Math. 201 Final Exam. (Time: 140 minutes) N. Nahlus & H. Yamani

Jan 28 , 2004

Name .... **I.D** .... **Circle** *your* **section number**

(Sec 9 at 1W) (sec 10 at 12:30T) (sec 11 at 2T) (sec 12 at 3:30T)

Part 1 (50 %) **10** multiple choice problems (No Penalty)

1. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1.07}}$. Then the series is

A) Conditionally convergent
B) Absolutely convergent
C) Divergent

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2. Consider the series $\sum_{n=1}^{\infty} (-1)^n \left(\frac{3^n + 5}{n!}\right)^n$. Then the series is

A) Conditionally convergent
B) Absolutely convergent
C) Divergent

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3. Consider the series $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$ and $\sum_{n=2}^{\infty} (-1)^n \frac{n!\,(n+5)!}{(2n)!}$. Then

A) 1^{st} series diverges & 2^{nd} series diverges
B) 1^{st} series converges & 2^{nd} series diverges
C) 1^{st} series diverges & 2^{nd} series converges
D) 1^{st} series converges & 2^{nd} series converges

4. The domain of convergence of $\sum_{n=2}^{\infty} (-1)^n \frac{(x-1)^n}{3^n n \ln n}$ is

A) $-2 \le x < 4$ B) $-2 < x \le 4$
C) $-2 < x < 4$ D) $-2 \le x \le 4$
E) $x = 1$ & $x = 4$
F) None of the above

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5. The Maclaurin series of $f(x) = \int_0^x \frac{1 - \cos t^{3/2}}{t} dt$ is

A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n\,!} \frac{x^{3n}}{3n}$
B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n\,!} \frac{x^{3n-1}}{3n-1}$
C) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n\,!} \frac{x^{3n+1}}{3n+1}$
D) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n\,!} \frac{x^{6n}}{6n}$
E) None of the above

Do not forget to integrate !

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6. Suppose $z = f(\frac{x-y}{3y})$ where f is a differentiable function. Then $x\frac{\partial z}{\partial x} = ky\frac{\partial z}{\partial y}$ where

A) $k = -3$ B) $k = 1/3$
C) $k = 3$ D) $k = -1$
E) $k = 1$ F) $k = -1/3$
G) None of the above

7. Let $f(x,y) = e^{-3xy+5}$. Then its critical point is

A) a local max.
B) a local min.
C) a saddle point.

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8. Consider the paraboloid $x^2 + y^2 - 4z = 1$ and the sphere $x^2 + y^2 + z^2 = 3$.
Then the ***tangent planes*** to both surfaces at the intersection point (1, 1, 1) are

A) perpendicular
B) parallel
C) neither perpendicular nor parallel.

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9. Given that $F(x,y,z) = 8$. If the components of ∇F are never zero, then

$$\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \quad \& \quad \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial y} \quad \text{are}$$

A) $+1$ & $\dfrac{\partial z}{\partial y}$ resp. B) $+1$ & $-\dfrac{\partial z}{\partial y}$ resp.

C) -1 & $\dfrac{\partial z}{\partial y}$ resp. D) -1 & $-\dfrac{\partial z}{\partial y}$ resp. E) None of the above

10. The value of the double integral $\int_0^2 \int_{y/2}^1 3ye^{x^3}\,dx\,dy$ is

A) 7/2 (e-1)
B) 9/2 (e-1)
C) 2(e-1)
D) 8(e-1)
E) 25/2 (e-1)

Part II (50 %) (Subjective)

11. (5 %) Find the area of the surface cut from the bottom of the paraboloid $z = 2x^2 + 2y^2$ by the plane $z = 8$. (Grading: 4pts for setting it up & changing it to polar)

12. (7 %) Use Green's Theorem to find $\oint_C (2xy^3 + x)dx + 4x^2y^2dy$

where C (traversed counterclock wise) is the boundary of the "triangular region in the 1[st] quadrant enclosed by the x-axis, x=1 and $y = x^2$

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13. (5 %) Set up (*but do not evaluate*) the double integral(s) in **polar** coordinates to find the area of the "triangular" region in the first quadrant bounded by $y=3x^2$, $x=0$ & $x+y = 4$. **Hint**: The point (1, 3) is a corner point of the region.

14. (8 %)

(i) Show that $\mathbf{F} = (x^2 - y)\mathbf{i} - (x + y^2)\mathbf{j}$ is a conservative vector field

(ii) Find a potential function for **F**

(iii) Evaluate $\oint_C (x^2 - y)dx - (x + y^2)dy$ where C is the line segment from (0, 1) to (2,0).

15. (5 %) Set up (*but do not evaluate*) the triple integral(s) in **Spherical** coordinates to find the volume and in the first octant of the surface inside the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 29$

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16. (5%) Set up (*but do not evaluate*) the triple integral(s) in **Cylinderical** coordinates to find the volume in the 1^{st} octant common to the cylinders $x^2 + y^2 = 4$ and $x^4 + z^2 = 1$.

17. (5 %) Consider the transformation $u = x - xy$ & $v = xy$

(so $x = u + v$ & $y = \ldots\ldots$)

(i) Show that the Jacobian $J = \dfrac{\partial(x, y)}{\partial(u, v)} = \dfrac{1}{u + v}$.

(ii) Use the above transformation to find $\iint_R x\, dy dx$ where R is the region bounded by the curves $x - xy = 1$, $x - xy = 5$, $xy = 2$, $xy = 3$

18. (5 %) Use Lagrange multipliers to find the points closest to the origin on the hyperbolic cylinder $x^2 - y^2 = 1$

19. (5 %) Find **by inspection** potential functions for the following conservative fields

(i) $F = \dfrac{-y}{x^2+y^2}\,\mathrm{i} + \dfrac{x}{x^2+y^2}\,\mathrm{j}$ for $(x, y) \neq (0,0)$

(ii) $F = \dfrac{x}{x^2+y^2}\,\mathrm{i} + \dfrac{y}{x^2+y^2}\,\mathrm{j}$ for $(x, y) \neq (0,0)$

(iii) $F = \dfrac{x}{(x^2+y^2)^2}\,\mathrm{i} + \dfrac{y}{(x^2+y^2)^2}\,\mathrm{j}$ for $(x, y) \neq (0,0)$

Box your answers