NDU - Faculty of Engineering
EEN 344
Communication Systems I

Exam 2
Time: 1 hr 10 min

Name: ______________________ **ID #:** ______________________

Remark: *Adjust your time. Do not waste too much time on parts that you cannot solve quickly.*

Problem I (20 Points)

An FM modulator is followed by an ideal bandpass filter having a center frequency of 500 Hz and a bandwidth of 70 Hz. The gain of the filter is 1 throughout its passband. The unmodulated carrier is given by (10)cos(1000πt) and the message signal is m(t) = (10)cos(20πt). The frequency sensitivity is $k_f = 8$ Hz/V.

1. Find the peak frequency deviation in Hz.

$$\Delta f = K_f A_m = \left(8\ \frac{Hz}{V}\right)(10\ V) = 80\ Hz$$

2. Find the peak phase deviation in radians. $\phi(t) = \frac{K A_m}{2\pi f_m} \sin 2\pi f_m t$

$$\text{Max}\ |\phi(t)| = \frac{K A_m}{2\pi f_m} = \beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

$$\Rightarrow \beta = \frac{80}{10} = 8\ rad.$$

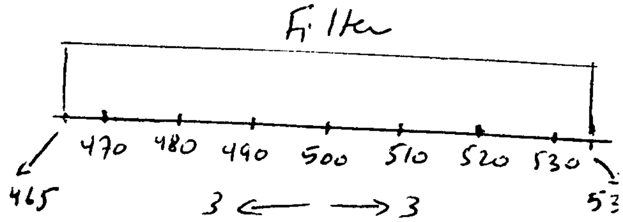
3. Is this signal narrowband or wideband? Justify your answer.

$\beta = 8 \Rightarrow \beta >> 0.3 \Rightarrow$ Wideband.

4. Find the power at the filter input and the filter output.

$$\text{Power at the filter input} = P_x = \frac{A_c^2}{2} = \frac{10^2}{2} = 50W$$

$$f_c = 500 \text{ Hz} \qquad f_m = 10 \text{ Hz}$$



$$P_r = J_0^2(\beta) + 2\sum_{n=1}^{3} J_n^2(\beta)$$

$$= J_0^2(8) + 2\left[J_1^2(8) + J_2^2(8) + J_3^2(8)\right]$$

$$= (0.172)^2 + 2\left[0.235^2 + (-0.113)^2 + (-0.291)^2\right]$$

$$= 0.334934 \qquad P_{out} = P_r P_x = (0.334934)(50W)$$

$$= 16.75 \text{ W}.$$

5. Find the bandwidth of the bandpass filter such that the power ratio is 0.99.

↑ change this for next exam.

We need to find n_{max}

$P_r \geq 0.99$ For $n = 14$

$$P_r = 0.98 + 2\left[0.061^2 + 0.026^2 + 0.01^2 + 0.003^2 + 0.001^2\right]$$

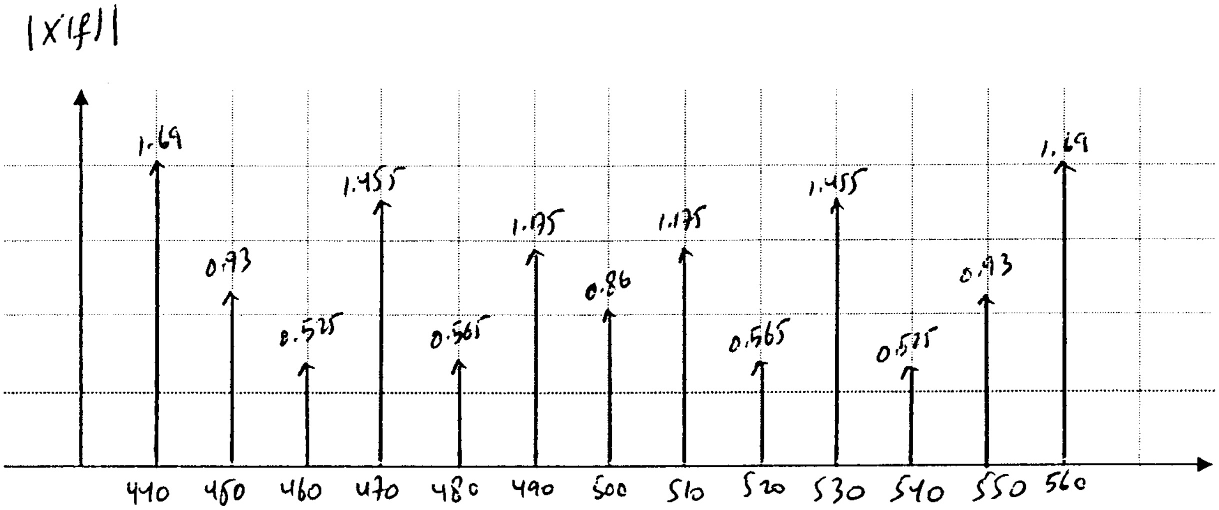
$$= 0.989 \simeq 0.99$$

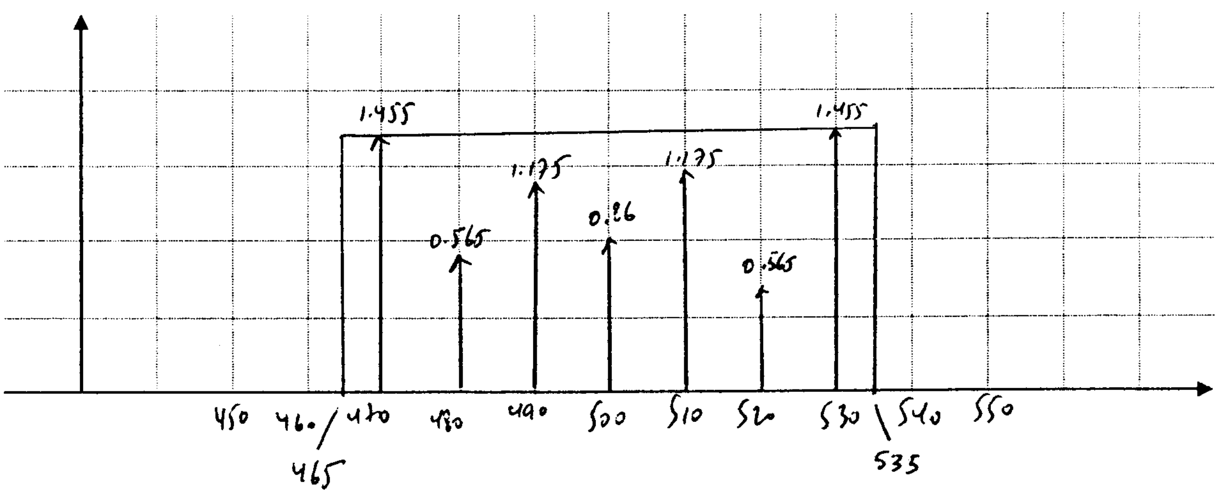
In the table on page 24; $J_{14}(8)$ is the max J_n given.

$$\Rightarrow B_T = 2 n_{max} f_m$$

$$= (2)(14)(10) = 280 \text{ Hz}$$

6. Draw the single-sided spectrum of the signal at the filter input and the filter output. Label the amplitude and frequency of each spectral component.





$$X(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right]$$

$$= 5 \sum_{n}^{\infty} J_n(8)\left[\delta(f - 500 - 10n)\right)$$

Problem II (20 Points)

For each of the cases below, a PM transmitter uses a carrier frequency $f_c = 1000$ Hz and a phase sensitivity of $K = 10$ rad/V.

Find for each of the cases: m(t), the phase deviation, and the frequency deviation.

1. $x(t) = A_c \cos[2\pi(1000)t + 10t^2]$
2. $x(t) = A_c \cos[2\pi(1000)t + 10\sqrt{t}\,]$
3. $x(t) = A_c \cos[2\pi(1100)t]$
4. $x(t) = A_c \cos[2\pi(500)t^2]$

$f_c = 1000\ Hz \quad K = 10\ \frac{rad}{V}$

PM :

$x(t) = A_c \cos\left[2\pi f_c t + K\, m(t)\right]$

1. $x(t) = A_c \cos\left[2\pi(1000)t + 10t^2\right]$

$K\, m(t) = 10t^2 \Rightarrow m(t) = \frac{10t^2}{10} = t^2$

Phase Dev. : $\phi(t) = K\, m(t) = 10t^2$

Freq. Dev. : $\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}(10t^2) = \frac{20t}{2\pi} = \frac{10t}{\pi}$

2. $x(t) = A_c \cos\left(2\pi(1000t) + 10\sqrt{t}\right)$

$K\, m(t) = 10\sqrt{t} \Rightarrow m(t) = \sqrt{t}$

Phase Dev. : $\phi(t) = 10\sqrt{t}$

Freq. Dev. : $\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{d1}{2\pi}\frac{d}{dt}\left(10\sqrt{t}\right) = \frac{5}{2\pi\sqrt{t}}$

3. $x(t) = A_c \cos\left[2\pi(1100)t\right]$

$$= A_c \cos\left[2\pi(1000)t + 2\pi(100)t\right]$$

$$K\,m(t) = 2\pi(100)t \Rightarrow m(t) = 20\pi t$$

Phase Dev. : $\phi(t) = 200\pi t$

Freq. Dev. : $\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\left(200\pi t\right)$

$$= \frac{200\pi}{2\pi} = 100 \text{ Hz}$$

4. $x(t) = A_c \cos\left[2\pi(500)t^2\right]$

$$= A_c \cos\left[2\pi(500)t^2 + 2\pi(1000)t - 2\pi(1000)t\right]$$

$$= A_c \cos\left[2\pi(1000)t + 2\pi(500)t^2 - 2\pi(1000)t\right]$$

$$K\,m(t) = 2\pi(500)t^2 - 2\pi(1000)t$$

$$\Rightarrow m(t) = 100\pi t^2 - 200\pi t$$

Phase Dev. : $\phi(t) = 1000\pi t^2 - 2000\pi t$

$$\frac{d\phi(t)}{} = \frac{1}{}\frac{d}{}\left(1000\pi t^2 - 2000\pi t\right)$$

Problem III (20 Points)

1. Plot the spectrum of a PAM wave produced by the modulating signal

$$m(t) = 4\cos^2(2\pi f_m t)$$

assuming a modulation frequency $f_m = 0.25$ Hz, sampling period $T_s = 0.5$ s, and a pulse duration $T = 0.4$ s.

You are required to find M(f), H(f), S(f), and then plot the amplitude of S(f).

Given: $\cos^2 A = (1 + \cos 2A)/2$

$h(t) = \text{rect}(t/T)$

$$m(t) = 4\cos^2 2\pi f_m t$$

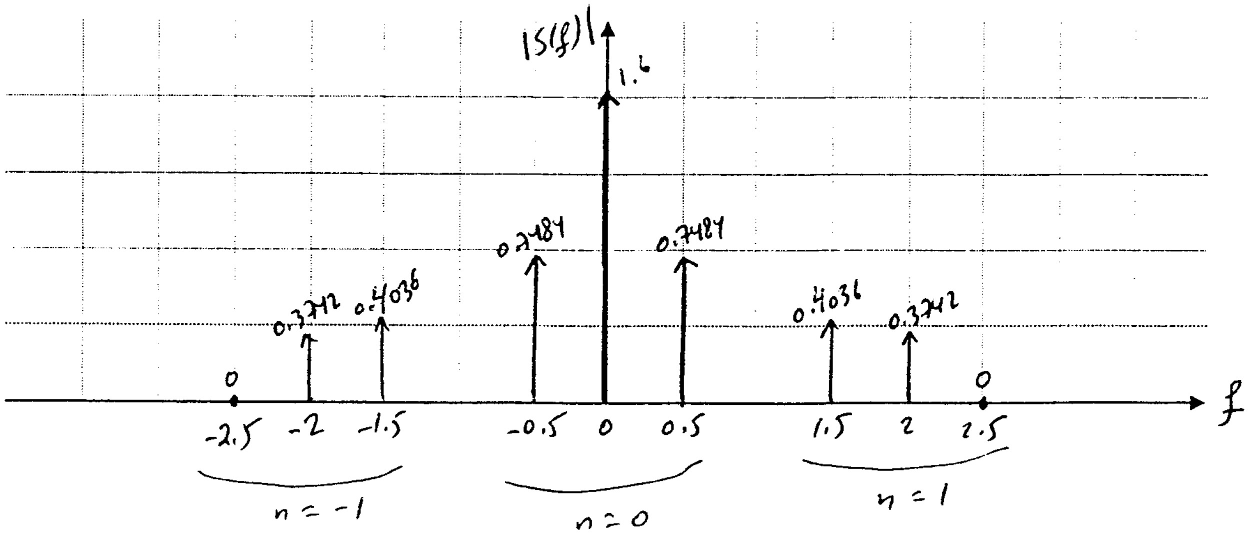
$$= 4\left(\tfrac{1}{2}\right)\left(1+\cos 4\pi f_m t\right) = 2\left(1+\cos 4\pi f_m t\right)$$

$$M(f) = 2\delta(f) + \delta(f-2f_m) + \delta(f+2f_m) \quad \text{where } f_m = 0.25\text{ Hz}$$

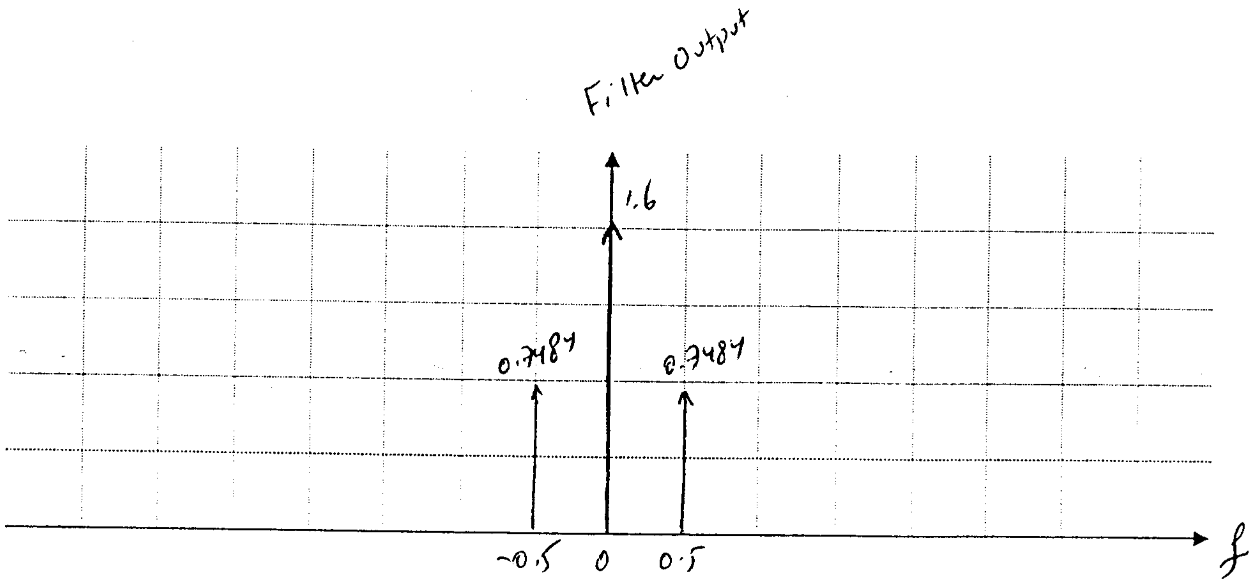
$$H(f) = T\,\text{sinc}(fT) = 0.4\,\text{sinc}(0.4f)$$

$$S(f) = f_s \sum_{n=-\infty}^{\infty} M(f - nf_s)\,H(f) \qquad f_s = \frac{1}{T_s} = 2\text{ Hz}$$

$$= 2\sum_{n=-\infty}^{\infty}\left[2\delta(f-2n) + \delta(f-2n-0.5) + \delta(f-2n+0.5)\right] \times 0.4\,\text{sinc}(0.4f)$$



2. Using an ideal reconstruction filter, plot the spectrum of the filter output. Compare this result with the output that would be obtained if there were no aperture effect.



Output of filter : $Y(f)$ → 3 center delta functions

$$Y(f) = 1.6\,\delta(f) + 0.7484\left[\delta(f-0.5) + \delta(f+0.5)\right]$$

$$y(t) = 1.6 + 1.4968 \cos(2\pi)(0.5)t$$

$$= 1.6 + 1.4968 \cos 4\pi f_m t$$

$$= 1.6\left[1 + 0.9355 \cos 4\pi f_m t\right]$$

0.9355 is very close to 1 ⇒ The aperture effect is very small.

with no aperture effect :

$$Y(f) = 4\,\delta(f) + 2\left[\delta(f-0.5) + \delta(f+0.5)\right]$$

Problem IV (10 Points) Extra Credit

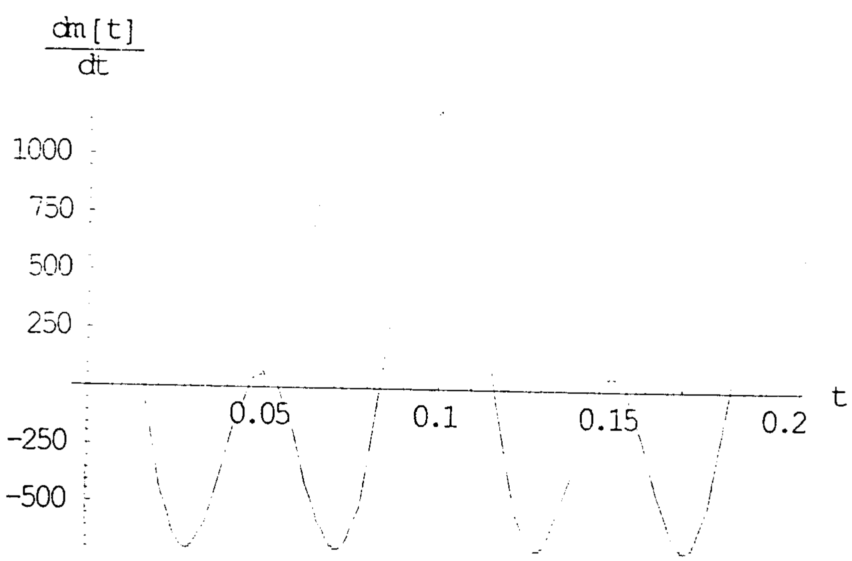
Consider a delta modulator that has the following message signal:

$$m(t) = 9\sin 2\pi(10)t + 5\sin 2\pi(20)t$$

Determine the condition for the sampling frequency required to prevent slope overload, assuming that the impulse weights δ_o are 0.05π.

Given: $\frac{dm(t)}{dt}$ is plotted in the following figure:

Note: The graph is given to you so that you know where the maxima are. You are not supposed to read the maximum off the graph. An exact value of the maximum slope of m(t) ($\max\left|\frac{dm(t)}{dt}\right|$) is required.



$$\frac{dm(t)}{dt} = 180\pi \cos 20\pi t + 200\pi \sin 40\pi t$$

$$\text{At } t=0 \Rightarrow \text{Max}\left|\frac{dm(t)}{dt}\right| = 180\pi + 200\pi = 380\pi$$

$$\frac{\delta_o}{T_s} \geq 380\pi \Rightarrow \frac{T_s}{\delta_o} \leq (380\pi)^{-1}$$

$$T_s \leq (380\pi)^{-1}(\delta_o)$$

$$T_s \leq (380\pi)^{-1}(0.05\pi) \Rightarrow T_s \leq 131.6\,\mu s.$$