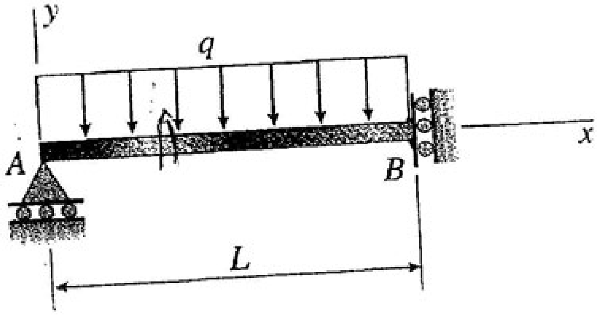
Check #1

Name:

MEN 302
EXAM 1

54 60

1) (20 pts) The beam shown in the figure has a roller support at A and a guided support at B. The guided support permits vertical movement only. The beam is under a uniform loading q. Use the second order differential equation to derive the equation of the deflection curve and determine the deflection at end B.



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- lets take a cut at $0 < x < L$ from A.

but ΣF_y of the Hole system at equilibrium is zero.

$\Sigma F_y = 0 \Rightarrow R_A - q \times L = 0.$

$\Rightarrow \underline{R_A = qL}$ ✓

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and $\Sigma M_A = 0 \Rightarrow M - R_A \times x + q x \times \frac{x}{2} = 0$

$\therefore M - qLx + q\frac{x^2}{2} = 0.$

$$\underline{EI\, y'' = M = qLx - \frac{qx^2}{2}}$$

$\therefore EI\, y' = \frac{qLx^2}{2} - \frac{q}{2}\frac{x^3}{3} = \frac{qLx^2}{2} - \frac{qx^3}{6} + C_1$

integrating $\underline{y'}$ to y

$$\underline{EI\, y = \frac{qLx^3}{2 \times 3} - \frac{qx^4}{6 \times 4} + C_1 x + C_2.}$$

for $x=0$ y (The deflection) $= 0$ ✓

$$0 = \frac{1}{EI}\left(C_2 \right) \Rightarrow \underline{C_2 = 0}$$

for $x=0$ $y' = 0$. X

$$0 = \frac{1}{EI}\left(C_1 \right) \Rightarrow \underline{C_1 = 0}$$

$$\therefore \boxed{y = \frac{1}{EI}\left(\frac{qLx^3}{6} - \frac{qx^4}{24} \right)}$$

$\therefore$ y_{atB} for $x = L$.

$$y_{atB} = \frac{1}{EI}\left(\frac{qL \times L^3}{6} - \frac{qL^4}{24} \right) = \frac{1}{EI}\left(\frac{qL^4}{6} - \frac{qL^4}{24} \right)$$

$$= \frac{1}{EI}\left(\frac{4qL^4 - qL^4}{24} \right) = \frac{1}{EI}\left(\frac{3qL^4}{24} \right) = \underline{\frac{1}{EI}\left(\frac{qL^4}{8} \right)}$$

X

2) (30 pts) An aircraft fuel tank, which may be considered to be a thin-walled cylindrical pressure vessel, has a mean radius $r = 300mm$ and a thickness $t = 6mm$. It is being hoisted by two cables into the position shown. Assuming that the vessel weighs $102 kg/m$ and contains fuel which exerts an internal pressure of $0.5MPa$, perform a complete stress analysis using the Von Mises criterion at points A and B on the cross-section, to calculate the factor of safety against yielding at section a-a located at the middle of the tank. For each point, show the stresses on a properly oriented element. Assume that the yield strength of the material is $360MPa$. Note that for a hollow circular cross-section, the maximum bending shear stress is $\frac{2V}{A}$ where V is the shear force and A is the area of the cross-section.

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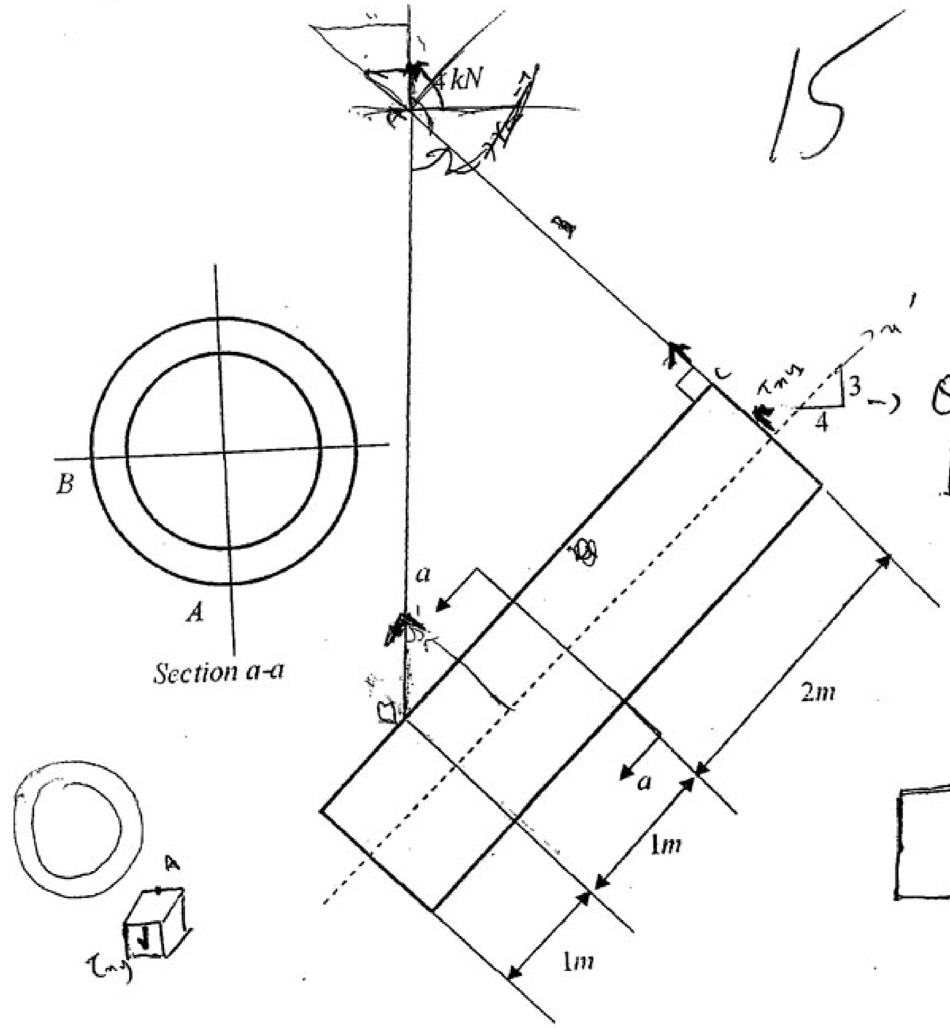

$\rightarrow \theta = \tan^{-1}\frac{3}{4}$

$\boxed{\theta = 36.869}$

4 KN along $x'y'$ = 4 KN $\sin(90-53.1) = 4 \times \sin(36.864)$

$= \boxed{2.4 \text{ KN} \doteq \tau_{x'y'}}$

No external force.

$P = 0.5 Mpa = 500000 Pa$.
$r = 0.3 m$.
$t = 0.006 m$.

$\delta_{y'_1} = \frac{Pr}{t} = \boxed{25 Mpa}$

$\delta_{x'_1} = \frac{Pr}{2t} = \frac{25}{2} = \boxed{12.5 Mpa}$

$A = \pi r^2$

$\tau_{x'y'} = \frac{2.4 KN}{A} = \frac{2.4}{282600} = 8.49.10^{-6}$.

Why using transformation equations for stress ?

$\delta_{x'}$
$12.5 = \frac{1}{2}(\delta_x + \delta_y) + \frac{1}{2}(\delta_x - \delta_y)\cos 2\theta + \tau_{xy} \sin 2\theta$

$12.5 = \frac{1}{2}(\delta_x + \delta_y) + \frac{1}{2}(\delta_x - \delta_y)\cos(73.74) + \tau_{xy} \sin(73.74)$

$12.5 = 0.5\delta_x + 0.5\delta_y + (0.5 \times 0.28)\delta_x - (0.5 \times 0.28)\delta_y + \tau_{xy} \times 0.96$

$12.5 = \delta_x(0.5 + 0.14) + \delta_y(0.5 - 0.14) + 0.96\tau_{xy}$.

$\boxed{12.5 = 0.64\delta_x + 0.36\delta_y + 0.96\tau_{xy}}$ ①. Why this?

$\delta_{y'} = \frac{1}{2}(\delta_x + \delta_y) - \frac{1}{2}(\delta_x - \delta_y)\cos 2\theta - \tau_{xy} \sin 2\theta$.

$25 = 0.5\delta_x + 0.5\delta_y - [0.5 \cdot 0.28\delta_x - 0.5 \times 0.28\delta_y] - 0.96\tau_{xy})$.

$\boxed{25 = \delta_x(\underbrace{0.5 - 0.14}_{0.36}) + \delta_y(\underbrace{0.5 + 0.14}_{0.64}) - 0.96\tau_{xy}}$ ②

$\tau_{x'y'} = -\frac{1}{2}(\delta_x - \delta_y)\sin 2\theta + \tau_{xy}\cos 2\theta$.

$2.4 = -0.5 \times 0.96\delta_x + 0.5 \times 0.96\delta_y + 0.28\tau_{xy}$.

$8.49.10^{-6} = \boxed{\frac{2.4}{A} = -0.48\delta_x + 0.48\delta_y + 0.28\tau_{xy}}$ ③.

3 equations and 3 unknowns. We calculate and found ?

$\delta_x = ~~+16.696~~ \ 17$

$\delta_y = +20.~~804~~$

$\tau_{xy} = -~~0.2~~6$

X

using the principal stresses equations

$\delta_{1,2} = \frac{\delta_x + \delta_y}{2} \pm \sqrt{\left(\frac{\delta_x - \delta_y}{2}\right)^2 + \tau_{xy}^2}$