(2)

FINAL EXAMINATION

MATH 201

January 29, 2005; 3:00-5:00 P.M.

Name: Signature:

Student number:

Section number (Encircle): 17 18 19 20

Instructors (Encircle): Dr. H. Yamani Mrs. M. Jurdak Prof. A. Lyzzaik

Instructions:

- No calculators are allowed.
- There are two types of questions:

PART I consists of four work-out problems. Give a detailed solution for each of these problems.

PART II consists of twelve multiple-choice questions each with **exactly one correct answer**. Circle the appropriate answer for each of these problems.

Grading policy:

- 10 points for each problem of **PART I**.
- 5 points for each problem of **PART II**.
- 0 point for no, wrong, or more than one answer of **PART II**.

GRADE OF PART I/40:

GRADE OF PART II/60:

TOTAL GRADE/100:

Part I(1). Use the change of variables $u = x - y$ and $v = x + y$ to evaluate the integral

$$\iint_R (x-y)^2 \cos^2(x+y)\, dx\, dy$$

over the square region R bounded by the lines $x - y = 1$, $x - y = -1$, $x + y = 1$, $x + y = 3$.

Part I(2). Find the absolute maximum and minimum values of the function

$$f(x, y) = 2x^2 - xy + y^2 - 7x$$

on the square region $R = \{(x, y) : 0 \leq x, y \leq 3\}$.

Part I(3). Find the volume of the "ice cream cone" C bounded by the cone $\phi = \pi/6$ and the sphere $\rho = 2a\cos\phi$ of radius a and tangent to the $xy-$plane at the origin.

Part I(4). Use Lagrange multipliers to find the point on the plane $2x + 3y + 4z = 12$ at which the function $f(x, y, z) = 4x^2 + y^2 + 5z^2$ has its least value.

Part II

Part II(1). If $f(x,y) = \dfrac{x^4+y^2}{x^4+x^2y+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0)=0$, then

(a) f is continuous at (0,0).

(b) $\lim_{(x,y)\to(0,0)} f(x,y) = 1$.

(c) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

(d) $\lim_{(x,y)\to(0,0)} f(x,y) = 2/3$.

(e) None of the above.

Part II(2). The Maclaurin series of the integral

$$\int_0^x \sqrt{1+t^3}\, dt \quad \text{is}$$

(a) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)!\,n} x^{3n+1}$.

(b) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{n!\,(3n+1)} x^{3n+1}$.

(c) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)!} x^{3n+1}$.

(d) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)} x^{3n+1}$.

(e) None of the above.

Part II(3). The series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}\,(\ln n)^{10}}$$

(a) converges absolutely.

(b) converges conditionally.

(c) diverges.

(d) converges conditionally and absolutely.

(e) None of the above.

Part II(4). The value of the integral

$$\int_0^1 \int_y^{\sqrt{y}} e^{y/x}\, dx\, dy \quad \text{is}$$

(a) $-1 + e/2$.

(b) $-1 + e$.

(c) $-1 + e/4$.

(d) $-1 + e/3$.

(e) None of the above.

Part II(5). The interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n4^n} \quad \text{is}$$

(a) $[-2, 6[$.

(b) $]-2, 6[$.

(c) $[-2, 6]$.

(d) $]-2, 6]$.

(e) None of the above.

Part II(6). The value of the double integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy \quad \text{is}$$

(a) $(\pi \ln 2)/2$.

(b) $\pi \ln 2$.

(c) $(\pi \ln 2)/4$.

(d) $(\pi \ln 2)/3$.

(e) None of the above.

Part II(7). The polynomial that approximates the function

$$F(x) = \int_0^x \frac{\sin t}{t}\, dt$$

throughout the interval $[0, 1/2]$ with an error of magnitude less than 10^{-3} is

(a) $x - x^3/12$.

(b) $x - x^3/6$.

(c) $x - x^3/18$.

(d) $x - x^3/24$.

(e) None of the above.

Part II(8). The series $\sum_{n=1}^{\infty} a_n$ converges if

(a) $a_n = 1/n^{\ln 2}$.

(b) $a_n = (1/n)\ln(1 + 1/n)$.

(c) $a_n < b_n$ and the series $\sum_{n=1}^{\infty} b_n$ converges.

(d) $a_n = n \sin(1/n)$.

(e) $\lim_{n\to\infty} \sqrt[n]{|a_n|} \geq 1$.

Part II(9). Parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $P(1, 1, 1)$ are

(a) $x = 1 + t,\ y = 1 + 2t,\ z = 1 + t,\ -\infty < t < \infty$.

(b) $x = 1 - t,\ y = 1 - 2t,\ z = 1 + t,\ -\infty < t < \infty$.

(c) $x = 1 + t,\ y = 1 - 2t,\ z = 1 - t,\ -\infty < t < \infty$.

(d) $x = 1 + t,\ y = 1 - 2t,\ z = 1 + t,\ -\infty < t < \infty$.

(e) None of the above.

Part II(10). A triple integral in cylindrical coordinates for the volume of the solid cut out from the sphere $x^2+y^2+z^2=4$ by the cylinder $x^2+y^2=2y$ is

(a) $\int_0^{\pi}\int_0^{2\cos\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta.$

(b) $\int_0^{\pi}\int_0^{2\sin\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta.$

(c) $\int_0^{\pi/2}\int_0^{2\sin\theta}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta.$

(d) $\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta.$

(e) None of the above.

Part II(11). By using Green's theorem, the value of the line integral

$$\oint_C (x+y)\,dx + (y+x^2)\,dy,$$

where C is the positively-directed boundary of the region bounded by the circles $x^2+y^2=1$ and $x^2+y^2=4$, is

(a) $-\pi$.

(b) 0.

(c) -3π.

(d) -2π.

(e) None of the above.

Part II(12). The only one **TRUE** statement of the following is

(a) The field $(y\sin z)\mathbf{i}+(x\sin z)\mathbf{j}+(xy\cos z)\mathbf{k}$ has potential function $xy\sin z\cos z$.

(b) The field $(e^x\cos y)\,\mathbf{i}+(e^x\sin y)\,\mathbf{j}$ is conservative.

(c) The differential $3x^2\,dx+2xy^2\,dy$ is exact.

(d) The value of the line integral $\int_{(2,0,1/2)}^{(0,2,1)} y\,dx+x\,dy+4\,dz$ is 2.

(e) If C is a simple closed curve, then $\oint_C y\,dx+x\,dy\neq 0$.