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12/11/09

EEN 360 Modern Control Systems
Fall 2009

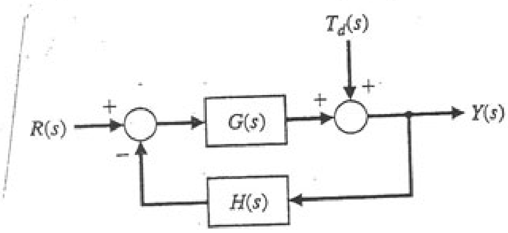
Midterm 2 (25 points)
Five problems (5 points each)
Open books only (closed notes, homework solutions, etc.)

Time limit: 55 minutes

Score: 19

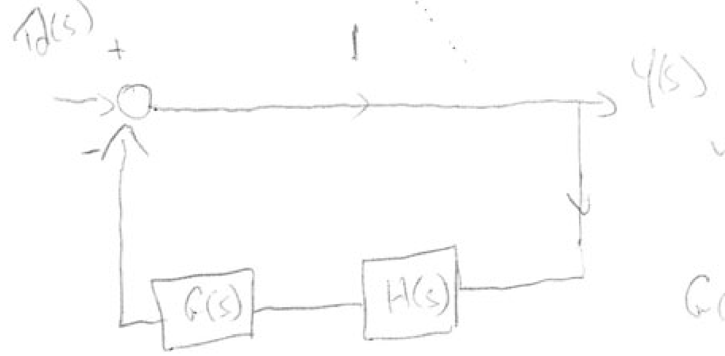
You must show your work to receive credit.

1. Consider the following closed-loop control system.
   (a) Derive the transfer function: TF = $Y(s)/T_d(s)$.
   (b) Determine the steady-state error of the output due to a unit step disturbance input, that is, let $T_d(s) = 1/s$.



$$G(s) = \frac{K}{s+10} \quad \text{and} \quad H(s) = \frac{14}{s^2+5s+6}.$$

a/



$$G(s)H(s) = \frac{K}{s+10} \times \frac{14}{s^2+5s+6}$$

$$= \frac{14K}{(s+10)(s^2+5s+6)}$$

$$TF = \frac{Y(s)}{T_d(s)} = \frac{1}{1 + G(s)H(s)}$$

$$\Rightarrow TF = \frac{1}{1 + \frac{14K}{(s+10)(s^2+5s+6)}} = \frac{(s+10)(s^2+5s+6)}{(s+10)(s^2+5s+6) + 14K}$$

b/ $e_{ss} = \lim_{s \to 0} s\,E(s)$

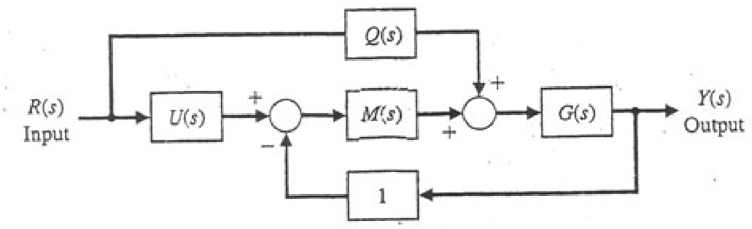
~~$E(s) = Y_d(s) - Y(s)$~~

$$E(s) = \frac{1}{s} \times \frac{(s+10)(s^2+5s+6)}{(s+10)(s^2+5s+6)+14K}$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{s}{s} * \frac{(s+10)(s^2+5s+6)}{(s+10)(s^2+5s+6)+14K}$$

$$= \frac{(10)(6)}{(10)(6)+14K} = \frac{60}{60+14K} = \frac{60}{6+1.4K}$$

2. A control system has the following diagram.
   (a) Determine the transfer function T(s)= Y(s)/R(s)
   (b) Calculate the sensitivity, S^T_G using equation 4.16



a/ $T(s) = Y(s)/R(s) = \dfrac{\sum_k P_k \Delta_k}{\Delta}$

$P_1 = Q(s)\,G(s)$

$P_2 = U(s)\,M(s)\,G(s)$

$T(s) = \dfrac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \dfrac{P_1 + P_2}{\Delta}$

$\Delta = 1 + M(s)\,G(s)$

$T(s) = \dfrac{Q(s)\,G(s) + U(s)\,M(s)\,G(s)}{1 + M(s)\,G(s)}$

$$S_G^T = S_G^N - S_G^D$$

$$S_G^N = \frac{\partial N}{\partial G} \frac{G(s)}{N}$$

$$\frac{\partial N}{\partial G} = \left(Q(s) + U(s)M(s)\right)$$

$$S_Q^N = \frac{Q(s)G(s) + U(s)M(s)G(s)}{Q(s)G(s) + U(s)M(s)G(s)} = 1$$

$$S_G^D = \frac{M(s) \times G(s)}{1+M(s)G(s)} = \frac{M(s)G(s)}{1+M(s)G(s)}$$

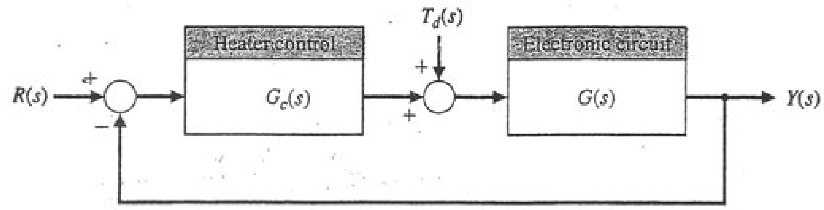
$$S_G^T = 1 - \frac{M(s)G(s)}{1+M(s)G(s)}$$

(5)

3. A control system has the following block diagram. $G_c(s)$ and $G(s)$ are given by

$$G_c(s) = K/(s+10) \qquad \text{and} \qquad G(s) = 2000/(s^2 + 6s + 5)$$

Derive the sensitivity of the system to K, S_K^T.



$$T.F. = \frac{\frac{K}{s+10}\,\frac{2000}{s^2+6s+5}}{1+\frac{K}{s+10}\,\frac{2000}{s^2+6s+5}} = \frac{2000K}{(s+10)(s^2+6s+5)+2000K}$$

$$= \frac{2000K}{s^3+6s^2+5s+10s^2+60s+50+2000K} = \frac{2000K}{s^3+16s^2+65s+50+2000K}$$

$$S_K^T = \frac{\partial T}{\partial K}\,\frac{K}{T} = \frac{2000(s^3+16s^2+65s+50+2000K)+4000000K}{s^3+16s^2+65s+50+2000K}$$

$$\frac{\partial T}{\partial K}$$

$$\Rightarrow S_K^T = \frac{\partial T}{\partial K}\,\frac{K}{T} = \frac{2000\left[(s^3+16s^2+65s+50+2000K)+4000000K\right]K}{2000K}$$

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$$= s^3+16s^2+65s+50+4002000K$$

(6)

4. A closed loop transfer function is given by

$$TF = 2(s + 6) / [s^2 + 4s + 9]$$

(a) Find the time response, y(t), for a step input r(t) = A, for t>0.
(b) Determine the percent overshoot of the response.

a/ $TF = \frac{2(s+6)}{s^2+4s+9} = \frac{Y(s)}{R(s)}$

$\Rightarrow Y(s) = \frac{2s+12}{s^2+4s+9} \frac{A}{s} = \frac{2As+12A}{s(s^2+4s+9)}$

roots: $s_1 = 0$

$s_2 = -2+2.2j$

$s_3 = -2-2.2j$

$\Rightarrow Y(s) = \frac{2As+12A}{s(s^2+4s+9)} = \frac{B}{s} + \frac{C}{s+2-2.2j} + \frac{D}{s+2+2.2j}$

$B = \left.\frac{2As+12A}{s^2+4s+9}\right|_0 = \frac{12A}{9} = 4/3\ A$

$$C = \frac{2As + 12A}{s(s+2+2.2j)}\Bigg|_{s=-2+2.2j} = \frac{2A(-2+2.2j)+12A}{(-2+2.2j)(4.4j)}$$

$$= \frac{2A(-4+2.2j)}{-8.8j - 9.68} = 0.697A\angle -71.0$$

$$D = \frac{2As + 12A}{s(s+2-2.2j)}\Bigg|_{s=-2-2.2j} = \frac{2A(-4-2.2j)}{(-2-2.2j)(-4.4j)}$$

$$= \frac{2A(-4-2.2j)}{8.8j - 9.68} = 0.697A\angle 71.08^\circ$$

$$Y(s) = \frac{4/3A}{s} + \frac{0.697A\angle -71.08^\circ}{s+2-2.2j} + \frac{0.697A\angle 7}{s+2+2.2j}$$

$$Y(t) = 4/3A + 1.394e^{-2t}\cos(2.2t - 71.08^\circ)\ A$$

(7)

b/ $\frac{2(s+6)}{s^2+4s+9} = \frac{\frac{w_n^2}{a}(s+a)}{s^2+2\xi w_n s+w_n^2}$

5. A unity negative feedback control system has the following plant transfer function:

$G(s) = [\ 6(s+5)\ ] / [\ s\ (s+1)\ (s+3)\ (s+10)\ ]$

Determine the steady state error for a ramp input, At.

$$\frac{A}{s^2} \cdot \frac{6(s+5)}{[s(s+1)(s+3)(s+10)]} = \frac{6A(s+5)}{s^3(s+1)(s+3)(s+10)}$$

For $N=0 \Rightarrow e_{ss} = \infty$

For $N=1 \Rightarrow e_{ss} = \frac{A}{K_v}$ /

$K_v = \lim_{s \to 0} s(s+5)\,6 =$

?

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