4.1

4.1 The velocity field of a flow is given by $V = (5z - 3)\hat{\mathbf{i}} + (x + 4)\hat{\mathbf{j}} + 4y\hat{\mathbf{k}}$ ft/s, where x, y, and z are in feet. Determine the fluid speed at the origin ($x = y = z = 0$) and on the x axis ($y = z = 0$).

$u = 5z - 3 \,,\; v = x + 4 \,,\; w = 4y$

Thus, at the origin $u = -3$, $v = 4$, $w = 0$ so that

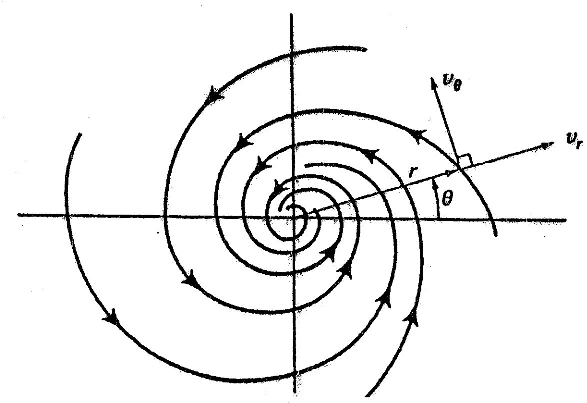
$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(-3)^2 + 4^2} = \underline{\underline{5 \text{ ft/s}}}$$

Similarly, on the x axis $u = -3$, $v = x + 4$, $w = 0$ so that

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(-3)^2 + (x+4)^2} = \underline{\underline{\sqrt{x^2 + 8x + 25}}} \text{ ft/s, where } x \sim \text{ft}$$

4.2

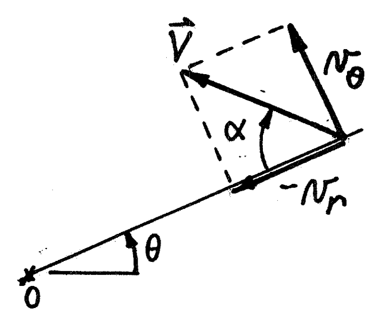
4.2 A flow can be visualized by plotting the velocity field as velocity vectors at representative locations in the flow as shown in Video V4.1 and Fig. E4.1. Consider the velocity field given in polar coordinates by $v_r = -10/r$ and $v_\theta = 10/r$. This flow approximates a fluid swirling into a sink as shown in Fig. P4.2. Plot the velocity field at locations given by $r = 1$, 2, and 3 with $\theta = 0$, 30, 60, and 90 deg.



■ FIGURE P4.2

With $v_r = -10/r$ and $v_\theta = 10/r$ then

$$V = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-10/r)^2 + (10/r)^2} = \frac{14.14}{r}$$



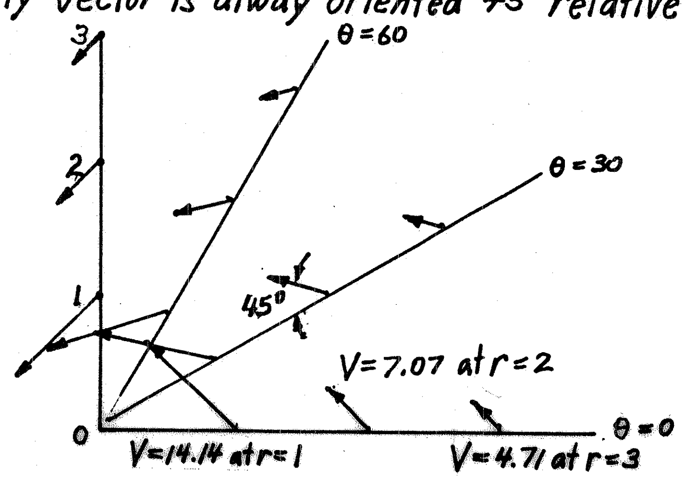
The angle α between the radial direction and the velocity vector is given by

$$\tan\alpha = \frac{v_\theta}{-v_r} = \frac{10/r}{-(-10/r)} = 1$$

Thus, $\alpha = 45°$ for any r, θ

(i.e. the velocity vector is alway oriented 45° relative to radial lines)

Note: V is independent of θ.



4.3

4.3 The velocity field of a flow is given by $\mathbf{V} = 20y/(x^2 + y^2)^{1/2}\hat{\mathbf{i}} - 20x/(x^2 + y^2)^{1/2}\hat{\mathbf{j}}$ ft/s, where x and y are in feet. Determine the fluid speed at points along the x axis; along the y axis. What is the angle between the velocity vector and the x axis at points $(x, y) = (5, 0), (5, 5)$, and $(0, 5)$?

$$u = \frac{20y}{(x^2+y^2)^{1/2}}, \quad v = -\frac{20x}{(x^2+y^2)^{1/2}}$$

Thus, $V = \sqrt{u^2 + v^2}$ or

$$V = \left[\frac{400x^2 + 400y^2}{(x^2+y^2)}\right]^{1/2} = \underline{\underline{20\ \frac{ft}{s}\ \text{for any } x, y}}$$

Also,

$$\tan\theta = \frac{v}{u} = \frac{\dfrac{-20x}{(x^2+y^2)^{1/2}}}{\dfrac{20y}{(x^2+y^2)^{1/2}}}$$

or

$$\tan\theta = -\frac{x}{y}$$

Thus, for $(x,y) = (5,0)$

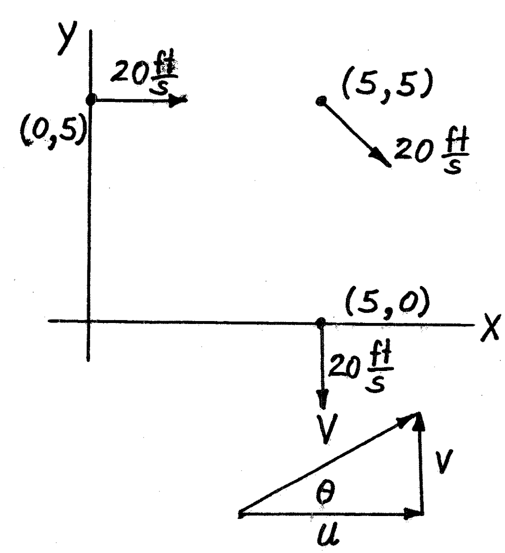
$\tan\theta = -\infty$ or $\theta = \underline{\underline{-90^\circ}}$

for $(x,y) = (5,5)$

$\tan\theta = -1$ or $\theta = \underline{\underline{-45^\circ}}$

for $(x,y) = (0,5)$

$\tan\theta = 0$ or $\theta = \underline{\underline{0^\circ}}$



4.4

4.4 The components of a velocity field are given by $u = x + y$, $v = xy^3 + 16$, and $w = 0$. Determine the location of any stagnation points ($\mathbf{V} = 0$) in the flow field.

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(x+y)^2 + (xy^3+16)^2} = 0$$

or

$u = x + y = 0$ so that $x = -y$

and

$v = xy^3 + 16 = 0$ so that $xy^3 = -16$

Hence, $(-y)y^3 = -16$, or $y = 2$

Therefore, $V = 0$ at $\underline{\underline{x = -2,\ y = 2}}$

4.5

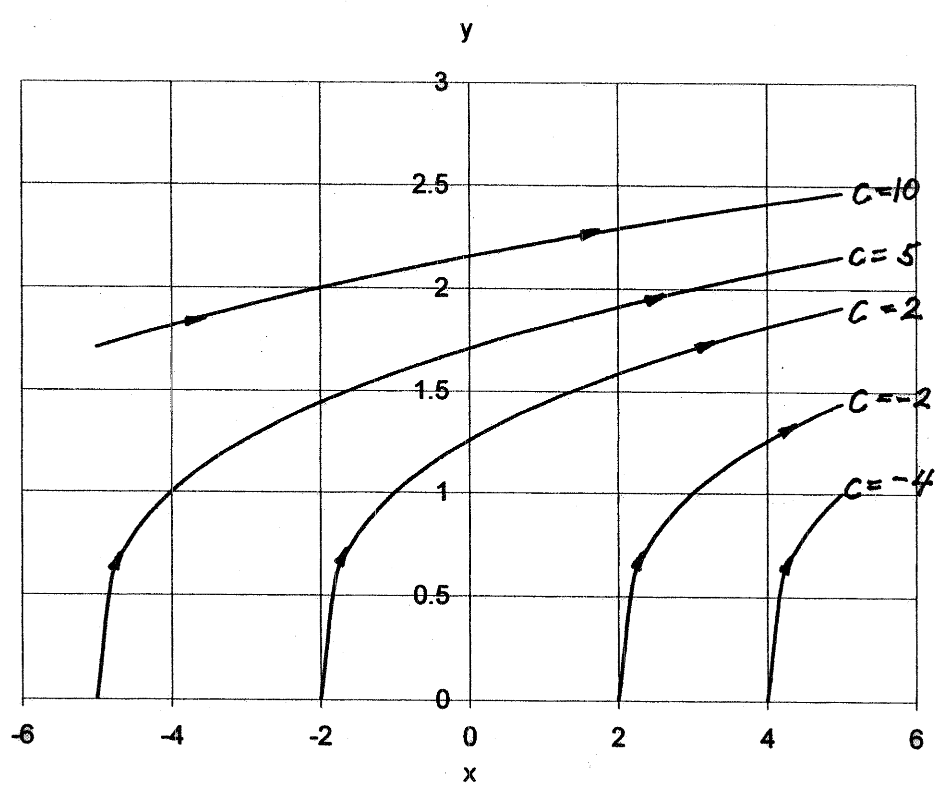
4.5 The x and y components of velocity for a two-dimensional flow are $u = 9y^2$ ft/s and $v = 3$ ft/s, where x is in feet. Determine the equation for the streamlines and graph representative streamlines in the upper half plane.

$u = 9y^2$ and $v = 3$ so that the streamlines are given by

$\frac{dy}{dx} = \frac{v}{u} = \frac{3}{9y^2}$ or $\int 3y^2 dy = \int dx$

Thus, $y^3 = x + C$, where C is a constant

Representative streamlines corresponding to different values of C are shown below. Note: Since $v = 3 > 0$ the flow is in the direction indicated.



4.6

4.6 Show that the streamlines for a flow whose velocity components are $u = c(x^2 - y^2)$ and $v = -2cxy$, where c is a constant, are given by the equation $x^2y - y^3/3 =$ constant. At which point (points) is the flow parallel to the y axis? At which point (points) is the fluid stationary?

$u = c(x^2 - y^2)$, $v = -2cxy$

Streamlines given by $y = f(x)$ are such that $\frac{dy}{dx} = \frac{v}{u}$

Consider the function $x^2y - \frac{y^3}{3} = \text{const.}$ (1)

Note: It is not easy to write this explicitly as $y = f(x)$

However, we can differentiate Eq. (1) to give

$$2xy\,dx + x^2 dy - y^2 dy = 0 \text{ , or}$$

$$(x^2 - y^2)\,dy + 2xy\,dx = 0$$

Thus, the lines in the x-y plane given by Eq. (1) have a slope

$\frac{dy}{dx} = \frac{-2xy}{(x^2-y^2)}$ or for any constant c, $\frac{dy}{dx} = \underline{\underline{\frac{-2cxy}{c(x^2-y^2)} \equiv \frac{v}{u}}}$

i.e. the function $x^2y - \frac{y^3}{3} = \text{const.}$ represents the streamlines of the given flow.

The flow is parallel to the x-axis when $\frac{dy}{dx} = 0$, or $v = 0$.

This occurs when either $x = 0$ or $y = 0$, i.e., the x-axis or the y-axis

The flow is parallel to the y-axis when $\frac{dy}{dx} = \infty$, or $u = 0$.

This occurs when $\underline{\underline{x = \pm y}}$

The fluid has zero velocity at $\underline{\underline{x = y = 0}}$

4.7

4.7 A velocity field is given by $\mathbf{V} = x\hat{\mathbf{i}} + x(x-1)(y+1)\hat{\mathbf{j}}$, where u and v are in ft/s and x and y are in feet. Plot the streamline that passes through $x = 0$ and $y = 0$. Compare this streamline with the streakline through the origin.

$u = x$, $v = x(x-1)(y+1)$ where the streamlines are obtained from

$$\frac{dy}{dx} = \frac{v}{u} = \frac{x(x-1)(y+1)}{x} = (x-1)(y+1)$$

or

$$\int \frac{dy}{(y+1)} = \int (x-1)\,dx$$ which when integrated gives

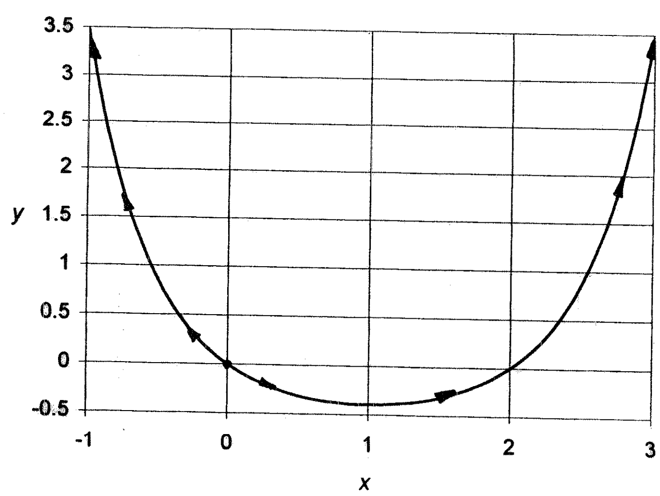
$$\ln(y+1) = \tfrac{1}{2}x^2 - x + C \quad \text{, where } C \text{ is a constant} \qquad (1)$$

For the streamline that passes through the origin $x=y=0$ the value of C is found from Eq. (1) as

$\ln(1) = C$, or $C = 0$

Thus, $\ln(y+1) = \frac{1}{2}x^2 - x$ or $\underline{\underline{y = e^{(\frac{1}{2}x^2 - x)} - 1}}$

This streamline is plotted below.

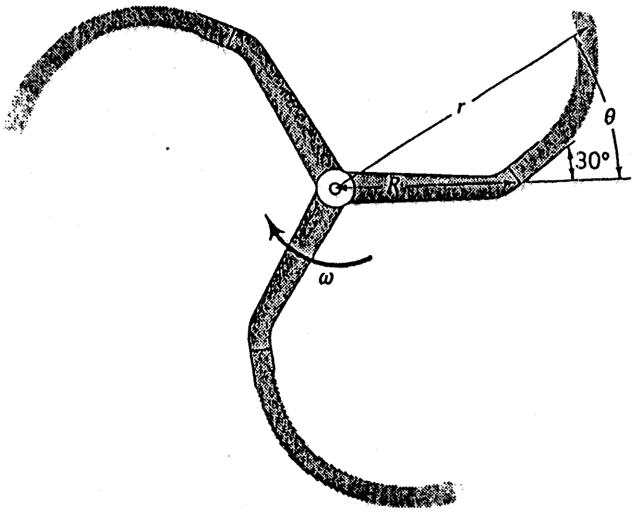


Note: The streamline is symmetrical about its low point of $x=1$, $y=-0.393$. At $x=y=0$ the velocity is 0. For $x<0$, $u<0$ and for $x>0$, $u>0$. Thus, the fluid flows from the origin $(x=y=0)$.

Since the flow is steady, <u>streaklines are the same as streamlines.</u>

4.8

4.8 Water flows from a rotating lawn sprinkler as shown in Video V4.6 and Figure P4.8. The end of the sprinkler arm moves with a speed of ωR, where $\omega = 10$ rad/s is the angular velocity of the sprinkler arm and $R = 0.5$ ft is its radius. The water exits the nozzle with a speed of $V = 10$ ft/s relative to the rotating arm. Gravity and the interaction between the air and the water are negligible. **(a)** Show that the pathlines for this flow are straight radial lines. *Hint:* Consider the direction of flow (relative to the stationary ground) as the water leaves the sprinkler arm. **(b)** Show that at any given instant the stream of water that came from the sprinkler forms an arc given by $r = R + (V_a/\omega)\theta$, where the



■ FIGURE P4.8

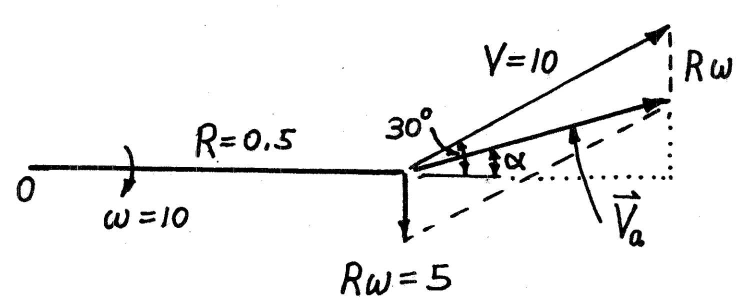
(a) Water leaves the nozzle with a velocity of $V = 10$ ft/s at an angle of $30°$ relative to the radial direction — for an observer riding on the sprinkler arm. This is the relative velocity. As shown in the sketch, the sprinkler arm has a circumferencial velocity of $R\omega = 0.5\text{ ft}\,(10\text{ rad/s}) = 5$ ft/s. The absolute velocity, $\vec{V}_a$, as observed by a person standing on the lawn is the vector sum of relative velocity and the nozzle velocity.

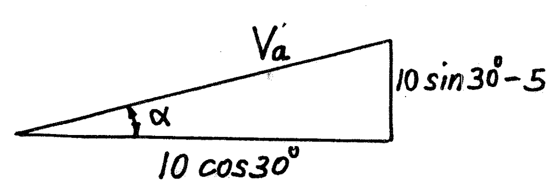
From the geometry of the figure:

$$\tan\alpha = \frac{10\sin 30° - 5}{10\cos 30°} = 0$$

That is $\alpha = 0$

i.e., the absolute water velocity is in the radial direction. Since there is no force acting on the water after it leaves, the water particles continue to move in the radial direction. Thus, the pathlines are straight radial lines.



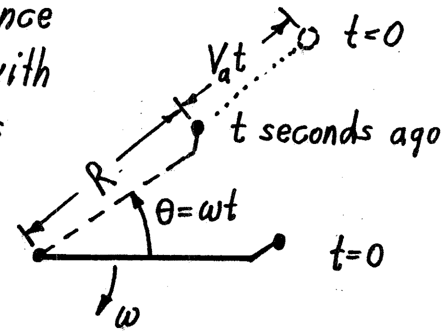


(b) The shape of the water stream at a given instant (i.e. a "snap shot" of the water) can be obtained as follows. Consider the water stream emanating from the end of the nozzle at $r=R$ and $\theta = 0$ at time $t=0$

(con't)

4.8 (con't)

A particle in this stream that left from the nozzle t seconds ago did so when the nozzle was at $\theta = \omega t$. Since the particles in straight, radial paths with speed V_a (see part (a)), this particle is at a distance of $r = R + V_a t$ from the origin.



Thus, the stream shape is

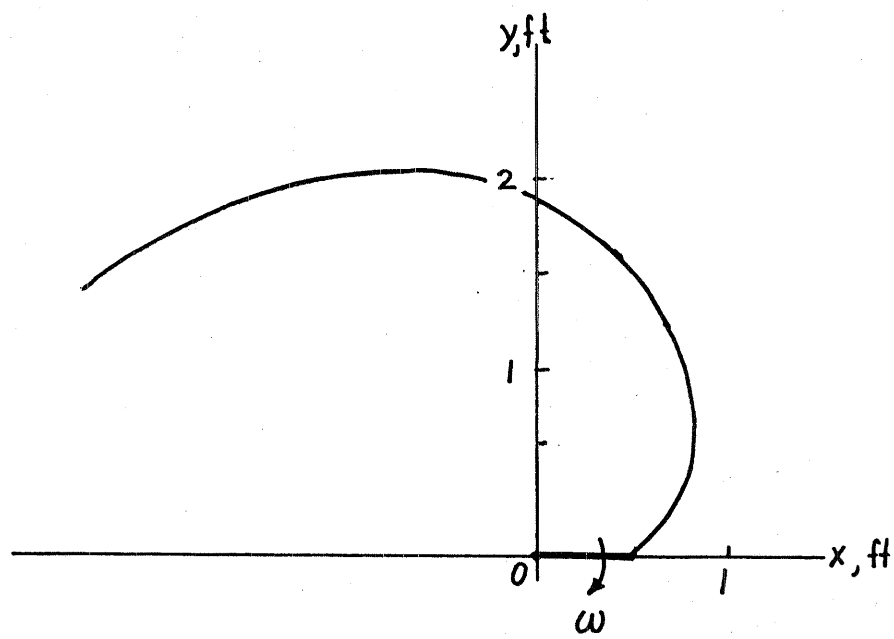
$r = R + V_a t$ and $\theta = \omega t$, or by eliminating t

$$\underline{\underline{r = R + \left(\frac{V_a}{\omega}\right)\theta}}$$

For the given data with $V_a = V\cos 30° = (10\,\frac{ft}{s})\cos 30° = 8.66\,\frac{ft}{s}$ (see part (a)) and $\omega = 10\,rad/s$ this becomes

$r = 0.5 + 0.866\,\theta$, where $r \sim ft$ and $\theta \sim rad$.

This stream shape is plotted below.



Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

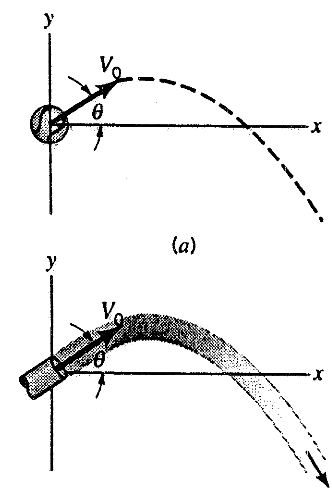
***4.9**

*4.9 Consider a ball thrown with initial speed V_0 at an angle of θ as shown in Fig. P4.9*a*. As discussed in beginning physics, if friction is negligible the path that the ball takes is given by

$$y = (\tan \theta)x - [g/(2\, V_0^2 \cos^2 \theta)]x^2$$

That is, $y = c_1 x + c_2 x^2$, where c_1 and c_2 are constants. The path is a parabola. The pathline for a stream of water leaving a small nozzle is shown in Fig. P4.9*b* and Video V4.3. The coordinates for this water stream are given in the following table. (a) Use the given data to determine appropriate values for c_1 and c_2 in the above equation and, thus, show that these water particles also follow a parabolic pathline. (b) Use your values of c_1 and c_2 to determine the speed of the water, V_0, leaving the nozzle.

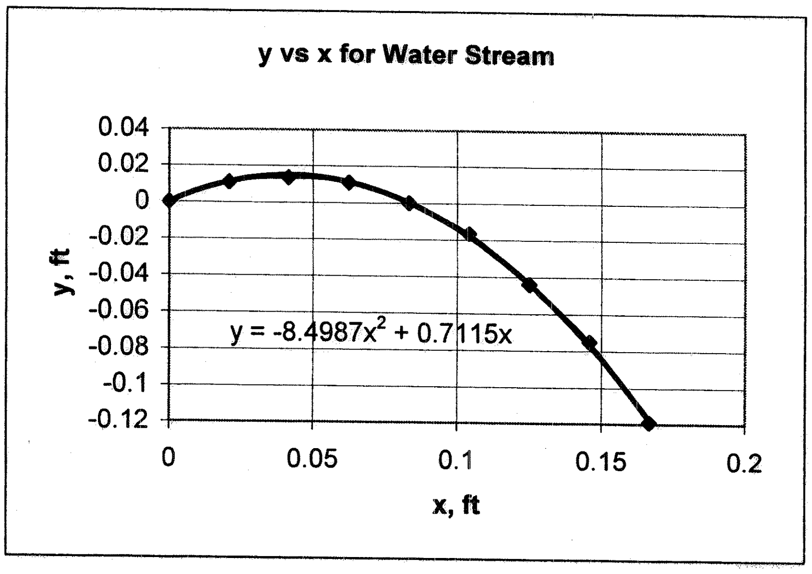
x, in.	*y*, in.
0	0
0.25	0.13
0.50	0.16
0.75	0.13
1.0	0.00
1.25	−0.20
1.50	−0.53
1.75	−0.90
2.00	−1.43



(*b*)

■ FIGURE P4.9

An EXCEL Program was used to plot the x-y data and to fit a second order curve to the data. The results are shown below.



Thus, with $y = c_1 x + c_2 x^2$ it follows that

$c_1 = 0.7115 = \tan\theta$ or $\theta = 35.4^\circ$

and

$$c_2 = -8.4987 = -\frac{g}{2V_0^2\cos^2\theta}$$

or

$$V_0^2 = \frac{32.2}{2(8.4987)\cos^2(35.4^\circ)} = 2.85\,\frac{ft^2}{s^2}$$

Thus, $\underline{\underline{V_0 = 1.69\,\frac{ft}{s}}}$

4.10

4.10 The x and y components of a velocity field are given by $u = x^2y$ and $v = -xy^2$. Determine the equation for the streamlines of this flow and compare with those in Example 4.2. Is the flow in this problem the same as that in Example 4.2? Explain.

Streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$

or $\frac{dy}{y} = -\frac{dx}{x}$ which can be integrated as:

$\int \frac{dy}{y} = -\int \frac{dx}{x}$ Thus, $\ln y = -\ln y + \tilde{C}$, where $\tilde{C}$ is a constant.

Thus, $\underline{\underline{xy = C}}$

Note: These streamlines are the same shape (same "flow pattern") as in Example 4.2 — but the velocity fields are different. However, the ratios $\frac{v}{u}$ are the same:

$$\frac{v}{u} = -\frac{xy^2}{x^2y} = -\frac{y}{x}$$

and

$$\frac{v}{u} = \frac{(V_0/\ell)(-y)}{(V_0/\ell)(x)} = -\frac{y}{x}$$

4.11

4.11 A flow in the x–y plane is given by the following velocity field: $u = 3$ and $v = 6$ m/s for $0 < t < 20$ s; $u = -4$ and $v = 0$ m/s for $20 < t < 40$ s. Dye is released at the origin ($x = y = 0$) for $t = 0$. **(a)** Draw the pathline at $t = 30$ s for two particles that were released from the origin—one released at $t = 0$ and the other released at $t = 20$ s. **(b)** On the same graph draw the streamlines at times $t = 10$ s and $t = 30$ s.

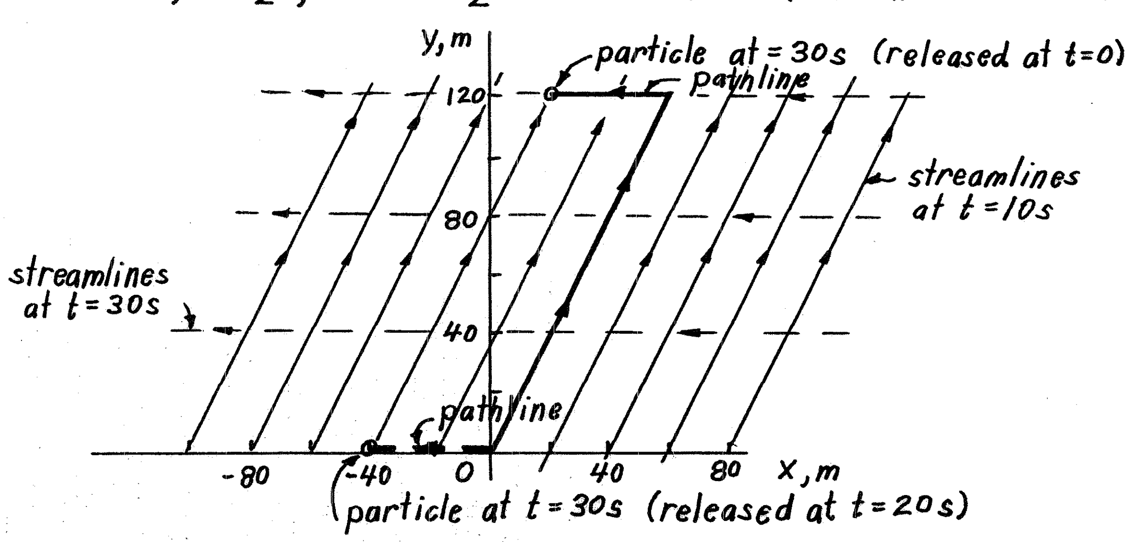
(a) For the particle released at $t=0$, $u = 3\frac{m}{s}$ and $v = 6\frac{m}{s}$ for $0<t<20s$. During this time the flow is steady and the pathline has a slope $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$. At $t=0$, $x=y=0$ and at $t=20$, $x = (3\frac{m}{s})(20s) = 60m$ and $y = (6\frac{m}{s})(20s) = 120m$

For $20<t<30$, $u = -4\frac{m}{s}$ and $v=0$, so that the flow is steady and the pathline has a slope of $\frac{dy}{dx} = 0$. The particle moves from $x = 60m$ to $x = 60 + (-4\frac{m}{s})(30-20)s = +20m$, but keeps the $y = 120m$ location during $20<t<30s$. This pathline is shown in the figure below.

For the particle released at the origin at $t = 20s$ it follows that $u = -4\frac{m}{s}$ and $v=0$. Thus, the corresponding pathline extends from $X=0$ to $X = (-4\frac{m}{s})(30-20)s = -40m$ at $t=30s$. This pathline is shown in the figure below.

(b) At $t = 10s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = \frac{6}{3} = 2$ or $y = 2X + C_1$, where $C_1 = const.$

At $t=30s$, streamlines are given by $\frac{dy}{dx} = \frac{v}{u} = 0$ or $y = C_2$, where $C_2 = const.$ These lines are shown below.



4.12

4.12 In addition to the customary horizontal velocity components of the air in the atmosphere (the "wind"), there often are vertical air currents (thermals) caused by buoyant effects due to uneven heating of the air as indicated in Fig. P4.12. Assume that the velocity field in a certain region is approximated by $u = u_0$, $v = v_0(1 - y/h)$ for $0 < y < h$, and $u = u_0$, $v = 0$ for $y > h$. Plot the shape of the streamline that passes through the origin for values of $u_0/v_0 = 0.5$, 1, and 2.

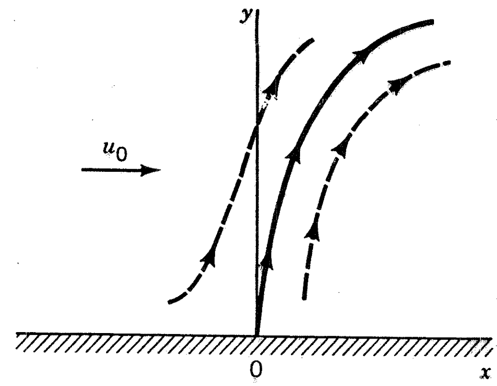


FIGURE P4.12

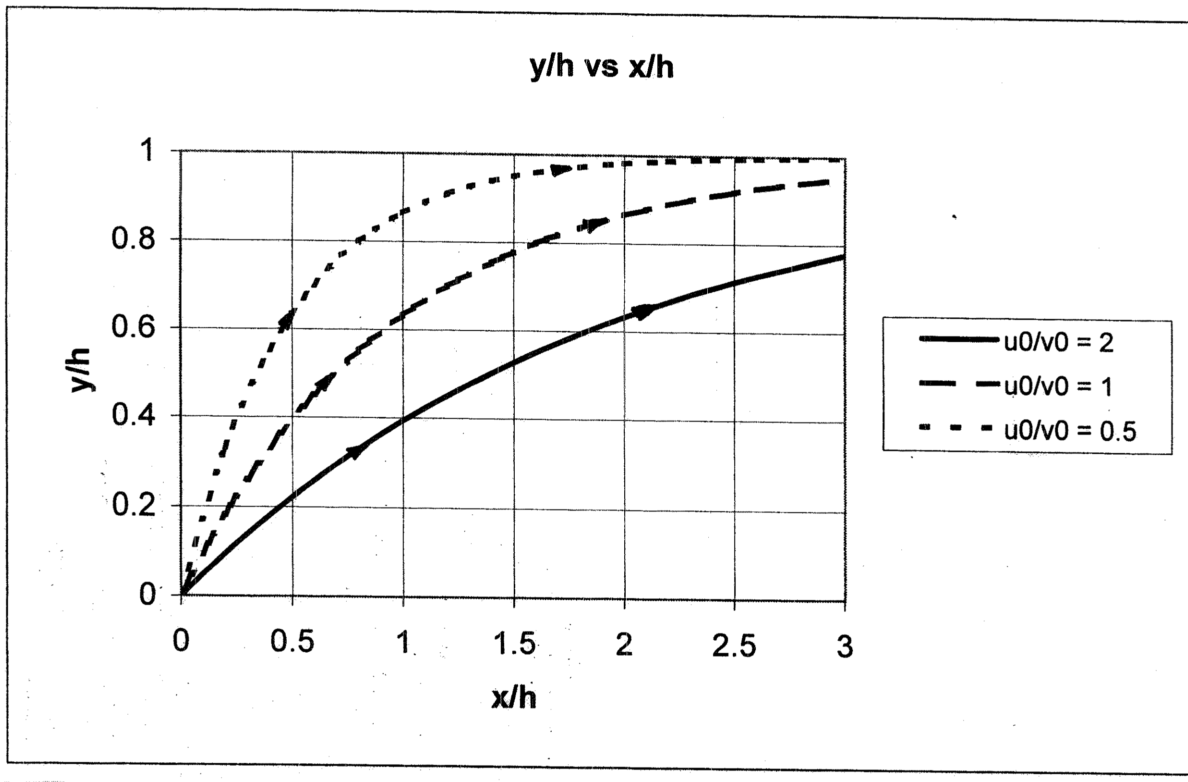
$u = u_0$, $v = V_0\left(1-\frac{y}{h}\right)$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{V_0\left(1-\frac{y}{h}\right)}{u_0} \quad \text{or} \quad \int_0^y \frac{dy}{\left(1-\frac{y}{h}\right)} = \frac{V_0}{u_0}\int_0^x dx$$

Thus, $-h\ln\left(1-\frac{y}{h}\right) = \frac{V_0}{u_0}x$ Note: The lower limits of integration ($x=0$, $y=0$) insure that this equation is for the streamline through the origin.

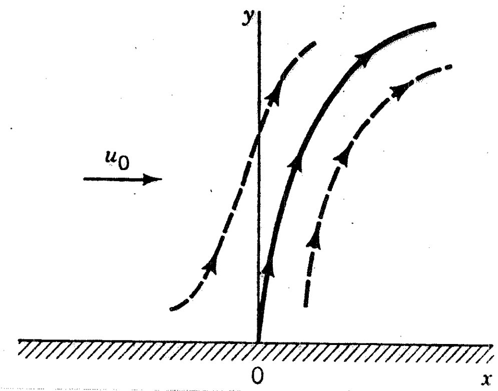
This streamline

$$x = -h\left(\frac{u_0}{V_0}\right)\ln\left(1-\frac{y}{h}\right)$$ is plotted below.



4.13*

4.13* Repeat Problem 4.12 using the same information except that $u = u_0 y/h$ for $0 \le y \le h$ rather than $u = u_0$. Use values of $u_0/v_0 = 0, 0.1, 0.2, 0.4, 0.6, 0.8$, and 1.0.



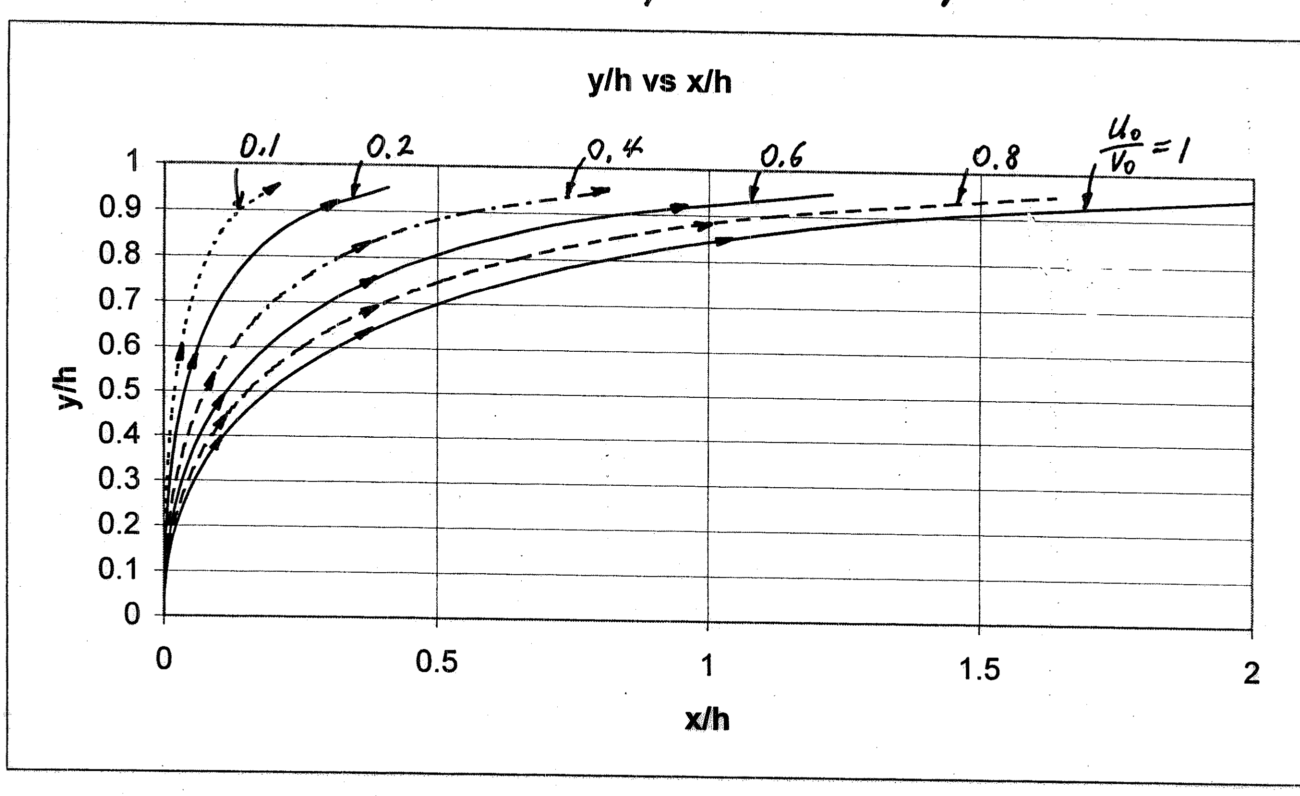
$u = \frac{u_0 y}{h}$, $v = v_0\left(1 - \frac{y}{h}\right)$ for $0<y<h$ so that streamlines for $y<h$ are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{v_0\left(1-\frac{y}{h}\right)}{u_0\frac{y}{h}} = \frac{v_0}{u_0}\frac{(h-y)}{y}$$ or with $x=0$ when $y=0$

$$\int_0^y \frac{y}{(h-y)}\,dy = \int_0^x \frac{v_0}{u_0}\,dx$$ This integrates to give

$$-y - h\ln(h-y) + h\ln(h) = \frac{v_0}{u_0}x \quad \text{or} \quad \frac{x}{h} = \left(\frac{u_0}{v_0}\right)\left[\ln\left(\frac{h}{h-y}\right) - \frac{y}{h}\right]$$

This streamline is plotted below for $0 \le \frac{y}{h} \le 1$, with $\frac{u_0}{v_0} = 0, 0.1, 0.2, 0.4, 0.6, 0.8$, and 1.0 The values were calculated and plotted using an EXCEL Program.



4.14

4.14 A velocity field is given by $u = cx^2$ and $v = cy^2$, where c is a constant. Determine the x and y components of the acceleration. At what point (points) in the flow field is the acceleration zero?

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (cx^2)(2cx) = \underline{\underline{2c^2x^3}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = (cy^2)(2cy) = \underline{\underline{2c^2y^3}}$$

Thus, $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = 0$ at $\underline{\underline{(x,y) = (0,0)}}$

4.15

4.15 Determine the acceleration field for a three-dimensional flow with velocity components $u = -x$, $v = 4x^2y^2$, and $w = x - y$.

$u = -x$, $v = 4x^2y^2$, and $w = x - y$ so that

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= 0 + (-x)(-1) + 4x^2y^2(0) + (x-y)(0) = x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x-y)(0)$$
$$= -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y - 1)$$

and

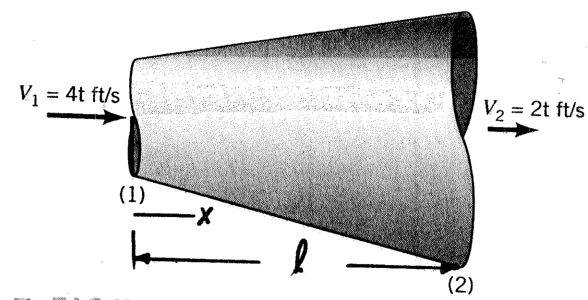
$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= 0 + (-x)(1) + (4x^2y^2)(-1) + (x-y)(0)$$
$$= -x - 4x^2y^2$$

Thus,

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$
$$= \underline{\underline{x\hat{\imath} + 8x^2y^2(4x^2y-1)\hat{\jmath} - (x+4x^2y^2)\hat{k}}}$$

4.17

4.17 The velocity of air in the diverging pipe shown in Fig. P4.17 is given by $V_1 = 4t$ ft/s and $V_2 = 2t$ ft/s, where t is in seconds. **(a)** Determine the local acceleration at points (1) and (2). **(b)** Is the average convective acceleration between these two points negative, zero, or positive? Explain.



■ FIGURE P4.17

a) $\left.\frac{\partial u}{\partial t}\right|_{(1)} = 4\frac{ft}{s^2}$ and $\left.\frac{\partial u}{\partial t}\right|_{(2)} = 2\frac{ft}{s^2}$

b) convective acceleration along the pipe $= u\frac{\partial u}{\partial x}$ where $u > 0$. At any time, t, $V_2 < V_1$. Thus, between (1) and (2)

$$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} < 0$$

Hence, $u\frac{\partial u}{\partial x} < 0$ or the average convective acceleration is negative.

4.18

4.18 Water flows in a pipe so that its velocity triples every 20 s. At $t = 0$ it has $u = 5$ ft/s. That is, $\mathbf{V} = u(t)\hat{\mathbf{i}} = 5\,(3^{t/20})\hat{\mathbf{i}}$ ft/s. Determine the accleration when $t = 0$, 10, and 20 s.

$u = 5(3^{t/20})$, $v = 0$, $w = 0$

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = \frac{\partial u}{\partial t}\hat{i}$ since $\vec{V}\cdot\nabla\vec{V} \equiv 0$ because $\vec{V}$ is not a function of $x, y,$ or z.

Since $\frac{\partial u}{\partial t} = 5\left[3^{t/20}\ln(3)\,\frac{1}{20}\right] = 0.275\,(3^{t/20})\,\frac{ft}{s^2}$ with $t \sim s$

it follows that

$\vec{a} = \underline{\underline{0.275\hat{i}\,\frac{ft}{s^2}}}$ at $t=0$

$\vec{a} = \underline{\underline{0.476\hat{i}\,\frac{ft}{s^2}}}$ at $t=10\,s$

and

$\vec{a} = \underline{\underline{0.825\hat{i}\,\frac{ft}{s^2}}}$ at $t=20\,s$

4.19

4.19 When a valve is opened, the velocity of water in a certain pipe is given by $u = 10(1 - e^{-t})$, $v = 0$, and $w = 0$, where u is in ft/s and t is in seconds. Determine the maximum velocity and maximum acceleration of the water.

$V = \sqrt{u^2 + v^2 + w^2} = 10(1 - e^{-t})$ so that $\frac{dV}{dt} = 10e^{-t} > 0$ for all t

Thus, $V_{max} = V\big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s}}}$

Also, $\vec{a} = a_x\hat{i}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$ with $\frac{\partial u}{\partial x} = 0$

Thus, $a_x = \frac{\partial u}{\partial t} = 10e^{-t}$, so that $a_{x_{max}} = a_x\big|_{t=\infty} = \underline{\underline{10\,\frac{ft}{s^2}}}$

4.20

4.20 The velocity of the water in the pipe shown in Fig. P4.20 is given by $V_1 = 0.50t$ m/s and $V_2 = 1.0t$ m/s, where t is in seconds. Determine the local acceleration at points (1) and (2). Is the average convective acceleration between these two points negative, zero, or positive? Explain.

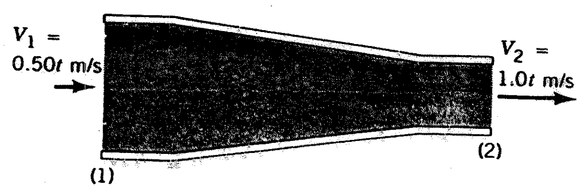


FIGURE P4.20

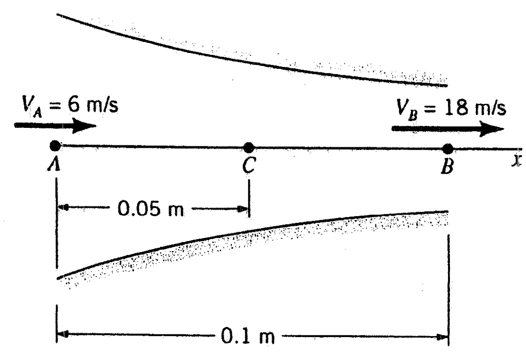
$$\frac{\partial V_1}{\partial t} = 0.5 \ \frac{m}{s^2}$$

$$\frac{\partial V_2}{\partial t} = 1.0 \ \frac{m}{s^2}$$

Since $V_2 > V_1$ it follows that $\frac{\partial V}{\partial x} > 0$. Also, $V > 0$ so that the convective acceleration, $V\frac{\partial V}{\partial x}$, is positive.

4.21

4.21 The fluid velocity along the x axis shown in Fig. P4.21 changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.



■ FIGURE P4.21

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x)$, $v = 0$, and $w = 0$

this becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = u\frac{\partial u}{\partial x}\,\hat{i} \qquad (1)$$

Since u is a linear function of x, $u = C_1 x + C_2$ where the constants C_1, C_2 are given as: $u_A = 6 = C_2$

and $u_B = 18 = 0.1\,C_1 + C_2$

or $C_1 = 120$, $C_2 = 6.$

Thus, $u = (120x + 6)\,\frac{m}{s}$ with $x \sim m$

From Eq. (1)

$$\vec{a} = u\frac{\partial u}{\partial x}\hat{i} = (120x + 6)\,\frac{m}{s}\left(120\,\frac{m}{m\cdot s}\right)\hat{i}$$

or

for $x_A = 0$, $\underline{\underline{\vec{a}_A = 720\,\hat{i}\,\frac{m}{s^2}}}$

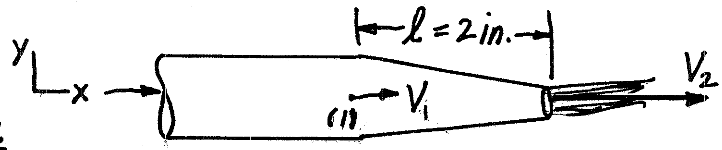
for $x_B = 0.05\,m$, $\underline{\underline{\vec{a}_B = 1440\,\hat{i}\,\frac{m}{s^2}}}$

and

for $x_C = 0.1\,m$, $\underline{\underline{\vec{a}_C = 2160\,\hat{i}\,\frac{m}{s^2}}}$

4.22

4.22 Water flows in a garden hose with a velocity of 5 ft/s, travels through a 2-in.-long nozzle, and exits the nozzle with a velocity of 40 ft/s. Estimate the average acceleration of the water as it flows through the nozzle.



$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= 0 + u\frac{\partial u}{\partial x} + 0 + 0$$

The average acceleration can thus be estimated by using

$$u \approx \frac{V_1 + V_2}{2} = \frac{5\,\frac{ft}{s} + 40\,\frac{ft}{s}}{2} = 22.5\,\frac{ft}{s}$$

and

$$\frac{\partial u}{\partial x} \approx \frac{V_2 - V_1}{\ell} = \frac{40\,\frac{ft}{s} - 5\,\frac{ft}{s}}{(2/12)ft} = 210\,/s$$

to obtain:

$$a_x = u\frac{\partial u}{\partial x} \approx 22.5\,\frac{ft}{s}\,(210/s) = \underline{\underline{4730\,\frac{ft}{s^2}}}$$

Note: This acceleration is equal to $4730\,\frac{ft}{s^2} / 32.2\,\frac{ft}{s^2} = 147$ times the acceleration of gravity (i.e., $a_x/g = 147$).

4.23

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

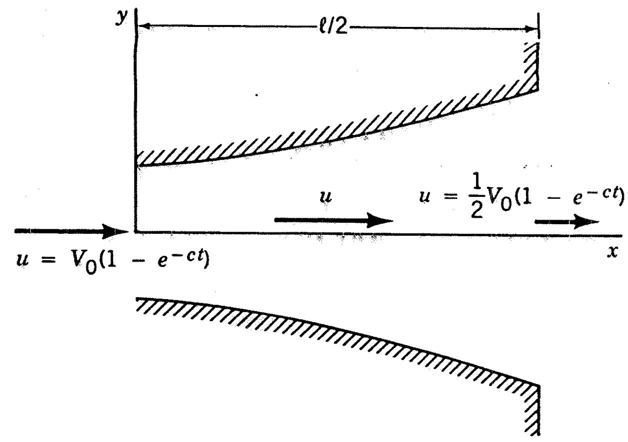


FIGURE P4.23

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$ this becomes

$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = a_x\hat{i}$, where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$$a_x = V_0\left(1 - \frac{x}{\ell}\right)c\,e^{-ct} + V_0^2(1 - e^{-ct})^2\left(1 - \frac{x}{\ell}\right)\left(-\frac{1}{\ell}\right)$$

or

$$\underline{\underline{a_x = V_0\left(1 - \frac{x}{\ell}\right)\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right]}}$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1 - e^{-c})^2 = 0$ The solution (root) of this equation is $\underline{\underline{c = 0.490\ \frac{1}{s}}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.

4.24

4.24 A fluid flows along the x axis with a velocity given by $\mathbf{V} = (x/t)\hat{\mathbf{i}}$, where x is in feet and t in seconds. **(a)** Plot the speed for $0 \leq x \leq 10$ ft and $t = 3$ s. **(b)** Plot the speed for $x = 7$ ft and $2 \leq t \leq 4$ s. **(c)** Determine the local and convective acceleration. **(d)** Show that the acceleration of any fluid particle in the flow is zero. **(e)** Explain physically how the velocity of a particle in this unsteady flow remains constant throughout its motion.

(a) $u = \frac{x}{t}\ \frac{ft}{s}$ so at $t = 3\text{ s}$, $u = \frac{x}{3}\ \frac{ft}{s}$

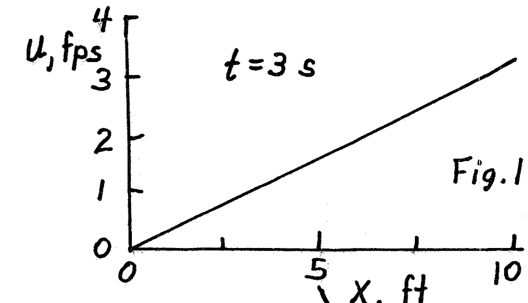


Fig.1

(b) For $x = 7$ ft, $u = \frac{7}{t}\ \frac{ft}{s}$

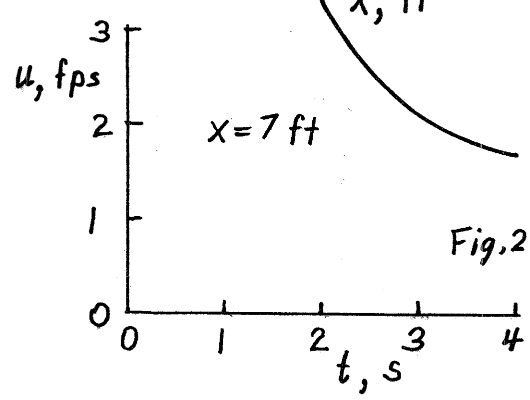


Fig.2

(c) $\frac{\partial u}{\partial t} = \underline{\underline{-\frac{x}{t^2}}}$ and $u\frac{\partial u}{\partial x} = \frac{x}{t}\left(\frac{1}{t}\right) = \underline{\underline{\frac{x}{t^2}}}$

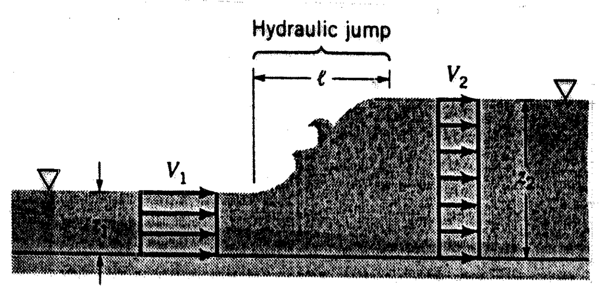
(d) For any fluid particle $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ which with $v = 0$, $w = 0$ becomes

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = \left(-\frac{x}{t^2} + \frac{x}{t^2}\right)\hat{\imath} \equiv 0$$

(e) The particles flow into areas of higher velocity (see Fig.1), but at any given location the velocity is decreasing in time (see Fig.2). For the given velocity field the local and convective accelerations are equal and opposite, giving zero acceleration throughout.

4.25

4.25 A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Fig. P4.25 and **Video V10.6**. In a relatively short distance (thickness = ℓ) the liquid depth changes from z_1 to z_2, with a corresponding change in velocity from V_1 to V_2. If $V_1 =$ 1.20 ft/s, $V_2 = 0.30$ ft/s, and $\ell = 0.02$ ft, estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many g's deceleration does this represent?



■ FIGURE P4.25

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ so with $\vec{V} = u(x)\hat{i}$, $\vec{a} = a_x\hat{i} = u\frac{\partial u}{\partial x}\hat{i}$

Without knowing the actual velocity distribution, $u = u(x)$, the acceleration can be approximated as

$$a_x = u\frac{\partial u}{\partial x} \approx \frac{1}{2}(V_1+V_2)\frac{(V_2-V_1)}{\ell} = \frac{1}{2}(1.20+0.30)\frac{ft}{s}\,\frac{(0.30-1.20)\frac{ft}{s}}{0.02\,ft}$$

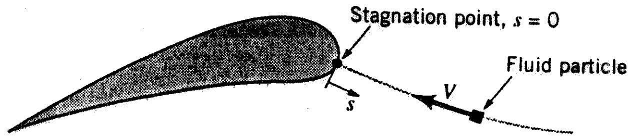
$$= \underline{\underline{-33.8\,\frac{ft}{s^2}}}$$

Thus, $\frac{|a_x|}{g} = \frac{33.8\,\frac{ft}{s^2}}{32.2\,\frac{ft}{s^2}} = \underline{\underline{1.05}}$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

4.26

4.26 A fluid particle flowing along a stagnation streamline, as shown in Video V4.5 and Fig. P4.26, slows down as it approaches the stagnation point. Measurements of the dye flow in the video indicate that the location of a particle starting on the stagnation streamline a distance $s = 0.6$ ft upstream of the stagnation point at $t = 0$ is given approximately by $s = 0.6e^{-0.5t}$, where t is in seconds and s is in ft. (a) Determine the speed of a fluid particle as a function of time, $V_{particle}(t)$, as it flows along the steamline. (b) Determine the speed of the fluid as a function of position along the streamline, $V = V(s)$. (c) Determine the fluid acceleration along the streamline as a function of position, $a_s = a_s(s)$.



■ FIGURE P4.26

(a) With $s = 0.6\,e^{-0.5t}$ it follows that

$$V_{particle} = \frac{ds}{dt} = 0.6\,(-0.5)\,e^{-0.5t} = \underline{\underline{-0.3\,e^{-0.5t}}} \text{ ft/s}$$

(b) From part (a),

$$V = (-0.5)\left[0.6\,e^{-0.5t}\right] \quad \text{where } s = 0.6\,e^{-0.5t}$$

Thus,

$$V = (-0.5)[s], \quad \text{or} \quad V = \underline{\underline{-0.5s}} \text{ ft/s where } s \sim \text{ft}$$

(c) For steady flow, $a_s = V\frac{dV}{ds}$

Thus, with $V = -0.5s$ and $\frac{dV}{ds} = -0.5$,

$$a_s = (-0.5s)(-0.5) = \underline{\underline{0.25s \text{ ft/s}^2}} \text{ where } s \sim \text{ft}$$

Note: For $s > 0$, a_s is positive — the particle's acceleration is to the right. Since the particle is moving to the left, a positive a_s for this case implies that the particle is decelerating (as it must be for this stagnation point flow).

4.27

4.27 A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. If the flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$ m, determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2).

With $u = ax + b$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$ can be written as

$$\vec{a} = a_x \hat{i} \quad \text{where} \quad a_x = u \frac{\partial u}{\partial x}. \qquad (1)$$

Since $u = V_1 = 10 \frac{m}{s}$ at $x = 0$ and $u = V_2 = 25 \frac{m}{s}$ at $x = 1$ we obtain

$$10 = 0 + b$$

$$25 = a + b \quad \text{so that } a = 15 \text{ and } b = 10$$

That is, $u = (15x + 10) \frac{m}{s}$, where $x \sim m$, so that from Eq.(1)

$$a_x = (15x + 10) \frac{m}{s} \left(15 \frac{1}{s}\right) = \underline{\underline{(225x + 150) \frac{m}{s^2}}}$$

Note: The local acceleration is zero, $\underline{\underline{\frac{\partial \vec{V}}{\partial t} = 0}}$, and the convective acceleration is $u \frac{\partial u}{\partial x} \hat{i} = \underline{\underline{(225x + 150) \hat{i} \frac{m}{s^2}}}$

At $x = 0$, $\vec{a} = \underline{\underline{150 \hat{i} \frac{m}{s^2}}}$; at $x = 1\,m$, $\vec{a} = \underline{\underline{375 \hat{i} \frac{m}{s^2}}}$

4.28

4.28 A company makes cars that are shipped to be sold throughout the country. At a dealership near the factory the cars cost \$20,000. At other dealerships the cost is higher because of shipping charges which are \$0.50 per mile. Determine the price of a new car at a location 800 miles from the factory and the rate of increase (\$ per hour) in the car price as it is being transported to that location on a truck traveling 55 mph on the highway. Explain your answer in terms of the material derivative.

Let x = distance from the factory, c = cost of the car, and c_0 = cost of the car at the factory. Thus, with r = rate per mile for shipping,

$c = c_0 + rx = 20{,}000 + 0.5x$, where $c \sim \$$ and $r \sim \$/mi$.

Hence, with $x = 800\,mi$

$C_{800} = 20{,}000 + 0.5(800) = \underline{\underline{\$20{,}400}}$

and

$\frac{Dc}{Dt}$ = rate of increase of cost $= \frac{dc_0}{dt} + r\frac{dx}{dt}$

where $\frac{dc_0}{dt} = 0$ and $\frac{dx}{dt} = V = 55\,mph$

Thus,

$\frac{Dc}{Dt} = \$0.5/mi.\left(55\frac{mi}{hr}\right) = \underline{\underline{\$27.5/hr.}}$

In terms of the material derivative,

$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = 0 + \left(55\frac{mi}{hr}\right)(\$0.5/mi) = \$27.5/hr.$

4.29

4.29 Repeat Problem 4.27 with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1/\partial t = 20$ m/s^2 and $\partial V_2/\partial t = 60$ m/s^2.

With $u = u(x,t)$, $v=0$, and $w=0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$\vec{a} = a_x\,\hat{\imath}$ where $a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$, with $u = a(t)x + b(t)$. (1)

At the given time $(t=t_o)$ $u = V_1 = 10\frac{m}{s}$ at $x=0$ and $u = V_2 = 25\frac{m}{s}$ at $x=1m$

Thus, $10 = 0 + b(t_o)$

$25 = a(t_o) + b(t_o)$ so that $a(t_o) = 15$ and $b(t_o) = 10$

Also at $t=t_o$, $\frac{\partial u}{\partial t} = \frac{\partial V_1}{\partial t} = 20\,\frac{m}{s^2}$ at $x=0$

and $\frac{\partial u}{\partial t} = \frac{\partial V_2}{\partial t} = 60\,\frac{m}{s^2}$ at $x=1m$ Note: These are local accelerations at time $t=t_o$

The convective acceleration at $x=0$ (Eq. (1)) is

$$u\frac{\partial u}{\partial x} = (ax+b)(a) = (15(0)+10)\frac{m}{s}(15\,\frac{1}{s}) = \underline{\underline{150\,\frac{m}{s^2}}}$$

while at $x=1$ it is

$$u\frac{\partial u}{\partial x} = (15(1)+10)\frac{m}{s}(15\,\frac{1}{s}) = \underline{\underline{375\,\frac{m}{s^2}}}$$

The fluid acceleration at $t=t_o$ is

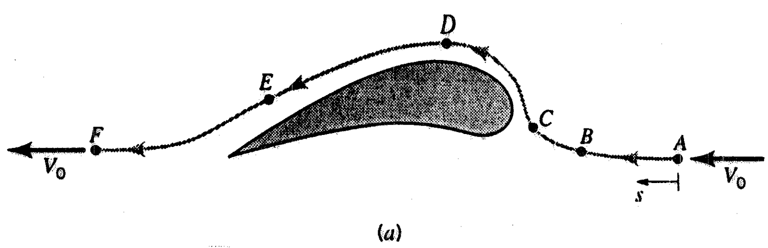
$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = (20+150)\hat{\imath}\,\frac{m}{s^2} = \underline{\underline{170\,\hat{\imath}\,\frac{m}{s^2}\text{ at } x=0}}$$

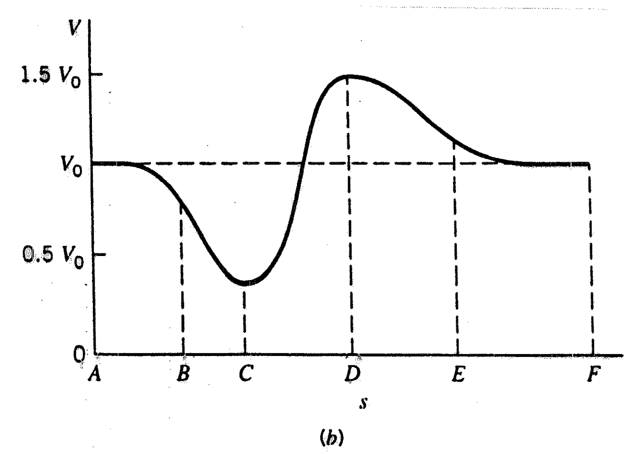
and

$$\vec{a} = (60+375)\hat{\imath}\,\frac{m}{s^2} = \underline{\underline{435\,\hat{\imath}\,\frac{m}{s^2}\text{ at } x=1m}}$$

4.30

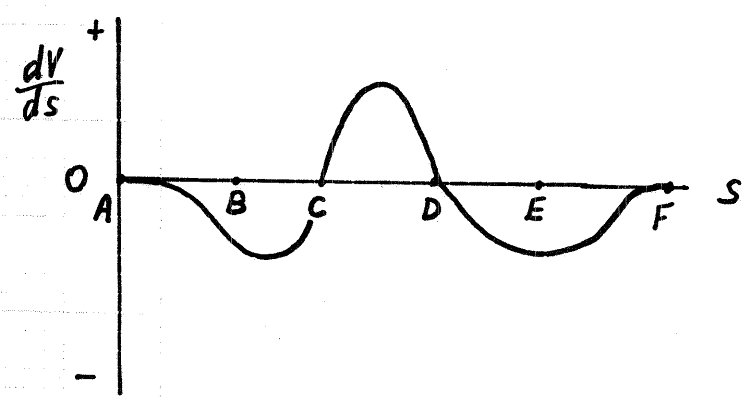
4.30 An incompressible fluid flows past a turbine blade as shown in Fig. P4.30*a* and **Video V4.5**. Far upstream and downstream of the blade the velocity is V_0. Measurements show that the velocity of the fluid along streamline *A–F* near the blade is as indicated in Fig. P4.30*b*. Sketch the streamwise component of acceleration, a_s, as a function of distance, *s*, along the streamline. Discuss the important characteristics of your result.



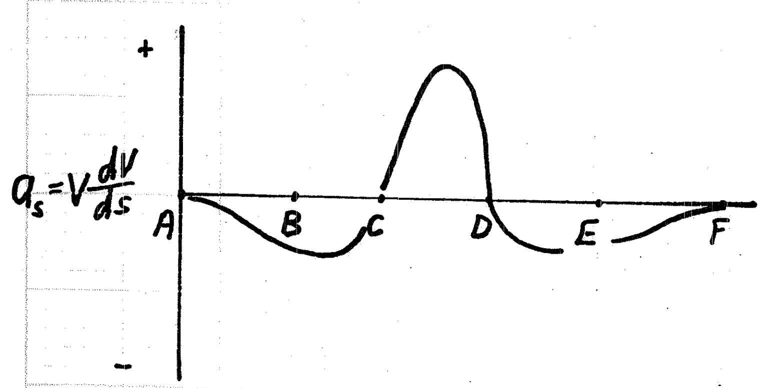


■ FIGURE P4.30

$a_s = V\frac{dV}{ds}$ where from the figure of $V = V(s)$ the function $\frac{dV}{ds}$ has the following shape.



Hence, the product $V\frac{dV}{ds}$ has the shape shown below.



The fluid decelerates from A to C, accelerates from C to D, and the decelerates again from D to F. The net acceleration from A to F is zero (i.e., $V_A = V_0 = V_F$).

4.31

4.31* Air flows steadily through a variable area pipe with a velocity of $\mathbf{V} = u(x)\hat{\mathbf{i}}$ ft/s, where the approximate measured values of $u(x)$ are given in the table. Plot the acceleration as a function of x for $0 \le x \le 12$ in. Plot the acceleration if the flowrate is increased by a factor of N (i.e., the values of u are increased by a factor of N), for $N = 2, 4, 10$.

x (in.)	u (ft/s)	x (in.)	u (ft/s)
0	10.0	7	20.1
1	10.2	8	17.4
2	13.0	9	13.5
3	20.1	10	11.9
4	28.3	11	10.3
5	28.4	12	10.0
6	25.8	13	10.0

Since $u = u(x)$, $v = 0$, and $w = 0$ it follows that $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ simplifies to $\vec{a} = a_x \hat{i}$ where $a_x = u\frac{\partial u}{\partial x}$ (1)
The values u are given in the table; the corresponding values of $\frac{\partial u}{\partial x}$ can be obtained by an approximate numerical differentiation. The results are shown below for the given date (i.e. with $N=1$).

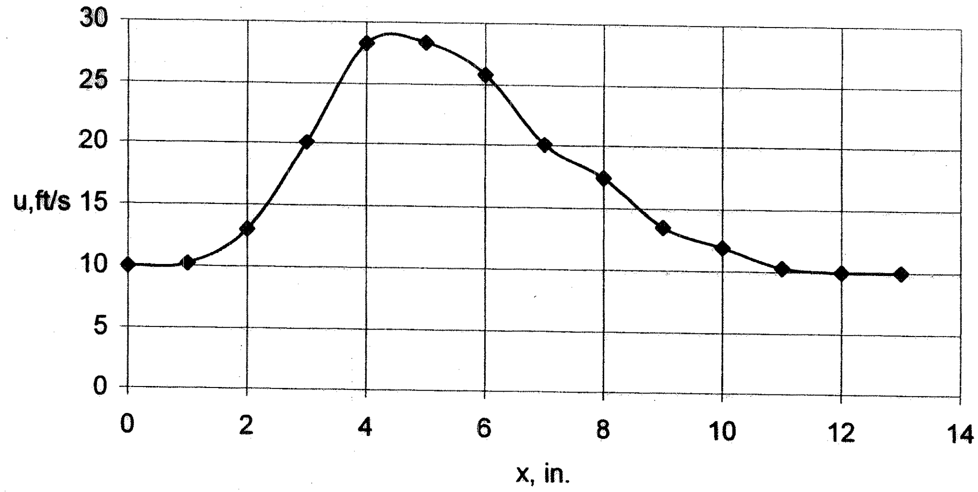
Note that since $a_x = u\frac{\partial u}{\partial x}$ it follows that and increase in velocity from u to Nu increases the acceleration from a_x to $N^2 a_x$

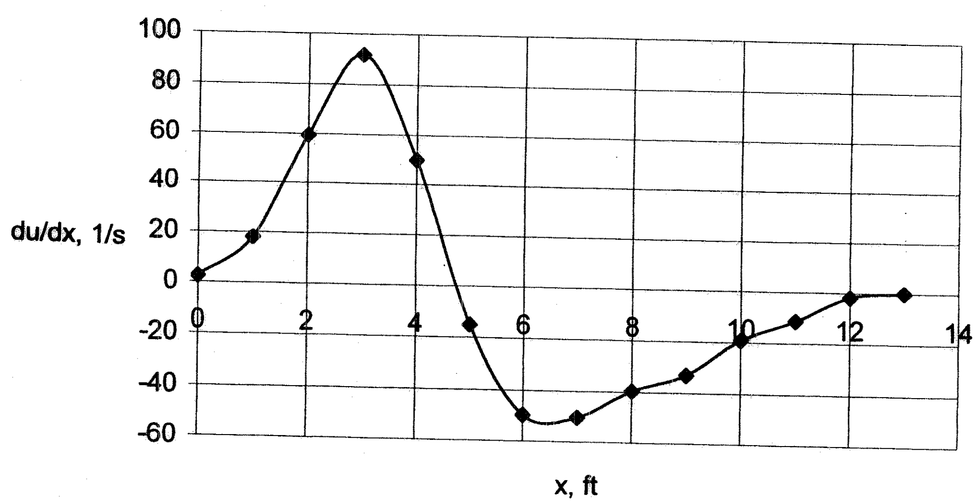
x, in.	u, ft/s	du/dx, 1/s	u du/dx
0	10	2.4	24
1	10.2	18	184
2	13	59.4	772
3	20.1	91.8	1845
4	28.3	49.8	1409
5	28.4	-15	-426
6	25.8	-49.8	-1285
7	20.1	-50.4	-1013
8	17.4	-39.6	-689
9	13.5	-33	-446
10	11.9	-19.2	-228
11	10.3	-11.4	-117
12	10	-1.8	-18
13	10	0	0

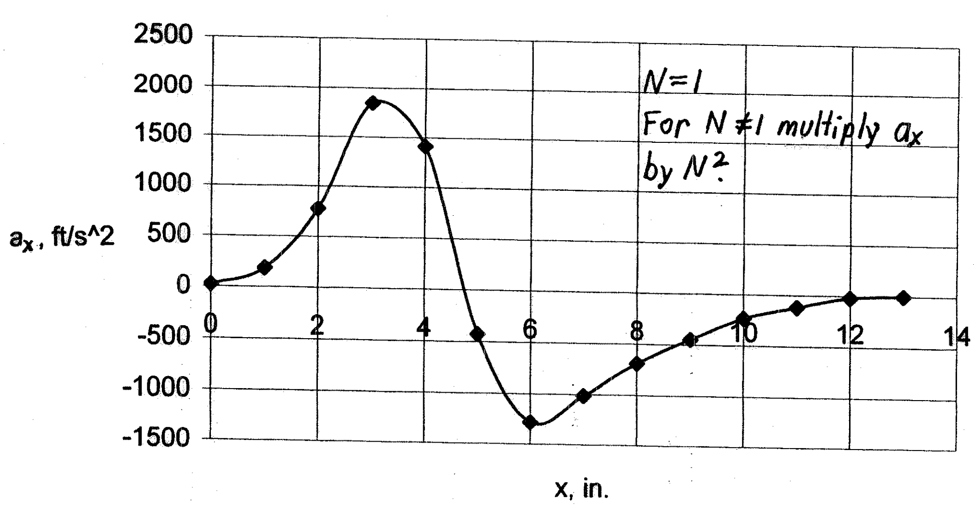
The results are plotted on the next page.

(con't)

***4.31** (con't)







4.32

4.32 Water flows steadily through a 30-ft-long pipe from a hot water heater to a faucet in the bathroom. The velocity is 10 ft/s. At the outlet of the water heater the temperature is a constant 180 °F. Because of heat transfer between the pipe and the cooler surroundings, the water temperature at the faucet outlet is a constant 150 °F. Determine the time rate of change of the temperature of the water as it flows through the pipe. Assume the temperature gradient along the length of the pipe is constant.

$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$, where $u = 10\frac{ft}{s}$ and $\frac{\partial T}{\partial x} = \frac{T_2 - T_1}{\Delta x} = \frac{150°F - 180°F}{30ft}$

Thus, $\frac{\partial T}{\partial x} = 1\ °F/ft$ so that with $\frac{\partial T}{\partial t} = 0$ because the flow is steady,

$\frac{DT}{Dt} = u\frac{\partial T}{\partial x} = 10\frac{ft}{s}(1\ °F/ft) = \underline{\underline{10\frac{°F}{s}}}$

4.33*

4.33* As is indicated in Fig. P4.33, the speed of exhaust in a car's exhaust pipe varies in time and distance because of the periodic nature of the engine's operation and the damping effect with distance from the engine. Assume that the speed is given by $V = V_0[1 + ae^{-bx}\sin(\omega t)]$, where $V_0 = 8$ fps, $a = 0.05$, $b = 0.2\ \text{ft}^{-1}$, and $\omega = 50$ rad/s. Calculate and plot the fluid acceleration at $x = 0, 1, 2, 3, 4$, and 5 ft for $0 \le t \le \pi/25$ s.

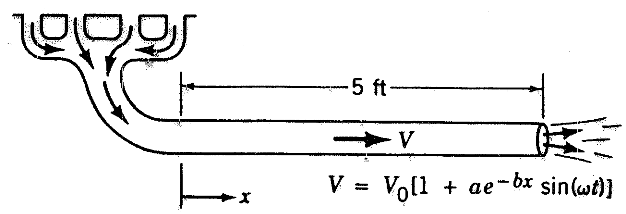


FIGURE P4.33

Since $u = u(x,t)$, $v = 0$, and $w = 0$ it follows that

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} = a_x\hat{i}\ , \quad \text{where} \quad a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} \qquad (1)$$

Thus, with $u = V_0[1 + a e^{-bx}\sin(\omega t)]$ Eq. (1) gives

$$a_x = V_0 a\omega e^{-bx}\cos(\omega t) + V_0[1 + a e^{-bx}\sin(\omega t)]V_0 a(-b)e^{-bx}\sin(\omega t)$$

$$= V_0 a e^{-bx}[\omega\cos(\omega t) - V_0 b\sin(\omega t)(1 + a e^{-bx}\sin(\omega t))]$$

With $V_0 = 8\,\frac{ft}{s}$, $a = 0.05$, $b = 0.2\,\frac{1}{ft}$, and $\omega = 50\,\frac{rad}{s}$ this becomes

$$a_x = 0.4\,e^{-0.2x}[50\cos(50t) - 1.6\sin(50t)(1 + 0.05\,e^{-0.2x}\sin(50t))]\ \frac{ft}{s^2} \qquad (2)$$

where $t \sim s$ and $x \sim ft$

Plot a_x from Eq. (2) for $0 \le t \le \frac{\pi}{25}$ s with $x = 0, 1, 2, 3, 4$, and 5 ft.

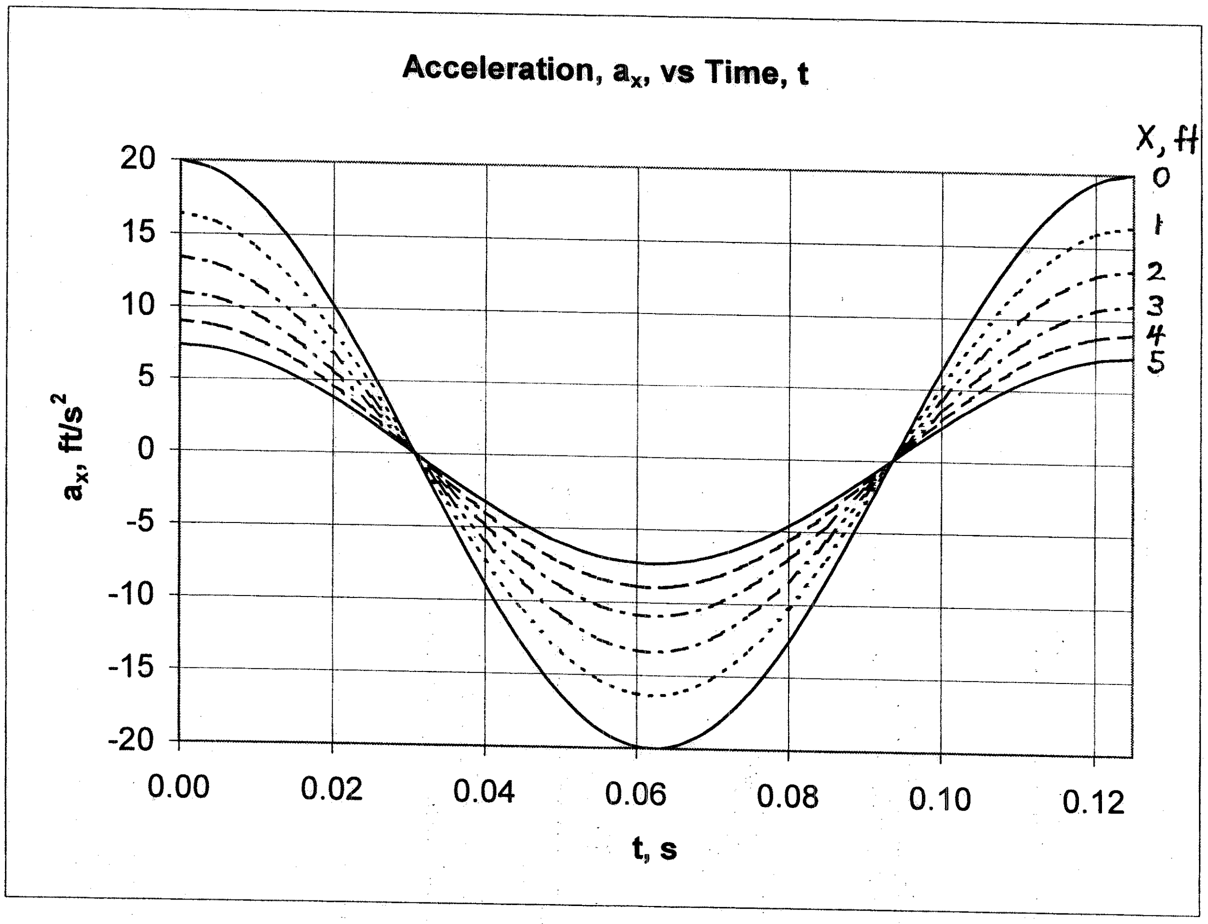
An Excel Program was used to calculate a_x from Eq. (2). The results are shown on the next page.

(con't)

4.33* (con't)

Acceleration at various x locations, ft/s^2

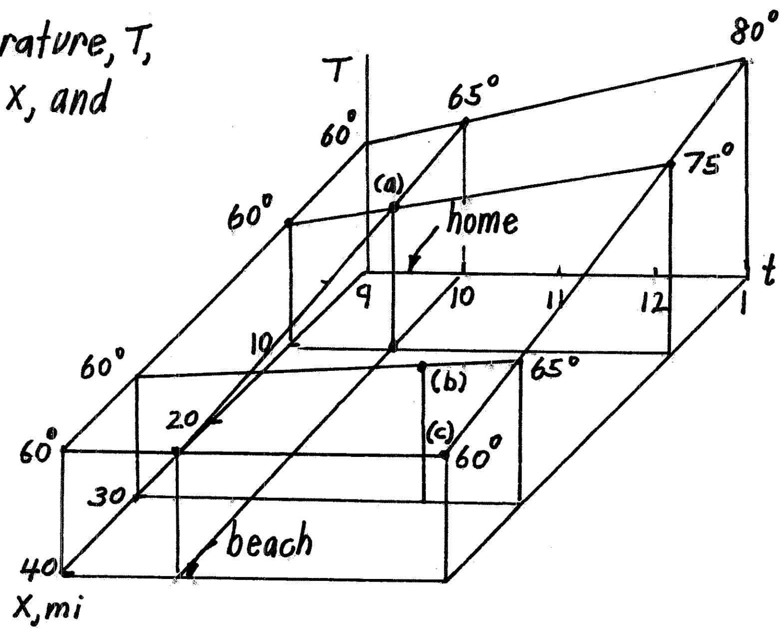
t, s	x = 0 ft	x = 1 ft	x = 2 ft	x= 3 ft	x = 4 ft	x = 5 ft
0.000	20.00	16.37	13.41	10.98	8.99	7.36
0.005	19.22	15.73	12.88	10.55	8.64	7.07
0.010	17.24	14.11	11.56	9.46	7.75	6.34
0.015	14.18	11.61	9.51	7.79	6.38	5.22
0.020	10.24	8.39	6.87	5.63	4.61	3.77
0.025	5.67	4.65	3.81	3.12	2.55	2.09
0.030	0.74	0.61	0.51	0.42	0.34	0.28
0.035	-4.23	-3.46	-2.83	-2.31	-1.89	-1.55
0.040	-8.93	-7.31	-5.98	-4.90	-4.01	-3.28
0.045	-13.08	-10.71	-8.76	-7.17	-5.87	-4.81
0.050	-16.42	-13.44	-11.00	-9.01	-7.37	-6.04
0.055	-18.73	-15.34	-12.56	-10.28	-8.42	-6.89
0.060	-19.89	-16.29	-13.33	-10.92	-8.94	-7.32
0.065	-19.81	-16.22	-13.28	-10.87	-8.90	-7.29
0.070	-18.51	-15.15	-12.41	-10.16	-8.32	-6.81
0.075	-16.06	-13.14	-10.76	-8.81	-7.21	-5.90
0.080	-12.61	-10.32	-8.45	-6.91	-5.66	-4.63
0.085	-8.37	-6.85	-5.61	-4.59	-3.76	-3.07
0.090	-3.62	-2.96	-2.42	-1.98	-1.62	-1.32
0.095	1.36	1.12	0.92	0.75	0.62	0.51
0.100	6.26	5.13	4.20	3.44	2.82	2.31
0.105	10.77	8.82	7.22	5.92	4.84	3.97
0.110	14.61	11.96	9.80	8.02	6.57	5.38
0.115	17.54	14.36	11.76	9.63	7.88	6.45
0.120	19.38	15.87	12.99	10.64	8.71	7.13
0.125	20.01	16.38	13.41	10.98	8.99	7.36



4.34

4.34 A bicyclist leaves from her home at 9 A.M. and rides to a beach 40 mi away. Because of a breeze off the ocean, the temperature at the beach remains 60 °F throughout the day. At the cyclist's home the temperature increases linearly with time, going from 60 °F at 9 A.M. to 80 °F by 1 P.M. The temperature is assumed to vary linearly as a function of position between the cyclist's home and the beach. Determine the rate of change of temperature observed by the cyclist for the following conditions: **(a)** as she pedals 10 mph through a town 10 mi from her home at 10 A.M.; **(b)** as she eats lunch at a rest stop 30 mi from her home at noon; **(c)** as she arrives enthusiastically at the beach at 1 P.M., pedaling 20 mph.

From the given data the temperature, T, varies as a function of location, x, and time, t, as shown in the figure.



Thus, $\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}$

(a) At $x = 10\ mi$ and $t = 10\ am$,

$$\frac{\partial T}{\partial t} = \frac{(75° - 60°)}{4\ hr} = \frac{15}{4}\ °/hr$$

and

$$\frac{\partial T}{\partial x} = \frac{(60° - 65°)}{40 mi} = -\frac{1}{8}\ °/mi$$

Thus, with $u = 10\ mi/hr$,

$$\frac{DT}{Dt} = \frac{15}{4}\ °/hr + 10\frac{mi}{hr}\left(-\frac{1}{8}\ °/mi\right)$$

$$= \underline{\underline{2.5\ °/hr}}$$

(b) At noon with $u = 0$ (resting) and $\frac{\partial T}{\partial t} = \frac{(65° - 60°)}{4\ hr} = \frac{5}{4}\ °/hr$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t} = \frac{5}{4}\ °/hr = \underline{\underline{1.25\ °/hr}}$$

(c) Upon arival at the beach with $u = 20\ mph$, $\frac{\partial T}{\partial t} = 0$, and $\frac{\partial T}{\partial x} = \frac{(60° - 80°)}{40 mi}$

$$= -0.5\ °/mi$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = u\frac{\partial T}{\partial x} = 20\frac{mi}{hr}(-0.5\ °/mi) = \underline{\underline{-10°/hr}}$$

4.35

4.35 The temperature distribution in a fluid is given by $T = 10x + 5y$, where x and y are the horizontal and vertical coordinates in meters and T is in degrees centigrade. Determine the time rate of change of temperature of a fluid particle traveling (a) horizontally with $u = 20$ m/s, $v = 0$ or (b) vertically with $u = 0$, $v = 20$ m/s.

$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$, where $\frac{\partial T}{\partial t} = 0$

Thus, if $u = 20\,\frac{m}{s}$ and $v = 0$, then $\frac{DT}{Dt} = u\frac{\partial T}{\partial x} = (20\,\frac{m}{s})(10\,\frac{°C}{m}) = \underline{\underline{200\,\frac{°C}{s}}}$

and

if $u = 0$ and $v = 20\,\frac{m}{s}$, then $\frac{DT}{Dt} = v\frac{\partial T}{\partial y} = (20\,\frac{m}{s})(5\,\frac{°C}{m}) = \underline{\underline{100\,\frac{°C}{s}}}$

4.36

4.36 Water flows over the crest of a dam with speed V as shown in Fig. P4.36. Determine the speed if the magnitude of the normal acceleration at point (1) is to equal the acceleration of gravity, g.

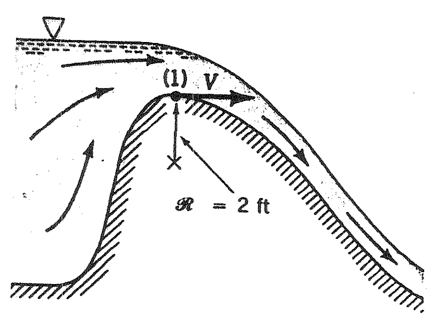
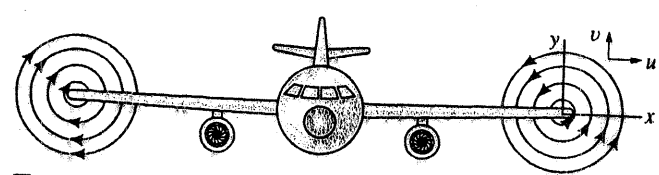


FIGURE P4.36

$a_n = \frac{V^2}{\mathcal{R}}$ or with $a_n = 32.2\,\frac{ft}{s^2}$, $V = \sqrt{a_n \mathcal{R}} = \sqrt{(32.2\,\frac{ft}{s^2})(2\,ft)}$
$= \underline{\underline{8.02\,\frac{ft}{s}}}$

4.37

4.37 As shown in **Video V4.2** and Fig. P4.37, a flying airplane produces swirling flow near the end of its wings. In certain circumstances this flow can be approximated by the velocity field $u = -Ky/(x^2 + y^2)$ and $v = Kx/(x^2 + y^2)$, where K is a constant depending on various parameter associated with the airplane (i.e., its weight, speed) and x and y are measured from the center of the swirl. **(a)** Show that for this flow the velocity is inversely proportional to the distance from the origin. That is, $V = K/(x^2 + y^2)^{1/2}$. **(b)** Show that the streamlines are circles.



■ FIGURE P4.37

(a) $$V = \sqrt{u^2 + v^2} = \left[\frac{(-Ky)^2}{(x^2+y^2)^2} + \frac{(Kx)^2}{(x^2+y^2)^2}\right]^{1/2} = \frac{K}{\sqrt{x^2+y^2}}$$

or

$$\underline{\underline{V = \frac{K}{r}}}, \text{ where } r = \sqrt{x^2+y^2}$$

(b) Streamlines are given by $$\frac{dy}{dx} = \frac{v}{u} = \frac{\dfrac{Kx}{(x^2+y^2)}}{\dfrac{-Ky}{(x^2+y^2)}} = -\frac{x}{y}$$

Thus,

$y\,dy = -x\,dx$ which when integrated gives

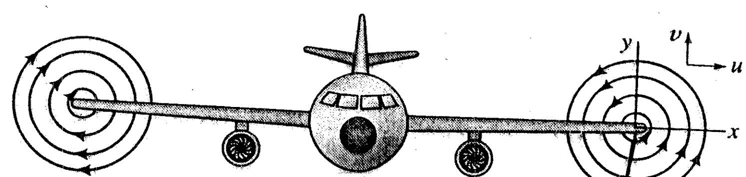
$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$, where C_1 is a constant.

or

$$\underline{\underline{x^2 + y^2 = \text{Constant}}}$$

4.38

4.38 Assume that the streamlines for the wingtip vortices from an airplane (see Fig. P4.37 and Video V4.2) can be approximated by circles of radius r and that the speed is $V = K/r$, where K is a constant. Determine the streamline acceleration, a_s, and the normal acceleration, a_n, for this flow.



■ FIGURE P4.37

$\mathcal{R} = r$

$a_s = V\frac{dV}{ds}$ where since $V = \frac{K}{r}$, $\frac{dV}{ds} = 0$

Thus,

$$a_s = \underline{\underline{0}}$$

Also,

$$a_n = \frac{V^2}{\mathcal{R}} = \frac{(K/r)^2}{r} = \underline{\underline{\frac{K}{r^3}}}$$

4.39

4.39 A fluid flows past a sphere with an upstream velocity of $V_0 = 40$ m/s as shown in Fig. P4.39. From a more advanced theory it is found that the speed of the fluid along the front part of the sphere is $V = \frac{3}{2}V_0 \sin\theta$. Determine the streamwise and normal components of acceleration at point A if the radius of the sphere is $a = 0.20$ m.

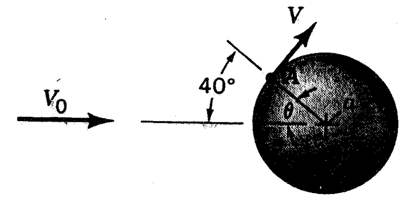


FIGURE P4.39

$$V = \tfrac{3}{2} V_0 \sin\theta = \tfrac{3}{2}\left(40\,\tfrac{m}{s}\right)\sin\theta = 60 \sin\theta \;\tfrac{m}{s} \qquad (1)$$

$$a_n = \frac{V^2}{R} = \frac{(60 \sin 40°)^2 \frac{m^2}{s^2}}{0.2m} = \underline{\underline{7440 \,\tfrac{m}{s^2}}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = (60 \sin\theta)\frac{\partial V}{\partial s}, \quad \text{where } \frac{\partial V}{\partial s} = \frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$$

From Eq. (1), $\frac{\partial V}{\partial \theta} = 60 \cos\theta$

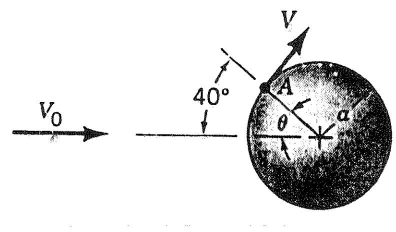
Also $s = a\theta = 0.2\,\theta$ m, where $\theta \sim$ rad, so that $\frac{\partial \theta}{\partial s} = \frac{1}{0.2m}$

Thus, for $\theta = 40°$

$$a_s = \left(60 \sin 40° \tfrac{m}{s}\right)\left(60 \cos 40° \tfrac{m}{s}\right)\left(\tfrac{1}{0.2m}\right) = \underline{\underline{8860 \,\tfrac{m}{s^2}}}$$

4.40*

4.40* For flow past a sphere as discussed in Problem 4.39, plot a graph of the streamwise acceleration, a_s, the normal acceleration, a_n, and the magnitude of the acceleration as a function of θ for $0 \le \theta \le 90°$ with $V_0 = 50$ ft/s and $a = 0.1$, 1.0, and 10 ft. Repeat for $V_0 = 5$ ft/s. At what point is the acceleration a maximum; a minimum?



$$a_n = \frac{V^2}{\mathcal{R}} = \frac{\left(\frac{3}{2}V_0 \sin\theta\right)^2}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin^2\theta}} \qquad (1)$$

and $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}$, where $\frac{\partial V}{\partial \theta} = \frac{3}{2}V_0\cos\theta$ and $s = a\theta$ or $\frac{\partial \theta}{\partial s} = \frac{1}{a}$

Thus,

$$a_s = \left(\frac{3}{2}V_0 \sin\theta\right)\left(\frac{3}{2}V_0\cos\theta\right)\frac{1}{a} = \underline{\underline{\frac{9V_0^2}{4a}\sin\theta\cos\theta}} \qquad (2)$$

Hence the magnitude of the acceleration is

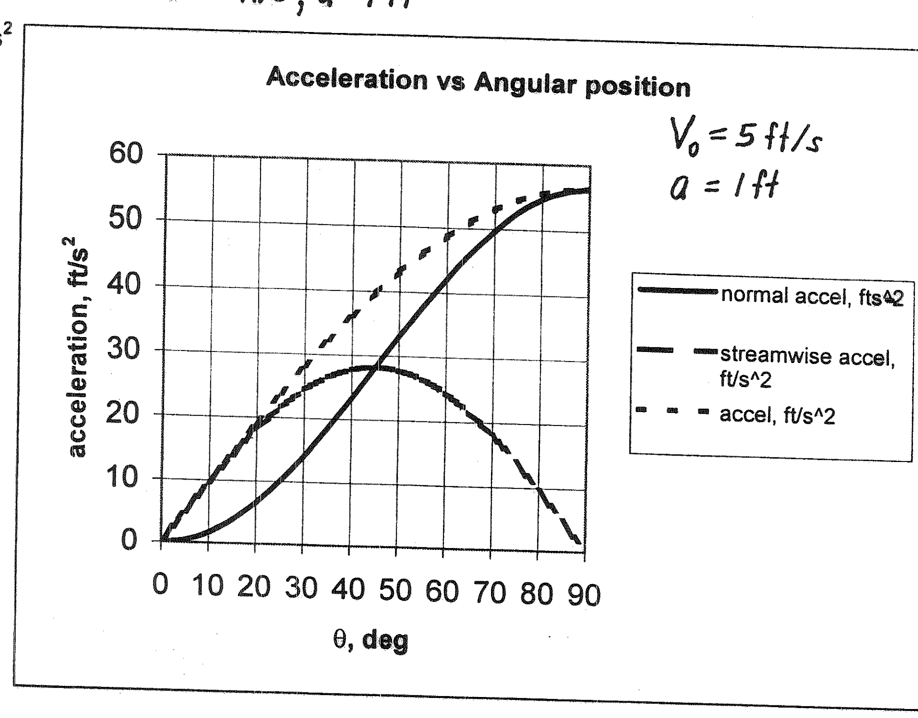
$$|\vec{a}| = \sqrt{a_n^2 + a_s^2} = \frac{9V_0^2}{4a}\sqrt{\sin^4\theta + \sin^2\theta\cos^2\theta} = \frac{9V_0^2}{4a}\sin\theta\sqrt{\sin^2\theta + \cos^2\theta}$$

or

$$(3) \qquad |\vec{a}| = \underline{\underline{\frac{9V_0^2}{4a}\sin\theta}} \qquad \text{Thus, } \underline{\underline{|\vec{a}|_{min} = 0 \text{ at } \theta = 0, \; |\vec{a}|_{max} = \frac{9V_0^2}{4a} \text{ at } \theta = 90°}}$$

An Excel Program was used to calculate a_s, a_n, and a from Eqns. (1), (2), and (3). The results are shown below. The results for other values are similar if the factor V_0^2/a is accounted for. The following data is for $V_0 = 5$ ft/s, $a = 1$ ft

θ, deg	a_n, ft/s^2	a_s, ft/s^2	a, ft/s^2
0	0.0	0.0	0.0
5	0.4	4.9	4.9
10	1.7	9.6	9.8
15	3.8	14.1	14.6
20	6.6	18.1	19.2
25	10.0	21.5	23.8
30	14.1	24.4	28.1
35	18.5	26.4	32.3
40	23.2	27.7	36.2
45	28.1	28.1	39.8
50	33.0	27.7	43.1
55	37.7	26.4	46.1
60	42.2	24.4	48.7
65	46.2	21.5	51.0
70	49.7	18.1	52.9
75	52.5	14.1	54.3
80	54.6	9.6	55.4
85	55.8	4.9	56.0
90	56.3	0.0	56.3



4.41

4.41 A fluid flows past a circular cylinder of radius a with an upstream speed of V_0 as shown in Fig. P4.41. A more advanced theory indicates that if viscous effects are negligible, the velocity of the fluid along the surface of the cylinder is given by $V = 2V_0 \sin\theta$. Determine the streamline and normal components of acceleration on the surface of the cylinder as a function of V_0, a, and θ.

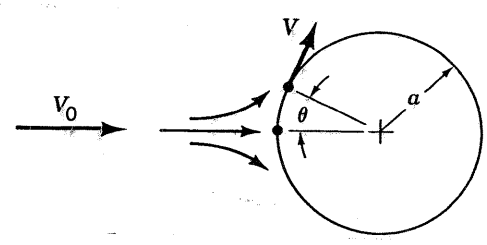


FIGURE P4.41

$$a_n = \frac{V^2}{R} = \frac{(2V_0 \sin\theta)^2}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin^2\theta}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}, \quad \text{where } \frac{\partial V}{\partial \theta} = 2V_0\cos\theta \text{ and } s = a\theta$$

$$\text{or } \frac{\partial \theta}{\partial s} = \frac{1}{a}$$

Thus,

$$a_s = (2V_0 \sin\theta)(2V_0\cos\theta)\frac{1}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin\theta\cos\theta}}$$

4.42*

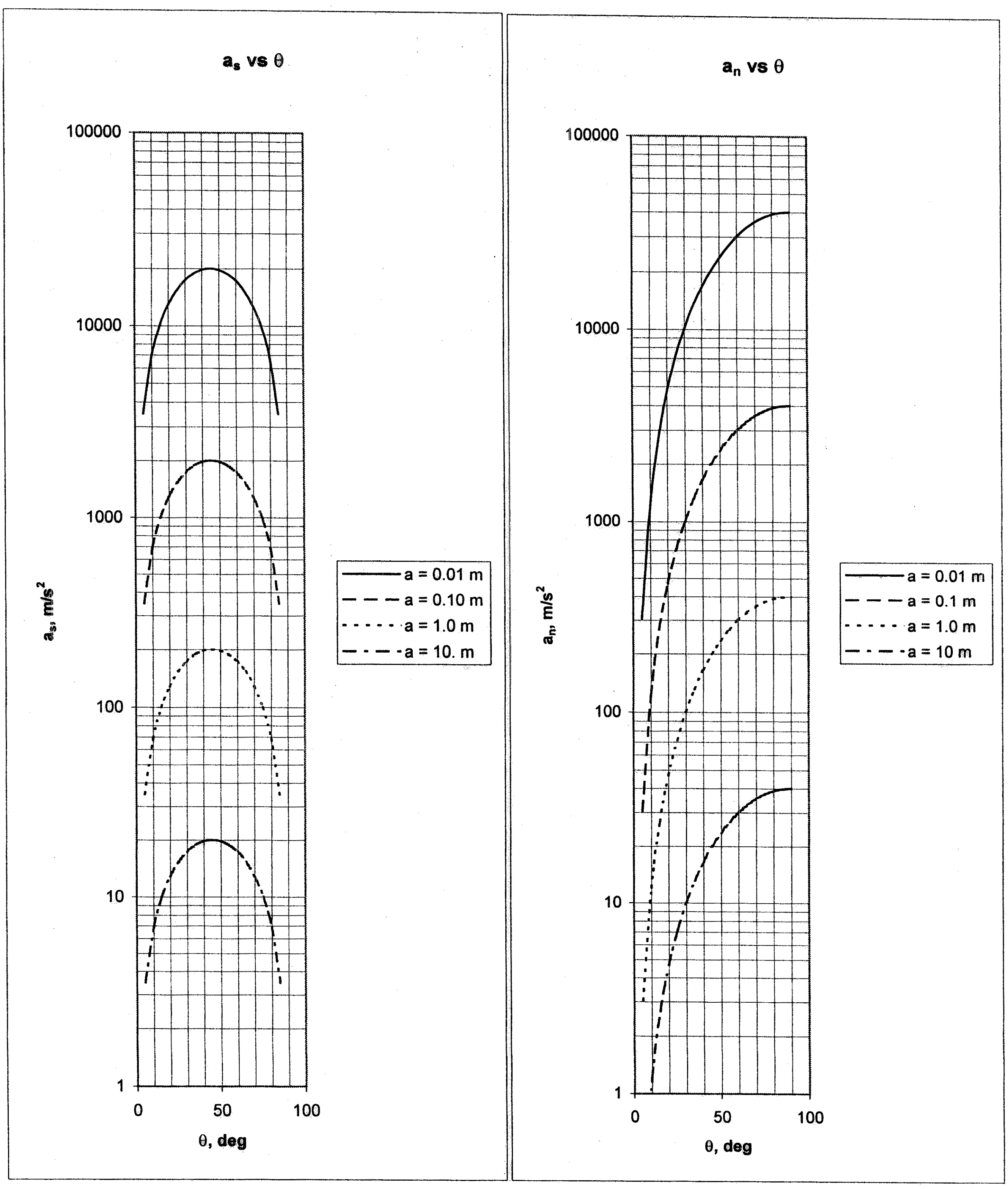
4.42* Use the results of Problem 4.41 to plot graphs of a_s and a_n for $0 \le \theta \le 90°$ with $V_0 = 10$ m/s and $a = 0.01$, 0.10, 1.0, and 10.0 m.

From Problem 4.41, $a_n = \frac{4V_0^2}{a}\sin^2\theta$ and $a_s = \frac{4V_0^2}{a}\sin\theta\cos\theta$. These results with $V_0 = 10\frac{m}{s}$ and $a = 0.01, 0.10, 1.0,$ and $10.0\,m$ are plotted below.

	a = 0.01 m	a = 0.10 m	a = 1.0 m	a = 10 m	a = 0.10 m	a = 0.10 m	a = 1.0 m	a = 10 m
θ, deg	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2
0	0	0	0	0.00	0	0	0	0.00
5	3473	347	35	3.47	304	30	3	0.30
10	6840	684	68	6.84	1206	121	12	1.21
15	10000	1000	100	10.00	2679	268	27	2.68
20	12856	1286	129	12.86	4679	468	47	4.68
25	15321	1532	153	15.32	7144	714	71	7.14
30	17321	1732	173	17.32	10000	1000	100	10.00
35	18794	1879	188	18.79	13160	1316	132	13.16
40	19696	1970	197	19.70	16527	1653	165	16.53
45	20000	2000	200	20.00	20000	2000	200	20.00
50	19696	1970	197	19.70	23473	2347	235	23.47
55	18794	1879	188	18.79	26840	2684	268	26.84
60	17321	1732	173	17.32	30000	3000	300	30.00
65	15321	1532	153	15.32	32856	3286	329	32.86
70	12856	1286	129	12.86	35321	3532	353	35.32
75	10000	1000	100	10.00	37321	3732	373	37.32
80	6840	684	68	6.84	38794	3879	388	38.79
85	3473	347	35	3.47	39696	3970	397	39.70
90	0	0	0	0.00	40000	4000	400	40.00

(con't)

4.42* (con't)



a_s vs θ
100000
10000
1000
100
10
1
0
50
100
θ, deg
a_s, m/s²
a = 0.01 m
a = 0.10 m
a = 1.0 m
a = 10. m
a_n vs θ
100000
10000
1000
100
10
1
0
50
100
θ, deg
a_n, m/s²
a = 0.01 m
a = 0.1 m
a = 1.0 m
a = 10 m

4.43

4.43 Determine the x and y components of acceleration for the flow given in Problem 4.6. If $c > 0$, is the particle at point $x = x_0 > 0$ and $y = 0$ accelerating or decelerating? Explain. Repeat if $x_0 < 0$.

Since $u = c(x^2 - y^2)$ and $v = -2cxy$ it follows that

$\vec{a} = a_x \hat{i} + a_y \hat{j}$, where

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = c(x^2-y^2)(2cx) + (-2cxy)(-2cy)$$

or

$$\underline{\underline{a_x = 2c^2 x(x^2+y^2)}}$$

and

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = c(x^2-y^2)(-2cy) + (-2cxy)(-2cx)$$

or

$$\underline{\underline{a_y = 2c^2 y(x^2+y^2)}}$$

For $x = x_0$ and $y = 0$ we obtain:

$u = cx_0^2$, $v = 0$

and

$a_x = 2c^2 x_0^3$, $a_y = 0$

Thus, with $c > 0$ and $x_0 > 0$ it follows that $u > 0$, $a_x > 0$; i.e., the fluid is <u>accelerating.</u>

With $c > 0$ and $x_0 < 0$ it follows that $u > 0$, $a_x < 0$; i.e., the fluid is <u>decelerating.</u>

4.44

4.44 When flood gates in a channel are opened, water flows along the channel downstream of the gates with an increasing speed given by $V = 4(1 + 0.1t)$ ft/s, for $0 \le t \le 20$ s, where t is in seconds. For $t > 20$ s the speed is a constant $V = 12$ ft/s. Consider a location in the curved channel where the radius of curvature of the streamlines is 50 ft. For $t = 10$ s determine **(a)** the component of acceleration along the streamline, **(b)** the component of acceleration normal to the streamline, and **(c)** the net acceleration (magnitude and direction). Repeat for $t = 30$ s.

$V = 4(1+0.1t)$ ft/s for $0 \le t \le 20s$ and $V = 12$ ft/s for $t > 20s$

$a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$ where $\frac{\partial V}{\partial s} = 0$

Thus,

$a_s = \frac{\partial V}{\partial t}$ and $a_n = \frac{V^2}{R}$, where $R = 50ft$

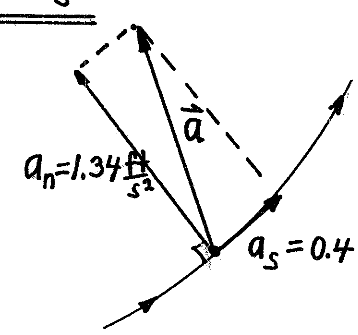
(1) For $t = 10s$:

(a) $a_s = \frac{\partial V}{\partial t} = 4(0.1) = \underline{\underline{0.4\frac{ft}{s^2}}}$

(b) $a_n = V^2/R = [4(1+0.1(10))]^2 ft^2/s^2/(50ft) = \underline{\underline{1.28\,ft/s^2}}$

and

(c) $a = (a_n^2 + a_s^2)^{1/2} = [(0.4\frac{ft}{s^2})^2 + (1.28\frac{ft}{s^2})^2]^{1/2} = \underline{\underline{1.34\frac{ft}{s^2}}}$



(2) For $t = 30s$:

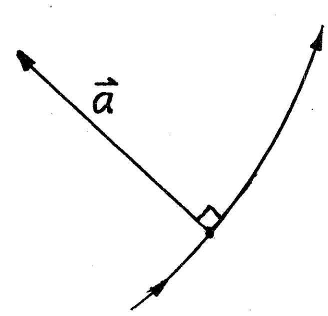
(a) Since $V = 12$ ft/s = constant, $\frac{\partial V}{\partial t} = 0$ and $\frac{\partial V}{\partial s} = 0$ so that

$a_s = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = \underline{\underline{0}}$

(b) $a_n = V^2/R = (12\,ft/s)^2/(50ft) = \underline{\underline{2.88\frac{ft}{s^2}}}$

and

(c) $a = (a_n^2 + a_s^2)^{1/2} = a_n = \underline{\underline{2.88\frac{ft}{s^2}}}$



4.45

4.45 Water flows steadily through the funnel shown in Fig. P4.45. Throughout most of the funnel the flow is approximately radial (along rays from O) with a velocity of $V = c/r^2$, where r is the radial coordinate and c is a constant. If the velocity is 0.4 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

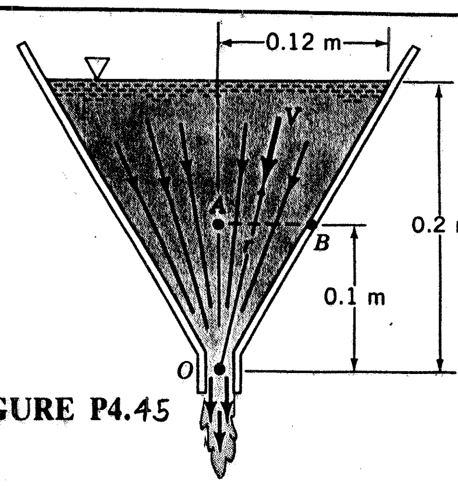


FIGURE P4.45

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r^2}$

Since $V = 0.4\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = Vr^2 = (0.4\frac{m}{s})(0.1m)^2 = 4\times10^{-3}\frac{m^3}{s}$, or $V = \frac{4\times10^{-3}}{r^2}\frac{m}{s}$, where $r \sim m$

Thus,

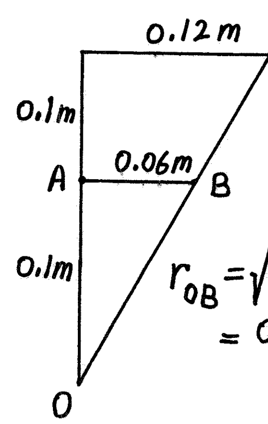
$$a_s = -\left(\frac{c}{r^2}\right)\left(-\frac{2c}{r^3}\right) = \frac{2c^2}{r^5}$$

At point A:

$$a_s = \frac{2(4\times10^{-3}\frac{m^3}{s})^2}{(0.1m)^5} = \underline{\underline{3.20\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{2(4\times10^{-3}\frac{m^3}{s})^2}{(0.1167m)^5} = \underline{\underline{1.48\frac{m}{s^2}}}$$



$r_{OB} = \sqrt{(0.1)^2 + (0.06)^2} = 0.1167m$

4.46

4.46 Water flows through the slit at the bottom of a two-dimensional water trough as shown in Fig. P4.46. Throughout most of the trough the flow is approximately radial (along rays from O) with a velocity of $V = c/r$, where r is the radial coordinate and c is a constant. If the velocity is 0.04 m/s when $r = 0.1$ m, determine the acceleration at points A and B.

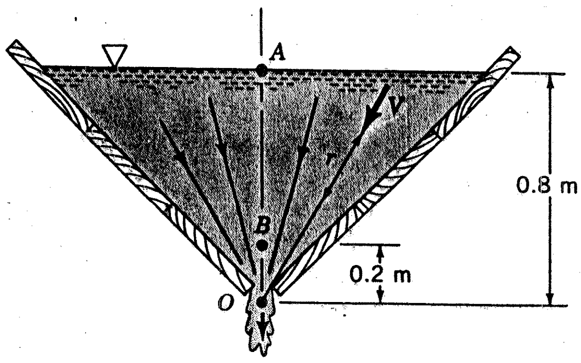


FIGURE P4.46

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$, where $V = \frac{c}{r}$

Since $V = 0.04\frac{m}{s}$ when $r = 0.1m$ it follows that

$c = Vr = (0.04\frac{m}{s})(0.1m) = 4\times10^{-3}\frac{m^2}{s}$, or $V = \frac{4\times10^{-3}}{r}\frac{m}{s}$, where $r \sim m$

Thus,

$$a_s = -\left(\frac{c}{r}\right)\left(-\frac{c}{r^2}\right) = \frac{c^2}{r^3}$$

At point A:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.8m)^3} = \underline{\underline{3.13\times10^{-5}\frac{m}{s^2}}}$$

At point B:

$$a_s = \frac{(4\times10^{-3}\frac{m^2}{s})^2}{(0.2m)^3} = \underline{\underline{2.00\times10^{-3}\frac{m}{s^2}}}$$

4.47

4.47 Air flows from a pipe into the region between two parallel circular disks as shown in Fig. P4.47. The fluid velocity in the gap between the disks is closely approximated by $V = V_0 R/r$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for $r = 1$, 2, or 3 ft if $V_0 = 5$ ft/s and $R = 3$ ft.

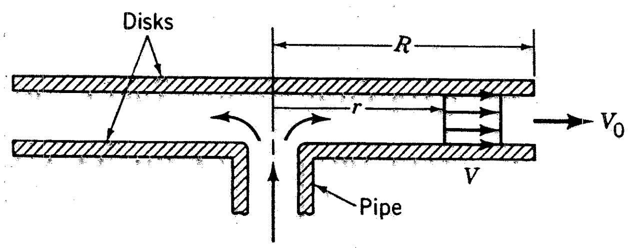


FIGURE P4.47

$\vec{a} = a_n \hat{n} + a_s \hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e., the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial r}$, where $V = \frac{V_0 R}{r}$

Since $V_0 = 5\,\frac{ft}{s}$ and $R = 3\,ft$, $V = \frac{15}{r}\,\frac{ft}{s}$, where $r \sim ft$

Thus,

$$a_s = \left(\frac{V_0 R}{r}\right)\left(-\frac{V_0 R}{r^2}\right) = -\frac{V_0^2 R^2}{r^3} = -\frac{(5\,\frac{ft}{s})^2 (3\,ft)^2}{r^3\,ft^3} = -\frac{225}{r^3}\,\frac{ft}{s^2},\quad r \sim ft$$

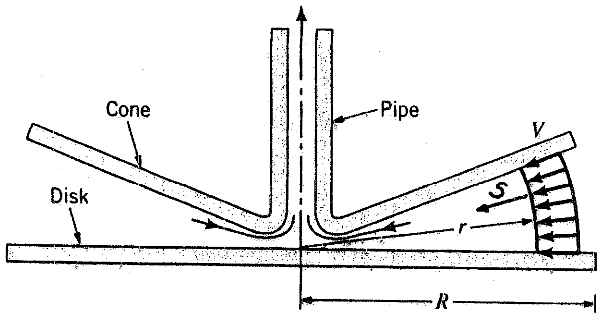
At $r = 1\,ft$, $a_s = \underline{\underline{-225\,\frac{ft}{s^2}}}$

At $r = 2\,ft$, $a_s = \underline{\underline{-28.1\,\frac{ft}{s^2}}}$

At $r = 3\,ft$, $a_s = \underline{\underline{-8.33\,\frac{ft}{s^2}}}$

4.48

4.48 Air flows into a pipe from the region between a circular disk and a cone as shown in Fig. P4.48. The fluid velocity in the gap between the disk and the cone is closely approximated by $V = V_0R^2/r^2$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for $r = 0.5$ and 2 ft if $V_0 = 5$ ft/s and $R = 2$ ft.



■ FIGURE P4.48

$\vec{a} = a_n\hat{n} + a_s\hat{s}$, where $a_n = \frac{V^2}{\mathcal{R}} = 0$ since $\mathcal{R} = \infty$ (i.e. the streamlines are straight)

Also, $a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r}$ since r and s are pointed in opposite directions.

Thus, with $V = V_0R^2/r^2$ it follows that

$a_s = -(V_0R^2/r^2)(-2V_0R^2/r^3) = 2V_0^2R^4/r^5$

$= 2(5\,ft/s)^2(2\,ft)^4/r^5 = 800/r^5\ \frac{ft}{s^2}$, where $r \sim ft$

At $r = 0.5$ ft, $a_s = 800/(0.5)^5\ \frac{ft}{s^2} = \underline{\underline{25{,}600\ \frac{ft}{s^2}}}$

At $r = 2$ ft, $a_s = 800/(2.0)^5\ \frac{ft}{s^2} = \underline{\underline{25\ \frac{ft}{s^2}}}$

4.49

4.49 Water flows through a duct of square cross section as shown in Fig. P4.49 with a constant, uniform velocity of $V = 20$ m/s. Consider fluid particles that lie along line A–B at time $t = 0$. Determine the position of these particles, denoted by line A'–B', when $t = 0.20$ s. Use the volume of fluid in the region between lines A–B and A'–B' to determine the flowrate in the duct. Repeat the problem for fluid particles originally along line C–D; along line E–F. Compare your three answers.

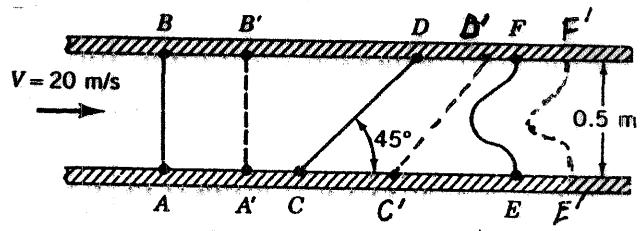


FIGURE P4.49

Since V is constant in time and space, all particles on line AB move a distance $\ell = V\Delta t = (20\,\frac{m}{s})(0.2s) = 4m$ from $t=0$ to $t=0.2s$

Thus, the volume of $ABA'B'$ is $\forall_{ABA'B'} = (0.5m)^2(4m) = 1.00\ m^3$

so that

$$Q = \frac{\forall_{ABA'B'}}{\Delta t} = \frac{1.00\ m^3}{0.2s} = \underline{\underline{5.0\ \frac{m^3}{s}}}$$

Similarly from $t=0$ to $t=0.2s$ the fluid along lines CD and EF move to $C'D'$ and $E'F'$, respectively. Also, $\forall_{CDC'D'} = \forall_{EFE'F'} = \forall_{ABA'B'}$ so that we obtain $Q = \frac{\forall}{\Delta t} = 5.0\frac{m^3}{s}$ regardless which line we consider.

4.50

4.50 Repeat Problem 4.49 if the velocity profile is linear from 0 to 20 m/s across the duct as shown in Fig. P4.50.

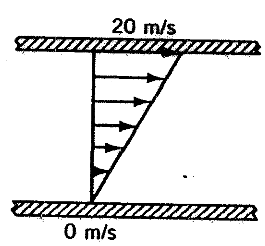


FIGURE P4.50

From $t=0$ to $t=0.1s$ the particle initially at B travels a distance $\ell_B = V_B\Delta t = (20\,\frac{m}{s})(0.1s) = 2m$ as show. Particle A remain fixed since $V_A = 0$. Since the velocity profile is linear, line AB remains straight, but "tilts" as indicated. Thus, the volume of fluid crossing the initial line AB is $\forall_{ABB'} = \frac{1}{2}\ell_B A = \frac{1}{2}(2m)(0.5m)^2 = 0.25m^3$ so that

$$Q = \frac{\forall_{ABB'}}{\Delta t} = \frac{0.25\,m^3}{0.1s} = \underline{\underline{2.5\,\frac{m^3}{s}}}$$

Since $\forall_{CDD'} = \forall_{EFF'} = V_{ABB'}$ it follows that the same value of Q is obtained regardless which volume is used.

4.51

4.51 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.51 and Video V10.5. The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

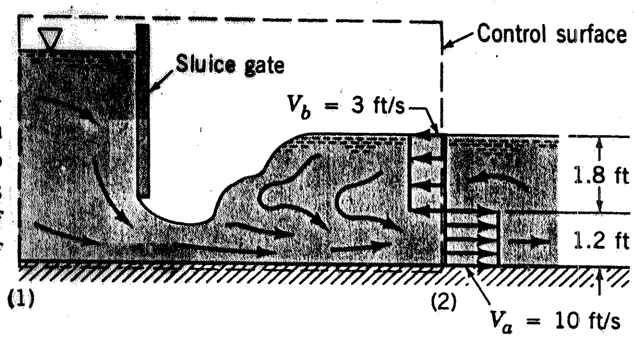


FIGURE P4.51

$$Q = V_a A_a - V_b A_b = (10\ \tfrac{ft}{s})(1.2\,ft)(20\,ft) - (3\ \tfrac{ft}{s})(1.8\,ft)(20\,ft)$$
$$= \underline{\underline{132\ \tfrac{ft^3}{s}}}$$

4.52

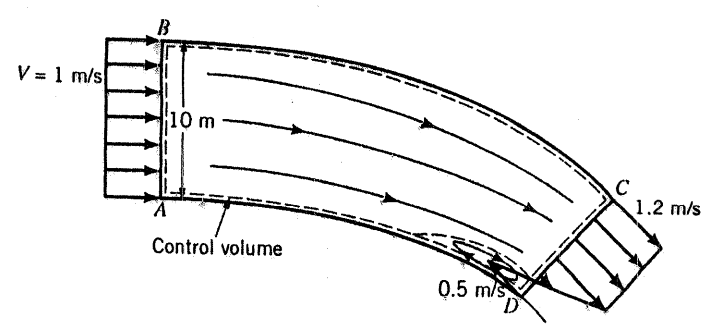
4.52 At time $t = 0$ the valve on an initially empty (perfect vacuum, $\rho = 0$) tank is opened and air rushes in. If the tank has a volume of $\forall_0$ and the density of air within the tank increases as $\rho = \rho_\infty(1 - e^{-bt})$, where b is a constant, determine the time rate of change of mass within the tank.

For $t \geq 0$, $\rho = \rho_0[1 - e^{-bt}]$ so that M = mass of air in tank $= \rho\forall_0 = \rho_0\forall_0[1 - e^{-bt}]$

Thus, $\frac{dM}{dt} = \underline{\underline{\rho_0 \forall_0 b\, e^{-bt}}}$

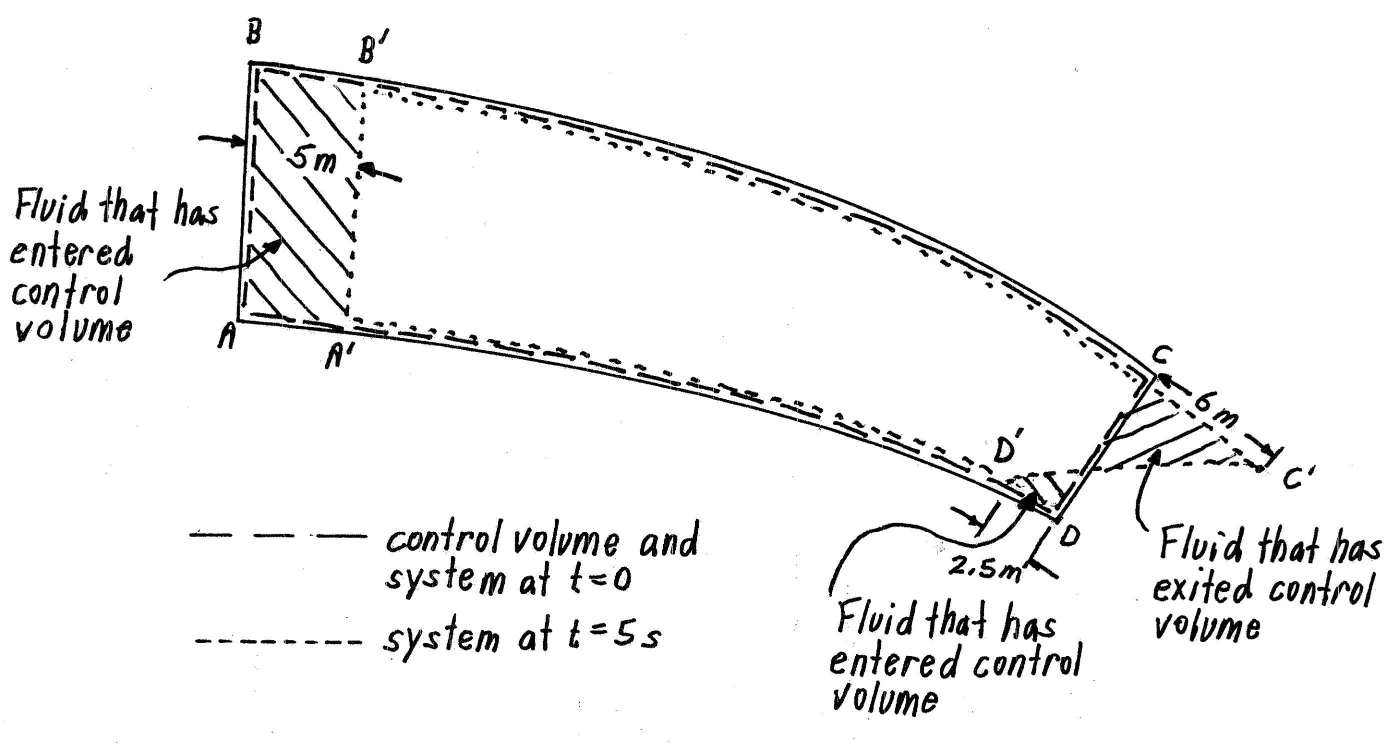
4.54

4.54 Water enters the bend of a river with the uniform velocity profile shown in Fig. P4.54. At the end of the bend there is a region of separation or reverse flow. The fixed control volume $ABCD$ coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 5$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.54

Since the distance the fluid travels in time $\delta t = 5s$ is $\ell = V\delta t$, the fluid at A-B when $t=0$ has traveled $\ell = (1 m/s)(5s) = 5m$ when $t = \delta t = 5s$. This is shown in the figure below. Similarly, the fluid across C-D at $t=0$ has moved as indicated when $t = \delta t = 5s$. Thus, the boundary of the system at $t = 5s$ are as show in the figure below. The fluid that entered and exited the control volume in that time period is also shown.



4.55

4.55 A layer of oil flows down a vertical plate as shown in Fig. P4.55 with a velocity of $\mathbf{V} = (V_0/h^2)(2hx - x^2)\hat{\mathbf{j}}$ where V_0 and h are constants. **(a)** Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. **(b)** Determine the flowrate across surface AB. Assume the width of the plate is b. (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.6.)

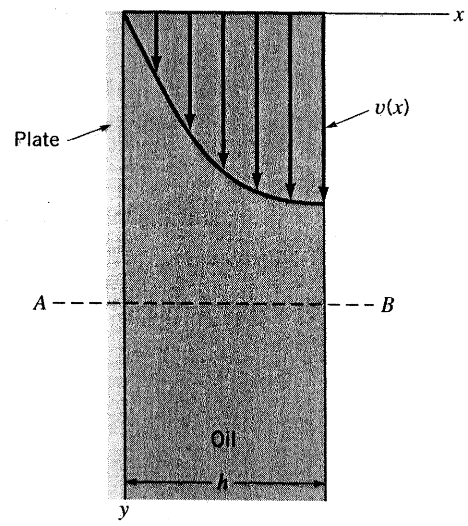


FIGURE P4.55

a) $v = \frac{V_0}{h^2}(2hx - x^2)$

Thus,

$$v\Big|_{x=0} = \frac{V_0}{h^2}(0-0) = 0 \quad \text{and}$$

$$\tau\Big|_{x=h} = \mu \frac{dv}{dx}\Big|_{x=h} = \mu \frac{V_0}{h^2}\left[2h - 2x\right]_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

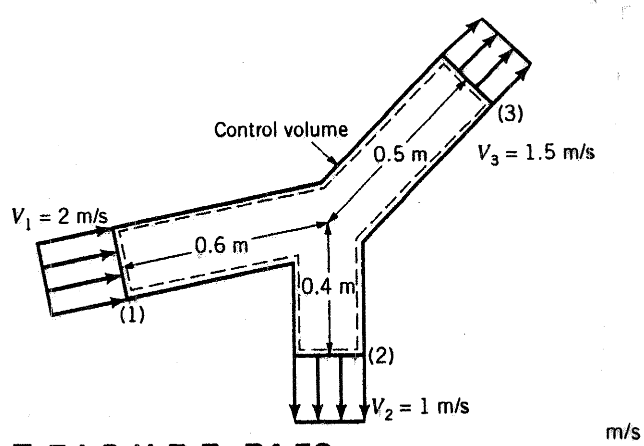
b) $$Q_{AB} = \int v\, dA = \int_{x=0}^{x=h} v\, b\, dx = \int_0^h \frac{V_0}{h^2}(2hx - x^2)\, b\, dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2}\left[hx^2 - \frac{1}{3}x^3\right]_0^h = \underline{\underline{\frac{2}{3}V_0 h b}}$$

4.56

4.56 Water flows in the branching pipe shown in Fig. P4.56 with uniform velocity at each inlet and outlet. The fixed control volume indicated coincides with the system at time $t = 20$ s. Make a sketch to indicate **(a)** the boundary of the system at time $t = 20.2$ s, **(b)** the fluid that left the control volume during that 0.2-s interval, and **(c)** the fluid that entered the control volume during that time interval.



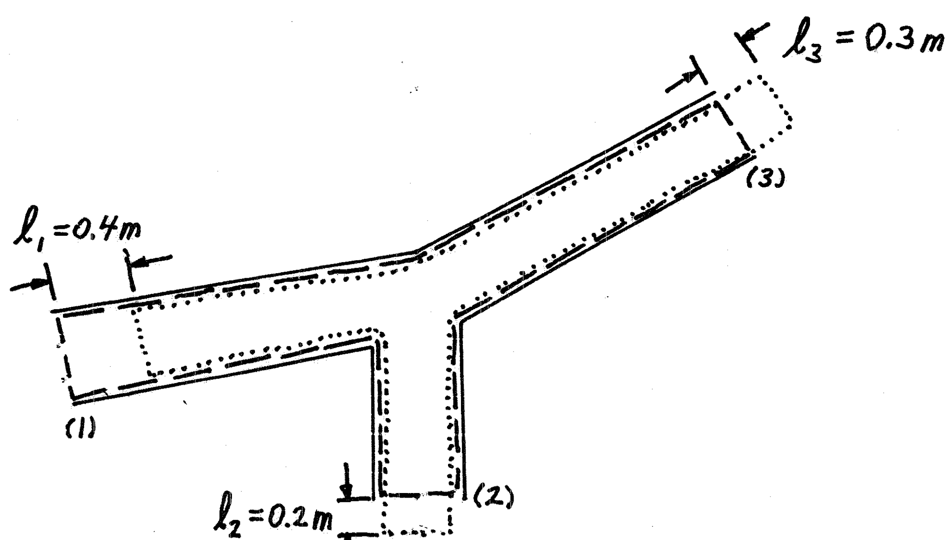
■ FIGURE P4.56

Since V is constant, the fluid travels a distance $\ell = V\delta t$ in time Δt. Thus, $\ell_1 = V_1\,\delta t = (2\,\frac{m}{s})(20. - 20)s = 0.4\,m$

$\ell_2 = V_2\,\delta t = (1\,\frac{m}{s})(20. - 20)s = 0.2\,m$

and $\ell_3 = V_3\,\delta t = (1.5\,\frac{m}{s})(20. - 20)s = 0.3\,m$

The system at $t = 20.2s$ and the fluid that has entered or exited the control volume are indicated in the figure below.

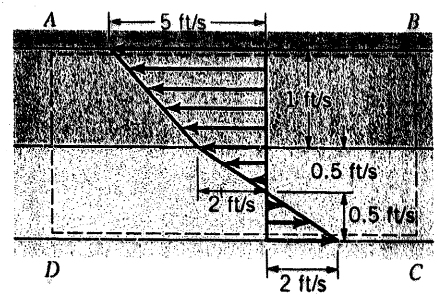


— — — control volume and system at $t = 20s$

.... system at $t = 20.2s$

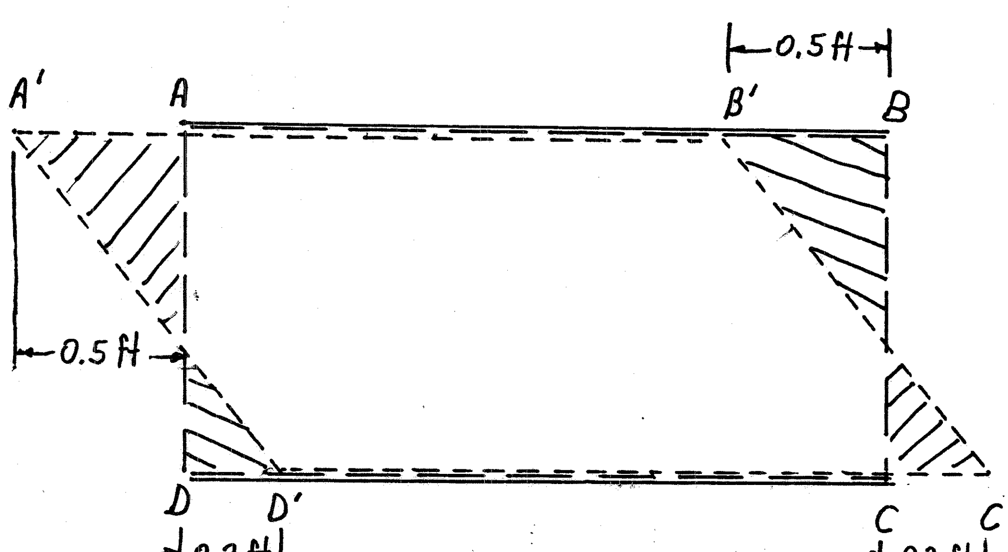
4.57

4.57 Two liquids with different densities and viscosities fill the gap between parallel plates as shown in Fig. P4.57. The top plate moves to the left with a speed of 5 ft/s; the bottom plate moves to the right with a speed of 2 ft/s. The velocity profile consists of two linear segments as indicated. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 0.1$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.57

From $t=0$ to $t=0.1\,s$ the bottom plate (and the liquid that sticks to it) moves $\ell = V\Delta t = (2\,ft/s)(0.1\,s) = 0.2\,ft$ to the right. Similarly, the top plate moves to the left a distance $\ell = V\Delta t = (5\,ft/s)(0.1\,s) = 0.5\,ft$. The fluid along lines A-D and B-C also move distances given by $\ell = V\Delta t$. For example, the interface between the 2 liquids moves a distance $\ell = V\Delta t = (2\,ft/s)(0.1\,s) = 0.2\,ft$. The fluid layer 0.5 ft above the bottom plate has $V=0$ and, therefore, does not move. The corresponding displacement of the fluid originally along A-D and B-C is shown in the figure below.



— — — control volume and system at $t=0$

- - - - - - - system at $t = 0.1\,s$

//// fluid that exited control volume

\\\\ fluid that entered control volume

4.58

4.58 Water is squirted from a syringe with a speed of V = 5 m/s by pushing in the plunger with a speed of V_p = 0.03 m/s as shown in Fig. P4.58. The surface of the deforming control volume consists of the sides and end of the cylinder and the end of the plunger. The system consists of the water in the syringe at t = 0 when the plunger is at section (1) as shown. Make a sketch to indicate the control surface and the system when t = 0.5 s.

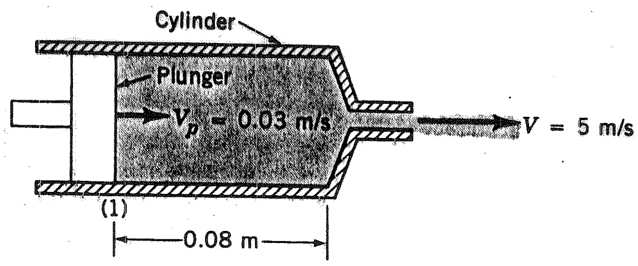
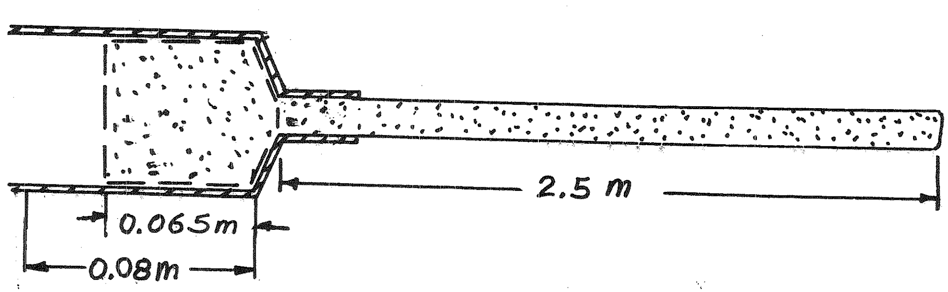


FIGURE P4.58

During the t = 0.5s time interval the plunger moves $\ell_1 = V_p\,\delta t = 0.015\,m$ and the water initially at the exit moves $\ell_2 = V\delta t = 2.5\,m$. The corresponding control surfaces and systems at $t=0$ and $t=0.5s$ shown in the figure below.

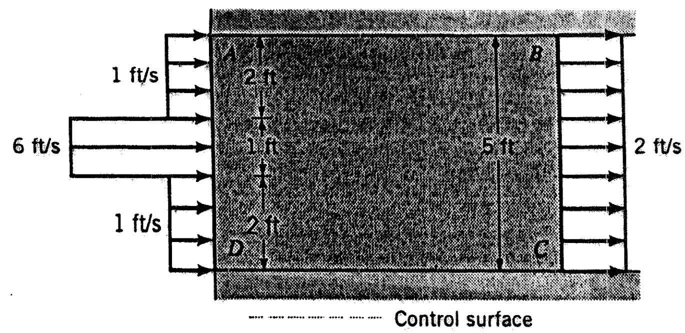


— — — control volume at t=0.5s

system at t=0.5s

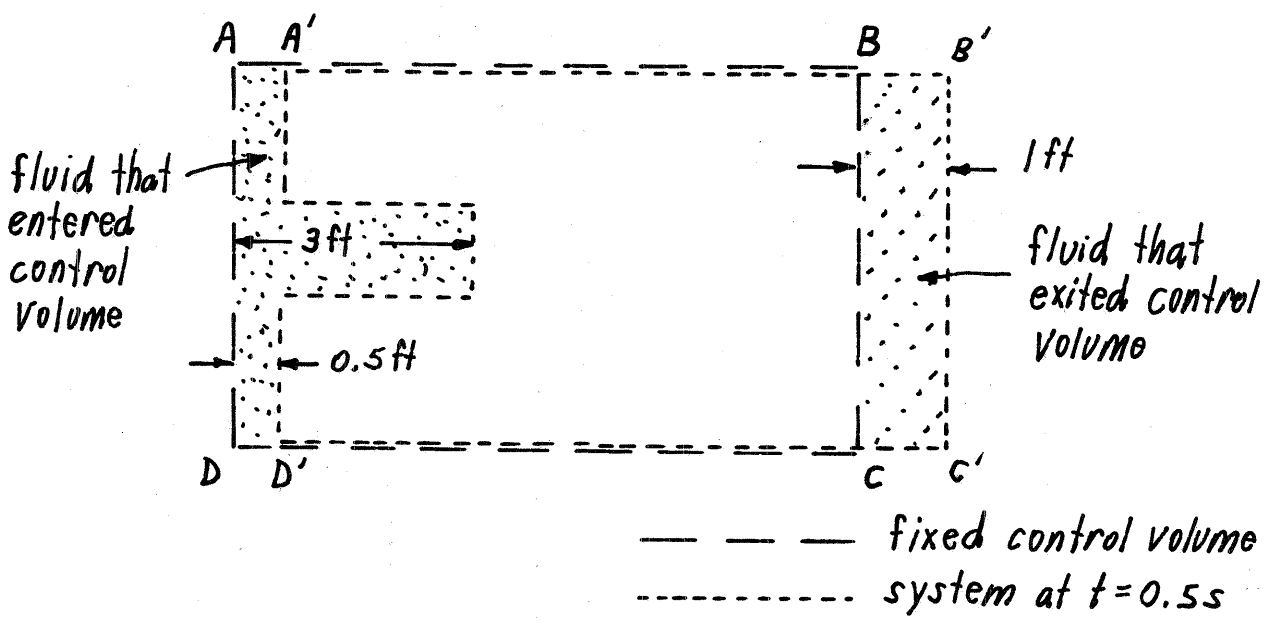
4.59

4.59 Water enters a 5-ft-wide, 1-ft-deep channel as shown in Fig. P4.59. Across the inlet the water velocity is 6 ft/s in the center portion of the channel and 1 ft/s in the remainder of it. Farther downstream the water flows at a uniform 2 ft/s velocity across the entire channel. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 0.5$ s and (b) the fluid that has entered and exited the control volume in that time period.



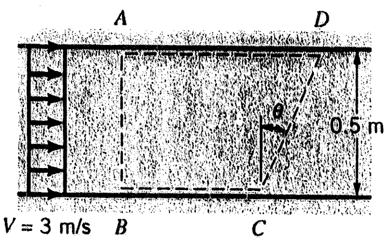
■ FIGURE P4.59

During the $t = 0.5$ s time interval the fluid that was along line BC at time $t=0$ has moved to the right a distance $\ell = V\,t = 2\,\frac{ft}{s}\,(0.5s) = 1\,ft$. Similarly, portions of the fluid along line AD have moved $\ell = 1\,\frac{ft}{s}\,(0.5s) = 0.5\,ft$ and $\ell = 6\,\frac{ft}{s}\,(0.5s) = 3\,ft$. This assumes the $1\,\frac{ft}{s}$ and $6\,\frac{ft}{s}$ fluid streams do not mix or intermingle during the 0.5 s time interval. See figure below.



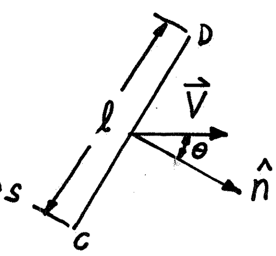
4.60

4.60 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.60 with a uniform velocity of 3 m/s. (a) Directly integrate **Eq. 4.16** with $b = 1$ to determine the mass flowrate (kg/s) across section CD of the control volume. (b) Repeat part (a) with $b = 1/\rho$, where ρ is the density. Explain the physical interpretation of the answer to part (b).



■ FIGURE P4.60

a) $\dot{B}_{out} = \int_{CS_{out}} \rho b \, \vec{V}\cdot\hat{n} \, dA$ (1)



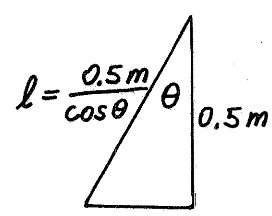
With $b = 1$ and $\vec{V}\cdot\hat{n} = V\cos\theta$ this becomes

$\dot{B}_{out} = \int_{CD} \rho V \cos\theta \, dA = \rho V\cos\theta \int_{CD} dA$

$= \rho V \cos\theta \, A_{CD}$, where $A_{CD} = \ell \, (2m)$

$= \left(\frac{0.5\,m}{\cos\theta}\right)(2m)$

$= \left(\frac{1}{\cos\theta}\right) m^2$



Thus, with $V = 3\,m/s$,

$\dot{B}_{out} = (3\,\frac{m}{s})\cos\theta \left(\frac{1}{\cos\theta}\right) m^2 (999\,\frac{kg}{m^3}) = \underline{\underline{3000\,\frac{kg}{s}}}$

b) With $b = 1/\rho$ Eq. (1) becomes

$\dot{B}_{out} = \int_{CD} \vec{V}\cdot\hat{n} \, dA = \int_{CD} V\cos\theta \, dA = V\cos\theta \, A_{CD}$

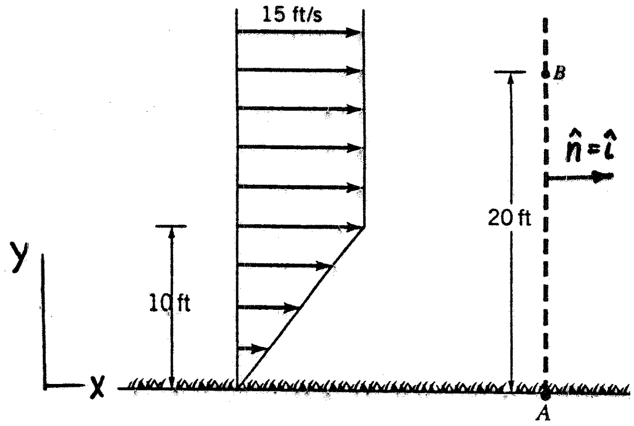
$= (3\,\frac{m}{s})\cos\theta\left(\frac{1}{\cos\theta}\right) m^2 = \underline{\underline{3.00\,\frac{m^3}{s}}}$

With $b = 1/\rho = \frac{1}{\left(\frac{mass}{Vol}\right)} = \frac{vol}{mass}$ it follows that "B = volume" (i.e., $b = \frac{B}{mass}$) so that $\int \vec{V}\cdot\hat{n}\,dA = \dot{B}_{out}$ represents the volume flowrate (m^3/s) from the control volume.

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

4.61

4.61 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.61. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface $A–B$, which is of unit depth into the paper.



■ FIGURE P4.61

$$\vec{\dot{B}}_{AB} = \int_{AB} \rho \vec{b}\, \vec{V}\cdot\hat{n}\, dA = \int_{AB} \rho \vec{V}\, \vec{V}\cdot\hat{n}\, dA = \rho \int_{y=0}^{y=20\,ft} (V\hat{i})[(V\hat{i})\cdot\hat{i}](1\,ft)\,dy$$

$$= \rho\hat{i}\int_0^{20} V^2\,dy$$

But, $V = \frac{15}{10}y\ \frac{ft}{s}$ for $0 \le y \le 10\,ft$ (i.e., $V=0$ at $y=0$; $V=15\frac{ft}{s}$ at $y=10$)
and $V = 15\frac{ft}{s}$ for $y \ge 10\,ft$

Thus,

$$\vec{\dot{B}}_{AB} = \rho\hat{i}\left[\int_0^{10}\left(\frac{15}{10}y\right)^2 dy + \int_{10}^{20}(15)^2 dy\right] = \rho\hat{i}\left[2.25\,\frac{y^3}{3}\Big|_0^{10} + 225\,y\Big|_0^{10}\right]$$

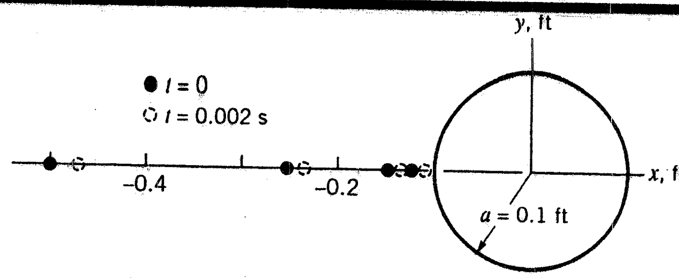
$$= 0.00238\,\frac{slugs}{ft^3}\left[750\,\frac{ft^4}{s^2} + 2250\,\frac{ft^4}{s^2}\right]\hat{i}$$

$$= 7.14\hat{i}\ \frac{slug\ ft}{s^2}$$

4.62

4.62 (See, **"Follow those particles,"** Section 4.1.) Two photographs of four particles in a flow past a sphere are superposed as shown in Fig. P4.62. The time interval between the photos is $\Delta t = 0.002$ s. The locations of the particles, as determined from the photos, are shown in the table. **(a)** Determine the fluid velocity for these particles. **(b)** Plot a graph to compare the results of part (a) with the theoretical velocity which is given by $V = V_0(1 + a^3/x^3)$, where a is the sphere radius and V_0 is the fluid speed far from the sphere.

Particle	x at $t = 0$ s (ft)	x at $t = 0.002$ s (ft)
1	−0.500	−0.480
2	−0.250	−0.232
3	−0.140	−0.128
4	−0.120	−0.112



■ FIGURE P4.62

The fluid velocity (which is assumed to be the same as the particle velocity) can be estimated by

$V = \Delta x/\Delta t$

Thus, for particle (1): $V_1 = [-0.480\text{ ft} - (-0.500\text{ ft})]/(0.002\text{ s}) = 10$ ft/s

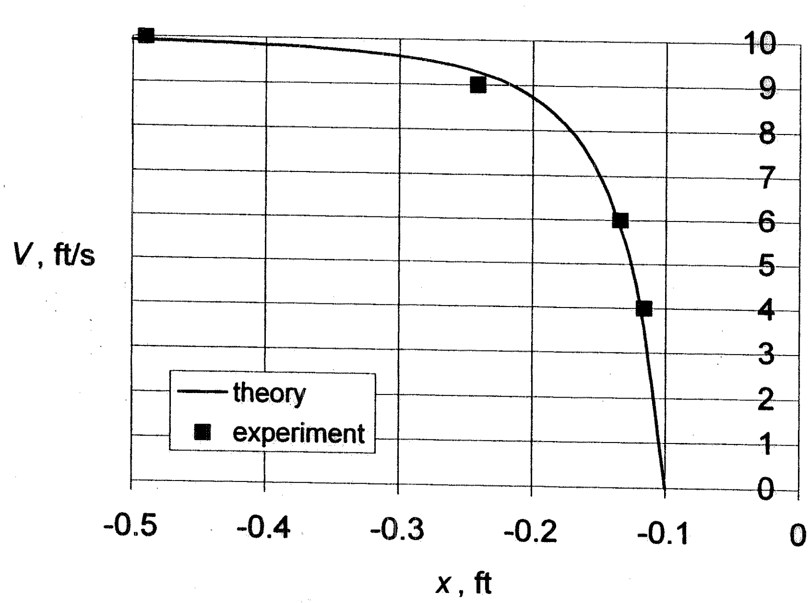
During to 0.002 s time interval the average location of the particle was

$x = [(-0.480\text{ ft}) + (-0.500\text{ ft})] = -0.0490$ ft

By similar calculations the following experimental results were obtained:

Particle	x, ft	V, ft/s
1	−0.490	10
2	−0.241	9
3	−0.134	6
4	−0.116	4

These experimental results and the theoretical results ($V = V_0(1 + a^3/x^3)$, where $V_0 = 10$ ft/s and $a = 0.1$ ft) are plotted in the figure below.



4.63

4.63 (See, "**Winds on Earth and Mars,**" Section 4.1.4.) A 10-ft-diameter dust devil that rotates one revolution per second travels across the Martian surface (in the x-direction) with a speed of 5 ft/s. Plot the pathline etched on the surface by a fluid particle 10 ft from the center of the dust devil for time $0 \le t \le 3$ s. The particle position is given by the sum of that for a stationary swirl $[x = 10 \cos(2\pi t), y = 10 \sin(2\pi t)]$ and that for a uniform velocity ($x = 5t$, y = constant), where x and y are in feet and t is in seconds.

The path line is given by

$x = 10 \cos(2\pi t) + 5t$
and
$y = 10 \sin(2\pi t)$, where $x \sim ft$, $y \sim ft$, and $t \sim s$

This path is plotted for $0 \le t \le 3\,s$ below.