(A) (1)

Math 201–Final Exam (Fall 04) B. Shayya

- Please write your **section number** on your booklet.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on pages **1** and **2** of the booklet.)

(24 pts) Show that the differential form in the following integral is exact, then evaluate the integral.

$$\int_{(5,0,9)}^{(1,\pi,0)} (2x\cos y + yz)\,dx + (xz - x^2\sin y)\,dy + (xy)\,dz$$

Problem 2 (answer on pages **3** and **4** of the booklet.)

(24 pts) Find the maximum and minimum values of $f(x,y,z) = xyz$ on the sphere $x^2+y^2+z^2 = 1$.

Problem 3 (answer on pages **5** and **6** of the booklet.)

Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2+y^2+z^2 = 4$, and on the sides by the cylinder $x^2+y^2 = 1$.

(i) (8 pts) Set up the triple integrals in cylindrical coordinates that give the volume of D using the order of integration $dz\,r dr\,d\theta$. Then find the volume of D.

(ii) (6 pts) Set up the limits of integration for evaluating the integral of a function $f(x,y,z)$ over D as an iterated triple integral in the order $dy\,dz\,dx$.

(iii) (12 pts) Set up the triple integrals in spherical coordinates that give the volume of D using the order of integration $d\phi\,d\rho\,d\theta$.

Problem 4 (answer on pages **7** and **8** of the booklet.)

(25 pts) Integrate $g(x,y,z) = z$ over the surface of the prism cut from the first octant by the planes $z = x$, $z = 2 - x$, and $y = 2$.

Problem 5 (answer on pages **9**, **10**, and **11** of the booklet.)

Let S be the cone $z = 1 - \sqrt{x^2+y^2}$, $0 \le z \le 1$, and let C be its base (i.e. C is the unit circle in the xy-plane). Find the counterclockwise circulation of the field

$$F(x,y,z) = x^2y\mathbf{i} + 2y^3z\mathbf{j} + 3z\mathbf{k}$$

around C

(a) (12 pts) directly,

(b) (8 pts) using Green's theorem, and

(c) (14 pts) using Stokes' theorem (i.e. by evaluating the flux of curl F outward through S).

Problem 6 (answer on pages **12** and **13** of the booklet.)

(25 pts) Let R be the region in the xy-plane bounded by the lines $y = 0$, $y = x$, $x + y = 4$, and $x + y = 9$. Use the transformation

$$x = uv, \qquad y = (1-u)v$$

to rewrite

$$\int\int_R \frac{1}{\sqrt{x+y}}\,dx\,dy$$

as an integral over an appropriate region G in the uv-plane. Then evaluate the uv-integral over G.

Problem 7 (answer on page **14** of the booklet.)
(6 pts each) Determine which of the following series converge, and which diverge.

$$(a)\ \sum_{n=1}^{\infty} \sqrt{n}\ln\left(1+\frac{1}{n^{2.1}}\right) \qquad (b)\ \sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{1.2}} \qquad (c)\ \sum_{n=1}^{\infty} n(\sqrt[n]{n}-1)$$

Problem 8 (answer on pages **15** and **16** of the booklet.)
(i) (6 pts) Use Taylor's theorem to prove that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad (-\infty < x < \infty).$$

(ii) (6 pts) Approximate

$$\int_0^{0.1} e^{-x^2}\,dx$$

with an error of magnitude less than 10^{-5}.

(iii) (6 pts) Show that

$$\int_0^{\infty} e^{-\pi x^2}\,dx = \frac{1}{2}.$$

(Hint. If $I = \int_0^\infty e^{-\pi x^2}dx$, then $I^2 = \int_0^\infty \int_0^\infty e^{-\pi(x^2+y^2)}dxdy$.)

(iv) (6 pts) Let E be the error resulting from the approximation

$$\int_0^{100} e^{-\pi x^2}\,dx \approx \frac{1}{2}.$$

Show that

$$|E| < \frac{e^{-5000\pi}}{2}.$$