01-750 CEN325
26-11-09

Notre Dame University
Date : 16/11/2009

Exam1: Shallow Foundations – CEN 325

Problem 1

Refer to the following soil profile:

1) Draw the total stress, pore pressure and effective stress profiles with depth for this case (assume $\gamma_w = 1\ T/m^3$).
2) Compute the consolidation settlement from an additional foundation load which would occur in the first clay layer.

The maximum past pressure (σ'_c) at 15m is 35 T/m^2 and the final actual stress at this depth after the foundation load has been applied is 25 T/m^2.

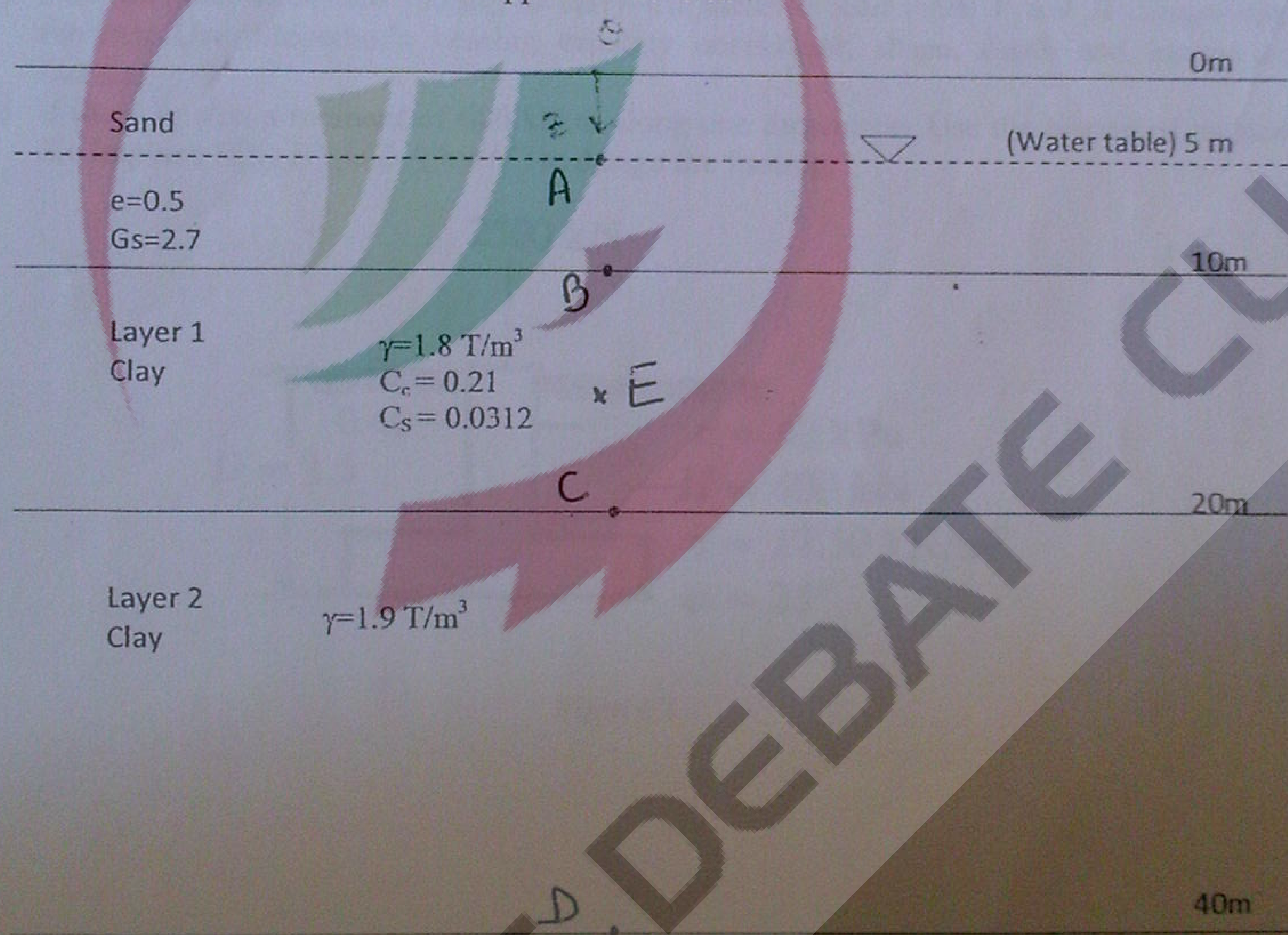


Figure 1

Problem 2

A footing of 1.5 m wide by 3 m long is founded on a stratum of sand. The vertical load is uniformly distributed; there is no horizontal load. The value of Df is 1 m, the water table is at the base of the footing, and the friction angle from the triaxial test is 30°. For the sand above the base of the footing, the water content is 20%, and the unit weight is 17 kN/m^3. The specific gravity of the particles of sand is 2.67:

1) Compute the net allowable load, Q in kPa, using the Hansen equations (for the bearing capacity coefficients and for the depth and shape factors). Use a safety factor of 3.
2) The problem remains the same as in part 1 except that the footing rests on overconsolidated clay at a depth of 1 m (i.e. D_f=1m). Assume the same unit weights as before and determine the undrained shear strength of the clay to yield the value of the net allowable load found in part 1.

Problem 3

Referring to figure 2 with a safety factor SF=5:

1) Find the size of square footing to carry the inclined load (with V and H components shown). Use Meyerhof's bearing capacity coefficient, shape, depth and inclination factors.
2) If there is also a moment of 600 kN.m along one dimension. Use the Meyerhof method (i.e. replace B by B', and A by A') to design the footing.

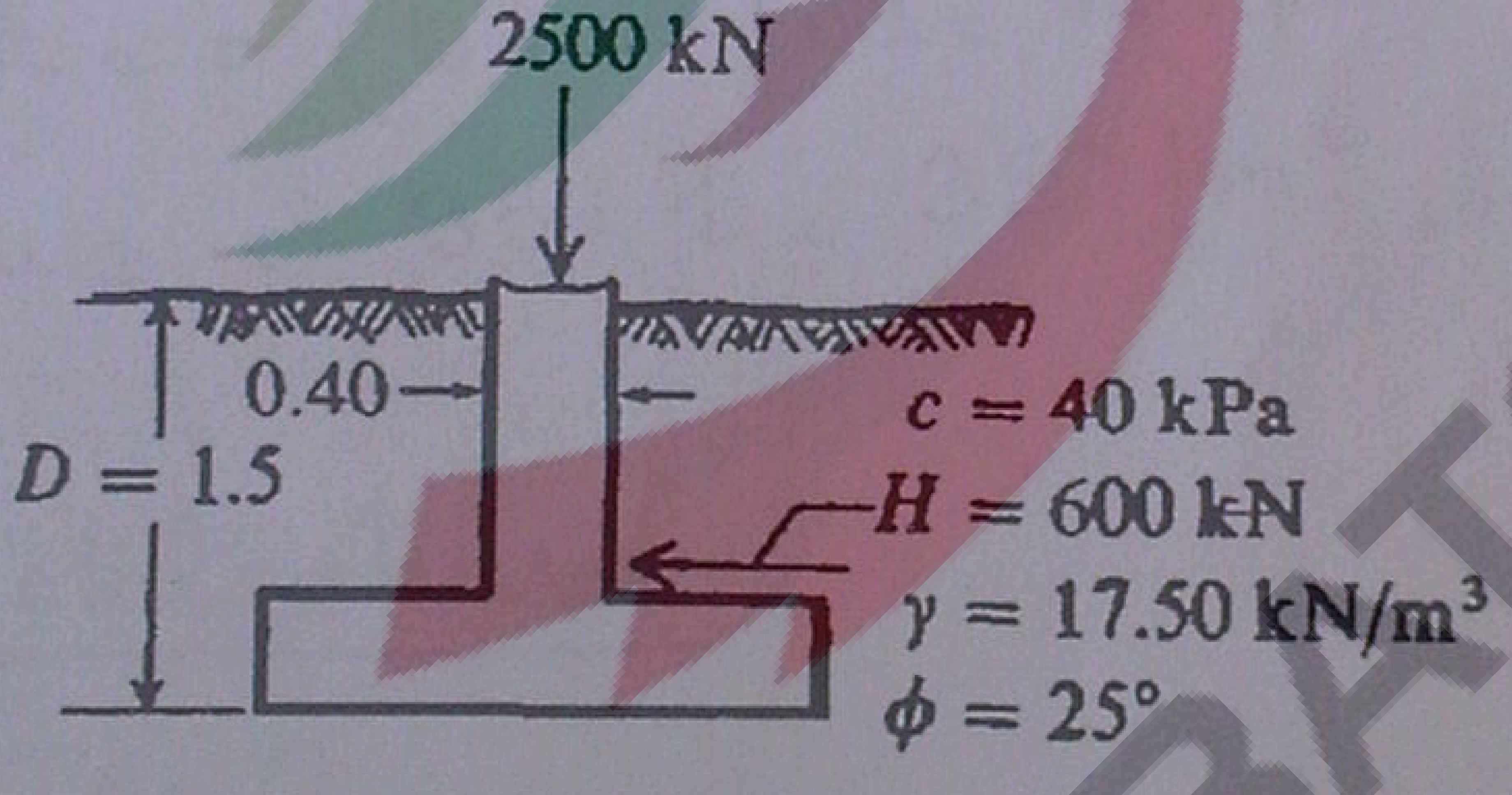


Figure 2

CEN 325 : Exam 1 - shallow foundations

Fall 2009 - Key

* Problem 1 (30)

1°) * Pore pressure :

(3)
$$\begin{cases} u_O = 0 \\ u_A = 0 \\ u_B = \gamma_w \times 5 = 5 \times 1 = 5\,T/m^2 \\ u_C = (5+10) \times 1 = 15\,T/m^2 \\ u_D = 35 \times 1 = 35\,T/m^2 \end{cases}$$

* Total stresses :

$\sigma_O = 0$

$\sigma_A = 5 \times \gamma_{d\,sand}$

(2) $\gamma_{d\,sand} = \dfrac{G_s \gamma_w}{1+e} = \dfrac{2.7 \cdot 1}{1+0.5} = 1.8\,T/m^3$

(2) $\gamma_{sat\,sand} = \dfrac{G_s \gamma_w + e\gamma_w}{1+e} = \dfrac{2.7 \times 1 + 0.5 \times 1}{1+0.5} = 2.13\,T/m$

(4)
$$\begin{cases} \sigma_A = 5 \times 1.8 = 9\,T/m^2 \\ \sigma_B = 5 \times 1.8 + 5 \times 2.13 = 19.65\,T/m^2 \quad (1) \\ \sigma_C = 5 \times 1.8 + 5 \times 2.13 + 1.8 \times 10 = 37.65\,T/m^2 \\ \sigma_D = 5 \times 1.8 + 5 \times 2.13 + 1.8 \times 10 + 20 \times 1.9 = 75.65\,T/m \end{cases}$$

* Effective stresses :

$\sigma' = \sigma - u$

(4)
$$\begin{cases} \sigma'_0 = 0 \\ \sigma'_A = \sigma_A = 9\,T/m^2 \\ \sigma'_B = 19.65 - 5 = 14.65\,T/m^2 \\ \sigma'_C = 37.65 - 15 = 22.65\,T/m^2 \quad (1) \\ \sigma'_D = 75.65 - 35 = 40.65\,T/m^2 \end{cases}$$

2°) $\sigma'_c = 35\,T/m^2$ at 15m

$\sigma'_f = \sigma'_0 + \Delta\sigma'$

(1) $\sigma'_{fE} = \sigma'_{0E} + \Delta\sigma' = 25\,T/m^2$

(3) $\Delta\sigma' = 25 - \sigma'_{0E} = 25 - \underbrace{\left[5\times1.8 + 5\times(2.13-1) + 5\times(1.8-1)\right]}_{18.65\,T/m^2}$

$\Delta\sigma' = 6.35\,T/m^2$ (1)

$\sigma'_{0E} = 18.65 < \sigma'_c = 35\,T/m^2$ (2)

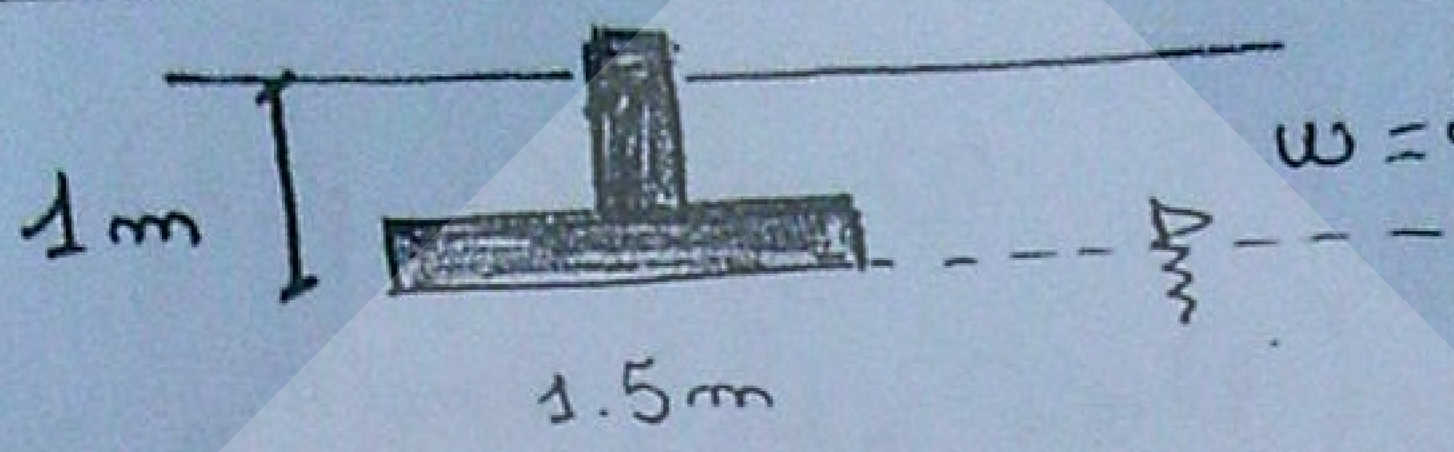
$\Longrightarrow$ overconsolidated clay.

and $\sigma'_{0E} + \Delta\sigma' = 25 < 35 \Longrightarrow$ Case I. (2)

(2) $S_c = \dfrac{C_s H_c}{1+e_0} \log \dfrac{\sigma'_0 + \Delta\sigma}{\sigma'_0} = \dfrac{0.0312\times10}{1+0.2} \log \dfrac{25}{18.65}$

$S_c = 0.0329$ m $\quad \boxed{S_c \simeq 33\ mm}$ (2)

Problem 2 (35)



$w = 20\%$ $\gamma = 17\ kN/m^3$

$G_s = 2.67$

$\varphi = 30°$

(3p) 1°) **Find γ_{sat} :**

$$\gamma_{sat} = \frac{G_s \gamma_w + e\gamma_w}{1+e}$$

Find e : $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{\gamma}{1+w}$ (1)

$$\Rightarrow 1+e = (1+w)\, G_s \frac{\gamma_w}{\gamma}$$

$$\Rightarrow e = (1+w)\, G_s \frac{\gamma_w}{\gamma} - 1$$

$e = 0.85$ (2)

$$\begin{cases} \gamma_{sat} = (G_s + e)\dfrac{\gamma_w}{1+e} = 18.66\ kN/m^3 \quad (2) \\ \gamma' = \gamma_{sat} - \gamma_w = 18.66 - 9.81 = 8.85\ kN/m^3 \quad (1) \end{cases}$$

$$q_{ult} = c N_c F_{cd} F_{cs} + q N_q F_{qd} F_{qs} + \frac{1}{2}\gamma B N_\gamma F_{\gamma d} F_{\gamma s}$$ (1)

* Hansen factors :

$$\begin{cases} N_c = (N_q - 1)\cot g\,\varphi \\ N_q = e^{\pi \tan\varphi}\left(\dfrac{1+\sin\varphi}{1-\sin\varphi}\right) \\ N_\gamma = 1.5\, N_c \tan^2\varphi \end{cases}$$

$\varphi = 30° \Longrightarrow$ $N_c = 30.14$ (1)

$N_q = 18.4$ (1)

$N_\gamma = 15.07$ (1)

* Shape factors :

$$\begin{cases} F_{cs} = 1 + \dfrac{B}{L}\dfrac{N_q}{N_c} = 1.305 \quad (1) \\ F_{qs} = 1 + \dfrac{B}{L}\tan\varphi = 1.29 \quad (1) \\ F_{\gamma s} = 1 - 0.4\dfrac{B}{L} = 0.8 \quad (1) \end{cases}$$

* Depth factors :

$$\frac{D_f}{B} = \frac{1}{1.5} < 1 \quad \begin{cases} F_{cd} = 1 + 0.4\dfrac{D_f}{B} = 1.266 \quad (1) \\ F_{qd} = 1 + 2\tan\varphi(1-\sin\varphi)^2\dfrac{D_f}{B} = 1.19 \quad (1) \\ F_{\gamma d} = 1 \quad (1) \end{cases}$$

* <u>Water table.</u>

$d = D_f$

$q = \gamma D_f$ (1)

replace γ by $\bar{\gamma} = \gamma'_{(1)} + 0$

} Case II and Case III are the same

$q_{ult} = 0 + 17 \times 1 \times 18.4 \times 1.192 \times 1.29$

$+ \frac{1}{2} \times 8.85 \times 1.5 \times 15.07 \times 0.8 \times 1$ (2)

$q_{ult} = 560.2 \ kN/m^2$

$Q_{all\,net} = \frac{(q_{ult} - q)A}{FS} = \frac{(560.2 - 17 \times 1) \times 1.5 \times 3}{3}$ (2)

$Q_{all\,net} = 814.8 \ kN$ (1)

(2P) 2°)

$q_{ult} = c_u N_c F_{cd} F_{cs} + q N_q F_{qd} F_{qs}$ (1)

$N_\gamma = 0$ (1) for $\varphi = 0$ $N_q = 1$ (1) $N_c = 5.14$ (1)

$814.8 = \left(\frac{q_{ult} - q}{FS}\right) \times A$

$q_{ult} = \frac{814.8 \times 3}{1.5 \times 3} + \overset{17 \times 1}{\tilde{q}} = 560.2 \ kN/m^2$

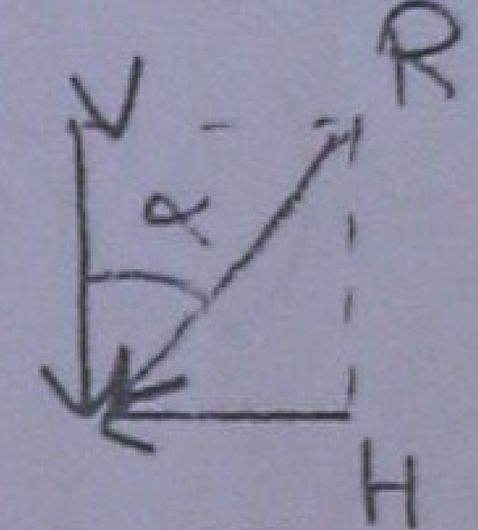
$$\begin{cases} F_{cs} = 1 + \frac{1.5}{3}\,\frac{1}{5.14} = 1.097 \ (1) & F_{qs} = 1 \ (1) \\ F_{cd} = 1.266 \ (1) & F_{qd} = 1 \ (1) \end{cases}$$

$$560.2 = C_u \times 5.14 \times 1.097 \times 1.266 + 17 \times 1 \times 1 \times 1 \quad (4)$$

$$\boxed{C_u = 76.1 \ kN/m^2}$$

Problem 3 (35)

(24p) 1°) * Inclination :



(2) $\tan\alpha = \frac{H}{V} \Longrightarrow \alpha = 13.5°$

$$\begin{cases} F_{ci} = F_{qi} = \left(1 - \frac{\alpha°}{90}\right)^2 = \left(1 - \frac{13.5}{90}\right)^2 = 0.7225 \quad (1\tfrac{1}{2}) \\ F_{\gamma i} = \left(1 - \frac{\alpha°}{\varphi°}\right)^2 = \left(1 - \frac{13.5}{25}\right)^2 = 0.2116 \quad (1\tfrac{1}{2}) \end{cases}$$

$\varphi = 25° \xrightarrow{\text{Meyerhoff}}$ $N_c = 20.72$ (1)

$N_q = 10.66$ (1)

$N_\gamma = 6.77$ (1)

* Shape factors

$\varphi \geq 10°$ $F_{cs} = 1 + 0.2 \frac{B}{L} \tan^2(45 + \frac{\varphi}{2})$ (1)

$F_{qs} = F_{\gamma s} = 1 + 0.1 \frac{B}{L} \tan^2(45 + \frac{\varphi}{2})$ (1)

square $B = L$

$F_{cs} = 1 + 0.2 \tan^2(45 + \frac{25}{2}) = 1.493$ (1/2)

$F_{qs} = F_{\gamma s} = 1 + 0.1 \tan^2(45 + \frac{25}{2}) = 1.246$ (1/2)

* (1) Depth factors:

$\varphi \geq 10°$ $F_{cd} = 1 + 0.2 \frac{\overset{1.5}{D_f}}{B} \tan(45 + \frac{\varphi}{2}) = 1 + \frac{0.471}{B}$ (1 1/2)

$F_{qd} = F_{\gamma d} = 1 + 0.1 \frac{\overset{1.5}{D_f}}{B} \tan(45 + \frac{\varphi}{2}) = 1 + \frac{0.235}{B}$ (1 1/2)

$Q_{all} = \frac{q_{ult} \times A}{3} = 2500$ (1)

$\frac{2500 \times 3}{B^2} = c N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$

(5)

$\frac{2500 \times 3}{B^2} = 40 \times 20.72 \times 1.493 \left(1 + \frac{0.471}{B}\right) \times 0.7225$

$+ 17.5 \times 1.5 \times 10.66 \times 1.246 \times \left(1 + \frac{0.235}{B}\right) \times 0.7225$

$+ 0.5 \times 17.5 \times B \times 6.77 \times 1.246 \times \left(1 + \frac{0.235}{B}\right) \times 0.2116$

By trial and error for $B =$ m

$Q_{ult} \simeq 12602{,}13\ kN > 2500 \times 5 = 12500\ kN$

so $\boxed{B = 3.15m}$ (4)

p) 2°)

$M = 600\ kN.m$

$e_B = \frac{M}{Q=V} = \frac{600}{2500} = 0.24m$ (1)

$B' = B - 2 \times 0.24 = B - 0.48$ (1)

$A' = B' \times \underset{=B}{L} = (B - 0{,}48) \times B$ (1)

* inclination factors are the same as 1)

$\begin{cases} F_{qi} = F_{ci} = 0{,}7225 \\ F_{\gamma i} = 0{,}46 \end{cases}$ (1)

* Depth factors are the same as 1)

$\begin{cases} F_{cd} = 1 + 0{,}471/B \\ F_{qd} = F_{\gamma d} = 1 + 0.235/B \end{cases}$ (1)

* Shape factors → B by B'

$$F_{cs} = 1 + 0.2\frac{(B - 0.48)}{B}\tan^2\left(45 + \frac{25}{2}\right)$$

$$F_{qs} = F_{\gamma s} = 1 + 0.1\left(\frac{B - 0.48}{B}\right)\tan^2\left(45 + \frac{25}{2}\right)$$

$$\begin{cases} F_{cs} = 1 + \frac{0.49}{B}(B - 0.48) & (1\tfrac{1}{2}) \\ F_{qs} = F_{\gamma s} = 1 + \frac{0.246}{B}(B - 0.48) & (1\tfrac{1}{2}) \end{cases}$$

$$\frac{2500 \times 3}{(B - 0.48) \times B} = 40 \times 20.72 \times 0.7225 \times \left(1 + \frac{0.471}{B}\right) \times \left(1 + \frac{0.49}{B}(B - 0.48)\right)$$

$$+ 17.5 \times 1.5 \times 10.66 \times 0.7225 \times \left(1 + \frac{0.235}{B}\right) \times \left(1 + \frac{0.246}{B}(B - 0.48)\right)$$

(2)

$$+ 0.5 \times 17.5 \times (B - 0.48) \times 6.77 \times 0.2116 \times \left(1 + \frac{0.235}{B}\right)\left(1 + \frac{0.246}{B}(B - 0.48)\right)$$

Trial and error

for $B = 3.4\ m$ (1) — $Q_{ult} = 12686 > 2500 \times 5$

OK