}(t) = y_I(t) + j\,y_Q(t)$$

$$= -j\,x(t).$$