American University of Beirut

Calculus and Analytic Geometry

Spring 2004

Final Exam

Date: Monday, May 31, 2004 - 3:00 pm to 5:00 pm

Instructors: Ms Sylvana Jaber, Mr Zadour Khachadourian and Dr. Mohamed Kobeissi

Name: ....

ID #: ....

Section:	4 (Ms Jaber)	5 (Mr Khachadourian)	6 (Mr Khachadourian)
	T 3:30-4:20 pm	T 2:00-2:50 pm	T 12:30-1:20 pm

This is **NOT** an open-book exam. Your exam should have 11 pages including this one, and there are 8 questions totaling 100 points. You can continue each exercise on the reverse side if needed.

Question	Grade
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	

Good luck

Exercise 1 *[12 points]*: Determine if the following series converge or diverge. **Justify your answers**

a. $\displaystyle\sum_{n=1}^{+\infty} \ln\left(1 - \frac{1}{n^2}\right)$

b. $\displaystyle\sum_{n=1}^{+\infty} n \sin\frac{1}{n}$

c. $\displaystyle\sum_{n=1}^{+\infty} e^{-n} \cos n$

d. $\displaystyle\sum_{n=2}^{+\infty} \frac{1}{\sqrt{n}\,\ln^5 n}$

Exercise 2 *[10 points]*: Evaluate the integral $\int_0^2 \int_0^1 \int_0^{1-x^2} \frac{\sin y}{\sqrt{1-y}} \, dydxdz.$

Exercise 3 *[16 points]*: Let V be the volume of the smaller region cut from the bottom of the cone $z = \sqrt{x^2 + y^2}$ by the plane $z = 3$.

a. Express V as an iterated triple integral in spherical coordinates (that is, set up the limits of integration but do not evaluate the resulting integral).

b. Express V as an iterated triple integral in cylindrical coordinates, then evaluate the resulting integral to find V.

c. Find the plane $z = c$ that divides the region into two parts of equal volume.

Exercise 4 *[14 points]*: Find the absolute minimum and maximum for the function $f(x, y) = x^2 + 4y^2 - y$ on the region $R = \{(x, y); x^2 + 4y^2 \leq 1, y \geq 0\}$.

Exercise 5 *[10 points]:* Let V be the volume of the region whose vertices are $(0, 0, 0)$, $(1, 2, 0)$, $(1, 2, 1)$, $(0, 2, 0)$ and $(0, 2, 1)$.

Express V as an iterated triple integrals in the following orders

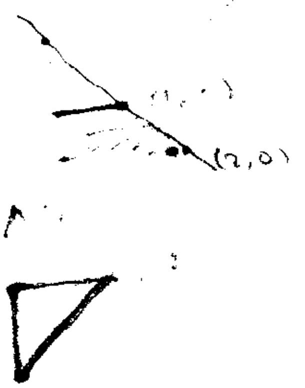
a. order dzdxdy (do not evaluate the integral)

b. order dydzdx, then evaluate the integral

Exercise 6 *[12 points]*: Evaluate the integral

$$\int_0^1 \int_y^{2-y} e^{(x-y)/(x+y)}\, dx\, dy$$

by applying the transformation $u = x - y$ and $v = x + y$.



$x = \frac{u+v}{2}$

Exercise 7 *[16 points]:*

a. Find the value of the line integral

$$\int_{(1,0,0)}^{(2,\pi/2,1)} (3x^2 + 2xz^2)dx + (7z\cos y)dy + (2x^2z + 7\sin y + e^z)dz$$

b. Find the counterclockwise circulation of the field $F(x, y) = (x + e^x \sin y)\mathrm{i} + (x + e^x \cos y)\mathrm{j}$ around the triangle whose vertices are (0,0), (1,0) and (0,1)

Exercise 8 *[10 points]:*

a. Show that $\lim_{(x,y)\to(0,0)} \cos\left(\frac{\pi x y^2}{x^2+y^4}\right)$ does not exist.

b. Let $\omega = f(x, y)$ be a differentiable function where $x = r/s$ and $y = s/r$. Use the chain rule to find $\frac{\partial w}{\partial r}$ at $(r, s) = (1, 2)$.