

- 1) (35%) Use the method of separation of variables to find the solution of the 1-dimensional heat flow IBVP in a finite length rod:

$$\begin{aligned} \text{PDE: } u_t &= u_{xx} & 0 < x < 1, & & 0 < t < \infty \\ \text{BCs: } u_x(0, t) &= u_x(1, t) = 0 & 0 < x < 1, & & 0 < t < \infty \\ \text{IC : } u(x, 0) &= \cos \pi x & 0 \leq x \leq 1, & & 0 < t < \infty \end{aligned}$$

- 2) (30%) Use the Fourier Transform to find the solution of the 1-dimensional heat flow IVP in an infinite rod:

$$\begin{aligned} \text{PDE: } u_t &= \alpha^2 u_{xx} & -\infty < x < \infty, & & 0 < t < \infty \\ \text{IC : } u(x, 0) &= \phi(x) & -\infty < x < \infty \end{aligned}$$

- 3) (35%) Use the Laplace Transform to solve the IBVP:

$$\begin{aligned} \text{PDE: } u_{xx} - u_{tt} &= -\sin \pi x & 0 < x < 1, & & 0 < t < \infty \\ \text{ICs: } u(x, 0) &= u_t(x, 0) = 0 & 0 \leq x \leq 1, & & 0 < t < \infty \\ \text{BCs: } u(0, t) &= u(1, t) = 0 & 0 < t < \infty \end{aligned}$$

- 1) Solve the 1-d heat flow problem

$$\begin{aligned} \text{PDE: } u_t &= \alpha^2 u_{xx} & 0 < x < 1, & & 0 < t < \infty \\ \text{BC: } u(0, t) &= 0 & 0 < t < \infty \\ \text{BE: } u(1, t) &= 0 & 0 < t < \infty \\ \text{IC: } u(x, 0) &= \phi(x) & 0 \leq x \leq 1 \end{aligned}$$

- 2) Solve the IBVP

$$\begin{aligned} \text{PDE: } u_t &= u_{xx} & 0 < x < 1, & & 0 < t < \infty \\ \text{BC: } u(0, t) &= 0 & 0 < t < \infty \\ \text{BC: } u(1, t) &= \cos t & 0 < t < \infty \\ \text{IC: } u(x, 0) &= x & 0 < x < 1 \end{aligned}$$

By (a) transforming it to one with homogeneous BCs

(b) solving the resulting problem by expanding it in terms of eigenfunctions