NDU - Faculty of Engineering
EEN 344
Communication Systems I

Final Exam
Time: 2 Hours

Name: ______________________ **ID #:** ______________________

Remark: Adjust your time. Do not waste too much time on parts that you cannot solve quickly.

Problem I: FM Modulation (35 Points)

Consider a single tone sinusoidal message $m(t)$ with amplitude $A_m = 1\ V$ which modulates the frequency of a carrier wave according to:

$$x(t) = 25\cos[2\pi f_c t + 3\sin(10000\pi t)]$$

1. Find the instantaneous frequency of $x(t)$. **(5 Pts)**
2. Find the single tone message $m(t)$. **(5 Pts)**
3. Find the peak frequency deviation. **(5 Pts)**
4. Find the bandwidth of $x(t)$ using Carson's rule. **(5 Pts)**
5. Calculate the power ratio and the power output of the modulator assuming a bandpass filter is used which will pass the carrier frequency component and in addition two frequency components on each side of the carrier frequency. **(5 Pts)**
6. The FM signal is transmitted through a channel with additive white noise. The power spectral density of the noise signal $n(t)$ is $S_n(f) = N_o/2 = 10^{-3}$ (W/Hz). Calculate the signal to noise ratio $(SNR)_{FM}$. **(5 Pts)**
7. Pre-emphasis and de-emphasis filters are used. Calculate the $3\text{-}dB$ frequency f_o of the de-emphasis filter such that the signal to noise ratio is improved by $6\ dB$. **(5 Pts)**

1. $\boxed{f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}} = f_c + k_f\, m(t)$

$$= \frac{1}{2\pi}\left[2\pi f_c + 3(10000)(\pi)\cos(10000\pi t)\right]$$

$$= f_c + 15000\cos(10000\pi t)$$

2. $A_m = 1\ V$

$$\Rightarrow m(t) = \cos 10000\pi t$$

$$f_m = 5000\ Hz = 5\ KHz$$

3. $\Delta f = \max |f(t) - f_c|$

$= \max |15000 \cos(10000 \pi t)|$

$$\boxed{\Delta f = 15000 \text{ Hz} = 15 \text{ KHz}}$$

4. $B_T = 2 f_m (\beta + 1) = 2(5 \text{ KHz})(\beta + 1)$

But $\beta = 3 \Rightarrow B_T = 2(5 \text{ KHz})(3+1)$

$= 40 \text{ KHz}$

$\because \beta = \frac{\Delta f}{f_m} = \frac{15000}{5000}$

5. $P_r = J_0^2(\beta) + 2 \sum_{n=1}^{2} J_n^2(\beta)$

$= J_0^2(3) + 2\left[J_1^2(3) + J_2^2(3)\right]$

$= (-0.26)^2 + 2\left[0.339^2 + 0.486^2\right]$

$= 0.769834$

$P_T = \frac{A_c^2}{2} = \frac{25^2}{2} = 312.5 \text{ W}$

$P_{out} = P_r P_T = (0.769834)(312.5 \text{ W})$

$= 240.573125 \text{ W}.$

6. $S_n(f) = \frac{N_0}{2} = 10^{-3}\ W/Hz \qquad W = f_m = 5\,KHz$

$P_T = 312.5\ W \qquad \langle m_n^2(t) \rangle = \frac{1}{2} \qquad K_f = 15000\ \frac{Hz}{V}$

$$(SNR)_{FM} = 3\left(\frac{K_f}{W}\right)^2 \langle m_n^2(t)\rangle \frac{P_T}{N_0 W}$$

$$= 3\left(\frac{15K}{5K}\right)^2 \left(\frac{1}{2}\right) \frac{312.5}{(2\times10^{-3})(5000)}$$

$$= 421.875$$

$$(SNR)_{FM\,dB} = 10\log(SNR)_{FM} = 26.252\ dB$$

7. $(SNR)_{Demp.\,dB} = 26.252 + 6 = 32.252\ dB$

$$10\log(SNR)_{Demp.} = 32.252\ dB \Rightarrow (SNR)_{Demp.} = 10^{\frac{32.252}{10}}$$

$$(SNR)_{Demp.} = 10^{3.2252} = 1679.577$$

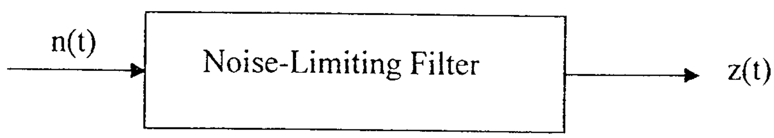
$$(SNR)_{Demp.} = \left(\frac{K_f}{f_0}\right)^2 \langle m_n^2(t)\rangle \frac{P_T}{N_0 W} = 1679.577$$

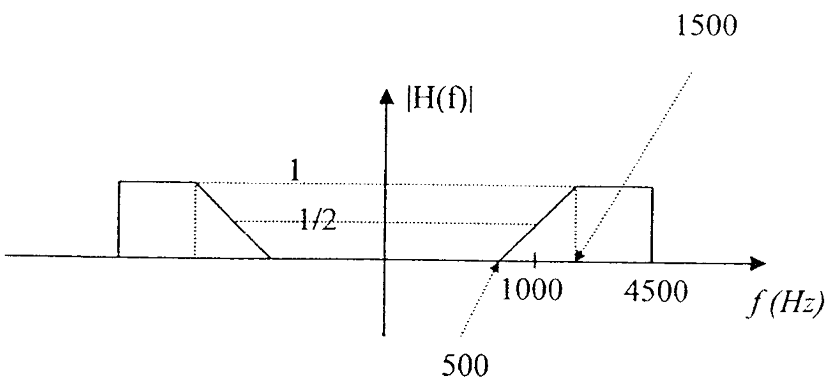
$$\left(\frac{K_f}{f_0}\right)^2 \left(\frac{1}{2}\right) \frac{312.5}{(2\times10^{-3})(5000)} = 1679.577$$

$$\left(\frac{K_f}{f_0}\right)^2 = 107.493 \qquad \frac{K_f}{f_0} = 10.368 \Rightarrow f_0 = \frac{15\,KHz}{10.368}$$

blem II: Noise Power (35 Points)

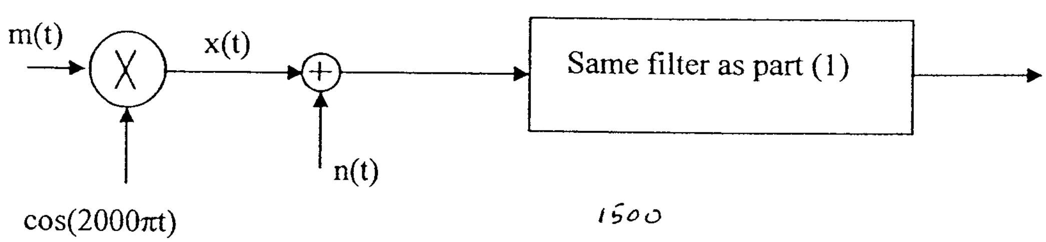
Consider a noise signal n(t) generated by an arbitrary communication channel. Assume that the noise signal is white with a power spectral density of $N_o/2$. Suppose that we use a noise-limiting filter with frequency response H(f) to reduce the noise as shown in the following figure:





Hint: $|H(f)| = 10^{-3}f - 0.5$ $\quad\quad$ $500\ Hz \le f \le 1500\ Hz$

1. Calculate the noise power; in terms of N_o, at the output of the noise-limiting filter. **(15 Pts)**
2. Now consider the system shown in the figure below. Let $m(t) = 10[1 + \cos(1000\pi t)]$ V.
   a) Find x(t). Given: $\cos(A)\cos(B) = (1/2)[\cos(A-B) + \cos(A+B)]$ **(5 Pts)**
   b) Find X(f), and then deduce the portion of X(f) that will pass through the noise-limiting filter. **(5 Pts)**
   c) Calculate the signal power at the output of the filter. Assume a 1 Ω load. **(5 Pts)**
   d) Calculate the signal to noise ratio in dB at the output of the filter. Note that the noise power is the same as part (1). Take $N_o = 3\times10^{-3}$ (W/Hz). **(5 Pts)**



1. Noise Power $= 2\left(\frac{N_o}{2}\right)\int_{500}^{1500}\left(10^{-3}f - 0.5\right)^2 df + 2\left(\frac{N_o}{2}\right)(4500-1500)$

$$= N_o\int^{1500}\left(10^{-6}f^2 - 10^{-3}f + 0.25\right)df + 3000\,N_o$$

$$\text{Noise Power} = N_0 \left[10^{-6}\frac{f^3}{3} - 10^{-3}\frac{f^2}{2} + 0.25f \right]_{500}^{1500}$$

$$+ 3000\,N_0 .$$

$$= N_0 \left[10^{-6}\frac{1500^3}{3} - 10^{-3}\frac{1500^2}{2} + (0.25)(1500) - 10^{-6}\frac{500^3}{3} + 10^{-3}\frac{500^2}{2} - (0.25)(500) \right] + 3000 N_0$$

$$= N_0 \left[1125 - 1125 + 375 - \frac{125}{3} - 125 + 125 \right]$$

$$+ 3000 N_0$$

$$= N_0 \left[375 - \frac{125}{3} \right] + 3000 N_0 = \frac{1000}{3} N_0 + 3000 N_0$$

$$= \frac{10000}{3} N_0 .$$

2. a) $x(t) = m(t) \cos 2000\pi t$

$$= 10\left[1 + \cos 1000\pi t\right] \cos 2000\pi t$$

$$= 10 \cos 2000\pi t + 10 \cos 1000\pi t \cos 2000\pi t$$

$$= 10 \cos 2000\pi t + 5 \cos 1000\pi t$$

$$+ 5 \cos 3000\pi t$$

b. $X(f) = 5\,\delta(f-1000) + 5\,\delta(f+1000)$

$$+ 2.5\,\delta(f-500) + 2.5\,\delta(f+500)$$

$$+ 2.5\,\delta(f-1500) + 2.5\,\delta(f+1500)$$

$$H(f) = \begin{cases} 0 & f = \pm 500 \text{ HZ} \\ \tfrac{1}{2} & f = \pm 1000 \text{ HZ} \\ 1 & f = \pm 1500 \text{ HZ} \end{cases}$$

Signal Output$(f) = X(f)\,H(f)$

$$= \frac{5}{2}\,\delta(f-1000) + \frac{5}{2}\,\delta(f+1000)$$

$$+ \frac{5}{2}\,\delta(f-1500) + \frac{5}{2}\,\delta(f+1500)$$

Signal Output $(t) = 5\cos 2000\pi t + 5\cos 3000\pi t$

c. Signal Power $= 5^2\left(\frac{1}{2}\right) + 5^2\left(\frac{1}{2}\right) = 25$ W.

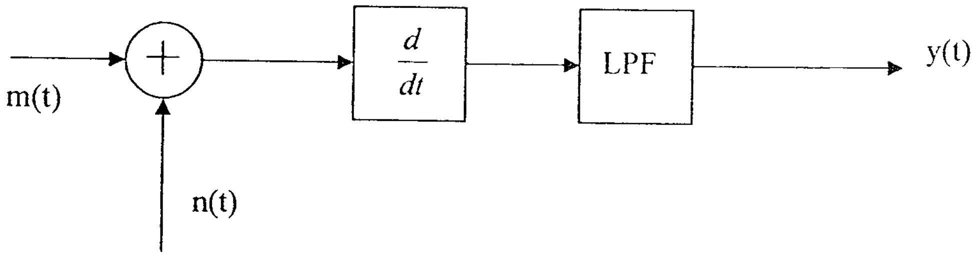
d. $$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{25\text{ W}}{\frac{10000}{3} N_0}$$

$$= \frac{25\text{ W}}{\frac{10000}{3}(3\times 10^{-3})} = \frac{25}{10} = 2.5$$

$$\text{SNR}_{dB} = 3.9794 \text{ dB}.$$

Problem III: Signal to Noise Ratio (30 Points)

Consider the following communication system:



The message signal m(t) is given as: $m(t) = (10/\pi)\sin(200\pi t)$.
The lowpass filter has a unity gain and a bandwidth W, where W = 200 Hz. The noise signal n(t) is white with a power spectral density of $S_n(f) = N_o/2 = 10^{-3}$ (W/Hz).
Find the signal to noise ratio in dB of the output signal y(t).

$$m(t) = \frac{10}{\pi} \sin 200\pi t$$

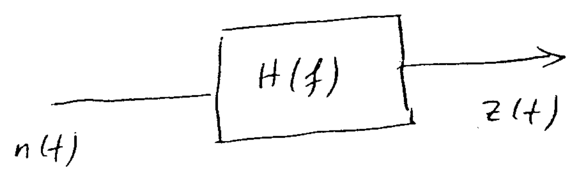
$$\frac{dm(t)}{dt} = \frac{10}{\pi}(200\pi) \cos 200\pi t$$

$$= 2000 \cos 200\pi t$$

This signal will pass thru the LPF.

$$\text{Signal Power} = (2000)^2\left(\frac{1}{2}\right) = 2 \times 10^6 \text{ W.}$$

$$\frac{d}{dt} \xrightarrow{\mathcal{F}} j2\pi f \qquad H(f) = j2\pi f$$



$$S_z(f) = |H(f)|^2 S_n(f) = (2\pi f)^2 \frac{N_o}{2}.$$

$$\text{Noise Power} = \int_{-200}^{200} 4\pi^2 f^2 \frac{N_0}{2}\, df$$

$$= 2\pi^2 N_0 \int_{-200}^{200} f^2\, df$$

$$= 4\pi^2 N_0 \int_{0}^{200} f^2\, df$$

$$= 4\pi^2 N_0 \left[\frac{f^3}{3}\right]_0^{200}$$

$$= \frac{4}{3}\pi^2 N_0 (200)^3 = \frac{4}{3}\pi^2 (2\times 10^{-3})(200)^3$$

$$= 210\,551.5606 \text{ W}$$

$$SNR = \frac{2\times 10^6}{210551.5606} = 9.49886$$

$$SNR_{dB} = 9.7767 \text{ dB} = 10 \log 9.49886$$