MEChanical of meterieals (2)

17/11/2005
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Name:

MEN 302
EXAM 1

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1) (30 pts) An engineer wishes to determine the shearing strength of a certain epoxy cement. The problem is to devise a test specimen such that the joint is subjected to pure shear. The joint shown in the figure, in which two bars are offset at an angle θ so as to keep the loading force F centroidal with the straight shanks, seems to accomplish this purpose. Using the contact area A and designating S_{su} as the ultimate shearing strength, the engineer obtains

$$S_{su} = \frac{F}{A}\cos\theta$$

The engineer's supervisor, in reviewing the test results, says the expression should be

$$S_{su} = \frac{F}{A}\left(1+\frac{1}{4}\tan^2\theta\right)^{1/2}\cos\theta$$

What is your position? Justify your answer.

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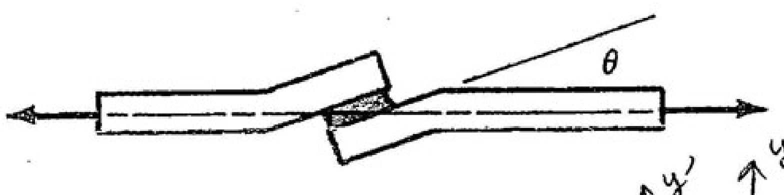
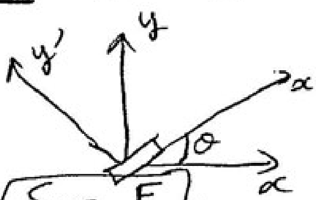




The joint is subjected to pure shear ⇒
No normal stress; $\sigma_{x'} = \sigma_{y'} = 0$. X

$\boxed{\sigma_x = \frac{F}{A}}$

$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$\boxed{\sigma_{x'} = 0 = \frac{F}{A}\cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta}$

$\boxed{\sigma_{y'} = 0 = \frac{F/A + \sigma_y}{2} - \frac{F/A - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta;}$ X

and we Know that $\sigma_x + \sigma_y = \cancel{\sigma_{x'}}^{0} + \cancel{\sigma_{y'}}^{0} \Rightarrow \boxed{\sigma_x = -\sigma_y}$

$\sigma_{x'} = 0 = \frac{F}{A}\cos^2\theta - \frac{F}{A}\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$\frac{F}{A}(\cos^2\theta - \sin^2\theta) = 2\tau_{xy}\sin\theta\cos\theta;$

$\tau_{xy} = \frac{-F}{2A}\frac{(-\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta} \Rightarrow \tau_{xy} = \frac{-F}{2A}\frac{(\cos 2\theta)}{2A\sin\theta\cos\theta} = \frac{-F}{2A}\frac{\cos 2\ldots}{\sin\ldots}$

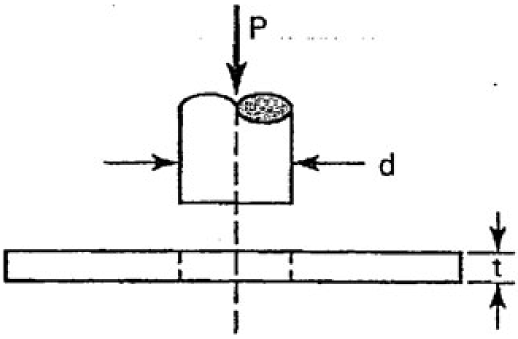
$$\delta_{1,2} = \cancel{\frac{F/A - F/A}{2}} \pm \sqrt{\left(\frac{\frac{F}{A} + \frac{F}{A}}{2}\right)^2 + \left(\frac{F^2 \cos^2 2\theta}{4A^2 \sin^2 2\theta}\right)};$$

$$\delta_{1,2} = \sqrt{\frac{F^2}{A^2} + \frac{F^2}{4A^2}\frac{\cos^2 2\theta}{\sin^2 2\theta}};$$

$$\delta_{1,2} = \frac{F}{A}\sqrt{1 + \frac{1}{4}\frac{\cos^2 2\theta}{\sin^2 2\theta}}$$

$$\boxed{\delta_{1,2} = \frac{F}{A}\sqrt{1 + \frac{1}{4}\tan^2 \theta}\,\cos\theta;}$$

2) (20 pts) A plate, t meters thick, is fabricated of a material having ultimate strengths in tension and compression of σ_u and σ'_u Pa, respectively. Calculate the force P required to punch a hole of d meters in diameter through the plate using the Mohr-Coulomb failure criterion. Assume that the shear force is uniformly distributed through the thickness of the plate.



σ_u = tension ; σ'_u = Compression;

Area of the hole = $\pi \frac{d^2}{4}$;

$$\boxed{\frac{\sigma_1}{\sigma_u} - \frac{\sigma_2}{\sigma'_u} = 1}$$

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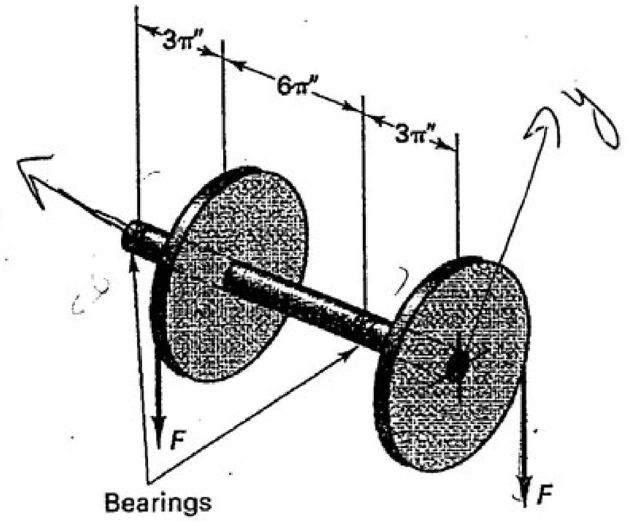
3) (30 pts) Two pulleys of 4π - *in* radius are attached to a 2 - *in* diameter solid shaft, which is supported by bearings, as shown. If the yield stress in shear is $6000\ psi$, determine the largest magnitude of the forces F that can be applied using the Tresca criterion.



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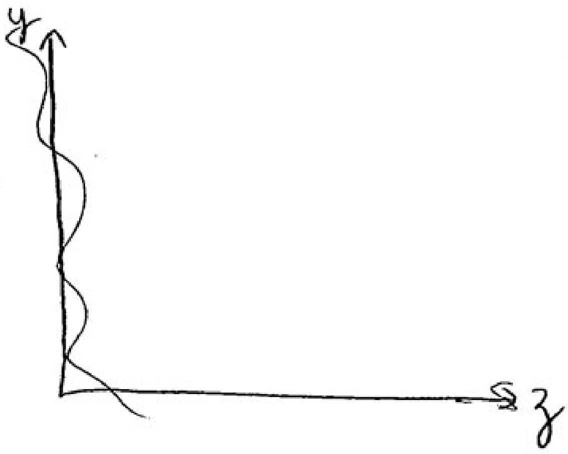
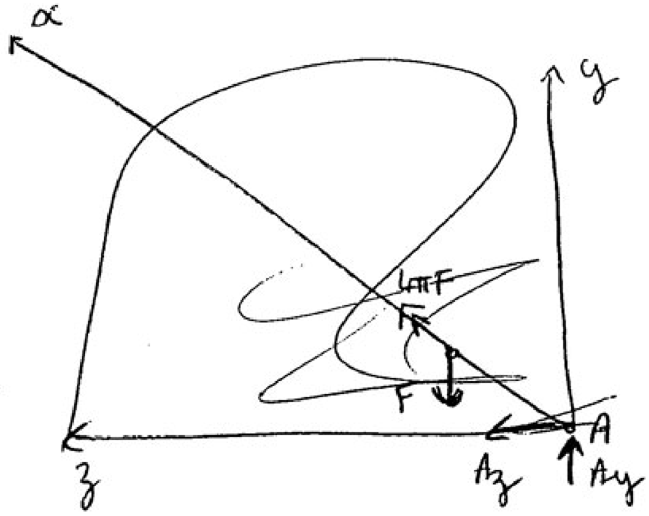
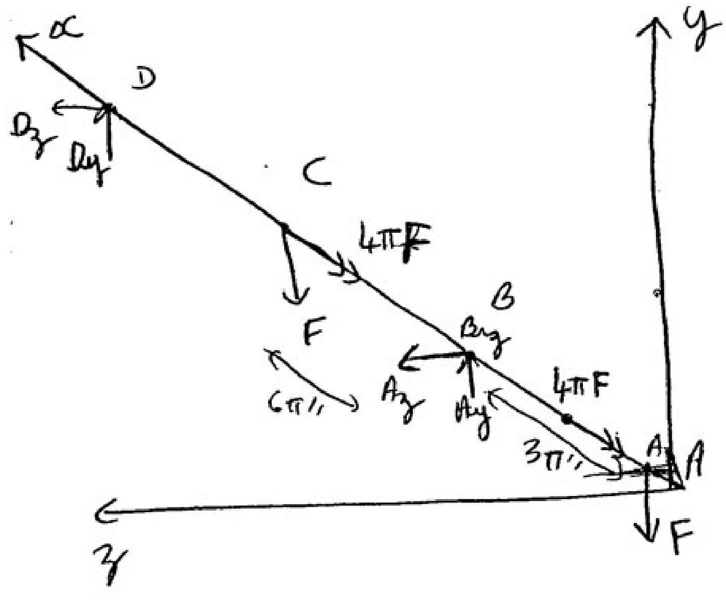






Assuming proper alignement of bearings;

$$\sum F_y = 0 \Rightarrow \boxed{A_y + D_y = 2F} \quad ①;$$

$$\sum F_z = 0 \Rightarrow \boxed{A_z + D_z = 0} \quad ②$$

$$\sum M_y = 0 \Rightarrow -D_z(12\pi) - A_z(3\pi) = 0;$$

$$\boxed{D_z(12\pi) = A_z(3\pi)} \quad ③.$$

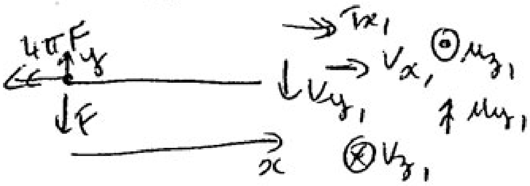
$$\sum M_z = 0 \Rightarrow -F(9\pi) + A_y(3\pi) + D_y(12\pi) = 0;$$

$$\boxed{12\pi(D_y) + 3\pi A_y = 9\pi F.} \quad ④$$

Substituting: $\cancel{\frac{12\pi D_z}{3\pi} + D_z = 0}$;

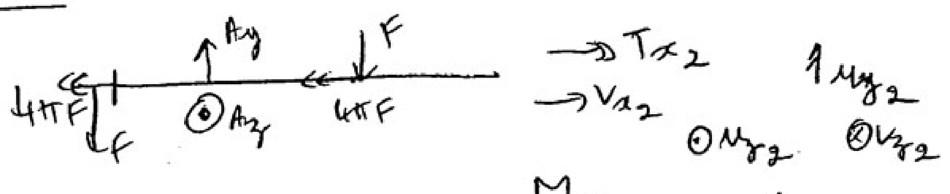
Determining V_{max}, T_{max} and M_{max} in the shaft:

1) $0 \leq x \leq 3\pi''$:



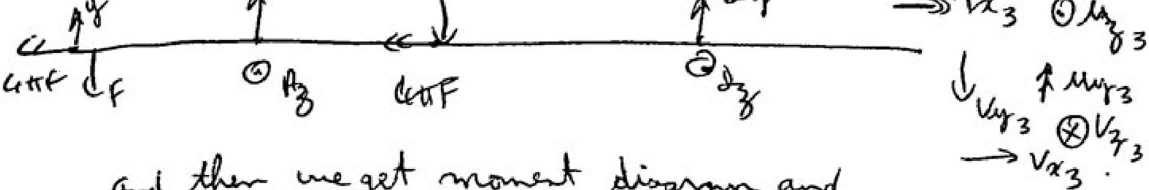
$M_{z_1} = ?$ X

2) $3\pi'' \leq x \leq 9\pi''$



M_{z_2} X

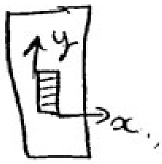
3) $9\pi'' \leq x \leq 12\pi''$

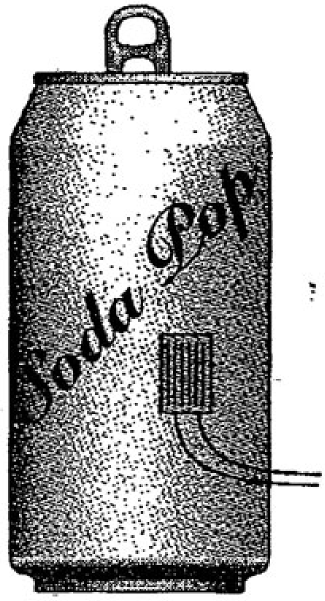


and then we get moment diagram and shear force diagram.

M_{z_3} X

4) (20 pts) A stain gage is installed in the longitudinal direction on the surface of an aluminum beverage can, as shown. The radius-to-thickness ratio of the can is 200. When the lid of the can is popped open, the strain changes by $\varepsilon_o = 170 \times 10^{-6}$. What was the pressure p in the can assuming that $E = 10^7\ psi$ and $\nu = 0.33$.





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$\frac{r}{t} = 200;\quad \epsilon_o = 170\times10^{-6};\quad E = 10^7 psi;\quad \nu = 0.33.$

$\sigma_x = \frac{p}{2}\frac{r}{t} = \frac{200p}{2} = 100p;$

$\sigma_y = p\frac{r}{t} = 200p;$

$\epsilon_o = 170\times10^{-6} = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta;$

$170\times10^{-6} = \frac{\cancel{\epsilon_x}+\epsilon_y}{2} + \frac{-\cancel{\epsilon_x}+\epsilon_y}{2} + 0$

$170\times10^{-6} = \frac{\epsilon_y}{2} + \frac{\epsilon_y}{2} = \epsilon_y \Rightarrow \boxed{\epsilon_y = 170\times10^{-6}}$ ✓

* $\cancel{\epsilon_y = -\nu\frac{\sigma_x}{E} \Rightarrow E\times\epsilon_y = -\nu\,\sigma_x}$

$\cancel{10^7\times170\times10^{-6} = -0.33\times\frac{p}{2}\times\frac{r}{t}}$

$\cancel{1700 = 100p} \Rightarrow p = 17$

$$\epsilon_y = \frac{\delta_y}{E} - \nu \frac{\delta_x}{E}$$

$$\epsilon_y \times E = 200P - 0.33(100P)$$

$$1700 = 170P \Rightarrow \boxed{P = 10 \text{ psi.}}$$