

FINAL EXAM.; MATH 201

January 28, 2004



Name:

Signature:

Student number:

Section number (Encircle):

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Instructor: Prof. Abdallah Lyzzaik

1. Instructions:

- Calculators are allowed.
- There are two types of questions: PART I consists of six subjective questions, and PART II consists of seven multiple-choice questions of which each has exactly one correct answer.

• GIVE DETAILED SOLUTIONS FOR THE PROBLEMS OF PART I IN THE PROVIDED SPACE AND CIRCLE THE APPROPRIATE ANSWER FOR EACH PROBLEM OF PART II.

2. Grading policy:

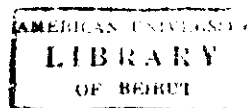
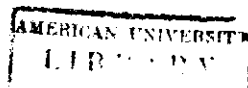
- 12 points for each problem of PART I.
- 4 points for each problem of PART II: 0 point for no answer, -1 for a wrong answer or more than one answer of PART II.

GRADE OF PART I/72:

GRADE OF PART II/28:

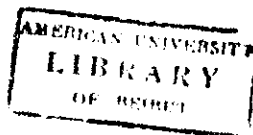
TOTAL GRADE/100:

Part I (1). Find the absolute maximum and minimum values attained by the function $f(x, y) = xy - x - y + 3$ on the triangular region R in the xy -plane with vertices $(0, 0)$, $(2, 0)$, and $(0, 4)$.



Part I (2). Evaluate the integral

$$\int_0^2 \int_{y/2}^1 y e^{x^3} dx dy.$$



Part I (3). Set up a triple integral (without evaluating it) in cylindrical coordinates for the volume of the solid bounded by the xy -plane, the cylinder $r^2 = \cos 2\theta$, and the sphere $x^2 + y^2 + z^2 = 1$.

Part I (4). Evaluate the integral

$$\iint_R \sin\left(\frac{y-x}{y+x}\right) dx dy,$$

where R is the trapezoid in the xy -plane with vertices $(1, 1)$, $(2, 2)$, $(4, 0)$, and $(2, 0)$, by making the change of variables: $u = y - x$, $v = y + x$.

4:
2

du, dv

u, v

Part I (5). Evaluate the line integral

$$\oint_C 3xy \, dx + 2x^2 \, dy,$$

where C is the boundary of the region R bounded above by the line $y = x$ and below by the parabola $y = x^2 - 2x$. Interpret this integral in terms of vector fields.

Part I (6). Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(2x-1)^n}{\ln n}.$$

State where the series converges absolutely and conditionally.

Part II

1. If $f(x, y) = 2x^2y/(x^4 + y^2)$, then

- (a) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$.
- (b) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$.
- (c) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 2$.
- (d) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist.
- (e) None of the above.

2. An estimate of the integral

$$\int_0^1 \frac{1 - \cos x}{x^2} dx$$

with an error less than $1/(6!5)$ is

- (a) $1/2! + 1/(4!3)$.
- (b) $1/2! - 1/(4!3) + 1/(6!5)$.
- (c) $-1/2! + 1/(4!3) - 1/(6!5)$.
- (d) $1/2! - 1/(4!3)$.
- (e) None of the above.

3. The function defined by

$$f(x, y) = \tan \left(\frac{x^3 - y^3}{x^2 + y^2} \right)$$

for $(x, y) \neq (0, 0)$, and $f(0, 0) = 0$

- (a) is continuous at $(0, 0)$.
- (b) has no limit at $(0, 0)$.
- (c) has a limit at $(0, 0)$ but is discontinuous at $(0, 0)$.
- (d) is bounded in the xy -plane.
- (e) None of the above.

4. If $w = f(x, y)$ where $x = e^r \cos \theta$ and $y = e^r \sin \theta$, then

(a) $w_{xx} + w_{yy} = w_{rr} + w_r/r + w_{\theta\theta}/r^2$.

(b) $w_{xx} + w_{yy} = -w_{rr} + w_r/r + w_{\theta\theta}/r^2$.

(c) $w_{xx} + w_{yy} = w_{rr} + w_r/r - w_{\theta\theta}/r^2$.

(d) $w_{xx} + w_{yy} = w_{rr} - w_r/r + w_{\theta\theta}/r^2$.

(e) None of the above.

5. An equation of the tangent plane to the surface with equation $z^3 + xz - y^2 = 1$ at the point $(1, 3, 2)$ is

(a) $2x + 6y + 13z = 10$.

(b) $2x + 6y - 13z = 10$.

(c) $2x - 6y + 13z = -10$.

(d) $2x - 6y + 13z = 10$.

(e) None of the above.

6. The volume of the solid bounded by the cylinder $y = x^2$ and the planes $y + z = 4$ and $z = 0$ is given by the triple integral

(a) $\int_0^4 \int_0^{4-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx dy dz$.

(b) $2 \int_0^4 \int_{\sqrt{y}}^2 \int_0^{4-y} dz dx dy$.

(c) $\int_{-2}^2 \int_0^{x^2} \int_0^{4-y} dz dy dx$.

(d) $\int_0^4 \int_0^{4-z} \int_{-\sqrt{y}}^{\sqrt{y}} dx dy dz$.

(e) None of the above.

7. The function $f(x, y) = x^3 + 3xy - y^3$ admits

- (a) a saddle point and a local minimum value.
- (b) no saddle point and no local minimum value.
- (c) a local minimum value and no saddle point.
- (d) a saddle point and no local minimum value.
- (e) None of the above.