Name:

MEN 302
EXAM 2
SUMMER 2004

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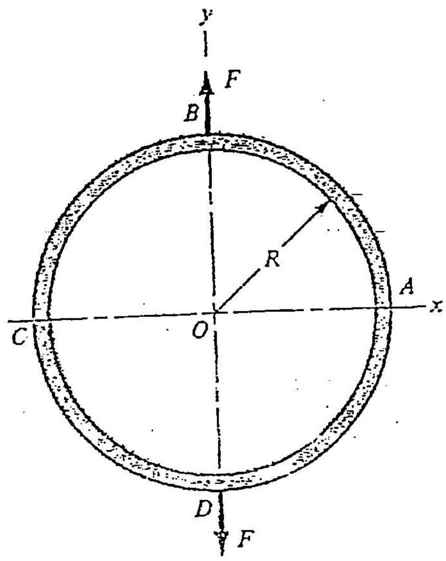
1) (30 pts) A thin ring is loaded by forces F as shown. Assuming that the entire strain energy in the ring is due to bending only, determine the maximum bending moment in the ring.



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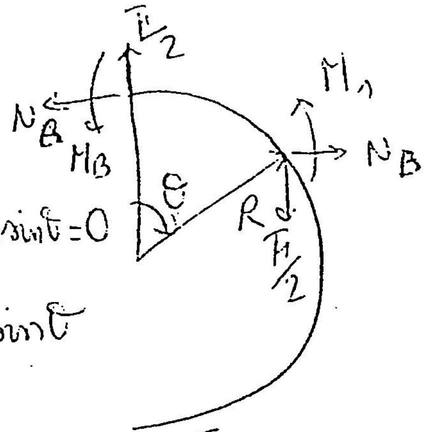
$\delta_{B.x} = 0 = \theta_B$

$0 \leq \theta \leq \pi/2$

1) Let $M_B = Q$ ✓



$M_1 + Q + N_B(R - R\cos\theta) - \frac{F}{2} R\sin\theta = 0$

$M_1 = -Q - N_B R(1 - \cos\theta) + \frac{F}{2} R\sin\theta$

$\frac{\partial M_1}{\partial Q} = -1$

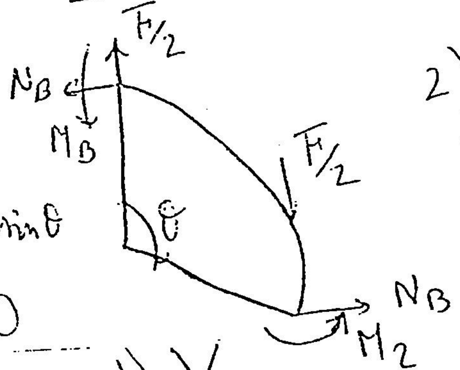
1) $N_B = Q$ X

$\cancel{M_1 = M_B - QR(1-\cos\theta)}$

$\frac{\partial M_1}{\partial Q} = -R(1 - \cos\theta)$

$\pi/2 \leq \theta \leq \pi$

2) ~~Let $M_B = Q$~~ $M_B = Q$



2) $N_B = Q$

$M_2 = -M_B + \frac{F}{2}R - QR(1$

$\frac{\partial M_2}{\partial Q} = -R(1 - \cos\theta$

$M_2 - \frac{F}{2} R(1 - \sin\theta) + Q - \frac{F}{2} R\sin\theta$

$+ N_B R(1 - \cos\theta) = 0$

$\boxed{M_2 = -Q + \frac{F}{2} R - N_B R(1 - \cos\theta)}$ X

$$\delta_{Bx} = 0 = -\frac{1}{EI}\int_0^{\pi/2}\left(-M_B - \phi R(1-\cos\theta) + \frac{F}{2}R\sin\theta\right)R^2(1-\cos\theta)\,d\theta$$

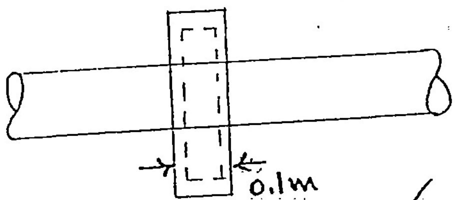
$$-\frac{1}{EI}\int_{\pi/2}^{\pi}\left(-M_B + \frac{F}{2}R - \phi R(1-\cos\theta)\right)R_2(1-\cos\theta)\,d\theta$$

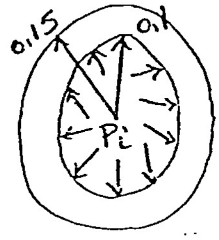
We set $\phi = N_B$ and then integrate to find:

$$-\cancel{\int_0^{\pi/2} M_B - N_B R(1-\cos\theta)) + \frac{F}{2}R\sin t}$$

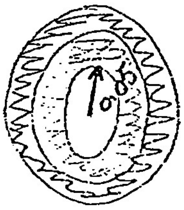
$\cancel{H = \phi R}$ $M_B = \frac{FR}{4}$ and $N_B = \frac{P}{2\pi}$

2) (20 pts) A gear of inner and outer radii 0.1 and 0.15 m, respectively, is shrunk onto a hollow shaft of inner radius 0.05 m. The maximum tangential stress induced in the gear wheel is 0.21 MPa. The length of the gear wheel parallel to the shaft axis is 0.1 m. Assuming a coefficient of static friction of 0.2, at the common surface, determine the torque transmitted by the gear without slip.





gear wheel



tact pressure at $r = r_i = 0.1m$: $P_i = \dfrac{\sigma_\theta (b^2 - a^2)}{a^2\left(1 + \dfrac{b^2}{a^2}\right)} = \dfrac{(0.21)(0.15^2 - 0.1^2)}{(0.1)^2\left(1 + \dfrac{0.15^2}{0.1^2}\right)} = 0.08\ MPa$

rce N_r due to P_i : $N_r = (0.08)(2\pi)(0.1)(0.1) = 5.03\ KN$

rque transmitted by the gear : $T = \mu_s N_r \cdot r_{shaft} = (0.2)(5.03)(0.05) = 0.05\ KN.m$

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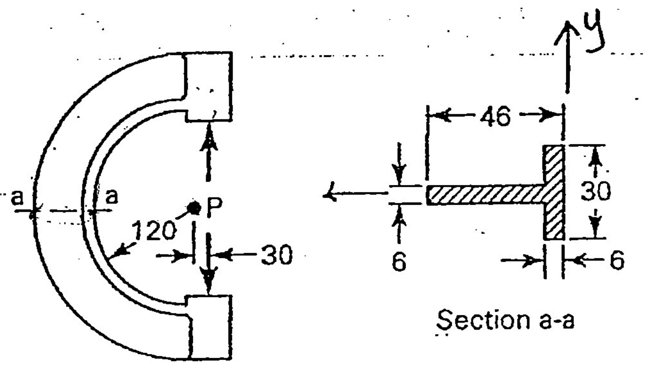
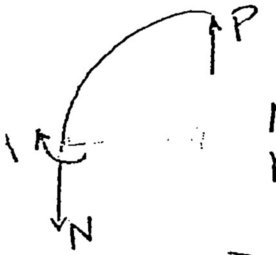
3) (30 pts) The allowable stress in tension and compression of the clamp body shown is $80 MPa$. Calculate the maximum permissible load the member can resist. All dimensions are in millimeters.





$N = P$

$M = P(0.15 + \bar{y})$

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$$\bar{y} = \frac{3(30)(6) + 26(6)(40)}{(30)(6) + (40)(6)} = 16.14 \text{ mm}$$

$\Rightarrow M = (0.15 + 0.0161)P = 0.166P$

$$\bar{r} = 120 + \frac{(30)(6)^2 + 2(6)(6)(40) + (6)(40)^2}{2[(6)(40) + (30)(6)]} = 136.14 \text{ mm}$$

$$?= \frac{(30)(6) + (6)(40)}{(30)\ln\left[\frac{(120+6)}{120}\right] + (6)\ln\left[\frac{(166)}{(120+6)}\right]} = 134.7 \text{ mm}$$

$= \frac{M(R-r)}{rA(\bar{r}-R)}$ due to bending $\Rightarrow \sigma_{Compression} = \frac{(0.166)P(0.1347-0.166)}{(0.166)(0.42\times10^{-3})(0.13614-0.1347)}$

$= -51.75P$ KPa

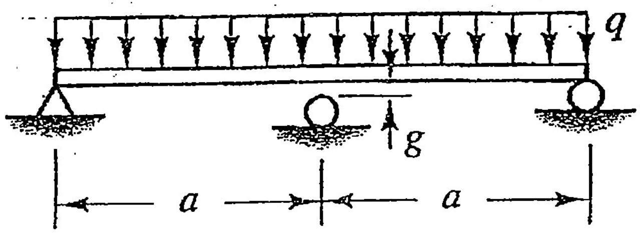
$$\sigma_{tension} = \frac{(0.166)P(0.1347-0.12)}{(0.12)(0.42\times10^{-3})(0.13614-0.1347)} = 33.6P \text{ KPa}$$

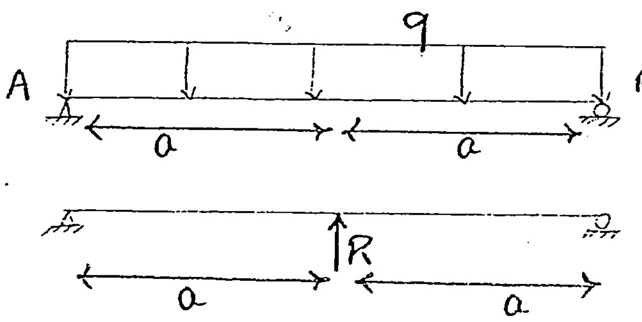
$_{total} = -51.75P + \frac{P}{0.42\times10^{-3}} = -49.37P$ KPa

$_{total} = 33.6P + \frac{P}{0.42\times10^{-3}} = 36P$ KPa

$\Rightarrow P = \frac{80}{\dots} = 1620$ MN

4) (20 pts) The uniform simply-supported beam shown makes contact with the central support only after the gap g is closed by application of the uniformly distributed load q. Determine the reaction of the central support.





$$\delta_{max_1} = \delta(a) = \frac{-5q(2a)^4}{384EI}$$

$$\delta_{max_2} = \delta(a) = \frac{R(2a)^3}{48EI}$$

$$g = \delta_{max_1} + \delta_{max_2} = \frac{-5q(16a^4)}{384EI} + \frac{8a^3R}{48EI}$$

$$\frac{8a^3R}{48EI} = g + \frac{5q(16a^4)}{384EI}$$

$$\Rightarrow R = \frac{48EIg}{8a^3} + \frac{10qa}{8} = \frac{6EIg}{a^3} + \frac{5qa}{4} = \frac{24EIg + 5qa^4}{4a^3}$$

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