

American University of Beirut
MATH 201
Calculus and Analytic Geometry III
Spring 2005



Final Exam

Name:

ID #:

Exercise 1 [9 points] Discuss whether the following series converges or diverges:

a) $\sum_{n=1}^{+\infty} \frac{n\sqrt{n}}{1+n^2}$

b) $\sum_{n=1}^{+\infty} \frac{1.3.5.7\dots(2n-1)}{(2n)!}$

c) $\sum_{n=0}^{+\infty} \frac{e^n}{3^{n(n+1)}}$

Exercise 2 [10 points] Find the interval of convergence of the power series $\sum_{n=1}^{+\infty} \frac{1}{\sqrt{n^2+1}} \left(x - \frac{1}{2}\right)^n$
(be sure to check at the end points)

Exercise 3 a) [5 points] Find an equation of the tangent plane and a parametric equation for the normal line to the surface $z = 3x^2y - 5x + 1$ at the point $(2, 1, 3)$.

b) [6 points] Suppose that $z = e^{xy}$ and that $x = 2u + v, y = u/v$. Use the chain rule to find $\partial z / \partial u$ and $\partial z / \partial v$.

(give the expressions in terms in u and v)

c) [5 points] Prove or disprove: $g(x, y) = \frac{x^2y^2}{x^4 + 3y^4}$ can be extended by continuity at $(0, 0)$.
Justify.

Exercise 4 [15 points] Find the absolute minimum and maximum values of

$$f(x, y) = 3xy - 6x - 3y + 7$$

on the closed triangular region R with vertices $(0, 0)$, $(3, 0)$, and $(0, 5)$.

Exercise 5 [10 points] Evaluate the double integral

$$\int_0^2 \int_0^{\sqrt{1-(y-1)^2}} \frac{x}{x^2 + y^2} dx dy$$

by converting it to a double integral in polar coordinates.

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Exercise 6 [10 points] Let V be the volume of the tetrahedron bounded by the coordinate planes and the plane $z = 4 - 4x - 2y$. Express V as iterated triple integral in cartesian coordinates in the order:

a) $dx dz dy$ (do not evaluate the resulting integral).

b) $dz dy dx$, then evaluate the resulting integral.

Exercise 7 [15 points] Let V be the volume of the region R cut from the cone $z = \sqrt{x^2 + y^2}$ by the two planes $z = 1$ and $z = 2$.

a) Sketch the region of integration.

b) Express V as iterated triple integral in cylindrical coordinates (do not evaluate the resulting integral).

c) Express V as iterated triple integral in spherical coordinates, then evaluate the resulting integral.

Exercise 8 [15 points]

a) Find the work done by the force field defined by

$$F(x, y, z) = 4yi + 2xzj + 3yk$$

acting on an object as it moves along the line segment from $(1, 1, 1)$ to $(2, 2, 1)$.

b) Evaluate the line integral $\int_{(3, -2, 0)}^{(1, 0, \pi)} (2x \cos z - x^2) dx + (z - 2y) dy + (y - x^2 \sin z) dz$.