mechanics of merenial (2)

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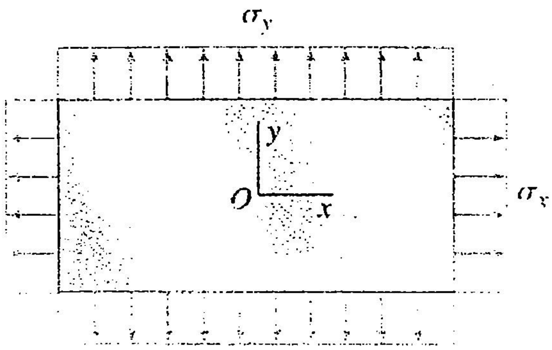
MEN 302
EXAM 1
SPRING 2004

96 good

1) (10 pts) A magnesium plate in biaxial stress is subjected to tensile stresses $\sigma_x = 30\,MPa$ and $\sigma_y = 15\,MPa$. The corresponding strains in the plate are $\varepsilon_x = 550 \times 10^{-6}$ and $\varepsilon_y = 100 \times 10^{-6}$. Determine Poisson's ratio ν and the modulus of elasticity of the material E.



$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x}{E} - \nu \dfrac{\sigma_y}{E} \\ \varepsilon_y = \dfrac{\sigma_y}{E} - \nu \dfrac{\sigma_x}{E} \end{cases} \qquad \begin{cases} E = \dfrac{\sigma_x}{\varepsilon_x} - \nu \dfrac{\sigma_y}{\varepsilon_x} \\ E = \dfrac{\sigma_y}{\varepsilon_y} - \nu \dfrac{\sigma_x}{\varepsilon_y} \end{cases}$$

$$\begin{cases} E = 5.45 \times 10^{10} - \nu (2.7 \times 10^{10}) \\ E = 1.5 \times 10^{11} - \nu (3 \times 10^{11}) \end{cases}$$

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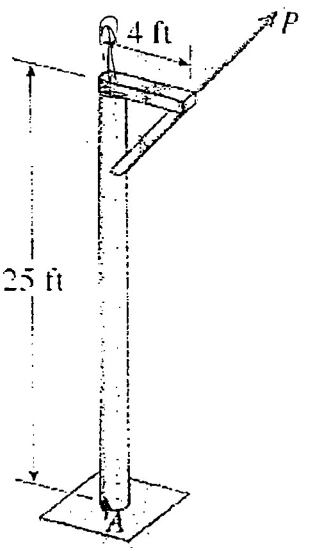
$$\begin{cases} E + \nu(2.7 \times 10^{10}) = 5.45 \times 10^{10} \\ E + \nu(3 \times 10^{11}) = 1.5 \times 10^{11} \end{cases}$$

$$\begin{cases} \boxed{E = 4.5 \times 10^{10}\ MPa} \ ✓ \\ \boxed{\nu = 0.348} \ ✓ \end{cases}$$

2) (20 pts) A post having a hollow circular cross section supports a horizontal load P [redacted] acting at the end of an arm that is $4\ ft$ long. The height of the post is $25\ ft$, and its section modulus (ratio of moment of inertia to distance from neutral axis to extreme fiber) is $S = \frac{I}{c} = 10\ in^3$. If the yield stress in tension and the yield stress in shear are, respectively, $S_y = 16000\ psi$ and $S_{sy} = 6000\ psi$, calculate, using both the Tresca and the Von Mises criteria, the largest permissible value of the load P by investigating the state of stress at point A, which is located at one of the poles of the cross section, as shown.



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At point A we have;

σ_x due to bending;

$$\sigma_x = \frac{Mc}{I} = \frac{M}{S} = \frac{P \times 25\,ft}{10\ in^3} = \boxed{2.5\,P \text{ [redacted] } Psi = \sigma_x}$$

ft/in³

τ_{xy} due to Torsion

$$\tau_{xy} = \frac{Tr}{J} = \frac{4P \times c}{2I} = \frac{2P}{S} = \boxed{0.2P\ \ Psi = \tau_{xy}}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.5P - 0}{2} \pm \sqrt{\left(\frac{2.5P - 0}{2}\right)^2 + (0.2P)^2}$$

$$= 1.25P \pm \sqrt{1.5625P^2 + 0.04P^2}$$

$$= 1.25P \pm 1.266P$$

X

$$\Longrightarrow \begin{cases} \sigma_1 = 2{,}516P \ (psi) \\ \sigma_2 = -0{,}016P \ (psi) \end{cases}$$

Using Tresca criteria:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{S_{cy}}{2n} \qquad (n = F.S. = \cancel{1})$$

$$\frac{2{,}516P + 0{,}016P}{2} = \frac{6000}{2}$$

$$2{,}532P = 6000$$

$$\boxed{P = 2368{,}66 \ lb}$$

Using Von Mises criteria:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{yT}^2$$

$$(2{,}532P)^2 + (-0{,}016P)^2 + (-2{,}516P)^2 = 2(16000)^2$$

$$12{,}74P^2 = 2(16000)^2$$

$$\boxed{P = 6338 \ lb}$$

So the largest value for P is: $\boxed{P = 2368{,}66 \ lb}$

3) (20 pts) A thin-walled cylindrical tank has a mean radius of $450\,mm$ and a wall thickness of $8.0\,mm$. The material has an ultimate strength of $680\,MPa$ in tension and an endurance limit of $240\,MPa$. The internal pressure cycles between $1.7\,MPa$ and $3.4\,MPa$. Determine the factor of safety against fatigue failure using the Von Mises criterion together with the Soderberg equation. $\rightarrow u_{ys}$

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$$\begin{cases} \sigma_{x_{min}} = \frac{Pr}{2t} = \frac{1{,}7\times10^6 \times 0{,}45}{2\times(0{,}008)} = 47{,}8\ MPa \\ \sigma_{x_{max}} = 2\times 47{,}8 = 85{,}6\ MPa \end{cases}$$

$$\begin{cases} \sigma_{y_{min}} = \frac{Pr}{t} = \frac{1{,}7\times10^6\times 0{,}45}{(0{,}008)} = 85{,}6\ MPa \\ \sigma_{y_{max}} = 2\times 85{,}6 = 181{,}25\ MPa \end{cases}$$

using sodeberg criterion;

$$\begin{cases} \sigma_{xa} = \frac{1}{2}(\sigma_{x_{max}} - \sigma_{x_{min}}) = 23{,}8\ MPa \\ \sigma_{x_m} = \frac{1}{2}(\sigma_{x_{max}} + \sigma_{x_{min}}) = 71{,}7\ MPa \end{cases}$$

$$\begin{cases} \sigma_{y_a} = \frac{1}{2}(\sigma_{y_{max}} - \sigma_{y_{min}}) = 47{,}8\ MPa \\ \sigma_{y_m} = \frac{1}{2}(\sigma_{y_{max}} + \sigma_{y_{min}}) = 143{,}4\ MPa \end{cases}$$

using Von Mises criterion;

$$(\sigma_{x_m} - \sigma_{y_m})^2 + (\sigma_{y_m})^2 + (\sigma_{x_m})^2 + 0 = 2\,\sigma_{em}^2$$

$$5140{,}88 + 20565{,}56 + 5140{,}88 = 2\,\sigma_{em}^2$$

$$\boxed{\sigma_{em} = 124{,}18\ MPa}$$

$$(\sigma_{xa} - \sigma_{ya})^2 + (\sigma_{ya})^2 + (\sigma_{xa})^2 = 2\,\sigma_{ea}^2$$

$$\boxed{\sigma_{ea} = 41{,}4\ MPa}$$

using soderberg again

$$\frac{\sigma_{ea}}{\sigma_a} + \frac{\sigma_{em}}{\sigma_u} = \frac{1}{F.S.}$$

$$\frac{41{,}4}{240} + \frac{124{,}18}{680} = \frac{1}{F.S.}$$

$$0{,}1725 + 0{,}1826 = \frac{1}{F.S.}$$

$$\Longrightarrow \boxed{F.S. = 2{,}815}$$