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FINAL EXAMINATION

MATH 201

January 29, 2005; 3:00-5:00 P.M.

Name:

Signature:

Student number:

Section number (Encircle): 17 18 19 20

Instructors (Encircle): Dr. H. Yamani Mrs. M. Jurdak Prof. A. Lyzzaik

Instructions:

- No calculators are allowed.
- There are two types of questions:

PART I consists of four work-out problems. Give a detailed solution for each of these problems.

PART II consists of twelve multiple-choice questions each with **exactly one correct answer**. Circle the appropriate answer for each of these problems.

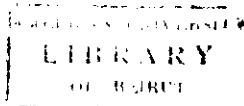
Grading policy:

- 10 points for each problem of **PART I**.
- 5 points for each problem of **PART II**.
- 0 point for no, wrong, or more than one answer of **PART II**.

GRADE OF PART I/40:

GRADE OF PART II/60:

TOTAL GRADE/100:



Part I(1). Use the change of variables $u = x - y$ and $v = x + y$ to evaluate the integral

$$\iint_R (x - y)^2 \cos^2(x + y) \, dx \, dy$$

over the square region R bounded by the lines $x - y = 1$, $x - y = -1$, $x + y = 1$, $x + y = 3$.

Part I(2). Find the absolute maximum and minimum values of the function

$$f(x, y) = 2x^2 - xy + y^2 - 7x$$

on the square region $R = \{(x, y) : 0 \leq x, y \leq 3\}$.



Part I(3). Find the volume of the "ice cream cone" C bounded by the cone $\phi = \pi/6$ and the sphere $\rho = 2a \cos \phi$ of radius a and tangent to the xy -plane at the origin.



Part I(4). Use Lagrange multipliers to find the point on the plane $2x + 3y + 4z = 12$ at which the function $f(x, y, z) = 4x^2 + y^2 + 5z^2$ has its least value.





Part II

Part II(1). If $f(x, y) = \frac{x^4 + y^2}{x^4 + x^2y + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$,

then

- (a) f is continuous at $(0, 0)$.
- (b) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$.
- (c) $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist.
- (d) $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 2/3$.
- (e) None of the above.

Part II(2). The Maclaurin series of the integral

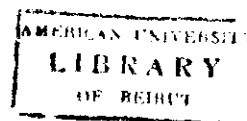
$$\int_0^x \sqrt{1+t^3} dt \text{ is}$$

- (a) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)! n} x^{3n+1}$.
- (b) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{n! (3n+1)} x^{3n+1}$.
- (c) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)!} x^{3n+1}$.
- (d) $\sum_{n=0}^{\infty} \frac{(1/2)(1/2-1)(1/2-2)\cdots(1/2-n+1)}{(3n+1)} x^{3n+1}$.
- (e) None of the above.

Part II(3). The series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n} (\ln n)^{10}}$$

- (a) converges absolutely.
- (b) converges conditionally.
- (c) diverges.
- (d) converges conditionally and absolutely.
- (e) None of the above.



Part II(4). The value of the integral

$$\int_0^1 \int_y^{\sqrt{y}} e^{y/x} dx dy \text{ is}$$

- (a) $-1 + e/2$.
- (b) $-1 + e$.
- (c) $-1 + e/4$.
- (d) $-1 + e/3$.
- (e) None of the above.

Part II(5). The interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n4^n} \text{ is}$$

- (a) $[-2, 6[$.
- (b) $] - 2, 6[$.
- (c) $[-2, 6]$.
- (d) $] - 2, 6]$.
- (e) None of the above.

Part II(6). The value of the double integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2} dx dy \text{ is}$$

- (a) $(\pi \ln 2)/2$.
- (b) $\pi \ln 2$.
- (c) $(\pi \ln 2)/4$.
- (d) $(\pi \ln 2)/3$.
- (e) None of the above.

Part II(7). The polynomial that approximates the function



$$F(x) = \int_0^x \frac{\sin t}{t} dt$$

throughout the interval $[0, 1/2]$ with an error of magnitude less than 10^{-3} is

- (a) $x - x^3/12$.
- (b) $x - x^3/6$.
- (c) $x - x^3/18$.
- (d) $x - x^3/24$.
- (e) None of the above.

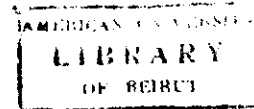
Part II(8). The series $\sum_{n=1}^{\infty} a_n$ converges if

- (a) $a_n = 1/n^{\ln 2}$.
- (b) $a_n = (1/n) \ln(1 + 1/n)$.
- (c) $a_n < b_n$ and the series $\sum_{n=1}^{\infty} b_n$ converges.
- (d) $a_n = n \sin(1/n)$.
- (e) $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \geq 1$.

Part II(9). Parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $P(1, 1, 1)$

are

- (a) $x = 1 + t, y = 1 + 2t, z = 1 + t, -\infty < t < \infty$.
- (b) $x = 1 - t, y = 1 - 2t, z = 1 + t, -\infty < t < \infty$.
- (c) $x = 1 + t, y = 1 - 2t, z = 1 - t, -\infty < t < \infty$.
- (d) $x = 1 + t, y = 1 - 2t, z = 1 + t, -\infty < t < \infty$.
- (e) None of the above.



Part II(10). A triple integral in cylindrical coordinates for the volume of the solid cut out from the sphere $x^2 + y^2 + z^2 = 4$ by the cylinder $x^2 + y^2 = 2y$ is

(a) $\int_0^\pi \int_0^{2\cos\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta.$

(b) $\int_0^\pi \int_0^{2\sin\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta.$

(c) $\int_0^{\pi/2} \int_0^{2\sin\theta} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta.$

(d) $\int_0^\pi \int_0^{2\sin\theta} \int_0^{\sqrt{4-r^2}} r \, dz \, dr \, d\theta.$

(e) None of the above.

Part II(11). By using Green's theorem, the value of the line integral

$$\oint_C (x + y) \, dx + (y + x^2) \, dy,$$

where C is the positively-directed boundary of the region bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$, is

(a) $-\pi.$

(b) $0.$

(c) $-3\pi.$

(d) $-2\pi.$

(e) None of the above.

Part II(12). The only one **TRUE** statement of the following is

(a) The field $(y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$ has potential function $xy \sin z \cos z.$

(b) The field $(e^x \cos y)\mathbf{i} + (e^x \sin y)\mathbf{j}$ is conservative.

(c) The differential $3x^2 \, dx + 2xy^2 \, dy$ is exact.

(d) The value of the line integral $\int_{(2,0,1/2)}^{(0,2,1)} y \, dx + x \, dy + 4 \, dz$ is 2.

(e) If C is a simple closed curve, then $\oint_C y \, dx + x \, dy \neq 0.$