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**Name:** **Signature:** **Student Number:**



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**Mathematics 201**
**Final Examination, January 29, 2005, 15:00 – 17:30**

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This exam consists of 16 multiple choice problems and 2 workout problems.
In each multiple choice problem you must write down your choice for the answer. You will get 5 pts. if your choice is correct and 0 pts. if your choice is wrong, or when you do not write anything, or when you write more than one answer.
In each workout problem you must supply a complete solution to the problem on the page containing the problem in this booklet.
Blue booklets will **not** be graded. They are for your rough work only.
No graphing or programmable calculators are allowed during the exam.
No questions are allowed during the exam.
Do not separate pages in your booklets.

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*Good Luck!*

**Part 1: 16 multiple choice problems. Each of them is worth 5 pts. There is no penalty for wrong answers.**

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Problem 1. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n \cdot n! \cdot n! \cdot (2x-3)^n}{(2n+1)!}$ .

(a) 1 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 2. Find the limit of $f(x,y) = \dfrac{e^{x+y} - e^y}{\sin(xy)}$ when $(x,y) \to (0,2)$ .

(a) $-\frac{1}{2}e^2$ (b) $e$ (c) $\frac{1}{2}e^2$ (d) the limit does not exist
(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 3. Decide which claim is true concerning the function $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$ and the point $P\,(-\frac{5}{3}, 0)$.

(a) $f$ has a local minimum at $P$ (b) $f$ has a local maximum at $P$

(c) $f$ has a saddle point at $P$ (d) $P$ is not a critical point of $f$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 4. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ for the transformation $x = u^2 + v^2$, $y = uv$ .

(a) $-2u^2 - 2v^2$ (b) $2u + 2v$ (c) $2u^2 - 2v^2$ (d) $2u^2 + 2v^2$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 5. Evaluate $\int\int_R |x|\, dA$ where $R$ is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ .

(a) $\frac{14}{3}\pi$ (b) $2\pi \ln 2$ (c) $\frac{15}{2}\pi$ (d) $\frac{56}{3}\pi$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 6. Decide which claim is true concerning the following two series

$$[1] \ \sum_{n=1}^{\infty} \frac{\ln n}{n} \cdot \cos n\pi \quad \text{and} \quad [2] \ \sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n} .$$

(a) [1] diverges and [2] converges (b) both series converge

(c) [1] converges and [2] diverges (d) both series diverge

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 7. The function $f(x) = \begin{cases} 1 & \text{when } -\pi < x < 0 \\ \sin x & \text{when } 0 < x < \pi \end{cases}$ is expanded into its Fourier series whose sum $F(x)$ is also the periodic extension of $f(x)$. Find $F(4\pi)$ .

(a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 8. Evaluate the following improper integral $\int_0^2 \int_0^\infty (x+y) \cdot e^{-(x+y)}\, dx\, dy$ .

(a) $2 - 5e^{-2}$ (b) $2 - 4e^{-1}$ (c) $2$ (d) this integral diverges

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 9. Find the maximum value of $f(x,y,z) = 2x+z+2$ on the sphere $x^2+y^2+z^2 = 5$ .

(a) $7$ (b) $2+3\sqrt{5}$ (c) $6$ (d) $7-2\sqrt{5}$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 10. Use Maclaurin series to find the derivative $f^{(2005)}(0)$ when $f(x) = x \cdot e^{-x^2}$ .

(a) $\frac{2005!}{668!}$ (b) $0$ (c) $\frac{2005!}{1001}$ (d) $\frac{2005!}{1002!}$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 11. Evaluate $\int_C (x+y)dx + zdy + (x+1)dz$ when $C$ is the straight-line segment from $P_0\,(-1,1,0)$ to $P_1\,(1,2,3)$.

(a) $2$ (b) $\frac{15}{2}$ (c) $10$ (d) $-1$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 12. Let $L$ be the tangent line to the curve of intersection of the surfaces

$2x^2+4y^2+z^2 = 10$ and $x^2+y^2+z = 4$ at the point $P\,(1,1,2)$. Then $L$ also passes through the following point $Q\,(x,y,z)$.

(a) $(1,0,2)$ (b) $(1,3,2)$ (c) $(1,0,4)$ (d) $(0,0,0)$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 13. The following is a term of the binomial series generated by the function $f(x) = (1+x)^{\frac{3}{4}}$ .

(a) $-\frac{14}{81}x^3$ (b) $\frac{5}{128}x^3$ (c) $\frac{7}{128}x^3$ (d) $-\frac{16}{81}x^3$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 14. Find $\frac{\partial z}{\partial s}$ at the point $P_0\,(s_0, t_0) = (0,1)$ when $z = f(x,y)$, $x = st$ and $y = s^2 + t^2$.

(a) $2\frac{\partial z}{\partial y}$ (b) $\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y}$ (c) $\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y}$ (d) $\frac{\partial z}{\partial x}$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 15. The surface $\rho = 2\sin\phi(\cos\theta + \sin\theta)$ is a sphere. Find its radius.

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $2\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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Problem 16. Find the volume of the region in the first octant bounded by the coordinate planes, the plane $x + z = 1$ and the surface $y = x^2 + 2z + 1$ .

(a) $\frac{2}{3}$ (b) $\frac{11}{12}$ (c) $\frac{5}{12}$ (d) $\frac{7}{12}$

(e) none of the above

**Your answer** (write A, B, C, D or E):

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**Part 2: Two workout problems. Each problem is worth 10 pts.**

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Problem 17. Consider the vector field $\vec{F}(x,y,z) = (2x+y)\vec{\imath} + (x+z)\vec{\jmath} + y\vec{k}$ .

(a) Verify that this field is conservative.

(b) Find the potential function $f(x,y,z)$ for $\vec{F}$.

(c) Use the result of (b) to evaluate the integral $\int_c \vec{F} \cdot d\vec{r}$ where $C$ is a smooth curve whose initial point is $(3,-2,1)$ and the terminal point is $(-1,2,0)$.

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Your **complete** solution

– include all essential details; continue on the reverse side of this page if needed:

Problem 18. Let $R$ be the triangular region with vertices at the points $(0,0)$, $(2,1)$ and $(1,2)$ in the $xy$-plane. Consider the integral $\int\int_R 4x\,dA$ .

(a) Set the limits of integration in both rectangular orders $dx\,dy$ and $dy\,dx$ .

(b) Set the limits of integration in polar coordinates.

(c) Evaluate the integral.

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Your **complete** solution

– include all essential details; continue on the reverse side of this page if needed: