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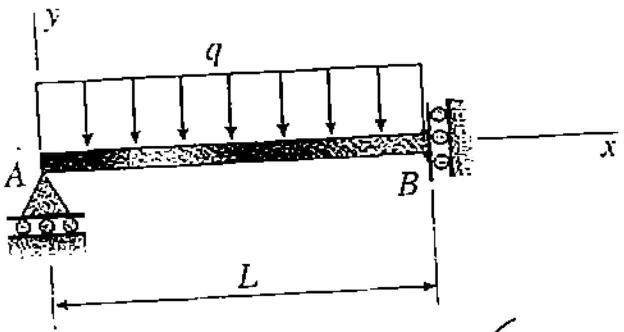
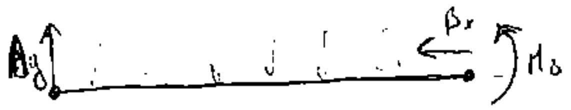
98 good

MEN 302
EXAM 1
SPRING 2006

1) (20 pts) The beam shown in the figure has a roller support at A and a guided support at B. The guided support permits vertical movement only. The beam is under a uniform loading q. Use the second order differential equation to derive the equation of the deflection curve and determine the deflection at end B.





20

$\Sigma F_x = 0$ $\boxed{B_x = 0}$

$\Sigma F_y: 0$ $-qL + A_y = 0$ $\boxed{A_y = qL}$

$\Sigma M_B = 0$ $M_B + qL\left(\frac{L}{2}\right) - qL(L) = 0$ $\boxed{M_B = qL^2 - \frac{qL^2}{2} = \frac{qL^2}{2}}$

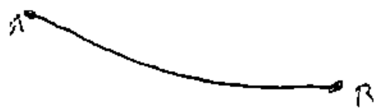
$0 < x \leq L$

$$M + qx\left(\frac{x}{2}\right) - A_y x = 0 \qquad M = qLx - \frac{q}{2}x^2$$

$$EI\, y'' = M = qLx - \frac{q}{2}x^2$$

$$EI\, y' = \frac{q}{2}Lx^2 - \frac{q}{6}x^3 + C_1 \quad ①$$

$$EI\, y = \frac{q}{6}Lx^3 - \frac{q}{24}x^4 + C_1 x + C_2 \quad ②$$

The deflection has this form 

$\Rightarrow \quad y(0) = 0 \quad (3)$

$\qquad y'(L) = 0 \quad (4)$

Replace (4) in (1)

$$\frac{q}{2}L^3 - \frac{q}{6}L^3 + C_1 = 0 \qquad C_1 = -\frac{q}{3}L^3$$

Replace (3) in (2)

$$C_2 = 0$$

$$\boxed{y = \frac{1}{EI}\left(\frac{q}{6}Lx^3 - \frac{q}{24}x^4 - \frac{q}{3}L^3 x\right)}$$ eq. of the deflection

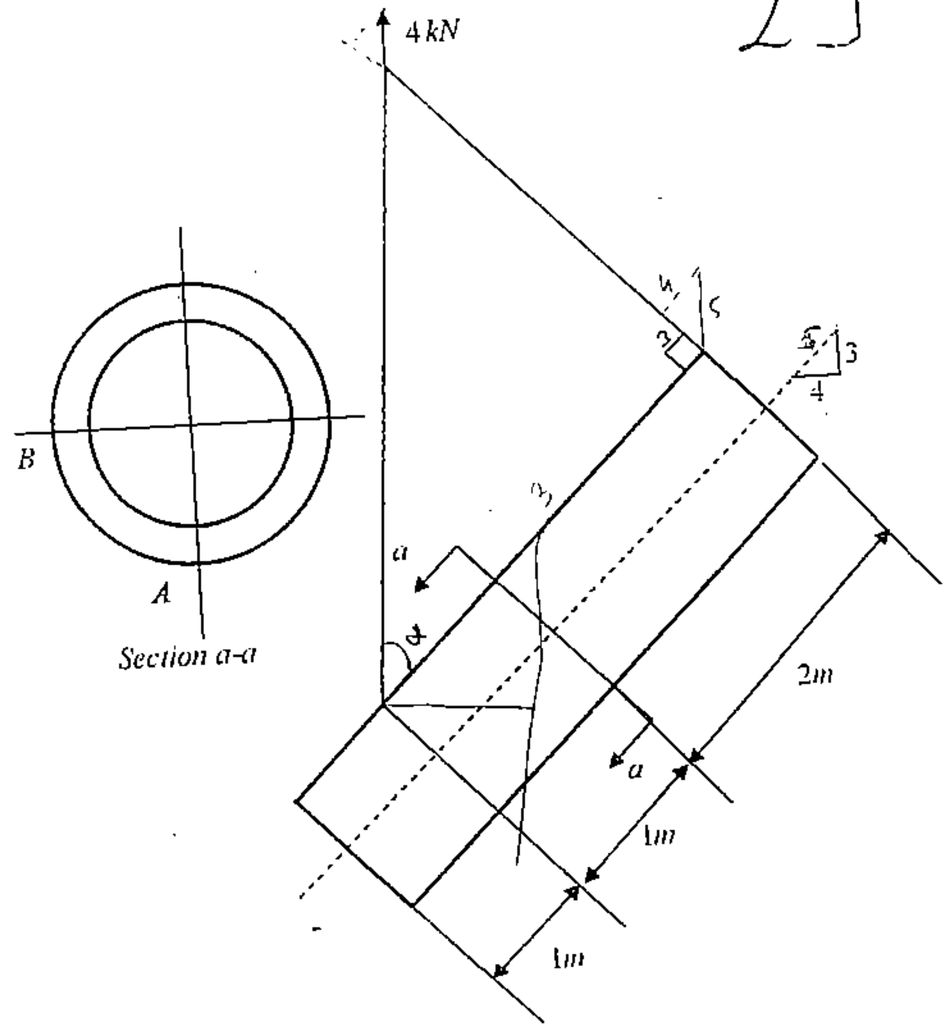
at B : $x = L$

$$y = \frac{1}{EI}\left(\frac{q}{6}L^4 - \frac{q}{24}L^4 - \frac{q}{3}L^4\right)$$

$$y = -\frac{5}{24\,EI}\,qL^4$$

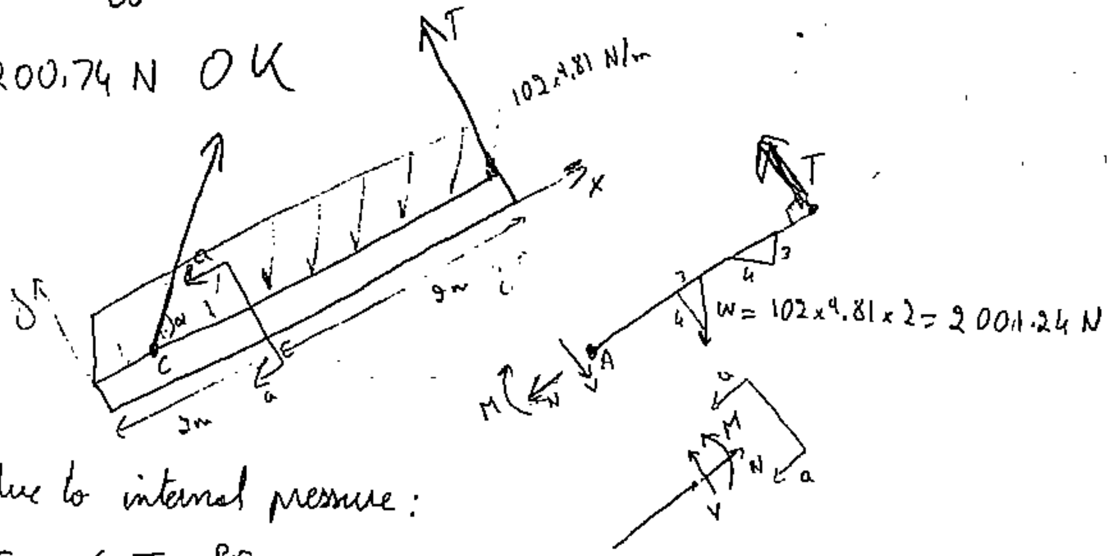
2) (30 pts) An aircraft fuel tank, which may be considered to be a thin-walled cylindrical pressure vessel, has a mean radius $r = 300mm$ and a thickness $t = 6mm$. It is being hoisted by two cables into the position shown. Assuming that the vessel weighs $102 kg/m$ and contains fuel which exerts an internal pressure of $0.5MPa$, perform a complete stress analysis using the Von Mises criterion at points A and B on the cross-section, to calculate the factor of safety against yielding at section a-a located at the middle of the tank. For each point, show the stresses on a properly oriented element. Assume that the yield strength of the material is $360MPa$. Note that for a hollow circular cross-section, the maximum bending shear stress is $\frac{2V}{A}$ where V is the shear force and A is the area of the cross-section.

29



$\Sigma M_C = 0$ $T \times 3 = \underbrace{(102 \times 9.81 \times 3)}_{W} \times \frac{4}{5} \times 1.5$

$T = 1200.74\ N$ OK



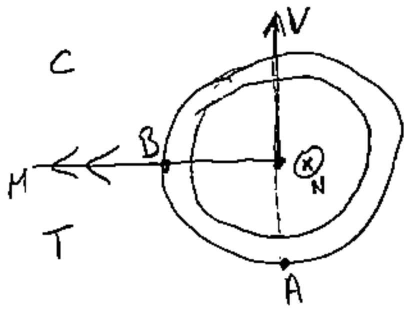
σ's due to internal pressure:

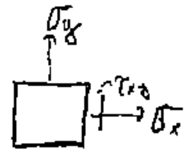
$\sigma_x = \frac{Pr}{2t}$ ✓ $\sigma_y = \frac{Pr}{t}$

At A: $\Sigma F_x = 0$ $\quad -N - W \times \frac{3}{5} = 0$ $\quad N = -1200\ N$ (Compression)

ΣF_y $\quad -V - W \times \frac{4}{5} + T = 0$ $\quad V = -2001.24 \times \frac{4}{5} + 1200.74$
$= 4002\ N$

ΣM_A $\quad -M - W \times \frac{4}{5} \times 1 + T \times 2 = 0$ $\quad M = -2001.24 \times \frac{4}{5} + 1200.74 \times 2$
$= 800\ N{\cdot}m$





A $\quad \tau_{xy} = 0$

At pt. A: No shear stress

Stress due to M: $\sigma_{xM} = \frac{MC}{I}$ ✓
Tension

$C = (300 + 3) \times 10^{-3} = 303 \times 10^{-3}$

$I = \frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{2}\left[(303 \times 10^{-3})^4 - (297 \times 10^{-3})^4\right]$

$= 1.02 \times 10^{-3}\ m^4$

$\sigma_{xM} = \frac{800 \times 303 \times 10^{-3}}{1.02 \times 10^{-3}} = 237.8\ kPa$ OK

$\sigma_{xN} = \frac{-1200}{A} = \frac{-1200}{2\pi r t} = 106.2k$

$\sigma_x = \sigma_{xp} + \sigma_{xM} + \sigma_{xN} = \frac{0.5 \times 10^3 \times 300 \times 10^{-3}}{2 \times 6 \times 10^{-3}} + 237.8 + 106.2 = 12\,631.6\ kPa$ OK

$\sigma_y = \frac{Pr}{t} = \frac{0.5\times10^3 \times 300\times10^{-3}}{6\times10^{-3}} = 25\,000$ KPa ✓

Since there is no shear σ_y & σ_x ✓ are the principle stresses

Using $(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2 = 2\,\sigma_{yp}^2$

$\sigma_p = 30.655$ MPa

Factor of Safety: $\frac{\sigma_p}{360} = .85$ X

At pt. B:

$\sigma_{xM} = 0$ ✓

$\sigma_x = -\sigma_{xN} + \sigma_{xP} = -106.2 + 12500 = 12393$ KPa ✓

$\sigma_y = 25\,000$ KPa ✓

OK

τ_{xy} is the max. in the cross section $= \frac{2V}{A} = \frac{2\times 400\times10^{-3}}{2\pi r t} = 70.77$ KPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{12393 + 25000}{2} \pm \sqrt{\left(\frac{12393+25000}{2}\right)^2 + 70.77^2}$$

$\sigma_1 = 25$ MPa $\qquad \sigma_2 = -12.39$ MPa

$$(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2 = 2\sigma_{yp}^2$$

$\sigma_{yp} = 21.65$ ✓

$F.S = \frac{21.65}{360}$ ✓