Notre Dame University
Computer Science Department

CSC 311 Theory of Computation
Homework 2

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1. (25 pts) Use the closure properties of regular languages to give the state diagram of DFAs recognizing the following languages
   (a) $L = \{w \in \{0,1\}^* : w$ begins with 1 and ends with $0\}$
   (b) $L = \{w \in \{0,1\}^* : w$ has exactly two 0's and at least two 1's$\}$
   (c) $L = \{w \in \{0,1\}^* : w$ has an even number of 0's and each 0 is followed by a $1\}$
   (d) $L = \{w \in \{0,1\}^* : w$ starts with a 0 and has odd length or starts with a 1 and has even length$\}$
   (e) $L = \{w \in \{0,1\}^* : w$ does not contain $1010\}$
2. (10 pts) Construct a DFA to recognize the language of all binary numbers which are multiple of 5. In other words, $L = \{w \in \{0,1\}^* : w$ is a binary number and it is multiple of $5\}$. Example: $101 \in L$ and $1010 \in L$ but $110 \notin L$.
3. (25 pts) Give the state diagram of NFAs recognizing the following languages
   (a) $L = \{w \in \{0,1\}^* : w$ ends with $00\}$
   (b) $L = \{w \in \{0,1\}^* : w$ contains the substring $0101\}$
   (c) $L = \{w \in \{0,1\}^* : w$ does not contain $1\}$. The NFA should contain **one state only**.
   (d) $L = \{w \in \{a,b\}^* : w$ contains 0 or more a's followed by 0 or more b's$\}$
   (e) $L = \{w \in \{0,1\}^* :$ the final symbol of w has occurred at least once before in $w\}$
4. (15 pts) Use the subset construction to construct DFA's that recognize the same language as in exercises 3a and 3b.
5. (15 pts) Let Σ be an alphabet and $D = (Q, \Sigma, \delta, s, F)$ be a finite automaton. Use the definition of the extended function $\hat{\delta}$ to show that $\hat{\delta}(q, ax) = \hat{\delta}(\delta(q,a), x)$, $a \in \Sigma, x \in \Sigma^*, q \in Q$
6. (10 pts) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA such that $\exists a \in \Sigma$ with the property $\delta(q,a) = q$ **for all** $q \in Q$. Show that either $\{a\}^* \subseteq L(M)$ or $\{a\}^* \cap L(M) = \emptyset$.
7. **(extra credit)Consider the language $L_k = \{w \in \{0,1\}^* :$ the k^{th} symbol of w from the right is a $1\}$. Prove that any DFA that recognizes L_k must have at least 2^k states. (Hint: do it by contradiction).