Name:

MEN 302
EXAM 2
SPRING 2006

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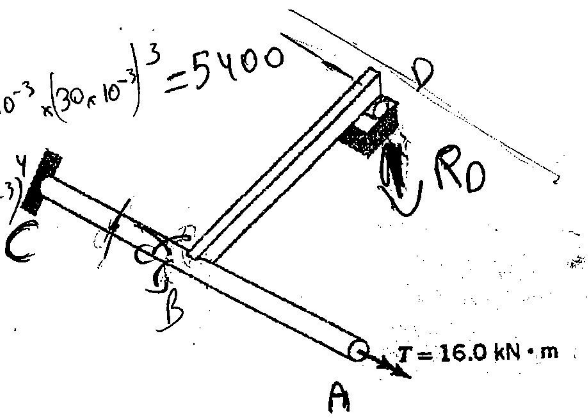
1) (20 pts) A steel torsion member has a length of $3.0m$ and diameter of $120mm$ $(E = 200GPa, G = 77.5GPa)$. It is fixed at one end. A steel beam of rectangular cross section $120mm$ by $30.0mm$, is welded perpendicularly to the torsion member at its midsection, as shown. The beam is supported by a roller located $2.0m$ from the welded section. The free end of the member is subjected to a torque $T = 16kN \cdot m$. Determine the reaction at the roller by Castigliano's theorem.

$EI = 200\times10^{9} \times \frac{1}{12}\, 12\times10^{-3} \times (30\times10^{-3})^{3} = 5400$

$GJ = 77.5\times10^{9} \times \frac{\pi}{2}(60\times10^{-3})^{4}$
$= 1577707.8$



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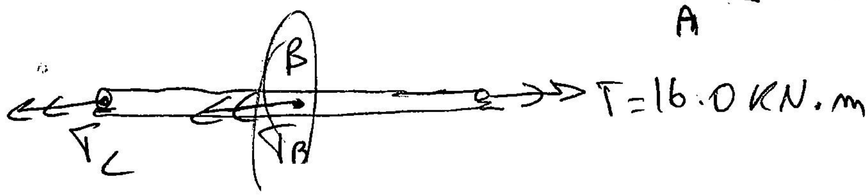


$\cancel{T_A = Q}$

$0 \le x < 1.5$ $\quad T_1 = Q \qquad \frac{\partial T_1}{\partial Q} = 1$

$T_C = Q - T_B$

$T_B = Q - T_C$

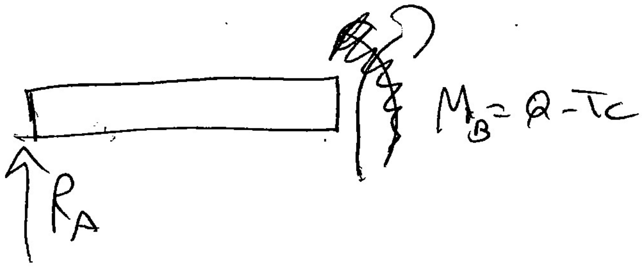
$1.5 \le x < 3 \quad \boxed{T_2 = T_C}$

$= Q - T_B$

$$\boxed{T_2 = Q - T_B}$$

$$\frac{\partial T_2}{\partial Q} = 1$$

On the beam;



$\cancel{\Sigma M_B = Q - T_C}$

$M_B + 2R_A = 0$

$Q - T_C + 2R_A = 0$

$$\boxed{R_A = \frac{T_C - Q}{2}}$$

$0 \le x < 2\,m \qquad M = R_A x \qquad \frac{\partial M}{\partial Q} = -\frac{Q}{2}$

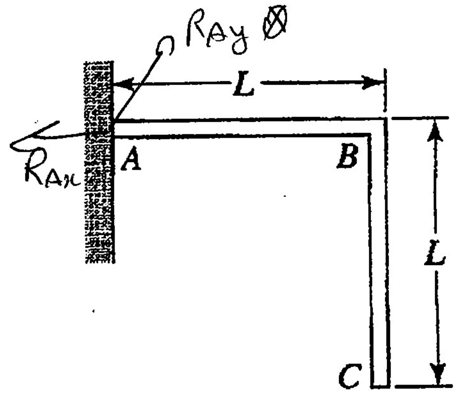
$$\boxed{M = \frac{T_C - Q}{2} x}$$

$\cancel{R_A = 0} = \frac{1}{GJ}\int_0^{1.5} Q\cdot(1)\cdot dx + \frac{1}{GJ}\int_{1.5}^{3} (Q - T_B)(1)\cdot dx + \frac{1}{EI}\int^{2}\left(\frac{T_C - Q}{2}\right)x\left(-\frac{x}{2}\right.$

$0 = \frac{16\times 1.5}{GJ} + \frac{1}{GJ}\left(16\times 3 - T_B\times 3 - 16\times 1.5 + T_B\times 1.5\right) + \frac{1}{EI}\int_0^2 -\frac{T_C x}{4} + \frac{Q^2}{4}x$

$0 = 1.5\times 10^{-5} + 1.5\times 10^{-5} + \frac{T_B\times 1.5}{GJ} + \frac{64 T_C}{8}\times\frac{1}{EI} + 0.02$

2) (15 pts) Member ABC lies in the plane of the paper, has a uniform circular cross-section, and is subjected to a uniform load $w\ \left(\frac{N}{mm}\right)$ that acts perpendicular to the plane of ABC. Use Castigliano's theorem to determine the deflection of point C perpendicular to ABC. Assume that all material and cross-sectional properties are given.



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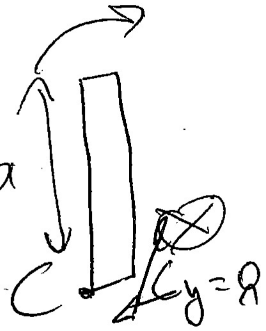
~~R_A~~ ⊗ R_A ⊗

R_C ⊗

M

Let ~~$R_{Ay} = Q$~~ $\boxed{R_C = Q}$

$wxL = F$



~~Section BC:~~

~~$M_1 = Qx \quad \frac{\partial M_1}{\partial Q} = x$~~

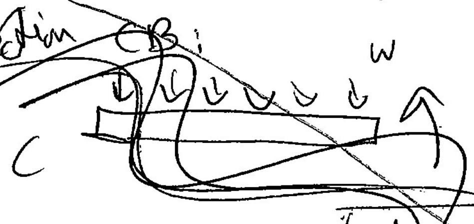
~~$\Sigma F_y = 0$~~ ~~$R_{Ay} = Q$~~

~~$R_C = Q$~~ If we Section BC:

$$\boxed{M_1 = Qx + \frac{wx^2}{2}}$$

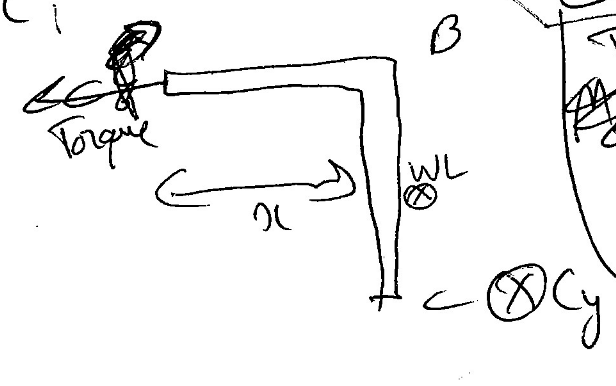
$$\boxed{\frac{\partial M_1}{\partial Q} = x}$$

~~Section CB:~~



~~$M_2 = \frac{wx^2}{2} + Qx$~~

Section ABC:



$$\boxed{\text{Torque} = wLx + Qx + \frac{wx^2}{2}}$$

$$\frac{\partial T}{\partial Q} = x$$

$$\delta_C = \frac{1}{EI}\int_0^L \left[\cancel{Q}x + \frac{Wx^2}{2}\right]\left[x\right] dx + \frac{1}{GJ}\int_0^L \left[WLx + \cancel{Qx} + \frac{Wx^2}{2}\right]x$$

Set $Q = 0$ and integrate

3) (15 pts) A cylinder of inner radius a and outer radius na, where n is an integer, has been designed to resist a specific internal pressure, but reboring becomes necessary. Calculate the new radius r required so that the maximum tangential stress does not exceed the previous value by more than $\Delta\sigma_\theta$, while the internal pressure is the same as before. Give your answer in terms of σ_θ, $\Delta\sigma_\theta$, n and a.

$a = a \qquad \boxed{b = na}$

$$\Delta r = \frac{a^2 p_i r}{E(b^2 - a^2)}\left[(1-\nu) + (1+\nu)\frac{b^2}{r^2}\right]$$

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$$\sigma_{\theta_1 max} = \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) \qquad b = na$$

$$\sigma_{\theta_2 max} = \frac{r^2 p_i}{b^2 - r^2}\left(1 + \frac{b^2}{r^2}\right)$$ where r is the new radius

$$\sigma_{\theta_2 max} - \sigma_{\theta_1 max} = \Delta\sigma_\theta \quad ✓$$

$$\frac{r^2 p_i}{b^2 - r^2}\left(1 + \frac{b^2}{r^2}\right) - \frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \Delta\sigma_\theta$$

$$\sigma_{\theta_2 max} = \Delta\sigma_\theta + \sigma_\theta \quad ✓$$

but $\underbrace{u + a}_{r} = \frac{a^3 p_i}{E(b^2 - a^2)}\left[(1-\nu) + (1+\nu)\frac{b^2}{a^2}\right] + a \qquad \boxed{\begin{matrix} r = a \\ b = na \end{matrix}}$

but here r = new radius $= u$

$$\frac{(u - a)E(b^2 - a^2)}{a^3\left[(1-\nu) + (1+\nu)\frac{b^2}{a^2}\right]} = p_i$$

$$\sigma_{\theta_2 max} = \Delta\sigma_\theta + \sigma_\theta$$

$$\frac{a^2 p_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right) = \Delta\sigma_\theta + \sigma_\theta$$

replace $P_i = \dfrac{(u-a)E(b^2-a^2)}{a^3\left[(1-\nu)+(1+\nu)b^2\right]}$