American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Fall 2005-2006

Final Exam

Name: .... ID #: ....

good luck

Exercise 1 *(10 points)* Given the surface $z = x^2 - 4xy + y^3 + 4y - 2$ containing the point $P(1, -1, -2)$

a. Find an equation of the tangent plane to the surface at P.

b. Find an equation of the normal line to the surface at P.

Exercise 2 *(15 points)*

a. Which of the following **series** converges and which diverges? justify.

i. $\displaystyle\sum_{n=0}^{+\infty} \frac{e^n}{1+e^{2n}}$

ii. $\displaystyle\sum_{n=1}^{+\infty} \frac{1-\cos n}{n^{\ln n}}$

iii. $\displaystyle\sum_{n=1}^{+\infty} \frac{n}{10+n^2}$

b. Find the radius of convergence of the series $\displaystyle\sum_{n=0}^{+\infty} \frac{x^{2n}}{2^n}$, then find its sum.

Exercise 3 *(10 points)*

a. If $w = f(x, y)$ is differentiable and $x = r + s, y = r - s$, show that

$$\frac{\partial w}{\partial r} \times \frac{\partial w}{\partial s} = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$$

b. Prove or disprove: The function $f(x, y) = \dfrac{x^2 y}{2x^4 + 3y^2}$ can be extended by continuity at $(0, 0)$.

Exercise 4 *(10 points)* Find the absolute minimum and maximum values of the function $f(x, y) = 4x - 8xy + 2y + 1$ on the triangular plate whose vertices are $(0, 0), (0, 1)$ and $(1, 0)$.

Exercise 5 *(10 points)* Evaluate the integral $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \sin\left(y^3\right)\, dydx$.

Exercise 6 *(10 points)* Let $I = \int\int\int_G xyz\ dV$ where G is the solid in the first octant that is bounded by the parabolic cylinder $z = 2 - x^2$ and the planes $z = 0, y = x$, and $y = 0$.

a. Express I as an iterated triple integral in the order $dzdydx$, then evaluate the resulting integral.

b. Express I as an iterated triple integral in the order $dxdzdy$ *(do not evaluate the integral).*

Exercise 7 *(15 points)* Rewrite the following triple integral

$$J = \int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2\ dzdxdy$$

a. in the order $dxdzdy$ *(do not evaluate the integral).*

b. in cylindrical coordinates *(do not evaluate the integral).*

c. in spherical coordinates, then evaluate the resulting integral.

Exercise 8 *(20 points)*

a. Evaluate

$$\int_{(0,0,1)}^{(1,\pi/2,e)} (\ln z + e^x \sin y)dx + e^x \cos y dy + (x/z - z)dz$$

b. The outward flux of a field $F = M\mathbf{i} + N\mathbf{j}$ across a simple closed curve C equals the double integral of $div\mathbf{F}$ over the region R enclosed by C:

$$\oint_C M dy - N dx = \int\int_R \Big(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\Big) dA \qquad (1)$$

Find the outward flux of the field $F = 2xy\mathbf{i} + x^2\mathbf{j}$ across the curve C in the first quadrant, bounded by the parabola $y = x^2$ and the line $y = 1$.

i. by using the line integral in the left side of equation (1)

ii. by using the double integral in the right side of equation (1)