(2)

American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2004

Final Exam

Name: .... ID #: ....

Exercise 1 a) *[2 points]*: Find $\lim_{n\to+\infty} \frac{\ln^n n}{n}$.

(hint: this is the same sequence of quiz 1)

b) *[3 points]*: What can you say about the series $\sum_{n=1}^{+\infty} \frac{\ln^n n}{n^2}$? Justify.

Exercise 2 a) *[3 points each]* Discuss whether the following series converges or diverges.

i) $\sum_{n=1}^{+\infty}(-1)^n \frac{\sin(n\frac{\pi}{2})}{n^2}$

ii) $\sum_{n=0}^{+\infty} \frac{1}{2^{n^2}}$

iii) $\sum_{n=1}^{+\infty} n^2 \sin(1/n)\tan(1/n)$

b) *[8 points]* Find the interval of convergence of the power series $\sum_{n=1}^{+\infty} \frac{\ln n}{n}(x+2)^n$

(be sure to check at the end points)

Exercise 3 a) *[3 points]* Find the equation of the tangent plane to the surface $z = x^5 - x^2y^2 + 4$ at the point $(1, 1, 4)$.

b) *[5 points]* Suppose that the equation $x^5 - x^2y^2 + 2yz - 8 = 0$ defines x as a function of y and z. Find the values of $\frac{\partial x}{\partial y}$ and $\frac{\partial x}{\partial z}$ at the point $(1, 1, 4)$.

c) *[3 points]* Prove or disprove: $g(x, y) = \frac{x^6y^4}{x^2+y^2}$ can be extended by continuity at $(0, 0)$. Justify.

Exercise 4 *[15 points]* Find the absolute minimum and maximum values of the function $f(x, y) = x^2 + y^2 + 2x - y + 1$ on the domain R defined by $\{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4 \text{ and } y \geq 0\}$.

Exercise 5 *[10 points]* Evaluate $I = \int_0^1 \int_z^1 \int_0^x e^{x^3}\, dydxdz$.

Exercise 6 *[12 points]* Let V be the volume of the region R bounded laterally by the cylinder $x^2 + (y-1)^2 = 1$, from above by the cone $z = \sqrt{x^2 + y^2}$, and from below by the xy-plane.

a) Sketch the region of integration.

b) Express V as iterated triple integral in cartesian coordinates in the order $dzdxdy$ *(do not evaluate the resulting integral).*

c) Express V as iterated triple integral in cylindrical coordinates, then evaluate the resulting integral.

Exercise 7 *[15 points]* Use the transformation $u = y/x, v = xy$ to find

$$\int\int_R e^{xy}\, dA$$

over the region R in the first quadrant enclosed by the lines $y = x, y = x/2$ and the hyperbola $y = 1/x$ and $y = 2/x$.
(sketch both regions of integration)

Exercise 8 *[15 points]*

a) Write the complete statement of Green's Theorem.

b) Find $\oint_C (x^2 - y)dx + xdy$, where C is the closed curve positively directed given by the equations $x^2 + y^2 = 4$, and $y \geq 0$.

i) By evaluating directly the line integral.

ii) By using Green's Theorem quoted above.