Notre Dame University-Louaize
Computer Science Department
CSC 311 Theory of Computation
Exam 1, Fall 2015

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

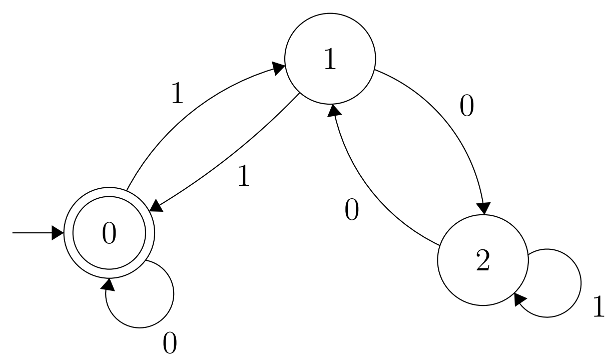
Name and ID: ______________________________

This exam has 6 pages, 5 questions for a total of 100 points.

1. (25 points) Find a DFA that recognizes each of the following languages

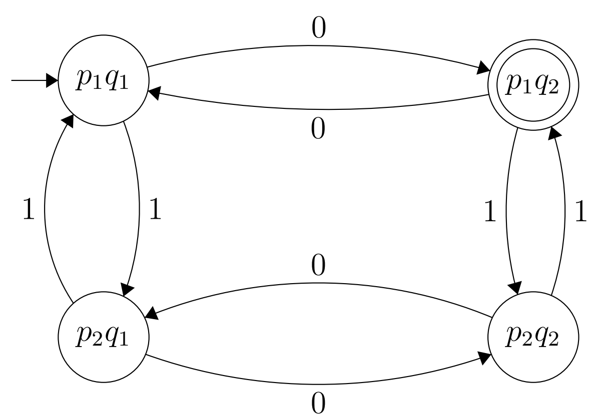
   (a) $L = \{w \in \{0,1\}^* \mid w$ is a binary number that is multiple of 3$\}$

Solution:



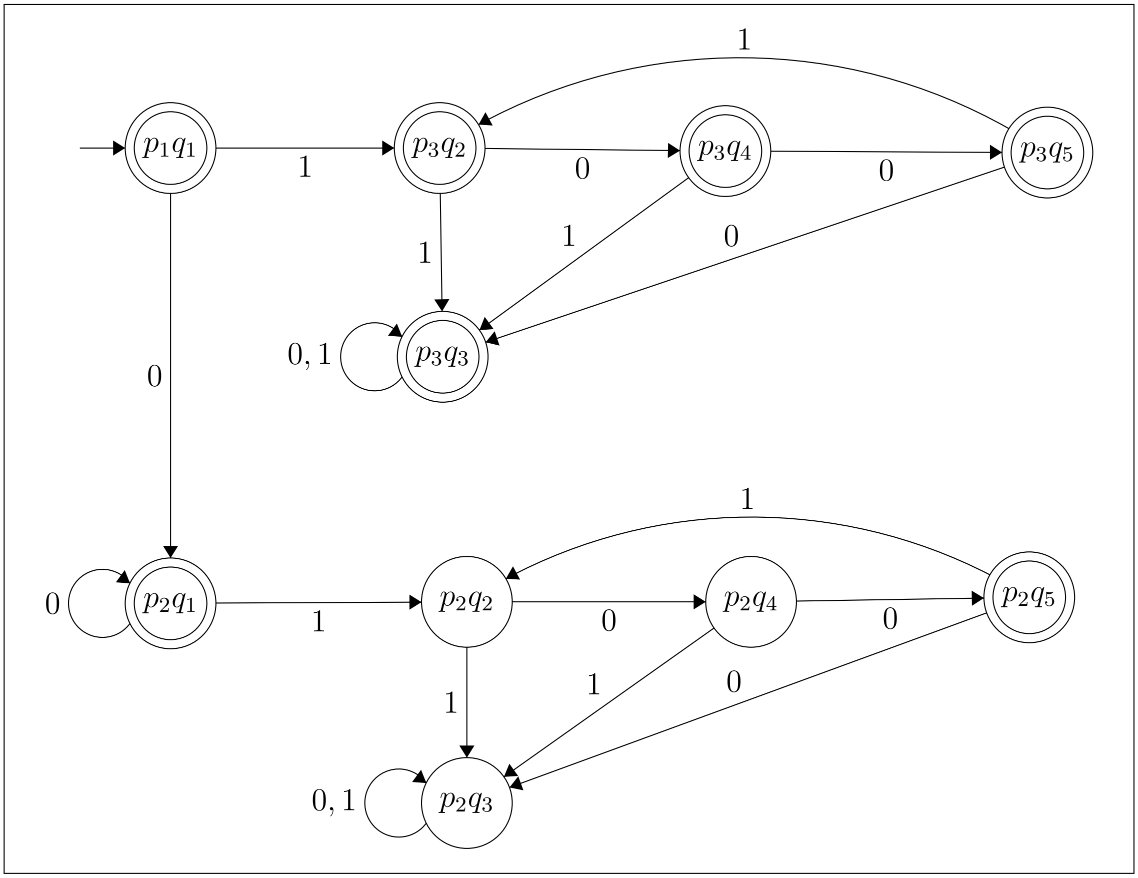
   (b) $L = \{w \in \{0,1\}^* \mid w$ has even number of 1's and an odd number of 0's $\}$

Solution:



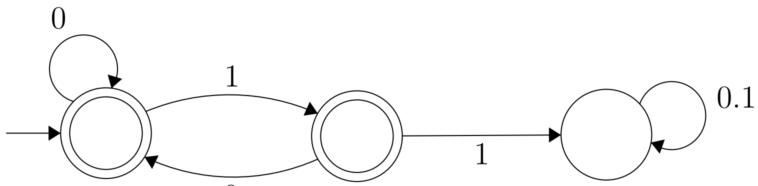
   (c) $L = \{w \in \{0,1\}^* \mid w$ starts with 1 or every 1 is followed by exactly two 0's.$\}$

Solution:



(d) $L = \{w \in \{0,1\}^* \mid w$ has no two consecutive 1's$\}$

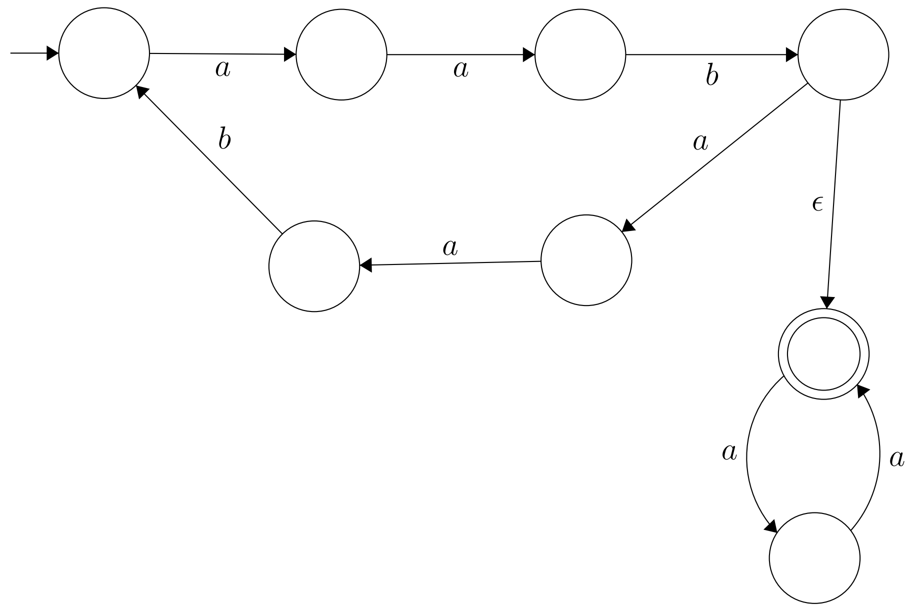
Solution:



2. (25 points) Construct an NFA or ϵ-NFA that recognizes the following languages

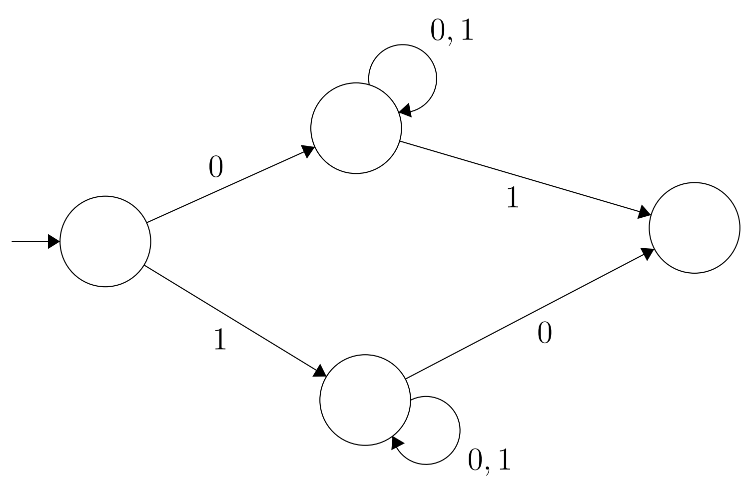
   (a) $L = \{w \in \{a, b\}^* \mid w = (aab)^n a^m$ with n odd and m even$\}$

Solution:



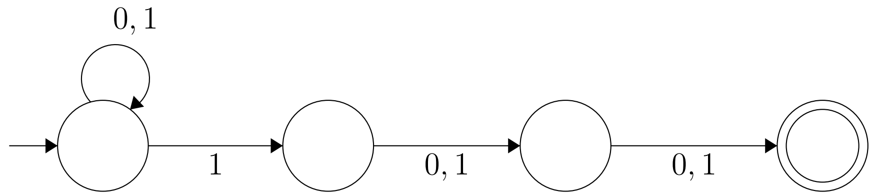
   (b) $L = \{w \in \{a, b\}^* \mid w$ ends with a different symbol then the one it starts with$\}$

Solution:



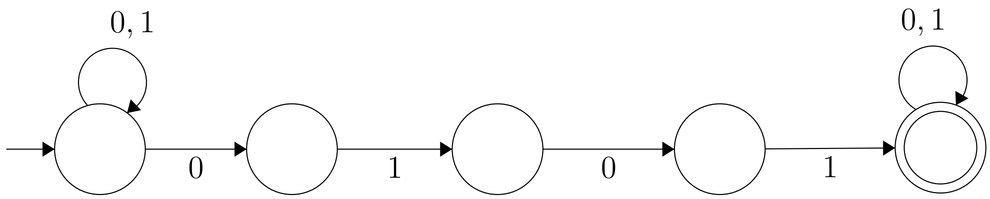
   (c) $L = \{w \in \{0, 1\}^* \mid$ the third symbol from the right is 1$\}$

Solution:



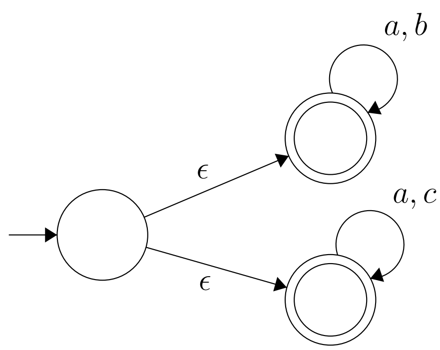
(d) $L = \{w \in \{0,1\}^* \mid$ contains the substring $0101\}$

Solution:



(e) $L = \{w \in \{a,b,c\}^* \mid w$ does not contain b or does not contain c $\}$

Solution:



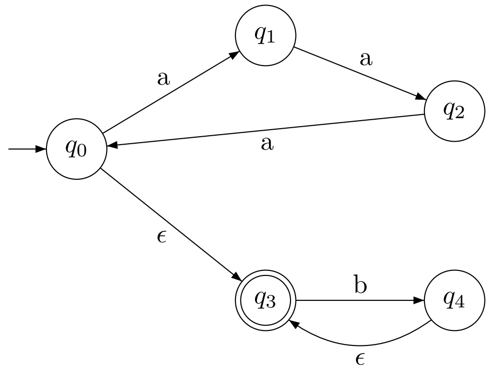


Figure 1: ϵ-NFA

3. (35 points) Consider the ϵ-NFA N shown in Figure 1

   (a) What is the ϵ-closure of all states?

   Solution: $\mathcal{E}(q_0) = \{q_0, q_3\}$
   $\mathcal{E}(q_1) = \{q1\}$
   $\mathcal{E}(q_2) = \{q_2\}$
   $\mathcal{E}(q_3) = \{q_3\}$
   $\mathcal{E}(q_4) = \{q_3, q_4\}$

   (b) What states are reached by N when the input is

   Solution: input b reaches $\{q_3, q_4\}$. Input abb reaches $\emptyset$

   1. b
   2. abb

   (c) is bbb accepted? Explain

   Solution: yes

   (d) is aaab accepted? Explain

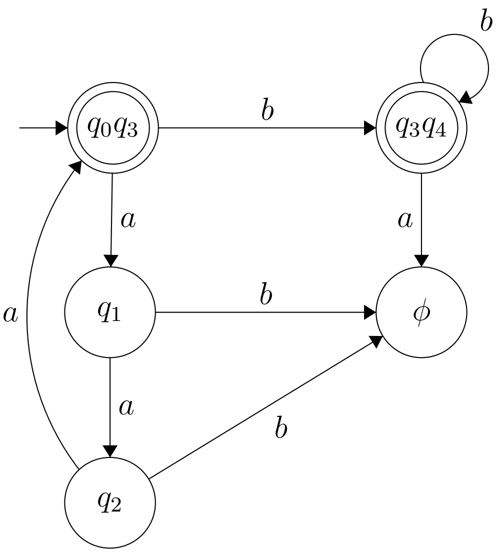
   Solution: yes

   (e) Describe $L(N)$.

   Solution: $L(N) = (aaa)^*b^*$

(f) Construct an equivalent DFA.

Solution:



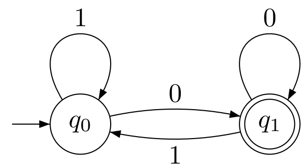


Figure 2: DFA that accepts strings that ends with a 0

4. (5 points) Consider the DFA M shown in Figure 2 and let $L = \{w \in \{0,1\}^* \mid w \text{ ends with a } 0\}$. Prove formally that $L \subseteq L(M)$. (Hint: recall that $L(M) = \{w \mid \hat{\delta}(q_0, w) \in F\}$)

Solution: Suppose that $x \in L$ then we can write $x = y0$ and therefore

$$\begin{aligned}\hat{\delta}(q_0, y0) &= \delta(\hat{\delta}(q_0, x), 0) \text{ by def} \\ &= \delta(q_0 \mid q_1, 0) = q_1\end{aligned}$$

where we used the notation $q_0 \mid q_1$ to mean that either q_0 or q_1

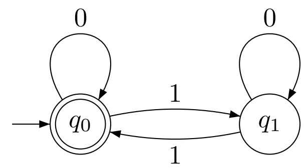


Figure 3: DFA that accepts strings with even number of 1's

5. (10 points) Consider the DFA M shown in Figure 3 and let $L = \{w \in \{0,1\}^* \mid w$ has even number of 1's$\}$. Prove formally that $L \subseteq L(M)$. (Hint: recall that $L(M) = \{w \mid \hat{\delta}(q_0, w) \in F\}$)

Solution: We prove the above by induction on the number of ones in the input string. Base case: Consider a string w that has 2 ones then we can write $w = 0^k10^l10^m$. Computing we get

$$\begin{aligned}
\delta(q_0, w) &= \delta(q_0, 0^k10^l10^m) \\
&= \hat{\delta}(\hat{\delta}(\hat{\delta}(\hat{\delta}(\hat{\delta}(q_0, 0^k), 1), 0^l), 1), 0^m) \\
&= \hat{\delta}(\hat{\delta}(\hat{\delta}(\hat{\delta}(q_0, 1), 0^l), 1), 0^m) \\
&= \hat{\delta}(\hat{\delta}(\hat{\delta}(q_1, 0^l), 1), 0^m) \\
&= \hat{\delta}(\hat{\delta}(q_1, 1), 0^m) \\
&= \hat{\delta}(q_0, 0^m) \\
&= q_0
\end{aligned}$$

Therefore all strings having two one's are accepted.
Hypothesis: assume that all string with n ones, n even, are accepted.
Induction: Let w be a string that has $n + 2$ ones. Then we can write $w = xy$ where x has n ones and y has 2 ones. then

$$\begin{aligned}
\hat{\delta}(q_0, w) &= \delta(\hat{\delta}(q_0, x), y) \\
&= \delta(q_0, y) \text{ by hypothesis} \\
&= q_0 \text{ by the base case}
\end{aligned}$$