

NDU MAT 335 Exam# 1 Wednesday 25 March 2009
Duration: 50 minutes

1.(35%) Consider the IBVP

$$\begin{aligned} \text{PDE: } u_t &= u_{xx} - u + x & 0 < x < 1, 0 < t < \infty \\ \text{BCs: } u(0,t) &= 0 & 0 < t < \infty \\ & u(1,t) = 1 & 0 < t < \infty \\ \text{IC: } u(x,0) &= 0 & 0 \leq x \leq 1 \end{aligned}$$

- a) Let $u(x,t) = x + e^{-t} \cdot w(x,t)$. Reformulate the above IBVP in terms of w .
- b) Use the method of separation of variables to find w . **Show all details.**

2.(30%) a) Use the Fourier transform to solve the IVP

$$\begin{aligned} \text{PDE: } u_t &= \alpha^2 u_{xx} - \beta u & -\infty < x < \infty, 0 < t < \infty \\ \text{IC: } u(x,0) &= \phi(x) & -\infty < x < \infty \end{aligned}$$

Show all details. (α, β are positive constants.)

- b) What is the solution in the special case $\phi(x) = e^{-x^2}$?

3.(35%) Solve the following nonhomogeneous PDE problem

$$\begin{aligned} \text{PDE: } u_t &= \alpha^2 u_{xx} + \sin 2\pi x & 0 < x < 1, 0 < t < \infty \\ \text{BCs: } u(0,t) &= 0 & 0 < t < \infty \\ & u(1,t) = 0 & 0 < t < \infty \\ \text{IC: } u(x,0) &= \sin 3\pi x & 0 \leq x \leq 1 \end{aligned}$$

Show all details.