Bachnak
11/06/09

EEN 360 Modern Control Systems
Fall 2009

Midterm 1 (25 points)
Five problems (5 points each)
Open books only (closed notes, homework solutions, etc.)

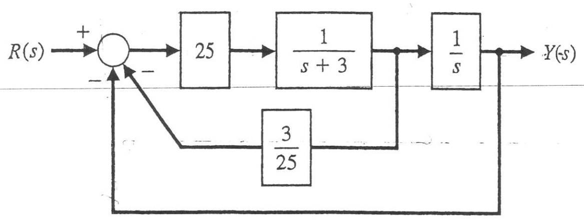
Time limit: 55 minutes

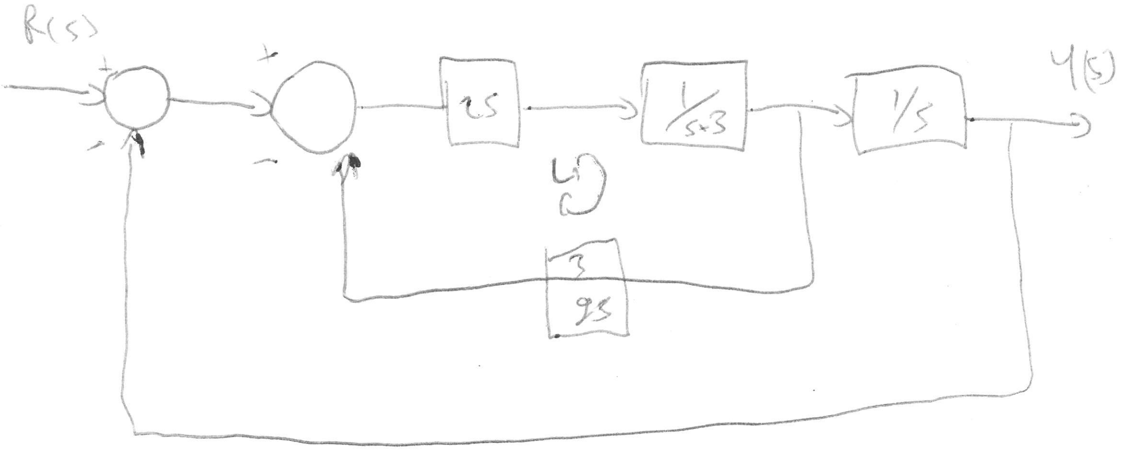
-6

Score: 19

You must show your work to receive credit.

1. A system has the following block diagram. Determine the TF = Y(s)/R(s). Simplify your answer as much as possible.





no change
wrong
answer
not only
6 → 3

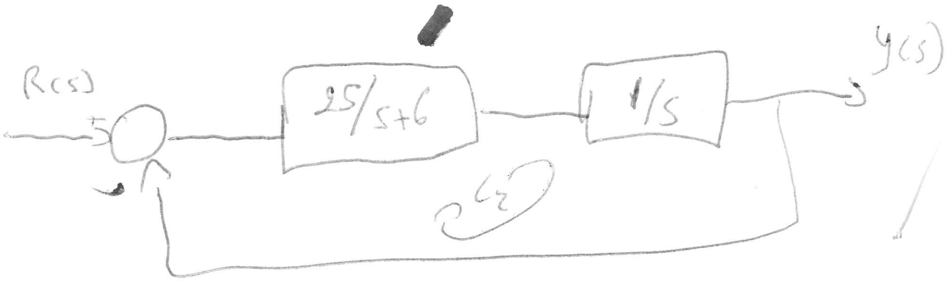
$L_1: G = 25 \times \frac{1}{s+3} = \frac{25}{s+3}$

$\frac{G}{1 \mp GH}$

$H = 3/25$

$$\Rightarrow \frac{\frac{25}{s+3}}{1 + \frac{3}{s+3}} = \frac{25}{s+3+3} = \frac{25}{s+6}$$ ✓

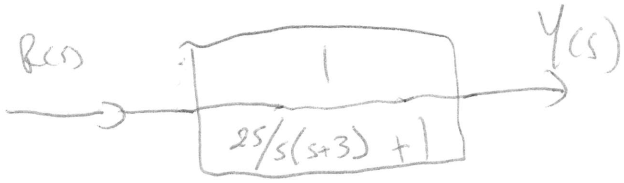
-1.5



$L_2: G = \dfrac{25}{s(s+3)}$, $H = 1$

$$\Rightarrow \frac{\frac{25}{s(s+3)}}{1 + \frac{25}{s(s+3)}} = \frac{1}{\frac{25}{s(s+3)} + 1} = \frac{1}{\frac{25 + s^2 + 6s}{s(s+3)}}$$

Wrong



X

$$\boxed{\frac{Y(s)}{R(s)} = \frac{25}{s^2 + 6s + 25}}$$

X

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{1}{\frac{25}{s(s+3)} + 1} = ?$$

$\cancel{s^2 + 25}$

$=$

2. For electromechanical systems that require large power amplification, rotary amplifiers are often used [8,19]. An amplidyne is a power amplifying rotary amplifier. An amplidyne and a servomotor are shown in Figure P2.11. Obtain the transfer function $\theta(s)/V_c(s)$, [redacted] Assume $v_d = k_2 i_q$ and $v_q = k_1 i_c$.

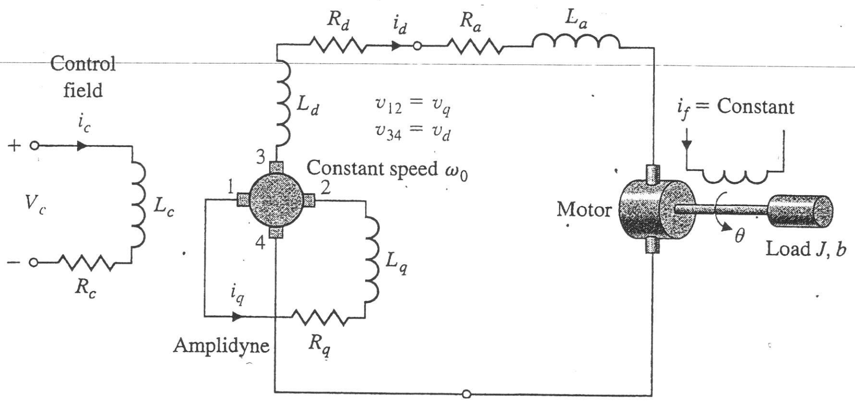


FIGURE P2.11 Amplidyne and armature-controlled motor.

$$\frac{\theta(s)}{V_c(s)} = \frac{\theta(s)}{V_d(s)} \frac{V_d(s)}{V_c(s)}$$

$$\frac{V_d(s)}{V_c(s)} = \frac{K/(R_c R_q)}{(s\tau_c+1)(s\tau_q+1)} = \frac{K/(R_c R_q)}{\left(s\frac{L_c}{R_c}+1\right)\left(s\frac{L_q}{R_q}+1\right)}$$

$$\frac{\theta(s)}{V_d(s)} = \frac{K_m}{s\left[(R_a+L_a s)(Js+b) + K_b K_m\right]}$$

$$\frac{\theta(s)}{V_c(s)} = \frac{K_m K/R_c R_q}{\left[s\left[(R_a+L_a s)(Js+b) + K_b K_m\right]\right]\left[\left(s\frac{L_c}{R_c}+1\right)\left(s\frac{L_q}{R_q}+1\right)\right]}$$

3. A system is represented by Equation (3.16), where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

(a) Find the matrix $\Phi(t)$. (b) For the initial conditions $x_1(0) = x_2(0) = 1$, find $\mathbf{x}(t)$.

$$\phi(s) = (sI - A)^{-1}$$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{\Delta} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} \qquad \Delta = s^2$$

$$\phi(s) = \begin{bmatrix} 1/s & 1/s^2 \\ 0 & 1/s \end{bmatrix}$$

$$\phi(t) = \mathcal{L}^{-1}\{\phi_s\} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

$$X(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+t \\ 1 \end{bmatrix}$$

4

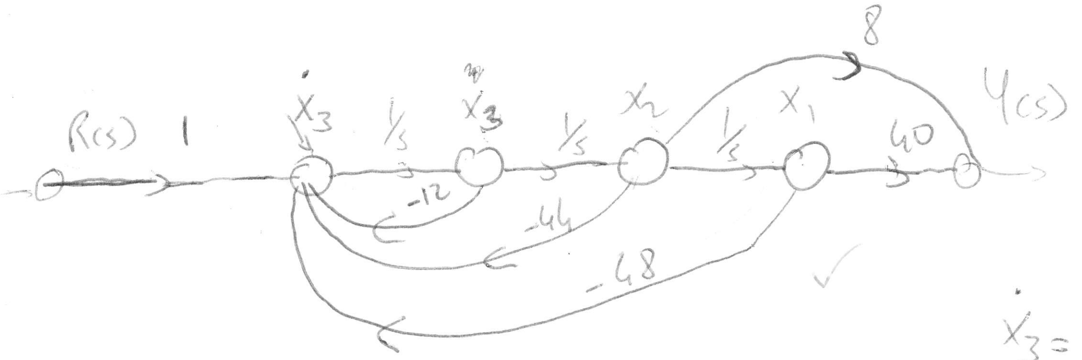
4. A system is described by its transfer function

$$\frac{Y(s)}{R(s)} = T(s) = \frac{8(s+5)}{s^3 + 12s^2 + 44s + 48}.$$

(a) Determine the phase variable canonical form.

Draw a signal flow graph

$$T(s) = \frac{8s+40}{s^3+12s^2+44s+48} = \frac{8s^{-2}+40s^{-3}}{1+12s^{-1}+44s^{-2}+48s^{-3}}$$



$X_3 = \dot{X}_2$

$\dot{X}_1 = X_2$

$\dot{X}_3 = -12X_3 - 44X_2 - 48X_1$

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -48 & -44 & -12 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 8 & 40 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

5. A multi-loop block diagram is shown in Figure E3.9. The state variables are denoted by x_1 and x_2. (a) Determine a state variable representation of the closed-loop system where the output is denoted by $y(t)$ and the input is $r(t)$.

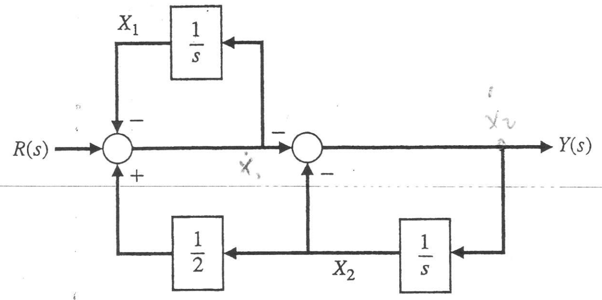


FIGURE E3.9 Multi-loop feedback control system.

-3.5

$\dot{x}_1 = -x_1 + 1/2$ X

$\dot{x}_2 = x_2 + \dot{x}_1 = x_2 - x_1 + 1/2$

$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ X

X

$y(t) = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$