*9.3

*9.3 The pressure distribution on the 1-m-diameter circular disk in Fig. P9.3 is given in the table. Determine the drag on the disk.

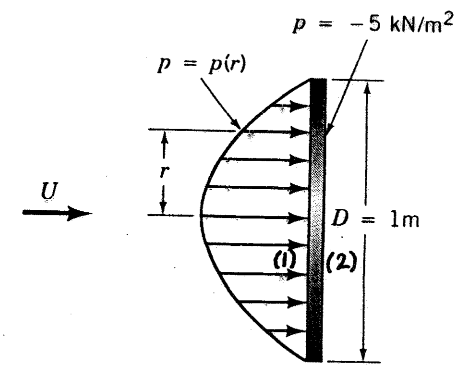


FIGURE P9.3

$$\mathcal{D} = \int_1 p\,dA - \int_2 p\,dA = \int_{r=0}^{r=\frac{D}{2}} p\,(2\pi r\,dr) - p_2\,\frac{\pi}{4}D^2, \text{ since } dA = 2\pi r\,dr$$

Thus,

$$\mathcal{D} = 2\pi\int_0^{0.5\,m} p\,r\,dr - \left(-5\,\frac{kN}{m^2}\right)\frac{\pi}{4}(1\,m^2) = 2\pi\int_0^{0.5} p\,r\,dr + 3.93\ kN$$

where $p \sim \frac{kN}{m^2}$, $r \sim m$

Evaluate the integral numerically using the following integrand:

r, m	pr, kN/m
0	0
0.05	0.214
0.10	0.406
0.15	0.558
0.20	0.620
0.25	0.695
0.30	0.711
0.35	0.662
0.40	0.564
0.45	0.333
0.50	0.000

r (m)	p (kN/m²)
0	4.34
0.05	4.28
0.10	4.06
0.15	3.72
0.20	3.10
0.25	2.78
0.30	2.37
0.35	1.89
0.40	1.41
0.45	0.74
0.50	0.0

Using a standard numerical integration technique with the above integrand gives $\mathcal{D} = \underline{\underline{5.43\ kN}}$

9.4

9.4 A 0.10 m-diameter circular cylinder moves through air with a speed U. The pressure distribution on the cylinder's surface is approximated by the three straight line segments shown in Fig. P9.4. Determine the drag coefficient on the cylinder. Neglect shear forces.

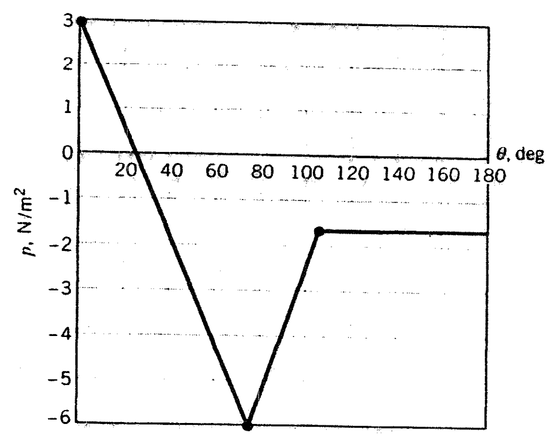
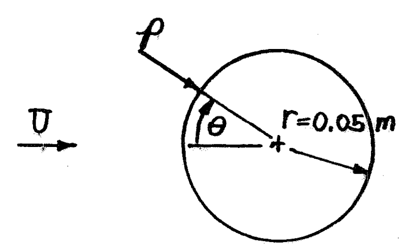


FIGURE P9.4

$$\mathcal{D}_p = \int p\, b r \cos\theta \, d\theta = br \int p \cos\theta \, d\theta$$

or

$$\mathcal{D}_p = 2br \int_{\theta=0}^{\pi} p \cos\theta \, d\theta$$



$dA = b\, r d\theta$

$b = \text{length}$

Break up the integration into the following three segments:

1) $0 \le \theta \le 70° = 1.222 \text{ rad}$ where

$$p = -7.39\,\theta + 3 \ \frac{N}{m^2}, \text{ where } \theta \sim \text{rad.}$$

i.e. $p\big|_{\theta=0} = 3$ and $p\big|_{\theta=1.222} = -6$

2) $70° \le p \le 100°$ or $1.222 \le \theta \le 1.745$ rad where

$$p = 8.59\,\theta - 16.5 \ \frac{N}{m^2}, \text{ where } \theta \sim \text{rad}$$

i.e. $p\big|_{\theta=1.222} = -6$ and $p\big|_{\theta=1.745} = -1.5$

and

3) $100° \le p \le 180°$ or $1.745 \le \theta \le 3.14$ rad where

$$p = -1.5 \ \frac{N}{m^2}$$

Thus,

$$\mathcal{D}_p = 2br\left[\int_0^{70°} p\cos\theta\, d\theta + \int_{70°}^{100°} p\cos\theta\, d\theta + \int_{100°}^{180°} p\cos\theta\, d\theta\right] = 2br\left[I_1 + I_2 + I_3\right] \quad (1)$$

where

(con't)

9.4	(con't)

$$I_1 = \int_0^{1.222} (-7.39\theta + 3)\cos\theta \, d\theta = \Big[-7.39(\cos\theta + \theta\sin\theta) + 3\sin\theta\Big]_0^{1.222} = -0.791$$

$$I_2 = \int_{1.222}^{1.745} (8.59\theta - 16.5)\cos\theta \, d\theta = \Big[8.59(\cos\theta + \theta\sin\theta - 16.5\sin\theta\Big]_{1.222}^{1.745} = -0.260$$

and

$$I_3 = \int_{1.745}^{3.14} (-1.5)\cos\theta \, d\theta = -1.5\sin\theta\Big|_{1.745}^{3.14} = 1.477$$

Hence,

$$\mathcal{D}_p = 2br[0.791 - 0.260 + 1.477] = 0.852\,br$$

or with

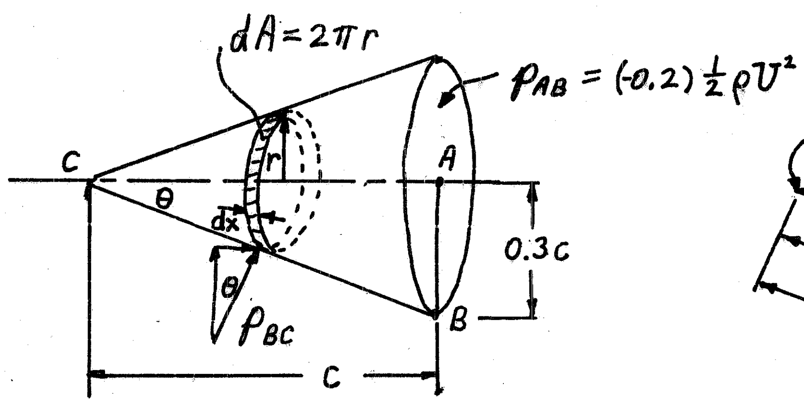
$$C_D = \frac{\mathcal{D}_p}{\frac{1}{2}\rho U^2 A} = \frac{0.852\,br}{\frac{1}{2}\rho U^2(2br)} = \frac{0.426}{\frac{1}{2}\rho U^2}$$

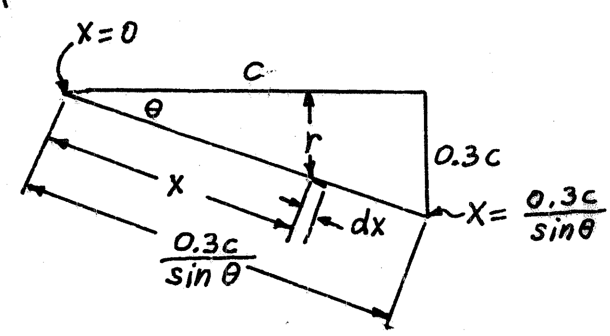
But the pressure at $\theta = 0$, the stagnation point, is $3\,\frac{N}{m^2}$.
Thus, $\frac{1}{2}\rho U^2 = 3\,\frac{N}{m^2}$ so that

$$C_D = \frac{0.426}{3} = \underline{\underline{0.142}}$$

9.5

9.5 Repeat Problem 9.1 if the object is a cone made by rotating the triangle $A-B-C$ about the horizontal axis $A-C$ rather than the wedge-shaped bar.





$$\mathcal{D} = \tfrac{1}{2}\rho U^2 A C_D \text{ where } C_D = 0.5 \text{ and } A = \pi(0.3c)^2 = 0.09\pi c^2 \qquad (1)$$

Also,

$$\mathcal{D} = -p_{AB}A + \int p_{BC}\sin\theta\, dA \text{ where } dA = 2\pi r\,dx = 2\pi(x\sin\theta)\,dx$$

Hence,

$$\mathcal{D} = -(-0.2)\tfrac{1}{2}\rho U^2(0.09\pi c^2) + \int_{x=0}^{x=0.3c/\sin\theta} p_{BC}\sin\theta\left[2\pi(x\sin\theta)dx\right] \qquad (2)$$

or with p_{BC} = average pressure = constant Eq. (2) becomes

$$\mathcal{D} = (0.2)\tfrac{1}{2}\rho U^2(0.09\pi c^2) + p_{BC}\sin^2\theta\,(2\pi)\int_{x=0}^{x=0.3c/\sin\theta} x\,dx$$

$$= (0.2)\tfrac{1}{2}\rho U^2(0.09\pi c^2) + p_{BC}\sin^2\theta\,(2\pi)(0.3c/\sin\theta)^2/2$$

or

$$\mathcal{D} = \left[(0.2)\tfrac{1}{2}\rho U^2 + p_{BC}\right](0.09\pi c^2) \qquad (3)$$

By combining Eqs. (1) and (3) we obtain

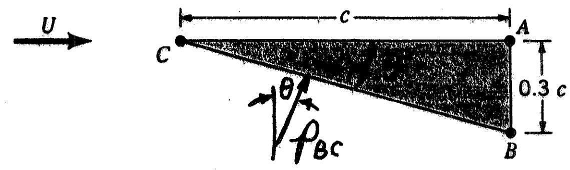
$$\tfrac{1}{2}\rho U^2(0.09\pi c^2)(0.5) = \left[(0.2)\tfrac{1}{2}\rho U^2 + p_{BC}\right](0.09\pi c^2)$$

which gives

$$p_{BC} = \underline{\underline{(0.3)\tfrac{1}{2}\rho U^2}}$$

9.1

9.1 The wedge-shaped bar shown in Fig. P9.1 moves through a fluid with its top (surface $A-C$) parallel to the free stream. The drag coefficient is 0.5 and the pressure on the base (surface $A-B$) is less than the free stream pressure by an amount equal to 20% of the dynamic pressure. Viscous effects are negligible. Determine the average pressure on surface $B-C$.



■ FIGURE P9.1

(a) $\mathcal{D} = \frac{1}{2}\rho U^2 A\, C_D$ where $A = A_{AB}$ and $C_D = 0.5$

Also, $\mathcal{D} = -p_{AB} A_{AB} + p_{BC} A_{BC} \sin\theta$, where $p_{AB} = (-0.2)\frac{1}{2}\rho U^2$

and $A_{BC} = A_{AB}/\sin\theta$

Hence,

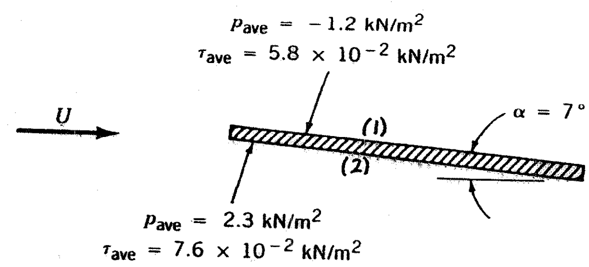
$$-(-0.2)\tfrac{1}{2}\rho U^2 A_{AB} + p_{BC}\frac{A_{AB}}{\sin\theta}\sin\theta = \tfrac{1}{2}\rho U^2 A_{AB}\,(0.5)$$

or

$$(0.2)\tfrac{1}{2}\rho U^2 + p_{BC} = (0.5)\tfrac{1}{2}\rho U^2 \quad \text{or} \quad \underline{\underline{p_{BC} = (0.3)\tfrac{1}{2}\rho U^2}}$$

9.2

9.2 The average pressure and shear stress acting on the surface of the 1-m-square flat plate are as indicated in Fig. P9.2. Determine the lift and drag generated. Determine the lift and drag if the shear stress is neglected. Compare these two sets of results.



■ **FIGURE P9.2**

Since $\int p\, dA = p_{ave} A$ and $\int \tau_w\, dA = \tau_{ave} A$ it follows that

$$\mathcal{D} = -p_1 A_1 \sin\alpha + p_2 A_2 \sin\alpha + \tau_1 A_1 \cos\alpha + \tau_2 A_2 \cos\alpha$$

or with $A_1 = A_2 = 1\, m^2$ and $\alpha = 7°$,

$$\mathcal{D} = A_1 \sin\alpha\,(p_2 - p_1) + A_1 \cos\alpha\,(\tau_1 + \tau_2)$$
$$= (1\,m^2)\sin 7°\,(2.3-(-1.2))\,\frac{kN}{m^2} + (1\,m^2)\cos 7°\,(5.8\times10^{-2} + 7.6\times10^{-2})\,\frac{kN}{m^2}$$
$$= 0.427\ kN + 0.133\ kN = \underline{\underline{0.560\ kN}}$$

Note, if shear stress is neglected $\mathcal{D} = \underline{\underline{0.427\ kN}}$ (i.e., $\tau_1 = \tau_2 = 0$)

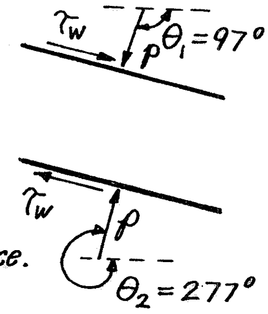
Also, $\mathcal{L} = -p_1 A_1 \cos\alpha + p_2 A_2 \cos\alpha - \tau_1 A_1 \sin\alpha - \tau_2 A_2 \sin\alpha$

or

$$\mathcal{L} = A_1 \cos\alpha\,(p_2 - p_1) - A_1 \sin\alpha\,(\tau_1 + \tau_2)$$
$$= (1\,m^2)\cos 7°\,(2.3-(-1.2))\,\frac{kN}{m^2} - (1\,m^2)\sin 7°\,(5.8\times10^{-2} + 7.6\times10^{-2})\,\frac{kN}{m^2}$$
$$= 3.47\ kN - 0.0163\ kN = \underline{\underline{3.45\ kN}}$$

Note, if shear stress is neglected $\mathcal{L} = \underline{\underline{3.47\ kN}}$

Note: If the general expressions $\mathcal{D} = \int p \cos\theta\, dA + \int \tau_w \sin\theta\, dA$ and $\mathcal{L} = -\int p \sin\theta\, dA + \int \tau_w \cos\theta\, dA$ are use, be careful about the signs involved. On the upper surface $\theta_1 = 97°$ and p and τ_w are positive as indicated in the figure. On the lower surface $\theta_2 = 277°$ and p and τ_w are positive as indicated in the lower figure.



For example, with this notation $\tau_w < 0$ on the lower surface.

$$\mathcal{L} = -(-1.2\,\tfrac{kN}{m^2})\sin 97°(1\,m^2) - (2.3\,\tfrac{kN}{m^2})\sin 277°(1\,m^2)$$
$$+ (5.8\times10^{-2}\,\tfrac{kN}{m^2})\cos 97°(1\,m^2) + (-7.6\times10^{-2}\,\tfrac{kN}{m^2})\cos 277°(1\,m^2)$$
$$= 3.45\ kN$$, as obtained above.

9.6

9.6 A 5-ft-long porpoise swims with a speed of 30 ft/s. Would a boundary layer type flow be developed along the sides of the animal? Explain.

$Re = \frac{U\ell}{\nu}$, or with $\ell = 5\,ft$, $U = 30\,\frac{ft}{s}$, and $\nu = 1.21 \times 10^{-5}\,\frac{ft^2}{s}$
(i.e., 60°F water)

$Re = \frac{(5\,ft)(30\,\frac{ft}{s})}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 1.24 \times 10^{7}$ This Reynolds number is large enough to ensure boundary layer flow. ($Re \approx 1000$ is often assumed to be the lower limit.)

9.7

9.7 Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar; turbulent? Explain.

Animal	Speed	Re
(a) large whale	10 m/s	300,000,000
(b) flying duck	20 m/s	300,000
(c) large dragonfly	7 m/s	30,000
(d) invertebrate larva	1 mm/s	0.3
(e) bacterium	0.01 mm/s	0.00003

Inertia important if $Re \gtrsim 1$ (i.e whale, duck, dragonfly)

Viscous effects dominate if $Re \lesssim 1$ (i.e larva, bacterium)

Boundary layer flow becomes turbulent for Re on the order of 10^5 to 10^6. (i.e. whale and perhaps the duck)

The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.

9.9

9.9 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

$Re = \frac{UD}{\nu} < 1$ or $U < \frac{\nu}{D}$ if viscous effects are to be important throughout the flow.

For standard air $\nu = 1.57 \times 10^{-4} \frac{ft^2}{s}$

Thus,

$U < \frac{1.57 \times 10^{-4}}{D}$, where D is the diameter in feet.

object	D, ft	U, $\frac{ft}{s}$
twig	2.08×10^{-2}	7.54×10^{-3}
hair	3.33×10^{-4}	0.471
smokestack	6	2.62×10^{-5}

9.10

9.10 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.
Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12\times10^{-3}\,m}{\sqrt{1.3\,m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m, \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

9.11

9.11 If the upstream velocity of the flow in Problem 9.10 is U = 1.5 m/s, determine the kinematic viscosity of the fluid.

For laminar flow $\delta = 5\sqrt{\frac{\nu x}{U}}$, or $\nu = \frac{U\delta^2}{25\,x}$

Thus,

$$\nu = \frac{(1.5\,\frac{m}{s})(12\times10^{-3}\,m)^2}{25\,(1.3\,m)} = \underline{\underline{6.65\times10^{-6}\,\frac{m^2}{s}}}$$

9.12

9.12 Water flows past a flat plate with an upstream velocity of $U = 0.02$ m/s. Determine the water velocity a distance of 10 mm from the plate at distances of $x = 1.5$ m and $x = 15$ m from the leading edge.

From the Blasius solution for boundary layer flow on a flat plate, $u = U f'(\eta)$, where η, the similarity variable, is $\eta = y\sqrt{\frac{U}{\nu x}}$. Values of $f'(\eta)$ are given in Table 9.1.

Since $Re_x = \frac{Ux}{\nu} = \frac{(0.02\frac{m}{s})(15m)}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5$ is less than the critical $Re_{x_{cr}} = 5\times10^5$, it follows that the boundary layer flow is laminar.

At $x_1 = 1.5m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_1 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(1.5m)}} = 1.091$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.2647 + \frac{(0.3938-0.2647)}{(1.2-0.8)}(1.091-0.8) = 0.359$$

Hence,

$$u_1 = Uf'(\eta_1) = (0.02\frac{m}{s})(0.359) = \underline{\underline{0.00718\frac{m}{s}}}$$

Similarly, at $x_2 = 15m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_2 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(15m)}} = 0.345$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.0 + \frac{(0.1328-0.0)}{(0.8-0.4)}(0.345-0.0) = 0.1145$$

Hence,

$$u_2 = Uf'(\eta_2) = (0.02\frac{m}{s})(0.1145) = \underline{\underline{0.00229\frac{m}{s}}}$$

9.13*	(con'd)

Also, the momentum thickness, Θ, is

$$\Theta = \int_0^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = 4.94 \int_{y=0}^{0.0268\,m} \sqrt{h}\left(1 - 4.94\sqrt{h}\right) dy \; .$$

Numerical integration of the tabulated data gives $\Theta = \underline{\underline{2.23 \times 10^{-3}\,m}}$

y, m	$(1 - 4.94\sqrt{h})$	$\sqrt{h}\,(1 - 4.94\sqrt{h})$
0	1	0
0.0021	0.491	0.0506
0.0043	0.282	0.0410
0.0064	0.210	0.0335
0.0107	0.109	0.0197
0.0150	0.0511	0.00981
0.0193	0.0194	0.00386
0.0236	0.00584	0.00118
0.0268	0	0
0.0293	0	0
0.0327	0	0

9.13*

*9.13 A Pitot tube connected to a water-filled U-tube manometer is used to measure the total pressure within a boundary layer. Based on the data given in the table below, determine the boundary layer thickness, δ, the displacement thickness, δ^*, and the momentum thickness, Θ.

y (mm), Distance above Plate	h (mm), Manometer Reading
0	0
2.1	10.6
4.3	21.1
6.4	25.6
10.7	32.5
15.0	36.9
19.3	39.4
23.6	40.5
26.8	41.0
29.3	41.0
32.7	41.0

From the Bernoulli equation, with $\rho_{air} \ll \rho_{H_2O}$ it follows that

$$p_1 + \tfrac{1}{2}\rho_{air} V_1^2 = p_2 + \tfrac{1}{2}\rho_{air} V_2^2, \text{ where}$$

$V_1 = u$, $V_2 = 0$, $p_1 = 0$, and $p_2 = \gamma_{H_2O}\, h$

Thus,

$$u = \sqrt{\frac{2\gamma_{H_2O} h}{\rho_{air}}} = \sqrt{\frac{2\left(9800 \frac{N}{m^3}\right) h \; m}{1.23 \frac{kg}{m^3}}}$$

or

$$u = 126.2\sqrt{h}, \text{ where } h \sim m,\ u \sim \frac{m}{s}$$

For $y > 26.8\,mm$ we see that $h = 41.0\,mm$

Thus, $U = 126.2\sqrt{(0.041)} = 25.55 \frac{m}{s}$

For $y = 23.6\,mm$, $u = 126.2\sqrt{(0.0405)}$
$= 25.40 \frac{m}{s}$

or $\frac{u}{U} = \frac{25.40}{25.55} = 0.994$

Thus, $\delta \approx \underline{\underline{23.6\ mm}}$

The displacement thickness, δ^*, is

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \quad \text{or since}$$

$$\frac{u}{U} = \frac{126.2\sqrt{h}}{25.55} = 4.94\sqrt{h} \quad \text{this becomes}$$

$$\delta^* = \int_{y=0}^{0.0268\,m} \left(1 - 4.94\sqrt{h}\right) dy$$

Numerical integration of the tabulated data gives $\delta^* = \underline{\underline{4.18 \times 10^{-3}\ m}}$

(con't)

9.14

9.14 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, δ^*. For air blowing past the flat plate shown in Fig. P9.14, plot the streamline A–B that passes through the edge of the boundary layer ($y = \delta_B$ at $x = \ell$) at point B. That is, plot $y = y(x)$ for streamline A–B. Assume laminar boundary layer flow.

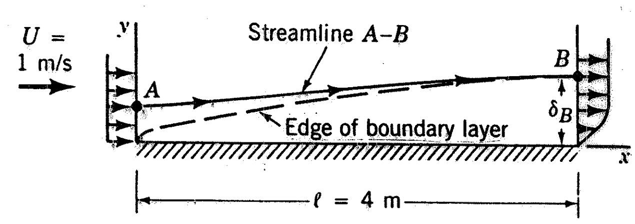
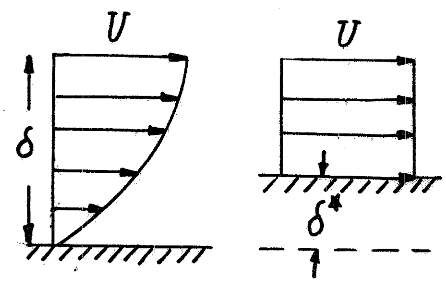


FIGURE P9.14

Since $Re_\ell = \frac{U\ell}{\nu} = \frac{(1\frac{m}{s})(4m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.74\times10^5 < 5\times10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta_B = 5\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.0382\,m$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with the plate displaced by an amount δ^*. Since there is no flow through the plate or streamline A-B,



$$Q_A = Q_B, \text{ or } U y_A = (\delta_B - \delta_B^*)U \qquad (1)$$

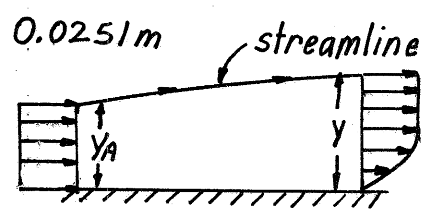
where $\delta^* = 1.721\sqrt{\frac{\nu x}{U}}$

or $\delta_B^* = 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.01315\,m$

Thus,

$$y_A = \delta_B - \delta_B^* = 0.0382\,m - 0.01315\,m = 0.0251\,m$$

Hence, for any x-location

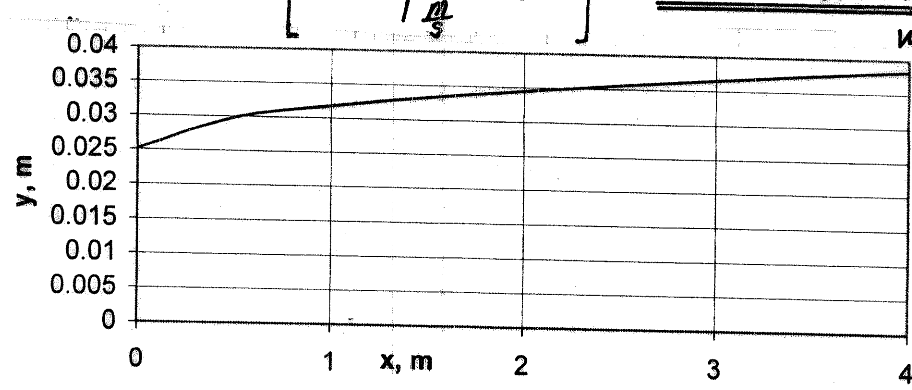


$Q_A = Q$ or $U y_A = U(y - \delta^*)$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\frac{\nu x}{U}}$

$$= 0.0251\,m + 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})\,x\,m}{1\frac{m}{s}}\right]^{\frac{1}{2}} = \underline{\underline{0.0251 + 6.58\times10^{-3}\sqrt{x}\ m,}}$$

where $x \sim m$



9.15

9.15 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.15. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \le x \le 10$ ft if U is to remain constant. Assume laminar flow.

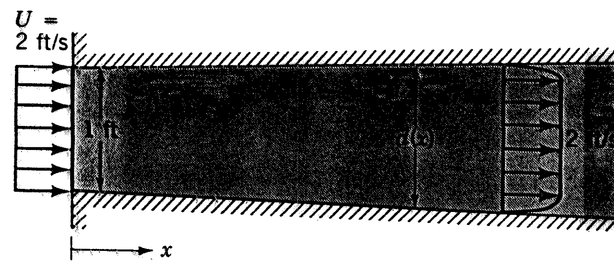


FIGURE P9.15

For incompressible flow $Q_o = Q(x)$ where Q_o = flowrate into the duct $= UA_o = (2\ \frac{ft}{s})(1\ ft^2) = 2\ \frac{ft^3}{s}$

and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_o = U(d-2\delta^*)^2 \quad \text{or} \quad d = 1\ ft + 2\delta^*, \tag{1}$$

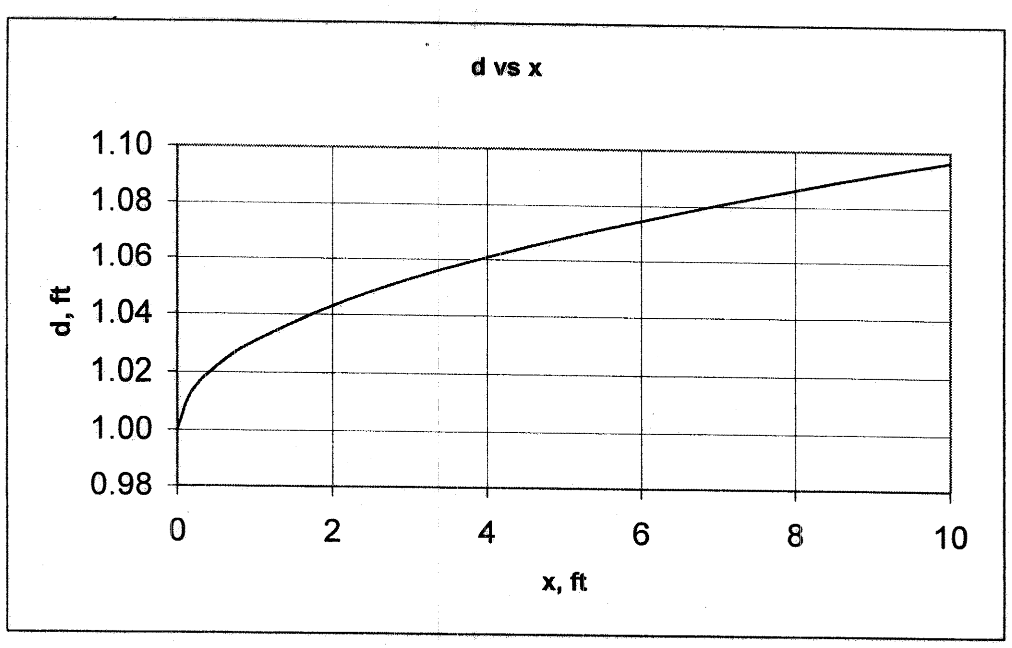
where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57 \times 10^{-4}\ \frac{ft^2}{s})\ x}{2\ \frac{ft}{s}}\right]^{1/2} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq. (1)

$$d = 1 + 0.0304\sqrt{x}\ ft$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.



9.16

9.16 A smooth, flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\,\frac{m^2}{s})\,x}{0.5\,\frac{m}{s}}} = 7.48\times10^{-3}\sqrt{x}\ m,\ \text{where } x\sim m$$

and

$$\tau_w = 0.332\,U^{3/2}\sqrt{\frac{\rho\mu}{x}} = 0.332\,(0.5\,\tfrac{m}{s})^{3/2}\sqrt{\frac{(999\,\frac{kg}{m^3})(1.12\times10^{-3}\,\frac{N\cdot s}{m^2})}{x}}$$

$$= \frac{0.124}{\sqrt{x}}\ \frac{N}{m^2},\ \text{where } x\sim m$$

Thus, at $x = 3$ m $\quad \delta = 7.48\times10^{-3}\sqrt{3} = \underline{\underline{0.0130\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{3}} = \underline{\underline{0.0716\ \frac{N}{m^2}}}$$

while at $x = 6$ m $\quad \delta = 7.48\times10^{-3}\sqrt{6} = \underline{\underline{0.0183\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{6}} = \underline{\underline{0.0506\ \frac{N}{m^2}}}$$

9.17

9.17 An atmospheric boundary layer is formed when the wind blows over the earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. P9.17, typical values are $n = 0.40$ for urban areas, $n = 0.28$ for woodland or suburban areas, and $n = 0.16$ for flat open country (Ref. 23). **(a)** If the velocity is 20 ft/s at the bottom of the sail on your boat ($y = 4$ ft), what is the velocity at the top of the mast ($y = 30$ ft)? **(b)** If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

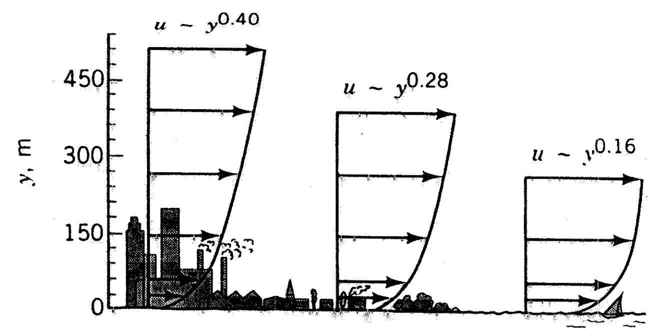


FIGURE P9.17

(a) $u = C\,y^{0.16}$, where C is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.16}$ or $u_2 = 20\,\dfrac{ft}{s}\left(\dfrac{30\,ft}{4\,ft}\right)^{0.16} = \underline{\underline{27.6\,\dfrac{ft}{s}}}$

(b) $u = \tilde{C}\,y^{0.4}$, where $\tilde{C}$ is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.40}$ or $u_2 = 10\,mph\left(\dfrac{60}{10}\right)^{0.4} = \underline{\underline{20.5\,mph}}$

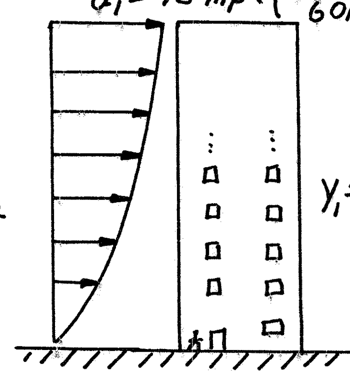
9.18

9.18 A 30-story office building (each story is 12 ft tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at hurricane strength (75 mph) at the top of the building. Use the atmospheric boundary layer information of Problem 9.17.

From Fig. P9.17 the boundary layer velocity profile is given by $u \sim y^{0.28}$, or $u = C\,y^{0.28}$, where C is a constant.

Thus, $\frac{u}{u_1} = \left(\frac{y}{y_1}\right)^{0.28}$

$u_1 = 75\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 110\,\frac{ft}{s}$



or

$u = 110\left(\frac{y}{360}\right)^{0.28}\,\frac{ft}{s}$ where $y \sim ft$

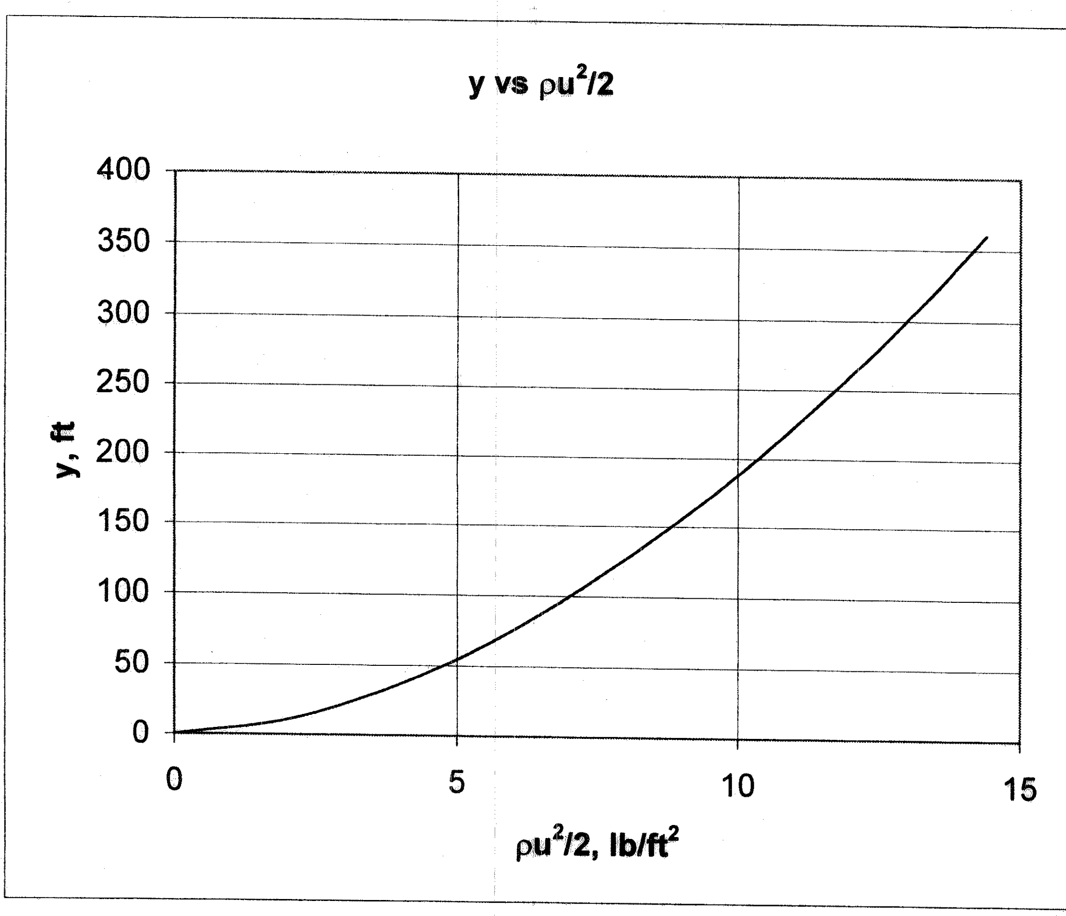
Hence,

$\frac{1}{2}\rho u^2 = \frac{1}{2}(2.38\times10^{-3}\,\frac{slug}{ft^3})\left[110\left(\frac{y}{360}\right)^{0.28}\frac{ft}{s}\right]^2$

or

$\frac{1}{2}\rho u^2 = 14.4\left(\frac{y}{360}\right)^{0.56}\,\frac{lb}{ft^2}$, where $y \sim ft$

This is plotted in the figure below.



9.19

9.19 The typical shape of small cumulous clouds is as indicated in Fig. P9.19. Based on boundary layer ideas, explain why it is clear that the wind is blowing from right to left as indicated.

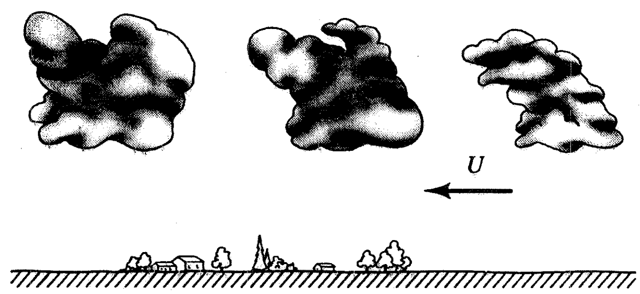


FIGURE P9.19

As indicated in Fig. P9.17, because of the atmospheric boundary layer the velocity of the wind generally increases with altitude. Thus, the top portions of a cloud travels faster than its base — the clouds tend to "tip" toward the direction of the wind. That is, the wind is from right to left.

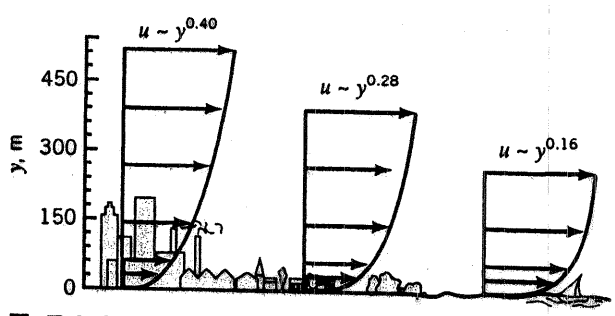


FIGURE P9.17

9.20

9.20 Show that for any function $f = f(\eta)$ the velocity components u and v determined by Eqs. 9.12 and 9.13 satisfy the incompressible continuity equation, Eq. 9.8.

Given $u = U f'(\eta)$, $v = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f'(\eta) - f(\eta))$

where $\eta = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y$ and $(\ )' \equiv \frac{d}{d\eta}$

Show that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any $f(\eta)$.

$\frac{\partial u}{\partial x} = U \frac{\partial f'}{\partial x} = U \frac{df'}{d\eta}\frac{\partial \eta}{\partial x}$, where $\frac{\partial \eta}{\partial x} = -\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}}} x^{-\frac{3}{2}}$

Thus,

$$\frac{\partial u}{\partial x} = -U f''\left[\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}} x^{\frac{3}{2}}}\right] = -\frac{U^{\frac{3}{2}} y f''}{2x^{\frac{3}{2}}\nu^{\frac{1}{2}}} \qquad (1)$$

and

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta \frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial y} - \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y}\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta f'' \frac{\partial \eta}{\partial y} - \frac{\partial \eta}{\partial y} f'\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\eta f'' \frac{\partial \eta}{\partial y}\right] \text{ , where } \frac{\partial \eta}{\partial y} = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$$

Hence,

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y f''\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} = \frac{U^{\frac{3}{2}} y f''}{2x^{\frac{3}{2}}\nu^{\frac{1}{2}}} \qquad (2)$$

By combining Eqs. (1) and (2) we see that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any function $f(\eta)$.

9.21

9.21 Show that by writing the velocity in terms of the similarity variable η and the function $f(\eta)$ the momentum equation for boundary layer flow on a flat plate (Eq. 9.9) can be written as the ordinary differential equation given by Eq. 9.14.

The governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{and} \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{2}$$

Consider $u = Uf'(\eta)$ and $v = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f' - f)$ where $(\ )' \equiv \frac{d}{d\eta}$ (2.1)

and $\eta = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y$

Thus, $\frac{\partial \eta}{\partial x} = -\frac{1}{2}\sqrt{\frac{U}{\nu}}\, y x^{-\frac{3}{2}} = -\frac{1}{2}\frac{\eta}{x}$ and $\frac{\partial \eta}{\partial y} = \sqrt{\frac{U}{\nu}}\, x^{-\frac{1}{2}}$ (3)

so that

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(Uf') = U\frac{\partial f'}{\partial x} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial x} = -\frac{1}{2}\frac{U}{x}\eta f'' \tag{4}$$

and

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\frac{\partial}{\partial y}(\eta f' - f) = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f'' + f' - f')\frac{\partial \eta}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\eta f''\sqrt{\frac{U}{\nu}}\, x^{-\frac{1}{2}}$$

or

$$\frac{\partial v}{\partial y} = \frac{1}{2}\frac{U}{x}\eta f'' \tag{5}$$

Thus, by using Eqs. (4) and (5) we see that Eq (1) is satisfied for any function $f(\eta)$.

Also, $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{\partial}{\partial \eta}(Uf')\left[\sqrt{\frac{U}{\nu}}\, x^{-\frac{1}{2}}\right] = \left(\frac{U^3}{\nu x}\right)^{\frac{1}{2}} f''$ (6)

and

$$\frac{\partial^2 u}{\partial y^2} = \left(\frac{U^3}{\nu x}\right)^{\frac{1}{2}}\frac{\partial f''}{\partial y} = \left(\frac{U^3}{\nu x}\right)^{\frac{1}{2}} f'''\frac{\partial \eta}{\partial y} = \frac{U^2}{\nu x} f''' \tag{7}$$

Thus, by using Eqs. (2.1), (6), and (7) with Eq. (2) we obtain

$$(Uf')\left(\frac{-1}{2}\frac{U}{x}\eta f''\right) + \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f' - f)\left(\frac{U^3}{\nu x}\right)^{\frac{1}{2}} f'' = \nu\frac{U^2}{\nu x} f'''$$

which simplifies to:

$$\underline{\underline{2f''' - ff'' = 0}}$$

From Eq. (2.1) the boundary conditions at $y = 0$ (i.e. $\eta = 0$) become

$u = 0 = Uf'(0)$ and $v = 0 = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(0 f'(0) - f(0))$

That is, $\underline{\underline{f(0) = 0 \text{ and } f'(0) = 0}}$

Similarly, as $y \to \infty$ (i.e., $\eta \to \infty$) we require $u \to U$. Thus, from Eq. (2.1) $\underline{\underline{f' \to 1 \text{ as } \eta \to \infty.}}$

9.22

9.22 An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of $Re_{xcr} = 5 \times 10^5$. If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

At 10,000 ft:

(a) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and from Table C.1, $\nu = \frac{\mu}{\rho} = \frac{3.534\times10^{-7}\,\frac{lb\cdot s}{ft^2}}{1.756\times10^{-3}\,\frac{slug}{ft^3}}$

$= 2.01\times10^{-4}\,\frac{ft^2}{s}$

Hence, with $Re_{x_{cr}} = 5\times10^5$,

$$x_{cr} = \frac{\nu\, Re_{x_{cr}}}{U} = \frac{\left(2.01\times10^{-4}\,\frac{ft^2}{s}\right)(5\times10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.171\,ft}}$$

At sea-level:

(b) $Re_{x_{cr}} = \frac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\frac{1\,hr}{3600\,s}\right)\left(\frac{5280\,ft}{mi}\right) = 587\,\frac{ft}{s}$

and $\nu = 1.57\times10^{-4}\,\frac{ft^2}{s}$

Hence,

$$x_{cr} = \frac{\nu\, Re_{cr}}{U} = \frac{\left(1.57\times10^{-4}\,\frac{ft^2}{s}\right)(5\times10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.134\,ft}}$$

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.

9.24

9.24 A laminar boundary layer velocity profile is approximated by $u/U = [2 - (y/\delta)](y/\delta)$ for $y \le \delta$, and $u = U$ for $y > \delta$. **(a)** Show that this profile satisfies the appropriate boundary conditions. **(b)** Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$.

(a) $\frac{u}{U} = g(Y) = 2Y - Y^2$ where $Y = y/\delta$

Thus, $\left.\frac{u}{U}\right|_{y=0} = 0$ as it must, $\left.\frac{u}{U}\right|_{y=\delta} = 2-1=1$ or $u=U$ at $y=\delta$ as it must.

Also, $\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$ so that $\left.\frac{du}{dy}\right|_{y=\delta} = U\left[\frac{2}{\delta} - \frac{2}{\delta}\right] = 0$

(b) From the momentum integral equation,

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

Thus,

$$C_1 = \int_0^1 (2Y - Y^2)(1-2Y+Y^2)\,dY = \int_0^1 (2Y - 5Y^2 + 4Y^3 - Y^4)\,dY$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15}$$

and

$$C_2 = (2-2Y)\Big|_{Y=0} = 2$$

so that

$$\delta = \sqrt{\frac{2\,(2)\,\nu x}{\frac{2}{15}U}} = \sqrt{\frac{30\,\nu x}{U}}$$

Hence, with $Re_x = \frac{Ux}{\nu}$,

$$\frac{\delta}{x} = \frac{\sqrt{30}}{\sqrt{Re_x}} = \underline{\underline{\frac{5.48}{\sqrt{Re_x}}}}$$

9.25

9.25 A laminar boundary layer velocity profile is approximated by the two straight-line segments indicated in Fig. P9.25. Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$, and wall shear stress, $\tau_w = \tau_w(x)$. Compare these results with those in Table 9.2.

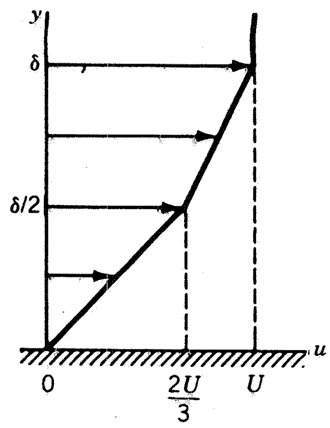


FIGURE P9.25

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0} \qquad (1)$$

and

$$\frac{u}{U} = g(Y) \text{ with } Y = \frac{y}{\delta},$$

For $0 \le Y < \frac{1}{2}$, $g = a_1 + b_1 Y$ with the constants a_1 and b_1 obtained from $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 0$ at $Y = 0$. Thus, $a_1 = 0$, $b_1 = \frac{4}{3}$

or $g = \frac{4}{3}Y$ for $0 \le Y < \frac{1}{2}$

Hence, $C_2 = \frac{4}{3}$ (2)

Similarly, for $\frac{1}{2} \le Y \le 1$, $g = a_2 + b_2 Y$ with $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 1$ at $Y = 1$

Thus,

$$\frac{2}{3} = a_2 + \frac{1}{2}b_2 \text{ and } 1 = a_2 + b_2 \text{ which give } a_2 = \frac{1}{3},\ b_2 = \frac{2}{3}$$

or

$$g = \frac{1}{3} + \frac{2}{3}Y \text{ for } \frac{1}{2} \le Y < 1$$

Hence,

$$C_1 = \int_0^1 g(1-g)\,dY = \int_0^{1/2} \frac{4}{3}Y\left(1 - \frac{4}{3}Y\right)dY + \int_{1/2}^1 \left(\frac{1}{3} + \frac{2}{3}Y\right)\left(1 - \frac{1}{3} - \frac{2}{3}Y\right)dY$$

$$= \frac{4}{9}\int_0^{1/2} (3Y - 4Y^2)\,dY + \frac{2}{9}\int_{1/2}^1 (1+2Y)(1-Y)\,dY$$ which upon integration gives

$C_1 = 0.1574$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$$\delta = \left[\frac{2\left(\frac{4}{3}\right)\nu x}{0.1574\, U}\right]^{1/2} = \underline{\underline{4.12\sqrt{\frac{\nu x}{U}}}} \text{ or } \frac{\delta\, Re_x^{1/2}}{x} = 4.12$$

Also,

$$\tau_w = \frac{\mu U}{\delta} C_2 = \underline{\underline{\frac{4\mu U}{3\delta}}} \text{ or } c_f = \frac{\sqrt{2C_1 C_2}}{\sqrt{Re_x}} = \frac{\sqrt{2(0.1574)\left(\frac{4}{3}\right)}}{\sqrt{Re_x}} = \frac{0.648}{\sqrt{Re_x}}$$

Compare these results to those in Table 9.2.

9.26*

9.26* An assumed dimensionless laminar boundary layer profile for flow past a flat plate is given in the table below. Use the momentum integral equation to determine $\delta = \delta(x)$. Compare your result with the exact Blasius solution result (see Table 9.2).

y/δ	u/U
0	0
0.080	0.133
0.16	0.265
0.24	0.394
0.32	0.517
0.40	0.630
0.48	0.729
0.56	0.811
0.64	0.876
0.72	0.923
0.80	0.956
0.88	0.976
0.96	0.988
1.00	1.000

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \quad \text{where } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

and

$$C_1 = \int_0^1 g(1-g)\,dY \quad \text{with } \frac{u}{U} = g(Y) \text{ and } Y = \frac{y}{\delta}$$

The value of C_2 can be approximated as $C_2 \approx \left.\frac{\Delta\left(\frac{u}{U}\right)}{\Delta\left(\frac{y}{\delta}\right)}\right|_{\frac{y}{\delta}=0} = \frac{0.133}{0.080} = 1.66$

and the value of C_1 can be obtained from numerical integration.

Y	g(1-g)
0	0
0.8	0.115
0.16	0.195
0.24	0.239
0.32	0.250
0.40	0.233
0.48	0.198
0.56	0.153
0.64	0.109
0.72	0.071
0.80	0.042
0.88	0.023
0.96	0.012
1.00	0

The result of the integration is

$$C_1 = \int_0^1 g(1-g)\,dY \approx 0.131 \quad \text{so that}$$

$$\delta = \left[\frac{2\nu x\,(1.66)}{U\,(0.131)}\right]^{\frac{1}{2}} = 5.03\left(\frac{\nu x}{U}\right)^{\frac{1}{2}}$$

or $\frac{\delta}{x} = \frac{5.03}{\sqrt{Re_x}}$, where $Re_x = \frac{\rho V x}{\mu}$

Note: The Blasius solution has 5, not 5.03

9.27*

9.27* For a fluid of specific gravity $SG = 0.86$ flowing past a flat plate with an upstream velocity of $U = 5$ m/s, the wall shear stress on a flat plate was determined to be as indicated in the table below. Use the momentum integral equation to determine the boundary layer momentum thickness, $\Theta = \Theta(x)$. Assume $\Theta = 0$ at the leading edge, $x = 0$.

Since $\tau_w = \rho U^2 \frac{d\Theta}{dx}$ it follows that $d\Theta = \frac{\tau_w}{\rho U^2}\,dx$

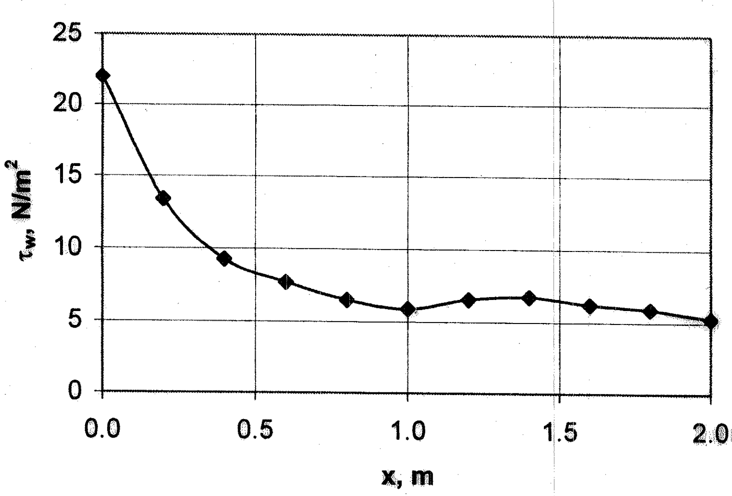
which can be integrated to give (using $\Theta = 0$ at $x = 0$)

$$\Theta = \frac{1}{\rho U^2}\int_0^x \tau_w\,dx = \frac{1}{(0.86)(1000\,\frac{kg}{m^3})(5\,\frac{m}{s})^2}\int_0^x \tau_w\,dx$$

or

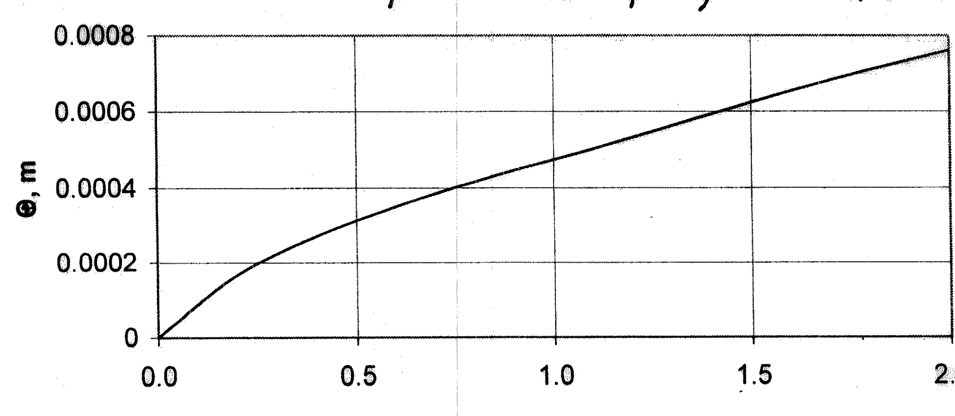
$$\Theta = 4.65\times10^{-5}\int_0^x \tau_w\,dx\ , \quad \text{where } \Theta \sim m,\ x \sim m, \text{ and } \tau_w \sim \frac{N}{m^2} \qquad (1)$$

For $0 \le x \le 2.0$ m, integrate Eq. (1) to determine Θ as a function of x. To do so, we need the value of τ_w at $x = 0$, which is not given in the table. Theoretically, $\tau_w = \infty$ at the leading. For our purposes, based on the extrapolated curve below, assume $\tau_w = 22\,\frac{N}{m^2}$ at $x = 0$



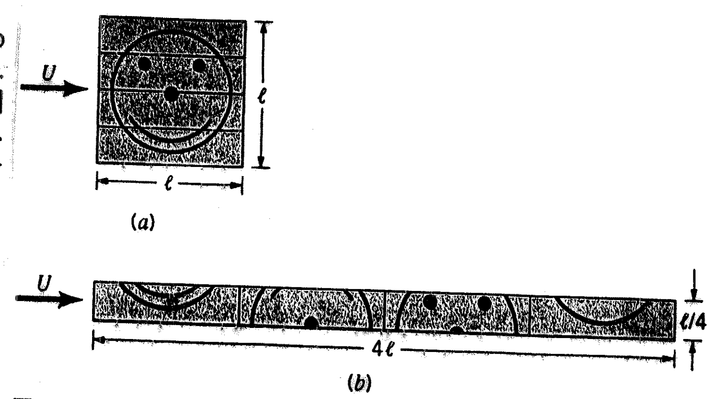
x (m)	τ_w (N/m²)
0	—
0.2	13.4
0.4	9.25
0.6	7.68
0.8	6.51
1.0	5.89
1.2	6.57
1.4	6.75
1.6	6.23
1.8	5.92
2.0	5.26

A standard numerical integration technique gives the following results.



9.28

9.28 The square flat plate shown in Fig. P9.28*a* is cut into four equal-sized prices and arranged as shown in Fig. P9.28*b*. Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow. Explain your answer physically.



■ FIGURE P9.28

For case (a):

$$\mathcal{D}_{f_a} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \text{ and } A = \ell^2$$

Thus,

$$\mathcal{D}_{f_a} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\ell^2 = 0.664\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b):

$$\mathcal{D}_{f_b} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \text{ and } A = (4\ell)\left(\frac{\ell}{4}\right) = \ell^2$$

Thus,

$$\mathcal{D}_{f_b} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}\ell^2 = \frac{1}{2}\left(0.664\rho U^{\frac{3}{2}}\sqrt{\nu}\,\ell^{\frac{3}{2}}\right) \qquad (2)$$

By comparing Eqs. (1) and (2) we see that

$$\underline{\underline{\mathcal{D}_{f_a} = 2.0\,\mathcal{D}_{f_b}}}$$

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.

9.29

9.29 The two flat plates shown in Fig. P9.29 are to have the same drag. Determine the upstream velocity U_b in terms of U_a and n. Assume laminar flow. Explain your answer physically.

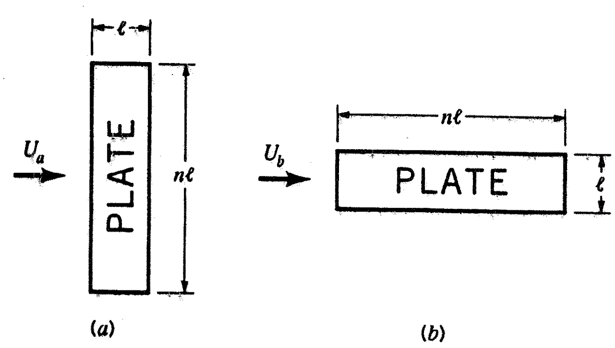


FIGURE P9.29

$$\mathcal{D}_a = \mathcal{D}_b$$

or

$$\tfrac{1}{2}\rho U_a^2 A_a C_{D_a} = \tfrac{1}{2}\rho U_b^2 A_b C_{D_b} \qquad (1)$$

where $A_a = A_b = (n\ell)\ell$ and $C_D = \dfrac{1.328}{\sqrt{Re}}$ or $C_{D_a} = \dfrac{1.328}{\sqrt{Re_a}}$, $C_{D_b} = \dfrac{1.328}{\sqrt{Re_b}}$

Thus, Eq. (1) gives

$$U_a^2 C_{D_a} = U_b^2 C_{D_b}$$

or

$$\frac{U_a^2}{\sqrt{Re_a}} = \frac{U_b^2}{\sqrt{Re_b}} \quad \text{where } Re_a = \frac{U_a \ell}{\nu} \text{ and } Re_b = \frac{U_b (n\ell)}{\nu}$$

Hence,

$$\frac{U_a^2}{\sqrt{U_a \ell/\nu}} = \frac{U_b^2}{\sqrt{U_b n\ell/\nu}}$$

or

$$U_a^{3/2} = U_b^{3/2}/n^{1/2}$$

Thus,

$$\underline{\underline{U_b = n^{1/3} U_a}}$$

9.30

9.30 If the drag on one side of a flat plate parallel to the upstream flow is $\mathfrak{D}$ when the upstream velocity is U, what will the drag be when the upstream velocity is $2U$; or $U/2$? Assume laminar flow.

For laminar flow $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$, where $C_{Df} = \dfrac{1.328}{\sqrt{\frac{U\ell}{\nu}}}$

Thus,

$$\mathcal{D} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}} A = 0.664\,\rho A \frac{\sqrt{\nu}}{\sqrt{\ell}}\, U^{3/2} \sim U^{3/2}$$

Hence,

$$\frac{\mathcal{D}_U}{\mathcal{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{2U} = 2.83\,\mathcal{D}_U}}$$

and

$$\frac{\mathcal{D}_U}{\mathcal{D}_{U/2}} \quad \frac{U^{3/2}}{(\frac{U}{2})^{3/2}} \quad \text{or} \quad \underline{\underline{\mathcal{D}_{U/2} = 0.354\,\mathcal{D}_U}}$$

9.31

9.31 Air flows past a parabolic-shaped flat plate oriented parallel to the free stream shown in Fig. P9.31. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar flow.

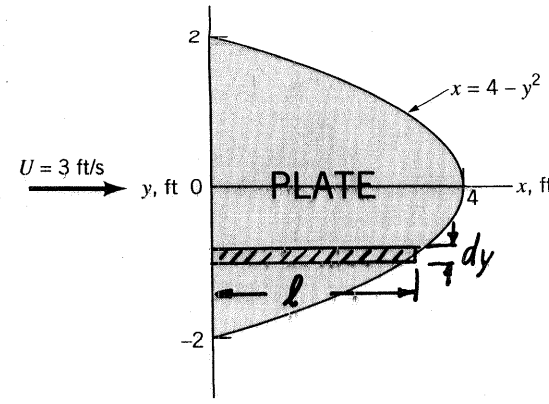


FIGURE P9.31

Treat each strip of thickness dy and length $\ell = \ell(y)$ as a small flat plate with drag $d\mathcal{D}$ where for laminar flow

$$d\mathcal{D} = C_{Df}\,\tfrac{1}{2}\rho U^2\,dA \quad \text{with } dA = \ell\,dy \text{ and } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{\rho U \ell}{\mu}}}$$

Thus,

$$d\mathcal{D} = \frac{1.328}{\sqrt{\frac{\rho U \ell}{\mu}}}\,\tfrac{1}{2}\rho U^2 \ell\,dy = 0.664\sqrt{\mu\rho}\,U^{3/2}\sqrt{\ell}\,dy$$

But $\ell = 4 - y^2$ so that

$$\mathcal{D} = \int d\mathcal{D} = \int_{y=-2}^{+2} 0.664\sqrt{\mu\rho}\,U^{3/2}\sqrt{4-y^2}\,dy$$

$$= 2(0.664)\sqrt{\mu\rho}\,U^{3/2}\int_0^2 \sqrt{4-y^2}\,dy$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}\left[y\sqrt{4-y^2} + 4\sin^{-1}\left(\tfrac{y}{2}\right)\right|_0^2\left(\tfrac{1}{2}\right)$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}\left[4\sin^{-1}(1)\right]\left(\tfrac{1}{2}\right)$$

$$= 1.328\sqrt{\mu\rho}\,U^{3/2}(2\pi)\left(\tfrac{1}{2}\right)$$

Note: The units on the integral are $ft^{3/2}$ (i.e. $2\pi \doteq L^{3/2}$)

Thus,

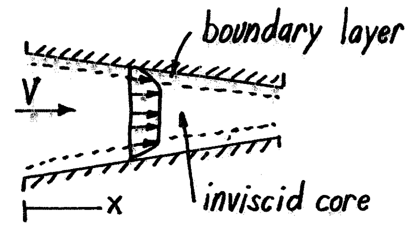
$$\mathcal{D} = 1.328\left[3.74\times10^{-7}\,\frac{lb\cdot s}{ft^2}\left(2.38\times10^{-3}\,\frac{slugs}{ft^3}\right)\right]^{1/2}\left(3\,\frac{ft}{s}\right)^{3/2}(\pi)\,ft^{3/2}$$

$$= \underline{\underline{6.47\times10^{-4}\,lb}}$$

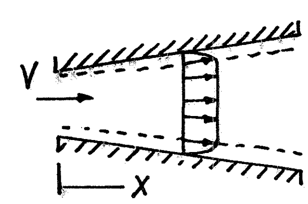
9.32

9.32 It is relatively easy to design an efficient nozzle to accelerate a fluid. Conversely, it is very difficult to build an efficient diffuser to decelerate a fluid without boundary layer separation and its subsequent inefficient flow behavior. Use the ideas of favorable and adverse pressure gradients to explain these facts.

For a nozzle as shown, the pressure decreases in the direction of flow and $\frac{dV}{dx}>0$. Thus the boundary layer fluid flows in a region of favorable pressure gradient, $\frac{\partial p}{\partial x}<0$. This reduces the chance of boundary layer separation. The fluid follows the nozzle contour with little difficulty, regardless of its shape.

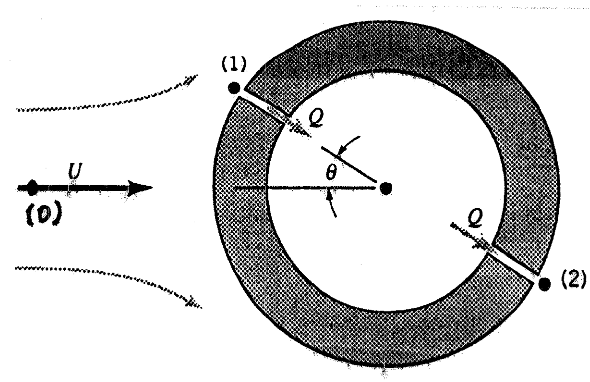


For a diffuser as shown, the pressure increases in the direction of flow and $\frac{dV}{dx}<0$. Thus the boundary layer fluid flows in a region of adverse pressure gradient, $\frac{\partial p}{\partial x}>0$. If the pressure gradient is too large (i.e., the diffuser angle too large), the boundary layer will separate and the diffuser will not function properly.



9.33

9.33 Two small holes are drilled opposite each other in a circular cylinder as shown in Fig. P9.33. Thus, when air flows past the cylinder, air will circulate through the interior of the cylinder at a rate of $Q = K(p_1 - p_2)$, where the constant K depends on the geometry of the passage connecting the two holes. It is assumed that the flow around the cylinder is not affected by either the presence of the two holes or the small flowrate through the passage. Let Q_0 denote the flowrate when $\theta = 0$. Plot a graph of Q/Q_0 as a function of θ for $0 \le \theta \le \pi/2$ if **(a)** the flow is inviscid, and **(b)** if the boundary layer on the cylinder is turbulent (see Fig. 9.17c for pressure data).



■ FIGURE P9.33

(a) For inviscid flow:

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2} = 1 - 4\sin^2\theta$$

Thus, $Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)]$

$$= K\left[\tfrac{1}{2}\rho U^2(1 - 4\sin^2\theta) - \tfrac{1}{2}\rho U^2(1 - 4\sin^2(\theta + \pi))\right]$$

but $\sin^2\theta = \sin^2(\theta + \pi)$

Hence, $\underline{\underline{Q = 0 \text{ for inviscid flow}}}$. Note: This is to be expected because of the symmetrical pressure distribution.

(b) For a turbulent boundary layer:

$$Q = K(p_1 - p_2) = K[(p_1 - p_0) - (p_2 - p_0)] = \tfrac{1}{2}\rho U^2 K[C_{p_1} - C_{p_2}]$$

where C_{p_1} is for θ and C_{p_2} is for $180 - \theta$ deg.

Obtain C_p data from Fig. 9.17.

Note: $C_p = 1$ for $\theta = 0$ and $C_p \approx -0.4$ for $\theta = 180$ deg

Thus, $Q_0 = Q\big|_{\theta = 0} = \tfrac{1}{2}\rho U^2 K[1 - (-0.4)] = 1.4(\tfrac{1}{2}\rho U^2 K)$

so that

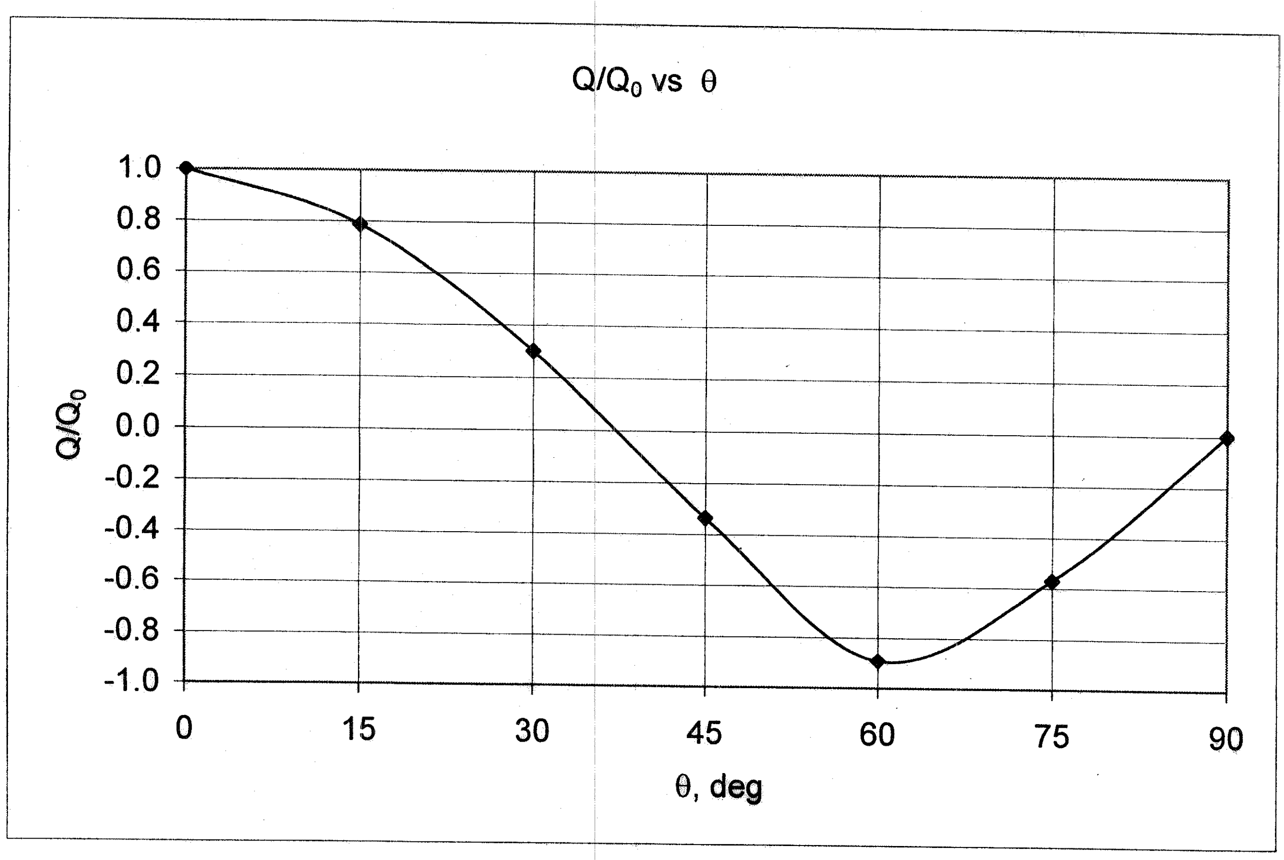
$$\frac{Q}{Q_0} = \frac{\frac{1}{2}\rho U^2 K[C_{p_1} - C_{p_2}]}{1.4(\frac{1}{2}\rho U^2 K)} = \frac{C_{p_1} - C_{p_2}}{1.4}$$

The results are tabulated and plotted below.

(con't)

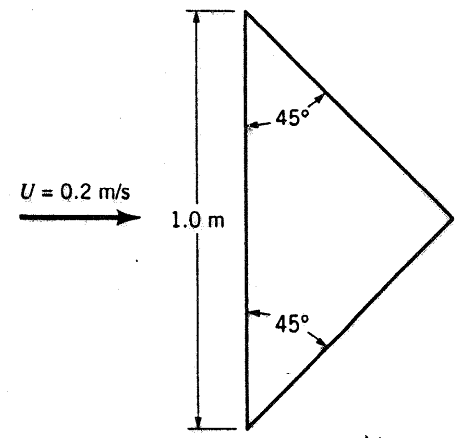
9.33 (con't)

θ, deg	180 - θ, deg	C_{p1}	C_{p2}	Q/Q_0
0	180	1.00	-0.40	1.00
15	165	0.70	-0.40	0.79
30	150	0.00	-0.42	0.30
45	135	-0.90	-0.43	-0.34
60	120	-1.70	-0.45	-0.89
75	105	-2.10	-1.30	-0.57
90	90	-1.90	-1.90	0.00

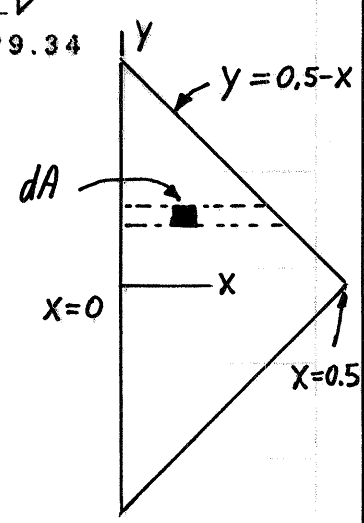


9.34

9.34 Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.34. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.



■ FIGURE P9.34



$$\mathscr{D} = \int \tau_w \, dA \quad \text{where} \quad \tau_w = 0.332\, U^{3/2} \sqrt{\frac{\rho \mu}{x}}$$

Thus,

$$\mathscr{D} = 0.332\, U^{3/2} \sqrt{\rho\mu} \int \frac{1}{\sqrt{x}}\, dA$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{x=0.5} \int_{y=0}^{y=0.5-x} \frac{dy\, dx}{\sqrt{x}}$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{0.5} \frac{0.5 - x}{\sqrt{x}}\, dx$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \left[0.5(2) x^{\frac{1}{2}} - \frac{2}{3} x^{3/2} \right]_0^{0.5}$$

$$= 0.664 \left(0.2 \frac{m}{s}\right)^{3/2} \sqrt{999 \frac{kg}{m^3} \left(1.12 \times 10^{-3} \frac{N \cdot s}{m^2}\right)} \left[\sqrt{0.5} - \frac{2}{3}(0.5)^{3/2} \right]$$

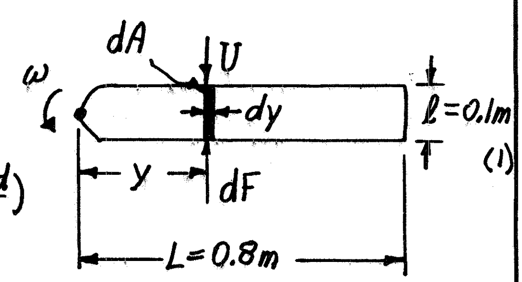
or

$$\mathscr{D} = \underline{\underline{0.0296\, N}}$$

9.36

9.36 A ceiling fan consists of five blades of 0.80-m length and 0.10-m width which rotate at 100 rpm. Estimate the torque needed to overcome the friction on the blades if they act as flat plates.

Let dM = torque from the drag on element dA of the blade



or

$$dM = (\mathcal{D}_{top} + \mathcal{D}_{bottom})y = 2(\tfrac{1}{2}\rho U^2 C_{Df}\, dA)y$$

where $U = \omega y$ and $\omega = 100\frac{rev}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$

or $\omega = 10.47\,\frac{rad}{s}$

The maximum Re_ℓ will occur at point (1) where $y = L$ or

$$Re_{\ell_1} = \frac{U\ell}{\nu} = \frac{\omega L \ell}{\nu} = \frac{(10.47\,\frac{rad}{s})(0.8m)(0.1m)}{1.46\times10^{-5}\,\frac{m^2}{s}} = 5.74\times10^4$$

Thus, at all points on the blade $Re_x < Re_{x\,cr} = 5\times10^5$ and the flow is laminar.

$$C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}$$

so that from Eq. (1)

$$dM = \rho U^2\,\frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\,(\ell\, dy)y = 1.328\,\rho U^{3/2}\sqrt{\nu\ell}\; y\,dy\text{, but with } U = \omega y$$

$$dM = 1.328\,\rho\,\omega^{3/2}\sqrt{\nu\ell}\; y^{5/2}\,dy$$

$$= 1.328\,(1.23\,\tfrac{kg}{m^3})\,(10.47\,\tfrac{rad}{s})^{3/2}\left[(1.46\times10^{-5}\,\tfrac{m^2}{s})(0.1m)\right]^{1/2} y^{5/2}\,dy$$

or

$$dM = 0.0669\; y^{5/2}\,dy \;\; N\cdot m\text{, where } y \sim m$$

Thus, the net torque on the four blades is

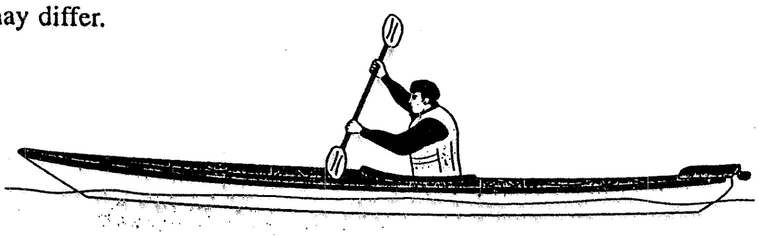
$$M = 5\int dM = 5\int_{y=0}^{0.8m} 0.0669\; y^{5/2}\,dy = 5(0.0669)\left(\tfrac{2}{7}\right) y^{7/2}\Big|_0^{0.8}$$

or

$$M = \underline{\underline{0.0438\; N\cdot m}}$$

9.37

9.37 As shown in Video V9.2 and Fig. P9.37*a*, a kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in Fig. P9.37*b*. Comment on reasons why the two sets of values may differ.



For a flat plate $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$ where (1)

$A = 17\,ft(2\,ft) = 34\,ft^2$ and C_{Df} is a function of $Re_\ell = \frac{U\ell}{\nu}$

Thus,

$$Re_\ell = \frac{17\,ft\,U}{1.21\times10^{-5}\,ft^2/s} = 1.40\times10^6\,U \quad (2)$$

Consider $1 \le U \le 8\,\frac{ft}{s}$, or $1.40\times10^6 \le Re_\ell \le 1.12\times10^7$

From Fig. 9.15 we see that in this Re_ℓ range the boundary layer flow is in the transitional range. Thus, from Table 9.3

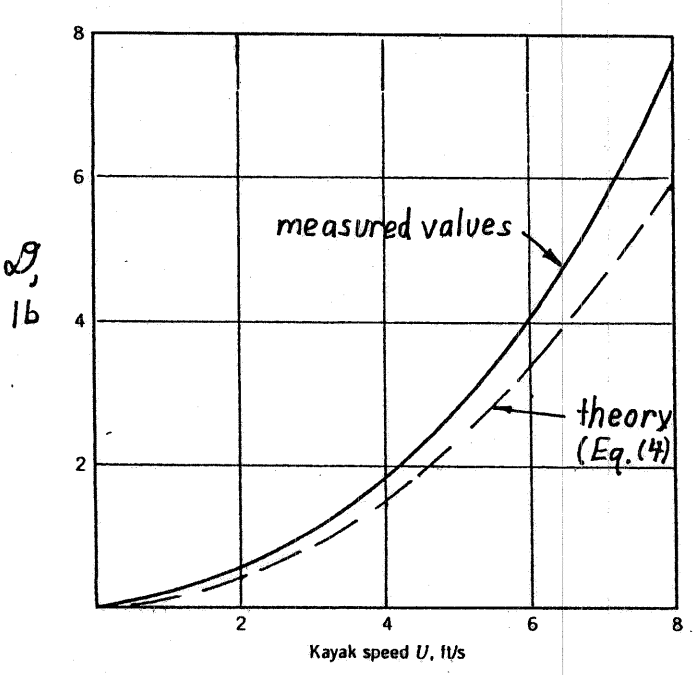
$$C_{Df} = 0.455/(\log Re_\ell)^{2.58} - 1700/Re_\ell \quad (3)$$

By combining Eqs. (1), (2), and (3):

$$\mathcal{D} = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)U^2 C_{Df}\,(34\,ft^2) \quad \text{or}$$

$$\mathcal{D} = 33.0\,U^2\left[0.455/(\log(1.40\times10^6 U))^{2.58} - 1700/(1.40\times10^6\,U)\right] \quad (4)$$

The results from this equation are plotted below.

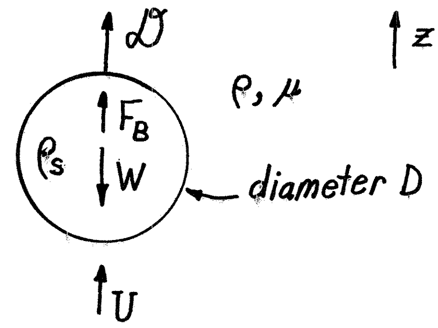


U, ft/s	$\mathcal{D}$, lb
1	0.0986
2	0.410
3	0.909
4	1.58
5	2.42
6	3.43
7	4.59
8	5.90

■ FIGURE P9.37

9.38

9.38 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, $\mathrm{Re} = \rho DU/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathcal{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathcal{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since $Re < 1$

$\mathcal{D} = 3\pi D U \mu$

Thus,

$$3\pi D U \mu + \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$$

which can be rearranged to give

$$\underline{\underline{\mu = \frac{g D^2(\rho_s - \rho)}{18\,U}}}$$

9.39

9.39 Determine the drag on a small circular disk of 0.01 ft diameter moving 0.01 ft/s through oil with a specific gravity of 0.87 and a viscosity 10,000 times that of water. The disk is oriented normal to the upstream velocity. By what percent is the drag reduced if the disk is oriented parallel to the flow?

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$, where $\rho = (0.87)(1.94\,\frac{slug}{ft^3}) = 1.688\,\frac{slugs}{ft^3}$ (1)

and $\mu = 10^4 \mu_{H_2O} = 10^4(2.34\times10^{-5}\,\frac{lb\cdot s}{ft^2}) = 0.234\,\frac{lb\cdot s}{ft^2}$

Thus, $Re = \frac{UD}{\nu} = \frac{\rho U D}{\mu} = \frac{(1.688\,\frac{slugs}{ft^3})(0.01\,\frac{ft}{s})(0.01\,ft)}{0.234\,\frac{lb\cdot s}{ft^2}} = 7.21\times10^{-4} \ll 1$

so that the low Re data of Table 9.4 is valid.

For the disk normal to the flow, $C_D = \frac{20.4}{Re} = \frac{20.4}{7.21\times10^{-4}} = 28{,}300$

so that from Eq.(1)

$\mathcal{D} = 28{,}300(\frac{1}{2})(1.688\,\frac{slugs}{ft^3})(0.01\,\frac{ft}{s})^2\frac{\pi}{4}(0.01\,ft)^2 = \underline{\underline{1.88\times10^{-4}\,lb}}$

If the disk is parallel to the flow, $C_D = \frac{13.6}{Re}$ so that

$$\frac{\mathcal{D}_{parallel}}{\mathcal{D}_{normal}} = \frac{C_{D\,parallel}}{C_{D\,normal}} = \frac{\left(\frac{13.6}{Re}\right)}{\left(\frac{20.4}{Re}\right)} = 0.667,$$ a $\underline{\underline{33.3\,\%\ \text{reduction}}}$

9.40

9.40 For small Reynolds number flows the drag coefficient of an object is given by a constant divided by the Reynolds number (see Table 9.4). Thus, as the Reynolds number tends to zero, the drag coefficient becomes infinitely large. Does this mean that for small velocities (hence, small Reynolds numbers) the drag is very large? Explain.

For a given object $C_D = \frac{C}{Re}$ (where the value of C depends on the shape of the object), provided $Re \lesssim 1$. Thus, as $Re \to 0$, $C_D \to \infty$.

However,

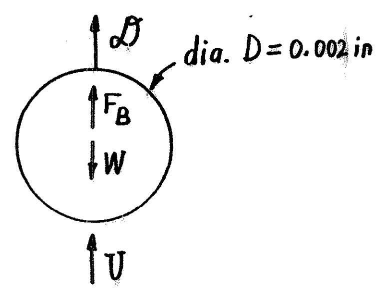
$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = \frac{C}{\left(\frac{\rho U \ell}{\mu}\right)}\frac{1}{2}\rho U^2 A \sim U$

That is, as $U \to 0$ (i.e. $Re \to 0$), then $\mathcal{D} \sim U$

Thus, does $C_D \to \infty$ mean that $\mathcal{D} \to \infty$? $\underline{\underline{No.}}$

9.41

9.41 A small spherical water drop of diameter 0.002 in. exists in the atmosphere at 5000-ft altitude. Will the drop rise or fall if it is in a thermal (an upward flowing column of air) having a speed of 4 ft/s? Repeat for speeds of 1 ft/s and 0.1 ft/s.



In stationary air the particle falls with speed U such that $\mathcal{D} + F_B = W$, where if $Re = \frac{UD}{\nu} < 1$ then

$$\mathcal{D} = \text{drag} = 3\pi D U \mu \quad \text{Also,} \quad W = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \text{weight} \qquad (1)$$

$$\text{and} \quad F_B = \gamma_{air} \forall = \gamma_{air} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = \text{buoyant force}$$

Since $\gamma_{air} << \gamma_{H_2O}$ we can neglect the buoyant force. That is, $\mathcal{D} = W$, or

$$3\pi D U \mu = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \quad \text{or} \quad U = \frac{\gamma_{H_2O} D^2}{18\mu}$$

At an altitude of 5000 ft, $\mu = 3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}$

so that

$$U = \frac{(62.4 \frac{lb}{ft^3})\left(\frac{0.002}{12} ft\right)^2}{18\left(3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} = 0.265 \frac{ft}{s}$$

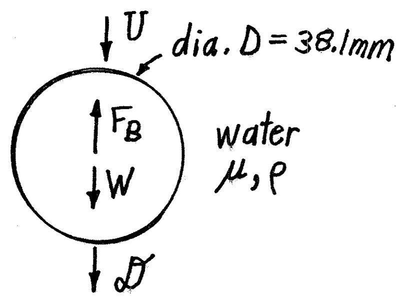
Thus, the drop will rise if the upward velocity is $4 \frac{ft}{s}$ or $1 \frac{ft}{s}$, but it will fall if it is $0.1 \frac{ft}{s}$.

Note: The above is correct if $Re < 1$. Since $Re = \frac{\rho U D}{\mu}$

or $Re = \frac{(2.048 \times 10^{-3} \frac{slug}{ft^3})(0.265 \frac{ft}{s})(\frac{0.002}{12} ft)}{3.637 \times 10^{-7} \frac{lb \cdot s}{ft^2}} = 0.249$ the low Re drag equation, Eq. (1), is valid.

9.42

9.42 A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.



For steady rise $\Sigma F_z = 0$

or

$$F_B = W + \mathcal{D}, \text{ where } \mathcal{D} = \text{drag} = C_D \tfrac{1}{2}\rho U^2 \tfrac{\pi}{4} D^2$$

$$W = \text{weight} = 0.0245\ N$$

$$F_B = \text{buoyant force} = \gamma \forall = \gamma \tfrac{4\pi}{3}\left(\tfrac{D}{2}\right)^3$$

Thus,

$$\gamma \tfrac{4\pi}{3}\left(\tfrac{D}{2}\right)^3 = W + C_D \tfrac{1}{2}\rho U^2 \tfrac{\pi}{4} D^2$$

or

$$\left(9.80\times10^3 \tfrac{N}{m^3}\right)\tfrac{4\pi}{3}\left(\tfrac{0.0381}{2}\right)^3 m^3 = 0.0245\ N + \tfrac{1}{2} C_D \left(999\ \tfrac{kg}{m^3}\right) U^2 \tfrac{\pi}{4}(0.0381\ m)^2$$

or

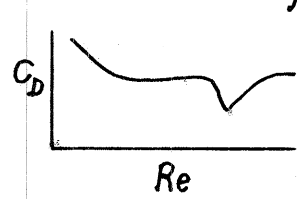
$$C_D U^2 = 0.455, \text{ where } U \sim \tfrac{m}{s} \qquad (1)$$

Also, $Re = \frac{UD}{\nu}$

or

$$Re = \frac{U\,(0.0381\,m)}{1.12\times10^{-6}\,\frac{m^2}{s}} = 3.40\times10^4\, U, \text{ where } U \sim \tfrac{m}{s} \qquad (2)$$

Finally, from Fig. 9.21:



(3)

Trial and error solution: Assume C_D; obtain U from Eq. (1), Re from Eq. (2); check C_D from Eq. (3), the graph.

Assume $C_D = 0.5 \longrightarrow U = 0.954\ \frac{m}{s} \longrightarrow Re = 3.24\times10^4 \longrightarrow C_D = 0.4 \neq 0.5$

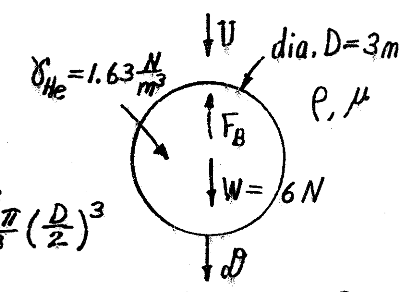
Assume $C_D = 0.4 \longrightarrow U = 1.06\ \frac{m}{s} \longrightarrow Re = 3.62\times10^4 \longrightarrow C_D = 0.4$ (checks)

Thus, $U = \underline{\underline{1.06\ \frac{m}{s}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.

*9.44

***9.44** A 3-m-diameter meteorological balloon that weighs 6 N when deflated is filled with helium having $\gamma = 1.63\ N/m^3$. Plot a graph of the rate at which it rises as a function of altitude for altitudes from sea level to its maximum altitude. Assume standard atmospheric conditions (Table C.2) and that the diameter remains constant.*



For steady rise $\sum F_z = 0$ or $F_B = W + \mathcal{D} + W_{He}$

where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$ and $F_B = \gamma \forall = \rho g \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

Thus,

$$\rho g \frac{4\pi}{3}\left(\frac{3}{2}m\right)^3 = 6\ N + C_D \frac{1}{2}\rho U^2 \frac{\pi}{4}(3\ m)^2 + \left(1.63 \frac{N}{m^3}\right)\frac{4\pi}{3}\left(\frac{3}{2}m\right)^3$$

or

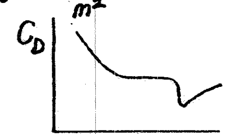
$$C_D U^2 = 4.00g - \frac{8.22}{\rho}, \quad \text{where } U \sim \frac{m}{s}, \ \rho \sim \frac{kg}{m^3} \qquad (1)$$

Also, $Re = \frac{\rho U D}{\mu}$

or

$$Re = \frac{3\rho U}{\mu}, \quad \text{where } \mu \sim \frac{N \cdot s}{m^2} \qquad (2)$$

Finally, from Fig. 9.23 (3)



For a given altitude obtain ρ, μ from Table C.2. Then a trial and error solution for U: Assume C_D; obtain U from Eq. (1), Re from Eq. (2); check C_D from Eq. (3), the graph.

Note: The maximum altitude will occur when $U = 0$ (i.e., $\mathcal{D} = 0$ so that $F_B = W + W_{He}$). From Eq. (2) this gives $\rho = \frac{2.06}{g} \approx 0.21 \frac{kg}{m^3}$

This occurs at an altitude of approximately $z \approx 15{,}000\ m$.

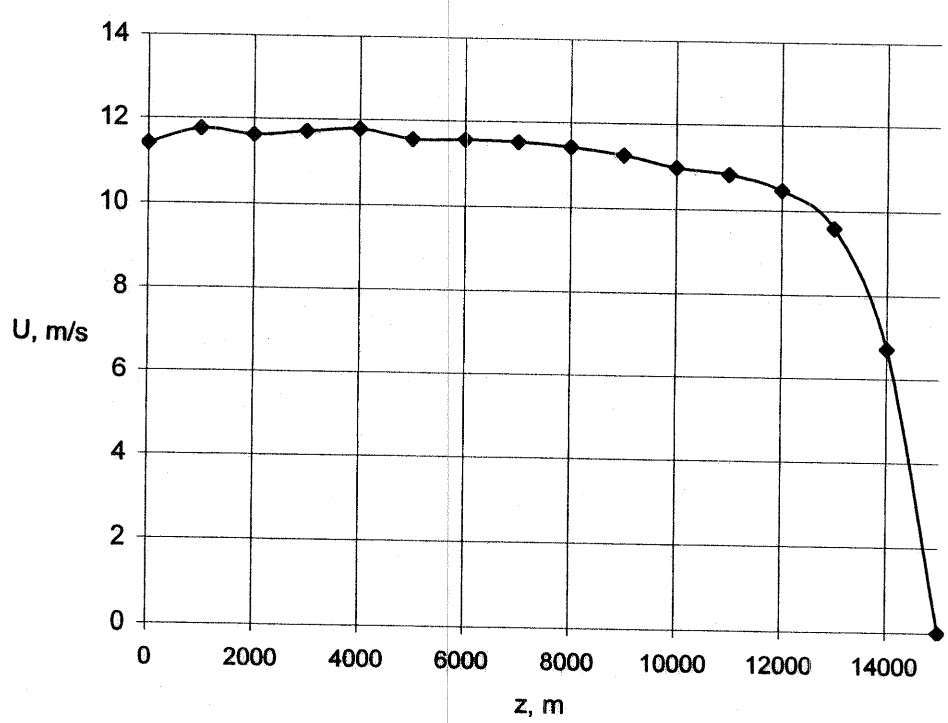
Since Fig. 9.23 is not in equation form, combine a computer/graph solution: a) select an altitude $z < 15{,}000\ m$; b) lookup ρ, μ (see table C.2 ; c) assume a value of C_D and calculate U from Eq. (1); d) calculate Re from Eq. (2) and lookup C_D in Fig. 9.23; e) compare new C_D value with the assumed one — iterate until they agree; f) back to step a). The results are shown below.

* In an actual weather balloon the diameter increases considerably as it rises (i.e., as the surrounding pressure decreases.

(con't)

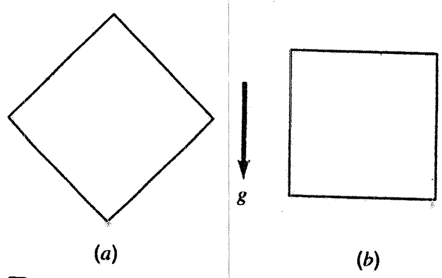
9.44 (con't)

z, m	U, m/s
0	11.41
1000	11.76
2000	11.62
3000	11.71
4000	11.78
5000	11.55
6000	11.56
7000	11.52
8000	11.42
9000	11.25
10000	10.97
11000	10.83
12000	10.47
13000	9.59
14000	6.72
15000	0

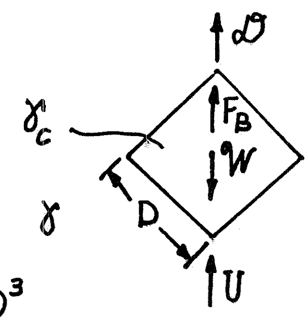


9.45

9.45 A 500-N cube of specific gravity $SG = 1.8$ falls through water at a constant speed U. Determine U if the cube falls (a) as oriented in Fig. P9.45*a*, (b) as oriented in Fig. P9.45*b*.



■ **FIGURE P9.50**



For steady fall, $\Sigma F = ma = 0$

or

$W = \mathcal{D} + F_B$, where $W = \text{weight} = 500\,N$ (1)

$F_B = \text{buoyant force} = \gamma D^3$

and $\mathcal{D} = \frac{1}{2}\rho U C_D A = \text{drag}$

But,

$W = \gamma_c D^3 = SG\,\gamma D^3$, or $500\,N = 1.8(9.80\times10^3 \frac{N}{m^3})D^3$

Thus, $D = 0.305\,m$ so that from Eq. (1)

$500\,N = \frac{1}{2}(999 \frac{kg}{m^3})U^2 C_D (0.305\,m)^2 + (9.80\times10^3 \frac{N}{m^3})(0.305\,m)^3$

or

$U^2 C_D = 4.78$ where $U \sim \frac{m}{s}$

(a) For case (a) $C_D = 0.80$ (see Fig. 9.29)

Hence, $U = \left(\frac{4.78}{0.80}\right)^{1/2} = \underline{\underline{2.44 \frac{m}{s}}}$

(b) For case (b) $C_D = 1.05$

Hence, $U = \left(\frac{4.78}{1.05}\right)^{1/2} = \underline{\underline{2.13 \frac{m}{s}}}$

9.46

9.46 A snowflake with a diameter of 0.15 in. is observed to fall through still air with a speed of 2.5 ft/s. The drag coefficient is assumed to be 1.5. **(a)** Estimate the weight of the snowflake. **(b)** Estimate the number of these snowflakes it would take, when melted, to fill a one-gallon jug with water.

(a) If the falling speed is constant, then the weight is balanced by the drag.

$$\mathcal{W} = \mathcal{D} = \frac{1}{2}\rho U^2 A C_D$$

$$= \frac{1}{2}\left(0.00237\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft}{s}\right)^2 \frac{\pi}{4}\left(\frac{0.15}{12}\ ft\right)^2 (1.5) = 1.36\times 10^{-6}\ slug\ \frac{ft}{s^2}$$

or

$$\mathcal{W} = \underline{\underline{1.36\times 10^{-6}\ lb}}$$

(b) Since 1 gallon $= 231\ in.^3 = 0.1337\ ft^3$ it follows that the weight of 1 gallon of water is $\mathcal{W}_{gal} = \mathcal{V}_{gal}\,\gamma$, or

$$\mathcal{W}_{gal} = 0.1337\ ft^3\left(62.4\ \frac{lb}{ft^3}\right) = 8.34\ lb$$

Thus if n = number of snowflakes that it takes to make a gallon of water,

$$n\,\mathcal{W}_{flake} = \mathcal{W}_{gal},$$

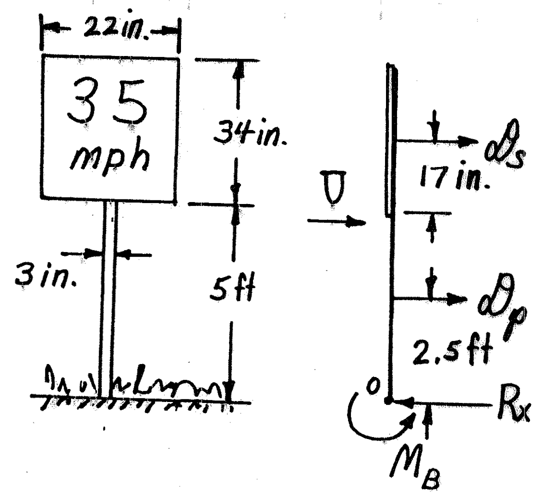
or

$$n = \frac{\mathcal{W}_{gal}}{\mathcal{W}_{flake}} = \frac{8.34\ lb}{1.36\times 10^{-6}\ lb} = \underline{\underline{6.13\times 10^{6}}}$$

That is, 6.13 million snowflakes per gallon!

9.47

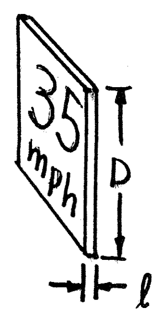
9.47 A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.6.) List any assumptions used in your calculations.



For equalibrium, $\Sigma M_0 = 0$ or

$$M_B = 2.5\,\text{ft}\,\mathcal{D}_p + \left(5 + \tfrac{17}{12}\right)\text{ft}\,\mathcal{D}_s \text{, where} \tag{1}$$

$\mathcal{D}_p$ = drag on the pole and $\mathcal{D}_s$ = drag on the sign



From Fig. 9.28 with $\ell/D < 0.1$ for the sign,

$C_{D_s} = 1.9$

From Fig. 9.19 if the post acts as a square rod with sharp corners $C_{D_p} = 2.2$ Thus, with $U = 30\,\text{mph} = 44\,\frac{\text{ft}}{\text{s}}$,

$$\mathcal{D}_s = \tfrac{1}{2}\rho U^2 C_{D_s} A_s = \tfrac{1}{2}\left(0.00238\,\tfrac{\text{slugs}}{\text{ft}^3}\right)\left(44\,\tfrac{\text{ft}}{\text{s}}\right)^2(1.9)\left(\tfrac{22\,(34)}{144}\,\text{ft}^2\right) = 22.7\,\text{lb}$$

and

$$\mathcal{D}_p = \tfrac{1}{2}\rho U^2 C_{D_p} A_p = \tfrac{1}{2}\left(0.00238\,\tfrac{\text{slugs}}{\text{ft}^3}\right)\left(44\,\tfrac{\text{ft}}{\text{s}}\right)^2(2.2)\left(\tfrac{3}{12}(5)\,\text{ft}^2\right) = 6.34\,\text{lb}$$

Thus, from Eq. (1):

$$M_B = 2.5\,\text{ft}\,(6.34\,\text{lb}) + \left(5 + \tfrac{17}{12}\right)\text{ft}\,(22.7\,\text{lb}) = \underline{\underline{162\ \text{ft}\cdot\text{lb}}}$$

9.51

9.51 If for a given vehicle it takes 20 hp to overcome aerodynamic drag while being driven at 65 mph, estimate the horsepower required at 75 mph.

$\mathcal{P} = \text{power} = U\mathcal{D} = U C_D \frac{1}{2}\rho U^2 A = C_D \frac{1}{2}\rho U^3 A$

Thus,

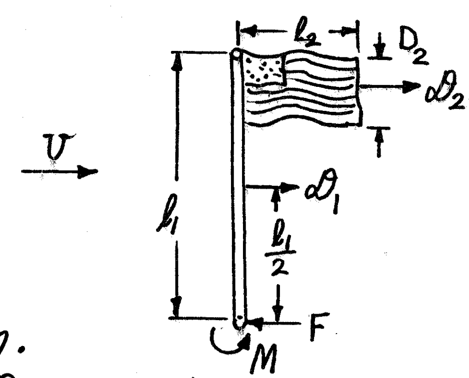
$$\frac{\mathcal{P}_{75}}{\mathcal{P}_{65}} = \frac{C_D \frac{1}{2}\rho (65)^3 A}{C_D \frac{1}{2}\rho (55)^3 A} = \left(\frac{75}{65}\right)^3 = 1.54,$$ provided the values of C_D are independent of U (i.e., Re).

Hence,

$\mathcal{P}_{75} = 1.54\,\mathcal{P}_5 = 1.54(20\,hp) = \underline{\underline{30.8\,hp}}$

9.49

9.49 Repeat Problem 9.48 if a 2-m by 2.5-m flag is attached to the top of the pole. See Fig. 9.30 for drag coefficient data for flags.



For equilibrium, $M = \frac{\ell_1}{2}\mathscr{D}_1 + \left(\ell_1 - \frac{D_2}{2}\right)\mathscr{D}_2$ (1)

where $\ell_1 = 20\,m$, $\ell_2 = 2.5\,m$, and $D_2 = 2\,m$.

From the solution to Problem 9.48, $\frac{\ell_1}{2}\mathscr{D}_1 = 7{,}080\ N\cdot m$ (2)

Also,

$\mathscr{D}_2 = C_D \frac{1}{2}\rho V^2 \ell_2 D_2$, where from Fig. 9.30 with $\frac{\ell_2}{D_2} = \frac{2.5}{2} = 1.25$

we obtain $C_D = 0.08$.

Thus,

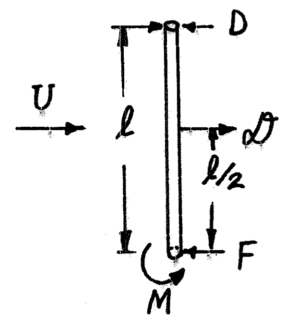
$\mathscr{D}_2 = 0.08\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(20\frac{m}{s}\right)^2(2.5\,m)(2\,m) = 98.4\,N$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$M = 7{,}080\ N\cdot m + (20\,m - 1\,m)(98.4\,N) = \underline{\underline{8{,}950\ N\cdot m}}$

9.48

9.48 Determine the moment needed at the base of 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.



For equilibrium, $M = \frac{\ell}{2}\mathcal{D}$ where (1)

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 \ell D$

Since $Re = \frac{UD}{\nu} = \frac{(20\frac{m}{s})(0.12m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.64\times10^5$, it follows from Fig. 9.21

that $C_D = 1.2$

Thus, $\mathcal{D} = 1.2\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(20\frac{m}{s})^2(20m)(0.12m) = 708N$

Hence, from Eq. (1)

$M = \frac{20m}{2}(708N) = \underline{\underline{7{,}080\ N\cdot m}}$

9.52

9.52 How much more power is required to peddle a bicycle at 15 mph into a 20-mph headwind than at 15 mph through still air? Assume a frontal area of 3.9 ft² and a drag coefficient of $C_D = 0.88$.

$\mathcal{P}$ = power = $U_B \mathcal{D}$ and $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$, where U_B = speed of the bike $= 15 \frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 22 \frac{ft}{s}$

and U = wind speed relative to bike.

Thus,

$$\mathcal{P} = \left(22 \frac{ft}{s}\right)(0.88)\left(\frac{1}{2}\right)\left(0.00238 \frac{slugs}{ft^3}\right) U^2 (3.9 \, ft^2) = 0.0898\, U^2 \ \frac{ft \cdot lb}{s} \quad (1)$$

with $U \sim \frac{ft}{s}$

a) With a 20 mph headwind, $U = (15 + 20)\frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 51.3 \frac{ft}{s}$

Thus,

$$\mathcal{P}_a = 0.0898\,(51.3)^2 = 236 \ \frac{ft \cdot lb}{s}$$

b) With still air, $U = 15\, mph = 22 \frac{ft}{s}$

Thus,

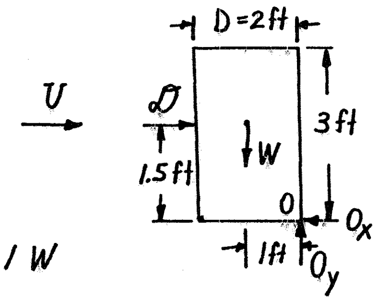
$$\mathcal{P}_b = 0.0898\,(22)^2 = 43.5 \ \frac{ft \cdot lb}{s}$$

Hence, need an additional power of $\mathcal{P}_a - \mathcal{P}_b = (236 - 43.5)\frac{ft \cdot lb}{s}\left(\frac{1\, hp}{550 \frac{ft \cdot lb}{s}}\right)$

$$= \underline{\underline{0.350 hp}}$$

9.53

9.53 Estimate the wind velocity necessary to knock over a 10-lb garbage can that is 3 ft tall and 2 ft in diameter. List your assumptions.



If the can is about to tip around corner O, then $\Sigma M_0 = 0$, or $1.5\mathscr{D} = 1\,W$

or $1.5\, C_D \frac{1}{2}\rho U^2 A = W$ A typical value of C_D for a cylinder is $C_D \approx 1$ (see Fig. 9.21)

Thus,

$(1.5\text{ ft})(1)(\frac{1}{2})(0.00238 \frac{\text{slugs}}{\text{ft}^3})\, U^2 (2\text{ ft})(3\text{ ft}) = 10\text{ft}\cdot\text{lb}$, where $U \sim \frac{\text{ft}}{\text{s}}$

or $U = \underline{\underline{30.6 \frac{\text{ft}}{\text{s}}}}$

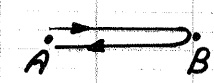
9.54

9.54 On a day without any wind, your car consumes x gallons of gasoline when you drive at a constant speed, U, from point A to point B and back to point A. Assume that you repeat the journey, driving at the same speed, on another day when there is a steady wind blowing from B to A. Would you expect your fuel consumption to be less than, equal to, or greater than x gallons for this windy round-trip? Support your answer with appropriate analysis.

Trip with the larger power lost due to aerodynamic drag will use the most gas. Let $(\ )_1$ mean "no wind" and $(\ )_2$ mean "wind".

(1) No wind:

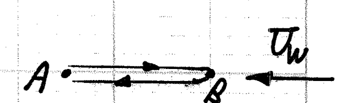
$$\mathcal{D}_1 = C_D \tfrac{1}{2}\rho U^2 A \quad \text{for both } A \rightarrow B \text{ and } B \rightarrow A$$



Thus,

$$\mathcal{P}_1 = \text{power} = U\mathcal{D}_1 = \tfrac{1}{2}\rho U^3 C_D A$$

(2) Wind (U_w = wind speed; assume $U_w < U$):



$$\mathcal{D}_2 = C_D \tfrac{1}{2}\rho (U+U_w)^2 A \quad \text{for } A \rightarrow B$$

$$\mathcal{D}_2 = C_D \tfrac{1}{2}\rho (U-U_w)^2 A \quad \text{for } B \rightarrow A$$

Thus,

$$\mathcal{P}_2 = \tfrac{1}{2}\rho (U+U_w)^2 U C_D A \quad \text{for } A \rightarrow B$$

$$\mathcal{P}_2 = \tfrac{1}{2}\rho (U-U_w)^2 U C_D A \quad \text{for } B \rightarrow A$$

Energy used $= \mathcal{P}t$, where t = time to go from $A \rightarrow B$ or $B \rightarrow A$

Thus,

$$E_1 = 2\left(\tfrac{1}{2}\rho U^3 C_D A\right)t \qquad \text{(Note: Factor of 2 for } A \rightarrow B + B \rightarrow A\text{)}$$

and

$$E_2 = \tfrac{1}{2}\rho (U+U_w)^2 U C_D A t + \tfrac{1}{2}\rho (U-U_w)^2 U C_D A t$$

Thus,

$$\frac{E_1}{E_2} = \frac{2U^3}{(U+U_w)^2 U + (U-U_w)^2 U} = \frac{2U^3}{2U^3 + 2U_w^2 U} = \frac{1}{1+(U_w/U)^2} < 1$$

Hence,

$$\frac{E_1}{E_2} < 1, \text{ i.e. } \underline{\underline{\text{more fuel needed when windy}}}$$

9.55

9.55 By paying close attention to design details, the drag coefficient for a typical car has been reduced over the years as indicated in Fig. 9.27. How fast can one drive a 2005 style car if it is to have the same aerodynamic drag as a 1940 model being driven at 65 mph. Assume that the frontal area of the 2005 model is 85% that of the 1940 model.

$$\mathscr{D}_{40} = \mathscr{D}_{05}$$

or

$$\frac{1}{2}\rho U_{40}^2 A_{40} C_{D40} = \frac{1}{2}\rho U_{05}^2 A_{05} C_{D05}, \quad \text{where } U_{40} = 65 \text{ mph},$$

$A_{05} = 0.85 A_{40}$ and from Fig. 9.27

$C_{D40} = 0.57$, $C_{D05} = 0.30$

Thus,

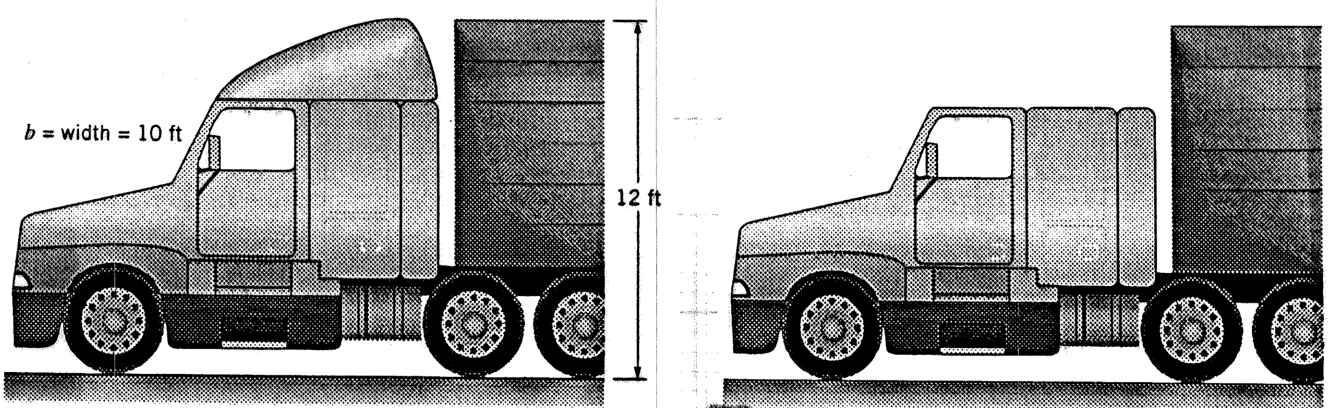
$$U_{40}^2 A_{40} C_{D40} = U_{05}^2 A_{05} C_{D05} \quad \text{or}$$

$$U_{05} = \left[\frac{A_{40} C_{D40}}{A_{05} C_{D05}}\right]^{1/2} U_{40} = \left[\frac{0.57}{0.85(0.30)}\right]^{1/2} (65 \text{ mph}) = \underline{\underline{97.2 \text{ mph}}}$$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.56

9.56 As shown in Video V9.8 and Fig. P9.56, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction of how many horsepoer needed at a highway speed of 65 mph?



(a) $C_D = 0.70$ (b) $C_D = 0.96$

■ FIGURE P9.56

$\mathcal{P}$ = power = $\mathcal{D}U$ where

$\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$

Thus, $\Delta\mathcal{P}$ = reduction in power

$= \mathcal{P}_b - \mathcal{P}_a$

$= \frac{1}{2}\rho U^3 A\left[C_{D_b} - C_{D_a}\right]$

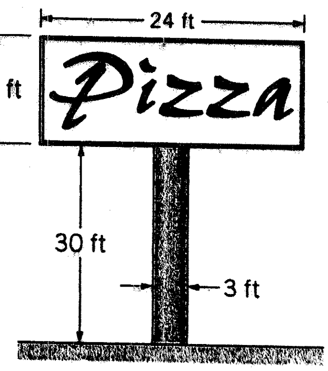
With $U = 65$ mph $= 95.3$ fps,

$$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left(95.3\,\frac{ft}{s}\right)^3(10\,ft)(12\,ft)\left[0.96-0.70\right]$$

$$= 32{,}100\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{58.4\,hp}}$$

9.57

9.57 The structure shown in Fig. P9.57 consists of a cylindrical support post to which a rectangular flat-plate sign is attached. Estimate the drag on the structure when a 50-mph wind blows against it.



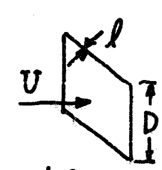
■ FIGURE P9.57

$\mathscr{D} = \mathscr{D}_{sign} + \mathscr{D}_{post}$, where $\mathscr{D}_{sign} = \frac{1}{2}\rho V^2 A_{sign} C_{D\,sign}$ and (1)

$\mathscr{D}_{post} = \frac{1}{2}\rho V^2 A_{post} C_{D\,post}$

Also, $A_{sign} = 12\,ft\,(24\,ft) = 288\,ft^2$ and

$A_{post} = 3\,ft\,(30\,ft) = 90\,ft^2$



From Fig. 9.28, for a thin flat plate with $\ell/D \lesssim 0.1$, $C_D = 1.9$

Thus, $C_{D\,sign} = 1.9$

Also, for the cylinder (post), $Re = \frac{VD}{\nu}$, where

$V = 50\frac{mi}{hr}\left(\frac{1\,hr}{(60)^2 s}\right)\frac{5280\,ft}{1\,mi} = 73.3\,\frac{ft}{s}$

so that,

$Re = \dfrac{(73.3\,\frac{ft}{s})(3\,ft)}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 1.40\times10^6$

Hence, from Fig. 9.21, $C_{D\,post} = 0.8$

By using the above data, Eq. (1) gives

$\mathscr{D} = \frac{1}{2}\rho V^2\left[A_{sign}C_{D\,sign} + A_{post}C_{D\,post}\right]$

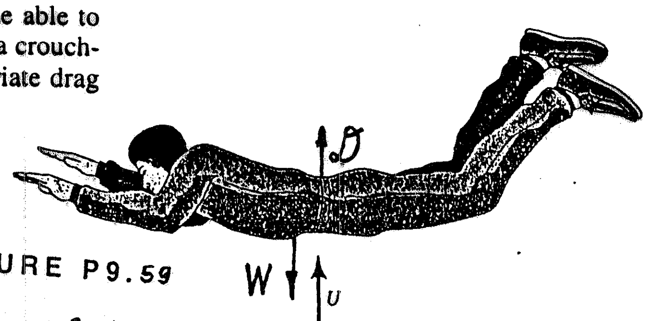
$= \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})(73.3\,\frac{ft}{s})^2\left[288\,ft^2(1.9) + 90\,ft^2(0.8)\right]$

or

$\mathscr{D} = \underline{\underline{3{,}960\,lb}}$

9.59

9.59 As shown in Video V9.5 and Fig. P9.59, a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) curls up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.



■ FIGURE P9.59

For equilibrium conditions

$W = \mathscr{D} = C_D \frac{1}{2} \rho U^2 A$

Assume $W = 160\ lb$ and $C_D A = 9\ ft^2$ (see Fig. 9.30)

Thus,

$160\ lb = (\frac{1}{2})(0.00238 \frac{slugs}{ft^3})\, U^2 (9\ ft^2)$ where $U \sim \frac{ft}{s}$

or

$U = (122 \frac{ft}{s})(\frac{1\ mi}{5280\ ft})(\frac{3600\ s}{1\ hr}) = \underline{\underline{83.2\ mph}}$

Note: If the skydiver "curled up into a ball", then $C_D A \approx 2.5\ ft$ (see Fig. 9.30) and $U = 158\ mph$

9.60*

9.60* The helium-filled balloon shown in Fig. P9.60 is to be used as a wind speed indicator. The specific weight of the helium is $\gamma = 0.011\ lb/ft^3$, the weight of the balloon material is 0.20 lb, and the weight of the anchoring cable is negligible. Plot a graph of θ as a function of U for $1 \le U \le 50$ mph. Would this be an effective device over the range of U indicated? Explain.

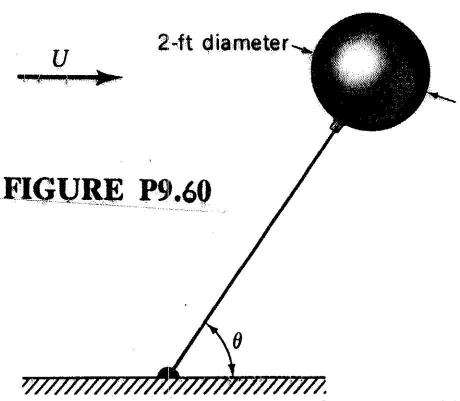


FIGURE P9.60

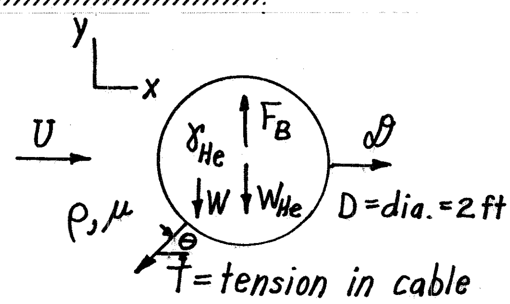
For the balloon to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathcal{D} = T\cos\theta$ or $T = \dfrac{\mathcal{D}}{\cos\theta}$

and $F_B = W + T\sin\theta + W_{He}$

which combine to give



$$F_B = W + \mathcal{D}\tan\theta + W_{He} \qquad (1)$$

But $W = 0.2\ lb$, $F_B = \rho g \forall = (7.65 \times 10^{-2} \frac{lb}{ft^3}) \frac{4\pi}{3}(\frac{2}{2}ft)^3 = 0.3204 lb$

and $W_{He} = \gamma_{He}\forall = (0.011 \frac{lb}{ft^3})\frac{4\pi}{3}(\frac{2}{2}ft)^3 = 0.0461\ lb$

Thus, Eq. (1) becomes

$0.3204 lb = 0.2\ lb + \mathcal{D}\tan\theta + 0.0461\ lb$

or

$\mathcal{D}\tan\theta = 0.0743\ lb$

Also, $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$= C_D U^2 (0.00238 \frac{slugs}{ft^3}) \frac{\pi}{8}(2 ft)^2$

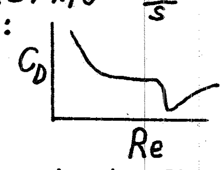
$= 0.00374 C_D U^2\ lb$, where $U \sim \frac{ft}{s}$

Hence,

$$0.00374 C_D U^2 \tan\theta = 0.0743 \quad \text{or} \quad \tan\theta = \frac{19.9}{C_D U^2} \qquad (2)$$

$$\text{Also, } Re = \frac{UD}{\nu} = \frac{(2\ ft)\,U}{1.57 \times 10^{-4} \frac{ft^2}{s}} \quad \text{or} \quad Re = 1.27 \times 10^4\, U \qquad (3)$$

and from Fig. 9.21:



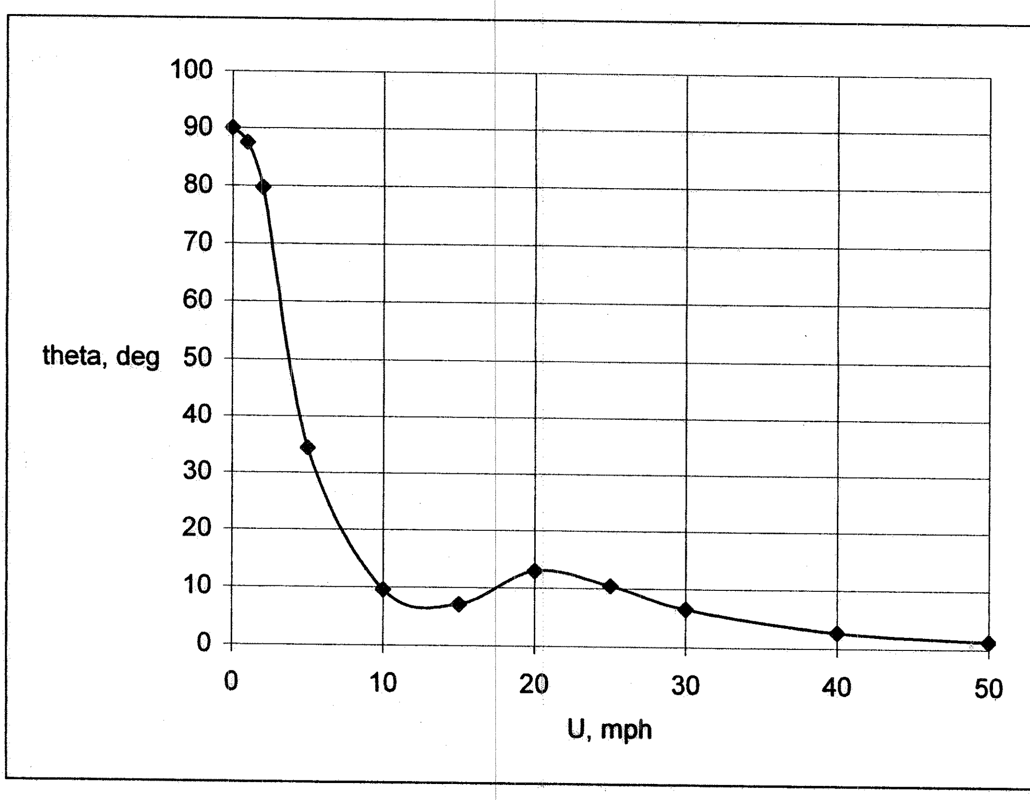
(4)

Thus, select various 1 mph ≤ U ≤ 50 mph (i.e., $1.47 \frac{ft}{s} \le U \le 73.3 \frac{ft}{s}$) and use Eqs. (2), (3), (4) to obtain θ. Plotted results are shown below.

(con't)

9.60* (con't)

U, mph	Re	CD	Θ, deg
0	0	---	90
1	12700	0.40	87.52
2	25400	0.42	79.71
5	63500	0.54	34.42
10	127000	0.55	9.55
15	190500	0.33	7.10
20	254000	0.10	13.02
25	317500	0.08	10.48
30	381000	0.09	6.52
40	508000	0.12	2.76
50	635000	0.16	1.32



Note: Because of the sudden change in C_D when the boundary layer becomes turbulent (at about 15 mph), the θ vs U curve is highly non-linear. In fact, for some values of θ there is more than one possible value of U. It would not work well as a wind speed indicator in this range.

9.61

9.61 A 0.30-m-diameter cork ball ($SG = 0.21$) is tied to an object on the bottom of a river as is shown in Fig. P9.61. Estimate the speed of the river current. Neglect the weight of the cable and the drag on it.

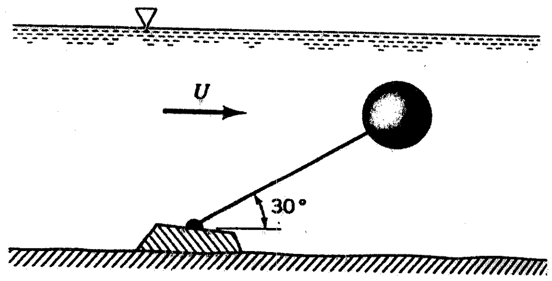
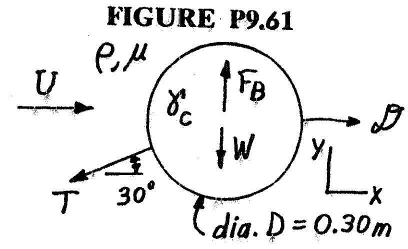


FIGURE P9.61



For the ball to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathcal{D} = T\cos 30°$ or $T = \dfrac{\mathcal{D}}{\cos 30°}$

and

$$F_B = W + T\sin 30°$$

Hence, $F_B = W + \mathcal{D}\tan 30°$, where $F_B = \rho g \forall = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{4\pi}{3}\left(\frac{0.30}{2}m\right)^3\right)$

$= 0.1385\ kN$

and

$W = \gamma_c \forall = \left(\frac{\gamma_c}{\gamma}\right)\gamma\forall = (SG)F_B$

$= 0.21(0.1385\ kN)$

$= 0.0291\ kN$

Thus,

$0.1385\ kN = 0.0291\ kN + \mathcal{D}\tan 30°$

or

$\mathcal{D} = 0.189\ kN$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = C_D U^2\left(\frac{1}{2}\right)\left(999\,\frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.3m)^2\right)$

$= 35.3\, C_D U^2\ N$, where $U \sim \frac{m}{s}$

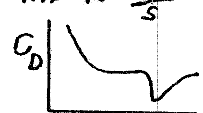
Hence

$35.3 C_D U^2 = 189$ or $C_D U^2 = 5.35$ (1)

Also, $Re = \dfrac{UD}{\nu} = \dfrac{(0.3m)U}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5\,U$ (2)

and

from Fig. 9.21 (3)



Trial and error solution for U: Assume C_D; calculate U from Eq.(1) and Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 3.27\,\frac{m}{s} \rightarrow Re = 8.76\times10^5 \rightarrow C_D = 0.15 \neq 0.5$

Assume $C_D = 0.15 \rightarrow U = 5.97\,\frac{m}{s} \rightarrow Re = 1.60\times10^6 \rightarrow C_D = 0.20 \neq 0.15$

Assume $C_D = 0.19 \rightarrow U = 5.31\,\frac{m}{s} \rightarrow Re = 1.42\times10^6 \rightarrow C_D = 0.19$ (checks)

Thus, $\underline{\underline{U = 5.31\,\frac{m}{s}}}$

9.62

9.62 Air flows past two equal sized spheres (one rough, one smooth) that are attached to the arm of a balance as is indicated in Fig. P9.62. With $U = 0$ the beam is balanced. What is the minimum air velocity for which the balance arm will rotate clockwise?

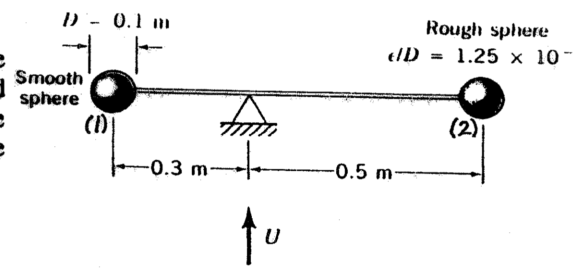


FIGURE P9.62

For clockwise rotation to start, $\Sigma M_0 < 0$

That is $0.3\,\mathscr{D}_1 \geq 0.5\,\mathscr{D}_2$, where $\mathscr{D}_1 = C_{D_1}\frac{1}{2}\rho U_1^2 A_1$ and

$\mathscr{D}_2 = C_{D_2}\frac{1}{2}\rho U_2^2 A_2$

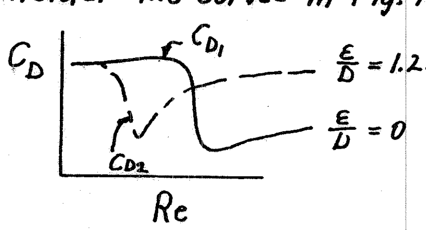
Thus,

$0.3\, C_{D_1}\frac{1}{2}\rho U_1^2 A_1 = 0.5\, C_{D_2}\frac{1}{2}\rho U_2^2 A_2$, or since $U_1 = U_2$ and $A_1 = A_2$

this gives

$$C_{D_2} = 0.6\, C_{D_1} \qquad (1)$$

Consider the curves in Fig. 9.25 with $\frac{\varepsilon}{D} = 0$ and $\frac{\varepsilon}{D} = 1.25 \times 10^{-2}$



Trial and error solution to find Re so that Eq. (1) is satisfied.

Assume $Re = 6\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.46$ or $\frac{C_{D2}}{C_{D_1}} = 0.92 \neq 0.6$

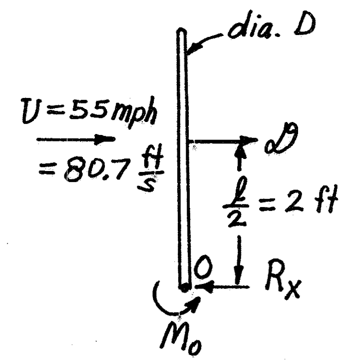
Assume $Re = 8\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.21$ or $\frac{C_{D2}}{C_{D_1}} = 0.42 \neq 0.6$

Assume $Re = 7\times10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.33$ or $\frac{C_{D2}}{C_{D_1}} = 0.66 \approx 0.6$

Thus, $Re \approx 7.1\times10^4 = \frac{UD}{\nu} = \frac{(0.1\,m)\,U}{1.46\times10^{-5}\,\frac{m^2}{s}}$ or $U \approx \underline{\underline{10.4\,\frac{m}{s}}}$

9.63

9.63 A radio antenna on a car consists of a circular cylinder $\frac{1}{4}$ in. in diameter and 4 ft long. Determine the bending moment at the base of the antenna if the car is driven 55 mph through still air.



For equilibrium, $\Sigma M_o = 0$, or $M_o = \frac{\ell}{2}\mathcal{D}$

where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

Since $Re = \frac{UD}{\nu} = \frac{(80.7\frac{ft}{s})(\frac{1}{48}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 1.07\times10^4$,

it follows from Fig. 9.21 that $C_D = 1.3$

Hence $\mathcal{D} = 1.3(\frac{1}{2})(0.00238\frac{slugs}{ft^3})(80.7\frac{ft}{s})^2(4ft)(\frac{1}{48}ft) = 0.840\ lb$

Thus, $M_o = (2\ ft)(0.840 lb) = \underline{\underline{1.68\ ft\cdot lb}}$

9.64

9.64 Determine the power needed to overcome the aerodynamic drag of the antenna in Problem 9.63.

$\mathcal{P} = \text{power} = U\mathcal{D} = U[C_D \tfrac{1}{2}\rho U^2 A]$

or

$\mathcal{P} = C_D \tfrac{1}{2}\rho U^3 A$, where $U = 55\,mph = 80.7\,\frac{ft}{s}$

Since $Re = \frac{UD}{\nu} = \frac{(80.7\,\frac{ft}{s})(\frac{1}{48}\,ft)}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 1.07\times10^4$, it follows from Fig. 9.21 that

$C_D = 1.3$

Hence,

$\mathcal{P} = 1.3(\tfrac{1}{2})(0.00238\,\frac{slugs}{ft^3})(80.7\,\frac{ft}{s})^3(4\,ft)(\frac{1}{48}\,ft) = 67.8\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$

$= \underline{\underline{0.123\,hp}}$

9.65

9.65 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

$\mathcal{D} = C_D \frac{1}{2} \rho V^2 A$, where $V = (55\, mph)\left(\frac{88 \frac{ft}{s}}{60\, mph}\right) = 80.7 \frac{ft}{s}$

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with $C_D \approx 1.1$ (see Fig. 9.29).

Thus,

$$\mathcal{D} = (1.1)\left(\frac{1}{2}\right)(0.00238)\left(80.7 \frac{ft}{s}\right)^2 \left(\frac{4}{12} ft\right)\left(\frac{6}{12} ft\right) = \underline{\underline{1.42\ lb}}$$

If your hand is normal to the the lift force is zero.

For $V = 550\, mph = 807 \frac{ft}{s}$ (i.e., a 10 fold increase in V) the drag will increase by a factor of 100 (i.e., $\mathcal{D} \sim V^2$), or $\mathcal{D} = \underline{\underline{142\ lb}}$

Note: We have assumed that C_D is not a function of V. That is, it is not a function of either $Re = \frac{VD}{\nu}$ or $Ma = \frac{V}{c}$.

9.67

9.67 A 2-mm-diameter meteor of specific gravity 2.9 has a speed of 6 km/s at an altitude of 50,000 m where the air density is 1.03×10^{-3} kg/m^3. If the drag coefficient at this large Mach number condition is 1.5, determine the deceleration of the meteor.

$$\mathcal{D} = ma \quad \text{where } m = \rho \forall = \rho \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = (2.9)\left(999 \frac{kg}{m^3}\right)\frac{4\pi}{3}\left(\frac{2 \times 10^{-3}\ m}{2}\right)^3$$
$$= 1.21 \times 10^{-5}\ kg$$

Also, $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$

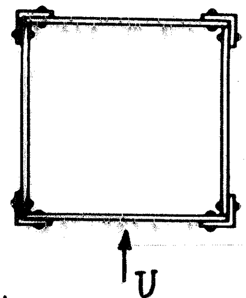
$$= 1.5\left(\frac{1}{2}\right)\left(1.03 \times 10^{-3} \frac{kg}{m^3}\right)\left(6 \times 10^3 \frac{m}{s}\right)^2 \frac{\pi}{4}\left(2 \times 10^{-3} m\right)^2 = 8.74 \times 10^{-2} N$$

Thus,

$$a = \frac{\mathcal{D}}{m} = \frac{8.74 \times 10^{-2} N}{1.21 \times 10^{-5} kg} = \underline{\underline{7220 \frac{m}{s^2}}}$$

9.68

9.68 A 30-ft-tall tower is constructed of equal 1-ft segments as is indicated in Fig. P9.68. Each of the four sides is similar. Estimate the drag on the tower when a 75-mph wind blows against it.



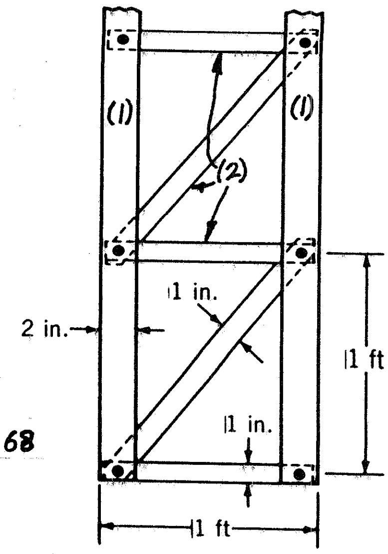


FIGURE P9.68

Assume no interference between the front and back portions of the tower. Also, neglect the drag on the sides of the tower. Hence, for thirty one-foot segments

$$\mathcal{D} = 30\left(\tfrac{1}{2}\rho U^2\right)\left[\left(C_{D_1}A_1 + C_{D_2}A_2\right)_{front} + \left(C_{D_1}A_1 + C_{D_2}A_2\right)_{back}\right] \qquad (1)$$

where

$$U = 75\,mph\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 110\,\frac{ft}{s}$$

From Fig. 9.28, $C_{D_{1_{front}}} = 1.98$, $C_{D_{1_{back}}} = 1.82$

$C_{D_2} \approx 1.9$

Thus, from Eq. (1)

$$\mathcal{D} = 30\left(\tfrac{1}{2}(0.00238\,\tfrac{slugs}{ft^3})(110\,\tfrac{ft}{s})^2\right)\Big[(1.98)(2)(1\,ft)\left(\tfrac{2}{12}\,ft\right)+(1.9)\left(\tfrac{1}{12}\,ft\right)\left(\tfrac{8+8+8\sqrt{2}}{12}\,ft\right)$$
$$+ (1.82)(2)(1\,ft)\left(\tfrac{2}{12}\,ft\right)+(1.9)\left(\tfrac{1}{12}\,ft\right)\left(\tfrac{8+8+8\sqrt{2}}{12}\,ft\right)\Big]$$

or

$$\underline{\underline{\mathcal{D} = 859\ lb}}$$

9.69

9.69 A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.69 and Video V3.1. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.

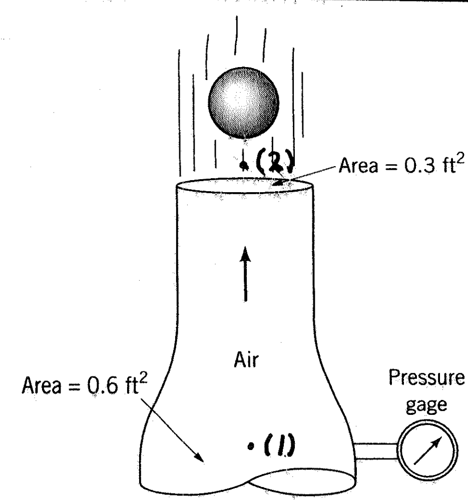
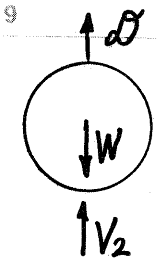


FIGURE P9.69

For equilibrium, $\mathcal{D} = W$ or

$C_D \frac{1}{2}\rho V_2^2 A = W$, where $A = \frac{\pi}{4}D^2$



Thus,

$$V_2 = \left[\frac{2W}{C_D \rho \pi D^2/4}\right]^{1/2}$$

$$= \left[\frac{8\,(0.14\,lb)}{0.5\left(0.00238\,\frac{slugs}{ft^3}\right)\pi\left(\frac{2}{12}\,ft\right)^2}\right]^{1/2} = 104\,\frac{ft}{s}$$

Also,

$$V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_1 = V_2 \frac{A_2}{A_1} = \left(104\,\frac{ft}{s}\right)\frac{0.3\,ft^2}{0.6\,ft^2} = 52.0\,\frac{ft}{s}$$

and

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \quad \text{where } p_2 = 0$$

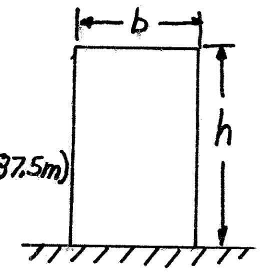
Thus,

$$p_1 = \frac{1}{2}\rho\left[V_2^2 - V_1^2\right] = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(104\,\frac{ft}{s}\right)^2 - \left(52.0\,\frac{ft}{s}\right)^2\right]$$

$$= \underline{\underline{9.65\,\frac{lb}{ft^2}}}$$

9.70

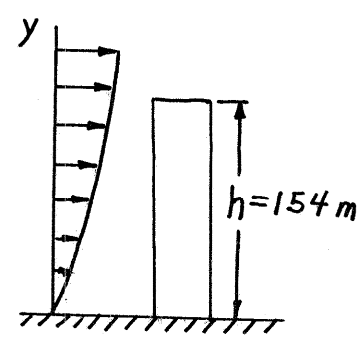
9.70 The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. **(a)** Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. **(b)** Repeat your calculations if the velocity profile against the building is a typical profile for an urban area (see Problem 9.17) and the wind speed halfway up the building is 20 m/s.



(a) $\mathscr{D} = C_D \frac{1}{2}\rho V^2 A = 1.3\left(\frac{1}{2}\right)\left(1.23\,\frac{kg}{m^3}\right)\left(20\,\frac{m}{s}\right)^2(154\,m)(87.5\,m)$

or

$\mathscr{D} = 4.31 \times 10^6\ N = \underline{\underline{4.31\ MN}}$



(b) For an urban area, $u = C\,y^{0.4}$

Thus, with $u = 20\,\frac{m}{s}$ at $y = \frac{h}{2} = 77\,m$

we obtain

$C = \frac{20}{77^{0.4}} = 3.52$, or $u = 3.52\,y^{0.4}$ with $u \sim \frac{m}{s}$, $y \sim m$

The total drag is

$$\mathscr{D} = \int d\mathscr{D} = \int C_D \tfrac{1}{2}\rho u^2\, dA = \tfrac{1}{2}\rho C_D \int_{y=0}^{y=154} (3.52\,y^{0.4})^2 (87.5)\, dy$$

or

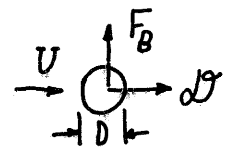
$$\mathscr{D} = \tfrac{1}{2}(1.23)(1.3)(3.52)^2(87.5)\int_0^{154} y^{0.8}\,dy = 867\left(\frac{1}{1.8}\right)(154)^{1.8} = 4.17 \times 10^6\ N$$

Thus,

$\mathscr{D} = \underline{\underline{4.17\ MN}}$

9.71

9.71 A 0.3-in.-diameter wire is draped across a river in which the water velocity is 3 ft/s. The length of the wire in the water is 420 ft. Determine the horizontal and vertical components of force of the water on the wire.



The horizontal force is equal to the drag, $\mathcal{D}$.

$\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$, where C_D is a function of the Reynolds number.

Thus, with

$$Re = \frac{\rho U D}{\mu} = \frac{1.94 \frac{slugs}{ft^3} (3\ ft/s)(0.3/12\ ft)}{2.34 \times 10^{-5}\ lb \cdot s/ft^2} = 6.22 \times 10^3$$ it follows from Fig. 9.21

that

$C_D \approx 1.3$

Hence,

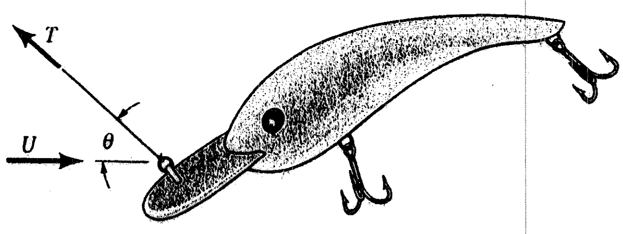
$$\mathcal{D} = 1.3 \left(\tfrac{1}{2}\right)\left(1.94 \tfrac{slugs}{ft^3}\right)\left(3 \tfrac{ft}{s}\right)^2 \left(\tfrac{0.3}{12} ft\right)(420 ft) = \underline{\underline{119\ lb}}$$

The vertical force is equal to the buoyant force, F_B.

$$F_B = \gamma \cdot \mathcal{V} = 62.4 \tfrac{lb}{ft^3} (420 ft) \tfrac{\pi}{4} \left(\tfrac{0.3}{12} ft\right)^2 = \underline{\underline{12.9\ lb}}$$

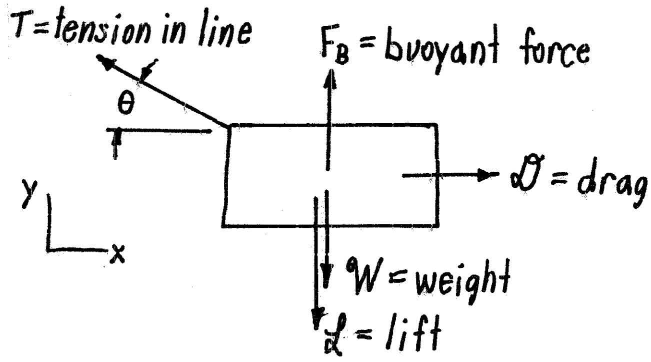
9.72

9.72 The fishing lure shown in Fig. P9.72 floats on the surface of the water when there is no tension, T, in the line. However, it is designed to sink when reeled in. If it is reeled in too fast (i.e., U is too large) it will sink to the bottom and get snagged. Show that the depth (or equivalently, θ) increases as U increases. Start with a free-body diagram of the lure including the tension in the line, the weight of the lure, the buoyant force, the lift



■ FIGURE P9.72

A free body diagram of the lure being pulled through the water is shown below. Note that the lift is shown acting downward, line an inverted wing or a spoiler on a race car.



Thus, $\Sigma F_x = 0$ and $\Sigma F_y = 0$ give

$T\cos\theta = \mathcal{D}$ (1)

and

$T\sin\theta = \mathcal{L} + \mathcal{W} - F_B$ (2)

Dividing Eq. (1) by Eq. (2) gives

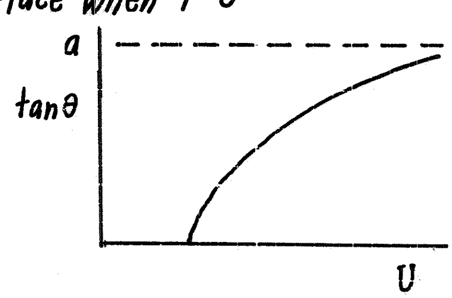
$$\tan\theta = \frac{\mathcal{L} + \mathcal{W} - F_B}{\mathcal{D}} = \frac{C_L \frac{1}{2}\rho U^2 A + \mathcal{W} - F_B}{C_D \frac{1}{2}\rho U^2 A} = \frac{C_L + (\mathcal{W} - F_B)/(\frac{1}{2}\rho U^2 A)}{C_D}$$

or

(3) $\tan\theta = a - \frac{b}{U^2}$, where $a = \frac{C_L}{C_D}$ and $b = \frac{2(F_B - \mathcal{W})}{C_D \rho A}$ are positive constants

(Note that $F_B > \mathcal{W}$ because the lure floats on the surface when $T=0$

From Eq. (3) the graph of $\tan\theta$ vs U looks as shown. Thus, as U increases $\tan\theta$ increases. That is, as U increases θ increases.



9.73

9.73 A regulation football is 6.78 in. in diameter and weighs 0.91 lb. If its drag coefficient is $C_D = 0.2$, determine its deceleration if it has a speed of 20 ft/s at the top of its trajectory.

$\mathscr{D} = ma$, where $m = \frac{W}{g} = \frac{0.91\ lb}{32.2\ \frac{ft}{s^2}} = 0.0283\ slugs$

and

$\mathscr{D} = C_D \frac{1}{2} \rho V^2 A = 0.2\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left(20\ \frac{ft}{s}\right)^2\left(\frac{\pi}{4}\left(\frac{6.78}{12}\ ft^2\right)\right) = 0.0239\ lb$

Thus,

$a = \frac{\mathscr{D}}{m} = \frac{0.0238\ lb}{0.0283\ slugs} = \underline{\underline{0.841\ \frac{ft}{s^2}}}$

9.75

9.75 The paint stirrer shown in Fig. P9.75 consists of two circular disks attached to the end of a thin rod that rotates at 80 rpm. The specific gravity of the paint is $SG = 1.1$ and its viscosity is $\mu = 2 \times 10^{-2}\ \text{lb}\cdot\text{s/ft}^2$. Estimate the power required to drive the mixer if the induced motion of the liquid is neglected.

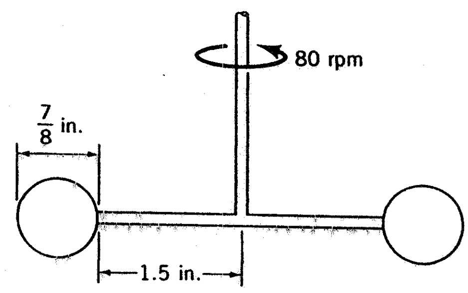


FIGURE P9.75

If we neglect the effects of the shaft and rod and consider the paint to be stationary, then

$M = 2\mathcal{D}R$, where M = torque to rotate shaft
and $\mathcal{D}$ = drag on one disk $= C_D \frac{1}{2}\rho U^2 A$

Also, $U = \omega R$ and $\mathcal{P}$ = power to rotate shaft $= M\omega$

Thus,

$$\mathcal{P} = 2\mathcal{D}R\,\omega = 2C_D \tfrac{1}{2}\rho(\omega R)^2 \tfrac{\pi}{4}D^2 R\,\omega$$

or

$$\mathcal{P} = \tfrac{\pi}{4}C_D\,\rho\,\omega^3 R^3 D^2 = \tfrac{\pi}{4}C_D\,\rho\,U^3 D^2 \quad \text{where } \rho = SG\,\rho_{H_2O} \qquad (1)$$

With $Re = \dfrac{\rho U D}{\mu} = \dfrac{SG\,\rho_{H_2O} U D}{\mu}$

where

$$U = \omega R = \left(80\,\frac{rev}{min}\right)\left(\frac{1\ min}{60\,s}\right)\left(\frac{2\pi\ rad}{1\ rev}\right)\left(\frac{1.5+\frac{7}{16}}{12}\,ft\right) = 1.353\,\frac{ft}{s}$$

we have

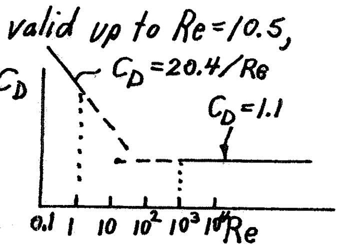
$$Re = \frac{(1.1)(1.94\,\frac{slugs}{ft^3})(1.353\,\frac{ft}{s})\left(\frac{7}{8(12)}\,ft\right)}{2\times 10^{-2}\,\frac{lb\cdot s}{ft^2}} = 10.5$$

For a circular disk, $C_D = 1.1$ if $Re > 10^3$ (see Fig. 9.29)

while $C_D = \dfrac{20.4}{Re}$ if $Re < 1$ (see Table 9.4) (2)

For this particular problem $1 < Re = 10.5 < 10^3$

Note: If the low Reynolds number result (Eq. (2)) is valid up to $Re = 10.5$,
then $C_D = \dfrac{20.4}{10.5} = 1.94$



To be on the conservative side (i.e., maximum power)
use the larger C_D — $C_D = 1.94$ From Eq. (1)

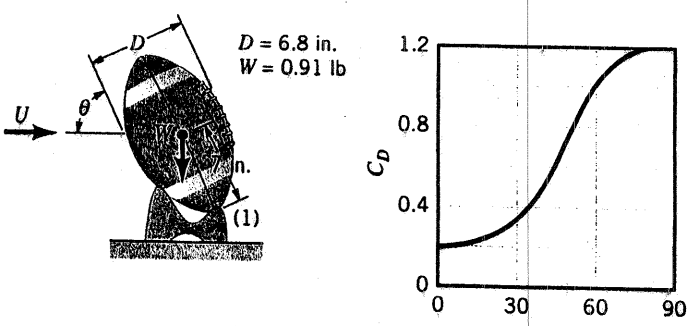
$$\mathcal{P} = \tfrac{\pi}{4}(1.94)(1.1)(1.94\,\tfrac{slugs}{ft^3})(1.353\,\tfrac{ft}{s})^3\left(\tfrac{7}{8(12)}\,ft\right)^2$$
$$= 0.0428\,\frac{ft\cdot lb}{s}$$

or

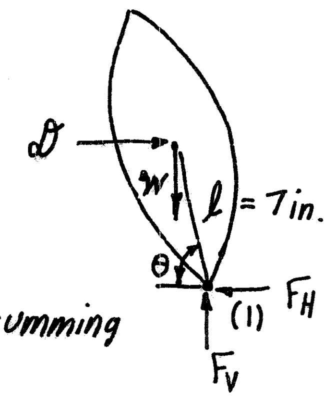
$$\mathcal{P} = \left(0.0428\,\frac{ft\cdot lb}{s}\right)\left(\frac{1\ hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{7.78\times 10^{-5}\ hp}}$$

9.77

9.77 The football shown in Fig. P9.77 can be blown from its kicking tee by pivoting freely about point (1). The drag coefficient (based on the cross-sectional area of the football, $\pi D^2/4$) is a function of its orientation, θ, as shown. Plot a graph of the wind speed needed to blow the football from the tee as a function of the angle θ.



■ FIGURE P9.77



From the free body diagram of the football, summing moments about the pivot point (1) gives

$\Sigma M_1 = 0$ or $W \ell \cos\theta = \mathcal{D} \ell \sin\theta$

Thus, $W = \mathcal{D} \tan\theta = C_D \frac{1}{2}\rho U^2 A \tan\theta$ which gives

$$U = \left[\frac{2W}{C_D \rho A \tan\theta} \right]^{1/2}$$

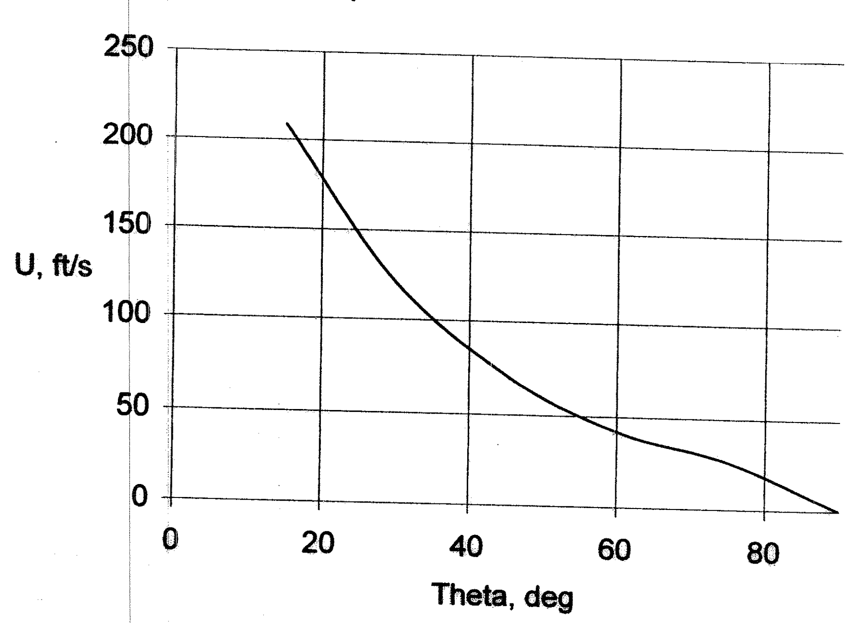
or

$$U = \left[\frac{2\,(0.91\,lb)}{C_D\,(0.00238\,slugs/ft^3)\frac{\pi}{4}\left(\frac{6.8}{12}\,ft\right)^2 \tan\theta} \right]^{1/2}$$

Hence, $U = 55.1\sqrt{1/(C_D \tan\theta)}$ (1)

Using values of C_D from Fig. P9.77, the following results are obtained:

θ, deg	drag coef, CD	U, ft/s
0	0.20	∞
15	0.26	208.8
30	0.36	120.9
45	0.60	71.1
60	1.00	41.9
75	1.17	26.4
90	1.20	0



9.78

9.78 An airplane tows a banner that is b = 0.8 m tall and ℓ = 25 m long at a speed of 150 km/hr. If the drag coefficient based on the area $b\ell$ is C_D = 0.06, estimate the power required to tow the banner. Compare the drag force on the banner with that on a rigid flat plate of the same size. Which has the larger drag force and why?

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $A = b\ell$.

Thus, with $C_D = 0.06$ and $U = (150 \frac{km}{hr})(\frac{1\,hr}{3600\,s})(\frac{1000\,m}{1\,km}) = 41.7 \frac{m}{s}$ this gives

$\mathcal{P} = (0.06)(\frac{1}{2})(1.23 \frac{kg}{m^3})(41.7 \frac{m}{s})^3(0.8m)(25m) = 53.5 \times 10^3\,W = \underline{\underline{53.5\,kW}}$

For a rigid flat plate

$\mathcal{P} = \mathcal{D}U = 2C_D \frac{1}{2}\rho U^3 b\ell$ (the factor of two is needed because the drag coefficient is based on the drag on one side of the plate)

With $Re_\ell = \frac{U\ell}{\nu} = \frac{(41.7\frac{m}{s})(25m)}{1.46 \times 10^{-5}\frac{m^2}{s}} = 7.14 \times 10^7$ we obtain from Fig. 9.15 a value of $C_D \approx 0.0025$ for a smooth plate.

Thus,

$\mathcal{P} = 2(0.0025)(\frac{1}{2})(1.23 \frac{kg}{m^3})(41.7 \frac{m}{s})^3(0.8m)(25m) = 4.46 \times 10^3\,W = \underline{\underline{4.46\,kW}}$

For the flat plate case the drag is relatively small because it is due entirely to shear (viscous) forces. Due to the "fluttering" of the banner, a good portion of its drag (and hence power) is a result of pressure forces. It is not as streamlined as a rigid flat plate.

9.79

9.79 By appropriate streamlining, the drag coefficient for an airplane is reduced by 18% while the frontal area remains the same. For the same power output, by what percentage is the flight speed increased?

$\mathcal{P} = \mathcal{D} V$, where $\mathcal{D} = C_D \frac{1}{2} \rho V^2 A$

Let $(\ )_o$ denote the original configuration and $(\ )_s$ the streamlied one.
Thus, with $\mathcal{P}_o = \mathcal{P}_s$ we obtain

$C_{D_o} \frac{1}{2} \rho_o V_o^2 A_o V_o = C_{D_s} \frac{1}{2} \rho_s V_s^2 A_s V_s$ or with $A_o = A_s$, $\rho_o = \rho_s$

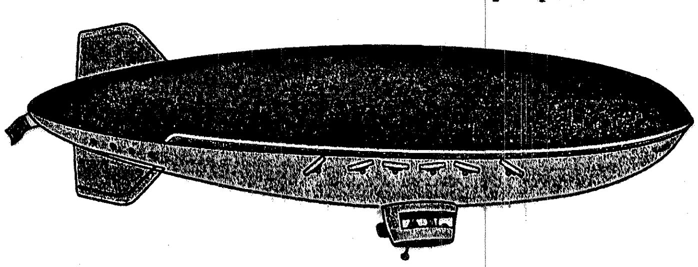
$V_o^3 C_{D_o} = V_s^3 C_{D_s}$ Thus, $\frac{V_s}{V_o} = \left[\frac{C_{Do}}{C_{Ds}}\right]^{\frac{1}{3}} = \left[\frac{C_{Do}}{C_{D_o} - 0.18 C_{D_o}}\right]^{\frac{1}{3}} = 1.0684$

i.e., a 6.84 % speed increase

Note: $\mathcal{P} \sim V^3 C_D$ so that $\delta \mathcal{P} = 3V^2 C_D \delta V + V^3 \delta C_D$. Thus, with
$\delta \mathcal{P} = 0$, this gives $\frac{\delta V}{V} = -\frac{1}{3} \frac{\delta C_D}{C_D} = -\frac{-0.18}{3} = +0.06 = 6\%$

9.80

9.80 The blimp shown in Fig. P9.80 is used at various athletic events. It is 128 ft long and has a maximum diameter of 33 ft. If its drag coefficient (based on the frontal area) is 0.060, estimate the power required to propel it **(a)** at its 35-mph cruising speed, or **(b)** at its maximum 55-mph speed.



■ **FIGURE P9.80**

$\mathcal{P} = \mathcal{D} U$ where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

Thus, with

$$\mathcal{D} = 0.060 \left(\tfrac{1}{2}\right)\left(0.00238 \frac{\text{slugs}}{\text{ft}^3}\right) U^2 \frac{\pi}{4} (33\ \text{ft})^2$$

$$= 0.0611\, U^2\ \text{lb},\ \text{where } U \sim \text{ft/s}$$

(a) Thus, with $U = 35 \frac{\text{mi}}{\text{hr}} \left(\frac{5280\ \text{ft/mi}}{3600\ \text{s/hr}}\right) = 51.3\ \text{ft/s}$,

$$\mathcal{D} = 0.0611 (51.3)^2 = 161\ \text{lb}$$

so that

$$\mathcal{P} = \mathcal{D} U = 161\ \text{lb}\ (51.3\ \text{ft/s}) \left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right) = \underline{\underline{15.0\ \text{hp}}}$$

(b) Similarly, with $U = 55\ \text{mph} = 80.7\ \text{ft/s}$,

$$\mathcal{D} = 0.0611 (80.7)^2 = 398\ \text{lb}$$

so that

$$\mathcal{P} = \mathcal{D} U = 398\ \text{lb}\ \left(80.7 \frac{\text{ft}}{\text{s}}\right) \left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right) = \underline{\underline{58.4\ \text{hp}}}$$

9.81

9.81 Estimate the power needed to overcome the aerodynamic drag of a person who runs at a rate of 100 yds in 10 s in still air. Repeat the calculations if the race is run into a 20-mph headwind; a 20-mph tailwind. Explain.

In still air $\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $U = \frac{300\,ft}{10\,s} = 30\frac{ft}{s}$.
From Fig. 9.30, $C_D A \approx 9\,ft^2$

Thus, $\mathcal{D} = (\frac{1}{2})(0.00238\frac{slugs}{ft^3})(30\frac{ft}{s})^2(9\,ft^2) = 9.64\,lb$

and $\mathcal{P} = (9.64\,lb)(30\frac{ft}{s})\left(\frac{1\,hp}{550\frac{ft\cdot lb}{s}}\right) = \underline{\underline{0.526\,hp}}$

Into a $20\,mph = 29.3\frac{ft}{s}$ headwind $\mathcal{P} = \mathcal{D}U_r$, where $U_r = 30\frac{ft}{s}$ = runner's speed
and
$\mathcal{D} = C_D \frac{1}{2}\rho(U_r + 29.3\frac{ft}{s})^2 A$ or

$\mathcal{D} = (\frac{1}{2})(0.00238\frac{slugs}{ft^3})(30+29.3)^2\frac{ft^2}{s^2}(9\,ft^2) = 37.7\,lb$

Thus, $\mathcal{P} = (37.7\,lb)(30\frac{ft}{s})\left(\frac{1\,hp}{550\frac{ft\cdot lb}{s}}\right) = \underline{\underline{2.06\,hp}}$

With a $20\,mph = 29.3\frac{ft}{s}$ tail wind the relative headwind that the runner feels is $U_r - 29.3\frac{ft}{s} = (30-29.3)\frac{ft}{s} = 0.7\frac{ft}{s}$

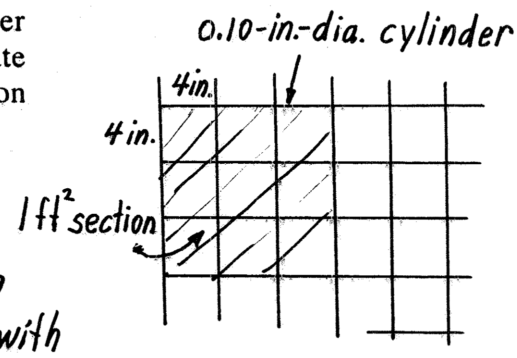
Thus,
$\mathcal{D} = (\frac{1}{2})(0.00238\frac{slugs}{ft^3})(0.7\frac{ft}{s})^2(9\,ft^2) = 0.00525\,lb$

Thus, $\mathcal{P} = (0.00525\,lb)(30\frac{ft}{s})\left(\frac{1\,hp}{550\frac{ft\cdot lb}{s}}\right) = \underline{\underline{0.000286\,hp}}$

Note: The tailwind essentially cancels the relative wind speed produced by the runner's forward motion.

9.83

9.83 A fishnet consists of 0.10-in.-diameter strings tied into squares 4 in. per side. Estimate the force needed to tow a 15 ft by 30 ft section of this net through seawater at 5 ft/s.



The net can be treated as one long 0.10-in.-diameter circular cylinder with

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$, where $U = 5\,\frac{ft}{s}$. Each $1\ ft^2$ section of the net contains 6 feet of string (do not count the edges twice). Thus, the total string length is approximately $\ell = (6\,\frac{ft}{ft^2})(15\,ft)(30\,ft) = 2700\,ft$

Also, since $\rho = 1.99\,\frac{slugs}{ft^3}$ and $\nu = 1.26 \times 10^{-5}\,\frac{ft^2}{s}$ (see Table 1.5)

$$Re = \frac{UD}{\nu} = \frac{(5\,\frac{ft}{s})(\frac{0.10}{12}\,ft)}{1.26\times 10^{-5}\,\frac{ft^2}{s}} = 3310$$

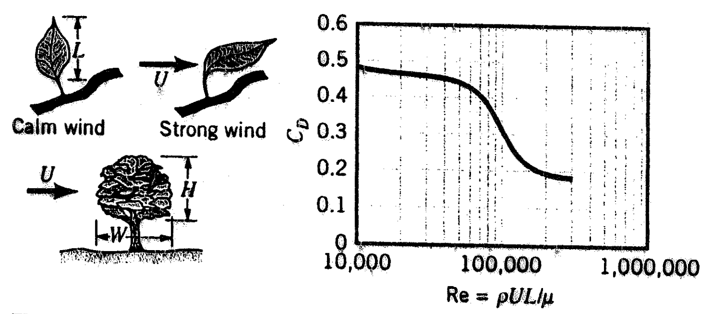
Hence, from Fig. 9.21 that $C_D = 1.1$

Thus,

$$\mathscr{D} = (1.1)(\tfrac{1}{2})(1.99\,\tfrac{slugs}{ft^3})(5\,\tfrac{ft}{s})^2(\tfrac{0.1}{12}\,ft)(2700\,ft) = \underline{\underline{616\ lb}}$$

9.84

9.84 As indicated in Fig. P9.84, the orientation of leaves on a tree is a function of the wind speed, with the tree becoming "more streamlined" as the wind increases. The resulting drag coefficient for the tree (based on the frontal area of the tree, HW) as a function of Reynolds number (based on the leaf length, L) is approximated as shown. Consider a tree with leaves of length $L = 0.3$ ft. What wind speed will produce a drag on the tree that is 6 times greater than the drag on the tree in a 15 ft/s wind?



■ FIGURE P9.84

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$ and $Re = \frac{\rho UL}{\mu}$

or

$\mathscr{D} = C_D \frac{1}{2}(0.00238)U^2 HW = 0.00119\ HW C_D U^2$ (1)

and

$Re = \dfrac{0.00238 \frac{slugs}{ft^3}\ U(0.3\ ft)}{3.74 \times 10^{-7}\ lb \cdot s/ft^2} = 1909\ U$, where $U \sim$ ft/s (2)

Thus, with $U = 15$ ft/s, $Re = 1909(15) = 28,600$ so that from Fig. P9.84,

$C_D = 0.46$ so

$\mathscr{D}_{15} = 0.00119\ HW(0.46)(15)^2 = 0.123\ HW$

For the drag 6 time as great, $\mathscr{D} = 6\mathscr{D}_{15} = 6(0.123\ HW) = 0.738\ HW$ (3)

Thus, from Eqs. (1) and (3):

$0.738\ HW = 0.00119\ HW\ C_D U^2$

or

$C_D U^2 = 621$ (4)

Trial and error solution:

Assume $C_D = 0.3$ so that from Eq. (4), $U = \sqrt{\frac{621}{0.3}} = 45.5$ ft/s and from Eq. (2)

$Re = 1909(45.5) = 86,900$. Thus, from Fig. P9.84, $C_D = 0.33 \neq 0.3$, the assumed value.

Try again. Assume $C_D = 0.33 \rightarrow U = 43.4$ ft/s $\rightarrow Re = 82,900 \rightarrow C_D = 0.36 \neq 0.33$

Try $C_D = 0.36 \rightarrow U = 41.5$ ft/s $\rightarrow Re = 79,300 \rightarrow C_D = 0.36$

Thus, $\underline{\underline{U = 41.5\ ft/s}}$

9.85

9.85 A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft^2. Determine the lift coefficient of this airplane for these conditions.

For equilibrium $\mathcal{L} = W = 1750\ lb$, where $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, with $U = (115\ mph)\dfrac{(88\ \frac{ft}{s})}{(60\ mph)} = 169\ \frac{ft}{s}$

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{1750\ lb}{\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(169\ \frac{ft}{s})^2(179\ ft^2)} = \underline{\underline{0.288}}$$

9.86

9.86 A light aircraft with a wing area of 200 ft^2 and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium $\mathcal{L} = W = 2000\ lb = C_L \frac{1}{2}\rho U^2 A$

or

$$2000\ lb = (0.40)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})U^2(200\ ft^2)$$

Hence,

$$U = 145\ \frac{ft}{s}$$

Also, $\mathcal{P}$ = power $= \mathcal{D}\,U$, where

$$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = (0.05)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(145\ \frac{ft}{s})^2(200\ ft^2) = 250\ lb$$

Note: This value of $\mathcal{D}$ could be obtained from

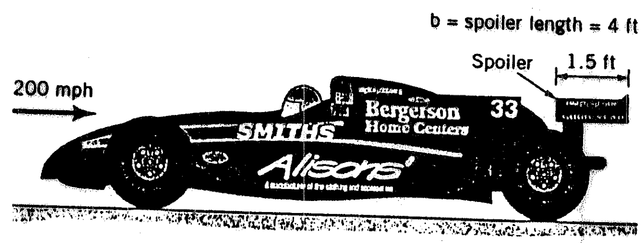
$$\frac{W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8,\ \text{or}\ \mathcal{D} = \frac{W}{8} = \frac{2000\ lb}{8} = 250\ lb$$

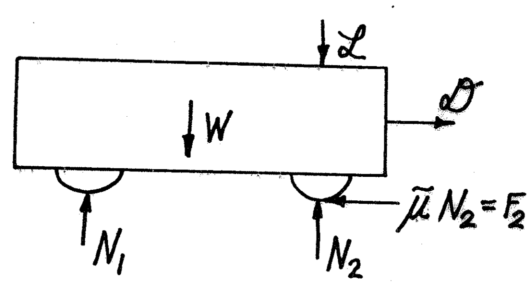
Thus,

$$\mathcal{P} = 250\ lb\,(145\ \frac{ft}{s}) = 3.63\times10^4\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right) = \underline{\underline{65.9\ hp}}$$

9.87

9.87 As shown in Video V9.9 and Fig. P9.87, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is $C_L = 1.1$, and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.





Tractive force $= F_2 = \tilde{\mu} N_2$

where $\tilde{\mu}$ = coefficient of friction = 0.6

Thus,

$\Delta F_2 = \tilde{\mu}\,\Delta N_2 = \tilde{\mu}\,\mathscr{L}$, where ΔF_2 is the increase in tractive force due to the (downward) lift.

Hence, with $U = 200\ mph = 293\ ft/s$,

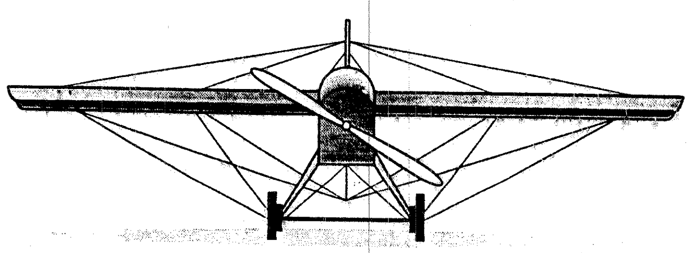
$$\mathscr{L} = \tfrac{1}{2}\rho U^2 C_L A = \tfrac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)\left(293\ \frac{ft}{s}\right)^2 (1.1)(1.5\ ft)(4\ ft) = 674\ lb,$$

and

$$\Delta F_2 = 0.6\,(674\ lb) = \underline{\underline{405\ lb}}$$

9.88

9.88 The wings of old airplanes are often strengthened by the use of wires that provided cross-bracing as shown in Fig. P9.88. If the drag coefficient for the wings was 0.020 (based on the planform area), determine the ratio of the drag from the wire bracing to that from the wings.



Speed: 70 mph
Wing area: 148 ft^2
Wire: length = 160 ft
diameter = 0.05 in.

■ FIGURE P9.88

$\mathscr{D}_{wing} = \frac{1}{2}\rho U^2 C_{D_{wing}} A_{wing}$

and

$\mathscr{D}_{wire} = \frac{1}{2}\rho U^2 C_{D_{wire}} A_{wire}$ so that

$$\frac{\mathscr{D}_{wire}}{\mathscr{D}_{wing}} = \frac{C_{D_{wire}} A_{wire}}{C_{D_{wing}} A_{wing}}, \text{ where } A_{wing} = 148\,ft^2,\ C_{D_{wing}} = 0.02$$

Also, $A_{wire} = \ell D = (160\,ft)\left(\frac{0.05}{12}\,ft\right) = 0.667\,ft^2$

and since

$$Re = \frac{UD}{\nu} = \frac{(70\,mph)\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right)\left(\frac{0.05}{12}\,ft\right)}{1.57\times10^{-4}\,\frac{ft^2}{s}} = 2720.$$

From Fig. 9.21, with $Re = 2720$ we obtain $C_D = 1.0$

Hence,

$$\frac{\mathscr{D}_{wire}}{\mathscr{D}_{wing}} = \frac{(1.0)(0.667\,ft^2)}{(0.02)(148\,ft^2)} = 0.225, \text{ or } \underline{\underline{22.5\%}}$$

9.89

9.89 The jet engines on a Boeing 757 must develop a certain amount of power to propel the airplane through the air with a speed of 570 mph at a cruising altitude of 35,000 ft. By what percent must the power be increased if the same airplane were to maintain its 570 mph flight speed at sea level?

$\mathcal{P}$ = power $= \mathcal{D}\,U = \frac{1}{2}\rho U^3 C_D A$

Let $(\;)_0$ and $(\;)_{35}$ denote conditions at sea level and 35,000 ft, respectively. Thus, $U_0 = U_{35}$ so that

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{\frac{1}{2}\rho_0 U_0^3 C_{D_0} A_0}{\frac{1}{2}\rho_{35} U_{35}^3 C_{D_{35}} A_{35}}$$ so if $A_0 = A_{35}$ and $C_{D_0} = C_{D_{35}}$, then

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{\rho_0}{\rho_{35}}$$, or with ρ values from Table C.1,

$$\frac{\mathcal{P}_0}{\mathcal{P}_{35}} = \frac{0.00238 \frac{slugs}{ft^3}}{0.000738 \frac{slugs}{ft^3}} = 3.22 = \underline{\underline{322\%\ \text{increase}}}$$

9.90

9.90 A wing generates a lift $\mathcal{L}$ when moving through sea-level air with a velocity U. How fast must the wing move through the air at an altitude of 35,000 ft with the same lift coefficient if it is to generate the same lift?

$\mathcal{L} = C_L \frac{1}{2} \rho U^2 A$ so with $\mathcal{L}$, C_L, and A constant

$$(\rho U^2)_{sea\ level} = (\rho U^2)_{35{,}000\,ft}$$

Hence,

$$U_{35{,}000\,ft} = \left(\frac{\rho_{sea\ level}}{\rho_{35{,}000\,ft}}\right)^{1/2} U_{sea\ level} = \left(\frac{2.38 \times 10^{-3}\ \frac{slug}{ft^3}}{7.38 \times 10^{-4}\ \frac{slug}{ft^3}}\right)^{1/2} U_{sea\ level}$$

or

$$U_{35{,}000\,ft} = 1.80\ U_{sea\ level}$$

***9.91**

9.91 Air blows over the flat-bottomed, two-dimensional object shown in Fig. P9.91. The shape of the object, $y = y(x)$, and the fluid speed along the surface, $u = u(x)$, are given in the table. Determine the lift coefficient for this object.

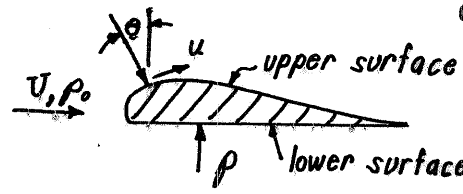
x (% c)	y (% c)	u/U
0	0	0
2.5	3.72	0.971
5.0	5.30	1.232
7.5	6.48	1.273
10	7.43	1.271
20	9.92	1.276
30	11.14	1.295
40	11.49	1.307
50	10.45	1.308
60	9.11	1.195
70	6.46	1.065
80	3.62	0.945
90	1.26	0.856
100	0	0.807

If viscous effects are negligible, then

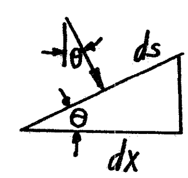
$$\mathcal{L} = \int_{lower} p\cos\theta dA - \int_{upper} p\cos\theta dA \qquad (1)$$

where from the Bernoulli equation

$$p + \tfrac{1}{2}\rho u^2 = p_0 + \tfrac{1}{2}\rho U^2 \qquad (2)$$



The effect of atmospheric pressure, p_0, drops out when the integration over the entire surface is performed.
With $\theta = 0$ on the lower surface and with $\cos\theta\, dA = \cos\theta\,(\ell\, ds) = \ell\, dx$, where ℓ = wing span, Eqs.(1) and (2) give



$$\mathcal{L} = \int_{lower}\left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell\, dx - \int_{upper}\left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell\, dx$$

or, since $u = U$ on the lower surface

$$\mathcal{L} = -\tfrac{1}{2}\rho\ell\int_{x=0}^{x=c}(U^2 - u^2)\,dx = \tfrac{1}{2}\rho U^2 \ell c\int_{x'=0}^{x'=1}\left[\left(\tfrac{u}{U}\right)^2 - 1\right]dx', \text{ where } x' = \frac{x}{c} \qquad (3)$$

Thus, since

$$C_L = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 A} = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 \ell c}$$

it follows from Eq.(3) that

$$C_L = \int_{x'=0}^{x'=1}\left[\left(\tfrac{u}{U}\right)^2 - 1\right]dx'$$

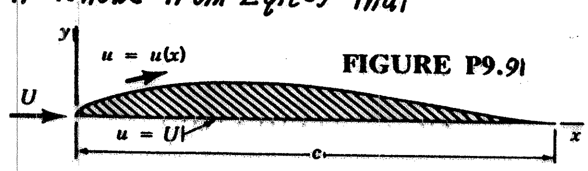


FIGURE P9.91

By using a standard numerical integration routine with the data given we obtain

$$\underline{\underline{C_L = 0.327}}$$

x'	$\left(\frac{u}{U}\right)^2 - 1$
0	-1.00
0.025	-0.0572
0.050	0.518
0.075	0.621
0.100	0.615
0.200	0.628
0.300	0.677
0.400	0.708
0.500	0.711
0.600	0.428
0.700	0.134
0.800	-0.107
0.900	-0.267
1.000	-0.349

9.93

9.93 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A = W$ (1)

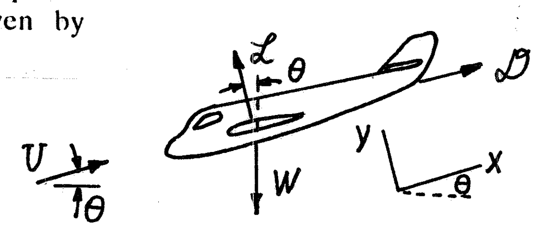
Let $(\ )_{100}$ denote conditions with 100 passengers and $(\ )_{372}$ with 372 passengers. Thus, with $C_{L\,100} = C_{L\,372}$, $A_{100} = A_{372}$, and $\rho_{100} = \rho_{372}$ Eq. (1) gives

$$\frac{\mathcal{L}_{100}}{\mathcal{L}_{372}} = \frac{U_{100}^2}{U_{372}^2} \quad \text{or} \quad U_{372} = U_{100}\left\{\frac{[580{,}000 + (372-100)(200)]\ lb}{580{,}000\ lb}\right\}^{1/2}, \text{ with } U_{100} = 140\ mph$$

Thus, $U_{372} = \underline{\underline{146\ mph}}$

9.94

9.94 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, θ, is given by $\tan\theta = C_D/C_L$.



For steady unpowered flight

$\Sigma F_x = 0$ gives $\mathcal{D} = W\sin\theta$

and

$\Sigma F_y = 0$ gives $\mathcal{L} = W\cos\theta$

Thus,

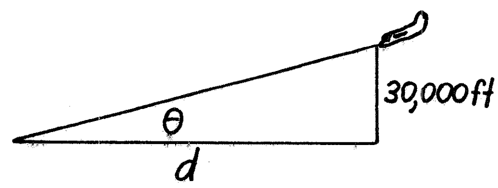
$$\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta, \text{ where } \frac{\mathcal{D}}{\mathcal{L}} = \frac{\frac{1}{2}\rho U^2 A C_D}{\frac{1}{2}\rho U^2 A C_L} = \frac{C_D}{C_L}$$

Hence, $\underline{\underline{\tan\theta = \frac{C_D}{C_L}}}$

9.95

9.95 If the lift coefficient for a Boeing 777 aircraft is 15 times greater than its drag coefficient, can it glide from an altitude of 30,000 ft to an airport 80 mi away if it loses power from its engines? Explain. (See Problem 9.94.)

From Problem 9.94, $\tan\theta = \frac{C_D}{C_L} = \frac{1}{15}$



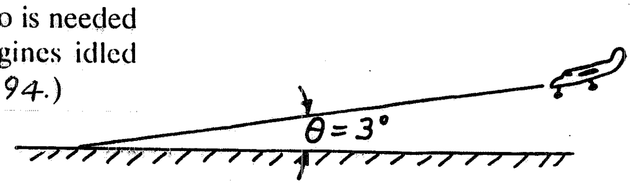
Hence,

$$\frac{30{,}000}{d} = \frac{1}{15}, \quad \text{or } d = 4.5\times10^5\ ft$$
$$= 85.2\ mi$$

Hence, $\underline{\underline{\text{the plane can glide 80 mi.}}}$

9.96

9.96 On its final approach to the airport an airplane flies on a flight path that is 3.0° relative to the horizontal. What lift-to-drag ratio is needed if the airplane is to land with its engines idled back to zero power? (See Problem 9.94.)



From Problem 9.94,

$$\tan\theta = \frac{C_D}{C_L}$$

or

$$\frac{C_D}{C_L} = \tan 3° = 0.0524 \qquad \underline{\underline{\frac{C_L}{C_D} = 19.1}}$$

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9.99

9.99 **The landing speed of an airplane such as the Space Shuttle is dependent on the air density. (See Video V9.1.)** By what percent must the landing speed be increased on a day when the temperature is 110 deg F compared to a day when it is 50 deg F? Assume the atmospheric pressure remains constant.

For equalibrium, lift = weight, or

$$\frac{1}{2}\rho U^2 C_L A = W$$

Thus, with constant W, C_L, and A,

$$(\rho U^2)_{T=110^\circ} = (\rho U^2)_{T=50^\circ} \quad \text{or}$$

$$U_{110^\circ} = \left(\frac{\rho_{50}}{\rho_{110}}\right)^{\frac{1}{2}} U_{50^\circ}$$

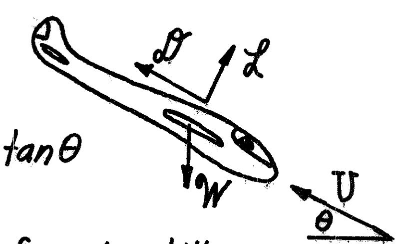
But $p = \rho R T$ so that $\dfrac{\rho_{50}}{\rho_{110}} = \dfrac{(p_{50}/RT_{50})}{(p_{110}/RT_{110})} = \dfrac{(460+110)}{(460+50)} = 1.1176$

Thus,

$$U_{110^\circ} = \sqrt{1.1176}\, U_{50^\circ} = 1.0572\, U_{50^\circ} \quad \text{or a } \underline{\underline{5.72\%\ \text{increase}}}$$

9.97

9.97 A sail plane with a lift-to-drag ratio of 25 flies with a speed of 50 mph. It maintains or increases its altitude by flying in thermals, columns of vertically rising air produced by buoyancy effects of nonuniformly heated air. What vertical airspeed is needed if the sail plane is to maintain a constant altitude?



With no vertical air motion the sailplane would glide with a slope angle θ, where since $\Sigma \vec{F} = 0$
$\mathcal{D} = W\sin\theta$ and $\mathcal{L} = W\cos\theta$. Hence, $\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta$
or since $\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$ and
$\mathcal{L} = \frac{1}{2}\rho U^2 C_L A$ it follows that $\tan\theta = \frac{C_D}{C_L}$. Therefore in still air the sailplane would loose altitude at a rate of $U\sin\theta$, where
$\theta = \tan^{-1}\left(\frac{C_D}{C_L}\right) = \tan^{-1}\left(\frac{1}{25}\right) = 2.29°$. Hence, an upward wind of $(50\,\text{mph})\sin 2.29°$
$= \underline{\underline{2.00\ \text{mph}}}$ will allow horizontal flight.

9.98

9.98 Over the years there has been a dramatic increase in the flight speed (U) and altitude (h), weight ($\mathcal{W}$), and wing loading ($\mathcal{W}/A$ = weight divided by wing area) of aircraft. Use the data given in the table below to determine the lift coefficient for each of the aircraft listed.

Aircraft	Year	$\mathcal{W}$, lb	U, mph	$\mathcal{W}/A$, lb/ft^2	h, ft
Wright Flyer	1903	750	35	1.5	0
Douglas DC-3	1935	25,000	180	25.0	10,000
Douglas DC-6	1947	105,000	315	72.0	15,000
Boeing 747	1970	800,000	570	150.0	30,000

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{2}{\rho U^2}\left(\frac{W}{A}\right)$$

Thus,

	ρ, $slugs/ft^3$	U, ft/s	W/A, lb/ft^2	C_L
Wright Flyer	2.38×10^{-3}	51.3	1.5	0.480
DC-3	1.76×10^{-3}	264	25.0	0.409
DC-6	1.50×10^{-3}	462	72.0	0.451
747	8.91×10^{-4}	836	150	0.482

9.103

9.103 Boundary Layer on a Flat Plate

Objective: A boundary layer is formed on a flat plate when air blows past the plate. The thickness, δ, of the boundary layer increases with distance, x, from the leading edge of the plate. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.103, to measure the boundary layer thickness.

Equipment: Wind tunnel; flat plate; boundary layer mouse consisting of ten Pitot tubes positioned at various heights, y, above the flat plate; inclined multiple manometer; measuring calipers; barometer, thermometer.

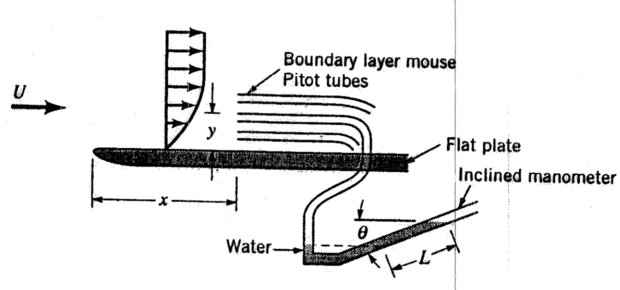
Experimental Procedure: Position the tips of the Pitot tubes of the boundary layer mouse a known distance, x, downstream from the leading edge of the plate. Use calipers to determine the distance, y, between each Pitot tube and the plate. Fasten the tubing from each Pitot tube to the inclined multiple manometer and determine the angle of inclination, θ, of the manometer board. Adjust the wind tunnel speed, U, to the desired value and record the manometer readings, L. Move the boundary layer mouse to a new distance, x, downstream from the leading edge of the plate and repeat the measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each distance, x, from the leading edge, use the manometer data to determine the air speed, u, as a function of distance, y, above the plate (see Eq. 3.13). That is, obtain $u = u(y)$ at various x locations. Note that both the wind tunnel test section and the open end of the manometer tubes are at atmospheric pressure.

Graph: Plot speed, u, as ordinates and distance from the plate, y, as abscissas for each location, x, tested.

Results: Use the $u = u(y)$ results to determine the approximate boundary layer thickness as a function of distance, $\delta = \delta(x)$. Plot a graph of boundary layer thickness as a function of distance from the leading edge. Note that the air flow within the wind tunnel is quite turbulent so that the measured boundary layer thickness is not expected to match the theoretical laminar boundary layer thickness given by the Blassius solution (see Eq. 9.15).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.103

(con't)

9.100

9.100 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight W = aircraft weight $= \mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, for given W, C_L, and A the dynamic pressure is constant, independent of altitude. That is

$$\left.\tfrac{1}{2}\rho U^2\right)_{10,000\,ft} = \left.\tfrac{1}{2}\rho U^2\right)_{30,000\,ft}, \text{ or } U_{30,000} = \left(\frac{\rho_{10,000}}{\rho_{30,000}}\right)^{\frac{1}{2}} U_{10,000}$$

Hence, $U_{30,000} > U_{10,000}$

Also, since the drag is $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ it follows that

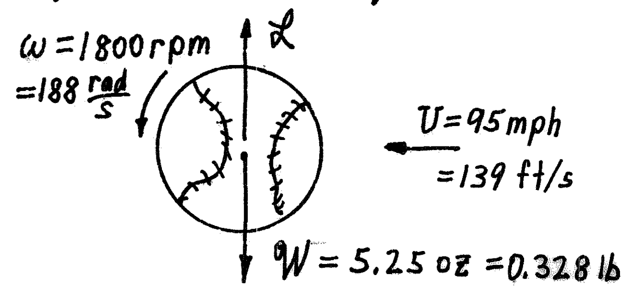
$$\mathcal{D}_{30,000} = \left. C_D \tfrac{1}{2}\rho U^2 A\right)_{30,000} = \left. C_D \tfrac{1}{2}\rho U^2 A\right)_{10,000} \text{ since } \tfrac{1}{2}\rho U^2_{30,000} = \tfrac{1}{2}\rho U^2_{10,000}$$

Hence, the aircrart can fly faster at high altituded with the same amount of drag ($\mathcal{D}_{30,000} = \mathcal{D}_{10,000}$)

9.102

9.102 For many years, hitters have claimed that some baseball pitchers have the ability to actually throw a rising fastball. Assuming that a top major leaguer pitcher can throw a 95-mph pitch and impart a 1800-rpm spin to the ball, is it possible for the ball to actually rise? Assume the baseball diameter is 2.9 in. and its weight is 5.25 oz.

If the lift produced on the spinning ball is greater than its weight the ball will rise.



$$\mathcal{L} = C_L \frac{1}{2}\rho U^2 A$$

where C_L is a function of $\frac{\omega D}{2U}$ as shown in Fig. 9.39.

Thus, with

$$\frac{\omega D}{2U} = \frac{(188\,\frac{rad}{s})(\frac{2.9}{12}\,ft)}{2\,(139\,ft/s)} = 0.163$$

$$C_L \approx 0.04$$

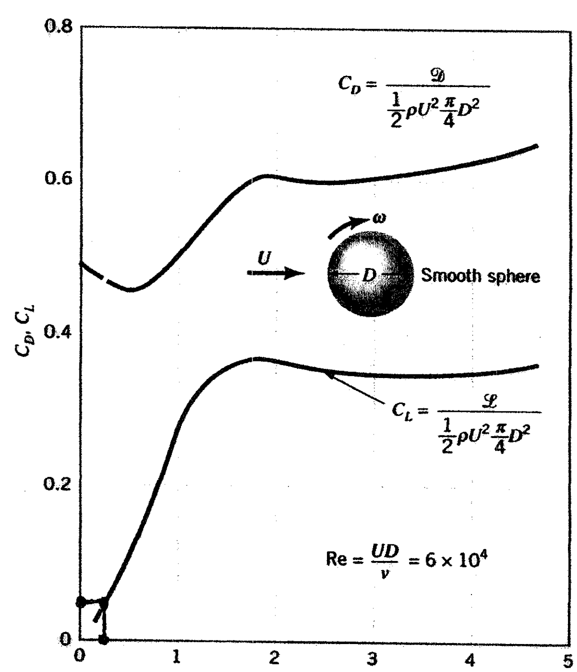
Hence, for the given conditions

$$\mathcal{L} = 0.04\left(\tfrac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left(139\,\frac{ft}{s}\right)^2 \times \frac{\pi}{4}\left(\frac{2.9}{12}\,ft\right)^2 = 0.0422\,lb$$

so that

$$\mathcal{L} = 0.0422\,lb < \mathcal{W} = 0.328\,lb$$

<u>The ball will not rise.</u>



■ FIGURE 9.39 Lift and drag coefficients for a spinning smooth sphere (Ref. 23).

Note: The above result is based on smooth-sphere data. The results for a baseball (with its rough surface containing seams) will probably give a somewhat larger lift because for a given angular velocity it can "drag" more air along as it spins.

9.103 (con't)

Solution for Problem 9.103: Boundary Layer on a Flat Plate

θ, deg	H_{atm}, in. Hg	T, deg F	γ_{H2O}, lb/ft^3
25	29.09	80	62.4

Data for x = 7.75 in.

y, in.	L, in.	u, ft/s
0.020	0.20	19.9
0.035	0.35	26.3
0.044	0.48	30.8
0.060	0.70	37.2
0.096	0.95	43.4
0.110	1.06	45.8
0.138	1.21	48.9
0.178	1.44	53.4
0.230	1.70	58.0
0.270	1.85	60.5

Data for x = 5.75 in.

y, in.	L, in.	u, ft/s
0.020	0.20	19.9
0.035	0.42	28.8
0.044	0.50	31.5
0.060	0.71	37.5
0.096	0.98	44.0
0.110	1.06	45.8
0.138	1.30	50.7
0.178	1.54	55.2
0.230	1.76	59.0
0.270	1.88	61.0

Data for x = 3.75 in.

y, in.	L, in.	u, ft/s
0.020	0.15	17.2
0.035	0.35	26.3
0.044	0.45	29.8
0.060	0.71	37.5
0.096	1.20	48.7
0.110	1.30	50.7
0.138	1.56	55.6
0.178	1.77	59.2
0.230	1.95	62.1
0.270	2.00	62.9

Data for x = 1.75 in.

y, in.	L, in.	u, ft/s
0.020	0.20	19.9
0.035	0.50	31.5
0.044	0.68	36.7
0.060	0.90	42.2
0.096	1.51	54.7
0.110	1.70	58.0
0.138	1.90	61.3
0.178	1.95	62.1
0.230	2.00	62.9
0.270	2.00	62.9

$\rho u^2/2 = \gamma_{H2O}*L \sin\theta$

where

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O}*H_{atm}$ = 847 lb/ft^3*(29.09/12 ft) = 2053 lb/ft^2

R = 1716 ft lb/slug deg R

T = 80 + 460 = 540 deg R

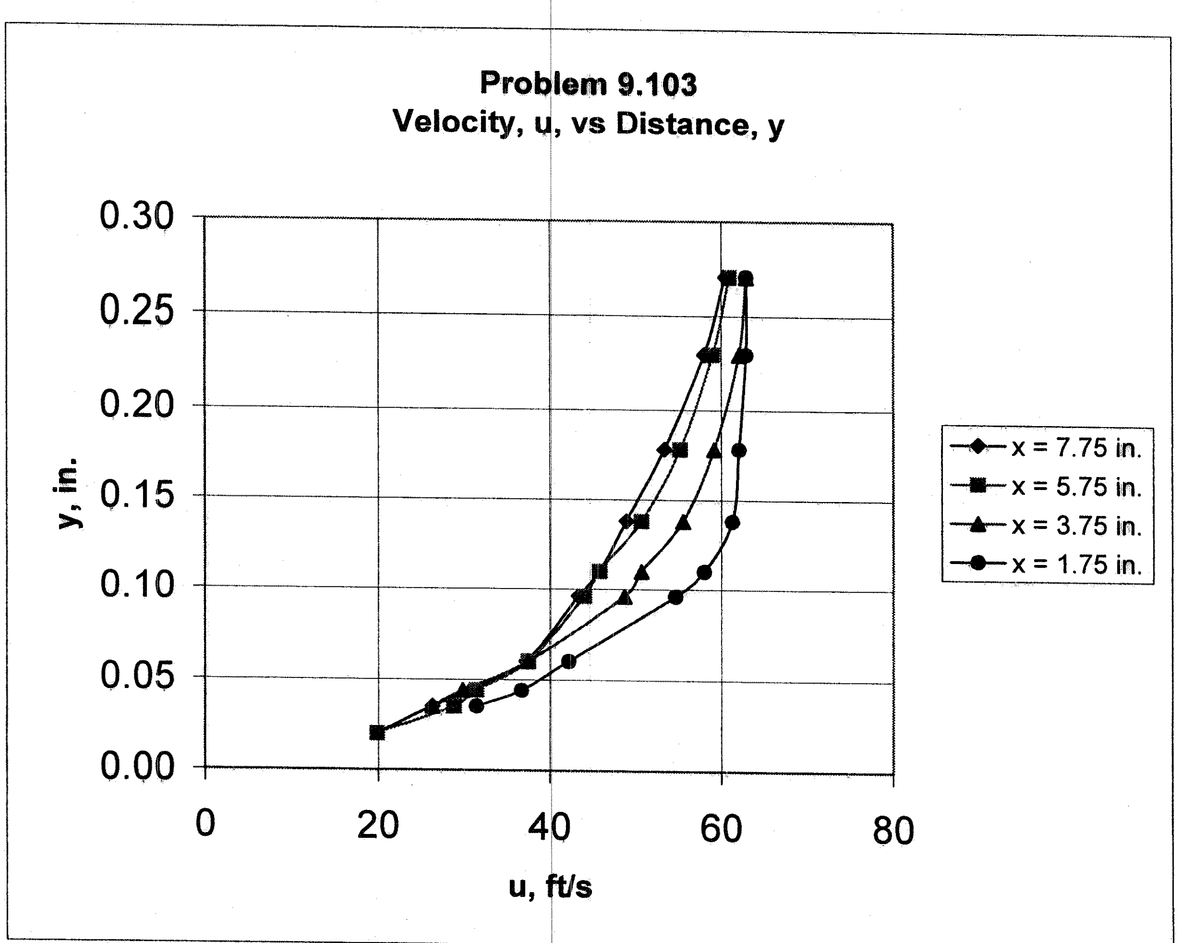
Thus, ρ = 0.00222 slug/ft^3

Approximate boundary layer thickness as obtained from the graph:

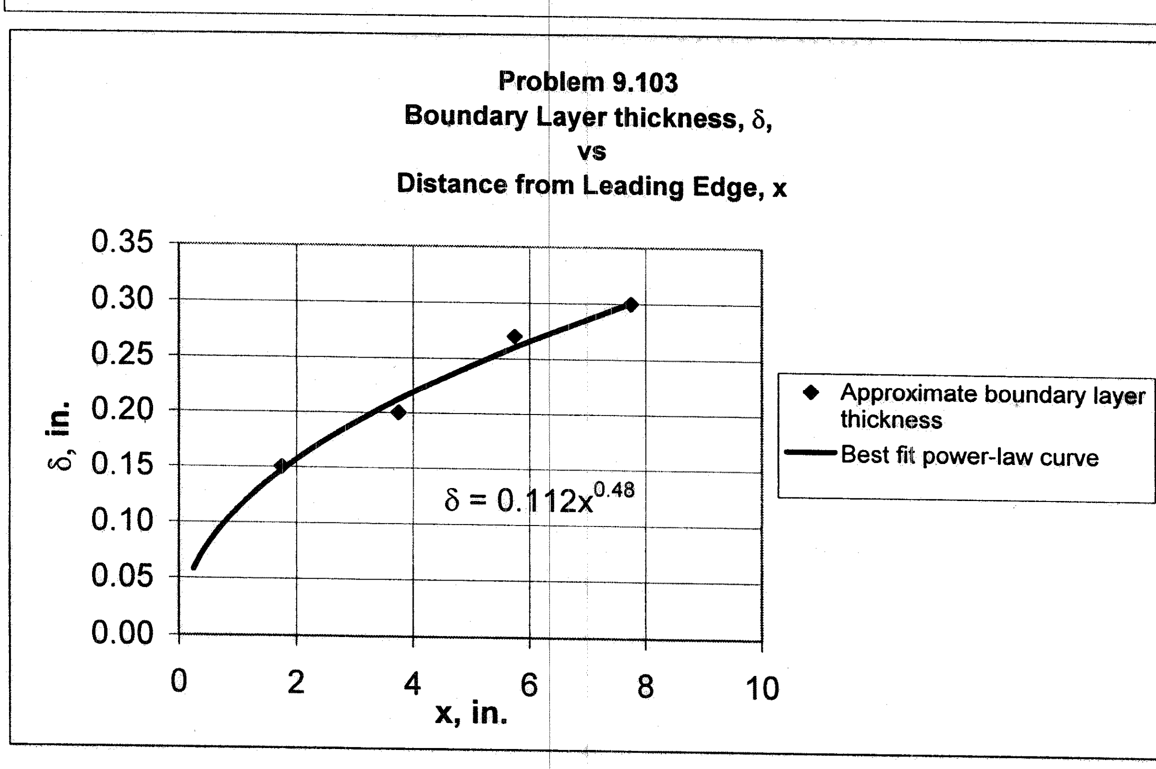
x, in.	δ, in.
1.75	0.15
3.75	0.20
5.75	0.27
7.75	0.30

(con't)

9.103 (con't)



Problem 9.103
Velocity, u, vs Distance, y
0.30
0.25
0.20
0.15
0.10
0.05
0.00
y, in.
0
20
40
60
80
u, ft/s
x = 7.75 in.
x = 5.75 in.
x = 3.75 in.
x = 1.75 in.



Problem 9.103
Boundary Layer thickness, δ,
vs
Distance from Leading Edge, x
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
δ, in.
0
2
4
6
8
10
x, in.
δ = 0.112x^0.48
Approximate boundary layer thickness
Best fit power-law curve

9.104

9.104 Pressure Distribution on a Circular Cylinder

Objective: Viscous effect within the boundary layer on a circular cylinder cause boundary layer separation, thereby causing the pressure distribution on the rear half of the cylinder to be different than that on the front half. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.104, to determine the pressure distribution on a circular cylinder.

Equipment: Wind tunnel; circular cylinder with 18 static pressure taps arranged equally from the front to the back of the cylinder; inclined multiple manometer; barometer; thermometer.

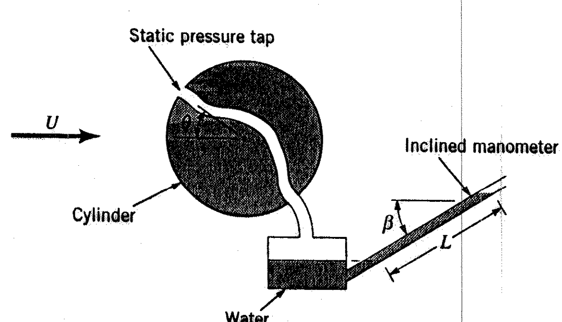
Experimental Procedure: Mount the circular cylinder in the wind tunnel so that a static pressure tap points directly upstream. Measure the angle, β, of the inclined manometer. Adjust the wind tunnel fan speed to give the desired free stream speed, U, in the test section. Attach the tubes from the static pressure taps to the multiple manometer and record the manometer readings, L, as a function of angular position, θ. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the data to determine the pressure coefficient, $C_p = (p - p_0)/(\rho U^2/2)$, as a function of position, θ. Here $p_0 = 0$ is the static pressure upstream of the cylinder in the free stream of the wind tunnel, and $p = \gamma_m L \sin\beta$ is the pressure on the surface of the cylinder.

Graph: Plot the pressure coefficient, C_p, as ordinates and the angular location, θ, as abscissas.

Results: On the same graph, plot the theoretical pressure coefficient, $C_p = 1 - 4\sin^2\theta$, obtained from ideal (inviscid) theory (see Section 6.6.3).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.104

(con't)

9.104 (con't)

Solution for Problem 9.104: Pressure Distribution on a Circular Cylinder

β, deg	H_{atm}, in. Hg	T, deg F	U, ft/s
25	29.97	75	47.9

		Experiment		Theory
θ, deg	L, in.	p, lb/ft^2	C_p	C_p
0	1.2	2.64	1.00	1.00
10	1.1	2.42	0.92	0.88
20	0.7	1.54	0.58	0.53
30	0.1	0.22	0.08	0.00
40	-0.6	-1.32	-0.50	-0.65
50	-1.6	-3.52	-1.33	-1.35
60	-2.4	-5.27	-2.00	-2.00
70	-3.1	-6.81	-2.58	-2.53
80	-3.0	-6.59	-2.50	-2.88
90	-2.7	-5.93	-2.25	-3.00
100	-2.7	-5.93	-2.25	-2.88
110	-2.6	-5.71	-2.17	-2.53
120	-2.6	-5.71	-2.17	-2.00
130	-2.6	-5.71	-2.17	-1.35
140	-2.6	-5.71	-2.17	-0.65
150	-2.6	-5.71	-2.17	0.00
160	-2.7	-5.93	-2.25	0.53
170	-2.7	-5.93	-2.25	0.88
180	-2.8	-6.15	-2.33	1.00

$p = \gamma_{H20}*L \sin\beta$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.97/12 ft) = 2115 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

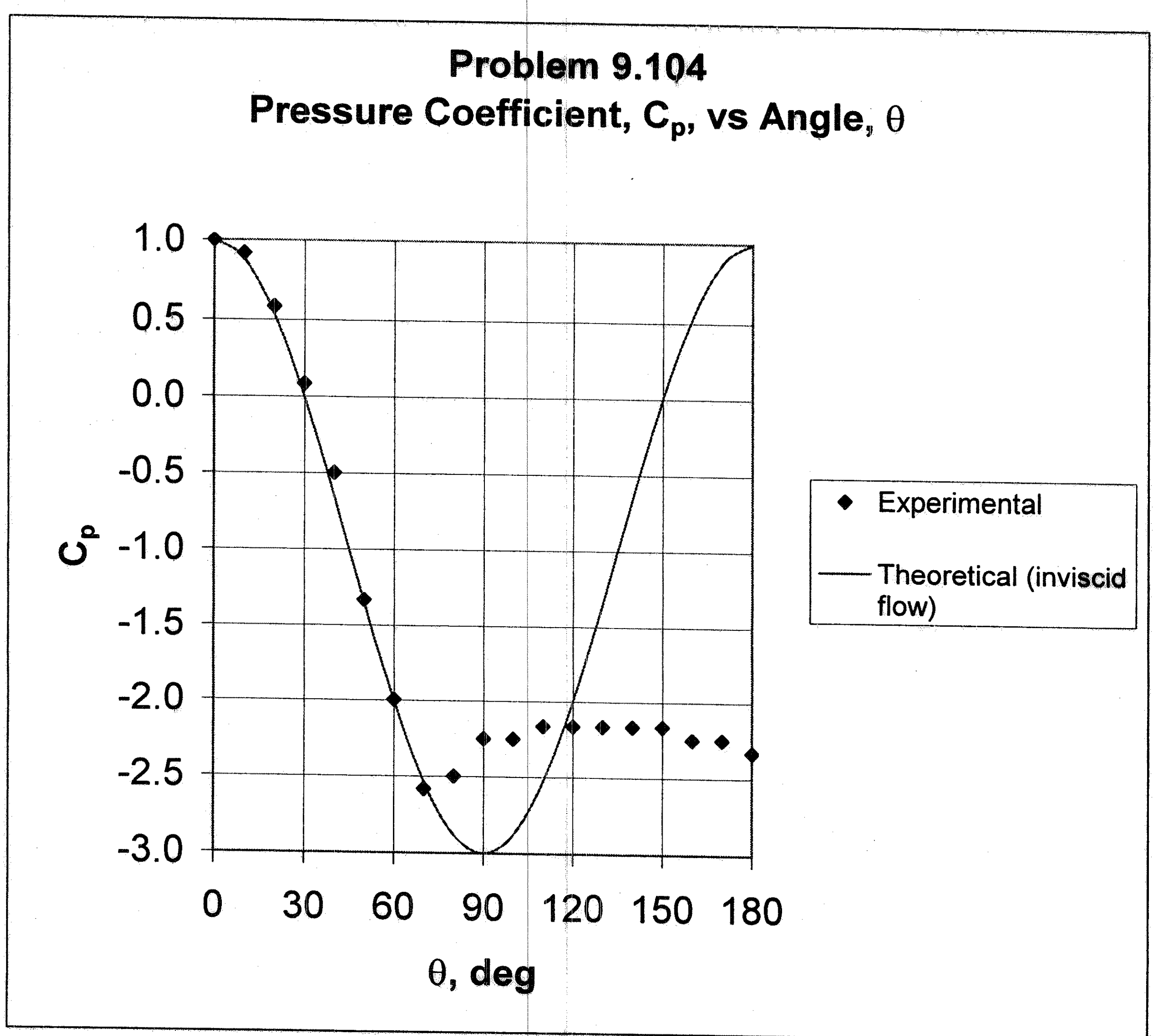
Thus, ρ = 0.00230 slug/ft^3

$C_p = p/(\rho U^2/2)$

Theory: $C_p = 1 - 4 \sin^2\theta$

(con't)

9.104 (con't)



Problem 9.104
Pressure Coefficient, C_p, vs Angle, θ
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
C_p
0
30
60
90
120
150
180
θ, deg
Experimental
Theoretical (inviscid flow)

9.105

9.105 (See "**Armstrong's aerodynamic bike and suit,**" Section 9.1.) By appropriate streamlining, the amount of power needed to peddle a bike can be lowered. How much must the drag coefficient for a bike and rider be reduced if the power a bike racer expends while riding 13 m/s is to be reduced by 10 watts? Assume the cross-sectional area of the bike and rider is 0.36 m^2.

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$ and $\mathcal{P} = \text{power} = \mathscr{D} U$

Thus,

$$\Delta\mathcal{P} = \frac{1}{2}\rho U^3 A\,(\Delta C_D)$$

or

$$10\text{ watts} = 10\,\frac{N\cdot m}{s} = \frac{1}{2}\left(1.23\,\frac{kg}{m^3}\right)\left(13\,\frac{m}{s}\right)^3(0.36\,m^2)\,\Delta C_D$$

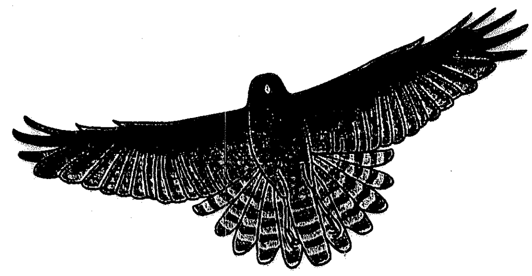
Hence,

$$\Delta C_D = \underline{\underline{0.0206}}$$

Note from Fig. 9.30 the value of the drag coeficient for a racing bile is about 0.88.

9.108

9.108 (See **"Learning from nature,"** Section 9.4.1.) As indicated in Fig. P9.108, birds can significantly alter their body shape and increase their planform area, A, by spreading their wing and tail feathers, thereby reducing their flight speed. If during landing the planform area is increased by 50% and the lift coefficient increased by 30% while all other parameters are held constant, by what percent is the flight speed reduced?



■ FIGURE P9.108

$$\mathcal{L} = C_L \tfrac{1}{2}\rho U^2 A$$

Let $(\,)_2$ denote landing conditions and $(\,)_1$ denote normal flight conditions.
Thus, with $\mathcal{L}_1 = \mathcal{L}_2$,

$$C_{L_1} \tfrac{1}{2}\rho U_1^2 A_1 = C_{L_2} \tfrac{1}{2}\rho U_2^2 A_2$$

or

$$U_2 = U_1 \sqrt{\frac{A_1}{A_2}} \sqrt{\frac{C_{L1}}{C_{L2}}} = U_1 \sqrt{\frac{A_1}{1.5A_1}} \sqrt{\frac{C_{L1}}{1.3C_{L_1}}}$$

or

$$U_2 = 0.716\, U_1$$

Hence,

$$\frac{U_2 - U_1}{U_1} = 0.716 - 1 = -0.284$$

i.e., a $\underline{\underline{28.4\%\ \text{reduction}}}$ in flight speed

9.109

9.109 (See "Why winglets?," Section 9.4.2.) It is estimated that by installing appropriately designed winglets on a certain airplane the drag coefficient will be reduced by 5%. For the same engine thrust, by what percent will the aircraft speed be increased by use of the winglets?

Let $(\ )_1$ denote without winglets and $(\ )_2$ with winglets.
Thus, since drag equals thrust and $\text{thrust}_1 = \text{thrust}_2$, it follows that

$$\mathcal{D}_1 = \mathcal{D}_2$$

or

$$C_{D_1} \tfrac{1}{2}\rho U_1^2 A_1 = C_{D_2} \tfrac{1}{2}\rho U_2^2 A_2$$

so that with $A_1 = A_2$,

$$U_2 = U_1\sqrt{\frac{C_{D1}}{C_{D2}}} = U_1\sqrt{\frac{C_{D1}}{0.95\,C_{D_1}}} = 1.0260\,U_1$$

Thus, a 2.60% increase in speed is realized.

9.106

9.106 (See **"Dimpled baseball bats,"** Section 9.3.3.) How fast must a 3.5-in.-diameter, dimpled baseball bat move through the air in order to take advantage of drag reduction produced by the dimples on the bat. Although there are differences, assume the bat (a cylinder) acts the same as a golf ball in terms of how the dimples affect the transition from a laminar to a turbulent boundary layer.

From Fig. 9.25, for a golf ball the dimples reduce drag for $Re = \frac{\rho U D}{\mu} \gtrsim 4 \times 10^4$

Thus, assume $Re = 4 \times 10^4$ for the bat so that

$$\frac{\rho U D}{\mu} = 4 \times 10^4$$

or

$$\frac{\left(0.00238 \frac{slugs}{ft^3}\right) U \left(\frac{3.5}{12} ft\right)}{\left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} = 4 \times 10^4$$

Thus,

$$U = 21.6 \frac{ft}{s}$$

9.107

9.107 (See "At 10,240 mpg it doesn't cost much to 'fill 'er up,'" Section 9.3.3.) (a) Determine the power it takes to overcome aerodynamic drag on a small (6 ft^2 cross section), streamlined ($C_D = 0.12$) vehicle traveling 15 mph. (b) Compare the power calculated in part (a) with that for a large (36 ft^2 cross-sectional area), nonstreamlined ($C_D = 0.48$) SUV traveling 65 mph on the interstate.

$\mathcal{P}$ = power $= U\mathcal{D}$, where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

so that

$$\mathcal{P} = C_D \frac{1}{2} \rho U^3 A$$

$$(a)\ \mathcal{P} = 0.12\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left[\left(15\ \frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (6\ ft^2)$$

$$= 9.12\ \frac{ft \cdot lb}{s}\left(\frac{1\ hp}{550\ ft \cdot lb/s}\right) = \underline{\underline{0.0166\ hp}}$$

$$(b)\ \mathcal{P} = 0.48\left(\frac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left[\left(65\ \frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (36\ ft^2)$$

$$= 17{,}800\ \frac{ft \cdot lb}{s}\left(\frac{1\ hp}{550\ ft \cdot lb/s}\right) = \underline{\underline{32.4\ hp}}$$