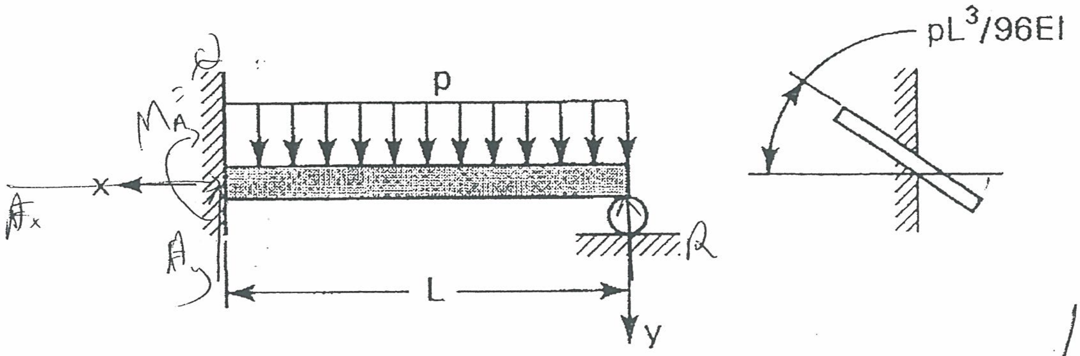
94 good

Name: _

m A

MEN 302
EXAM 2
SPRING 2007

1) (15 pts) The slope at the wall of a fixed beam is as shown in part (b) of the figure and is given by $\frac{pL^3}{96EI}$. Determine the force acting at the simple support, expressed in terms of p and L.



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$\sum F_x = 0 \Rightarrow A_x = 0$

$\sum F_y = 0 \Rightarrow A_y + R = pL.$

The beam is statically indeterminate to the 1st degree.

Let $Q = M_A$

$\sum M_A = 0 \Rightarrow Q + \frac{pL^2}{2} - RL = 0$

$\Rightarrow R = \frac{Q}{L} + \frac{pL}{2}$

$\boxed{A_y = \frac{pL}{2} - \frac{Q}{L}}$

OK

M V R p x

$M + \frac{px^2}{2} - Rx = 0$

$\Rightarrow M = Rx - \frac{px^2}{2} = \frac{Qx}{L} + \frac{pLx}{2} - \frac{px^2}{2}$

$0 \le x \le L$

$$\frac{\partial M}{\partial Q} = \frac{x}{L}$$

$$M \cdot \frac{\partial M}{\partial Q} = \frac{Qx^2}{L^2} + \frac{Px^2}{2} - \frac{Px^3}{2L}$$

Energy in the beam is due to bending only

$$\theta_A = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^L \frac{Qx^2}{L^2} + \frac{Px^2}{2} - \frac{Px^3}{2L}\,dx$$

OK

$$= \frac{1}{EI}\left[\frac{Qx^3}{3L^2} + \frac{Px^3}{6} - \frac{Px^4}{8L}\right]_0^L$$

$$= \frac{1}{EI}\left[\frac{QL}{3} + \frac{PL^3}{6} - \frac{PL^3}{8}\right] \qquad \therefore Q = M_A$$

$$= \frac{1}{\cancel{EI}}\left[\frac{M_A L}{3} + \frac{PL^3}{24}\right] = \frac{PL^3}{96\cancel{EI}}$$

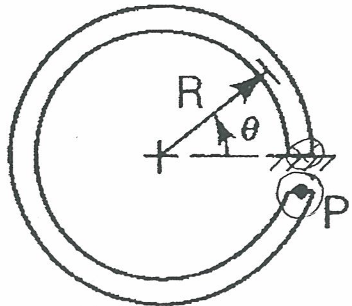
$$\frac{M_A L}{3} = -\frac{PL^3}{32}$$

$$\boxed{M_A = \frac{-3PL^2}{32}}$$ OK

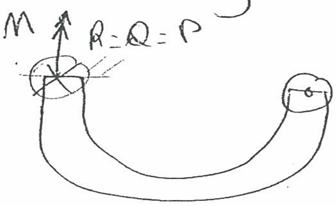
but $$\boxed{R = \frac{M_A}{L} + \frac{PL}{2} = \frac{-3PL}{32} + \frac{PL}{2} = \frac{13PL}{32}}$$

2) (15 pts) A cylindrical circular rod in the form of a split circular ring of radius R is fixed at one end. A force P is applied at the free end in a direction perpendicular to the plane of the ring. Using Castigliano's theorem and considering the energy due to bending only, calculate the deflection at the free end. $= N, M_T$

Set $Q = P$



Due to symmetry we'll only consider half a ring:



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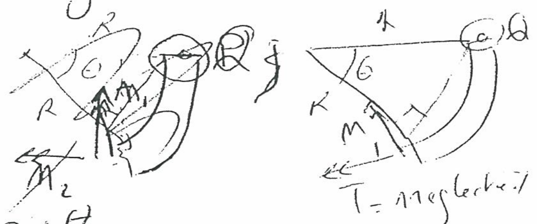
$M = 2Q \cdot R$

taking a ~~cut~~ cut and isolating the right part:

$\cancel{M_1 - Q}$



(+)↑

(+)↑ $M_1 - QR(\sin\theta) = 0$

$M_1 = QR(\sin\theta)$ ✓

~~$M_2 = QR\sin\theta$~~

$\frac{\partial M_1}{\partial Q} = R(\sin\theta)$ ✓ ~~$\frac{\partial M_2}{\partial Q} = R\sin\theta$~~

Applying Castigliano's Theorem. $(Q = P)$

$$\delta_P = \frac{2}{EI}\int_0^{\pi} PR^2(\sin\theta)^2 + \cancel{PR^2(\sin\theta)}\, d\theta$$

$$\delta_p = \frac{2PR^2}{EI}\int_0^{\pi} 1 - 2\cos\theta + \cos^2\theta \, d\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta) = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\delta = \frac{2PR^2}{EI}\int_0^{\pi} \frac{3}{2} - 2\cos\theta + \cos 2\theta \, d\theta$$

$$= \frac{2PR^2}{EI}\left[\frac{3\theta}{2} - 2\sin\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi}$$

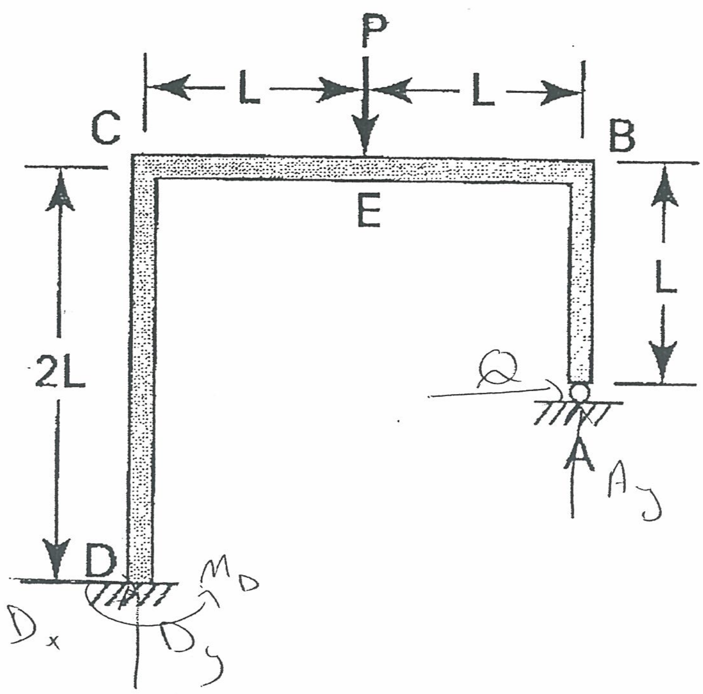
$$\boxed{\delta_p = \frac{3\pi PR^2}{EI}}$$

$$\delta_p = \frac{2PR^2}{EI}\int_0^{\pi} \sin^2\theta \, d\theta = \frac{2PR^2}{EI}\int_0^{\pi} \frac{1 - \cos 2\theta}{2} \, d\theta$$

$$= \frac{2PR^2}{EI}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$

$$\boxed{\delta_p = \frac{\pi PR^2}{EI}}$$

3) (20 pts) A frame of constant flexural rigidity EI carries a concentrated load P at point E. Using Castigliano's theorem, determine the horizontal displacement δ_h at support A. Consider the energy due to bending only.



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$\sum F_x = 0 \rightarrow D_x = 0$

$\sum F_y = 0 \Rightarrow D_y + A_y = P$

$+\circlearrowleft \sum M_D = 0 \Rightarrow -M_D + PL - A_y 2L = 0$

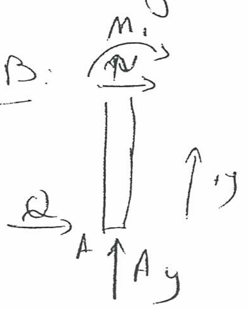
$M_D = PL - 2A_y L$

Putting ~~Q~~ a force $Q = 0$ at A as shown.

Since we must consider the Energy due to bending, we will not calculate V & N for all X-sections.

Member AB:

$0 < y < L$

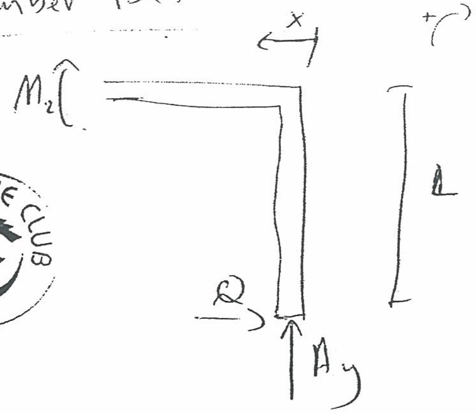


$M_1 = Qy$ ✓ $\dfrac{\partial M_1}{\partial Q} = y$

$\left(U_1 = \dfrac{1}{EI} \int_0^L Qy^2 \, dy \right)$

a) $0 < x < L$

Member BC,

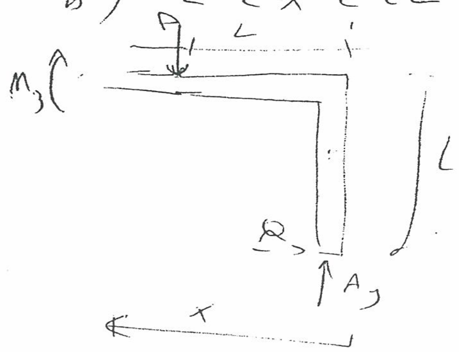


$$M_2 - A_y x - QL = 0$$

$$M_2 = QL + A_y x \quad ✓$$

$$\frac{\partial M_2}{\partial Q} = L$$

b) $L < x < 2L$



$$M_3 - A_y x - QL + P(x-L) = 0$$

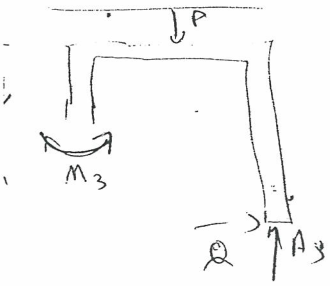
$$M_3 = QL + A_y x - Px + PL$$

$$\frac{\partial M_3}{\partial Q} = L \quad ✓$$

$$U_2 = \left[\int_0^L M_2 \frac{\partial M_2}{\partial Q}\,dx + \int_L^{2L} M_3 \frac{\partial M_3}{\partial Q}\,dx\right]\frac{1}{EI}$$

$$= \frac{1}{EI}\left[\int_0^L (QL^2 + A_y x L)\,dx + \int_L^{2L} (QL^2 + A_y x L - PxL + PL^2)\,dx\right]$$

Member CD $0 < y < 2L$



$$-M_3 + PL - A_y 2L + -Q(2L-y) = 0$$

$$M_3 = -Q(2L-y) - 2A_y L + PL$$

OK

$$\frac{\partial M_3}{\partial Q} = -(2L-y)$$

$$U_3 = \frac{1}{EI}\int_0^{2L} \left(4L^2Q - 4LyQ + y^2Q + 4A_y L^2 - 2A_y yL - 2PL^2 + PLy\right)dy$$

$$\delta_h = U_1 + U_2 + U_3 \quad \text{with } Q = 0 \quad ✓$$

$$\Rightarrow \delta_h = \frac{1}{EI}\left[\int 0\,dy + \int_0^L A_y x L\,dx + \int_L^{2L} A_y x L + P_x L - PL^2\,dx + \int_0^{2L} \left(4A_y L^2 - 2A_y yL - 2PL^2 + PLy\right)dy\right]$$

$$\delta_H = \frac{1}{EI}\left[\frac{A_y L^3}{2} + \frac{A_y 4L^3}{2} - \frac{A_y L^3}{2} + \frac{P4L^3}{2} - \frac{PL^3}{2} - 2PL^3 + PL^3 + 8A_y L^3 - \frac{8A_y L^3}{2} - 2PL^3 + \frac{4PL^3}{2}\right]$$

$$\cancel{= \frac{1}{EI}\left(6A_y L^3 - \frac{7PL^3}{2}\right)}$$

$$\delta_H = \frac{1}{EI}\left[2A_y L^3 - \frac{3}{2}PL^3\right]$$

Calculating A_y in function of P, L & EI.

Putting $A_y = Q$ & going over the same sections:

α $M_1 = 0 \Rightarrow U_1 = 0$

α $M_2 = Qx$, $\frac{\partial M_2}{\partial Q} = x$ $\Big\}$ $U_2 = \left[\frac{A_y L^3}{3}\right]\frac{1}{EI}$

α $M_3 = Qx - P(x-L)$, $\frac{\partial M_3}{\partial Q} = x$ $\Big\}$ $U_3 = \left(-\frac{A_y L^3}{3} + \frac{PL^3}{3} - \frac{PL^3}{2} + \frac{8A_y L^3}{3} - \frac{8PL^3}{3} + 2PL^3\right)\frac{1}{EI}$

α $M_4 = 2QL - PL$, $\frac{\partial M_4}{\partial Q} = 2L$ $\Big\}$ $U_4 = \left(8A_y L^3 - 4PL^3\right)\frac{1}{EI}$

$$\delta_{y_A} = 0 = U_1 + U_2 + U_3 + U_4$$

this yields $\Rightarrow A_y = \frac{29}{64}P$

Replacing in δ_H

we get

$$\boxed{\delta_H = -\frac{19L^3}{32EI}}$$

X