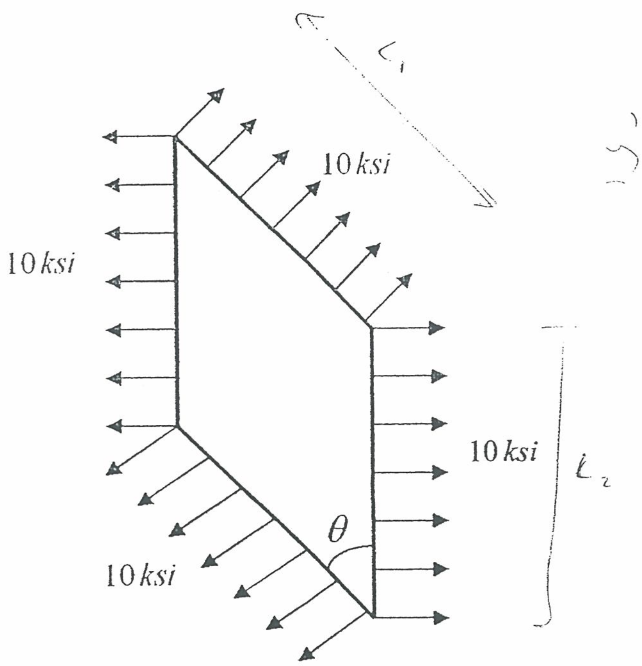
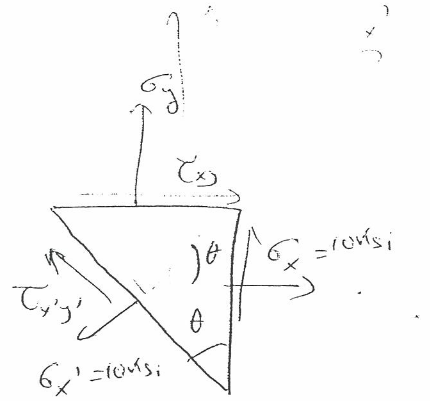
Name: 84

MEN 302
EXAM 1
SPRING 2007

1) (15 pts) A plate in the shape of a parallelogram has two opposite vertical edges, and a constant thickness throughout. It is loaded by normal tensile stresses of magnitude 10 *ksi* all around, as shown. Calculate the principal stresses.





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~~Resultant force~~

Normal stress on L_2 = 10 Ksi

Shear stress on L_2 = 0

$\hookrightarrow \tau_{xy} = 0$ ✓ (as stated by the prob)

$\tau_{x'y'} = 0$ (as stated)

$$\overset{0}{\cancel{\tau_{x'y'}}} = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \overset{0}{\cancel{\tau_{xy}}}\cos 2\theta$$ ✓

$\theta \neq 0 \Rightarrow \sin 2\theta \neq 0 \Rightarrow$

$\sigma_x - \sigma_y = 0 \Rightarrow \boxed{\sigma_x = \sigma_y = 10\ \text{Ksi}}$

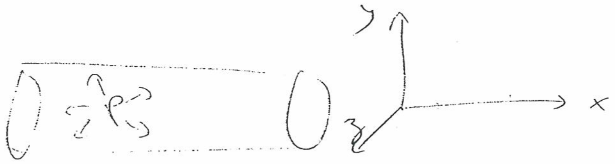
... principal

$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} \Rightarrow \sigma_{y'} = 10$ ksi

$\sigma_{x'}$ & $\sigma_{y'}$ can also be considered as the principal stresses.

THE DEBATE CLUB

2) (10 pts) A cylindrical thin-walled pressure vessel has an average radius of $59.5\,in$ and a thickness of $1\,in$. It contains an internal pressure of $120\,psi$. If $E = 29\times10^6\,psi$ and $\nu = 0.25$, calculate the elongation of the circumference.



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$$\sigma_x = \frac{Pr}{2t} = \frac{120 \cdot 59{,}5}{2 \cdot 1} = 3570\ psi$$

$$\sigma_y = \frac{Pr}{t} = \frac{120 \cdot 59{,}5}{1} = 7140\ psi = \sigma_z$$

X

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{29 \cdot 10^6}\left[7140 - 0{,}25\,(3570 + 7140)\right]$$

$$= \frac{4462{,}5}{29\,10^6} = 1{,}5388 \cdot 10^{-4}\ \frac{in}{in}$$

old circumf = $l = 2\pi r$

new circumference = new perimeter = $2\pi r' = l'$

$= 2\pi r(1 + \varepsilon_y)$

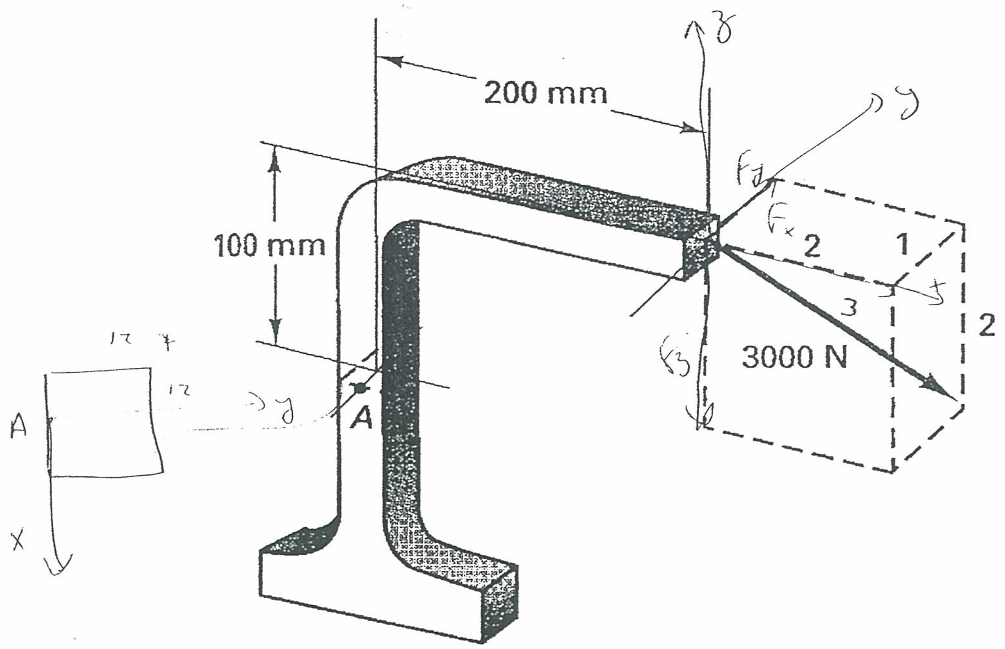
$$\Delta l_{circumference} = l' - l = 2\pi r(1 + \varepsilon_y) - 2\pi r$$

$$= 2\pi r\,\varepsilon_y$$

$$= 0{,}05753\ in.$$

X

(25 pts) A bent rectangular bar is subjected to an inclined force of $3000\,N$, as shown. The cross-section of the bar is $12 \times 12\,mm$. The tensile strength of the material is $S_y = 1800\,MPa$. Using the Tresca criterion at point A (located at the mid-side of the cross-section), calculate the factor of safety against yielding at point A. Show the state of stress on a stress element at point A. Note that the torsional shear stress for a square cross-section is given by $\tau_{Torsion} = \dfrac{T}{0.208b^3}$, where T is the torque and b is the side length of the cross-section.



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The 3000 N force can be written as a set of 3 forces!

$F_x = \frac{2}{3}\, 3000 = 2000\ N.$ ✓

$F_y = \frac{1}{3} \cdot 3000 = 1000\ N$ ✓

$F_z = -\frac{2}{3}\, 3000 = -2000\ N$ OK

No bending stress due to F_x since A is on the N.A. of the cross-section
Same for F_z ✓

$\sigma_z = \dfrac{F_z}{A} = \dfrac{-2000}{12 \cdot 12 \cdot 10^{-6}} = -13{,}\bar{8}\ MPa$ compressive

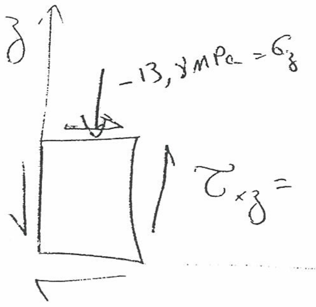
Since A is on the N.A, then it is under the shear force due to F_x. $\tau_{xz} = \dfrac{VQ}{It} = \dfrac{2000 \cdot 12 \cdot 10^{-3} \cdot 12 \cdot 10^{-3}}{\frac{12 \cdot 12^3 \cdot 10^{-12}}{12} \cdot 12 \cdot 10^{-3}} = \dfrac{2000}{144 \cdot 10^{-6}} = 13{,}\bar{8}\ MPa$ ✗

Due to F_y we have:

. shear stress (Torsion): $\tau_{xz} = \dfrac{F_y\, d}{0{,}208 \cdot b^3} = \dfrac{1000 \cdot 0{,}2}{0{,}208 \cdot (18 \cdot 10^{-3})^3} = 556{,}4\,\text{MPa}$ ✓

the shear stress is τ_{xz} since the y face is stress-free.

Since the y-~~flance~~ is a stress free ~~plane~~, our problem reduces to a 2-d stress problem



$\tau_{xz} = 556{,}4 + 13{,}8 = 570{,}2\,\text{MPa}$

$$\sigma_{1/2} = \frac{\sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$= 6{,}9 \pm \sqrt{47{,}61 + 325128}$

$= 6{,}9 \pm 570{,}24$

$= +577{,}24 = \sigma_1$ X

$-563{,}24 = \sigma_2$ X

. According to tresca criterium:

$|\sigma_1 - \sigma_2| = \sigma_{yp}$ OK

$\sigma_{yp} = 1140{,}48\,\text{MPa}$

$F.S. = \dfrac{\sigma_u}{\sigma_{yp}} = \cancel{\dfrac{1140{,}48}{1800}} = \dfrac{1800}{1140{,}48} = 1{,}58$ X